Package 'spatstat.sparse'

June 4, 2020

Version 1.1-0

Date 2020-06-02

Title Sparse Three-Dimensional Arrays and Linear Algebra Utilities

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Depends R (>= 3.3.0), stats, utils, methods, Matrix, abind, tensor

Imports spatstat.utils (>= 1.17-0)

Description Defines sparse three-dimensional arrays
and supports standard operations on them.
The package also includes utility functions for
matrix calculations that are common in
statistics, such as quadratic forms.

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URL http://www.spatstat.org

LazyData no

NeedsCompilation yes

ByteCompile true

BugReports https://github.com/spatstat/spatstat.sparse/issues

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Repository CRAN

Date/Publication 2020-06-04 14:20:02 UTC

R topics documented:

`spatstat.sparse-package`

The spatstat.sparse Package

Description

The **spatstat.sparse** package defines three-dimensional sparse arrays, and supports standard operations on them. It also provides some utility functions for matrix calculations such as quadratic forms.

Details

The **spatstat.sparse** package

- defines a class of sparse three-dimensional arrays and supports standard operations on them (see Section *Sparse 3D Arrays*).
- provides utility functions for matrix computations that are common in statistics, such as quadratic forms (see Section *Matrix Utilities*).

The code in **spatstat.sparse** was originally written for internal use within the **spatstat** package, but has now been removed and organised into a separate, stand-alone package which can be used for other purposes.

Sparse 3D Arrays

The main purpose of **spatstat.sparse** is to define a class of sparse three-dimensional arrays.

An array `A` is three-dimensional if it is indexed by three integer indices, so that `A[i,j,k]` specifies an element of the array. The array is called sparse if only a small fraction of the entries are non-zero. A sparse array can be represented economically by listing only the entries which are non-zero.

The **spatstat.sparse** package defines the class `sparse3Darray` of sparse three-dimensional arrays. These arrays can have numeric, integer, logical, or complex entries.

The package supports:

- creation of sparse arrays from raw data
- conversion to/from other data types
- array indexing, extraction of entries, assignment of new values

- arithmetic and logical operations
- tensor operations (generalising matrix multiplication)
- permutation of array dimensions
- binding of several arrays into a single array
- printing of sparse arrays.

The **spatstat.sparse** package uses the **Matrix** package to handle slices of three-dimensional arrays which are two-dimensional (sparse matrices) or one-dimensional (sparse vectors).

The main functions are:

`sparse3Darray`	Create a sparse 3D array
`as.sparse3Darray`	Convert other data to a sparse 3D array
`[.sparse3Darray`	Subset operator
`aperm.sparse3Darray`	Permute a sparse array
`Ops.sparse3Darray`	arithmetic and logical operators
`Complex.sparse3Darray`	complex operators
`Math.sparse3Darray`	standard mathematical functions
`Summary.sparse3Darray`	mean, maximum etc
`tensorSparse`	Tensor product
`as.array.sparse3Darray`	Convert sparse array to full array

The class `"sparse3Darray"` has methods for `anyNA`, `dim`, `dim<-`, `dimnames`, `dimnames<-` and `print`, documented in `methods.sparse3Darray`.

For other undocumented functions, see `spatstat.sparse-internal`.

Matrix Utilities

The package also includes some utilities for matrix calculations:

`sumouter`	sum of outer products of rows of a matrix
`quadform`	quadratic form involving rows of a matrix
`bilinearform`	bilinear form involving rows of a matrix
`matrixsqrt`	square root of a matrix
`matrixpower`	powers of a matrix

Licence

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Author(s)

Adrian Baddeley <Adrian.Baddeley@curtin.edu.au>, Rolf Turner <r.turner@auckland.ac.nz> and Ege Rubak <rubak@math.aau.dk>.

`aperm.sparse3Darray` *Transposition of Sparse Array*

Description

Transpose a sparse three-dimensional array by permuting its dimensions.

