Package 'set6'

July 27, 2020

Title R6 Mathematical Sets Interface

Version 0.1.8

Description An object-oriented package for mathematical sets, upgrading the current gold-standard {sets}. Many forms of mathematical sets are implemented, including (countably finite) sets, tuples, intervals (countably infinite or uncountable), and fuzzy variants. Wrappers extend functionality by allowing symbolic representations of complex operations on sets, including unions, (cartesian) products, exponentiation, and differences (asymmetric and symmetric).

LinkingTo Rcpp

Imports checkmate, Rcpp, R6, utils

Suggests knitr, testthat, devtools, rmarkdown

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URL https://xoopR.github.io/set6/, https://github.com/xoopR/set6

BugReports https://github.com/xoopR/set6/issues

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 $'SetWrapper_ComplementSet.R'\ 'SetWrapper_ExponentSet.R'$

'SetWrapper PowersetSet.R' 'SetWrapper ProductSet.R'

'SetWrapper_UnionSet.R' 'Set_ConditionalSet.R' 'Set_FuzzySet.R'

 $"Set_FuzzySet_FuzzyTuple.R" "Set_Interval.R" "setSymbol.R" "$

'Set_Interval_SpecialSet.R' 'Set_LogicalSet.R' 'Set_Tuple.R'

'Set_UniversalSet.R' 'asFuzzySet.R' 'asInterval.R' 'asSet.R'

'helpers.R' 'assertions.R' 'atomic_coercions.R'

'listSpecialSets.R' 'operation_cleaner.R'

'operation_powerset.R' 'operation_setcomplement.R'

'operation setintersect.R' 'operation setpower.R'

'operation_setproduct.R' 'operation_setsymdiff.R'

'operation_setunion.R' 'operators.R' 'set6-deprecated.R'

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set6-package

set6: R6 Mathematical Sets Interface

Description

set6 upgrades the {sets} package to R6. Many forms of mathematical sets are implemented, including (countably finite) sets, tuples, intervals (countably infinite or uncountable), and fuzzy variants. Wrappers extend functionality by allowing symbolic representations of complex operations on sets, including unions, (cartesian) products, exponentiation, and differences (asymmetric and symmetric).

Details

The main features of set6 are:

- Object-oriented programming, which allows a clear inheritance structure for Sets, Intervals, Tuples, and other variants.
- Set operations and wrappers for both explicit and symbolic representations for algebra of sets.

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• Methods for assertions and comparison checks, including subsets, equality, and containedness.

```
To learn more about set6, start with the set6 vignette:
```

```
vignette("set6","set6")
```

And for more advanced usage see the complete tutorials at

https://github.com/xoopR/set6

Author(s)

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See Also

Useful links:

- https://xoopR.github.io/set6/
- https://github.com/xoopR/set6
- Report bugs at https://github.com/xoopR/set6/issues

as.FuzzySet

Coercion to R6 FuzzySet/FuzzyTuple

Description

Coerces object to an R6 FuzzySet/FuzzyTuple

Usage

```
as.FuzzySet(object)
## S3 method for class 'numeric'
as.FuzzySet(object)
## S3 method for class 'list'
as.FuzzySet(object)
## S3 method for class 'matrix'
as.FuzzySet(object)
## S3 method for class 'data.frame'
as.FuzzySet(object)
## S3 method for class 'Set'
```

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```
as.FuzzySet(object)
## S3 method for class 'FuzzySet'
as.FuzzySet(object)
## S3 method for class 'Interval'
as.FuzzySet(object)
## S3 method for class 'ConditionalSet'
as.FuzzySet(object)
as.FuzzyTuple(object)
## S3 method for class 'numeric'
as.FuzzyTuple(object)
## S3 method for class 'list'
as.FuzzyTuple(object)
## S3 method for class 'matrix'
as.FuzzyTuple(object)
## S3 method for class 'data.frame'
as.FuzzyTuple(object)
## S3 method for class 'Set'
as.FuzzyTuple(object)
## S3 method for class 'FuzzySet'
as.FuzzyTuple(object)
## S3 method for class 'Interval'
as.FuzzyTuple(object)
## S3 method for class 'ConditionalSet'
as.FuzzyTuple(object)
```

Arguments

object object to coerce

Details

- as.FuzzySet.list Assumes list has two items, named elements and membership, and that they are ordered to be corresponding.
- as.FuzzySet.matrix Assumes first column corresponds to elements and second column corresponds to their respective membership.
- as.FuzzySet.data.frame First checks to see if one column is called elements and the other is called membership. If not then uses as.FuzzySet.matrix.

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- as.FuzzySet.Set Creates a FuzzySet by assuming Set elements all have membership equal to 1.
- as.FuzzySet.Interval First tries coercion via as.Set.Interval then uses as.FuzzySet.Set.

See Also

```
FuzzySet FuzzyTuple
Other coercions: as.Interval(), as.Set()
```

as.Interval

Coercion to R6 Interval

Description

Coerces object to an R6 Interval.

Usage

```
as.Interval(object)
## S3 method for class 'Set'
as.Interval(object)
## S3 method for class 'Interval'
as.Interval(object)
## S3 method for class 'list'
as.Interval(object)
## S3 method for class 'data.frame'
as.Interval(object)
## S3 method for class 'matrix'
as.Interval(object)
## S3 method for class 'numeric'
as.Interval(object)
## S3 method for class 'numeric'
as.Interval(object)
## S3 method for class 'ConditionalSet'
as.Interval(object)
```

Arguments

object to coerce

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Details

• as.Interval.list/as.Interval.data.frame - Assumes the list/data.frame has named items/columns: lower, upper, type, class.

- as.Interval.numeric If the numeric vector is a continuous interval with no breaks then coerces to an Interval with: lower = min(object), upper = max(object), class = "integer". Ordering is ignored.
- as.Interval.matrix Tries coercion via as.Interval.numeric on the first column of the matrix.
- as.Interval.Set First tries coercion via as.Interval.numeric, if possible wraps result in a Set
- as.Interval.FuzzySet Tries coercion via as.Interval.Set on the support of the FuzzySet.

See Also

Interval

Other coercions: as.FuzzySet(), as.Set()

as.Set

Coercion to R6 Set/Tuple

Description

Coerces object to an R6 Set/Tuple

Usage

```
as.Set(object)
## Default S3 method:
as.Set(object)
## S3 method for class 'numeric'
as.Set(object)
## S3 method for class 'list'
as.Set(object)
## S3 method for class 'matrix'
as.Set(object)
## S3 method for class 'data.frame'
as.Set(object)
## S3 method for class 'Set'
as.Set(object)
```

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```
## S3 method for class 'FuzzySet'
as.Set(object)
## S3 method for class 'Interval'
as.Set(object)
## S3 method for class 'ConditionalSet'
as.Set(object)
as.Tuple(object)
## Default S3 method:
as.Tuple(object)
## S3 method for class 'numeric'
as.Tuple(object)
## S3 method for class 'list'
as.Tuple(object)
## S3 method for class 'matrix'
as.Tuple(object)
## S3 method for class 'data.frame'
as.Tuple(object)
## S3 method for class 'FuzzySet'
as.Tuple(object)
## S3 method for class 'Set'
as.Tuple(object)
## S3 method for class 'Interval'
as.Tuple(object)
## S3 method for class 'ConditionalSet'
as.Tuple(object)
```

Arguments

object object to coerce

Details

- as.Set.default Creates a Set using the object as the elements.
- as.Set.list Creates a Set for each element in list.
- as.Set.matrix/as.Set.data.frame-Creates a Set for each column in matrix/data.frame.

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- as . Set . FuzzySet Creates a Set from the support of the FuzzySet.
- as.Set.Interval If the interval has finite cardinality then creates a Set from the Interval elements.

See Also

Set Tuple

Other coercions: as.FuzzySet(), as.Interval()

 ${\tt ComplementSet}$

Set of Complements

Description

ComplementSet class for symbolic complement of mathematical sets.

Details

The purpose of this class is to provide a symbolic representation for the complement of sets that cannot be represented in a simpler class. Whilst this is not an abstract class, it is not recommended to construct this class directly but via the set operation methods.

Super classes

```
set6::Set -> set6::SetWrapper -> ComplementSet
```

Active bindings

elements Returns the elements in the object.

length Returns the number of elements in the object.

addedSet For the ComplementSet wrapper, X-Y, returns the set X.

subtractedSet For the ComplementSet wrapper, X-Y, returns the set Y.

Methods

Public methods:

- ComplementSet\$new()
- ComplementSet\$strprint()
- ComplementSet\$contains()
- ComplementSet\$clone()

Method new(): Create a new ComplementSet object. It is not recommended to construct this class directly.

```
Usage:
```

```
ComplementSet$new(addset, subtractset, lower = NULL, upper = NULL, type = NULL)
```

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Arguments: addset Set to be subtracted from. subtractset Set to subtract. lower lower bound of new object. upper upper bound of new object. type closure type of new object. Returns: A new ComplementSet object. **Method** strprint(): Creates a printable representation of the object. Usage: ComplementSet\$strprint(n = 2)Arguments: n numeric. Number of elements to display on either side of ellipsis when printing. Returns: A character string representing the object. **Method** contains(): Tests if elements x are contained in self. Usage: ComplementSet\$contains(x, all = FALSE, bound = FALSE) Arguments: x Set or vector of Sets. all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test. bound logical Returns: If all == TRUE then returns TRUE if all x are contained in self, otherwise FALSE. If all == FALSE returns a vector of logicals corresponding to the length of x, representing if each

is contained in self. If bound == TRUE then an element is contained in self if it is on or within the (possibly-open) bounds of self, otherwise TRUE only if the element is within self or the bounds are closed.

Method clone(): The objects of this class are cloneable with this method.

```
Usage:
ComplementSet$clone(deep = FALSE)
Arguments:
deep Whether to make a deep clone.
```

See Also

Set operations: setunion, setproduct, setpower, setcomplement, setsymdiff, powerset, setintersect Other wrappers: ExponentSet, PowersetSet, ProductSet, UnionSet

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Complex

Set of Complex Numbers

Description

The mathematical set of complex numbers, defined as the the set of reals with possibly imaginary components. i.e.

$$a + bi : a, b \in R$$

where R is the set of reals.

Details

Unlike the other SpecialSets, Complex can be used to define an Interval. In this case where values can be complex, as opposed to reals or integers in Interval.

Super classes

```
set6::Set -> set6::Interval -> set6::SpecialSet -> Complex
```

Methods

Public methods:

- Complex\$new()
- Complex\$contains()
- Complex\$clone()

Method new(): Create a new Complex object.

```
Usage:
```

```
Complexnew(lower = -Inf + (0+0i), upper = Inf + (0+0i))
```

Arguments:

lower complex. Where to start the set.

upper complex. Where to end the set.

Returns: A new Complex object.

Method contains(): Tests to see if x is contained in the Set.

Usage:

```
Complex$contains(x, all = FALSE, bound = NULL)
```

Arguments:

x any. Object or vector of objects to test.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test. bound logical.

Details: x can be of any type, including a Set itself. x should be a tuple if checking to see if it lies within a set of dimension greater than one. To test for multiple x at the same time, then provide these as a list.

If all = TRUE then returns TRUE if all x are contained in the Set, otherwise returns a vector of logicals. For Intervals, bound is used to specify if elements lying on the (possibly open) boundary of the interval are considered contained (bound = TRUE) or not (bound = FALSE).

Returns: If all is TRUE then returns TRUE if all elements of x are contained in the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

The infix operator %inset% is available to test if x is an element in the Set, see examples.

Method clone(): The objects of this class are cloneable with this method.

Usage:

Complex\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

See Also

Other special sets: ExtendedReals, Integers, Naturals, NegIntegers, NegRationals, NegReals, PosIntegers, PosNaturals, PosRationals, PosReals, Rationals, Reals

ConditionalSet

Mathematical Set of Conditions

Description

A mathematical set defined by one or more logical conditions.

Details

Conditional sets are a useful tool for symbolically defining possibly infinite sets. They can be combined using standard 'and', &, and 'or', |, operators.

Super class

```
set6::Set -> ConditionalSet
```

Active bindings

```
condition Returns the condition defining the ConditionalSet.
```

class Returns argclass, see \$new.

elements Returns NA.

Methods

Public methods:

- ConditionalSet\$new()
- ConditionalSet\$contains()
- ConditionalSet\$equals()
- ConditionalSet\$strprint()
- ConditionalSet\$summary()
- ConditionalSet\$isSubset()
- ConditionalSet\$clone()

Method new(): Create a new ConditionalSet object.

Usage:

ConditionalSet\$new(condition, argclass = NULL)

Arguments:

condition function. Defines the set, see details.

argclass list. Optional list of sets that the function arguments live in, see details.

Details: The condition should be given as a function that when evaluated returns either TRUE or FALSE. Further constraints can be given by providing the universe of the function arguments as Sets, if these are not given then the UniversalSet is assumed. See examples.

Returns: A new ConditionalSet object.

Method contains(): Tests to see if x is contained in the Set.

Usage:

ConditionalSet\$contains(x, all = FALSE, bound = NULL)

Arguments:

x any. Object or vector of objects to test.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test. bound ignored, added for consistency.

Details: x can be of any type, including a Set itself. x should be a tuple if checking to see if it lies within a set of dimension greater than one. To test for multiple x at the same time, then provide these as a list.

If all = TRUE then returns TRUE if all x are contained in the Set, otherwise returns a vector of logicals.

An element is contained in a ConditionalSet if it returns TRUE as an argument in the defining function. For sets that are defined with a function that takes multiple arguments, a Tuple should be passed to x.

Returns: If all is TRUE then returns TRUE if all elements of x are contained in the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

The infix operator %inset% is available to test if x is an element in the Set, see examples.

