# Package 'stable'

| February 4, 2019  |
|---|
| Version 1.1.4   |
| <b>Title</b> Probability Functions and Generalized Regression Models for Stable Distributions   |
| Imports stabledist  |
| <b>Depends</b> R (>= 1.4), rmutil   |
| <b>Description</b> Density, distribution, quantile and hazard functions of a stable variate; generalized regression models for the parameters of a stable distribution. See the README for how to make equivalent calls to those of 'stabledist'. See github for Journal article. |
| License GPL (>= 2)  |
| <pre>URL http://www.commanster.eu/rcode.html</pre>  |
| BugReports https://github.com/swihart/stable/issues   |
| Encoding UTF-8  |
| LazyData true   |
| LazyLoad true   |
| RoxygenNote 6.1.0   |
| NeedsCompilation yes  |
| Author Bruce Swihart [cre, aut], Jim Lindsey [aut] (Jim created this package, Bruce is maintaining the CRAN version), Philippe Lambert [aut]  |
| Maintainer Bruce Swihart <bruce.swihart@gmail.com></bruce.swihart@gmail.com>  |
| Repository CRAN   |
| <b>Date/Publication</b> 2019-02-04 19:20:03 UTC   |
| R topics documented:  |
| dstable       2         Links       4         Parameter_Conversion       5  |

2 dstable

| stable.mode | <br> |  |
|-------------|------|--|
| stablereg   | <br> |  |

Index 13

dstable

Stable Distribution

#### Description

These functions provide information about the stable distribution with the location, the dispersion, the skewness and the tail thickness respectively modelled by the parameters loc, disp, skew and tail.

#### Usage

```
dstable(x, loc = 0, disp = 1/sqrt(2), skew = 0, tail = 2,
    npt = 501, up = 10, eps = 1e-06, integration = "Romberg")

pstable(q, loc = 0, disp = 1/sqrt(2), skew = 0, tail = 2,
    eps = 1e-06)

qstable(p, loc = 0, disp = 1/sqrt(2), skew = 0, tail = 2,
    eps = 1e-06)

rstable(n = 1, loc = 0, disp = 1/sqrt(2), skew = 0, tail = 2,
    eps = 1e-06)

hstable(x, loc = 0, disp = 1/sqrt(2), skew = 0, tail = 2,
    eps = 1e-06)
```

#### **Arguments**

| x, q            | vector of quantiles.  |
|-----------------|---|
| loc             | vector of (real) location parameters.   |
| disp            | vector of (positive) dispersion parameters.   |
| skew            | vector of skewness parameters (in [-1,1]).  |
| tail            | vector of parameters (in [0,2]) related to the tail thickness                                     |
| npt, up, integr | ation   |
|                 | $\label{eq:Aspectation} As \ detailed \ herein-only \ available \ when \ using \ \mbox{dstable}.$ |
| eps             | scalar giving the required precision in computation.  |
| p               | vector of probabilites.   |
| n               | number of observations.   |
|                 |   |

dstable 3

#### **Details**

dstable, pstable, qstable and hstable compute the density, the distribution, the quantile and the hazard functions of a stable variate. rstable generates random deviates with the prescribed stable distribution.

loc is a location parameter in the same way as the mean in the normal distribution: it can take any real value.

disp is a dispersion parameter in the same way as the standard deviation in the normal distribution: it can take any positive value.

skew is a skewness parameter: it can take any value in (-1,1). The distribution is right-skewed, symmetric and left-skewed when skew is negative, null or positive respectively.

tail is a tail parameter (often named the characteristic exponent): it can take any value in (0,2) (with tail=1 and tail=2 yielding the Cauchy and the normal distributions respectively when symmetry holds).

If loc, disp, skew, or tail are not specified they assume the default values of 0, 1/sqrt(2), 0 and 2 respectively. This corresponds to a normal variate with mean = 0 and variance  $= 1/2disp^2$ .

The stable characteristic function is given by

$$greekphi(t) = ilocat - disp|t|^{tail}[1 + iskew sign(t)greekomega(t, tail)]$$

where

$$greekomega(t,tail) = \frac{2}{\pi}LOG(ABS(t))$$

when tail=1, and

$$greekomega(t,tail) = tan(\frac{\pi tail}{2})$$

otherwise.

