# Package 'simts'

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```
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Title Time Series Analysis Tools
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Maintainer Stéphane Guerrier < stef.guerrier@gmail.com>
Description A system contains easy-to-use tools as a support for time series analysis courses. In par-
     ticular, it incorporates a technique called Generalized Method of Wavelet Mo-
     ments (GMWM) as well as its robust implementation for fast and robust parameter estima-
     tion of time series models which is described, for example, in Guer-
     rier et al. (2013) <doi: 10.1080/01621459.2013.799920>. More de-
     tails can also be found in the paper linked to via the URL below.
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License AGPL-3 | file LICENSE
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AIC.fitsimts

Akaike's Information Criterion

### **Description**

This function calculates AIC, BIC or HQ for a fitsimts object. This function currently only supports models estimated by the MLE.

#### Usage

```
## S3 method for class 'fitsimts' AIC(object, k = 2, ...)
```

### **Arguments**

object A fitsimts object.

k The penalty per parameter to be used; the default k = 2 is the classical AIC.

... Optionally more fitted model objects.

#### Value

AIC, BIC or HQ

### Author(s)

Stéphane Guerrier

```
set.seed(1)
n = 300
Xt = gen_gts(n, AR(phi = c(0, 0, 0.8), sigma2 = 1))
mod = estimate(AR(3), Xt)

# AIC
AIC(mod)

# BIC
AIC(mod, k = log(n))

# HQ
AIC(mod, k = 2*log(log(n)))
```

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Create an Autoregressive P [AR(P)] Process

AR

### **Description**

Sets up the necessary backend for the AR(P) process.

### Usage

```
AR(phi = NULL, sigma2 = 1)
```

### **Arguments**

phi A vector with double values for the  $\phi$  of an AR(P) process (see Note for details)

sigma2 A double value for the variance,  $\sigma^2$ , of an AR(P) process. (see Note for de-

tails).

#### Value

An S3 object with called ts.model with the following structure:

process.desc Used in summary: "AR-1", "AR-2", ..., "AR-P", "SIGMA2"

theta  $\phi_1, \phi_2, ..., \phi_p, \sigma^2$ 

plength Number of Parameters

desc "AR"

print String containing simplified model

**obj.desc** Depth of Parameters e.g. list(p,1)

starting Guess starting values? TRUE or FALSE (e.g. specified value)

#### Note

We consider the following model:

$$X_t = \sum_{j=1}^{p} \phi_j X_{t-1} + \varepsilon_t$$

, where  $\varepsilon_t$  is iid from a zero mean normal distribution with variance  $\sigma^2$ .

#### Author(s)

James Balamuta

```
AR(1) # Slower version of AR1()
AR(phi=.32, sigma=1.3) # Slower version of AR1()
AR(2) # Equivalent to ARMA(2,0).
```

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AR1

Definition of an Autoregressive Process of Order 1

#### **Description**

Definition of an Autoregressive Process of Order 1

### Usage

```
AR1(phi = NULL, sigma2 = 1)
```

#### **Arguments**

phi A double value for the parameter  $\phi$  (see Note for details).

sigma2 A double value for the variance parameter  $\sigma^2$  (see Note for details).

#### Value

An S3 object containing the specified ts.model with the following structure:

process.desc Used in summary: "AR1", "SIGMA2"

**theta** Parameter vector including  $\phi$ ,  $\sigma^2$ 

plength Number of parameters

print String containing simplified model

desc "AR1"

**obj.desc** Depth of Parameters e.g. list(1,1)

starting Find starting values? TRUE or FALSE (e.g. specified value)

#### Note

We consider the following AR(1) model:

$$X_t = \phi X_{t-1} + \varepsilon_t$$

, where  $\varepsilon_t$  is iid from a zero mean normal distribution with variance  $\sigma^2$ .

### Author(s)

James Balamuta

```
AR1()
AR1(phi=.32, sigma2 = 1.3)
```

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ar1\_to\_wv

AR(1) process to WV

### Description

This function computes the Haar WV of an AR(1) process

### Usage

```
ar1_to_wv(phi, sigma2, tau)
```

### **Arguments**

tau

A double that is the phi term of the AR(1) process phi A double corresponding to variance of AR(1) process sigma2 A vec containing the scales e.g.  $2^{\tau}$ 

#### **Details**

This function is significantly faster than its generalized counter part arma\_to\_wv.

#### Value

A vec containing the wavelet variance of the AR(1) process.

### **Process Haar Wavelet Variance Formula**

The Autoregressive Order 1 (AR(1)) process has a Haar Wavelet Variance given by:

$$\frac{2\sigma^{2}\left(4\phi^{\frac{\tau_{j}}{2}+1}-\phi^{\tau_{j}+1}-\frac{1}{2}\phi^{2}\tau_{j}+\frac{\tau_{j}}{2}-3\phi\right)}{\left(1-\phi\right)^{2}\left(1-\phi^{2}\right)\tau_{j}^{2}}$$

ARIMA

Create an Autoregressive Integrated Moving Average (ARIMA) Process

### Description

Sets up the necessary backend for the ARIMA process.

### Usage

```
ARIMA(ar = 1, i = 0, ma = 1, sigma2 = 1)
```

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### **Arguments**

| ar     | A vector or integer containing either the coefficients for $\phi$ 's or the process number $p$ for the Autoregressive (AR) term.   |
|--------|--|
| i      | An integer containing the number of differences to be done.  |
| ma     | A vector or integer containing either the coefficients for $\theta$ 's or the process number $q$ for the Moving Average (MA) term. |
| sigma2 | A double value for the standard deviation, $\sigma$ , of the ARIMA process.  |

#### **Details**

A variance is required since the model generation statements utilize randomization functions expecting a variance instead of a standard deviation like R.

#### Value

An S3 object with called ts.model with the following structure:

```
process.desc AR * p, MA * q

theta \sigma

plength Number of parameters

print String containing simplified model

obj.desc y desc replicated x times

obj Depth of parameters e.g. list(c(length(ar),length(ma),1))

starting Guess starting values? TRUE or FALSE (e.g. specified value)
```

#### Note

We consider the following model:

$$\Delta^i X_t = \sum_{j=1}^p \phi_j \Delta^i X_{t-j} + \sum_{j=1}^q \theta_j \varepsilon_{t-j} + \varepsilon_t$$

, where  $\varepsilon_t$  is iid from a zero mean normal distribution with variance  $\sigma^2$ .

#### Author(s)

James Balamuta

```
# Create an ARMA(1,2) process
ARIMA(ar=1,2)
# Creates an ARMA(3,2) process with predefined coefficients.
ARIMA(ar=c(0.23,.43,.59), ma=c(0.4,.3))
# Creates an ARMA(3,2) process with predefined coefficients and standard deviation
ARIMA(ar=c(0.23,.43,.59), ma=c(0.4,.3), sigma2 = 1.5)
```

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ARMA

Create an Autoregressive Moving Average (ARMA) Process

### **Description**

Sets up the necessary backend for the ARMA process.

### Usage

```
ARMA(ar = 1, ma = 1, sigma2 = 1)
```

### Arguments

| ar     | A vector or integer containing either the coefficients for $\phi$ 's or the process number $p$ for the Autoregressive (AR) term.   |
|--------|--|
| ma     | A vector or integer containing either the coefficients for $\theta$ 's or the process number $q$ for the Moving Average (MA) term. |
| sigma2 | A double value for the standard deviation, $\sigma$ , of the ARMA process.   |

#### **Details**

A variance is required since the model generation statements utilize randomization functions expecting a variance instead of a standard deviation like R.

### Value

An S3 object with called ts.model with the following structure:

```
process.desc AR * p, MA * q
theta \sigma
plength Number of Parameters
print String containing simplified model
obj.desc y desc replicated x times
obj Depth of Parameters e.g. list(c(length(ar),length(ma),1))
starting Guess Starting values? TRUE or FALSE (e.g. specified value)
```

#### Note

We consider the following model:

$$X_{t} = \sum_{j=1}^{p} \phi_{j} X_{t-j} + \sum_{j=1}^{q} \theta_{j} \varepsilon_{t-j} + \varepsilon_{t}$$

, where  $\varepsilon_t$  is iid from a zero mean normal distribution with variance  $\sigma^2$ .

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#### Author(s)

James Balamuta

### **Examples**

```
# Create an ARMA(1,2) process ARMA(ar=1,2) # Creates an ARMA(3,2) process with predefined coefficients.  \text{ARMA}(\text{ar}=\text{c}(0.23,.43,.59), \text{ ma}=\text{c}(0.4,.3))  # Creates an ARMA(3,2) process with predefined coefficients and standard deviation  \text{ARMA}(\text{ar}=\text{c}(0.23,.43,.59), \text{ ma}=\text{c}(0.4,.3), \text{ sigma2}=1.5)
```

ARMA11

Definition of an ARMA(1,1)

#### **Description**

Definition of an ARMA(1,1)

#### Usage

```
ARMA11 (phi = NULL, theta = NULL, sigma2 = 1)
```

### **Arguments**

phi A double containing the parameter  $\phi_1$  (see Note for details). theta A double containing the parameter  $\theta_1$  (see Note for details). sigma2 A double value for the parameter  $\sigma^2$  (see Note for details).

#### **Details**

A variance is required since the model generation statements utilize randomization functions expecting a variance instead of a standard deviation like R.

### Value

An S3 object with called ts.model with the following structure:

```
process.desc AR1, MA1, SIGMA2
theta \phi, \theta, \sigma^2
plength Number of Parameters: 3
print String containing simplified model
obj.desc Depth of Parameters e.g. list(c(1,1,1))
starting Guess Starting values? TRUE or FALSE (e.g. specified value)
```

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### Note

We consider the following model:

$$X_t = \phi X_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t,$$

where  $\varepsilon_t$  is iid from a zero mean normal distribution with variance  $\sigma^2$ .

#### Author(s)

James Balamuta

#### **Examples**

```
# Creates an ARMA(1,1) process with predefined coefficients.
ARMA11(phi = .23, theta = .1, sigma2 = 1)
# Creates an ARMA(1,1) process with values to be guessed on callibration.
ARMA11()
```

arma11\_to\_wv

ARMA(1,1) to WV

### Description

This function computes the WV (haar) of an Autoregressive Order 1 - Moving Average Order 1 (ARMA(1,1)) process.

#### Usage

```
arma11_to_wv(phi, theta, sigma2, tau)
```

### **Arguments**

phi A double corresponding to the autoregressive term. theta A double corresponding to the moving average term. sigma2 A double the variance of the process. tau A vec containing the scales e.g.  $2^{\tau}$ 

#### **Details**

This function is significantly faster than its generalized counter part arma\_to\_wv

#### Value

A vec containing the wavelet variance of the ARMA(1,1) process.

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#### **Process Haar Wavelet Variance Formula**

The Autoregressive Order 1 and Moving Average Order 1 (ARMA(1,1)) process has a Haar Wavelet Variance given by:

$$\nu_j^2\left(\phi, \theta, \sigma^2\right) = -\frac{2\sigma^2\left(-\frac{1}{2}(\theta+1)^2\left(\phi^2-1\right)\tau_j - (\theta+\phi)(\theta\phi+1)\left(\phi^{\tau_j} - 4\phi^{\frac{\tau_j}{2}} + 3\right)\right)}{\left(\phi-1\right)^3(\phi+1)\tau_j^2}$$

arma\_to\_wv

ARMA process to WV

#### **Description**

This function computes the Haar Wavelet Variance of an ARMA process

### Usage

### **Arguments**

| ar     | A vec containing the coefficients of the AR process |
|--------|---|
| ma     | A vec containing the coefficients of the MA process |
| sigma2 | A double containing the residual variance           |
| tau    | A vec containing the scales e.g. $2^{\tau}$         |

#### **Details**

The function is a generic implementation that requires a stationary theoretical autocorrelation function (ACF) and the ability to transform an ARMA(p,q) process into an MA $(\infty)$  (e.g. infinite MA process).

#### Value

A vec containing the wavelet variance of the ARMA process.

#### **Process Haar Wavelet Variance Formula**

The Autoregressive Order p and Moving Average Order q (ARMA(p,q)) process has a Haar Wavelet Variance given by:

$$\frac{\tau_{j}\left[1-\rho\left(\frac{\tau_{j}}{2}\right)\right]+2\sum_{i=1}^{\frac{\tau_{j}}{2}-1}i\left[2\rho\left(\frac{\tau_{j}}{2}-i\right)-\rho\left(i\right)-\rho\left(\tau_{j}-i\right)\right]}{\tau_{j}^{2}}\sigma_{X}^{2}$$

where  $\sigma_X^2$  is given by the variance of the ARMA process. Furthermore, this assumes that stationarity has been achieved as it directly

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australia

Quarterly Increase in Stocks Non-Farm Total, Australia

### **Description**

A dataset containing the quarterly increase in stocks non-farm total in Australia, with frequency 4 starting from September 1959 to March 1991 with a total of 127 observations.

#### Usage

australia

#### **Format**

A data frame with 127 rows and 2 variables:

Quarter year and quarter

Increase quarterly increase in stocks non-farm total

#### **Source**

```
Time Series Data Library (citing: Australian Bureau of Statistics) https://datamarket.com/data/set/22t0/quarterly-increase-in-stocks-non-farm-total-australia-sep-1959-media=22t0&display=line
```

auto\_corr

Empirical ACF and PACF

### Description

This function can estimate either the autocovariance / autocorrelation for univariate time series, or the partial autocovariance / autocorrelation for univariate time series.