Examples:

```
# Set of positives
 s = ConditionalSet$new(function(x) x > 0)
 s$contains(list(1,-1))
 # Set via equality
 s = ConditionalSet new(function(x, y) x + y == 2)
 s$contains(list(Set$new(2, 0), Set$new(0, 2)))
 # Tuples are recommended when using contains as they allow non-unique elements
 s = ConditionalSet\new(function(x, y) x + y == 4)
 \dontrun{
 s$contains(Set$new(2, 2)) # Errors as Set$new(2,2) == Set$new(2)
 }
 # Set of Positive Naturals
 s = ConditionalSet$new(function(x) TRUE, argclass = list(x = PosNaturals$new()))
 s$contains(list(-2, 2))
Method equals(): Tests if two sets are equal.
 Usage:
 ConditionalSet$equals(x, all = FALSE)
 Arguments:
 x Set or vector of Sets.
 all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.
 Details: Two sets are equal if they contain the same elements. Infix operators can be used for:
                                 Equal
                                 Not equal !=
 Returns: If all is TRUE then returns TRUE if all x are equal to the Set, otherwise FALSE. If all
 is FALSE then returns a vector of logicals corresponding to each individual element of x.
Method strprint(): Creates a printable representation of the object.
 Usage:
 ConditionalSet$strprint(n = NULL)
```

Arguments:

n ignored, added for consistency.

Returns: A character string representing the object.

Method summary(): See strprint.

Usage:
ConditionalSet\$summary(n = NULL)

Arguments:
n ignored, added for consistency.

```
Method isSubset(): Currently undefined for ConditionalSets.
       ConditionalSet$isSubset(x, proper = FALSE, all = FALSE)
      Arguments:
       x ignored, added for consistency.
       proper ignored, added for consistency.
       all ignored, added for consistency.
     Method clone(): The objects of this class are cloneable with this method.
       Usage:
       ConditionalSet$clone(deep = FALSE)
      Arguments:
       deep Whether to make a deep clone.
See Also
   Other sets: FuzzySet, FuzzyTuple, Interval, LogicalSet, Set, Tuple, UniversalSet
Examples
    # Set of Positive Naturals
   s \leftarrow ConditionalSet*new(function(x) TRUE, argclass = list(x = PosNaturals*new()))
    ## Method `ConditionalSet$contains`
   ## -----
    # Set of positives
   s = ConditionalSet$new(function(x) x > 0)
   s$contains(list(1,-1))
   # Set via equality
   s = ConditionalSet new(function(x, y) x + y == 2)
   s$contains(list(Set$new(2, 0), Set$new(0, 2)))
   # Tuples are recommended when using contains as they allow non-unique elements
    s = ConditionalSet new(function(x, y) x + y == 4)
   ## Not run:
   s$contains(Set$new(2, 2)) # Errors as Set$new(2,2) == Set$new(2)
```

s = ConditionalSet\$new(function(x) TRUE, argclass = list(x = PosNaturals\$new()))

End(Not run)

Set of Positive Naturals

s\$contains(list(-2, 2))

equals equals

contains

contains Operator

Description

Operator for \$contains methods. See Set\$contains for full details. Operators can be used for:

Name	Description	Operator
Contains	x contains y	y \$inset\$ x

Usage

```
x %inset% y
```

Arguments

х, у

Set

equals

equals Operator

Description

Operator for \$equals methods. See Set\$equals for full details. Operators can be used for:

Name	Description	Operator
Equal	x equals y	==
Not Equal	x does not equal y	! =

Usage

```
## S3 method for class 'Set'
x == y
## S3 method for class 'Set'
x != y
```

Arguments

х, у

Set

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ExponentSet

Set of Exponentiations

Description

ExponentSet class for symbolic exponentiation of mathematical sets.

Details

The purpose of this class is to provide a symbolic representation for the exponentiation of sets that cannot be represented in a simpler class. Whilst this is not an abstract class, it is not recommended to construct this class directly but via the set operation methods.

Super classes

```
set6::Set -> set6::SetWrapper -> set6::ProductSet -> ExponentSet
```

Active bindings

power Returns the power that the wrapped set is raised to.

Methods

Public methods:

- ExponentSet\$new()
- ExponentSet\$strprint()
- ExponentSet\$contains()
- ExponentSet\$clone()

Method new(): Create a new ExponentSet object. It is not recommended to construct this class directly.

```
Usage:
ExponentSet$new(set, power)

Arguments:
set Set to wrap.
power numeric. Power to raise Set to.

Returns: A new ExponentSet object.
```

Method strprint(): Creates a printable representation of the object.

```
Usage:

ExponentSet$strprint(n = 2)

Arguments:

n numeric. Number of elements to display on either side of ellipsis when printing.

Returns: A character string representing the object.
```

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Method contains(): Tests if elements x are contained in self.

Usage:

ExponentSet\$contains(x, all = FALSE, bound = FALSE)

Arguments:

x Set or vector of Sets.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test. bound logical

Returns: If all == TRUE then returns TRUE if all x are contained in self, otherwise FALSE. If all == FALSE returns a vector of logicals corresponding to the length of x, representing if each is contained in self. If bound == TRUE then an element is contained in self if it is on or within the (possibly-open) bounds of self, otherwise TRUE only if the element is within self or the bounds are closed.

Method clone(): The objects of this class are cloneable with this method.

Usage:

ExponentSet\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

See Also

Set operations: setunion, setproduct, setpower, setcomplement, setsymdiff, powerset, setintersect

Other wrappers: ComplementSet, PowersetSet, ProductSet, UnionSet

ExtendedReals

Set of Extended Real Numbers

Description

The mathematical set of extended real numbers, defined as the union of the set of reals with $\pm \infty$. i.e.

$$R \cup -\infty, \infty$$

where R is the set of reals.

Super classes

```
set6::Set -> set6::Interval -> set6::SpecialSet -> set6::Reals -> ExtendedReals
```

Methods

Public methods:

• ExtendedReals\$new()

• ExtendedReals\$clone()

Method new(): Create a new ExtendedReals object.

Usage:

ExtendedReals\$new()

Returns: A new ExtendedReals object.

Method clone(): The objects of this class are cloneable with this method.

Usage:

ExtendedReals\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

See Also

Other special sets: Complex, Integers, Naturals, NegIntegers, NegRationals, NegReals, PosIntegers, PosNaturals, PosRationals, PosReals, Rationals, Reals

FuzzySet

Mathematical Fuzzy Set

Description

A general FuzzySet object for mathematical fuzzy sets, inheriting from Set.

Details

Fuzzy sets generalise standard mathematical sets to allow for fuzzy relationships. Whereas a standard, or crisp, set assumes that an element is either in a set or not, a fuzzy set allows an element to be in a set to a particular degree, known as the membership function, which quantifies the inclusion of an element by a number in [0, 1]. Thus a (crisp) set is a fuzzy set where all elements have a membership equal to 1. Similarly to Sets, elements must be unique and the ordering does not matter, to establish order and non-unique elements, FuzzyTuples can be used.

Super class

```
set6::Set -> FuzzySet
```

Methods

```
Public methods:
```

```
FuzzySet$new()
FuzzySet$strprint()
FuzzySet$membership()
FuzzySet$alphaCut()
FuzzySet$support()
FuzzySet$core()
FuzzySet$inclusion()
FuzzySet$equals()
FuzzySet$isSubset()
```

• FuzzySet\$clone()

Arguments:

Method new(): Create a new FuzzySet object.

```
Usage:
FuzzySet$new(
    ...,
    elements = NULL,
    membership = rep(1, length(elements)),
    class = NULL
)
```

... Alternating elements and membership, see details.

elements Elements in the set, see details.

membership Corresponding membership of the elements, see details.

class Optional string naming a class that if supplied gives the set the typed property.

Details: FuzzySets can be constructed in one of two ways, either by supplying the elements and their membership in alternate order, or by providing a list of elements to elements and a list of respective memberships to membership, see examples. If the class argument is non-NULL, then all elements will be coerced to the given class in construction, and if elements of a different class are added these will either be rejected or coerced.

Returns: A new FuzzySet object.

Method strprint(): Creates a printable representation of the object.

```
Usage:
FuzzySet$strprint(n = 2)
Arguments:
```

n numeric. Number of elements to display on either side of ellipsis when printing.

Returns: A character string representing the object.

Method membership(): Returns the membership, i.e. value in [0, 1], of either the given element(s) or all elements in the fuzzy set.

Usage:

FuzzySet\$membership(element = NULL)

Arguments:

element element or list of element in the set, if NULL returns membership of all elements

Details: For FuzzySets this is straightforward and returns the membership of the given element(s), however in FuzzyTuples when an element may be duplicated, the function returns the membership of all instances of the element.

Returns: Value, or vector of values, in [0, 1]

Examples:

```
f = FuzzySet$new(1, 0.1, 2, 0.5, 3, 1)
f$membership()
f$membership(2)
```

Method alphaCut(): The alpha-cut of a fuzzy set is defined as the set

$$A_{\alpha} = \{x \in F | m \ge \alpha\}$$

where x is an element in the fuzzy set, F, and m is the corresponding membership.

Usage:

```
FuzzySet$alphaCut(alpha, strong = FALSE, create = FALSE)
```

Arguments:

alpha numeric in [0, 1] to determine which elements to return

strong logical, if FALSE (default) then includes elements greater than or equal to alpha, otherwise only strictly greater than

create logical, if FALSE (default) returns the elements in the alpha cut, otherwise returns a crisp set of the elements

Returns: Elements in FuzzySet or a Set of the elements.

Examples:

```
f = FuzzySet$new(1, 0.1, 2, 0.5, 3, 1)
# Alpha-cut
f$alphaCut(0.5)

# Strong alpha-cut
f$alphaCut(0.5, strong = TRUE)

# Create a set from the alpha-cut
f$alphaCut(0.5, create = TRUE)
```

Method support(): The support of a fuzzy set is defined as the set of elements whose membership is greater than zero, or the strong alpha-cut with $\alpha = 0$,

$$A_{\alpha} = \{x \in F | m > 0\}$$

where x is an element in the fuzzy set, F, and m is the corresponding membership.

Usage:

```
FuzzySet$support(create = FALSE)
```

Arguments:

create logical, if FALSE (default) returns the support elements, otherwise returns a Set of the support elements

Returns: Support elements in fuzzy set or a Set of the support elements.

Examples:

```
f = FuzzySet$new(0.1, 0, 1, 0.1, 2, 0.5, 3, 1)
f$support()
f$support(TRUE)
```

Method core(): The core of a fuzzy set is defined as the set of elements whose membership is equal to one, or the alpha-cut with $\alpha = 1$,

$$A_{\alpha} = \{x \in F : m \geq 1\}$$

where x is an element in the fuzzy set, F, and m is the corresponding membership.

Usage:

```
FuzzySet$core(create = FALSE)
```

Arguments:

create logical, if FALSE (default) returns the core elements, otherwise returns a Set of the core elements

Returns: Core elements in FuzzySet or a Set of the core elements.

Examples:

```
f = FuzzySet$new(0.1, 0, 1, 0.1, 2, 0.5, 3, 1)
f$core()
f$core(TRUE)
```

Method inclusion(): An element in a fuzzy set, with corresponding membership m, is:

- Included If m=1
- Partially Included If 0 < m < 1
- Not Included If m=0

Usage:

FuzzySet\$inclusion(element)

Arguments:

element element or list of elements in fuzzy set for which to get the inclusion level

Details: For FuzzySets this is straightforward and returns the inclusion level of the given element(s), however in FuzzyTuples when an element may be duplicated, the function returns the inclusion level of all instances of the element.

Returns: One of: "Included", "Partially Included", "Not Included"

Examples:

```
f = FuzzySet$new(0.1, 0, 1, 0.1, 2, 0.5, 3, 1)
f$inclusion(0.1)
f$inclusion(1)
f$inclusion(3)
```

Method equals(): Tests if two sets are equal.

Usage:

FuzzySet\$equals(x, all = FALSE)

Arguments:

x Set or vector of Sets.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

Details: Two fuzzy sets are equal if they contain the same elements with the same memberships. Infix operators can be used for:

Equal == Not equal !=

Returns: If all is TRUE then returns TRUE if all x are equal to the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

Method isSubset(): Test if one set is a (proper) subset of another

Usage:

FuzzySet\$isSubset(x, proper = FALSE, all = FALSE)

Arguments:

x any. Object or vector of objects to test.

proper logical. If TRUE tests for proper subsets.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

Details: If using the method directly, and not via one of the operators then the additional boolean argument proper can be used to specify testing of subsets or proper subsets. A Set is a proper subset of another if it is fully contained by the other Set (i.e. not equal to) whereas a Set is a (non-proper) subset if it is fully contained by, or equal to, the other Set. Infix operators can be used for:

Subset <
Proper Subset <=
Superset >
Proper Superset >=

Returns: If all is TRUE then returns TRUE if all x are subsets of the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

Method clone(): The objects of this class are cloneable with this method.

