Package ‘tensorregress’

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Title Generalized Tensor Regression with Covariates on Multiple Modes

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Description Implement the generalized tensor regression in Xu, Hu and Wang (2019) <arXiv:1910.09499>. Solve tensor-response regression given covariates on multiple modes with alternating updating algorithm.

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R topics documented:

`as.tensor` *Tensor Conversion*

Description

Create a `Tensor-class` object from an `array`, `matrix`, or `vector`.

Usage

```
as.tensor(x, drop = FALSE)
```

Arguments

`x`	an instance of `array`, `matrix`, or `vector`
`drop`	whether or not modes of 1 should be dropped

Value

a `Tensor-class` object

Examples

```
#From vector
vec <- runif(100); vecT <- as.tensor(vec); vecT
#From matrix
mat <- matrix(runif(1000),nrow=100,ncol=10)
matT <- as.tensor(mat); matT
#From array
indices <- c(10,20,30,40)
arr <- array(runif(prod(indices)), dim = indices)
arrT <- as.tensor(arr); arrT
```

dim-methods *Mode Getter for Tensor*

Description

Return the vector of modes from a tensor

Usage

```
## S4 method for signature 'Tensor'
dim(x)
```

Arguments

x	the Tensor instance

Details

dim(x)

Value

an integer vector of the modes associated with x

Examples

```
tnsr <- rand_tensor()
dim(tnsr)
```

fold *General Folding of Matrix*

Description

General folding of a matrix into a Tensor. This is designed to be the inverse function to unfold-methods, with the same ordering of the indices. This amounts to following: if we were to unfold a Tensor using a set of row_idx and col_idx, then we can fold the resulting matrix back into the original Tensor using the same row_idx and col_idx.

Usage

```
fold(mat, row_idx = NULL, col_idx = NULL, modes = NULL)
```

Arguments

mat	matrix to be folded into a Tensor
row_idx	the indices of the modes that are mapped onto the row space
col_idx	the indices of the modes that are mapped onto the column space
modes	the modes of the output Tensor

Details

This function uses aperm as the primary workhorse.

Value

Tensor object with modes given by modes

References

T. Kolda, B. Bader, "Tensor decomposition and applications". SIAM Applied Mathematics and Applications 2009, Vol. 51, No. 3 (September 2009), pp. 455-500. URL: https://www.jstor.org/stable/25662308.

See Also

unfold-methods

Examples

```
tnsr <- new('Tensor',3L,c(3L,4L,5L),data=runif(60))
matT3<-unfold(tnsr,row_idx=2,col_idx=c(3,1))
identical(fold(matT3,row_idx=2,col_idx=c(3,1),modes=c(3,4,5)),tnsr)
```

---

hosvd *(Truncated-)Higher-order SVD*

Description

Higher-order SVD of a K-Tensor. Write the K-Tensor as a (m-mode) product of a core Tensor (possibly smaller modes) and K orthogonal factor matrices. Truncations can be specified via ranks (making them smaller than the original modes of the K-Tensor will result in a truncation). For the mathematical details on HOSVD, consult Lathauwer et. al. (2000).

Usage

```
hosvd(tnsr, ranks = NULL)
```

Arguments

tnsr	Tensor with K modes
ranks	a vector of desired modes in the output core tensor, default is tnsr@modes

Details

A progress bar is included to help monitor operations on large tensors.

Value

a list containing the following:

`Z` core tensor with modes speficied by `ranks`

`U` a list of orthogonal matrices, one for each mode

`est` estimate of `tnsr` after compression

`fnorm_resid` the Frobenius norm of the error `fnorm(est-tnsr)` - if there was no truncation, then this is on the order of mach_eps * fnorm.

Note

The length of `ranks` must match `tnsr@num_modes`.

References

L. Lathauwer, B.Moor, J. Vandewalle, "A multilinear singular value decomposition". Journal of Matrix Analysis and Applications 2000, Vol. 21, No. 4, pp. 1253–1278.