#### Usage

```
auto_corr(x, lag.max = NULL, pacf = FALSE, type = "correlation",
  demean = TRUE, robust = FALSE)
```

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#### **Arguments**

| X       | A vector or ts object (of length $N > 1$ ).  |
|---------|--|
| lag.max | An integer indicating the maximum lag up to which to compute the empirical ACF / PACF.   |
| pacf    | A boolean indicating whether to output the PACF. If it's TRUE, then the function will only estimate the empirical PACF. If it's FALSE (the default), then the function will only estimate the empirical ACF. |
| type    | A character string giving the type of acf to be computed. Allowed values are "correlation" (the default) and "covariance".   |
| demean  | A boolean indicating whether the data should be detrended (TRUE) or not (FALSE). Defaults to TRUE.   |
| robust  | A boolean indicating whether a robust estimator should be used (TRUE) or not (FALSE). Defaults to FALSE. This only works when the function is estimating ACF.  |

### **Details**

lagmax default is 10\*log10(N/m) where N is the number of observations and m is the number of time series being compared. If lagmax supplied is greater than the number of observations N, then one less than the total will be taken (i.e. N - 1).

### Value

```
An array of dimensions N \times 1 \times 1.
```

### Author(s)

Yuming Zhang

### **Examples**

```
m = auto_corr(datasets::AirPassengers)
m = auto_corr(datasets::AirPassengers, pacf = TRUE)
```

best\_model

Select the Best Model

# Description

This function retrieves the best model from a selection procedure.

### Usage

```
best_model(x, ic = "aic")
```

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#### Arguments

| X  | An object of class select_arma, select_ar or select_ma.                       |
|----|---|
| ic | A string indicating the type of criterion to use in selecting the best model. |
|    | Supported criteria include "aic" (AIC), "bic" (BIC) and "hq" (HQ).            |

### **Examples**

```
set.seed(18)
xt = gen_arima(N=100, ar=0.3, d=1, ma=0.3)
x = select_arima(xt, d=1L)
best_model(x, ic = "aic")

set.seed(19)
xt = gen_mal(100, 0.3, 1)
x = select_ma(xt, q.min=2L, q.max=5L)
best_model(x, ic = "bic")

set.seed(20)
xt = gen_arma(100, c(.3,.5), c(.1), 1, 0)
x = select_arma(xt, p.min = 1L, p.max = 4L, q.min = 1L, q.max = 3L)
best_model(x, ic = "hq")
```

check

Diagnostics on Fitted Time Series Model

### Description

This function can perform (simple) diagnostics on the fitted time series model. It can output 6 diagnostic plots to assess the model, including (1) residuals plot, (2) histogram of distribution of standardized residuals, (3) Normal Q-Q plot of residuals, (4) ACF plot, (5) PACF plot, (6) Box test results.

### Usage

```
check(model = NULL, resids = NULL, simple = FALSE)
```

### Arguments

```
model A fitsimts, lm or gam object.

resids A vector of residuals for diagnostics.

simple A boolean indicating whether to return simple diagnostic plots or not.
```

## Author(s)

Stéphane Guerrier and Yuming Zhang

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#### **Examples**

```
Xt = gen_gts(300, AR(phi = c(0, 0, 0.8), sigma2 = 1))
model = estimate(AR(3), Xt)
check(model)

check(resids = rnorm(100))

Xt = gen_gts(1000, SARIMA(ar = c(0.5, -0.25), i = 0, ma = 0.5, sar = -0.8,
si = 1, sma = 0.25, s = 24, sigma2 = 1))
model = estimate(SARIMA(ar = 2, i = 0, ma = 1, sar = 1, si = 1, sma = 1, s = 24),
Xt, method = "rgmwm")
check(model)
check(model, simple=TRUE)
```

compare\_acf

Comparison of Classical and Robust Correlation Analysis Functions

### **Description**

Compare classical and robust ACF of univariate time series.

### Usage

```
compare_acf(x, lag.max = NULL, demean = TRUE, show.ci = TRUE,
alpha = 0.05, plot = TRUE, ...)
```

### **Arguments**

| X       | A vector or "ts" object (of length $N > 1$ ).  |
|---------|--|
| lag.max | A integer indicating the maximum lag up to which to compute the ACF and PACF functions.  |
| demean  | A bool indicating whether the data should be detrended (TRUE) or not (FALSE). Defaults to TRUE.  |
| show.ci | A bool indicating whether to compute and show the confidence region. Defaults to ${\tt TRUE.}$   |
| alpha   | A double indicating the level of significance for the confidence interval. By default $alpha = 0.05$ which gives a 1 - $alpha = 0.95$ confidence interval. |
| plot    | A bool indicating whether a plot of the computed quantities should be produced. Defaults to ${\tt TRUE}.$  |
|         | Additional parameters.   |

### Author(s)

Yunxiang Zhang

corr\_analysis 17

### **Examples**

```
# Estimate both the ACF and PACF functions
compare_acf(datasets::AirPassengers)
```

corr\_analysis

Correlation Analysis Functions

### Description

Correlation Analysis function computes and plots both empirical ACF and PACF of univariate time series.

#### Usage

```
corr_analysis(x, lag.max = NULL, type = "correlation", demean = TRUE,
    show.ci = TRUE, alpha = 0.05, plot = TRUE, ...)
```

### **Arguments**

| X       | A vector or "ts" object (of length $N > 1$ ).  |
|---------|--|
| lag.max | A integer indicating the maximum lag up to which to compute the ACF and PACF functions.  |
| type    | A character string giving the type of acf to be computed. Allowed values are "correlation" (the default) and "covariance".                                 |
| demean  | A bool indicating whether the data should be detrended (TRUE) or not (FALSE). Defaults to TRUE.  |
| show.ci | A bool indicating whether to compute and show the confidence region. Defaults to ${\tt TRUE.}$   |
| alpha   | A double indicating the level of significance for the confidence interval. By default $alpha = 0.05$ which gives a 1 - $alpha = 0.95$ confidence interval. |
| plot    | A bool indicating whether a plot of the computed quantities should be produced. Defaults to ${\tt TRUE}.$  |
|         | Additional parameters.   |

### Value

Two array objects (ACF and PACF) of dimension  $N \times S \times S$ .

### Author(s)

Yunxiang Zhang

```
# Estimate both the ACF and PACF functions
corr_analysis(datasets::AirPassengers)
```

deriv\_2nd\_ar1

```
derivative_first_matrix

Analytic D matrix of Processes
```

### **Description**

This function computes each process to WV (haar) in a given model.

#### Usage

```
derivative_first_matrix(theta, desc, objdesc, tau)
```

### Arguments

theta A vec containing the list of estimated parameters. desc A vector<string> containing a list of descriptors. objdesc A field<vec> containing a list of object descriptors. tau A vec containing the scales e.g.  $2^{\tau}$ 

#### **Details**

Function returns the matrix effectively known as "D"

#### Value

A matrix with the process derivatives going down the column

### Author(s)

James Joseph Balamuta (JJB)

deriv\_2nd\_ar1 Analytic second derivative matrix for AR(1) process

# Description

Calculates the second derivative for the AR(1) process and places it into a matrix form. The matrix form in this case is for convenience of the calculation.

### Usage

```
deriv_2nd_ar1(phi, sigma2, tau)
```

deriv\_2nd\_arma11

#### **Arguments**

phi A double corresponding to the phi coefficient of an AR(1) process. sigma2 A double corresponding to the error term of an AR(1) process. tau A vec containing the scales e.g.  $2^{\tau}$ 

#### Value

A matrix with the first column containing the second partial derivative with respect to  $\phi$  and the second column contains the second partial derivative with respect to  $\sigma^2$ 

#### **Process Haar WV Second Derivative**

Taking the second derivative with respect to  $\phi$  yields:

$$\frac{\partial^2}{\partial \phi^2} \nu_j^2 \left( \phi, \sigma^2 \right) = \frac{2\sigma^2 \left( \left( \phi^2 - 1 \right) \tau_j \left( 2(\phi(7\phi + 4) + 1)\phi^{\frac{\tau_j}{2} - 1} - (\phi(7\phi + 4) + 1)\phi^{\tau_j - 1} + 3(\phi + 1)^2 \right) + \left( \phi^2 - 1 \right)^2 \tau_j^2 \left( \phi^2 - 1 \right) \tau$$

Taking the second derivative with respect to  $\sigma^2$  yields:

$$\frac{\partial^2}{\partial \sigma^4} \nu_j^2 \left( \sigma^2 \right) = 0$$

Taking the derivative with respect to  $\phi$  and  $\sigma^2$  yields:

$$\frac{\partial^2}{\partial \phi \partial \sigma^2} \nu_j^2 \left( \phi, \sigma^2 \right) = \frac{2 \left( \left( \phi^2 - 1 \right) \tau_j \left( \phi^{\tau_j} - 2 \phi^{\frac{\tau_j}{2}} - \phi - 1 \right) - \left( \phi (3\phi + 2) + 1 \right) \left( \phi^{\tau_j} - 4 \phi^{\frac{\tau_j}{2}} + 3 \right) \right)}{(\phi - 1)^4 (\phi + 1)^2 \tau_j^2}$$

#### Author(s)

James Joseph Balamuta (JJB)

deriv\_2nd\_arma11 Analytic D matrix for ARMA(1,1) process

#### **Description**

Obtain the second derivative of the ARMA(1,1) process.

#### Usage

```
deriv_2nd_armal1(phi, theta, sigma2, tau)
```

### **Arguments**

phi A double corresponding to the phi coefficient of an ARMA(1,1) process. theta A double corresponding to the theta coefficient of an ARMA(1,1) process. sigma2 A double corresponding to the error term of an ARMA(1,1) process. tau A vec containing the scales e.g.  $2^{\tau}$ 

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#### Value

A matrix with:

- The **first** column containing the second partial derivative with respect to  $\phi$ ;
- The **second** column containing the second partial derivative with respect to  $\theta$ ;
- The **third** column contains the second partial derivative with respect to  $\sigma^2$ .
- The **fourth** column contains the partial derivative with respect to  $\phi$  and  $\theta$ .
- The **fiveth** column contains the partial derivative with respect to  $\sigma^2$  and  $\phi$ .
- The **sixth** column contains the partial derivative with respect to  $\sigma^2$  and  $\theta$ .

#### **Process Haar WV Second Derivative**

Taking the second derivative with respect to  $\phi$  yields:

$$\frac{\partial^{2}}{\partial\phi^{2}}\nu_{j}^{2}\left(\phi,\theta,\sigma^{2}\right) = \frac{2\sigma^{2}}{\left(\phi-1\right)^{5}\left(\phi+1\right)^{3}\tau_{j}^{2}} \begin{pmatrix} (\phi-1)^{2}\left((\phi+1)^{2}\left(\theta^{2}\phi+\theta\phi^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\theta+\phi)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{2}}-12(\phi+\phi)^{2}+\theta+\phi\right)\tau_{j}^{2}\left(\phi^{\frac{\tau_{j}}{$$

Taking the second derivative with respect to  $\theta$  yields:

$$\frac{\partial^2}{\partial \theta^2} \nu_j^2 \left( \phi, \theta, \sigma^2 \right) = \frac{2\sigma^2 \left( \left( \phi^2 - 1 \right) \tau_j + 2\phi \left( \phi^{\tau_j} - 4\phi^{\frac{\tau_j}{2}} + 3 \right) \right)}{\left( \phi - 1 \right)^3 \left( \phi + 1 \right) \tau_j^2}$$

Taking the second derivative with respect to  $\sigma^2$  yields:

$$\frac{\partial^2}{\partial \sigma^4} \nu_j^2 \left( \phi, \theta, \sigma^2 \right) = 0$$

Taking the derivative with respect to  $\sigma^2$  and  $\theta$  yields:

$$\frac{\partial}{\partial \theta} \frac{\partial}{\partial \sigma^2} \nu_j^2 \left( \phi, \theta, \sigma^2 \right) = \frac{2}{\left( \phi - 1 \right)^3 \left( \phi + 1 \right) \tau_j^2} \left( \left( \theta + 1 \right) \left( \phi^2 - 1 \right) \tau_j + \left( 2\theta \phi + \phi^2 + 1 \right) \left( \phi^{\tau_j} - 4\phi^{\frac{\tau_j}{2}} + 3 \right) \right)$$