Usage:

FuzzySet\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

See Also

Other sets: ConditionalSet, FuzzyTuple, Interval, LogicalSet, Set, Tuple, UniversalSet

Examples

```
# Different constructors
FuzzySet$new(1, 0.5, 2, 1, 3, 0)
FuzzySetnew(elements = 1:3, membership = c(0.5, 1, 0))
# Crisp sets are a special case FuzzySet
# Note membership defaults to full membership
FuzzySet$new(elements = 1:5) == Set$new(1:5)
f <- FuzzySet$new(1, 0.2, 2, 1, 3, 0)
f$membership()
f$alphaCut(0.3)
f$core()
f$inclusion(0)
f$membership(0)
f$membership(1)
## -----
## Method `FuzzySet$membership`
## -----
f = FuzzySet$new(1, 0.1, 2, 0.5, 3, 1)
f$membership()
f$membership(2)
## -----
## Method `FuzzySet$alphaCut`
f = FuzzySet$new(1, 0.1, 2, 0.5, 3, 1)
# Alpha-cut
f$alphaCut(0.5)
# Strong alpha-cut
f$alphaCut(0.5, strong = TRUE)
# Create a set from the alpha-cut
f$alphaCut(0.5, create = TRUE)
## Method `FuzzySet$support`
## -----
f = FuzzySet$new(0.1, 0, 1, 0.1, 2, 0.5, 3, 1)
f$support()
f$support(TRUE)
## -----
## Method `FuzzySet$core`
## -----
f = FuzzySet$new(0.1, 0, 1, 0.1, 2, 0.5, 3, 1)
```

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```
f$core()
f$core(TRUE)

## ------
## Method `FuzzySet$inclusion`
## -----
f = FuzzySet$new(0.1, 0, 1, 0.1, 2, 0.5, 3, 1)
f$inclusion(0.1)
f$inclusion(1)
f$inclusion(3)
```

FuzzyTuple

Mathematical Fuzzy Tuple

Description

A general FuzzyTuple object for mathematical fuzzy tuples, inheriting from FuzzySet.

Details

Fuzzy tuples generalise standard mathematical tuples to allow for fuzzy relationships. Whereas a standard, or crisp, tuple assumes that an element is either in a tuple or not, a fuzzy tuple allows an element to be in a tuple to a particular degree, known as the membership function, which quantifies the inclusion of an element by a number in [0, 1]. Thus a (crisp) tuple is a fuzzy tuple where all elements have a membership equal to 1. Similarly to Tuples, elements do not need to be unique and the ordering does matter, FuzzySets are special cases where the ordering does not matter and elements must be unique.

Super classes

```
set6::Set -> set6::FuzzySet -> FuzzyTuple
```

Methods

Public methods:

- FuzzyTuple\$equals()
- FuzzyTuple\$isSubset()
- FuzzyTuple\$alphaCut()
- FuzzyTuple\$clone()

Method equals(): Tests if two sets are equal.

```
Usage:
FuzzyTuple$equals(x, all = FALSE)
Arguments:
x Set or vector of Sets.
```

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all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

Details: Two fuzzy sets are equal if they contain the same elements with the same memberships and in the same order. Infix operators can be used for:

Equal == Not equal !=

Returns: If all is TRUE then returns TRUE if all x are equal to the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

Method isSubset(): Test if one set is a (proper) subset of another

Usage:

FuzzyTuple\$isSubset(x, proper = FALSE, all = FALSE)

Arguments:

x any. Object or vector of objects to test.

proper logical. If TRUE tests for proper subsets.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

Details: If using the method directly, and not via one of the operators then the additional boolean argument proper can be used to specify testing of subsets or proper subsets. A Set is a proper subset of another if it is fully contained by the other Set (i.e. not equal to) whereas a Set is a (non-proper) subset if it is fully contained by, or equal to, the other Set.

Infix operators can be used for:

Subset <
Proper Subset <=
Superset >
Proper Superset >=

Returns: If all is TRUE then returns TRUE if all x are subsets of the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

Method alphaCut(): The alpha-cut of a fuzzy set is defined as the set

$$A_{\alpha} = \{x \in F | m \ge \alpha\}$$

where x is an element in the fuzzy set, F, and m is the corresponding membership.

Usage:

FuzzyTuple\$alphaCut(alpha, strong = FALSE, create = FALSE)

Arguments:

alpha numeric in [0, 1] to determine which elements to return

strong logical, if FALSE (default) then includes elements greater than or equal to alpha, otherwise only strictly greater than

create logical, if FALSE (default) returns the elements in the alpha cut, otherwise returns a crisp set of the elements

Returns: Elements in FuzzyTuple or a Set of the elements.

Method clone(): The objects of this class are cloneable with this method.

Usage:

FuzzyTuple\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

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See Also

Other sets: ConditionalSet, FuzzySet, Interval, LogicalSet, Set, Tuple, UniversalSet

Examples

```
# Different constructors
FuzzyTuple$new(1, 0.5, 2, 1, 3, 0)
FuzzyTuplenew(elements = 1:3, membership = c(0.5, 1, 0))
# Crisp sets are a special case FuzzyTuple
# Note membership defaults to full membership
FuzzyTuple$new(elements = 1:5) == Tuple$new(1:5)
f <- FuzzyTuple$new(1, 0.2, 2, 1, 3, 0)
f$membership()
f$alphaCut(0.3)
f$core()
f$inclusion(0)
f$membership(0)
f$membership(1)
# Elements can be duplicated, and with different memberships,
# although this is not necessarily sensible.
FuzzyTuple$new(1, 0.1, 1, 1)
# More important is ordering.
FuzzyTuple$new(1, 0.1, 2, 0.2) != FuzzyTuple$new(2, 0.2, 1, 0.1)
FuzzySet$new(1, 0.1, 2, 0.2) == FuzzySet$new(2, 0.2, 1, 0.1)
```

Integers

Set of Integers

Description

The mathematical set of integers, defined as the set of whole numbers. i.e.

$$\dots, -3, -2, -1, 0, 1, 2, 3, \dots$$

Super classes

```
set6::Set -> set6::Interval -> set6::SpecialSet -> Integers
```

Methods

Public methods:

- Integers\$new()
- Integers\$clone()

Method new(): Create a new Integers object.

```
Usage:
```

```
Integers$new(lower = -Inf, upper = Inf, type = "()")
```

Arguments:

lower numeric. Where to start the set. Advised to ignore, used by child-classes.

upper numeric. Where to end the set. Advised to ignore, used by child-classes.

type character Set closure type. Advised to ignore, used by child-classes.

Returns: A new Integers object.

Method clone(): The objects of this class are cloneable with this method.

Usage:

Integers\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

See Also

Other special sets: Complex, ExtendedReals, Naturals, NegIntegers, NegRationals, NegReals, PosIntegers, PosNaturals, PosRationals, PosReals, Rationals, Reals

Interval

Mathematical Finite or Infinite Interval

Description

A general Interval object for mathematical intervals, inheriting from Set. Intervals may be open, closed, or half-open; as well as bounded above, below, or not at all.

Details

The Interval class can be used for finite or infinite intervals, but often Sets will be preferred for integer intervals over a finite continuous range. Use Complex to define an interval with complex values.

Super class

```
set6::Set -> Interval
```

Active bindings

length If the Interval is countably finite then returns the number of elements in the Interval, otherwise Inf. See the cardinality property for the type of infinity.

elements If the Interval is finite then returns all elements in the Interval, otherwise NA.

Methods

Public methods:

```
• Interval$new()
```

- Interval\$strprint()
- Interval\$equals()
- Interval\$contains()
- Interval\$isSubset()
- Interval\$isSubinterval()
- Interval\$clone()

Method new(): Create a new Interval object.

```
Usage:
Interval$new(
  lower = -Inf,
  upper = Inf,
  type = c("[]", "(]", "[)", "()"),
  class = "numeric",
  universe = ExtendedReals$new()
)
```

Arguments:

lower numeric. Lower limit of the interval.

upper numeric. Upper limit of the interval.

type character. One of: '()', '(]', '[)', '[]', which specifies if interval is open, left-open, right-open, or closed.

class character. One of: 'numeric', 'integer', which specifies if interval is over the Reals or Integers.

universe Set. Universe that the interval lives in, default Reals.

Details: Intervals are constructed by specifying the Interval limits, the boundary type, the class, and the possible universe. The universe differs from class as it is primarily used for the setcomplement method. Whereas class specifies if the interval takes integers or numerics, the universe specifies what range the interval could take.

Returns: A new Interval object.

Method strprint(): Creates a printable representation of the object.

```
Usage:
Interval$strprint(...)
Arguments:
... ignored, added for consistency.
Returns: A character string representing the object.
```

Method equals(): Tests if two sets are equal.

```
Usage:
Interval$equals(x, all = FALSE)
```

Arguments:

x Set or vector of Sets.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

Details: Two Intervals are equal if they have the same: class, type, and bounds. Infix operators can be used for:

```
Equal == Not equal !=
```

Returns: If all is TRUE then returns TRUE if all x are equal to the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

Examples:

```
Interval$new(1,5) == Interval$new(1,5)
Interval$new(1,5, class = "integer") != Interval$new(1,5,class="numeric")
```

Method contains(): Tests to see if x is contained in the Set.

Usage:

```
Interval$contains(x, all = FALSE, bound = FALSE)
```

Arguments:

x any. Object or vector of objects to test.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test. bound logical.

Details: x can be of any type, including a Set itself. x should be a tuple if checking to see if it lies within a set of dimension greater than one. To test for multiple x at the same time, then provide these as a list.

If all = TRUE then returns TRUE if all x are contained in the Set, otherwise returns a vector of logicals. For Intervals, bound is used to specify if elements lying on the (possibly open) boundary of the interval are considered contained (bound = TRUE) or not (bound = FALSE).

Returns: If all is TRUE then returns TRUE if all elements of x are contained in the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

The infix operator %inset% is available to test if x is an element in the Set, see examples.

Examples:

```
s = Set$new(1:5)

# Simplest case
s$contains(4)
8 %inset% s

# Test if multiple elements lie in the set
s$contains(4:6, all = FALSE)
s$contains(4:6, all = TRUE)

# Check if a tuple lies in a Set of higher dimension
```

```
s2 = s * s
s2$contains(Tuple$new(2,1))
c(Tuple$new(2,1), Tuple$new(1,7), 2) %inset% s2
```

Method isSubset(): Test if one set is a (proper) subset of another

Usage:

```
Interval$isSubset(x, proper = FALSE, all = FALSE)
```

Arguments:

x any. Object or vector of objects to test.

proper logical. If TRUE tests for proper subsets.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

Details: If using the method directly, and not via one of the operators then the additional boolean argument proper can be used to specify testing of subsets or proper subsets. A Set is a proper subset of another if it is fully contained by the other Set (i.e. not equal to) whereas a Set is a (non-proper) subset if it is fully contained by, or equal to, the other Set.

When calling isSubset on objects inheriting from Interval, the method treats the interval as if it is a Set, i.e. ordering and class are ignored. Use isSubinterval to test if one interval is a subinterval of another.

Infix operators can be used for:

Subset <
Proper Subset <=
Superset >>
Proper Superset >=

Returns: If all is TRUE then returns TRUE if all x are subsets of the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

Examples:

```
Interval$new(1,3) < Interval$new(1,5)
Set$new(1,3) < Interval$new(0,5)</pre>
```

Method isSubinterval(): Test if one interval is a (proper) subinterval of another

Usage:

```
Interval$isSubinterval(x, proper = FALSE, all = FALSE)
```

Arguments:

```
x Set or list
```

proper If TRUE then tests if x is a proper subinterval (i.e. subinterval and not equal to) of self, otherwise FALSE tests if x is a (non-proper) subinterval.

all If TRUE then returns TRUE if all x are subintervals, otherwise returns a vector of logicals.

Details: If x is a Set then will be coerced to an Interval if possible. \$isSubinterval differs from \$isSubset in that ordering and class are respected in \$isSubinterval. See examples for a clearer illustration of the difference.

Returns: If all is TRUE then returns TRUE if all x are subsets of the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

```
Examples:
    Interval$new(1,3)$isSubset(Set$new(1,2)) # TRUE
    Interval$new(1,3)$isSubset(Set$new(2, 1)) # TRUE
    Interval$new(1,3, class = "integer")$isSubinterval(Set$new(1, 2)) # TRUE
    Interval$new(1,3)$isSubinterval(Set$new(1, 2)) # FALSE
    Interval$new(1,3)$isSubinterval(Set$new(2, 1)) # FALSE

    Reals$new()$isSubset(Integers$new()) # TRUE
    Reals$new()$isSubinterval(Integers$new()) # FALSE

Method clone(): The objects of this class are cloneable with this method.

    Usage:
    Interval$clone(deep = FALSE)

    Arguments:
    deep Whether to make a deep clone.
```

See Also

Other sets: ConditionalSet, FuzzySet, FuzzyTuple, LogicalSet, Set, Tuple, UniversalSet

Examples

```
# Set of Reals
Interval$new()
# Set of Integers
Interval$new(class = "integer")
# Half-open interval
i <- Interval$new(1, 10, "(]")</pre>
i\$contains(c(1, 10))
i$contains(c(1, 10), bound = TRUE)
# Equivalent Set and Interval
Set$new(1:5) == Interval$new(1, 5, class = "integer")
# SpecialSets can provide more efficient implementation
Interval$new() == ExtendedReals$new()
Interval$new(class = "integer", type = "()") == Integers$new()
## Method `Interval$equals`
## -----
Interval new(1,5) == Interval new(1,5)
Interval$new(1,5, class = "integer") != Interval$new(1,5,class="numeric")
## -----
## Method `Interval$contains`
## -----
```

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```
s = Set$new(1:5)
# Simplest case
s$contains(4)
8 %inset% s
# Test if multiple elements lie in the set
s$contains(4:6, all = FALSE)
s$contains(4:6, all = TRUE)
# Check if a tuple lies in a Set of higher dimension
s2 = s * s
s2$contains(Tuple$new(2,1))
c(Tuple$new(2,1), Tuple$new(1,7), 2) %inset% s2
## -----
## Method `Interval$isSubset`
## -----
Interval$new(1,3) < Interval$new(1,5)</pre>
Set$new(1,3) < Interval$new(0,5)
## Method `Interval$isSubinterval`
## -----
Interval$new(1,3)$isSubset(Set$new(1,2)) # TRUE
Interval$new(1,3)$isSubset(Set$new(2, 1)) # TRUE
Interval$new(1,3, class = "integer")$isSubinterval(Set$new(1, 2)) # TRUE
Interval$new(1,3)$isSubinterval(Set$new(1, 2)) # FALSE
Interval$new(1,3)$isSubinterval(Set$new(2, 1)) # FALSE
Reals$new()$isSubset(Integers$new()) # TRUE
Reals$new()$isSubinterval(Integers$new()) # FALSE
```

isSubset

isSubset Operator

Description

Operator for \$isSubset methods. See Set\$isSubset for full details. Operators can be used for:

Name	Description	Operator	
Subset	x is a subset of y	x <= y	
Proper Subset	x is a proper subset of y	x < y	
Superset	x is a superset of y	x >= y	
Proper Superset	x is a proper superset of y	x > y	

listSpecialSets 35

Usage

```
## S3 method for class 'Set'
x < y
## S3 method for class 'Set'
x <= y
## S3 method for class 'Set'
x > y
## S3 method for class 'Set'
x >= y
```

Arguments

x, y Set

 ${\tt listSpecialSets}$

Lists Implemented R6 Special Sets

Description

Lists special sets that can be used in Set.