See Also

`tucker`

Examples

```
tnsr <- rand_tensor(c(6,7,8))
hosvdD <-hosvd(tnsr)
hosvdD$fnorm_resid
hosvdD2 <-hosvd(tnsr,ranks=c(3,3,4))
hosvdD2$fnorm_resid
```

`kronecker_list` *List Kronecker Product*

Description

Returns the Kronecker product from a list of matrices or vectors. Commonly used for n-mode products and various Tensor decompositions.

Usage

```
kronecker_list(L)
```

Arguments

`L`	list of matrices or vectors

Value

matrix that is the Kronecker product

Examples

```
smalllizt <- list('mat1' = matrix(runif(12),ncol=4),
'mat2' = matrix(runif(12),ncol=4),
'mat3' = matrix(runif(12),ncol=4))
dim(kronecker_list(smalllizt))
```

`rand_tensor` *Tensor with Random Entries*

Description

Generate a Tensor with specified modes with iid normal(0,1) entries.

Usage

```
rand_tensor(modes = c(3, 4, 5), drop = FALSE)
```

Arguments

`modes`	the modes of the output Tensor
`drop`	whether or not modes equal to 1 should be dropped

Value

a Tensor object with modes given by `modes`

Note

Default `rand_tensor()` generates a 3-Tensor with modes `c(3,4,5)`.

Examples

```
rand_tensor()
rand_tensor(c(4,4,4))
rand_tensor(c(10,2,1),TRUE)
```

sele_rank *Rank selection*

Description

Estimate the Tucker rank of coefficient tensor based on BIC criterion. The choice of BIC aims to balance between the goodness-of-fit for the data and the degree of freedom in the population model.

Usage

```
sele_rank(
  tsr,
  X_covar1 = NULL,
  X_covar2 = NULL,
  X_covar3 = NULL,
  rank_range,
  Nsim = 10,
  cons = "non",
  lambda = 0.1,
  alpha = 1,
  solver = "CG",
  dist
)
```

Arguments

`tsr`	response tensor with 3 modes
`X_covar1`	covariate on first mode
`X_covar2`	covariate on second mode
`X_covar3`	covariate on third mode
`rank_range`	a matrix containing rank candidates on each row
`Nsim`	max number of iterations if update does not convergence
`cons`	the constraint method, "non" for without constraint, "vanilla" for global scale down at each iteration, "penalty" for adding log-barrier penalty to object function.
`lambda`	penalty coefficient for "penalty" constraint
`alpha`	max norm constraint on linear predictor
`solver`	solver for solving object function when using "penalty" constraint, see "details"
`dist`	distribution of response tensor, see "details"

Details

For rank selection, recommend using non-constraint version.

Constraint `penalty` adds log-barrier regularizer to general object function (negative log-likelihood). The main function uses solver in function "optim" to solve the objective function. The "solver" passes to the argument "method" in function "optim".

`dist` specifies three distributions of response tensor: binary, poisson and normal distributions.

Value

a list containing the following:

`rank` a vector with selected rank with minimal BIC

`result` a matrix containing rank candidate and its loglikelihood and BIC on each row

Examples

```
seed=24
dist='binary'
data=sim_data(seed, whole_shape = c(20,20,20),
core_shape=c(3,3,3),p=c(5,5,5),dist=dist, dup=5, signal=4)
rank_range = rbind(c(3,3,3),c(3,3,2),c(3,2,2),c(2,2,2),c(3,2,3))
re = sele_rank(data$tsr[[1]],data$X_covar1,data$X_covar2,data$X_covar3,
 rank_range = rank_range,Nsim=10,cons = 'non',dist = dist)
```

---

sim_data *Simulation of tensor regression models*

Description

Generate response tensors with multiple covariates under different simulation models, specifically for tensors with 3 modes