Usage

```
   ## S3 method for class 'sparse3Darray'
aperm(a, perm = NULL, resize = TRUE, ...)
```

Arguments

`a`	A sparse three-dimensional array (object of class `"sparse3Darray"`).
`perm`	The subscript permutation vector, a permutation of the integers `1:3`.
`resize`	Logical value specifying whether the dimensions and dimnames of the array should also be adjusted, by permuting them according to the permutation.
`...`	Ignored.

Details

The function `aperm` is generic. This is the method for the class `"sparse3Darray"` of sparse three-dimensional arrays.

Value

Another sparse three-dimensional array (object of class `"sparse3Darray"`).

Author(s)

Adrian Baddeley <Adrian.Baddeley@curtin.edu.au>, Rolf Turner <r.turner@auckland.ac.nz> and Ege Rubak <rubak@math.aau.dk>.

See Also

`sparse3Darray`, `tensorSparse`.

Examples

```
M <- sparse3Darray(i=1:4, j=sample(1:4, replace=TRUE),
                   k=c(1,2,1,2), x=1:4, dims=c(5,7,2))
dim(M)
P <- aperm(M, c(3,1,2))
dim(P)
```

`as.array.sparse3Darray` *Convert Sparse Array to Full Array*

Description

Convert a sparse three-dimensional array to a full three-dimensional array.

Usage

```
## S3 method for class 'sparse3Darray'
as.array(x, ...)
```

Arguments

`x`	Sparse three-dimensional array (object of class `"sparse3Darray"`).
`...`	Ignored.

Details

This is a method for the generic `as.array` for sparse three-dimensional arrays (class `"sparse3Darray"`). It converts the sparse three-dimensional array `x` into an `array` representing the same data.

Value

An array (class `"array"`) with the same dimensions as `x` and the same type of entries as `x`.

Author(s)

Adrian Baddeley <Adrian.Baddeley@curtin.edu.au>, Rolf Turner <r.turner@auckland.ac.nz> and Ege Rubak <rubak@math.aau.dk>.

See Also

`sparse3Darray`, `as.sparse3Darray`

Examples

```
M <- sparse3Darray(i=1:3, j=c(3,1,2), k=4:2,
                   x=runif(3), dims=rep(4, 3))
V <- as.array(M)
```

`as.sparse3Darray` *Convert Data to a Sparse Three-Dimensional Array*

Description

Convert other kinds of data to a sparse three-dimensional array.

Usage

```
as.sparse3Darray(x, ...)
```

Arguments

`x`	Data in another format (see Details).
`...`	Ignored.

Details

This function converts data in various formats into a sparse three-dimensional array (object of class `"sparse3Darray"`).

The argument `x` can be

- a sparse three-dimensional array (class `"sparse3Darray"`)
- an `array`
- a `matrix`, which will be interpreted as an array with dimension `c(dim(x),1)`
- a sparse matrix (inheriting class `"sparseMatrix"` in the **Matrix** package) which will be interpreted as an array with dimension `c(dim(x),1)`
- a vector of atomic values, which will be interpreted as an array of dimension `c(length(x),1,1)`
- a sparse vector (inheriting class `"sparseVector"` in the **Matrix** package) which will be interpreted as an array of dimension `c(x@length,1,1)`
- a list of matrices with the same dimensions, which will be interpreted as slices `A[,,k]` of an array `A`
- a list of sparse matrices (each inheriting class `"sparseMatrix"` in the **Matrix** package) with the same dimensions, which will be interpreted as slices `A[,,k]` of an array `A`.

Value

Sparse three-dimensional array (object of class `"sparse3Darray"`).

Author(s)

Adrian Baddeley <Adrian.Baddeley@curtin.edu.au>, Rolf Turner <r.turner@auckland.ac.nz> and Ege Rubak <rubak@math.aau.dk>.