Usage

```
listSpecialSets(simplify = FALSE)
```

Arguments

simplify

logical. If FALSE (default) returns data.frame of set name and symbol, otherwise set names as characters.

Value

Either a list of characters (if simplify is TRUE) or a data. frame of SpecialSets and their traits.

Examples

```
listSpecialSets()
listSpecialSets(TRUE)
```

36 LogicalSet

LogicalSet

Set of Logicals

Description

The LogicalSet is defined as the Set containing the elements TRUE and FALSE.

Super class

```
set6::Set -> LogicalSet
```

Methods

Public methods:

- LogicalSet\$new()
- LogicalSet\$clone()

Method new(): Create a new LogicalSet object.

Usage:

LogicalSet\$new()

Details: The Logical set is the set containing TRUE and FALSE.

Returns: A new LogicalSet object.

Method clone(): The objects of this class are cloneable with this method.

Usage:

LogicalSet\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

See Also

```
Other sets: ConditionalSet, FuzzySet, FuzzyTuple, Interval, Set, Tuple, UniversalSet
```

Examples

```
1 <- LogicalSet$new()
print(1)
l$contains(list(TRUE, 1, FALSE))</pre>
```

Naturals 37

Naturals

Set of Natural Numbers

Description

The mathematical set of natural numbers, defined as the counting numbers. i.e.

 $0, 1, 2, \dots$

Super classes

```
set6::Set -> set6::Interval -> set6::SpecialSet -> Naturals
```

Methods

Public methods:

- Naturals\$new()
- Naturals\$clone()

Method new(): Create a new Naturals object.

Usage:

Naturals\$new(lower = 0)

Arguments:

lower numeric. Where to start the set. Advised to ignore, used by child-classes.

Returns: A new Naturals object.

Method clone(): The objects of this class are cloneable with this method.

Usage:

Naturals\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

See Also

Other special sets: Complex, ExtendedReals, Integers, NegIntegers, NegRationals, NegReals, PosIntegers, PosNaturals, PosRationals, PosReals, Rationals, Reals

NegIntegers NegIntegers

NegIntegers

Set of Negative Integers

Description

The mathematical set of negative integers, defined as the set of negative whole numbers. i.e.

$$\dots, -3, -2, -1, 0$$

Super classes

```
set6::Set -> set6::Interval -> set6::SpecialSet -> set6::Integers -> NegIntegers
```

Methods

Public methods:

- NegIntegers\$new()
- NegIntegers\$clone()

Method new(): Create a new NegIntegers object.

Usage:

NegIntegers\$new(zero = FALSE)

Arguments:

zero logical. If TRUE, zero is included in the set.

Returns: A new NegIntegers object.

Method clone(): The objects of this class are cloneable with this method.

Usage:

NegIntegers\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

See Also

Other special sets: Complex, ExtendedReals, Integers, Naturals, NegRationals, NegReals, PosIntegers, PosNaturals, PosRationals, PosReals, Rationals, Reals

NegRationals 39

NegRationals

Set of Negative Rational Numbers

Description

The mathematical set of negative rational numbers, defined as the set of numbers that can be written as a fraction of two integers and are non-positive. i.e.

$$\frac{p}{q} : p, q \in Z, p/q \le 0, q \ne 0$$

where Z is the set of integers.

Details

The \$contains method does not work for the set of Rationals as it is notoriously difficult/impossible to find an algorithm for determining if any given number is rational or not. Furthermore, computers must truncate all irrational numbers to rational numbers.

Super classes

```
set6::Set -> set6::Interval -> set6::SpecialSet -> set6::Rationals -> NegRationals
```

Methods

Public methods:

- NegRationals\$new()
- NegRationals\$clone()

Method new(): Create a new NegRationals object.

Usage:

NegRationals\$new(zero = FALSE)

Arguments:

zero logical. If TRUE, zero is included in the set.

Returns: A new NegRationals object.

Method clone(): The objects of this class are cloneable with this method.

Usage:

NegRationals\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

See Also

Other special sets: Complex, ExtendedReals, Integers, Naturals, NegIntegers, NegReals, PosIntegers, PosNaturals, PosRationals, PosReals, Rationals, Reals

NegReals NegReals

NegReals

Set of Negative Real Numbers

Description

The mathematical set of negative real numbers, defined as the union of the set of negative rationals and negative irrationals. i.e.

$$I^- \cup Q^-$$

where I^- is the set of negative irrationals and Q^- is the set of negative rationals.

Super classes

```
set6::Set -> set6::Interval -> set6::SpecialSet -> set6::Reals -> NegReals
```

Methods

Public methods:

- NegReals\$new()
- NegReals\$clone()

Method new(): Create a new NegReals object.

Usage:

NegReals\$new(zero = FALSE)

Arguments:

zero logical. If TRUE, zero is included in the set.

Returns: A new NegReals object.

Method clone(): The objects of this class are cloneable with this method.

Usage:

NegReals\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

See Also

Other special sets: Complex, ExtendedReals, Integers, Naturals, NegIntegers, NegRationals, PosIntegers, PosNaturals, PosRationals, PosReals, Rationals, Reals

PosIntegers 41

PosIntegers

Set of Positive Integers

Description

The mathematical set of positive integers, defined as the set of positive whole numbers. i.e.

 $0, 1, 2, 3, \dots$

Super classes

```
set6::Set -> set6::Interval -> set6::SpecialSet -> set6::Integers -> PosIntegers
```

Methods

Public methods:

- PosIntegers\$new()
- PosIntegers\$clone()

Method new(): Create a new PosIntegers object.

Usage:

PosIntegers\$new(zero = FALSE)

Arguments:

zero logical. If TRUE, zero is included in the set.

Returns: A new PosIntegers object.

Method clone(): The objects of this class are cloneable with this method.

Usage:

PosIntegers\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

See Also

Other special sets: Complex, ExtendedReals, Integers, Naturals, NegIntegers, NegRationals, NegReals, PosNaturals, PosRationals, Reals

42 PosNaturals

PosNaturals

Set of Positive Natural Numbers

Description

The mathematical set of positive natural numbers, defined as the positive counting numbers. i.e.

 $1, 2, 3, \dots$

Super classes

```
set6::Set -> set6::Interval -> set6::SpecialSet -> set6::Naturals -> PosNaturals
```

Methods

Public methods:

- PosNaturals\$new()
- PosNaturals\$clone()

Method new(): Create a new PosNaturals object.

Usage:

PosNaturals\$new()

Returns: A new PosNaturals object.

Method clone(): The objects of this class are cloneable with this method.

Usage:

PosNaturals\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

See Also

Other special sets: Complex, ExtendedReals, Integers, Naturals, NegIntegers, NegRationals, NegReals, PosIntegers, PosRationals, PosReals, Rationals, Reals

PosRationals 43

PosRationals

Set of Positive Rational Numbers

Description

The mathematical set of positive rational numbers, defined as the set of numbers that can be written as a fraction of two integers and are non-negative. i.e.

$$\frac{p}{q} : p, q \in Z, p/q \ge 0, q \ne 0$$

where Z is the set of integers.

Details

The \$contains method does not work for the set of Rationals as it is notoriously difficult/impossible to find an algorithm for determining if any given number is rational or not. Furthermore, computers must truncate all irrational numbers to rational numbers.

Super classes

```
set6::Set -> set6::Interval -> set6::SpecialSet -> set6::Rationals -> PosRationals
```

Methods

Public methods:

- PosRationals\$new()
- PosRationals\$clone()

Method new(): Create a new PosRationals object.

Usage:

PosRationals\$new(zero = FALSE)

Arguments:

zero logical. If TRUE, zero is included in the set.

Returns: A new PosRationals object.

Method clone(): The objects of this class are cloneable with this method.

Usage:

PosRationals\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

See Also

Other special sets: Complex, ExtendedReals, Integers, Naturals, NegIntegers, NegRationals, NegReals, PosIntegers, PosNaturals, PosReals, Rationals, Reals

44 PosReals

PosReals

Set of Positive Real Numbers

Description

The mathematical set of positive real numbers, defined as the union of the set of positive rationals and positive irrationals. i.e.

$$I^+ \cup Q^+$$

where I^+ is the set of positive irrationals and Q^+ is the set of positive rationals.

Super classes

```
set6::Set -> set6::Interval -> set6::SpecialSet -> set6::Reals -> PosReals
```

Methods

Public methods:

- PosReals\$new()
- PosReals\$clone()

Method new(): Create a new PosReals object.

Usage:

PosReals\$new(zero = FALSE)

Arguments:

zero logical. If TRUE, zero is included in the set.

Returns: A new PosReals object.

Method clone(): The objects of this class are cloneable with this method.

Usage:

PosReals\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

See Also

Other special sets: Complex, ExtendedReals, Integers, Naturals, NegIntegers, NegRationals, NegReals, PosIntegers, PosNaturals, PosRationals, Rationals, Reals

powerset 45

powerset

Calculate a Set's Powerset

Description

Calculates and returns the powerset of a Set.

Usage

```
powerset(x, simplify = FALSE)
```

Arguments

x Set

simplify

logical, if TRUE then tries to simplify the result to a Set otherwise creates an object of class PowersetSet.

Details

A powerset of a set, S, is defined as the set of all subsets of S, including S itself and the empty set.

Value

Set

See Also

```
Other operators: setcomplement(), setintersect(), setpower(), setproduct(), setsymdiff(), setunion()
```

Examples

```
# simplify = FALSE is default
powerset(Set$new(1, 2))
powerset(Set$new(1, 2), simplify = TRUE)

# powerset of intervals
powerset(Interval$new())

# powerset of powersets
powerset(powerset(Reals$new()))
powerset(powerset(Reals$new()))$properties$cardinality
```

46 PowersetSet

PowersetSet

Set of Powersets

Description

PowersetSet class for symbolic powerset of mathematical sets.

Details

The purpose of this class is to provide a symbolic representation for the powerset of sets that cannot be represented in a simpler class. Whilst this is not an abstract class, it is not recommended to construct this class directly but via the set operation methods.

Super classes

```
set6::Set -> set6::SetWrapper -> set6::ProductSet -> PowersetSet
```

Methods

Public methods:

- PowersetSet\$new()
- PowersetSet\$strprint()
- PowersetSet\$contains()
- PowersetSet\$isSubset()
- PowersetSet\$clone()

Method new(): Create a new PowersetSet object. It is not recommended to construct this class directly.

```
Usage:
```

PowersetSet\$new(set)

Arguments:

set Set to wrap.

Returns: A new PowersetSet object.

Method strprint(): Creates a printable representation of the object.

Usage:

PowersetSet\$strprint(n = 2)

Arguments:

n numeric. Number of elements to display on either side of ellipsis when printing.

Returns: A character string representing the object.

Method contains(): Tests if elements x are contained in self.

Usage:

PowersetSet\$contains(x, all = FALSE, bound = NULL)

Arguments:

- x Set or vector of Sets.
- x Set or vector of Sets.
- all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.
- all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test. bound logical

Returns: If all == TRUE then returns TRUE if all x are contained in self, otherwise FALSE. If all == FALSE returns a vector of logicals corresponding to the length of x, representing if each is contained in self. If bound == TRUE then an element is contained in self if it is on or within the (possibly-open) bounds of self, otherwise TRUE only if the element is within self or the bounds are closed.

Method isSubset(): Tests if x is a (proper) subset of self.

Usage:

PowersetSet\$isSubset(x, proper = FALSE, all = FALSE)

Arguments:

- x Set or vector of Sets.
- x Set or vector of Sets.

proper logical. If TRUE tests for proper subsets.

- all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.
- all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

Returns: If all == TRUE then returns TRUE if all x are (proper) subsets of self, otherwise FALSE. If all == FALSE returns a vector of logicals corresponding to the length of x, representing if each is a (proper) subset of self.

Method clone(): The objects of this class are cloneable with this method.

Usage:

PowersetSet\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

See Also

Set operations: setunion, setproduct, setpower, setcomplement, setsymdiff, powerset, setintersect

Other wrappers: ComplementSet, ExponentSet, ProductSet, UnionSet

48 ProductSet

ProductSet

Set of Products

Description

ProductSet class for symbolic product of mathematical sets.

Details

The purpose of this class is to provide a symbolic representation for the product of sets that cannot be represented in a simpler class. Whilst this is not an abstract class, it is not recommended to construct this class directly but via the set operation methods.

Super classes

```
set6::Set -> set6::SetWrapper -> ProductSet
```

Active bindings

length Returns the number of elements in the object.

Methods

Public methods:

- ProductSet\$new()
- ProductSet\$strprint()
- ProductSet\$contains()
- ProductSet\$clone()

Method new(): Create a new ProductSet object. It is not recommended to construct this class directly.

```
Usage:
ProductSet$new(
    setlist,
    lower = NULL,
    upper = NULL,
    type = NULL,
    cardinality = NULL
)
Arguments:
setlist list of Sets to wrap.
lower lower bound of new object.
upper upper bound of new object.
type closure type of new object.
```

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cardinality Either an integer, "Aleph0", or a beth number. If NULL then calculated automatically (recommended).

Returns: A new ProductSet object.

Method strprint(): Creates a printable representation of the object.

Usage:

ProductSet\$strprint(n = 2)

Arguments:

n numeric. Number of elements to display on either side of ellipsis when printing.

Returns: A character string representing the object.

Method contains(): Tests if elements x are contained in self.