The characteristic function is inverted using Fourier's transform to obtain the corresponding stable density. This inversion requires the numerical evaluation of an integral from 0 to  $\infty$ . Two algorithms are proposed for this. The default is Romberg's method (integration="Romberg") which is used to evaluate the integral with an error bounded by eps. The alternative method is Simpson's integration (integration="Simpson"): it approximates the integral from 0 to  $\infty$  by an integral from 0 to up with npt points subdividing (O, up). These three extra arguments – integration, up and npt – are only available when using dstable. The other functions are all based on Romberg's algorithm.

#### **Functions**

• dstable: density

• pstable: cdf

• qstable: quantiles

• rstable: random deviates

• hstable: hazard

#### Author(s)

Philippe Lambert (Catholic University of Louvain, Belgium, <phlambert@stat.ucl.ac.be>)
Jim Lindsey

4 Links

#### References

Lambert, P. and Lindsey, J.K. (1999) Analysing financial returns using regression models based on non-symmetric stable distributions. Applied Statistics, 48, 409-424.

#### See Also

stablereg to fit generalized nonlinear regression models for the stable distribution parameters.

#### **Examples**

```
par(mfrow=c(2,2))
x < - seq(-5,5,by=0.1)
# Influence of loc (location)
plot(x,dstable(x,loc=-2,disp=1/sqrt(2),skew=-0.8,tail=1.5),
 type="l",ylab="",main="Varying LOCation")
lines(x,dstable(x,loc=0,disp=1/sqrt(2),skew=-0.8,tail=1.5))
lines(x,dstable(x,loc=2,disp=1/sqrt(2),skew=-0.8,tail=1.5))
# Influence of disp (dispersion)
plot(x,dstable(x,loc=0,disp=0.5,skew=0,tail=1.5),
  type="l",ylab="",main="Varying DISPersion")
lines(x,dstable(x,loc=0,disp=1/sqrt(2),skew=0,tail=1.5))
lines(x,dstable(x,loc=0,disp=0.9,skew=0,tail=1.5))
# Influence of skew (skewness)
plot(x,dstable(x,loc=0,disp=1/sqrt(2),skew=-0.8,tail=1.5),
 type="l",ylab="",main="Varying SKEWness")
lines(x,dstable(x,loc=0,disp=1/sqrt(2),skew=0,tail=1.5))
lines(x,dstable(x,loc=0,disp=1/sqrt(2),skew=0.8,tail=1.5))
# Influence of tail (tail)
plot(x,dstable(x,loc=0,disp=1/sqrt(2),skew=0,tail=0.8),
 type="1",ylab="",main="Varying TAIL thickness")
lines(x,dstable(x,loc=0,disp=1/sqrt(2),skew=0,tail=1.5))
lines(x,dstable(x,loc=0,disp=1/sqrt(2),skew=0,tail=2))
stabledist::dstable(x=1, 1, 0)
```

Links Links

#### Description

Link and inverse functions for use in stablereg

Parameter\_Conversion 5

#### Usage

```
loc_g(x)
loc_h(x)
disp_g(x)
disp_h(x)
skew_g(x)
skew_h(x)
tail_g(x)
tail_h(x)
```

#### Arguments

x the function argument

### Description

sd2s has stabledist parameter inputs and returns stable parameters. s2sd has stable parameter inputs and returns stabledist parameters.

#### Usage

```
sd2s(alpha, beta, gamma, delta, pm = 1)
s2sd(tail, skew, disp, loc, pm = 1)
```

#### Arguments

| alpha | the stabledist 'alpha'  |
|-------|---|
| beta  | the stabledist 'beta'   |
| gamma | the stabledist 'gamma'  |
| delta | the stabledist 'delta'  |
| pm    | default 1; currently only value supported. the stabledist parameterization 'pm' |
| tail  | the stable 'tail' analogous to 'alpha'  |
| skew  | the stable 'skew' analogous to 'beta'   |
| disp  | the stable 'disp' analogous to 'gamma'  |
| loc   | the stable 'loc' analogous to 'delta'   |

#### **Details**

This is a generic function: methods can be defined for it directly or via the Summary group generic. For this to work properly, the arguments ... should be unnamed, and dispatch is on the first argument.