Usage

```
sim_data(
  seed,
  whole_shape = c(20, 20, 20),
  core_shape = c(3, 3, 3),
  p = c(3, 3, 0),
  dist,
  dup,
  signal,
  block = rep(FALSE, 3)
)
```

Arguments

seed	a random seed for generating data
whole_shape	a vector containing dimension of the tensor
core_shape	a vector containing Tucker rank of the coefficient tensor
p	a vector containing numbers of covariates on each mode, see "details"
dist	distribution of response tensor, see "details"
dup	number of simulated tensors from the same linear predictor
signal	a scalar controlling the max norm of the linear predictor
block	a vector containing boolean variables, see "details"

Details

By default non-positive entry in p indicates no covariate on the corresponding mode of the tensor.

dist specifies three distributions of response tensor: binary, poisson or normal distribution.

block specifies whether the coefficient factor matrix is a membership matrix, set to TRUE when utilizing the stochastic block model

Value

a list containing the following:

tsr a list of simulated tensors, with the number of replicates specified by dup

X_covar1 a matrix, covariate on first mode

X_covar2 a matrix, covariate on second mode

X_covar3 a matrix, covariate on third mode

W a list of orthogonal coefficient matrices - one for each mode, with the number of columns given by core_shape

G an array, core tensor with size specified by core_shape

C_ts an array, coefficient tensor, Tucker product of G,A,B,C

U an array, linear predictor,i.e. Tucker product of C_ts,X_covar1,X_covar2,X_covar3

Examples

```
seed = 34
dist = 'binary'
data=sim_data(seed, whole_shape = c(20,20,20), core_shape=c(3,3,3),
p=c(5,5,5),dist=dist, dup=5, signal=4)
```

`Tensor-class` *S4 Class for a Tensor*

Description

An S4 class for a tensor with arbitrary number of modes. The Tensor class extends the base "array" class to include additional tensor manipulation (folding, unfolding, reshaping, subsetting) as well as a formal class definition that enables more explicit tensor algebra.

Slots

num_modes number of modes (integer)

modes vector of modes (integer), aka sizes/extents/dimensions

data actual data of the tensor, which can be 'array' or 'vector'

Note

All of the decompositions and regression models in this package require a Tensor input.

Author(s)

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References

James Li, Jacob Bien, Martin T. Wells (2018). rTensor: An R Package for Multidimensional Array (Tensor) Unfolding, Multiplication, and Decomposition. Journal of Statistical Software, Vol. 87, No. 10, 1-31. URL: http://www.jstatsoft.org/v087/i10/.

See Also

`as.tensor`

`tensor_regress` *Generalized Tensor Regression*

Description

Tensor-response regression given covariates on multiple modes. Main function in the package. The function takes a response tensor, multiple covariate matrices, and a desired Tucker rank as input. The output is a constrained MLE for the coefficient tensor.

Usage

```
tensor_regress(
  tsr,
  X_covar1 = NULL,
  X_covar2 = NULL,
  X_covar3 = NULL,
  core_shape,
  Nsim = 20,
  cons = c("non", "vanilla", "penalty"),
  lambda = 0.1,
  alpha = 1,
  solver = "CG",
  dist = c("binary", "poisson", "normal")
)
```

Arguments

tsr	response tensor with 3 modes
X_covar1	covariate on first mode
X_covar2	covariate on second mode
X_covar3	covariate on third mode
core_shape	the Tucker rank of the regression coefficients
Nsim	max number of iterations if update does not convergence
cons	the constraint method, "non" for without constraint, "vanilla" for global scale down at each iteration, "penalty" for adding log-barrier penalty to object function
lambda	penalty coefficient for "penalty" constraint
alpha	max norm constraint on linear predictor
solver	solver for solving object function when using "penalty" constraint, see "details"
dist	distribution of the response tensor, see "details"

Details

Constraint `penalty` adds log-barrier regularizer to general object function (negative log-likelihood). The main function uses solver in function "optim" to solve the objective function. The "solver" passes to the argument "method" in function "optim".