Usage:

ProductSet\$contains(x, all = FALSE, bound = FALSE)

Arguments:

x Set or vector of Sets.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test. bound logical

Returns: If all == TRUE then returns TRUE if all x are contained in self, otherwise FALSE. If all == FALSE returns a vector of logicals corresponding to the length of x, representing if each is contained in self. If bound == TRUE then an element is contained in self if it is on or within the (possibly-open) bounds of self, otherwise TRUE only if the element is within self or the bounds are closed.

Method clone(): The objects of this class are cloneable with this method.

Usage:

ProductSet\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

See Also

Set operations: setunion, setproduct, setpower, setcomplement, setsymdiff, powerset, setintersect

Other wrappers: ComplementSet, ExponentSet, PowersetSet, UnionSet

50 Properties

Properties

Set Properties Class

Description

Used to store the properties of a Set. Though this is not an abstract class, it should never be constructed outside of the Set constructor.

Active bindings

```
closure Returns the closure of the Set. One of "open", "half-open", or "closed."
```

countability Returns the countability of the Set. One of "countably finite", "countably infinite", or "uncountable".

cardinality Returns the cardinality of the Set. Either an integer if the Set is countably finite, Aleph0 if countably infinite, or a Beth number.

empty Returns if the Set is empty or not. TRUE if the Set cardinality is 0, FALSE otherwise.

singleton Returns if the Set is a singleton or not. TRUE if the Set cardinality is 1, FALSE otherwise.

Methods

Public methods:

- Properties\$new()
- Properties\$print()
- Properties\$strprint()
- Properties\$clone()

Method new(): Creates a new Properties object.

```
Usage:
```

Properties\$new(closure = character(0), cardinality = NULL)

Arguments:

closure One of "open", "half-open", or "closed."

cardinality If non-NULL then either an integer, "Aleph0", or a Beth number.

Returns: A new Properties object.

Method print(): Prints the Properties list.

Usage:

Properties\$print()

Returns: Prints Properties list to console.

Method strprint(): Creates a printable representation of the Properties.

Usage:

Properties\$strprint()

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Returns: A list of properties.

Method clone(): The objects of this class are cloneable with this method.

Usage:

Properties\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

Rationals

Set of Rational Numbers

Description

The mathematical set of rational numbers, defined as the set of numbers that can be written as a fraction of two integers. i.e.

$$\frac{p}{q} \,:\, p,q \,\in\, Z, \; q \neq 0$$

where Z is the set of integers.

Details

The \$contains method does not work for the set of Rationals as it is notoriously difficult/impossible to find an algorithm for determining if any given number is rational or not. Furthermore, computers must truncate all irrational numbers to rational numbers.

Super classes

```
set6::Set -> set6::Interval -> set6::SpecialSet -> Rationals
```

Methods

Public methods:

- Rationals\$new()
- Rationals\$clone()

Method new(): Create a new Rationals object.

Usage:

Rationals\$new(lower = -Inf, upper = Inf, type = "()")

Arguments:

lower numeric. Where to start the set. Advised to ignore, used by child-classes.

upper numeric. Where to end the set. Advised to ignore, used by child-classes.

type character Set closure type. Advised to ignore, used by child-classes.

Returns: A new Rationals object.

Method clone(): The objects of this class are cloneable with this method.

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```
Usage:
```

Rationals\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

See Also

Other special sets: Complex, ExtendedReals, Integers, Naturals, NegIntegers, NegRationals, NegReals, PosIntegers, PosNaturals, PosRationals, PosReals, Reals

Reals

Set of Real Numbers

Description

The mathematical set of real numbers, defined as the union of the set of rationals and irrationals. i.e.

 $I \cup Q$

where I is the set of irrationals and Q is the set of rationals.

Super classes

```
set6::Set -> set6::Interval -> set6::SpecialSet -> Reals
```

Methods

Public methods:

- Reals\$new()
- Reals\$clone()

Method new(): Create a new Reals object.

Usage:

```
Reals$new(lower = -Inf, upper = Inf, type = "()")
```

Arguments:

lower numeric. Where to start the set. Advised to ignore, used by child-classes.

upper numeric. Where to end the set. Advised to ignore, used by child-classes.

type character Set closure type. Advised to ignore, used by child-classes.

Returns: A new Reals object.

Method clone(): The objects of this class are cloneable with this method.

Usage:

```
Reals$clone(deep = FALSE)
```

Arguments:

deep Whether to make a deep clone.

See Also

Other special sets: Complex, ExtendedReals, Integers, Naturals, NegIntegers, NegRationals, NegReals, PosIntegers, PosNaturals, PosRationals, PosReals, Rationals

Set

Mathematical Set

Description

A general Set object for mathematical sets. This also serves as the parent class to intervals, tuples, and fuzzy variants.

Details

Mathematical sets can loosely be thought of as a collection of objects of any kind. The Set class is used for sets of finite elements, for infinite sets use Interval. These can be expanded for fuzzy logic by using FuzzySets. Elements in a set cannot be duplicated and ordering of elements does not matter, Tuples can be used if duplicates or ordering are required.

Active bindings

properties Returns an object of class Properties, which lists the properties of the Set. Set properties include:

- empty is the Set empty or does it contain elements?
- singleton is the Set a singleton? i.e. Does it contain only one element?
- cardinality number of elements in the Set
- countability One of: countably finite, countably infinite, uncountable
- closure One of: closed, open, half-open

traits List the traits of the Set. Set traits include:

• crisp - is the Set crisp or fuzzy?

type Returns the type of the Set. One of: (), (], [), [], {}

max If the Set consists of numerics only then returns the maximum element in the Set. For open or half-open sets, then the maximum is defined by

$$upper - 1e - 15$$

min If the Set consists of numerics only then returns the minimum element in the Set. For open or half-open sets, then the minimum is defined by

$$lower + 1e - 15$$

upper If the Set consists of numerics only then returns the upper bound of the Set.

lower If the Set consists of numerics only then returns the lower bound of the Set.

class If all elements in the Set are the same class then returns that class, otherwise "ANY".

elements If the Set is finite then returns all elements in the Set as a list, otherwise "NA". universe Returns the universe of the Set, i.e. the set of values that can be added to the Set. range If the Set consists of numerics only then returns the range of the Set defined by

$$upper-lower$$

length If the Set is finite then returns the number of elements in the Set, otherwise Inf. See the cardinality property for the type of infinity.

Methods

Public methods:

- Set\$new()
- Set\$print()
- Set\$strprint()
- Set\$summary()
- Set\$contains()
- Set\$equals()
- Set\$isSubset()
- Set\$add()
- Set\$remove()
- Set\$clone()

Method new(): Create a new Set object.

```
Usage:
```

```
Set$new(..., universe = UniversalSet$new(), elements = NULL, class = NULL)
```

Arguments:

... any. Elements in the set.

universe Set. Universe that the Set lives in, i.e. elements that could be added to the Set. Default is the UniversalSet.

elements list. Alternative constructor that may be more efficient if passing objects of multiple classes.

class character. Optional string naming a class that if supplied gives the set the typed property.

Details: Sets are constructed by elements of any types (including R6 classes), excluding lists. Sets should be used within Sets instead of lists. The universe argument is useful for taking the absolute complement of the Set. If a universe isn't given then UniversalSet is assumed. If the class argument is non-NULL, then all elements will be coerced to the given class in construction, and if elements of a different class are added these will either be rejected or coerced.

Returns: A new Set object.

Method print(): Prints a symbolic representation of the Set.

Usage.

```
Set\print(n = 2)
```

Arguments:

n numeric. Number of elements to display on either side of ellipsis when printing.

Details: The function useUnicode() can be used to determine if unicode should be used when printing the Set. Internally print first calls strprint to create a printable representation of the Set.

Method strprint(): Creates a printable representation of the object.

```
Usage:
```

```
Set\$strprint(n = 2)
```

Arguments:

n numeric. Number of elements to display on either side of ellipsis when printing.

Returns: A character string representing the object.

Method summary(): Summarises the Set.

```
Usage:
```

```
Setsummary(n = 2)
```

Arguments:

n numeric. Number of elements to display on either side of ellipsis when printing.

Details: The function useUnicode() can be used to determine if unicode should be used when printing the Set. Summarised details include the Set class, properties, and traits.

Method contains(): Tests to see if x is contained in the Set.

Usage:

```
Set$contains(x, all = FALSE, bound = NULL)
```

Arguments:

x any. Object or vector of objects to test.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test. bound ignored, added for consistency.

Details: x can be of any type, including a Set itself. x should be a tuple if checking to see if it lies within a set of dimension greater than one. To test for multiple x at the same time, then provide these as a list.

If all = TRUE then returns TRUE if all x are contained in the Set, otherwise returns a vector of logicals.

Returns: If all is TRUE then returns TRUE if all elements of x are contained in the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

The infix operator %inset% is available to test if x is an element in the Set, see examples.

Examples:

```
s = Set$new(elements = 1:5)
# Simplest case
s$contains(4)
8 %inset% s
```

```
# Test if multiple elements lie in the set
 s$contains(4:6, all = FALSE)
 s$contains(4:6, all = TRUE)
 # Check if a tuple lies in a Set of higher dimension
 s2 = s * s
 s2$contains(Tuple$new(2,1))
 c(Tuple$new(2,1), Tuple$new(1,7), 2) %inset% s2
Method equals(): Tests if two sets are equal.
 Usage:
 Set\$equals(x, all = FALSE)
 Arguments:
 x Set or vector of Sets.
 all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.
 Details: Two sets are equal if they contain the same elements. Infix operators can be used for:
                                 Equal
                                  Not equal !=
 Returns: If all is TRUE then returns TRUE if all x are equal to the Set, otherwise FALSE. If all
 is FALSE then returns a vector of logicals corresponding to each individual element of x.
 Examples:
 # Equals
 Set$new(1,2)$equals(Set$new(5,6))
 Set$new(1,2)$equals(Interval$new(1,2))
 Set$new(1,2) == Interval$new(1,2, class = "integer")
 # Not equal
 !Set$new(1,2)$equals(Set$new(1,2))
 Set$new(1,2) != Set$new(1,5)
Method isSubset(): Test if one set is a (proper) subset of another
```

Usage:

```
Set$isSubset(x, proper = FALSE, all = FALSE)
```

Arguments:

x any. Object or vector of objects to test.

proper logical. If TRUE tests for proper subsets.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

Details: If using the method directly, and not via one of the operators then the additional boolean argument proper can be used to specify testing of subsets or proper subsets. A Set is a proper subset of another if it is fully contained by the other Set (i.e. not equal to) whereas a Set is a (non-proper) subset if it is fully contained by, or equal to, the other Set. Infix operators can be used for:

```
Subset <
Proper Subset <=
Superset >
Proper Superset >=
```

Returns: If all is TRUE then returns TRUE if all x are subsets of the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

Examples:

```
Set$new(1,2,3)$isSubset(Set$new(1,2), proper = TRUE)
Set$new(1,2) < Set$new(1,2,3) # proper subset

c(Set$new(1,2,3), Set$new(1)) < Set$new(1,2,3) # not proper
Set$new(1,2,3) <= Set$new(1,2,3) # proper</pre>
```

Method add(): Add elements to a set.

```
Usage:
Set$add(...)
Arguments:
```

... elements to add

Details: \$add is a wrapper around the setunion method with setunion(self, Set\$new(...)). Note a key difference is that any elements passed to ... are first converted to a Set, this important difference is illustrated in the examples by adding an Interval to a Set.

Additionally, \$add first coerces . . . to \$class if self is a typed-set (i.e. \$class != "ANY"), and \$add checks if elements in . . . live in the universe of self.

Returns: An object inheriting from Set.

```
Examples:
```

```
Set$new(1,2)$add(3)$print()
Set$new(1,2,universe = Interval$new(1,3))$add(3)$print()
\dontrun{
# errors as 4 is not in [1,3]
Set$new(1,2,universe = Interval$new(1,3))$add(4)$print()
}
# coerced to complex
Set$new(0+1i, 2i, class = "complex")$add(4)$print()
# setunion vs. add
Set$new(1,2)$add(Interval$new(5,6))$print()
Set$new(1,2) + Interval$new(5,6)
```

Method remove(): Remove elements from a set.

```
Usage:
Set$remove(...)
Arguments:
... elements to remove
```

Details: \$remove is a wrapper around the setcomplement method with setcomplement(self, Set\$new(...)). Note a key difference is that any elements passed to ... are first converted to a Set, this important difference is illustrated in the examples by removing an Interval from a Set.

Returns: If the complement cannot be simplified to a Set then a ComplementSet is returned otherwise an object inheriting from Set is returned.