Taking the derivative with respect to  $\sigma^2$  and  $\phi$  yields:

$$\frac{\partial}{\partial \phi} \frac{\partial}{\partial \sigma^{2}} \nu_{j}^{2} \left( \phi, \theta, \sigma^{2} \right) = \frac{2}{\left( \phi - 1 \right)^{4} \left( \phi + 1 \right)^{2} \tau_{j}^{2}} \begin{pmatrix} -(\phi - 1)(\phi + 1) \begin{pmatrix} -(\theta + \phi)(\theta \phi + 1)\tau_{j} \left( \phi^{\frac{\tau_{j}}{2}} - 2 \right) \phi^{\frac{\tau_{j}}{2}} - 1 \\ -\theta(\theta + \phi) \left( \phi^{\tau_{j}} - 4\phi^{\frac{\tau_{j}}{2}} + 3 \right) \\ -(\theta \phi + 1) \left( \phi^{\tau_{j}} - 4\phi^{\frac{\tau_{j}}{2}} + 3 \right) \\ -(\theta + 1)^{2} \phi \tau_{j} \end{pmatrix} + (\phi - 1) \left( -\frac{1}{2}(\theta + 1)^{2} \left( \phi^{2} - 1 \right) \tau_{j} - (\theta + \phi)(\theta \phi + 1) \left( \phi^{\tau_{j}} - 4\phi^{\frac{\tau_{j}}{2}} + 4\phi^{\frac{\tau_{j}}{2}} + 4\phi^{\frac{\tau_{j}}{2}} + 4\phi^{\frac{\tau_{j}}{2}} \right) \right) + (\phi - 1) \left( -\frac{1}{2}(\theta + 1)^{2} \left( \phi^{2} - 1 \right) \tau_{j} - (\theta + \phi)(\theta \phi + 1) \left( \phi^{\tau_{j}} - 4\phi^{\frac{\tau_{j}}{2}} + 4\phi^{\frac{\tau_{j}}{2}} \right) \right) + (\phi - 1) \left( -\frac{1}{2}(\theta + 1)^{2} \left( \phi^{2} - 1 \right) \tau_{j} - (\theta + \phi)(\theta \phi + 1) \left( \phi^{\tau_{j}} - 4\phi^{\frac{\tau_{j}}{2}} + 4\phi^{\frac{\tau_{j}}{2}} \right) \right) + (\phi - 1) \left( -\frac{1}{2}(\theta + 1)^{2} \left( \phi^{2} - 1 \right) \tau_{j} - (\theta + \phi)(\theta \phi + 1) \left( \phi^{\tau_{j}} - 4\phi^{\frac{\tau_{j}}{2}} + 4\phi^{\frac{\tau_{j}}{2}} \right) \right) \right) + (\phi - 1) \left( -\frac{1}{2}(\theta + 1)^{2} \left( \phi^{2} - 1 \right) \tau_{j} - (\theta + \phi)(\theta \phi + 1) \left( \phi^{\tau_{j}} - 4\phi^{\frac{\tau_{j}}{2}} + 4\phi^{\frac{\tau_{j}}{2}} \right) \right) \right) \right) + (\phi - 1) \left( -\frac{1}{2}(\theta + 1)^{2} \left( \phi^{2} - 1 \right) \tau_{j} - (\theta + \phi)(\theta \phi + 1) \left( \phi^{\tau_{j}} - 4\phi^{\frac{\tau_{j}}{2}} + 4\phi^{\frac{\tau_{j}}{2}} \right) \right) \right) \right)$$

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Taking the derivative with respect to  $\phi$  and  $\theta$  yields:

$$\frac{\partial}{\partial \theta} \frac{\partial}{\partial \phi} \nu_j^2 \left( \phi, \theta, \sigma^2 \right) = -\frac{2\sigma^2}{\left( \phi - 1 \right)^4 \left( \phi + 1 \right)^2 \tau_j^2} \left( \begin{array}{c} \tau_j \left( \begin{array}{c} 2(\theta + 1)(\phi - 1)(\phi + 1)^2 \\ +2\left( \phi^2 - 1 \right) \left( 2\theta \phi + \phi^2 + 1 \right) \phi^{\frac{\tau_j}{2} - 1} \\ -\left( \phi^2 - 1 \right) \left( 2\theta \phi + \phi^2 + 1 \right) \phi^{\tau_j - 1} \end{array} \right) + 2 \left( \theta(\phi(3\phi + 2) + 1) + \phi \left( \phi^2 + \phi + 3 \right) + 1 \right) \left( \phi^{\tau_j} - 4\phi^{\frac{\tau_j}{2}} + 3 \right) \right)$$

#### Author(s)

James Joseph Balamuta (JJB)

deriv\_2nd\_dr

Analytic second derivative matrix for drift process

### Description

To ease a later calculation, we place the result into a matrix structure.

### Usage

#### **Arguments**

tau

A vec containing the scales e.g.  $2^{\tau}$ 

#### Value

A matrix with the first column containing the second partial derivative with respect to  $\omega$ .

#### Author(s)

James Joseph Balamuta (JJB)

deriv\_2nd\_ma1

Analytic second derivative for MA(1) process

### **Description**

To ease a later calculation, we place the result into a matrix structure.

### Usage

```
deriv_2nd_ma1(theta, sigma2, tau)
```

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#### Arguments

theta A double corresponding to the theta coefficient of an MA(1) process.

sigma2 A double corresponding to the error term of an MA(1) process.

tau A vec containing the scales e.g.  $2^{\tau}$ 

#### Value

A matrix with the first column containing the second partial derivative with respect to  $\theta$ , the second column contains the partial derivative with respect to  $\theta$  and  $\sigma^2$ , and lastly we have the second partial derivative with respect to  $\sigma^2$ .

#### **Process Haar WV Second Derivative**

Taking the second derivative with respect to  $\theta$  yields:

$$\frac{\partial^2}{\partial \theta^2} \nu_j^2 \left( \theta, \sigma^2 \right) = \frac{2\sigma^2}{\tau_j}$$

Taking the second derivative with respect to  $\sigma^2$  yields:

$$\frac{\partial^2}{\partial \sigma^4} \nu_j^2 \left( \theta, \sigma^2 \right) = 0$$

Taking the first derivative with respect to  $\theta$  and  $\sigma^2$  yields:

$$\frac{\partial}{\partial \theta} \frac{\partial}{\partial \sigma^2} \nu_j^2 \left( \theta, \sigma^2 \right) = \frac{2(\theta + 1)\tau_j - 6}{\tau_j^2}$$

#### Author(s)

James Joseph Balamuta (JJB)

deriv\_ar1

Analytic D matrix for AR(1) process

### **Description**

Obtain the first derivative of the AR(1) process.

#### Usage

```
deriv_ar1(phi, sigma2, tau)
```

#### **Arguments**

phi A double corresponding to the phi coefficient of an AR(1) process.

sigma2 A double corresponding to the error term of an AR(1) process.

tau A vec containing the scales e.g.  $2^{\tau}$ 

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#### Value

A matrix with the first column containing the partial derivative with respect to  $\phi$  and the second column contains the partial derivative with respect to  $\sigma^2$ 

#### **Process Haar WV First Derivative**

Taking the derivative with respect to  $\phi$  yields:

$$\frac{\partial}{\partial \phi} \nu_j^2 \left( \phi, \sigma^2 \right) = \frac{2\sigma^2 \left( \left( \phi^2 - 1 \right) \tau_j \left( -2\phi^{\frac{\tau_j}{2}} + \phi^{\tau_j} - \phi - 1 \right) - \left( \phi \left( 3\phi + 2 \right) + 1 \right) \left( -4\phi^{\frac{\tau_j}{2}} + \phi^{\tau_j} + 3 \right) \right)}{\left( \phi - 1 \right)^4 \left( \phi + 1 \right)^2 \tau_j^2}$$

Taking the derivative with respect to  $\sigma^2$  yields:

$$\frac{\partial}{\partial \sigma^2} \nu_j^2 \left( \phi, \sigma^2 \right) = \frac{\left( \phi^2 - 1 \right) \tau_j + 2\phi \left( -4\phi^{\frac{\tau_j}{2}} + \phi^{\tau_j} + 3 \right)}{\left( \phi - 1 \right)^3 \left( \phi + 1 \right) \tau_j^2}$$

#### Author(s)

James Joseph Balamuta (JJB)

deriv\_arma11

Analytic D matrix for ARMA(1,1) process

#### **Description**

Obtain the first derivative of the ARMA(1,1) process.

#### Usage

```
deriv_armal1(phi, theta, sigma2, tau)
```

### **Arguments**

phi A double corresponding to the phi coefficient of an ARMA(1,1) process. theta A double corresponding to the theta coefficient of an ARMA(1,1) process. sigma2 A double corresponding to the error term of an ARMA(1,1) process. tau A vec containing the scales e.g.  $2^{\tau}$ 

#### Value

A matrix with:

- The **first** column containing the partial derivative with respect to  $\phi$ ;
- The **second** column containing the partial derivative with respect to  $\theta$ ;
- The **third** column contains the partial derivative with respect to  $\sigma^2$ .

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#### **Process Haar WV First Derivative**

Taking the derivative with respect to  $\phi$  yields:

Taking the derivative with respect to  $\theta$  yields:

$$\frac{\partial}{\partial \theta} \nu_j^2 \left( \phi, \theta, \sigma^2 \right) = \frac{2\sigma^2 \left( \left( \theta + 1 \right) \left( \phi^2 - 1 \right) \tau_j + \left( 2\theta \phi + \phi^2 + 1 \right) \left( \phi^{\tau_j} - 4\phi^{\frac{\tau_j}{2}} + 3 \right) \right)}{\left( \phi - 1 \right)^3 \left( \phi + 1 \right) \tau_j^2}$$

Taking the derivative with respect to  $\sigma^2$  yields:

$$\frac{\partial}{\partial \sigma^2} \nu_j^2 \left( \phi, \theta, \sigma^2 \right) = \frac{2\sigma^2 \left( \left( \phi^2 - 1 \right) \tau_j + 2\phi \left( \phi^{\tau_j} - 4\phi^{\frac{\tau_j}{2}} + 3 \right) \right)}{(\phi - 1)^3 (\phi + 1) \tau_j^2}$$

#### Author(s)

James Joseph Balamuta (JJB)

deriv\_dr

Analytic D matrix for Drift (DR) Process

# Description

Obtain the first derivative of the Drift (DR) process.

#### Usage

#### Arguments

omega A double that is the slope of the drift.

tau A vec containing the scales e.g.  $2^{\tau}$ 

#### Value

A matrix with the first column containing the partial derivative with respect to  $\omega$ .

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#### **Process Haar WV First Derivative**

Taking the derivative with respect to  $\omega$  yields:

$$\frac{\partial}{\partial\omega}\nu_j^2\left(\omega\right) = \frac{\tau_j^2\omega}{8}$$

**Note:** We are taking the derivative with respect to  $\omega$  and not  $\omega^2$  as the  $\omega$  relates to the slope of the process and not the processes variance like RW and WN. As a result, a second derivative exists and is not zero.

#### Author(s)

James Joseph Balamuta (JJB)

deriv\_ma1

Analytic D matrix for MA(1) process

### **Description**

Obtain the first derivative of the MA(1) process.

#### Usage

deriv\_ma1(theta, sigma2, tau)

### **Arguments**

theta A double corresponding to the theta coefficient of an MA(1) process.

sigma2 A double corresponding to the error term of an MA(1) process.

tau A vec containing the scales e.g.  $2^{\tau}$ 

#### Value

A matrix with the first column containing the partial derivative with respect to  $\theta$  and the second column contains the partial derivative with respect to  $\sigma^2$ 

### **Process Haar WV First Derivative**

Taking the derivative with respect to  $\theta$  yields:

$$\frac{\partial}{\partial \theta} \nu_j^2 \left( \theta, \sigma^2 \right) = \frac{\sigma^2 \left( 2 \left( \theta + 1 \right) \tau_j - 6 \right)}{\tau_i^2}$$

Taking the derivative with respect to  $\sigma^2$  yields:

$$\frac{\partial}{\partial \sigma^2} \nu_j^2 \left( \theta, \sigma^2 \right) = \frac{\left( \theta + 1 \right)^2 \tau_j - 6\theta}{\tau_j^2}$$

#### Author(s)

James Joseph Balamuta (JJB)

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deriv\_qn

Analytic D matrix for Quantization Noise (QN) Process

### Description

Obtain the first derivative of the Quantization Noise (QN) process.

### Usage

```
deriv_qn(tau)
```

### **Arguments**

tau

A vec containing the scales e.g.  $2^{\tau}$ 

#### Value

A matrix with the first column containing the partial derivative with respect to  $Q^2$ .

#### **Process Haar WV First Derivative**

Taking the derivative with respect to  $Q^2$  yields:

$$\frac{\partial}{\partial Q^2} \nu_j^2 \left( Q^2 \right) = \frac{6}{\tau_i^2}$$

### Author(s)

James Joseph Balamuta (JJB)

deriv\_rw

Analytic D matrix Random Walk (RW) Process

### **Description**

Obtain the first derivative of the Random Walk (RW) process.

### Usage

### **Arguments**

tau

A vec containing the scales e.g.  $2^{\tau}$ 

deriv\_wn

### Value

A matrix with the first column containing the partial derivative with respect to  $\gamma^2$ .

### **Process Haar WV First Derivative**

Taking the derivative with respect to  $\gamma^2$  yields:

$$\frac{\partial}{\partial \gamma^2} \nu_j^2 \left( \gamma^2 \right) = \frac{\tau_j^2 + 2}{12\tau_j}$$

#### Author(s)

James Joseph Balamuta (JJB)

deriv\_wn

Analytic D Matrix for a Gaussian White Noise (WN) Process

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### Description

Obtain the first derivative of the Gaussian White Noise (WN) process.

### Usage

deriv\_wn(tau)

### **Arguments**

tau

A vec containing the scales e.g.  $2^{\tau}$ 

### Value

A matrix with the first column containing the partial derivative with respect to  $\sigma^2$ .