#### Value

What you need. See examples.

#### **Examples**

```
q <- -1
# nolan pm=1 parameters:
a <- 1.3
b <- -0.4
c <- 2
d <- 0.75
s <- sd2s(alpha=a, beta=b, gamma=c, delta=d)
stable::pstable(q, tail = s$tail, skew=s$skew, disp = s$disp, loc = s$loc)
stabledist::pstable(q, alpha=a, beta=b, gamma=c, delta=d, pm=1)
sd <- s2sd(tail = s$tail, skew=s$skew, disp = s$disp, loc = s$loc)
stabledist::pstable(q, alpha=a, beta=b, gamma=c, delta=d, pm=1)
sd <- s2sd(tail = s$tail, skew=s$skew, disp = s$disp, loc = s$loc)
stabledist::pstable(q, alpha=sd$alpha, beta=sd$beta, gamma=sd$gamma, delta=sd$delta, pm=1)</pre>
```

Parameter\_Conversion\_Nolan\_pm1\_pm0

Easy conversion of parameters between stabledist and stable

#### Description

pm0\_to\_pm1 has stabledist parameter inputs for pm=0 and returns pm=1 equivalent parameterization. pm1\_to\_pm0 has stabledist parameter inputs for pm=1 and returns pm=0 equivalent parameterization.

#### Usage

```
pm0_to_pm1(a0, b0, c0, d0)
pm1_to_pm0(a1, b1, c1, d1)
```

#### **Arguments**

| a0 | ) | the stabledist 'alpha' for pm=0 in 'stabledist' |
|----|---|---|
| b0 | ) | the stabledist 'beta' for pm=0 in 'stabledist'  |
| c0 | ) | the stabledist 'gamma' for pm=0 in 'stabledist' |
| d0 | ) | the stabledist 'delta' for pm=0 in 'stabledist' |
| a1 |   | the stabledist 'alpha' for pm=1 in 'stabledist' |

stable.mode 7

| b1 | the stabledist 'beta' for pm=1 in 'stabledist'  |
|----|---|
| c1 | the stabledist 'gamma' for pm=1 in 'stabledist' |
| d1 | the stabledist 'delta' for pm=1 in 'stabledist' |

#### Value

What you need. See examples.

#### **Examples**

```
q <- -1
# nolan pm=1 parameters:
a1 <- 1.3
b1 <- -0.4
c1 <- 2
d1 <- 0.75
# Convert to nolan pm=0 parameters:
pm0 <- pm1_to_pm0(a1,b1,c1,d1)</pre>
a0 <- pm0$a0
b0 <- pm0$b0
c0 <- pm0$c0
d0 <- pm0$d0
# check:
stabledist::pstable(q, alpha=a1, beta=b1 , gamma=c1 , delta=d1, pm=1)
#> [1] 0.1965513
# only change delta=d0 for pm=0
stable dist::pstable (q, alpha=a1, beta=b1, gamma=c1, delta=d0, pm=0)\\
stabledist::pstable(q, alpha=a0, beta=b0 , gamma=c0 , delta=d0, pm=0)
#> [1] 0.1965513
stabledist::dstable(q, alpha=a1, beta=b1 , gamma=c1 , delta=d1, pm=1)
#> [1] 0.0572133
# only change delta=d0 for pm=0
stabledist::dstable(q, alpha=a1, beta=b1 \ , gamma=c1 \ , delta=d0, pm=0)\\
stabledist::dstable(q, alpha=a0, beta=b0 , gamma=c0 , delta=d0, pm=0)
#> [1] 0.0572133
```

stable.mode

Mode of a Stable Distribution

#### Description

This function gives a reliable approximation to the mode of a stable distribution with location, dispersion, skewness and tail thickness specified by the parameters loc, disp, skew and tail. tail must be in (1,2).