```
Examples:
```

```
Set$new(1,2,3)$remove(1,2)$print()
Set$new(1,Set$new(1),2)$remove(Set$new(1))$print()
Interval$new(1,5)$remove(5)$print()
Interval$new(1,5)$remove(4)$print()

# setcomplement vs. remove
Set$new(1,2,3)$remove(Interval$new(5,7))$print()
Set$new(1,2,3) - Interval$new(5,7)

Method clone(): The objects of this class are cloneable with this method.
Usage:
Set$clone(deep = FALSE)
Arguments:
deep Whether to make a deep clone.
```

See Also

Other sets: ConditionalSet, FuzzySet, FuzzyTuple, Interval, LogicalSet, Tuple, UniversalSet

Examples

```
# Simplest case
s$contains(4)
8 %inset% s
# Test if multiple elements lie in the set
s$contains(4:6, all = FALSE)
s$contains(4:6, all = TRUE)
# Check if a tuple lies in a Set of higher dimension
s2 = s * s
s2$contains(Tuple$new(2,1))
c(Tuple$new(2,1), Tuple$new(1,7), 2) %inset% s2
## Method `Set$equals`
## -----
# Equals
Set$new(1,2)$equals(Set$new(5,6))
Set$new(1,2)$equals(Interval$new(1,2))
Set$new(1,2) == Interval$new(1,2, class = "integer")
# Not equal
!Set$new(1,2)$equals(Set$new(1,2))
Set$new(1,2) != Set$new(1,5)
## Method `Set$isSubset`
## -----
Set$new(1,2,3)$isSubset(Set$new(1,2), proper = TRUE)
Set new(1,2) < Set new(1,2,3) # proper subset
c(Set$new(1,2,3), Set$new(1)) < Set$new(1,2,3) # not proper
Set$new(1,2,3) \le Set$new(1,2,3) # proper
## -----
## Method `Set$add`
## -----
Set$new(1,2)$add(3)$print()
Set$new(1,2,universe = Interval$new(1,3))$add(3)$print()
## Not run:
# errors as 4 is not in [1,3]
Set$new(1,2,universe = Interval$new(1,3))$add(4)$print()
## End(Not run)
# coerced to complex
Set$new(0+1i, 2i, class = "complex")$add(4)$print()
# setunion vs. add
Set$new(1,2)$add(Interval$new(5,6))$print()
```

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set6News

Show set6 NEWS.md File

Description

Displays the contents of the NEWS.md file for viewing set6 release information.

Usage

```
set6News()
```

Value

NEWS.md in viewer.

Examples

```
set6News()
```

setcomplement

Complement of Two Sets

Description

Returns the set difference of two objects inheriting from class Set. If y is missing then the complement of x from its universe is returned.

setcomplement 61

Usage

```
setcomplement(x, y, simplify = TRUE)
## S3 method for class 'Set'
setcomplement(x, y, simplify = TRUE)
## S3 method for class 'Interval'
setcomplement(x, y, simplify = TRUE)
## S3 method for class 'FuzzySet'
setcomplement(x, y, simplify = TRUE)
## S3 method for class 'FuzzyTuple'
setcomplement(x, y, simplify = TRUE)
## S3 method for class 'ConditionalSet'
setcomplement(x, y, simplify = TRUE)
## S3 method for class 'Reals'
setcomplement(x, y, simplify = TRUE)
## S3 method for class 'Rationals'
setcomplement(x, y, simplify = TRUE)
## S3 method for class 'Integers'
setcomplement(x, y, simplify = TRUE)
## S3 method for class 'ComplementSet'
setcomplement(x, y, simplify = TRUE)
## S3 method for class 'Set'
x - y
```

Arguments

x, y Set

simplify logical, if TRUE (default) returns the result in its simplest form, usually a Set or UnionSet, otherwise a ComplementSet.

Details

The difference of two sets, X, Y, is defined as the set of elements that exist in set X and not Y,

$$X - Y = \{z : z\epsilon X \quad and \quad \neg(z\epsilon Y)\}$$

The set difference of two ConditionalSets is defined by combining their defining functions by a negated 'and', !&, operator. See examples.

The complement of fuzzy and crisp sets first coerces fuzzy sets to crisp sets by finding their support.

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Value

An object inheriting from Set containing the set difference of elements in x and y.

See Also

```
Other operators: powerset(), setintersect(), setpower(), setproduct(), setsymdiff(), setunion()
```

Examples

```
# absolute complement
setcomplement(Set$new(1, 2, 3, universe = Reals$new()))
setcomplement(Set$new(1, 2, universe = Set$new(1, 2, 3, 4, 5)))
# complement of two sets
Set$new(-2:4) - Set$new(2:5)
setcomplement(Set$new(1, 4, "a"), Set$new("a", 6))
# complement of two intervals
Interval$new(1, 10) - Interval$new(5, 15)
Interval$new(1, 10) - Interval$new(-15, 15)
Interval$new(1, 10) - Interval$new(-1, 2)
# complement of mixed set types
Set$new(1:10) - Interval$new(5, 15)
Set$new(5, 7) - Tuple$new(6, 8, 7)
# FuzzySet-Set returns a FuzzySet
FuzzySet$new(1, 0.1, 2, 0.5) - Set$new(2:5)
# Set-FuzzySet returns a Set
Set$new(2:5) - FuzzySet$new(1, 0.1, 2, 0.5)
# complement of conditional sets
ConditionalSetnew(function(x, y, simplify = TRUE) x >= y) -
  ConditionalSetnew(function(x, y, simplify = TRUE) x == y)
# complement of special sets
Reals$new() - NegReals$new()
Rationals$new() - PosRationals$new()
Integers$new() - PosIntegers$new()
```

setintersect 63

Description

Returns the intersection of two objects inheriting from class Set.

Usage

```
## S3 method for class 'Interval'
setintersect(x, y)

## S3 method for class 'ConditionalSet'
setintersect(x, y)

## S3 method for class 'UnionSet'
setintersect(x, y)

## S3 method for class 'ComplementSet'
setintersect(x, y)

## S3 method for class 'ProductSet'
setintersect(x, y)

## S3 method for class 'ProductSet'
setintersect(x, y)

## S3 method for class 'Set'
x & y
```

Arguments

x, y Set

Details

The intersection of two sets, X, Y, is defined as the set of elements that exist in both sets,

$$X \cap Y = \{z : z \in X \quad and \quad z \in Y\}$$

In the case where no elements are common to either set, then the empty set is returned.

The intersection of two ConditionalSets is defined by combining their defining functions by an 'and', &, operator. See examples.

The intersection of fuzzy and crisp sets first coerces fuzzy sets to crisp sets by finding their support.

Value

A Set consisting of elements in both x and y.

See Also

```
Other operators: powerset(), setcomplement(), setpower(), setproduct(), setsymdiff(), setunion()
```

64 setpower

Examples

```
# intersection of two sets
Set$new(-2:4) & Set$new(2:5)
setintersect(Set$new(1, 4, "a"), Set$new("a", 6))
Set$new(1:4) & Set$new(5:7)
# intersection of two intervals
Interval$new(1, 10) & Interval$new(5, 15)
Interval$new(1, 2) & Interval$new(2, 3)
Interval$new(1, 5, class = "integer") &
  Interval$new(2, 7, class = "integer")
# intersection of mixed set types
Set$new(1:10) & Interval$new(5, 15)
Set$new(5, 7) & Tuple$new(6, 8, 7)
# Ignores membership of FuzzySet
FuzzySet$new(1, 0.1, 2, 0.5) & Set$new(2:5)
# intersection of conditional sets
ConditionalSetnew(function(x, y) x \ge y) &
  ConditionalSetnew(function(x, y) x == y)
ConditionalSetnew(function(x) x == 2) &
  ConditionalSetnew(function(y) y == 3)
# But be careful not to make an empty set
ConditionalSetnew(function(x) x == 2) &
  ConditionalSetnew(function(x) x == 3)
```

setpower

Power of a Set

Description

A convenience wrapper for the n-ary cartesian product of a Set by itself, possibly multiple times.

Usage

```
setpower(x, power, simplify = FALSE, nest = FALSE)
## S3 method for class 'Set'
x ^ power
```

setpower 65

Arguments

X	Set
power	power to raise set to, if "n" then a variable dimension set is created, see examples.
simplify	logical, if TRUE returns the result in its simplest (unwrapped) form, usually a Set, otherwise an ExponentSet.
nest	logical, if FALSE (default) returns the n-ary cartesian product, otherwise returns the cartesian product applied n times. Sets. See details and examples.

Details

See the details of setproduct for a longer discussion on the use of the nest argument, in particular with regards to n-ary cartesian products vs. 'standard' cartesian products.

Value

An R6 object of class Set or ExponentSet inheriting from ProductSet.

See Also

```
Other operators: powerset(), setcomplement(), setintersect(), setproduct(), setsymdiff(), setunion()
```

Examples

```
# Power of a Set
setpower(Set$new(1, 2), 3, simplify = FALSE)
setpower(Set$new(1, 2), 3, simplify = TRUE)
Set$new(1, 2)^3
# Power of an interval
Interval$new(2, 5)^5
Reals$new()^3
# Use tuples for contains
(PosNaturals$new()^3)$contains(Tuple$new(1, 2, 3))
# Power of ConditionalSet is meaningless
ConditionalSet$new(function(x) TRUE)^2
# Power of FuzzySet
FuzzySet$new(1, 0.1, 2, 0.5)^2
# Variable length
x \leftarrow Interval new(0, 1)^n"
x$contains(Tuple$new(0))
x$contains(Tuple$new(0, 1))
x$contains(Tuple$new(0, 1, 0, 0, 1, 1, 0))
x$contains(list(Tuple$new(0, 2), Tuple$new(1, 1)))
```

66 setproduct

setproduct

Cartesian Product of Sets

Description

Returns the cartesian product of objects inheriting from class Set.

Usage

```
setproduct(..., simplify = FALSE, nest = FALSE)
## S3 method for class 'Set'
x * y
```

Arguments

.. Sets

simplify logical, if TRUE returns the result in its simplest (unwrapped) form, usually a Set

otherwise a ProductSet.

nest logical, if FALSE (default) then will treat any ProductSets passed to ... as un-

wrapped Sets. See details and examples.

x, y Set

Details

The cartesian product of multiple sets, the 'n-ary Cartesian product', is often implemented in programming languages as being identical to the cartesian product of two sets applied recursively. However, for sets X,Y,Z,

$$XYZ \neq (XY)Z$$

This is accommodated with the nest argument. If nest == TRUE then X*Y*Z == (XY)Z, i.e. the cartesian product for two sets is applied recursively. If nest == FALSE then X*Y*Z == (XYZ) and the n-ary cartesian product is computed. As it appears the latter (n-ary product) is more common, nest = FALSE is the default. The N-ary cartesian product of N sets, X1, ..., XN, is defined as

$$X1...XN = (x1, ..., xN) : x1\epsilon X1 \cap ... \cap xN\epsilon XN$$

where (x1, ..., xN) is a tuple.

The product of fuzzy and crisp sets first coerces fuzzy sets to crisp sets by finding their support.

Value

Either an object of class ProductSet or an unwrapped object inheriting from Set.

See Also

```
Other operators: powerset(), setcomplement(), setintersect(), setpower(), setsymdiff(), setunion()
```

setsymdiff 67

Examples

```
# difference between nesting
Set$new(1, 2) * Set$new(2, 3) * Set$new(4, 5)
setproduct(Set$new(1, 2) * Set$new(2, 3), Set$new(4, 5), nest = FALSE) # same as above
setproduct(Set$new(1, 2) * Set$new(2, 3), Set$new(4, 5), nest = TRUE)
unnest_set <- setproduct(Set$new(1, 2) * Set$new(2, 3), Set$new(4, 5), nest = FALSE)
nest\_set \leftarrow setproduct(Set$new(1, 2) * Set$new(2, 3), Set$new(4, 5), nest = TRUE)
# note the difference when using contains
unnest_set$contains(Tuple$new(1, 3, 5))
nest_set$contains(Tuple$new(Tuple$new(1, 3), 5))
# product of two sets
Set$new(-2:4) * Set$new(2:5)
setproduct(Set$new(1, 4, "a"), Set$new("a", 6))
setproduct(Set$new(1, 4, "a"), Set$new("a", 6), simplify = TRUE)
# product of two intervals
Interval$new(1, 10) * Interval$new(5, 15)
Intervalnew(1, 2, type = "()") * Interval<math>new(2, 3, type = "(]")
Interval$new(1, 5, class = "integer") *
  Interval$new(2, 7, class = "integer")
# product of mixed set types
Set$new(1:10) * Interval$new(5, 15)
Set$new(5, 7) * Tuple$new(6, 8, 7)
FuzzySet$new(1, 0.1) * Set$new(2)
# product of FuzzySet
FuzzySetnew(1, 0.1, 2, 0.5) * Set<math>new(2:5)
# product of conditional sets
ConditionalSetnew(function(x, y) x >= y) *
  ConditionalSetnew(function(x, y) x == y)
# product of special sets
PosReals$new() * NegReals$new()
```

setsymdiff

Symmetric Difference of Two Sets

Description

Returns the symmetric difference of two objects inheriting from class Set.

Usage

```
setsymdiff(x, y, simplify = TRUE)
x %-% y
```

68 setunion

Arguments

x, y Set

simplify logical, if TRUE (default) returns the result in its simplest form, usually a Set or UnionSet, otherwise a ComplementSet.

Details

The symmetric difference, aka disjunctive union, of two sets, X, Y, is defined as the set of elements that exist in set X or in Y but not both,

$$\{z : (z\epsilon X \cup z\epsilon Y) \cap \neg (z\epsilon X \cap z\epsilon Y)\}\$$

The symmetric difference can also be expressed as the union of two sets minus the intersection. Therefore setsymdiff is written as a thin wrapper over these operations, so for two sets, A,B: $A \% B = (A \mid B) - (A \& B)$.

The symmetric difference of fuzzy and crisp sets first coerces fuzzy sets to crisp sets by finding their support.

Value

An object inheriting from Set containing the symmetric difference of elements in both x and y.

See Also

Other operators: powerset(), setcomplement(), setintersect(), setpower(), setproduct(), setunion()

Examples

setunion

Union of Sets

Description

Returns the union of objects inheriting from class Set.

setunion 69

Usage

```
setunion(..., simplify = TRUE)
## S3 method for class 'Set'
x + y
## S3 method for class 'Set'
x | y
```

Arguments

```
... Sets
simplify logical, if TRUE (default) returns the result in its simplest (unwrapped) form,
usually a Set, otherwise a UnionSet.
x, y
Set
```

Details

The union of N sets, X1, ..., XN, is defined as the set of elements that exist in one or more sets,

$$U = \{x : x \in X1 \quad or \quad x \in X2 \quad or \quad \dots \quad or \quad x \in XN\}$$

The union of multiple ConditionalSets is given by combining their defining functions by an 'or', |, operator. See examples.

The union of fuzzy and crisp sets first coerces fuzzy sets to crisp sets by finding their support.

Value

An object inheriting from Set containing the union of supplied sets.

See Also

```
Other operators: powerset(), setcomplement(), setintersect(), setpower(), setproduct(), setsymdiff()
```

Examples

```
# union of Sets

Set$new(-2:4) + Set$new(2:5)
setunion(Set$new(1, 4, "a"), Set$new("a", 6))
Set$new(1, 2) + Set$new("a", 1i) + Set$new(9)

# union of intervals

Interval$new(1, 10) + Interval$new(5, 15) + Interval$new(20, 30)
Interval$new(1, 2, type = "()") + Interval$new(2, 3, type = "(]")
Interval$new(1, 5, class = "integer") +
    Interval$new(2, 7, class = "integer")
```

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```
# union of mixed types

Set$new(1:10) + Interval$new(5, 15)
Set$new(1:10) + Interval$new(5, 15, class = "integer")
Set$new(5, 7) | Tuple$new(6, 8, 7)

# union of FuzzySet
FuzzySet$new(1, 0.1, 2, 0.5) + Set$new(2:5)

# union of conditional sets

ConditionalSet$new(function(x, y) x >= y) +
   ConditionalSet$new(function(x, y) x == y) +
   ConditionalSet$new(function(x) x == 2)

# union of special sets
PosReals$new() + NegReals$new()
Set$new(-Inf, Inf) + Reals$new()
```

SetWrapper

Abstract SetWrapper Class

Description

This class should not be constructed directly. Parent class to SetWrappers.