See Also

sparse3Darray

Examples

```
A <- array(c(1,3,0,0,0,0,0,4,0,2,0,5,
             0,0,1,0,0,0,1,0,0,0,1,0),
           dim=c(3,4,2))
#' array to sparse array
B <- as.sparse3Darray(A) # positive extent
#' list of matrices to sparse array
B <- as.sparse3Darray(list(A[,,1], A[,,2]))
#' matrix to sparse array
B1 <- as.sparse3Darray(A[,,1])
#' vector to sparse array
B11 <- as.sparse3Darray(A[,1,1])
```

Extract.sparse3Darray *Extract or Replace Entries in a Sparse Array*

Description

Extract or replace entries in a sparse three-dimensional array.

Usage

```
  ## S3 method for class 'sparse3Darray'
x[i, j, k, drop=TRUE, ...]
  ## S3 replacement method for class 'sparse3Darray'
x[i, j, k, ...] <- value
```

Arguments

x	Sparse three-dimensional array (object of class "sparse3Darray").
i,j,k	Subset indices for each dimension of the array. See Details.
value	Replacement value for the subset.
drop	Logical value indicating whether to return a lower-dimensional object (matrix or vector) when appropriate.
...	Ignored. This argument is required for compatibility with the generic function.

Details

These functions are defined for a sparse three-dimensional array x. They extract a designated subset of the array, or replace the values in the designated subset.

The function [.sparse3Darray is a method for the generic subset extraction operator [. The function [<-.sparse3Darray is a method for the generic subset replacement operator [<-.

These methods use the same indexing rules as the subset operator for full arrays:

- If i, j and k are integer vectors, the subset is the Cartesian product (i.e. all cells in the array identified by an entry of i, an entry of j and an entry of k).
- Some or all of the arguments i, j and k may be missing from the call; a missing index argument is interpreted as meaning that all possible values of that index are allowed.
- Arguments i, j and k may be logical vectors (with the value TRUE assigned to entries that should be included).
- Arguments i, j and k may be character vectors with entries matching the corresponding dimnames.
- Argument i may be an integer matrix with 3 columns (and the arguments j,k should be absent). Each row of the matrix contains the indices of one cell in the array.

If the designated subset lies within the array bounds, then the result of [will be a sparse three-dimensional array, sparse matrix or sparse vector. If drop=FALSE the result will always be three-dimensional; if drop=TRUE (the default) the result will be reduced to two or one dimensions when appropriate.

If the designated subset *does not* lie within the array bounds, then the result of [will be a full three-dimensional array, matrix or vector containing NA values at the positions that were outside the array bounds.

The result of [<- is always a sparse three-dimensional array. If the designated subset did not lie within the array bounds of x, then the array bounds will be extended (with a warning message).

Value

[.sparse3Darray returns either a sparse three-dimensional array (class "sparse3Darray"), a sparse matrix (class sparseMatrix in the **Matrix** package), a sparse vector (class sparseVector in the **Matrix** package), or in some cases a full array, matrix or vector.

[<-.sparse3Darray returns another sparse three-dimensional array.

Author(s)

Adrian Baddeley <Adrian.Baddeley@curtin.edu.au>, Rolf Turner <r.turner@auckland.ac.nz> and Ege Rubak <rubak@math.aau.dk>.

See Also

sparse3Darray, methods.sparse3Darray.

Examples

```
M <- sparse3Darray(i=1:4, j=sample(1:4, replace=TRUE),
                   k=c(1,2,1,2), x=1:4, dims=c(5,5,2))
dimnames(M) <- list(letters[1:5], LETTERS[1:5], c("yes", "no"))
M[ 3:4, , ]
M[ 3:4, 2:4, ]
M[ 4:3, 4:2, 1:2]
M[, 3, ]
```

`marginSumsSparse` *Margin Sums of a Sparse Matrix or Sparse Array*

Description

For a sparse matrix or sparse array, compute the sum of array entries for a specified margin or margins.