#### Usage

```
stable.mode(loc, disp, skew, tail)
```

8 stable.mode

#### Arguments

| loc  | vector of (real) location parameters.                          |
|------|--|
| disp | vector of (positive) dispersion parameters.                    |
| skew | vector of skewness parameters (in [-1,1]).                     |
| tail | vector of parameters (in [1,2]) related to the tail thickness. |

#### **Details**

loc is a location parameter in the same way as the mean in the normal distribution: it can take any real value.

disp is a dispersion parameter in the same way as the standard deviation in the normal distribution: it can take any positive value.

skew is a skewness parameter: it can take any value in (-1,1). The distribution is right-skewed, symmetric and left-skewed when skew is negative, null or positive respectively.

tail is a tail parameter (often named the characteristic exponent): it can take any value in (0,2) (with tail=1 and tail=2 yielding the Cauchy and the normal distributions respectively when symmetry holds).

The simplest empirical formula found to give a satisfactory approximation to the mode for values of tail in (1,2) is

$$loc+disp*a*skew*exp(-b*abs(skew))$$
 with 
$$a=1.7665114+1.8417675*tail-2.2954390*tail^2+0.4666749*tail^3$$
 and 
$$b=-0.003142967+632.4715*tail*exp(-7.106035*tail)$$

#### Value

A list of size 3 giving the mode, a and b.

#### Author(s)

Philippe Lambert (Catholic University of Louvain, Belgium, <phlambert@stat.ucl.ac.be>) and Jim Lindsey.

#### References

Lambert, P. and Lindsey, J.K. (1999) Analysing financial returns using regression models based on non-symmetric stable distributions. Applied Statistics, 48, 409-424.

#### See Also

stable for more details on the stable distribution.

stablereg to fit generalized linear models for the stable distribution parameters.

#### **Examples**

```
x <- seq(-5,5,by=0.1)
plot(x,dstable(x,loc=0,disp=1,skew=-1,tail=1.5),type="l",ylab="f(x)")
xhat <- stable.mode(loc=0,disp=1,skew=-1,tail=1.5)$ytilde
fxhat <- dstable(xhat,loc=0,disp=1,skew=-1,tail=1.5)
lines(c(xhat,xhat),c(0,fxhat),lty="dotted")</pre>
```

stablereg

Stable Generalized Regression Models

#### **Description**

stablereg fits user specified generalized linear and nonlinear regression models based on the stable distribution to (uncensored, right and/or left censored) data. This allows the location, the dispersion, the skewness and the tails of the fitted stable distribution to vary with explanatory variables.

#### Usage

```
stablereg(y = NULL, loc = 0, disp = 1, skew = 0, tail = 1.5,
  oloc = TRUE, odisp = TRUE, oskew = TRUE, otail = TRUE,
  noopt = FALSE, iloc = NULL, idisp = NULL, iskew = NULL,
  itail = NULL, loc_h = NULL, disp_h = NULL, skew_h = NULL,
  tail_h = NULL, weights = 1, exact = FALSE, delta = 1,
  envir = parent.frame(), integration = "Romberg", eps = 1e-06,
  up = 10, npoint = 501, hessian = TRUE, llik.output = FALSE,
  print.level = 0, ndigit = 10, steptol = 1e-05, gradtol = 1e-05,
  fscale = 1, typsize = abs(p0), stepmax = sqrt(p0 %*% p0),
  iterlim = 100)
```

#### **Arguments**

У

The response vector or a repeated data object. If the repeated data object contains more than one response variable, give that object in envir and give the name of the response variable to be used here.

For censored data, two columns with the second being the censoring indicator (1: uncensored, 0: right censored, -1: left censored.)

```
loc, loc_h, oloc, iloc
```

Describe the regression model fitted for the location parameter of the stable distribution, perhaps after transformation by the link function loc\_g (set to the identity by default. The inverse link function is denoted by loc\_h. Note that these functions cannot contain unknown parameters).

Two specifications are possible:

(1) loc is a linear or nonlinear language expression beginning with  $\sim$  or an R function, describing the regression function for the location parameter (after transformation by loc\_g, the link function).

iloc is a vector of initial conditions for the parameters in the regression for this parameter.

oloc is a boolean indicating if an optimization of the likelihood has to be carried out on these parameters. If oloc is set to TRUE, a default zero value is considered for the starting values iloc. But if no optimization is desired on the location parameters, i.e. when the likelihood has to be evaluated or optimized at a fixed location, then iloc has to be explicitly specified.