#### **Process Haar WV First Derivative**

Taking the derivative with respect to  $\sigma^2$  yields:

$$\frac{\partial}{\partial \sigma^2} \nu_j^2 \left( \sigma^2 \right) = \frac{1}{\tau_j}$$

### Author(s)

James Joseph Balamuta (JJB)

28 diag\_ljungbox

### Description

Performs the Box-Pierce test to assess the Null Hypothesis of Independence in a Time Series

### Usage

```
diag_boxpierce(x, order = NULL, stop_lag = 20, stdres = FALSE,
    plot = TRUE)
```

### Arguments

| X        | An arima or data set.   |
|----------|---|
| order    | An integer indicating the degrees of freedom. If 'x' is not a series of residuals, then set equal to 0. |
| stop_lag | An integer indicating the length of lags that should be calculated.                                     |
| stdres   | A boolean indicating whether to standardize the residualizes (e.g. $res/sd(res)$ ) or not.              |
| plot     | A logical. If TRUE (the default) a plot should be produced.   |

#### Author(s)

James Balamuta, Stéphane Guerrier, Yuming Zhang

### **Description**

Performs the Ljung-Box test to assess the Null Hypothesis of Independence in a Time Series

### Usage

```
diag_ljungbox(x, order = NULL, stop_lag = 20, stdres = FALSE,
    plot = TRUE)
```

### Arguments

| X        | An arima or data set.   |
|----------|---|
| order    | An integer indicating the degrees of freedom. If 'x' is not a series of residuals, then set equal to 0. |
| stop_lag | An integer indicating the length of lags that should be calculated.                                     |
| stdres   | A boolean indicating whether to standardize the residualizes (e.g. $res/sd(res)$ ) or not.              |
| plot     | A logical. If TRUE (the default) a plot should be produced.   |

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#### Author(s)

James Balamuta, Stéphane Guerrier, Yuming Zhang

| diad  | plot                               |
|-------|------------------------------------|
| uray_ | $_{\rm P}_{\rm T}_{\rm O}_{\rm C}$ |

Diagnostic Plot of Residuals

### **Description**

This function will plot 8 diagnostic plots to assess the model used to fit the data. These include: (1) residuals plot, (2) residuals vs fitted values, (3) histogram of distribution of standardized residuals, (4) Normal Q-Q plot of residuals, (5) ACF plot, (6) PACF plot, (7) Haar Wavelet Variance Representation, (8) Box test results.

### Usage

```
diag_plot(Xt = NULL, model = NULL, resids = NULL, std = FALSE)
```

### **Arguments**

Xt The data used to construct said model.

model A fitsimts, lm or gam object.

resids A vector of residuals for diagnostics.

std A boolean indicating whether we use standardized residuals for (1) residuals

plot and (8) Box test results.

#### Author(s)

Yuming Zhang

```
diag_portmanteau_ Portmanteau Tests
```

### **Description**

Performs the Portmanteau test to assess the Null Hypothesis of Independence in a Time Series

### Usage

```
diag_portmanteau_(x, order = NULL, stop_lag = 20, stdres = FALSE,
  test = "Ljung-Box", plot = TRUE)
```

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### **Arguments**

| X        | An arima or data set.  |
|----------|--|
| order    | An integer indicating the degrees of freedom. If 'x' is not a series of residuals, then set equal to $0$ . |
| stop_lag | An integer indicating the length of lags that should be calculated.  |
| stdres   | A boolean indicating whether to standardize the residualizes (e.g. $res/sd(res)$ ) or not.                 |
| test     | A string indicating whether to perform Ljung-Box test or Box-Pierce test.                                  |
| plot     | A logical. If TRUE (the default) a plot should be produced.  |
|          |  |

### Author(s)

James Balamuta, Stéphane Guerrier, Yuming Zhang

DR Create an Drift (DR) Process

### **Description**

Sets up the necessary backend for the DR process.

### Usage

```
DR(omega = NULL)
```

# Arguments

omega A double value for the slope of a DR process (see Note for details).

### Value

An S3 object with called ts.model with the following structure:

```
process.desc Used in summary: "DR"
theta slope
print String containing simplified model
plength Number of parameters
obj.desc y desc replicated x times
obj Depth of parameters e.g. list(1)
starting Guess starting values? TRUE or FALSE (e.g. specified value)
```

#### Note

We consider the following model:

 $Y_t = \omega t$ 

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### Author(s)

James Balamuta

### **Examples**

dr\_to\_wv

Drift to WV

# Description

This function compute the WV (haar) of a Drift process

### Usage

# Arguments

omega A double corresponding to the slope of the drift

tau A vec containing the scales e.g.  $2^{\tau}$ 

### Value

A vec containing the wavelet variance of the drift.

#### **Process Haar Wavelet Variance Formula**

The Drift (DR) process has a Haar Wavelet Variance given by:

$$\nu_j^2\left(\omega\right) = \frac{\tau_j^2 \omega^2}{16}$$

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estimate

Fit a Time Series Model to Data

### **Description**

This function can fit a time series model to data using different methods.

### Usage

```
estimate(model, Xt, method = "mle", demean = TRUE)
```

#### **Arguments**

model A time series model.

Xt A vector of time series data.

method A string indicating the method used for model fitting. Supported methods

include mle, yule-walker, gmwm and rgmwm.

demean A boolean indicating whether the model includes a mean / intercept term or

not.

### Author(s)

Stéphane Guerrier and Yuming Zhang

```
Xt = gen_gts(300, AR(phi = c(0, 0, 0.8), sigma2 = 1))
plot(Xt)
estimate (AR(3), Xt)
Xt = gen_gts(300, MA(theta = 0.5, sigma2 = 1))
plot(Xt)
estimate(MA(1), Xt, method = "gmwm")
Xt = gen_gts(300, ARMA(ar = c(0.8, -0.5), ma = 0.5, sigma2 = 1))
estimate(ARMA(2,1), Xt, method = "rgmwm")
Xt = gen_gts(300, ARIMA(ar = c(0.8, -0.5), i = 1, ma = 0.5, sigma2 = 1))
plot(Xt)
estimate(ARIMA(2,1,1), Xt, method = "mle")
Xt = gen_qts(1000, SARIMA(ar = c(0.5, -0.25), i = 0, ma = 0.5, sar = -0.8,
si = 1, sma = 0.25, s = 24, sigma2 = 1))
plot(Xt)
estimate(SARIMA(ar = 2, i = 0, ma = 1, sar = 1, si = 1, sma = 1, s = 24), Xt,
method = "rgmwm")
```

evaluate 33

| _    |     |                 |     |
|------|-----|-----------------|-----|
| 0772 | ווו | $\rightarrow t$ | - 0 |
|      |     |                 |     |

Evalute a time series or a list of time series models

### **Description**

This function calculates AIC, BIC and HQ or the MAPE for a list of time series models. This function currently only supports models estimated by the MLE.

### Usage

```
evaluate(models, Xt, criterion = "IC", start = 0.8, demean = TRUE,
    print = TRUE)
```

### **Arguments**

| models    | A time series model or a list of time series models.  |
|-----------|---|
| Xt        | A time series (i.e gts object).   |
| criterion | Either "IC" for AIC, BIC and HQ or "MAPE" for MAPE.   |
| start     | A numeric indicating the starting proportion of the data that is used for prediction (assuming criterion = "MAPE"). |
| demean    | A boolean indicating whether the model includes a mean / intercept term or not. $ \\$                               |
| print     | logical. If TRUE (the default) results are printed.   |

### Value

```
AIC, BIC and HQ or MAPE
```

### Author(s)

Stéphane Guerrier

```
\label{eq:set_seed} set.seed(18) \\ n = 300 \\ Xt = gen\_gts(n, AR(phi = c(0, 0, 0.8), sigma2 = 1)) \\ evaluate(AR(1), Xt) \\ evaluate(list(AR(1), AR(3), MA(3), ARMA(1,2), SARIMA(ar = 1, i = 0, ma = 1, sar = 1, si = 1, sma = 1, s = 12)), Xt) \\ evaluate(list(AR(1), AR(3)), Xt, criterion = "MAPE") \\ \\
```

34 gen\_ar1blocks

| gen  | ar1blocks |
|------|-----------|
| 9611 | alibiocks |

Generate AR(1) Block Process

### **Description**

This function allows us to generate a non-stationary AR(1) block process.

### Usage

```
gen_ar1blocks(phi, sigma2, n_total, n_block, scale = 10,
title = NULL, seed = 135, ...)
```

#### **Arguments**

| phi     | A double value for the autocorrection parameter $\phi$ .                                 |
|---------|--|
| sigma2  | A double value for the variance parameter $\sigma^2$ .                                   |
| n_total | An integer indicating the length of the simulated $AR(1)$ block process.                 |
| n_block | An integer indicating the length of each block of the $AR(1)$ block process.             |
| scale   | An integer indicating the number of levels of decomposition. The default value is $10. $ |
| title   | A string indicating the name of the time series data.                                    |
| seed    | An integer defined for simulation replication purposes.                                  |
|         | Additional parameters.   |

### Value

A vector containing the AR(1) block process.

### Note

This function generates a non-stationary AR(1) block process whose theoretical maximum overlapping allan variance (MOAV) is different from the theoretical MOAV of a stationary AR(1) process. This difference in the value of the allan variance between stationary and non-stationary processes has been shown through the calculation of the theoretical allan variance given in "A Study of the Allan Variance for Constant-Mean Non-Stationary Processes" by Xu et al. (IEEE Signal Processing Letters, 2017), preprint available: https://arxiv.org/abs/1702.07795.

#### Author(s)

Yuming Zhang and Haotian Xu

gen\_bi

#### **Examples**

```
Xt = gen_ar1blocks(phi = 0.9, sigma2 = 1,
n_total = 1000, n_block = 10, scale = 100)
plot(Xt)

Yt = gen_ar1blocks(phi = 0.5, sigma2 = 5, n_total = 800,
n_block = 20, scale = 50)
plot(Yt)
```

gen bi

Generate Bias-Instability Process

#### **Description**

This function allows to generate a non-stationary bias-instability process.

#### Usage

```
gen_bi(sigma2, n_total, n_block, title = NULL, seed = 135, ...)
```

# Arguments

| sigma2  | A double value for the variance parameter $\sigma^2$ .                          |
|---------|---|
| n_total | An integer indicating the length of the simulated bias-instability process.     |
| n_block | An integer indicating the length of each block of the bias-instability process. |
| title   | A string defining the name of the time series data.                             |
| seed    | An integer defined for simulation replication purposes.                         |
|         | Additional parameters.  |

### Value

A vector containing the bias-instability process.

#### Note

This function generates a non-stationary bias-instability process whose theoretical maximum overlapping allan variance (MOAV) is close to the theoretical MOAV of the best approximation of this process through a stationary AR(1) process over some scales. However, this approximation is not good enough when considering the logarithmic representation of the allan variance. Therefore, the exact form of the allan variance of this non-stationary process allows us to better interpret the signals characterized by bias-instability, as shown in "A Study of the Allan Variance for Constant-Mean Non-Stationary Processes" by Xu et al. (IEEE Signal Processing Letters, 2017), preprint available: https://arxiv.org/abs/1702.07795.

#### Author(s)

Yuming Zhang

36 gen\_gts

#### **Examples**

```
Xt = gen_bi(sigma2 = 1, n_total = 1000, n_block = 10)
plot(Xt)

Yt = gen_bi(sigma2 = 0.8, n_total = 800, n_block = 20,
title = "non-stationary bias-instability process")
plot(Yt)
```

gen\_gts

Simulate a simts TS object using a theoretical model

### Description

Create a gts object based on a time series model.

### Usage

```
gen_gts(n, model, start = 0, end = NULL, freq = 1, unit_ts = NULL,
  unit_time = NULL, name_ts = NULL, name_time = NULL)
```

#### **Arguments**

| n         | An integer containing the length of the time series.   |
|-----------|--|
| model     | A $\ensuremath{\text{ts.model}}$ or $\ensuremath{\text{simts}}$ object containing the available models in the simts package. |
| start     | A numeric that provides the time of the first observation.   |
| end       | A numeric that provides the time of the last observation.  |
| freq      | A numeric that provides the rate of samples. Default value is 1.   |
| unit_ts   | A string that contains the unit expression of the time series. Default value is ${\tt NULL}.$                                |
| unit_time | A string that contains the unit expression of the time. Default value is ${\tt NULL.}$                                       |
| name_ts   | A string that provides an identifier for the time series data. Default value is ${\tt NULL}.$                                |
| name_time | A string that provides an identifier for the time. Default value is ${\tt NULL.}$  |
|           |  |

#### **Details**

This function accepts either a ts.model object (e.g. AR1(phi = .3, sigma2 = 1) + WN(sigma2 = 1)) or a simts object.

#### Value

A gts object

gen\_lts 37

#### Author(s)

James Balamuta and Wenchao Yang

### **Examples**

```
# Set seed for reproducibility
set.seed(1336)
n = 1000
# AR1 + WN
model = AR1(phi = .5, sigma2 = .1) + WN(sigma2=1)
x = gen_gts(n, model)
plot(x)
# Reset seed
set.seed(1336)
# GM + WN
# Convert from AR1 to GM values
m = ar1_{to}gm(c(.5,.1),10)
\# Beta = 6.9314718, Sigma2_gm = 0.1333333
model = GM(beta = m[1], sigma2\_gm = m[2]) + WN(sigma2=1)
x2 = gen_gts(n, model, freq = 10, unit_time = 'sec')
plot(x2)
# Same time series
all.equal(x, x2, check.attributes = FALSE)
```

gen\_lts

Generate a Latent Time Series Object Based on a Model

## Description

Simulate a lts object based on a supplied time series model.