Usage

```
marginSumsSparse(X, MARGIN)
```

Arguments

`X`	A matrix, an array, a sparse matrix (of class `"sparseMatrix"` from the **Matrix** package) or a sparse three-dimensional array (of class `"sparse3Darray"` from the **spatstat.sparse** package).
`MARGIN`	Integer or integer vector specifying the margin or margins.

Details

This function computes the equivalent of `apply(X,MARGIN,sum)` for sparse matrices and arrays `X`. The argument `X` may be

- a matrix
- an array of any number of dimensions
- a sparse matrix (object inheriting class `"sparseMatrix"` in the **Matrix** package)
- a sparse three-dimensional array (of class `"sparse3Darray"` from the **spatstat.sparse** package).

In the first two cases, the computation is performed by calling `apply(X,MARGIN,sum)` and the result is a vector, matrix or array. In the last two cases, the result is a single value, a sparse vector, a sparse matrix, or a sparse three-dimensional array.

Value

A single value, vector, matrix, array, sparse vector (class `"sparseVector"` in the **Matrix** package), sparse matrix (class `"sparseMatrix"` in the **Matrix** package), or sparse three-dimensional array (class `"sparse3Darray"` from the **spatstat.sparse** package).

Author(s)

Adrian Baddeley <Adrian.Baddeley@curtin.edu.au>, Rolf Turner <r.turner@auckland.ac.nz> and Ege Rubak <rubak@math.aau.dk>.

See Also

`apply`

Examples

```
M <- sparse3Darray(i=1:3, j=c(3,1,2), k=4:2,
                   x=round(runif(3), 2), dims=rep(4, 3))
marginSumsSparse(M, 1:2)
marginSumsSparse(M, 1)
marginSumsSparse(M, integer(0)) # equivalent to sum(M)
```

`Math.sparse3Darray` *S3 Group Generic Methods for Sparse Three-Dimensional Arrays*

Description

Group generic methods which make it possible to apply the familiar mathematical operators and functions to sparse three-dimensional arrays (objects of class `"sparse3Darray"`). See Details for a list of implemented functions.

Usage

```
## S3 methods for group generics have prototypes:
Math(x, ...)
Ops(e1, e2)
Complex(z)
Summary(..., na.rm=FALSE)
```

Arguments

`x, z, e1, e2`	Sparse three-dimensional arrays (objects of class `"sparse3Darray"`). Alternatively `e1` or `e2` can be a single scalar, vector, sparse vector, matrix or sparse matrix.
`...`	further arguments passed to methods.
`na.rm`	Logical value specifying whether missing values should be removed.

Details

These group generics make it possible to perform element-wise arithmetic and logical operations with sparse three-dimensional arrays, or apply mathematical functions element-wise, or compute standard summaries such as the mean and maximum.

Below is a list of mathematical functions and operators which are defined for sparse 3D arrays.

1. Group `"Math"`:
   - `abs`, `sign`, `sqrt`,
     `floor`, `ceiling`, `trunc`,
     `round`, `signif`

- exp, log, expm1, log1p,
  cos, sin, tan,
  cospi, sinpi, tanpi,
  acos, asin, atan
  cosh, sinh, tanh,
  acosh, asinh, atanh
- lgamma, gamma, digamma, trigamma
- cumsum, cumprod, cummax, cummin

2. Group "Ops":
   - "+", "-", "*", "/", "^", "%%", "%/%"
   - "&", "|", "!"
   - "==", "!=", "<", "<=", ">=", ">"
3. Group "Summary":
   - all, any
   - sum, prod
   - min, max
   - range
4. Group "Complex":
   - Arg, Conj, Im, Mod, Re

Value

The result of group "Math" functions is another three-dimensional array of the same dimensions as x, which is sparse if the function maps 0 to 0, and otherwise is a full three-dimensional array.

The result of group "Ops" operators is another three-dimensional array of the same dimensions as e1 and e2, which is sparse if both e1 and e2 are sparse.

The result of group "Complex" functions is another sparse three-dimensional array of the same dimensions as z.

The result of group "Summary" functions is a logical value or a numeric value or a numeric vector of length 2.