(2) loc is a numeric expression (i.e. a scalar or a vector of the same size as the data vector y, or y[,1] when censoring is considered).

If oloc is set to TRUE, i.e. when an optimization of the likelihood has to be carried out on the location parameter, then the location parameter (after transformation by the link function loc\_g) is set to an unknown parameter with initial value equal to iloc[1] or loc[1] when iloc is not specified.

But when oloc is set to FALSE, i.e. when the likelihood has to be evaluated or optimized at a fixed location, then the transformed location is assumed to be equal to loc when it is of the same length as the data vector y (or y[,1] when censoring is considered), and to loc[1] otherwise.

Specification (1) is especially useful in ANOVA-like situations where the location is assumed to change with the levels of some factor variable.

disp, disp\_h, odisp, idisp

describe the regression model for the dispersion parameter of the fitted stable distribution, after transformation by the link function disp\_g (set to the log function by default). The inverse link function is denoted by disp\_h. Again these functions cannot contain unknown parameters. The same rules as above apply when specifying the generalized regression model for the dispersion parameter.

skew, skew\_h, oskew, iskew

describe the regression model for the skewness parameter of the fitted stable distribution, after transformation by the link function skew\_g (set to log{(1 + [.])/(1 - [.])} by default). The inverse link function is denoted by skew\_h. Again these functions cannot contain unknown parameters. The same rules as above apply when specifying the generalized regression model for the skewness parameter.

tail, tail\_h, otail, itail

describe the regression model considered for the tail parameter of the fitted stable distribution, after transformation by the link function tail\_g (set to log{([.] -1)/(2 - [.])} by default. The inverse link function is denoted by tail\_h. Again these functions cannot contain unknown parameters). The same rules as above apply when specifying the generalized regression model for the tail parameter.

When set to TRUE, it forces oloc, odisp, oskew and otail to FALSE, whatever the user choice for these last three arguments. It is especially useful when looking for appropriate initial values for the regression model parameters, before undertaking the optimization of the likelihood.

weights Weight vector.

If TRUE, fits the exact likelihood function for continuous data by integration over intervals of observation, i.e. interval censoring.

Scalar or vector giving the unit of measurement for each response value, set to unity by default. For example, if a response is measured to two decimals,

noopt

exact

delta

delta=0.01. If the response is transformed, this must be multiplied by the Jacobian. For example, with a log transformation, delta=1/y. (The delta values for the censored response are ignored.) The transformation cannot contain unknown parameters.

envir Environment in which model formulae are to be interpreted or a data object of

class, repeated, tccov, or tvcov; the name of the response variable should be

given in y. If y has class repeated, it is used as the environment.

integration, eps, up, npoint

integration indicates which algorithm must be used to evaluate the stable density when the likelihood is computed with exact set to FALSE. See the man

page on stable for extra information.

hessian Arguments controlling the optimization procedure nlm.

llik.output is TRUE when the likelihood has to be displayed at each iteration of the opti-

mization.

print.level Arguments controlling the optimization procedure nlm. ndigit Arguments controlling the optimization procedure nlm. Arguments controlling the optimization procedure nlm. steptol gradtol Arguments controlling the optimization procedure nlm. fscale Arguments controlling the optimization procedure nlm. typsize Arguments controlling the optimization procedure nlm. Arguments controlling the optimization procedure nlm. stepmax iterlim Arguments controlling the optimization procedure nlm.

#### Value

A list of class stable is returned. The printed output includes the -log-likelihood, the corresponding AIC, the maximum likelihood estimates, standard errors, and correlations. It also include all the relevant information calculated, including error codes.

#### Warning

Because of the numerical integrations involved, convergence can be very sensitive to the initial parameter values supplied and to the settings of the arguments controlling nlm. If nlm feeds extreme parameter values in the tails of the distribution to the likelihood function, the integration may hang for a long time.

#### Author(s)

Philippe Lambert (Catholic University of Louvain, Belgium, <phlambert@stat.ucl.ac.be>) and Jim Lindsey.

#### References

Lambert, P. and Lindsey, J.K. (1999) Analysing financial returns using regression models based on non-symmetric stable distributions. Applied Statistics 48, 409-424.