`dist` specifies three distributions of response tensor: binary, poisson and normal distribution.

Value

a list containing the following:

`W` a list of orthogonal coefficient matrices - one for each mode, with the number of columns given by `core_shape`

`G` an array, core tensor with the size specified by `core_shape`

C_ts an array, coefficient tensor, Tucker product of G,A,B,C

U linear predictor,i.e. Tucker product of C_ts,X_covar1,X_covar2,X_covar3

lglk a vector containing loglikelihood at convergence

sigma a scalar, estimated error variance (for Gaussian tensor) or dispersion parameter (for Bernoulli and Poisson tensors)

violate a vector listing whether each iteration violates the max norm constraint on the linear predictor, 1 indicates violation

Author(s)

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References

Z. Xu, J. Hu, and M. Wang, "Generalized Tensor Regression with Covariates on Multiple Modes". 2019. <arXiv:1910.09499>. URL: https://arxiv.org/abs/1910.09499

Examples

```
seed = 34
dist = 'binary'
data=sim_data(seed, whole_shape = c(20,20,20), core_shape=c(3,3,3),
p=c(5,5,5),dist=dist, dup=5, signal=4)
re = tensor_regress(data$tsr[[1]],data$X_covar1,data$X_covar2,data$X_covar3,
core_shape=c(3,3,3),Nsim=10, cons = 'non', dist = dist)
```

---

ttl *Tensor Times List*

Description

Contracted (m-Mode) product between a Tensor of arbitrary number of modes and a list of matrices. The result is folded back into Tensor.

Usage

```
ttl(tnsr, list_mat, ms = NULL)
```

Arguments

tnsr	Tensor object with K modes
list_mat	a list of matrices
ms	a vector of modes to contract on (order should match the order of list_mat)

Details

Performs `ttm` repeated for a single Tensor and a list of matrices on multiple modes. For instance, suppose we want to do multiply a Tensor object `tnsr` with three matrices `mat1`, `mat2`, `mat3` on modes 1, 2, and 3. We could do `ttm(ttm(ttm(tnsr,mat1,1),mat2,2),3)`, or we could do `ttl(tnsr,list(mat1,mat2,mat3),c(1,2,3))`. The order of the matrices in the list should obviously match the order of the modes. This is a common operation for various Tensor decompositions such as CP and Tucker. For the math on the m-Mode Product, see Kolda and Bader (2009).

Value

Tensor object with K modes

Note

The returned Tensor does not drop any modes equal to 1.

References

T. Kolda, B. Bader, "Tensor decomposition and applications". SIAM Applied Mathematics and Applications 2009, Vol. 51, No. 3 (September 2009), pp. 455-500. URL: https://www.jstor.org/stable/25662308

See Also

`ttm`

Examples

```
tnsr <- new('Tensor',3L,c(3L,4L,5L),data=runif(60))
lizt <- list('mat1' = matrix(runif(30),ncol=3),
'mat2' = matrix(runif(40),ncol=4),
'mat3' = matrix(runif(50),ncol=5))
ttl(tnsr,lizt,ms=c(1,2,3))
```

`ttm` *Tensor Matrix Product (m-Mode Product)*

Description

Contracted (m-Mode) product between a Tensor of arbitrary number of modes and a matrix. The result is folded back into Tensor.

Usage

```
ttm(tnsr, mat, m = NULL)
```

Arguments

`tnsr`	Tensor object with K modes
`mat`	input matrix with same number columns as the `m`th mode of `tnsr`
`m`	the mode to contract on

Details

By definition, the number of columns in `mat` must match the `m`th mode of `tnsr`. For the math on the m-Mode Product, see Kolda and Bader (2009).

Value

a Tensor object with K modes

Note

The `m`th mode of `tnsr` must match the number of columns in `mat`. By default, the returned Tensor does not drop any modes equal to 1.