#### Usage

```
gen_lts(n, model, start = 0, end = NULL, freq = 1, unit_ts = NULL,
  unit_time = NULL, name_ts = NULL, name_time = NULL,
  process = NULL)
```

## Arguments

| n     | An interger indicating the amount of observations generated in this function |  |
|-------|--|--|
| model | A ts.model or simts object containing one of the allowed models.             |  |
| start | A numeric that provides the time of the first observation.                   |  |
| end   | A numeric that provides the time of the last observation.                    |  |

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| freq      | A numeric that provides the rate/frequency at which the time series is sampled. The default value is 1.                    |
|-----------|--|
| unit_ts   | A string that contains the unit of measure of the time series. The default value is ${\tt NULL}.$                          |
| unit_time | $\boldsymbol{A}$ string that contains the unit of measure of the time. The default value is $\mathtt{NULL}.$               |
| name_ts   | $\boldsymbol{A}$ string that provides an identifier for the time series data. Default value is $\mathtt{NULL}.$            |
| name_time | A string that provides an identifier for the time. Default value is ${\tt NULL}.$  |
| process   | A vector that contains model names of each column in the data object where the last name is the sum of the previous names. |

#### **Details**

This function accepts either a ts.model object (e.g. AR1(phi = .3, sigma2 = 1) + WN(sigma2 = 1)) or a simts object.

#### Value

A lts object with the following attributes:

**start** The time of the first observation.

end The time of the last observation.

freq Numeric representation of the sampling frequency/rate.

unit A string reporting the unit of measurement.

name Name of the generated dataset.

process A vector that contains model names of decomposed and combined processes

## Author(s)

James Balamuta, Wenchao Yang, and Justin Lee

### **Examples**

```
# AR
set.seed(1336)
model = AR1(phi = .99, sigma2 = 1) + WN(sigma2 = 1)
test = gen_lts(1000, model)
plot(test)
```

gen\_nswn 39

| aen | nswn |
|-----|------|

Generate Non-Stationary White Noise Process

#### **Description**

This function allows to generate a non-stationary white noise process.

## Usage

```
gen_nswn(n_total, title = NULL, seed = 135, ...)
```

## Arguments

| n_total | An integer indicating the length of the simulated non-stationary white noise process. |
|---------|---|
| title   | A string defining the name of the time series data.                                   |
| seed    | An integer defined for simulation replication purposes.                               |
|         | Additional parameters.  |

#### Value

A vector containing the non-stationary white noise process.

#### Note

This function generates a non-stationary white noise process whose theoretical maximum overlapping allan variance (MOAV) corresponds to the theoretical MOAV of the stationary white noise process. This example confirms that the allan variance is unable to distinguish between a stationary white noise process and a white noise process whose second-order behavior is non-stationary, as pointed out in the paper "A Study of the Allan Variance for Constant-Mean Non-Stationary Processes" by Xu et al. (IEEE Signal Processing Letters, 2017), preprint available: https://arxiv.org/abs/1702.07795.

#### Author(s)

Yuming Zhang

### **Examples**

```
Xt = gen_nswn(n_total = 1000)
plot(Xt)

Yt = gen_nswn(n_total = 2000, title = "non-stationary
white noise process", seed = 1960)
plot(Yt)
```

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Create a Gauss-Markov (GM) Process

# Description

Sets up the necessary backend for the GM process.

### Usage

```
GM(beta = NULL, sigma2_gm = 1)
```

### **Arguments**

beta A double value for the  $\beta$  of an GM process (see Note for details).

sigma2\_gm A double value for the variance,  $\sigma_{gm}^2$ , of a GM process (see Note for details).

#### **Details**

When supplying values for  $\beta$  and  $\sigma_{gm}^2$ , these parameters should be of a GM process and NOT of an AR1. That is, do not supply AR1 parameters such as  $\phi$ ,  $\sigma^2$ .

Internally, GM parameters are converted to AR1 using the 'freq' supplied when creating data objects (gts) or specifying a 'freq' parameter in simts or simts.imu.

The 'freq' of a data object takes precedence over the 'freq' set when modeling.

#### Value

An S3 object with called ts.model with the following structure:

process.desc Used in summary: "BETA", "SIGMA2"

theta  $\beta$ ,  $\sigma_{qm}^2$ 

plength Number of parameters

**print** String containing simplified model

desc "GM"

**obj.desc** Depth of parameters e.g. list(1,1)

starting Guess starting values? TRUE or FALSE (e.g. specified value)

#### Note

We consider the following model:

$$X_t = e^{(-\beta)} X_{t-1} + \varepsilon_t$$

, where  $\varepsilon_t$  is iid from a zero mean normal distribution with variance  $\sigma^2(1-e^{2\beta})$ .

#### Author(s)

James Balamuta

GM

gmwm 41

## **Examples**

```
GM()
GM(beta=.32, sigma2_gm=1.3)
```

gmwm

Generalized Method of Wavelet Moments (GMWM) for IMUs, ARMA, SSM, and Robust

## Description

Performs estimation of time series models by using the GMWM estimator.

## Usage

```
gmwm(model, data, model.type = "ssm", compute.v = "auto",
  robust = FALSE, eff = 0.6, alpha = 0.05, seed = 1337, G = NULL,
  K = 1, H = 100, freq = 1)
```

## Arguments

| model      | A ts.model object containing one of the allowed models.   |
|------------|---|
| data       | A matrix or data.frame object with only column (e.g. $N\times 1), {\rm a}$ lts object, or a gts object.   |
| model.type | A string containing the type of GMWM needed: "imu" or "ssm".  |
| compute.v  | A string indicating the type of covariance matrix solver. Valid values are: "fast", "bootstrap", "diag" (asymptotic diag), "full" (asymptotic full). By default, the program will fit a "fast" model. |
| robust     | A boolean indicating whether to use the robust computation (TRUE) or not (FALSE). $ \label{eq:total_policy} $   |
| eff        | A double between 0 and 1 that indicates the efficiency.   |
| alpha      | A double between 0 and 1 that correspondings to the $\frac{\alpha}{2}$ value for the wavelet confidence intervals.  |
| seed       | An integer that controls the reproducibility of the auto model selection phase.   |
| G          | An integer to sample the space for $IMU$ and $SSM$ models to ensure optimal identitability.   |
| K          | An integer that controls how many times the bootstrapping procedure will be initiated.  |
| Н          | An integer that indicates how many different samples the bootstrap will be collect.   |
| freq       | A double that indicates the sampling frequency. By default, this is set to 1 and only is important if $\texttt{GM}()$ is in the model   |

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#### **Details**

This function is under work. Some of the features are active. Others... Not so much.

The V matrix is calculated by:  $diag \left[ (Hi - Lo)^2 \right]$ .

The function is implemented in the following manner: 1. Calculate MODWT of data with levels = floor(log2(data)) 2. Apply the brick.wall of the MODWT (e.g. remove boundary values) 3. Compute the empirical wavelet variance (WV Empirical). 4. Obtain the V matrix by squaring the difference of the WV Empirical's Chi-squared confidence interval (hi - lo)^2 5. Optimize the values to obtain  $\hat{\theta}$  6. If FAST = TRUE, return these results. Else, continue.

Loop k = 1 to K Loop h = 1 to H 7. Simulate xt under  $F_{\hat{\theta}}$  8. Compute WV Empirical END 9. Calculate the covariance matrix 10. Optimize the values to obtain  $\hat{\theta}$  END 11. Return optimized values.

The function estimates a variety of time series models. If type = "imu" or "ssm", then parameter vector should indicate the characters of the models that compose the latent or state-space model. The model options are:

```
"AR1" a first order autoregressive process with parameters (\phi, \sigma^2)
```

"GM" a guass-markov process  $(\beta, \sigma_{am}^2)$ 

"ARMA" an autoregressive moving average process with parameters  $(\phi_p, \theta_q, \sigma^2)$ 

"DR" a drift with parameter  $\omega$ 

"QN" a quantization noise process with parameter Q

"RW" a random walk process with parameter  $\sigma^2$ 

"WN" a white noise process with parameter  $\sigma^2$ 

If only an ARMA() term is supplied, then the function takes conditional least squares as starting values If robust = TRUE the function takes the robust estimate of the wavelet variance to be used in the GMWM estimation procedure.

#### Value

A gmwm object with the structure:

estimate Estimated Parameters Values from the GMWM Procedure

init.guess Initial Starting Values given to the Optimization Algorithm

wv.empir The data's empirical wavelet variance

ci.low Lower Confidence Interval

ci.high Upper Confidence Interval

orgV Original V matrix

V Updated V matrix (if bootstrapped)

omega The V matrix inversed

obj.fun Value of the objective function at Estimated Parameter Values

theo Summed Theoretical Wavelet Variance

decomp.theo Decomposed Theoretical Wavelet Variance by Process

gmwm\_imu 43

scales Scales of the GMWM Object

robust Indicates if parameter estimation was done under robust or classical

eff Level of efficiency of robust estimation

model.type Models being guessed

compute.v Type of V matrix computation

augmented Indicates moments have been augmented

alpha Alpha level used to generate confidence intervals

expect.diff Mean of the First Difference of the Signal

N Length of the Signal

G Number of Guesses Performed

H Number of Bootstrap replications

**K** Number of V matrix bootstraps

model ts.model supplied to gmwm

model.hat A new value of ts.model object supplied to gmwm

starting Indicates whether the procedure used the initial guessing approach

seed Randomization seed used to generate the guessing values

**freq** Frequency of data

gmwm\_imu

GMWM for (Robust) Inertial Measurement Units (IMUs)

#### **Description**

Performs the GMWM estimation procedure using a parameter transform and sampling scheme specific to IMUs.

#### Usage

```
gmwm_imu(model, data, compute.v = "fast", robust = F, eff = 0.6, ...)
```

### **Arguments**

| model     | Ats.model object containing one of the allowed models.   |
|-----------|--|
| data      | A matrix or data.frame object with only column (e.g. $N\times 1$ ), or a lts object, or a gts object.            |
| compute.v | A string indicating the type of covariance matrix solver. "fast", "bootstrap", "asymp.diag", "asymp.comp", "fft" |
| robust    | A boolean indicating whether to use the robust computation (TRUE) or not (FALSE).                                |
| eff       | A double between 0 and 1 that indicates the efficiency.  |
|           | Other arguments passed to the main gmwm function   |

44 gmwm\_imu

#### **Details**

This version of the gmwm function has customized settings ideal for modeling with an IMU object. If you seek to model with an Gauss Markov, GM, object. Please note results depend on the freq specified in the data construction step within the imu. If you wish for results to be stable but lose the ability to interpret with respect to freq, then use AR1 terms.

#### Value

A gmwm object with the structure:

estimate Estimated Parameters Values from the GMWM Procedure

init.guess Initial Starting Values given to the Optimization Algorithm

wv.empir The data's empirical wavelet variance

ci.low Lower Confidence Interval

ci.high Upper Confidence Interval

orgV Original V matrix

V Updated V matrix (if bootstrapped)

omega The V matrix inversed

obj.fun Value of the objective function at Estimated Parameter Values

theo Summed Theoretical Wavelet Variance

decomp.theo Decomposed Theoretical Wavelet Variance by Process

scales Scales of the GMWM Object

robust Indicates if parameter estimation was done under robust or classical

eff Level of efficiency of robust estimation

model.type Models being guessed

compute.v Type of V matrix computation

augmented Indicates moments have been augmented

alpha Alpha level used to generate confidence intervals

expect.diff Mean of the First Difference of the Signal

N Length of the Signal

G Number of Guesses Performed

**H** Number of Bootstrap replications

**K** Number of V matrix bootstraps

model ts.model supplied to gmwm

model.hat A new value of ts.model object supplied to gmwm

starting Indicates whether the procedure used the initial guessing approach

seed Randomization seed used to generate the guessing values

freq Frequency of data

gts 45

gts

Create a simts TS object using time series data

### **Description**

Takes a time series and turns it into a time series oriented object that can be used for summary and graphing functions in the simts package.

## Usage

```
gts(data, start = 0, end = NULL, freq = 1, unit_ts = NULL,
   unit_time = NULL, name_ts = NULL, name_time = NULL,
   data_name = NULL, Time = NULL, time_format = NULL)
```

### **Arguments**

| data        | A one-column matrix, data.frame, or a numeric vector.   |
|-------------|---|
| start       | A numeric that provides the time of the first observation.  |
| end         | A numeric that provides the time of the last observation.   |
| freq        | A numeric that provides the rate/frequency at which the time series is sampled. The default value is $1.$   |
| unit_ts     | A string that contains the unit of measure of the time series. The default value is $\mathtt{NULL}.$  |
| unit_time   | $\boldsymbol{A}$ string that contains the unit of measure of the time. The default value is $\mathtt{NULL}.$  |
| name_ts     | $\boldsymbol{A}$ string that provides an identifier for the time series data. Default value is $\mathtt{NULL}.$   |
| name_time   | A string that provides an identifier for the time. Default value is ${\tt NULL}.$   |
| data_name   | A string that contains the name of the time series data.  |
| Time        | A numeric or character vector containing the times of observations. Default value is NULL. See $\times$ object in as . Date function.   |
| time_format | A string specifiying the format of 'Time'. If not provided, 'Time' is assumed to be all integers. Default value is <code>NULL</code> . See format argument in as <code>.Date</code> function. |

### Value

A gts object

### Author(s)

James Balamuta and Wenchao Yang

imu\_time

#### **Examples**

```
m = data.frame(rnorm(50))
x = gts(m, unit_time = 'sec', name_ts = 'example')
plot(x)

x = gen_gts(50, WN(sigma2 = 1))
x = gts(x, freq = 100, unit_time = 'sec')
plot(x)
```

hydro

Mean Monthly Precipitation, from 1907 to 1972

### **Description**

Hydrology data that indicates a robust approach may be preferred to a classical approach when estimating time series.