#### See Also

lm, glm, stable and stable.mode.

#### **Examples**

```
## Share return over a 50 day period (see reference above)
# shares
y < -c(296,296,300,302,300,304,303,299,293,294,294,293,295,287,288,297,
305, 307, 307, 304, 303, 304, 304, 309, 309, 309, 307, 306, 304, 300, 296, 301, 298,
295, 295, 293, 292, 297, 294, 293, 306, 303, 301, 303, 308, 305, 302, 301, 297, 299)
# returns
ret <- (y[2:50]-y[1:49])/y[1:49]
# hist(ret, breaks=seq(-0.035,0.045,0.01))
day <- seq(0,0.48,by=0.01) # time measured in days/100
x \leftarrow seq(1, length(ret))-1
# Classic stationary normal model tail=2
print(z1 <- stablereg(y = ret, delta = 1/y[1:49],</pre>
loc = ^{1}, disp= ^{1}, skew = ^{1}, tail = tail_g(1.9999999),
iloc = 0, idisp = -3, iskew = 0, oskew = FALSE, otail = FALSE))
# Normal model (tail=2) with dispersion=disp_h(b0+b1*day)
print(z2 \leftarrow stablereg(y = ret, delta = 1/y[1:49], loc = \sim day,
disp = ^{-1}, skew = ^{-1}, tail = tail_g(1.999999), iloc = c(0.003,0),
idisp = -4.5, iskew = 0, oskew = FALSE, otail = FALSE))
# Stable model with loc(ation)=loc_h(b0+b1*day)
print(z3 <- stablereg(y = ret, delta = 1/y[1:49],</pre>
loc = ^{\sim}day, disp = ^{\sim}1, skew = ^{\sim}1, tail = ^{\sim}1,
iloc = c(0.001, -0.004), idisp = -4.8, iskew = 0, itail = 0.6))
# Stable model with disp(ersion)=disp_h(b0+b1*day)
print(z4 \leftarrow stablereg(y = ret, delta = 1/y[1:49],
loc = ~1, disp = ~day, skew = ~1, tail = ~1,
iloc = 0.003, idisp = c(-4.8,0), iskew = -0.03, itail = 1.6))
# Stable model with skew(ness)=skew_h(b0+b1*day)
# Evaluation at fixed parameter values (because noopt is set to TRUE)
print(z5 \leftarrow stablereg(y = ret, delta = 1/y[1:49],
loc = ^{-1}, disp = ^{-1}, skew = ^{-1}day, tail = ^{-1},
iloc = 5.557e-04, idisp = -4.957, iskew = c(2.811, -2.158),
itail = 1.57, noopt=TRUE))
# Stable model with tail=tail_h(b0+b1*day)
print(z6 \leftarrow stablereg(y = ret, delta = 1/y[1:49], loc = ret \sim 1,
disp = ^{-1}, skew = ^{-1}, tail = ^{-1}day, iloc = 0.002,
idisp = -4.8, iskew = -2, itail = c(2.4, -4), hessian=FALSE))
```

## **Index**

```
*Topic distribution
                                               skew_g (Links), 4
    stable.mode, 7
                                               skew_h (Links), 4
*Topic models
                                               stable.mode, 7
    stablereg, 9
                                               stablereg, 4, 9
                                               Summary, 6
aic.stable(stablereg), 9
                                               tail_g (Links), 4
tail_h (Links), 4
df.residual.stable(stablereg), 9
disp_g (Links), 4
disp_h (Links), 4
dstable, 2
fitted.stable (stablereg), 9
glm, 12
hstable (dstable), 2
Links, 4
lm, 12
loc_g (Links), 4
loc_h (Links), 4
nlm, 11
Parameter_Conversion, 5
Parameter_Conversion_Nolan_pm1_pm0, 6
pm0_to_pm1
        (Parameter_Conversion_Nolan_pm1_pm0),
        6
pm1_to_pm0
        (Parameter_Conversion_Nolan_pm1_pm0),
pstable (dstable), 2
qstable (dstable), 2
rstable (dstable), 2
s2sd (Parameter_Conversion), 5
sd2s (Parameter_Conversion), 5
```