References

T. Kolda, B. Bader, "Tensor decomposition and applications". SIAM Applied Mathematics and Applications 2009, Vol. 51, No. 3 (September 2009), pp. 455-500. URL: https://www.jstor.org/stable/25662308

See Also

`ttl`

Examples

```
tnsr <- new('Tensor',3L,c(3L,4L,5L),data=runif(60))
mat <- matrix(runif(50),ncol=5)
ttm(tnsr,mat,m=3)
```

`tucker` *Tucker Decomposition*

Description

The Tucker decomposition of a tensor. Approximates a K-Tensor using a n-mode product of a core tensor (with modes specified by `ranks`) with orthogonal factor matrices. If there is no truncation in all the modes (i.e. `ranks = tnsr@modes`), then this is the same as the HOSVD, `hosvd`. This is an iterative algorithm, with two possible stopping conditions: either relative error in Frobenius norm has gotten below `tol`, or the `max_iter` number of iterations has been reached. For more details on the Tucker decomposition, consult Kolda and Bader (2009).

Usage

```
tucker(tnsr, ranks = NULL, max_iter = 25, tol = 1e-05)
```

Arguments

`tnsr`	Tensor with K modes
`ranks`	a vector of the modes of the output core Tensor
`max_iter`	maximum number of iterations if error stays above `tol`
`tol`	relative Frobenius norm error tolerance

Details

Uses the Alternating Least Squares (ALS) estimation procedure also known as Higher-Order Orthogonal Iteration (HOOI). Intialized using a (Truncated-)HOSVD. A progress bar is included to help monitor operations on large tensors.

Value

a list containing the following:

`Z` the core tensor, with modes specified by `ranks`

`U` a list of orthgonal factor matrices - one for each mode, with the number of columns of the matrices given by `ranks`

`conv` whether or not `resid` < `tol` by the last iteration

`est` estimate of `tnsr` after compression

`norm_percent` the percent of Frobenius norm explained by the approximation

`fnorm_resid` the Frobenius norm of the error `fnorm(est-tnsr)`

`all_resids` vector containing the Frobenius norm of error for all the iterations

Note

The length of `ranks` must match `tnsr@num_modes`.

References

T. Kolda, B. Bader, "Tensor decomposition and applications". SIAM Applied Mathematics and Applications 2009, Vol. 51, No. 3 (September 2009), pp. 455-500. URL: https://www.jstor.org/stable/25662308

See Also

`hosvd`

Examples

```
tnsr <- rand_tensor(c(4,4,4,4))
tuckerD <- tucker(tnsr,ranks=c(2,2,2,2))
tuckerD$conv
tuckerD$norm_percent
plot(tuckerD$all_resids)
```

`unfold-methods` *Tensor Unfolding*

Description

Unfolds the tensor into a matrix, with the modes in `rs` onto the rows and modes in `cs` onto the columns. Note that `c(rs,cs)` must have the same elements (order doesn't matter) as `x@modes`. Within the rows and columns, the order of the unfolding is determined by the order of the modes. This convention is consistent with Kolda and Bader (2009).

Usage

```
unfold(tnsr, row_idx, col_idx)
```

Arguments

`tnsr`	the Tensor instance
`row_idx`	the indices of the modes to map onto the row space
`col_idx`	the indices of the modes to map onto the column space

Details

```
unfold(tnsr,row_idx=NULL,col_idx=NULL)
```

Value

matrix with `prod(row_idx)` rows and `prod(col_idx)` columns

References

T. Kolda, B. Bader, "Tensor decomposition and applications". SIAM Applied Mathematics and Applications 2009, Vol. 51, No. 3 (September 2009), pp. 455-500. URL: https://www.jstor.org/stable/25662308.

Examples

```
tnsr <- rand_tensor()
matT3<-unfold(tnsr,row_idx=2,col_idx=c(3,1))
```

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