#### Usage

hydro

#### **Format**

A time series object with frequency 12 starting at 1907 and going to 1972 for a total of 781 observations.

### Source

https://datamarket.com/data/set/22w1/mean-monthly-precipitation-1907-1972

imu\_time

Pulls the IMU time from the IMU object

## Description

Helper function for the IMU object to access rownames () with a numeric conversion.

### Usage

```
imu_time(x)
```

#### **Arguments**

Х

A imu object

### Value

A vector with numeric information.

is.gts 47

is.gts

Is simts Object

## Description

Is the object a gts, imu, or lts object?

## Usage

```
is.gts(x)
is.imu(x)
is.lts(x)
is.ts.model(x)
```

#### **Arguments**

Х

A gts, imu, lts object.

### **Details**

Uses inherits over is for speed.

#### Value

A logical value that indicates whether the object is of that class (TRUE) or not (FALSE).

### Author(s)

James Balamuta

lts

Generate a Latent Time Series Object from Data

## Description

Create a lts object based on a supplied matrix or data frame. The latent time series is obtained by the sum of underlying time series.

### Usage

```
lts(data, start = 0, end = NULL, freq = 1, unit_ts = NULL,
  unit_time = NULL, name_ts = NULL, name_time = NULL,
  process = NULL)
```

48 *MA* 

#### **Arguments**

| data      | A multiple-column matrix or data. frame. It must contain at least 3 columns of which the last represents the latent time series obtained through the sum of the previous columns. |
|-----------|---|
| start     | A numeric that provides the time of the first observation.  |
| end       | A numeric that provides the time of the last observation.   |
| freq      | A numeric that provides the rate/frequency at which the time series is sampled. The default value is 1.   |
| unit_ts   | A string that contains the unit of measure of the time series. The default value is NULL.   |
| unit_time | A string that contains the unit of measure of the time. The default value is ${\tt NULL}.$  |
| name_ts   | A string that provides an identifier for the time series data. Default value is ${\tt NULL}.$   |
| name_time | A string that provides an identifier for the time. Default value is NULL.   |
| process   | A vector that contains model names of each column in the data object where the last name is the sum of the previous names.  |

#### Value

Alts object

## Author(s)

Wenchao Yang and Justin Lee

## **Examples**

```
model1 = AR1(phi = .99, sigma2 = 1)
model2 = WN(sigma2 = 1)
col1 = gen_gts(1000, model1)
col2 = gen_gts(1000, model2)
testMat = cbind(col1, col2, col1+col2)
testLts = lts(testMat, unit_time = 'sec', process = c('AR1', 'WN', 'AR1+WN'))
plot(testLts)
```

MA

Create an Moving Average Q [MA(Q)] Process

## Description

Sets up the necessary backend for the MA(Q) process.

## Usage

```
MA(theta = NULL, sigma2 = 1)
```

*MA* 49

### **Arguments**

theta A double value for the parameter  $\theta$  (see Note for details).

sigma2 A double value for the variance parameter  $\sigma^2$  (see Note for details).

#### Value

An S3 object with called ts.model with the following structure:

process.desc Used in summary: "MA-1", "MA-2", ..., "MA-Q", "SIGMA2"

theta  $\theta_1, \theta_2, ..., \theta_q, \sigma^2$ 

plength Number of parameters

desc "MA"

print String containing simplified model

**obj.desc** Depth of parameters e.g. list(q,1)

starting Guess starting values? TRUE or FALSE (e.g. specified value)

#### Note

We consider the following model:

$$X_t = \sum_{j=1}^{q} \theta_j \varepsilon_{t-1} + \varepsilon_t$$

, where  $\varepsilon_t$  is iid from a zero mean normal distribution with variance  $\sigma^2$ .

## Author(s)

James Balamuta

#### **Examples**

```
MA(1) # One theta
MA(2) # Two thetas!

MA(theta=.32, sigma=1.3) # 1 theta with a specific value.
MA(theta=c(.3,.5), sigma=.3) # 2 thetas with specific values.
```

50 MA1

MA1

Definition of an Moving Average Process of Order 1

## Description

Definition of an Moving Average Process of Order 1

### Usage

```
MA1(theta = NULL, sigma2 = 1)
```

## Arguments

theta A double value for the parameter  $\theta$  (see Note for details). sigma2 A double value for the variance parameter  $\sigma^2$  (see Note for details).

### Value

An S3 object with called ts.model with the following structure:

```
process.desc Used in summary: "MA1", "SIGMA2" theta \theta, \sigma^2 plength Number of parameters print String containing simplified model desc "MA1" obj.desc Depth of parameters e.g. list(1,1) starting Guess starting values? TRUE or FALSE (e.g. specified value)
```

#### Note

We consider the following model:

$$X_t = \theta \varepsilon_{t-1} + \varepsilon_t$$

, where  $\varepsilon_t$  is iid from a zero mean normal distribution with variance  $\sigma^2$ .

### Author(s)

James Balamuta

### **Examples**

```
MA1()
MA1(theta = .32, sigma2 = 1.3)
```

ma1\_to\_wv 51

ma1\_to\_wv

Moving Average Order 1 (MA(1)) to WV

### **Description**

This function computes the WV (haar) of a Moving Average order 1 (MA1) process.

### Usage

```
mal_to_wv(theta, sigma2, tau)
```

### **Arguments**

theta A double corresponding to the moving average term.

sigma2 A double the variance of the process. tau A vec containing the scales e.g.  $2^{\tau}$ 

#### **Details**

This function is significantly faster than its generalized counter part arma\_to\_wv.

#### Value

A vec containing the wavelet variance of the MA(1) process.

#### **Process Haar Wavelet Variance Formula**

The Moving Average Order 1 (MA(1)) process has a Haar Wavelet Variance given by:

$$\nu_j^2\left(\theta,\sigma^2\right) = \frac{\left(\left(\theta+1\right)^2 \tau_j - 6\theta\right)\sigma^2}{\tau_j^2}$$

make frame

Default utility function for various plots titles

## Description

Adds title, grid, and required x- and y-axes.

#### **Usage**

```
make_frame(x_range, y_range, xlab, ylab, main = "", mar = c(5.1, 5.1,
1, 2.1), add_axis_x = TRUE, add_axis_y = TRUE, col_box = "black",
col_grid = "grey95", col_band = "grey95", col_title = "black",
add_band = TRUE, title_band_width = 0.09, grid_lty = 1)
```

52 make\_frame

### **Arguments**

| x_range   | A numeric providing the range of values for the x-axis.                        |  |
|---|--|--|
| y_range   | A numeric providing the range of values for the y-axis.                        |  |
| xlab  | A string that gives a title for the x-axis.                                    |  |
| ylab  | A string that gives a title for the y-axis.                                    |  |
| main  | A string that gives an overall title for the plot. Default is an empty string. |  |
| mar   | A vector indicating overall margin values for the plot.                        |  |
| add_axis_x A boolean indicating whether a x-axis should be added. |  |  |
| add_axis_y  | A boolean indicating whether a y-axis should be added.                         |  |
| col_box   | A string indicating the color for the title box.                               |  |
| col_grid  | A string indicating the color of the grid for the plot.                        |  |
| col_band  | and A string indicating the color of the band.                                 |  |
| col_title   | col_title A string indicating the color of the plot title.                     |  |
| add_band A boolean indicating whether there should be a band.     |  |  |
| title_band_width  |  |  |
|   | A double providing the value of the band width. Default is 0.09.               |  |
| grid_lty  | grid_lty A integer indicating the line type of the grid lines.                 |  |

### Value

Added title, grid, and axes.

#### Author(s)

Stephane Guerrier and Justin Lee

## **Examples**

MAPE 53

| MAPE | Median Absolute Prediction Error |  |
|------|----------------------------------|--|
|      |                                  |  |

### **Description**

This function calculates Median Absolute Prediction Error (MAPE), which assesses the prediction performance with respect to point forecasts of a given model. It is calculated based on one-step ahead prediction and reforecasting.

## Usage

```
MAPE (model, Xt, start = 0.8, plot = TRUE)
```

## Arguments

| model | A time series model.  |
|-------|---|
| Xt    | A vector of time series data.   |
| start | A numeric indicating the starting proportion of the data that is used for prediction. |
| plot  | A boolean indicating whether a model accuracy plot based on MAPE is returned or not.  |

### Value

The MAPE calculated based on one-step ahead prediction and reforecasting is returned along with its standard deviation.

### Author(s)

Stéphane Guerrier and Yuming Zhang

| np_boot_sd_med Bootstrap standard err | ror for the median |
|---------------------------------------|--------------------|
|---------------------------------------|--------------------|

### **Description**

Non-parametric bootstrap to obtain the standard of the median of iid data.

## Usage

```
np\_boot\_sd\_med(x, B = 5000)
```

### **Arguments**

x A vector of data.

B A numeric indicating the number of simulations.

54 plot.PACF

### Value

Bootstrap standard error for the median

plot.PACF

Plot Partial Auto-Covariance and Correlation Functions

### **Description**

The function plots the output of the theo\_pacf and auto\_corr functions (partial autocovariance or autocorrelation functions).

## Usage

```
## S3 method for class 'PACF'
plot(x, xlab = NULL, ylab = NULL, show.ci = TRUE,
   alpha = NULL, col_ci = NULL, transparency = NULL, main = NULL,
   parValue = NULL, ...)
```

### **Arguments**

| X            | A "PACF" object output from theo_pacf or auto_corr.  |
|--------------|--|
| xlab         | A string indicating the label of the x axis: the default name is 'Lags'.   |
| ylab         | A string indicating the label of the y axis: the default name is 'PACF'.   |
| show.ci      | A bool indicating whether to show the confidence region. Defaults to ${\tt TRUE.}$   |
| alpha        | A double indicating the level of significance for the confidence interval. By default $alpha = 0.05$ which gives a 1 - $alpha = 0.95$ confidence interval. |
| col_ci       | A string that specifies the color of the region covered by the confidence intervals (confidence region). $ \\$   |
| transparency | A double between 0 and 1 indicating the transparency level of the color defined in $\texttt{col\_ci}$ . Defaults to 0.25.                                  |
| main         | A string indicating the title of the plot. Default name is "Variable name PACF plot'.  |
| parValue     | A vector defining the margins for the plot.  |
| • • •        | Additional parameters  |

### Author(s)

Yunxiang Zhang and Yuming Zhang

plot.simtsACF 55

#### **Examples**

```
# Plot the Partial Autocorrelation
m = auto_corr(datasets::AirPassengers, pacf = TRUE)
plot(m)

# More customized CI
plot(m, xlab = "my xlab", ylab = "my ylab", show.ci = TRUE,
alpha = NULL, col_ci = "grey", transparency = 0.5, main = "my main")
```

plot.simtsACF

Plot Auto-Covariance and Correlation Functions

### **Description**

The function plots the output of the theo\_acf and auto\_corr functions (autocovariance or autocorrelation functions).

#### Usage

```
## S3 method for class 'simtsACF'
plot(x, xlab = NULL, ylab = NULL, show.ci = TRUE,
   alpha = NULL, col_ci = NULL, transparency = NULL, main = NULL,
   parValue = NULL, ...)
```

## Arguments

| X            | An "ACF" object output from theo_acf and auto_corr.  |
|--------------|--|
| xlab         | A string indicating the label of the x axis: the default name is 'Lags'.   |
| ylab         | A string indicating the label of the y axis: the default name is 'ACF'.  |
| show.ci      | A bool indicating whether to show the confidence region. Defaults to ${\tt TRUE.}$   |
| alpha        | A double indicating the level of significance for the confidence interval. By default $alpha = 0.05$ which gives a $1$ - $alpha = 0.95$ confidence interval. |
| col_ci       | A string that specifies the color of the region covered by the confidence intervals (confidence region). $ \\$   |
| transparency | A double between 0 and 1 indicating the transparency level of the color defined in $\texttt{col\_ci}$ . Defaults to 0.25.                                    |
| main         | A string indicating the title of the plot. Default name is "Variable name ACF plot'.   |
| parValue     | A vector defining the margins for the plot.  |
|              | Additional parameters  |
|              |  |

## Author(s)

Yunxiang Zhang, Stéphane Guerrier and Yuming Zhang

56 plot\_pred

#### **Examples**

```
# Calculate the Autocorrelation
m = auto_corr(datasets::AirPassengers)

# Plot with 95% CI
plot(m)

# Plot with 90% CI
plot(m, alpha = 0.1)

# Plot without 95% CI
plot(m, show.ci = FALSE)

# More customized CI
plot(m, xlab = "my xlab", ylab = "my ylab", show.ci = TRUE,
alpha = NULL, col_ci = "grey", transparency = 0.5, main = "my main")
```

plot\_pred

Plot Time Series Forecast Function

## Description

This function plots the time series output from a forecast method with approximate 68

## Usage

```
plot_pred(x, model, n.ahead, level = NULL, xlab = NULL, ylab = NULL,
    main = NULL, ...)
```

### **Arguments**

| X       | A gts object   |
|---------|--|
| model   | Ats model  |
| n.ahead | An integer indicating number of units of time ahead for which to make forecasts  |
| level   | A double or vector indicating confidence level of prediction interval. By default, it uses the levels of $0.50$ and $0.95$ . |
| xlab    | A string for the title of x axis   |
| ylab    | A string for the title of y axis   |
| main    | A string for the over all title of the plot  |
|         | Additional parameters  |

## Author(s)

Yuming Zhang

predict.fitsimts 57

#### **Description**

This function plots the time series forecast.