Author(s)

Adrian Baddeley <Adrian.Baddeley@curtin.edu.au>, Rolf Turner <r.turner@auckland.ac.nz> and Ege Rubak <rubak@math.aau.dk>.

See Also

sparse3Darray, tensorSparse

Examples

```
M <- sparse3Darray(i=1:4, j=sample(1:4, replace=TRUE),
                   k=c(1,2,1,2), x=1:4, dims=c(5,5,2))
negM <- -M
twoM <- M + M
```

```
Mplus <- M + 1 ## not sparse!
posM <- (M > 0)
range(M)
sinM <- sin(M)
cosM <- cos(M) ## not sparse!
expM1 <- expm1(M)
```

`matrixpower` *Power of a Matrix*

Description

Evaluate a specified power of a matrix.

Usage

```
matrixpower(x, power, complexOK = TRUE)
matrixsqrt(x, complexOK = TRUE)
matrixinvsqrt(x, complexOK = TRUE)
```

Arguments

`x`	A square matrix containing numeric or complex values.
`power`	A numeric value giving the power (exponent) to which `x` should be raised.
`complexOK`	Logical value indicating whether the result is allowed to be complex.

Details

These functions raise the matrix `x` to the desired power: `matrixsqrt` takes the square root, `matrixinvsqrt` takes the inverse square root, and `matrixpower` takes the specified power of `x`.

Up to numerical error, `matrixpower(x,2)` should be equivalent to `x %*% x`, and `matrixpower(x,-1)` should be equivalent to `solve(x)`, the inverse of `x`.

The square root `y <-matrixsqrt(x)` should satisfy `y %*% y = x`. The inverse square root `z <-matrixinvsqrt(x)` should satisfy `z %*% z = solve(x)`.

Computations are performed using the eigen decomposition (`eigen`).

Value

A matrix of the same size as `x` containing numeric or complex values.

Author(s)

Adrian Baddeley <Adrian.Baddeley@curtin.edu.au>.

See Also

`eigen`, `svd`

Examples

```
x <- matrix(c(10,2,2,1), 2, 2)
y <- matrixsqrt(x)
y
y %*% y
z <- matrixinvsqrt(x)
z %*% y
matrixpower(x, 0.1)
```

`methods.sparse3Darray` *Methods for Sparse Three-Dimensional Arrays*

Description

Methods for the class `"sparse3Darray"` of sparse three-dimensional arrays.

Usage

```
 ## S3 method for class 'sparse3Darray'
anyNA(x, recursive = FALSE)
 ## S3 method for class 'sparse3Darray'
dim(x)
 ## S3 replacement method for class 'sparse3Darray'
dim(x) <- value
 ## S3 method for class 'sparse3Darray'
dimnames(x)
 ## S3 replacement method for class 'sparse3Darray'
dimnames(x) <- value
 ## S3 method for class 'sparse3Darray'
print(x, ...)
```

Arguments

`x`	A sparse three-dimensional array (object of class `"sparse3Darray"`).
`value`	Replacement value (see Details).
`recursive,...`	Ignored.

Details

These are methods for the generics `anyNA`, `dim`, `dim<-`, `dimnames`, `dimnames<-` and `print` for the class `"sparse#Darray"` of sparse three-dimensional arrays.

For `dimnames(x) <-value`, the `value` should either be `NULL`, or a list of length 3 containing character vectors giving the names of the margins.

For `dim(x) <-value`, the `value` should be an integer vector of length 3 giving the new dimensions of the array. Note that this operation does not change the array positions of the non-zero entries (unlike `dim(x) <-value` for a full array). An error occurs if some of the non-zero entries would lie outside the new extent of the array.

Value

anyNA returns a single logical value.

dim returns an integer vector of length 3.

dimnames returns NULL, or a list of length 3 whose entries are character vectors.

dim<- and dimnames<- return a sparse 3D array.

print returns NULL, invisibly.