Super class

```
set6::Set -> SetWrapper
```

Active bindings

wrappedSets Returns the list of Sets that are wrapped in the given wrapper.

Methods

Public methods:

- SetWrapper\$new()
- SetWrapper\$equals()
- SetWrapper\$isSubset()
- SetWrapper\$clone()

Method new(): Create a new SetWrapper object. It is not recommended to construct this class directly.

Usage:

SetWrapper 71

```
SetWrapper$new(
    setlist,
    lower = NULL,
    upper = NULL,
    type = NULL,
    class = NULL,
    cardinality
  )
 Arguments:
  setlist List of Sets to wrap.
  lower Set. Lower bound of wrapper.
  upper Set. Upper bound of wrapper.
  type character. Closure type of wrapper.
  class character. Ignored.
  cardinality character or integer. Cardinality of wrapper.
 Returns: A new SetWrapper object.
Method equals(): Tests if x is equal to self.
  Usage:
  SetWrapper$equals(x, all = FALSE)
 Arguments:
  x Set or vector of Sets.
  x any. Object or vector of objects to test.
  all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.
  all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.
 Returns: If all == TRUE then returns TRUE if all x are equal to self, otherwise FALSE. If all
  == FALSE returns a vector of logicals corresponding to the length of x, representing if each is
  equal to self.
Method isSubset(): Tests if x is a (proper) subset of self.
  Usage:
  SetWrapper$isSubset(x, proper = FALSE, all = FALSE)
 Arguments:
  x Set or vector of Sets.
  x any. Object or vector of objects to test.
  proper logical. If TRUE tests for proper subsets.
  all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.
  all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.
 Returns: If all == TRUE then returns TRUE if all x are (proper) subsets of self, otherwise FALSE.
 If all == FALSE returns a vector of logicals corresponding to the length of x, representing if each
 is a (proper) subset of self.
```

Method clone(): The objects of this class are cloneable with this method.

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```
Usage:
SetWrapper$clone(deep = FALSE)
Arguments:
deep Whether to make a deep clone.
```

SpecialSet

Abstract Class for Special Sets

Description

The 'special sets' are the group of sets that are commonly used in mathematics and are thus given their own names.

Details

This is an abstract class and should not be constructed directly. Use listSpecialSets to see the list of implemented special sets.

Super classes

```
set6::Set -> set6::Interval -> SpecialSet
```

Methods

Public methods:

- SpecialSet\$new()
- SpecialSet\$strprint()
- SpecialSet\$clone()

Method new(): SpecialSet is an abstract class, the constructor cannot be used directly.

```
Usage:
```

```
SpecialSet$new(lower = -Inf, upper = Inf, type = "()", class = "numeric")
```

Arguments:

lower defines the lower bound of the interval.

upper defines the upper bound of the interval.

type defines the interval closure type.

class defines the interval class.

Method strprint(): Creates a printable representation of the object.

```
Usage:
```

```
SpecialSet$strprint(n = NULL)
```

Arguments:

n ignored, added for consistency.

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Returns: A character string representing the object.

Method clone(): The objects of this class are cloneable with this method.

Usage:

SpecialSet\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

testClosed

assert/check/test/Closed

Description

Validation checks to test if a given object is closed.

Usage

```
testClosed(object, errormsg = "This is not a closed set")
checkClosed(object, errormsg = "This is not a closed set")
assertClosed(object, errormsg = "This is not a closed set")
```

Arguments

object object to test

error message to overwrite default if check fails

Value

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

```
testClosed(Interval$new(1, 10, type = "[]"))
testClosed(Interval$new(1, 10, type = "(]"))
```

74 testClosedBelow

testClosedAbove

assert/check/test/ClosedAbove

Description

Validation checks to test if a given object is closedabove.

Usage

```
testClosedAbove(object, errormsg = "This is not a set closed above")
checkClosedAbove(object, errormsg = "This is not a set closed above")
assertClosedAbove(object, errormsg = "This is not a set closed above")
```

Arguments

object object to test

error message to overwrite default if check fails

Value

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

Examples

```
testClosedAbove(Interval$new(1, 10, type = "[]"))
testClosedAbove(Interval$new(1, 10, type = "[)"))
```

testClosedBelow

assert/check/test/ClosedBelow

Description

Validation checks to test if a given object is closedbelow.

Usage

```
testClosedBelow(object, errormsg = "This is not a set closed below")
checkClosedBelow(object, errormsg = "This is not a set closed below")
assertClosedBelow(object, errormsg = "This is not a set closed below")
```

testConditionalSet 75

Arguments

object object to test

error message to overwrite default if check fails

Value

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

Examples

```
testClosedBelow(Interval$new(1, 10, type = "[]"))
testClosedBelow(Interval$new(1, 10, type = "(]"))
```

testConditionalSet

assert/check/test/ConditionalSet

Description

Validation checks to test if a given object is an R6 ConditionalSet.

Usage

```
testConditionalSet(
  object,
  errormsg = "This is not an R6 ConditionalSet object"
)
checkConditionalSet(
  object,
  errormsg = "This is not an R6 ConditionalSet object"
)
assertConditionalSet(
  object,
  errormsg = "This is not an R6 ConditionalSet object"
)
```

Arguments

object object to test

error message to overwrite default if check fails

Value

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

76 testContains

Examples

```
testConditionalSet(Set$new(2, 3))
testConditionalSet(list(Set$new(2), Set$new(3)))
testConditionalSet(Tuple$new(2, 3))
testConditionalSet(Interval$new())
testConditionalSet(FuzzySet$new(2, 0.1))
testConditionalSet(FuzzyTuple$new(2, 0.1))
testConditionalSet(ConditionalSet$new(function(x) x == 0))
```

testContains

assert/check/test/Contains

Description

Validation checks to test if given elements are contained in a set.

Usage

```
testContains(
  object,
  elements,
  errormsg = "elements are not contained in the set"
)

checkContains(
  object,
  elements,
  errormsg = "elements are not contained in the set"
)

assertContains(
  object,
  elements,
  errormsg = "elements are not contained in the set"
)
```

Arguments

object object to test
elements elements to check
errormsg error message to overwrite default if check fails

Value

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

testCountablyFinite 77

Examples

```
testContains(Set$new(1,2,3), c(1,2))
testContains(Set$new(1,2,3), c(3,4))
```

testCountablyFinite

assert/check/test/CountablyFinite

Description

Validation checks to test if a given object is countablyfinite.

Usage

```
testCountablyFinite(object, errormsg = "This is not a countably finite set")
checkCountablyFinite(object, errormsg = "This is not a countably finite set")
assertCountablyFinite(object, errormsg = "This is not a countably finite set")
```

Arguments

object

object to test

errormsg

error message to overwrite default if check fails

Value

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

Examples

```
testCountablyFinite(Set$new(1,2,3))
testCountablyFinite(Interval$new(1,10))
```

testCrisp

assert/check/test/Crisp

Description

Validation checks to test if a given object is crisp.

78 testEmpty

Usage

```
testCrisp(object, errormsg = "This is not crisp.")
checkCrisp(object, errormsg = "This is not crisp.")
assertCrisp(object, errormsg = "This is not crisp.")
```

Arguments

object object to test

error message to overwrite default if check fails

Value

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

Examples

```
testCrisp(Set$new(1))
testCrisp(FuzzySet$new(1, 0.5))
```

testEmpty

assert/check/test/Empty

Description

Validation checks to test if a given object is empty.

Usage

```
testEmpty(object, errormsg = "This is not an empty set")
checkEmpty(object, errormsg = "This is not an empty set")
assertEmpty(object, errormsg = "This is not an empty set")
```

Arguments

object object to test

error message to overwrite default if check fails

Value

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

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Examples

```
testEmpty(Set$new())
testEmpty(Set$new(1))
```

testFinite

assert/check/test/Finite

Description

Validation checks to test if a given object is finite.

Usage

```
testFinite(object, errormsg = "This is not finite")
checkFinite(object, errormsg = "This is not finite")
assertFinite(object, errormsg = "This is not finite")
```

Arguments

object

object to test

errormsg

error message to overwrite default if check fails

Value

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

Examples

```
testFinite(Interval$new(1, 10, class = "integer"))
testFinite(Interval$new(1, 10, class = "numeric"))
```

testFuzzy

assert/check/test/Fuzzy

Description

Validation checks to test if a given object is fuzzy.

Usage

```
testFuzzy(object, errormsg = "This is not fuzzy.")
checkFuzzy(object, errormsg = "This is not fuzzy.")
assertFuzzy(object, errormsg = "This is not fuzzy.")
```

80 testFuzzySet

Arguments

object object to test

error message to overwrite default if check fails

Value

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

Examples

```
testFuzzy(FuzzySet$new(1, 0.5))
testFuzzy(Set$new(1))
```

testFuzzySet

assert/check/test/FuzzySet

Description

Validation checks to test if a given object is an R6 FuzzySet.

Usage

```
testFuzzySet(object, errormsg = "This is not an R6 FuzzySet object")
checkFuzzySet(object, errormsg = "This is not an R6 FuzzySet object")
assertFuzzySet(object, errormsg = "This is not an R6 FuzzySet object")
```

Arguments

object object to test

error message to overwrite default if check fails

Value

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

```
testFuzzySet(Set$new(2, 3))
testFuzzySet(list(Set$new(2), Set$new(3)))
testFuzzySet(Tuple$new(2, 3))
testFuzzySet(Interval$new())
testFuzzySet(FuzzySet$new(2, 0.1))
testFuzzySet(FuzzyTuple$new(2, 0.1))
testFuzzySet(ConditionalSet$new(function(x) x == 0))
```

testFuzzyTuple 81

testFuzzyTuple	assert/check/test/FuzzyTuple	
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Description

Validation checks to test if a given object is an R6 FuzzyTuple.

Usage

```
testFuzzyTuple(object, errormsg = "This is not an R6 FuzzyTuple object")
checkFuzzyTuple(object, errormsg = "This is not an R6 FuzzyTuple object")
assertFuzzyTuple(object, errormsg = "This is not an R6 FuzzyTuple object")
```

Arguments

object object to test
errormsg error message to overwrite default if check fails

Value

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

Examples

```
testFuzzyTuple(Set$new(2, 3))
testFuzzyTuple(list(Set$new(2), Set$new(3)))
testFuzzyTuple(Tuple$new(2, 3))
testFuzzyTuple(Interval$new())
testFuzzyTuple(FuzzySet$new(2, 0.1))
testFuzzyTuple(FuzzyTuple$new(2, 0.1))
testFuzzyTuple(ConditionalSet$new(function(x) x == 0))
```

testInterval assert/check/test/Interval

Description

Validation checks to test if a given object is an R6 Interval.

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Usage

```
testInterval(object, errormsg = "This is not an R6 Interval object")
checkInterval(object, errormsg = "This is not an R6 Interval object")
assertInterval(object, errormsg = "This is not an R6 Interval object")
```

Arguments

object object to test

error message to overwrite default if check fails

Value

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

Examples

```
testInterval(Set$new(2, 3))
testInterval(list(Set$new(2), Set$new(3)))
testInterval(Tuple$new(2, 3))
testInterval(Interval$new())
testInterval(FuzzySet$new(2, 0.1))
testInterval(FuzzyTuple$new(2, 0.1))
testInterval(ConditionalSet$new(function(x) x == 0))
```

testSet

assert/check/test/Set

Description

Validation checks to test if a given object is an R6 Set.

Usage

```
testSet(object, errormsg = "This is not an R6 Set object")
checkSet(object, errormsg = "This is not an R6 Set object")
assertSet(object, errormsg = "This is not an R6 Set object")
```

Arguments

object object to test

error message to overwrite default if check fails

testSetList 83

Value

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

Examples

```
testSet(Set$new(2, 3))
testSet(list(Set$new(2), Set$new(3)))
testSet(Tuple$new(2, 3))
testSet(Interval$new())
testSet(FuzzySet$new(2, 0.1))
testSet(FuzzyTuple$new(2, 0.1))
testSet(ConditionalSet$new(function(x) x == 0))
```

testSetList

assert/check/test/SetList

Description

Validation checks to test if a given object is an R6 SetList.

Usage

```
testSetList(object, errormsg = "One or more items in the list are not Sets")
checkSetList(object, errormsg = "One or more items in the list are not Sets")
assertSetList(object, errormsg = "One or more items in the list are not Sets")
```

Arguments

object object to test
errormsg error message to overwrite default if check fails

Value

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

```
testSetList(Set$new(2, 3))
testSetList(list(Set$new(2), Set$new(3)))
testSetList(Tuple$new(2, 3))
testSetList(Interval$new())
testSetList(FuzzySet$new(2, 0.1))
testSetList(FuzzyTuple$new(2, 0.1))
testSetList(ConditionalSet$new(function(x) x == 0))
```

84 testSubset

testSubset

assert/check/test/Subset

Description

Validation checks to test if given sets are subsets of a set.

Usage

```
testSubset(
  object,
  sets,
 proper = FALSE,
 errormsg = "sets are not subsets of the object"
)
checkSubset(
  object,
  sets,
  proper = FALSE,
 errormsg = "sets are not subsets of the object"
assertSubset(
 object,
  sets,
 proper = FALSE,
 errormsg = "sets are not subsets of the object"
)
```

Arguments

object object to test sets sets to check

proper logical. If TRUE tests for proper subsets.

error message to overwrite default if check fails

Value

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

```
testSubset(Set$new(1,2,3), Set$new(1,2))
testSubset(Set$new(1,2,3), Set$new(3,4))
```

testTuple 85

testTuple

assert/check/test/Tuple

Description

Validation checks to test if a given object is an R6 Tuple.

Usage

```
testTuple(object, errormsg = "This is not an R6 Tuple object")
checkTuple(object, errormsg = "This is not an R6 Tuple object")
assertTuple(object, errormsg = "This is not an R6 Tuple object")
```

Arguments

object object to test

error message to overwrite default if check fails

Value

If check passes then assert returns object invisibly and test/check return TRUE. If check fails, assert stops code with error, check returns an error message as string, and test returns FALSE.

Examples

```
testTuple(Set$new(2, 3))
testTuple(list(Set$new(2), Set$new(3)))
testTuple(Tuple$new(2, 3))
testTuple(Interval$new())
testTuple(FuzzySet$new(2, 0.1))
testTuple(FuzzyTuple$new(2, 0.1))
testTuple(ConditionalSet$new(function(x) x == 0))
```

Tuple

Mathematical Tuple

Description

A general Tuple object for mathematical tuples, inheriting from Set.

Details

Tuples are similar to sets, except that they drop the constraint for elements to be unique, and ordering in a tuple does matter. Tuples are useful for methods including \$contains that may require non-unique elements. They are also the return type of the product of sets. See examples.

86 Tuple

Super class

```
set6::Set -> Tuple
```

Methods

Public methods:

- Tuple\$equals()
- Tuple\$isSubset()
- Tuple\$clone()

Method equals(): Tests if two sets are equal.