### Usage

```
## S3 method for class 'fitsimts'
predict(object, n.ahead = 10, show_last = 100,
   level = NULL, xlab = NULL, ylab = NULL, main = NULL,
   plot = TRUE, ...)
```

### **Arguments**

| object    | A fitsimts object obtained from estimate function.  |
|-----------|---|
| n.ahead   | An integer indicating number of units of time ahead for which to make forecasts.  |
| show_last | A integer indicating the number of last observations to show in the forecast plot.                                      |
| level     | A double or vector indicating confidence level of prediction interval. By default, it uses the levels of 0.50 and 0.95. |
| xlab      | A string for the title of x axis.   |
| ylab      | A string for the title of y axis.   |
| main      | A string for the over all title of the plot.  |
| plot      | A logical value. logical. If $\mathtt{TRUE}(\text{the default})$ the predictions are plotted.                           |
|           | Additional arguments.   |
|           |   |

#### Author(s)

Stéphane Guerrier and Yuming Zhang

### **Examples**

```
Xt = gen_gts(300, AR(phi = c(0, 0, 0.8), sigma2 = 1))
model = estimate(AR(3), Xt)
predict(model)
predict(model, level = 0.95)

x = gts(as.vector(lynx), start = 1821, end = 1934, freq = 1,
unit_ts = bquote(paste(10^8," ",m^3)), name_ts = "Numbers",
unit_time = "year", data_name = "Annual Numbers of Lynx Trappings")
model = estimate(AR(1), x)
predict(model, n.ahead = 20)
predict(model, n.ahead = 20, level = 0.95)
```

58 QN

```
predict (model, n.ahead = 20, level = c(0.50, 0.80, 0.95))
```

predict.gmwm

Predict future points in the time series using the solution of the Generalized Method of Wavelet Moments

### **Description**

Creates a prediction using the estimated values of GMWM through the ARIMA function within R.

#### Usage

```
## S3 method for class 'gmwm'
predict(object, data.in.gmwm, n.ahead = 1, ...)
```

### **Arguments**

```
object A gmwm object
data.in.gmwm The data SAME EXACT DATA used in the GMWM estimation
n.ahead Number of observations to forecast
... Additional parameters passed to ARIMA Predict
```

#### Value

```
A predict.gmwm object with:
```

```
pred Predictionsse Standard Errorsresid Residuals from ARIMA ML Fit
```

QN

Create an Quantisation Noise (QN) Process

## **Description**

Sets up the necessary backend for the QN process.

## Usage

```
QN(q2 = NULL)
```

#### Arguments

q2

A double value for the  $Q^2$  of a QN process.

*q*n\_to\_wv

#### Value

An S3 object with called ts.model with the following structure:

```
process.desc Used in summary: "QN" theta Q^2 plength Number of parameters print String containing simplified model
```

print string containing simplified mode

desc y desc replicated x times

**obj.desc** Depth of parameters e.g. list(1)

starting Guess starting values? TRUE or FALSE (e.g. specified value)

### Author(s)

James Balamuta

### **Examples**

qn\_to\_wv

Quantisation Noise (QN) to WV

### **Description**

This function compute the Haar WV of a Quantisation Noise (QN) process

### Usage

## Arguments

q2 A double corresponding to variance of drift

tau A vec containing the scales e.g.  $2^{\tau}$ 

### Value

A vec containing the wavelet variance of the QN.

#### **Process Haar Wavelet Variance Formula**

The Quantization Noise (QN) process has a Haar Wavelet Variance given by:

$$\nu_j^2\left(Q^2\right) = \frac{6Q^2}{\tau_j^2}$$

60 rgmwm

| resid_plot | Plot the Distribution of (Standardized) Residuals |  |
|------------|---|--|
|            |   |  |

## Description

This function plots a histogram (with kernel density function and normal distribution) of the standardized residuals or a basic plot the (standardized) residuals, or both.

### Usage

```
resid_plot(res, std = FALSE, type = "hist", ...)
```

### **Arguments**

| res  | A vector of residuals.   |
|------|--|
| std  | A boolean indicating whether the residuals plot is for standardized residuals or original residuals.   |
| type | A string indicating either: "hist" (standardized residual histogram with superimposed kernel density estimator and normal distribution), "resid" (standard residual plot), or "both" |
|      | Additional parameters  |

## Author(s)

Yuming Zhang

| rgmwm | GMWM for Robust/Classical Comparison |  |
|-------|--------------------------------------|--|
| rgmwm | GMWM for Robust/Classical Comparison |  |

## Description

Creates a rgmwm object to compare the results generated by robust/classical method.

## Usage

```
rgmwm(model, data, eff = c(0.9, 0.8, 0.6), ...)
```

## Arguments

| model | Ats.model object containing one of the allowed models.  |
|-------|---|
| data  | A matrix or data.frame object with only one column (e.g. $N\times 1$ ), or a lts object, or a gts object. |
| eff   | A double vector between 0 and 1 that indicates the efficiency.  |
|       | Other arguments passed to the main gmwm function.   |

rtruncated\_normal 61

### **Details**

By default, the rgmwm function will fit a classical gmwm object. From there, the user has the ability to specify any eff that is less than or equal to 0.99.

## Value

A rgmwm object

rtruncated\_normal Truncated Normal Distribution Sampling Algorithm

## **Description**

Enables sampling from a truncated normal

### Usage

```
rtruncated_normal(n, mu, sigma, a, b)
```

### **Arguments**

| n     | An unsigned int indicating the number of observations to generate. \\ |
|-------|---|
| mu    | A double indicating the mean of the normal.                           |
| sigma | A double indicating the standard deviation of the normal.             |
| a     | A double that is the lower bound of the truncated normal.             |

b A double that is the upper bound of the truncated normal.

RW Create an Random Walk (RW) Process

### **Description**

Sets up the necessary backend for the RW process.

## Usage

```
RW(gamma2 = NULL)
```

### **Arguments**

gamma2 A double value for the variance  $\gamma^2$ 

62 RW2dimension

#### Value

An S3 object with called ts.model with the following structure:

process.desc Used in summary: "RW"

theta  $\sigma$ 

plength Number of parameters

print String containing simplified model

desc y desc replicated x times

**obj.desc** Depth of parameters e.g. list(1)

starting Guess starting values? TRUE or FALSE (e.g. specified value)

### Note

We consider the following model:

$$Y_t = \sum_{t=0}^{T} \gamma_0 * Z_t$$

where  $Z_t$  is iid and follows a standard normal distribution.

#### Author(s)

James Balamuta

### **Examples**

RW()
RW(gamma2=3.4)

RW2dimension

Function to Compute Direction Random Walk Moves

#### **Description**

The RW2dimension function computes direction random walk moves.

### Usage

```
RW2dimension(steps = 100, probs = c(0.25, 0.5, 0.75))
```

### **Arguments**

steps An integer that counts the number of steps of the random walk.

probs A vector of double that specifies the probabilities to choose each direction.

#### Author(s)

Stéphane Guerrier

rw\_to\_wv 63

### **Examples**

```
RW2dimension(steps = 50, probs = c(0.2, 0.5, 0.6))
```

rw\_to\_wv

Random Walk to WV

### **Description**

This function compute the WV (haar) of a Random Walk process

#### Usage

```
rw_to_wv(gamma2, tau)
```

#### Arguments

gamma2 A double corresponding to variance of RW

tau A vec containing the scales e.g.  $2^{\tau}$ 

#### Value

A vec containing the wavelet variance of the random walk.

#### **Process Haar Wavelet Variance Formula**

The Random Walk (RW) process has a Haar Wavelet Variance given by:

$$\nu_j^2 \left( \gamma^2 \right) = \frac{\left( \tau_j^2 + 2 \right) \gamma^2}{12\tau_j}$$

sales

Sales Dataset

## Description

This dataset contains the US monthly clothing retail sales in millions of dollars taken from 1992 to 2016 for a total of 302 observations.

### Usage

sales

#### **Format**

A dataframe with 302 rows and 1 variable:

#### **Source**

http://r-exercises.com/wp-content/uploads/2017/04/sales.csv

64 SARIMA

| SARIMA | Create<br>(SARIMA |  | Autoregressive | Integrated | Moving | Average |
|--------|-------------------|--|----------------|------------|--------|---------|
|        |                   |  |                |            |        |         |

## Description

Sets up the necessary backend for the SARIMA process.

### Usage

```
SARIMA(ar = 1, i = 0, ma = 1, sar = 1, si = 0, sma = 1, s = 12, sigma2 = 1)
```

#### **Arguments**

| ar     | A vector or integer containing either the coefficients for $\phi$ 's or the process number $p$ for the Autoregressive (AR) term.             |
|--------|--|
| i      | An integer containing the number of differences to be done.  |
| ma     | A vector or integer containing either the coefficients for $\theta$ 's or the process number $q$ for the Moving Average (MA) term.           |
| sar    | A vector or integer containing either the coefficients for $\Phi$ 's or the process number $P$ for the Seasonal Autoregressive (SAR) term.   |
| si     | An integer containing the number of seasonal differences to be done.   |
| sma    | A vector or integer containing either the coefficients for $\Theta$ 's or the process number $Q$ for the Seasonal Moving Average (SMA) term. |
| S      | An integer containing the seasonality.   |
| sigma2 | A double value for the standard deviation, $\sigma$ , of the SARMA process.  |

### **Details**

A variance is required since the model generation statements utilize randomization functions expecting a variance instead of a standard deviation unlike R.

#### Value

An S3 object with called ts.model with the following structure:

```
process.desc AR*p, MA*q, SAR*P, SMA*Q
theta \sigma
plength Number of parameters
desc Type of model
desc.simple Type of model (after simplification)
print String containing simplified model
obj.desc y desc replicated x times
obj Depth of Parameters e.g. list(c(length(ar), length(ma), length(sar), length(sma), 1, i, si))
starting Guess Starting values? TRUE or FALSE (e.g. specified value)
```

SARMA 65

#### Author(s)

James Balamuta

## **Examples**

```
# Create an SARIMA(1,1,2)\times(1,0,1) process SARIMA(ar = 1, i = 1, ma = 2, sar = 1, si = 0, sma =1) # Creates an SARMA(1,0,1)\times(1,1,1) process with predefined coefficients. SARIMA(ar=0.23, i = 0, ma=0.4, sar = .3, sma = .3)
```

SARMA

Create a Seasonal Autoregressive Moving Average (SARMA) Process

### **Description**

Sets up the necessary backend for the SARMA process.

### Usage

```
SARMA(ar = 1, ma = 1, sar = 1, sma = 1, s = 12, sigma2 = 1)
```

## Arguments

| ar     | A vector or integer containing either the coefficients for $\phi$ 's or the process number $p$ for the Autoregressive (AR) term.             |
|--------|--|
| ma     | A vector or integer containing either the coefficients for $\theta$ 's or the process number $q$ for the Moving Average (MA) term.           |
| sar    | A vector or integer containing either the coefficients for $\Phi$ 's or the process number $P$ for the Seasonal Autoregressive (SAR) term.   |
| sma    | A vector or integer containing either the coefficients for $\Theta$ 's or the process number $Q$ for the Seasonal Moving Average (SMA) term. |
| S      | A integer indicating the seasonal value of the data.   |
| sigma2 | A double value for the standard deviation, $\sigma$ , of the SARMA process.  |

#### **Details**

A variance is required since the model generation statements utilize randomization functions expecting a variance instead of a standard deviation unlike R.

savingrt savingrt

#### Value

```
An S3 object with called ts.model with the following structure:
```

```
process.desc AR * p, MA * q, SAR * P, SMA * Q
theta \sigma
plength Number of Parameters
print String containing simplified model
obj.desc y desc replicated x times
obj Depth of Parameters e.g. list(c(length(ar), length(ma), length(sar), length(sma), 1))
starting Guess Starting values? TRUE or FALSE (e.g. specified value)
```

#### Author(s)

James Balamuta

### **Examples**

```
# Create an SARMA(1,2)\times(1,1) process SARMA(ar = 1, ma = 2,sar = 1, sma =1)
# Creates an SARMA(1,1)\times(1,1) process with predefined coefficients. SARMA(ar=0.23, ma=0.4, sar = .3, sma = .3)
```

savingrt

Personal Saving Rate

#### **Description**

Personal saving as a percentage of disposable personal income (DPI), frequently referred to as "the personal saving rate," is calculated as the ratio of personal saving to DPI.