Author(s)

Adrian Baddeley <Adrian.Baddeley@curtin.edu.au>, Rolf Turner <r.turner@auckland.ac.nz> and Ege Rubak <rubak@math.aau.dk>.

See Also

sparse3Darray

Examples

```
M <- sparse3Darray(i=1:4, j=sample(1:4, replace=TRUE),
                   k=c(1,2,1,2), x=1:4, dims=c(5,5,2))

anyNA(M)
dim(M)
dimnames(M)
dimnames(M) <- list(letters[1:5], LETTERS[1:5], c("Yes", "No"))
print(M)
```

---

sparse3Darray *Create a Sparse Three-Dimensional Array*

Description

Create a sparse representation of a three-dimensional array.

Usage

```
sparse3Darray(i = integer(0), j = integer(0), k = integer(0),
              x = numeric(0),
              dims = c(max(i), max(j), max(k)), dimnames = NULL,
              strict = FALSE, nonzero = FALSE)
```

Arguments

i,j,k	Integer vectors of equal length (or length 1), specifying the cells in the array which have non-zero entries.
x	Vector (numeric, integer, logical or complex) of the same length as i, j and k, giving the values of the array entries that are not zero.
dims	Dimension of the array. An integer vector of length 3.
dimnames	Names for the three margins of the array. Either NULL or a list of three character vectors.
strict	Logical value specifying whether to enforce the rule that each entry in i,j,k,x refers to a different cell. If strict=TRUE, entries which refer to the same cell in the array will be reduced to a single entry by summing the x values. Default is strict=FALSE.
nonzero	Logical value specifying whether to remove any entries of x which equal zero.

Details

An array A is three-dimensional if it is indexed by three integer indices, so that A[i,j,k] specifies an element of the array. The array is called sparse if only a small fraction of the entries are non-zero. A sparse array can be represented economically by listing only the entries which are non-zero.

The **spatstat.sparse** package defines the class sparse3Darray of sparse three-dimensional arrays. These arrays can have numeric, integer, logical, or complex entries.

The function sparse3Darray creates an object of class "sparse3Darray". This object is essentially a list containing the vectors i,j,k,x and the arguments dims,dimnames.

The arguments i,j,k,x should be vectors of equal length identifying the cells in the array which have non-zero entries (indexed by i,j,k) and giving the values in these cells (given by x).

The default behaviour of sparse3Darray is to accept the arguments i,j,k,x without modifying them. This would allow some entries of x to be equal to zero, and would allow a cell in the array to be referenced more than once in the indices i,j,k.

If nonzero=TRUE, entries will be removed if the x value equals zero.

If strict=TRUE, entries which refer to the same cell in the array will be combined into a single entry by summing the x values.

Value

An object of class "sparse3Dvector".

Author(s)

Adrian Baddeley <Adrian.Baddeley@curtin.edu.au>, Rolf Turner <r.turner@auckland.ac.nz> and Ege Rubak <rubak@math.aau.dk>.

See Also

as.sparse3Darray

Examples

```
## creation by specifying nonzero elements
M <- sparse3Darray(i=1:3, j=c(3,1,2), k=4:2,
                   x=runif(3), dims=rep(4, 3))
M
## duplicate entries
Mn <- sparse3Darray(i=c(1,1,2), j=c(2,2,1), k=c(3,3,2),
                   x=runif(3), dims=rep(3, 3))
## cumulate entries in duplicate positions
Ms <- sparse3Darray(i=c(1,1,2), j=c(2,2,1), k=c(3,3,2),
                    x=runif(3), dims=rep(3, 3), strict=TRUE)
```

`sumouter` *Compute Quadratic Forms*

Description

Calculates certain quadratic forms of matrices.

Usage

```
sumouter(x, w=NULL, y=x)
quadform(x, v)
bilinearform(x, v, y)
```

Arguments

`x,y`	A matrix, whose rows are the vectors in the quadratic form.
`w`	Optional vector of weights
`v`	Matrix determining the quadratic form

Details

The matrices `x` and `y` will be interpreted as collections of row vectors. They must have the same number of rows. The entries of `x` and `y` may be numeric, integer, logical or complex values.