```
Usage:
```

```
Tuple = FALSE
```

Arguments:

x Set or vector of Sets.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

Details: An object is equal to a Tuple if it contains all the same elements, and in the same order. Infix operators can be used for:

```
Equal == Not equal !=
```

Returns: If all is TRUE then returns TRUE if all x are equal to the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

Examples:

```
Tuple$new(1,2) == Tuple$new(1,2)
Tuple$new(1,2) != Tuple$new(1,2)
Tuple$new(1,1) != Set$new(1,1)
```

Method isSubset(): Test if one set is a (proper) subset of another

Usage:

```
Tuple$isSubset(x, proper = FALSE, all = FALSE)
```

Arguments:

x any. Object or vector of objects to test.

proper logical. If TRUE tests for proper subsets.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

Details: If using the method directly, and not via one of the operators then the additional boolean argument proper can be used to specify testing of subsets or proper subsets. A Set is a proper subset of another if it is fully contained by the other Set (i.e. not equal to) whereas a Set is a (non-proper) subset if it is fully contained by, or equal to, the other Set.

When calling \$isSubset on objects inheriting from Interval, the method treats the interval as if it is a Set, i.e. ordering and class are ignored. Use \$isSubinterval to test if one interval is a subinterval of another.

Infix operators can be used for:

Tuple 87

```
Subset <
Proper Subset <=
Superset >
Proper Superset >=
```

An object is a (proper) subset of a Tuple if it contains all (some) of the same elements, and in the same order.

Returns: If all is TRUE then returns TRUE if all x are subsets of the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

Examples:

```
Tuplenew(1,2,3) < Tuple<math>new(1,2,3,4)
Tuplenew(1,3,2) < Tuple<math>new(1,2,3,4)
```

Method clone(): The objects of this class are cloneable with this method.

```
Usage:
```

Tuple\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

See Also

Other sets: ConditionalSet, FuzzySet, FuzzyTuple, Interval, LogicalSet, Set, UniversalSet

```
# Tuple of integers
Tuple$new(1:5)
# Tuple of multiple types
Tuple$new("a", 5, Set$new(1), Tuple$new(2))
# Each Tuple has properties and traits
t <- Tuple$new(1, 2, 3)
t$traits
t$properties
# Elements can be duplicated
Tuple$new(2, 2) != Tuple$new(2)
# Ordering does matter
Tuple$new(1, 2) != Tuple$new(2, 1)
## Method `Tuple$equals`
## -----
Tuple new(1,2) == Tuple new(1,2)
Tuple$new(1,2) != Tuple$new(1,2)
Tuplenew(1,1) != Set new(1,1)
```

88 UnionSet

```
## -----
## Method `Tuple$isSubset`
## -----

Tuple$new(1,2,3) < Tuple$new(1,2,3,4)

Tuple$new(1,3,2) < Tuple$new(1,2,3,4)
```

UnionSet

Set of Unions

Description

UnionSet class for symbolic union of mathematical sets.

Details

The purpose of this class is to provide a symbolic representation for the union of sets that cannot be represented in a simpler class. Whilst this is not an abstract class, it is not recommended to construct this class directly but via the set operation methods.

Super classes

```
set6::Set -> set6::SetWrapper -> UnionSet
```

Active bindings

elements Returns the elements in the object.

length Returns the number of elements in the object.

Methods

Public methods:

- UnionSet\$new()
- UnionSet\$strprint()
- UnionSet\$contains()
- UnionSet\$clone()

Method new(): Create a new UnionSet object. It is not recommended to construct this class directly.

```
Usage:
UnionSet$new(setlist, lower = NULL, upper = NULL, type = NULL)
Arguments:
setlist list of Sets to wrap.
lower lower bound of new object.
upper upper bound of new object.
```

UnionSet 89

type closure type of new object.

Returns: A new UnionSet object.

Method strprint(): Creates a printable representation of the object.

Usage:

UnionSetstrprint(n = 2)

Arguments:

n numeric. Number of elements to display on either side of ellipsis when printing.

Returns: A character string representing the object.

Method contains(): Tests if elements x are contained in self.

Usage:

UnionSet\$contains(x, all = FALSE, bound = FALSE)

Arguments:

x any. Object or vector of objects to test.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test. bound logical.

Returns: If all == TRUE then returns TRUE if all x are contained in self, otherwise FALSE. If all == FALSE returns a vector of logicals corresponding to the length of x, representing if each is contained in self. If bound == TRUE then an element is contained in self if it is on or within the (possibly-open) bounds of self, otherwise TRUE only if the element is within self or the bounds are closed.

Method clone(): The objects of this class are cloneable with this method.

Usage:

UnionSet\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

See Also

Set operations: setunion, setproduct, setpower, setcomplement, setsymdiff, powerset, setintersect

Other wrappers: ComplementSet, ExponentSet, PowersetSet, ProductSet

UniversalSet

Mathematical Universal Set

Description

The UniversalSet is defined as the Set containing all possible elements.

Details

The Universal set is the default universe to all sets, and is the largest possible set. The Universal set contains every single possible element. We denote the Universal set with V instead of U to avoid confusion with the union symbol. The Universal set cardinality is set to Inf where we assume Inf is greater than any Aleph or Beth numbers. The Universal set is also responsible for a few set paradoxes, to resolve these we use the following results:

Let V be the universal set, S be any non-universal set, and S the empty set, then

$$V \cup S = V$$

$$V \cap S = S$$

$$S - V = 0$$

$$V^n = V$$

$$P(V) = V$$

Super class

set6::Set -> UniversalSet

Methods

Public methods:

- UniversalSet\$new()
- UniversalSet\$equals()
- UniversalSet\$isSubset()
- UniversalSet\$contains()
- UniversalSet\$strprint()
- UniversalSet\$clone()

Method new(): Create a new UniversalSet object.

Usage:

UniversalSet\$new()

Details: The Universal set is the set containing every possible element.

Returns: A new UniversalSet object.

Method equals(): Tests if two sets are equal.

```
Usage:
```

UniversalSet\$equals(x, all = FALSE)

Arguments:

x Set or vector of Sets.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

Details: Only the UniversalSet is equal to itself.

Returns: If all is TRUE then returns TRUE if all x are equal to the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x. Infix operators can be used for:

Equal == Not equal !=

```
Examples:
```

```
# Equals
Set$new(1,2)$equals(Set$new(5,6))
Set$new(1,2)$equals(Interval$new(1,2))
Set$new(1,2) == Interval$new(1,2, class = "integer")
# Not equal
!Set$new(1,2)$equals(Set$new(1,2))
Set$new(1,2) != Set$new(1,5)
```

Method isSubset(): Test if one set is a (proper) subset of another

Usage:

```
UniversalSet$isSubset(x, proper = FALSE, all = FALSE)
```

Arguments:

x any. Object or vector of objects to test.

proper logical. If TRUE tests for proper subsets.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test.

Details: If using the method directly, and not via one of the operators then the additional boolean argument proper can be used to specify testing of subsets or proper subsets. A Set is a proper subset of another if it is fully contained by the other Set (i.e. not equal to) whereas a Set is a (non-proper) subset if it is fully contained by, or equal to, the other Set.

When calling \$isSubset on objects inheriting from Interval, the method treats the interval as if it is a Set, i.e. ordering and class are ignored. Use \$isSubinterval to test if one interval is a subinterval of another.

Infix operators can be used for:

Subset <
Proper Subset <=
Superset >>
Proper Superset >=

Every Set is a subset of a UniversalSet. No Set is a super set of a UniversalSet, and only a UniversalSet is not a proper subset of a UniversalSet.

Returns: If all is TRUE then returns TRUE if all x are subsets of the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

Examples:

```
Set$new(1,2,3)$isSubset(Set$new(1,2), proper = TRUE)
Set$new(1,2) < Set$new(1,2,3) # proper subset

c(Set$new(1,2,3), Set$new(1)) < Set$new(1,2,3) # not proper
Set$new(1,2,3) <= Set$new(1,2,3) # proper</pre>
```

Method contains(): Tests to see if x is contained in the Set.

Usage:

```
UniversalSet$contains(x, all = FALSE, bound = NULL)
```

Arguments:

x any. Object or vector of objects to test.

all logical. If FALSE tests each x separately. Otherwise returns TRUE only if all x pass test. bound ignored.

Details: x can be of any type, including a Set itself. x should be a tuple if checking to see if it lies within a set of dimension greater than one. To test for multiple x at the same time, then provide these as a list.

If using the method directly, and not via one of the operators then the additional boolean arguments all and bound. If all = TRUE then returns TRUE if all x are contained in the Set, otherwise returns a vector of logicals. For Intervals, bound is used to specify if elements lying on the (possibly open) boundary of the interval are considered contained (bound = TRUE) or not (bound = FALSE).

Returns: If all is TRUE then returns TRUE if all elements of x are contained in the Set, otherwise FALSE. If all is FALSE then returns a vector of logicals corresponding to each individual element of x.

The infix operator %inset% is available to test if x is an element in the Set, see examples. Every element is contained within the Universal set.

```
s = Set$new(1:5)

# Simplest case
s$contains(4)
8 %inset% s

# Test if multiple elements lie in the set
s$contains(4:6, all = FALSE)
s$contains(4:6, all = TRUE)

# Check if a tuple lies in a Set of higher dimension
s2 = s * s
s2$contains(Tuple$new(2,1))
c(Tuple$new(2,1), Tuple$new(1,7), 2) %inset% s2
```

```
Method strprint(): Creates a printable representation of the object.

Usage:
UniversalSet$strprint(n = 2)

Arguments:
n numeric. Number of elements to display on either side of ellipsis when printing.

Returns: A character string representing the object.

Method clone(): The objects of this class are cloneable with this method.

Usage:
```

UniversalSet\$clone(deep = FALSE)

Arguments:

deep Whether to make a deep clone.

See Also

Other sets: ConditionalSet, FuzzySet, FuzzyTuple, Interval, LogicalSet, Set, Tuple

```
u <- UniversalSet$new()</pre>
print(u)
u$contains(c(1, letters, TRUE, Set<math>new()), all = TRUE)
## -----
## Method `UniversalSet$equals`
## -----
# Equals
Set$new(1,2)$equals(Set$new(5,6))
Set$new(1,2)$equals(Interval$new(1,2))
Set$new(1,2) == Interval$new(1,2, class = "integer")
# Not equal
!Set$new(1,2)$equals(Set$new(1,2))
Set$new(1,2) != Set$new(1,5)
## Method `UniversalSet$isSubset`
## -----
Set$new(1,2,3)$isSubset(Set$new(1,2), proper = TRUE)
Set$new(1,2) < Set$new(1,2,3) # proper subset
c(Set$new(1,2,3), Set$new(1)) < Set$new(1,2,3) # not proper
Set$new(1,2,3) \le Set$new(1,2,3) # proper
## Method `UniversalSet$contains`
```

94 useUnicode

```
s = Set$new(1:5)

# Simplest case
s$contains(4)
8 %inset% s

# Test if multiple elements lie in the set
s$contains(4:6, all = FALSE)
s$contains(4:6, all = TRUE)

# Check if a tuple lies in a Set of higher dimension
s2 = s * s
s2$contains(Tuple$new(2,1))
c(Tuple$new(2,1), Tuple$new(1,7), 2) %inset% s2
```

useUnicode

Get/Set Unicode Printing Method

Description

Change whether unicode symbols should be used when printing sets.

Usage

```
useUnicode(use)
```

Arguments

use

logical, if TRUE unicode will be used in printing, otherwise simpler character strings. If missing the current setting is returned.

Details

Using unicode symbols makes the printing of sets and properties 'prettier', however may not work on all machines or versions of R. Therefore this function is used to decide whether unicode representations should be used, or standard alpha-numeric and special characters.

By default set6 starts with unicode printing turned on.

```
current <- useUnicode()
useUnicode(TRUE)
useUnicode()
useUnicode(current)</pre>
```

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