### Usage

```
savingrt
```

#### **Format**

A gts time series object with frequency 12 starting at 1959 and going to 2016 for a total of 691 observations.

#### Source

```
https://fred.stlouisfed.org/series/PSAVERT
```

select 67

select

Time Series Model Selection

#### **Description**

This function performs model fitting and calculates the model selection criteria to be plotted.

### Usage

```
select(model, Xt, include.mean = TRUE, criterion = "aic",
  plot = TRUE)
```

#### Arguments

model A time series model (only ARIMA are currently supported).

Xt A vector of time series data.

include.mean A boolean indicating whether to fit ARIMA with the mean or not.

criterion A string indicating which model selection criterion should be used (possible values: "aic" (default), "bic", "hq").

plot A boolean indicating whether a model selection plot is returned or not.

### Author(s)

Stéphane Guerrier and Yuming Zhang

## **Examples**

```
set.seed(763)
Xt = gen_gts(100, AR(phi = c(0.2, -0.5, 0.4), sigma2 = 1))
select(AR(5), Xt, include.mean = FALSE)

Xt = gen_gts(100, MA(theta = c(0.2, -0.5, 0.4), sigma2 = 1))
select(MA(5), Xt, include.mean = FALSE)

Xt = gen_gts(500, ARMA(ar = 0.5, ma = c(0.5, -0.5, 0.4), sigma2 = 1))
select(ARMA(5,3), Xt, criterion = "hq", include.mean = FALSE)
```

68 select\_arima

select\_arima

Run Model Selection Criteria on ARIMA Models

## Description

This function performs model fitting and calculates the model selection criteria to be plotted or used in best\_model function.

### Usage

```
select_arima(xt, p.min = 0L, p.max = 3L, d = 0L, q.min = 0L,
   q.max = 3L, include.mean = TRUE, plot = TRUE)

select_arma(xt, p.min = 0L, p.max = 3L, q.min = 0L, q.max = 3L,
   include.mean = TRUE, plot = TRUE)

select_ar(xt, p.min = 0L, p.max = 3L, include.mean = TRUE,
   plot = TRUE)

select_ma(xt, q.min = 0L, q.max = 3L, include.mean = TRUE,
   plot = TRUE)
```

#### **Arguments**

| xt           | A vector of univariate time series.                                   |
|--------------|---|
| p.min        | An integer indicating the lowest order of AR(p) process to search.    |
| p.max        | An integer indicating the highest order of $AR(p)$ process to search. |
| d            | An integer indicating the differencing order for the data.            |
| q.min        | An integer indicating the lowest order of MA(q) process to search.    |
| q.max        | An integer indicating the highest order of MA(q) process to search.   |
| include.mean | A bool indicating whether to fit ARIMA with the mean or not.          |
| plot         | A logical. If TRUE (the default) a plot should be produced.           |

### **Examples**

simple\_diag\_plot 69

```
Basic Diagnostic Plot of Residuals
simple_diag_plot
```

#### **Description**

This function will plot four diagnostic plots to assess how well the model fits the data. These plots are: (1) residuals plot, (2) histogram of (standardized) residuals, (3) normal Q-Q plot of residuals and (4) residuals vs fitted values plot.

## Usage

```
simple_diag_plot(Xt, model, std = FALSE)
```

### **Arguments**

Χt The original time series data. model The arima model fit to the data.

std A boolean indicating whether we use standardized residuals for the (1) resid-

uals plot and the (2) histogram of (standardized) residuals.

#### Author(s)

### Yuming Zhang

```
simplified_print_SARIMA
```

Simplify and print SARIMA model

### **Description**

Simplify and print SARIMA model

#### Usage

```
simplified_print_SARIMA(p, i, q, P, si, Q, s)
```

### **Arguments**

| р  | An integer denoting the length of ar.                                |
|----|--|
| i  | An integer containing the number of differences to be done.          |
| q  | An integer denoting the length of ma.                                |
| P  | An integer denoting the length of sma.                               |
| si | An integer containing the number of seasonal differences to be done. |
| Q  | An integer denoting the length of sar.                               |
| S  | An integer indicating the seasonal value of the data.                |

70 summary.fitsimts

### Value

```
An S3 object with the following structure:
```

```
print String containing simplified modelsimplified Type of model (after simplification)
```

### Author(s)

Stephane Guerrier

```
summary.fitsimts S\iota
```

Summary of fitsimts object

## Description

Displays summary information about fitsimts object

### Usage

```
## S3 method for class 'fitsimts'
summary(object, ...)
```

## Arguments

```
object A fitsimts object
```

. . . Other arguments passed to specific methods

## Value

Estimated parameters values with confidence intervals and standard errors.

## Author(s)

Stéphane Guerrier

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summary.gmwm

Summary of GMWM object

### **Description**

Displays summary information about GMWM object

### Usage

```
## S3 method for class 'gmwm'
summary(object, inference = NULL, bs.gof = NULL,
bs.gof.p.ci = NULL, bs.theta.est = NULL, bs.ci = NULL, B = 100,
...)
```

### **Arguments**

```
object A GMWM object

inference A value containing either: NULL (auto), TRUE, or FALSE
bs.gof A value containing either: NULL (auto), TRUE, FALSE
bs.gof.p.ci A value containing either: NULL (auto), TRUE, FALSE
bs.theta.est A value containing either: NULL (auto), TRUE, FALSE
bs.ci A value containing either: NULL (auto), TRUE, FALSE
bs.ci A value containing either: NULL (auto), TRUE, FALSE

An int that indicates how many bootstraps should be performed.

Other arguments passed to specific methods
```

#### Value

```
A summary.gmwm object with:

estimate Estimated Theta Values

testinfo Goodness of Fit Information

inference Inference performed? T/F

bs.gof Bootstrap GOF? T/F

bs.gof.p.ci Bootstrap GOF P-Value CI? T/F

bs.theta.est Bootstrap Theta Estimates? T/F

bs.ci Bootstrap CI? T/F

starting Indicates if program supplied initial starting values

seed Seed used during guessing / bootstrapping

obj.fun Value of obj.fun at minimized theta

N Length of Time Series
```

### Author(s)

JJB

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| t.heo | acf |
|-------|-----|
|       |     |

Theoretical Autocorrelation (ACF) of an ARMA process

### **Description**

This function computes the theoretical Autocorrelation (ACF) of an ARMA process.

### Usage

```
theo_acf(ar, ma = NULL, lagmax = 20)
```

### **Arguments**

ar A vector containing the AR coefficients.

ma A vector containing the MA coefficients.

lagmax An integer indicating the maximum lag up to which to compute the theoret-

ical ACF.

### Author(s)

Yuming Zhang

#### **Examples**

```
# Compute the theoretical ACF for an ARMA(1,0) (i.e. a first-order autoregressive model: ARC theo_acf(ar = -0.25, ma = NULL) # Computes the theoretical ACF for an ARMA(2, 1) theo_acf(ar = c(.50, -0.25), ma = 0.20, lagmax = 10)
```

theo\_pacf

Theoretical Partial Autocorrelation (PACF) of an ARMA process

### **Description**

This function computes the theoretical Partial Autocorrelation (PACF) of an ARMA process.

## Usage

```
theo_pacf(ar, ma = NULL, lagmax = 20)
```

### **Arguments**

ar A vector containing the AR coefficients.

ma A vector containing the MA coefficients.

lagmax An integer indicating the maximum lag up to which to compute the theoret-

ical PACF.

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#### Author(s)

Yuming Zhang

#### **Examples**

```
\# Computes the theoretical ACF for an ARMA(1,0) (i.e. a first-order autoregressive model: AF theo_pacf(ar = -0.25, ma = NULL, lagmax = 7) \# Computes the theoretical ACF for an ARMA(2, 1) theo_pacf(ar = c(.50, -0.25), ma = .20, lagmax = 10)
```

update.gmwm

Update (Robust) GMWM object for IMU or SSM

### **Description**

Provides a way to estimate different models over the previously estimated wavelet variance values and covariance matrix.

#### Usage

```
## S3 method for class 'gmwm'
update(object, model, ...)
```

### **Arguments**

object A gmwm object.

model A ts.model object containing one of the allowed models

... Additional parameters (not used)

## Value

A gmwm object with the structure:

estimate Estimated Parameters Values from the GMWM Procedure init.guess Initial Starting Values given to the Optimization Algorithm wv.empir The data's empirical wavelet variance ci.low Lower Confidence Interval ci.high Upper Confidence Interval orgV Original V matrix
V Updated V matrix (if bootstrapped)
omega The V matrix inversed
obj.fun Value of the objective function at Estimated Parameter Values theo Summed Theoretical Wavelet Variance

decomp.theo Decomposed Theoretical Wavelet Variance by Process

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scales Scales of the GMWM Object

robust Indicates if parameter estimation was done under robust or classical

eff Level of efficiency of robust estimation

model.type Models being guessed

compute.v Type of V matrix computation

augmented Indicates moments have been augmented

alpha Alpha level used to generate confidence intervals

expect.diff Mean of the First Difference of the Signal

N Length of the Signal

G Number of Guesses Performed

H Number of Bootstrap replications

**K** Number of V matrix bootstraps

model ts.model supplied to gmwm

model.hat A new value of ts.model object supplied to gmwm

starting Indicates whether the procedure used the initial guessing approach

seed Randomization seed used to generate the guessing values

freq Frequency of data

update.lts

Update Object Attribute

#### **Description**

Update the attributes of lts, gts and imu object

#### Usage

```
## S3 method for class 'lts'
update(object, type, new, keep.start = T, ...)
## S3 method for class 'gts'
update(object, type, new, keep.start = T, ...)
## S3 method for class 'imu'
update(object, type, new, ...)
```

#### **Arguments**

object Alts, gts or imu object

type A string that contains the attribute to be updated

new The updated value for the attribute

keep.start A boolean value that indicates whether 'start' or 'end' should remain the same

when 'freq' is updated

. . . Further arguments passed to or from other methods.

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#### **Details**

This function is able to update some attributes for gts, lts and imu objects. For lts object, the attributes that can be updated are 'start', 'end', 'freq', 'unit\_time', 'name\_ts' and 'process'. For gts object, the attributes that can be updated are 'start', 'end', 'freq', 'unit\_time' and 'name\_ts'. For imu object, the attributes that can be updated are 'axis', 'freq', 'unit\_time' and 'name\_ts'.

If one between 'start' and 'end' is updated, the other one will also be updated, since end-start == (N-1) / freq must be TRUE, where N is the number of observations in the object.

If 'freq' is updated, by default 'start' will remain the same, and 'end' will be updated at the same time, unless you set 'keep.start = F'.

If 'unit\_time' is updated, the old unit\_time will be replaced by the new one, and other attributes will remain the same. It is different from the unit time conversion feature.

#### Value

An object with the updated attribute.

### **Examples**

```
gts1 = gts(rnorm(50), freq = 1, unit_time = 'sec', name_ts = 'test1')
gts2 = update(gts1, 'unit_time', 'min')
attr(gts2, 'unit_time')
gts3 = update(gts1, 'name_ts', 'test2')
attr(gts3, 'name_ts')
```

value

Obtain the value of an object's properties

### **Description**

Used to access different properties of the gts, imu, or lts object.

### Usage

```
value(x, type)
## S3 method for class 'imu'
value(x, type)
```

#### **Arguments**

```
x A gts, imu, or lts object.type A string indicating the field to be retrieved.
```

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#### **Details**

To access information about imu properties use:

```
"accel" Returns the number of accelerometers
```

#### Value

The method will return a single numeric or string result depending on the slot being accessed.

#### Methods (by class)

• imu: Access imu object properties

#### Author(s)

James Balamuta

WN

Create an White Noise (WN) Process

### Description

Sets up the necessary backend for the WN process.

### Usage

```
WN(sigma2 = NULL)
```

### **Arguments**

sigma2

A double value for the variance,  $\sigma^2$ , of a WN process.

#### Value

An S3 object with called ts.model with the following structure:

```
process.desc Used in summary: "WN"
```

theta  $\sigma$ 

plength Number of Parameters

print String containing simplified model

desc y desc replicated x times

obj.desc Depth of Parameters e.g. list(1)

starting Guess Starting values? TRUE or FALSE (e.g. specified value)

<sup>&</sup>quot;gyro" Returns the number of gyroscopes

<sup>&</sup>quot;sensors" Returns total number of sensors

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### Note

In this process,  $Y_t$  is iid from a zero mean normal distribution with variance  $\sigma^2$ 

#### Author(s)

James Balamuta

### **Examples**

```
WN()
WN(sigma2=3.4)
```

wn\_to\_wv

Gaussian White Noise to WV

### **Description**

This function compute the Haar WV of a Gaussian White Noise process

## Usage

### **Arguments**

sigma2 A double corresponding to variance of WN

tau A vec containing the scales e.g.  $2^{\tau}$ 

### Value

A vec containing the wavelet variance of the white noise.

#### **Process Haar Wavelet Variance Formula**

The Gaussian White Noise (WN) process has a Haar Wavelet Variance given by:

$$\nu_j^2 \left( \sigma^2 \right) = \frac{\sigma^2}{\tau_j^2}$$