The command `sumouter` computes the sum of the outer products of corresponding row vectors, weighted by the entries of `w`:

$$M = \sum_i w_i x_i^\top y_i$$

where x_i is the `i`-th row of `x` and y_i is the `i`-th row of `y` (after removing any rows containing `NA` or other non-finite values). If `w` is missing, the weights will be taken as 1. The result is a $p \times q$ matrix where `p = ncol(x)` and `q = ncol(y)`.

The command `quadform` evaluates the quadratic form, defined by the matrix `v`, for each of the row vectors of `x`:

$$y_i = x_i V x_i^\top$$

The result y is a numeric vector of length n where n = nrow(x). If x[i,] contains NA or other non-finite values, then y[i] = NA.

The command bilinearform evaluates the more general bilinear form defined by the matrix v. Here x and y must be matrices of the same dimensions. For each row vector of x and corresponding row vector of y, the bilinear form is

$$z_i = x_i V y_i^\top$$

The result z is a numeric vector of length n where n = nrow(x). If x[i,] or y[i,] contains NA or other non-finite values, then z[i] = NA.

Value

A vector or matrix.

Author(s)

Adrian Baddeley <Adrian.Baddeley@curtin.edu.au> and Rolf Turner <r.turner@auckland.ac.nz>

Examples

```
x <- matrix(1:12, 4, 3)
dimnames(x) <- list(c("Wilma", "Fred", "Barney", "Betty"), letters[1:3])
x

sumouter(x)

w <- 4:1
sumouter(x, w)
v <- matrix(1, 3, 3)
quadform(x, v)

# should be the same as quadform(x, v)
bilinearform(x, v, x)

# See what happens with NA's
x[3,2] <- NA
sumouter(x, w)
quadform(x, v)
```

---

tensorSparse *Tensor Product of Sparse Vectors, Matrices or Arrays*

Description

Compute the tensor product of two vectors, matrices or arrays which may be sparse or non-sparse.

Usage

```
tensorSparse(A, B, alongA = integer(0), alongB = integer(0))
```

Arguments

A,B	Vectors, matrices, three-dimensional arrays, or objects of class sparseVector, sparseMatrix or sparse3Darray.
alongA	Integer vector specifying the dimensions of A to be collapsed.
alongB	Integer vector specifying the dimensions of B to be collapsed.

Details

This function is a generalisation, to sparse arrays, of the function tensor in the **tensor** package.

tensorSparse has the same syntax and interpretation as tensor. For example, if A and B are matrices, then tensor(A,B,2,1) is the matrix product A %*% B while tensor(A,B,2,2) is A %*% t(B).

This function tensorSparse handles sparse vectors (class "sparseVector" in the **Matrix** package), sparse matrices (class "sparseMatrix" in the **Matrix** package) and sparse three-dimensional arrays (class "sparse3Darray" in the **spatstat.sparse** package) in addition to the usual vectors, matrices and arrays.

The result is a sparse object if at least one of A and B is sparse. Otherwise, if neither A nor B is sparse, then the result is computed using tensor.

The main limitation is that the result cannot have more than 3 dimensions (because sparse arrays with more than 3 dimensions are not yet supported).

Value

Either a scalar, a vector, a matrix, an array, a sparse vector (class "sparseVector" in the **Matrix** package), a sparse matrix (class "sparseMatrix" in the **Matrix** package) or a sparse three-dimensional array (class "sparse3Darray" in the **spatstat.sparse** package).

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See Also

sparse3Darray, aperm.sparse3Darray

Examples

```
M <- sparse3Darray(i=1:4, j=sample(1:4, replace=TRUE),
                   k=c(1,2,1,2), x=1:4, dims=c(5,5,2))
A <- tensorSparse(M, M, 1:2, 2:1)
```

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