# Package 'restriktor'

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Title Restricted Statistical Estimation and Inference for Linear Models

Version 0.2-800

**Description** Allow for easy-to-use testing or evaluating of linear equality and inequality restrictions about parameters and effects in (generalized) linear statistical models.

**Depends** R(>= 3.0.0)

Imports boot, ic.infer, lavaan, MASS, mvtnorm, quadprog

**License** GPL ( $\geq 2$ )

LazyData yes

URL http://restriktor.org

NeedsCompilation no

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Suggests

**Repository** CRAN

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# **R** topics documented:

rriktor-package	. 2
gerManagement	. 4
ms	. 5
Test	. 6
Test-methods	. 12
TestC	. 13
TestF	. 15
TestLRT	
TestScore	. 26
TestWald	
Test_ceq	. 37

conTest_summary	 40
con_weights_boot	 43
Exam	 44
goric	
Hurricanes	
restriktor	 49
restriktor-methods	 56
ZelazoKolb1972	 59
	60

#### Index

restriktor-package Package for equality and inequality restricted estimation and hypothesis testing

#### Description

Package restriktor implements estimation, testing and evaluating of linear equality and inequality restriktions about parameters and effects for univariate and multivariate normal models and generalized linear models.

## Details

Package:	restriktor
Type:	Package
Version:	0.2-800
Date:	2020-02-25
License:	GPL (>=2)
LazyLoad:	yes

Function restriktor estimates the parameters of an univariate and multivariate linear model (lm), robust estimation of the linear model (rlm) or a generalized linear model (glm) subject to linear equality and/or inequality restriktions. The real work horses are the conLM, conMLM, the conRLM, and the conGLM functions. A major advantage of **restriktor** is that the constraints can be specified by a text-based description. This means that users do not have to specify the complex constraint matrix (comparable with a contrast matrix) themselves.

The function restriktor offers the possibility to compute (model robust) standard errors under the restriktions. The parameter estimates can also be bootstrapped, where bootstrapped standard errors and confidence intervals are available via the summary function. Moreover, the function computes the Generalized Order-restricted Information Criterion (GORIC), which is a modification of the AIC and a generalization of the ORIC.

The function iht (alias conTest) conducts restricted hypothesis tests. F, Wald/LRT and score teststatistics are available. The null-distribution of these test-statistics takes the form of a mixture of F-distributions. The mixing weights (a.k.a. chi-bar-square weights or level probabilities) can be computed using the multivariate normal distribution function with additional Monte Carlo steps or

#### restriktor-package

via a simulation approach. Bootstrap methods are available to calculate the mixing weights and to compute the p-value directly. Parameters estimates under the null- and alternative-hypothesis are available from the summary function.

The function goric (generalized order-restricted information criterion) computes GORIC values, weights and relative-weights or GORICA (generalized order-restricted information crittion approximation) values, weights and relative weights. The GORIC(A) values are comparable to the AIC values. The function offers the possibility to evaluate an order-restricted hypothesis against its complement, the unconstrained hypothesis or against a set of hypotheses. For now, only one order-restricted hypothesis can be evaluated against its complement but work is in progress to evaluate a set of order-restricted hypothesis against its complement.

The package makes use of various other R packages: **quadprog** is used for restricted estimation, **boot** for bootstrapping, **ic.infer** for computing the mixing weights based on the multivariate normal distribution, **lavaan** for parsing the constraint syntax.

## Value

The output of function restriktor belongs to S3 class conLM, conMLM, conRLM or conGLM.

The output of function conTest belongs to S3 class conTest.

These classes offer print and summary methods.

#### Acknowledgements

This package uses as an internal function the function nchoosek from **ic.infer**, which is originally from **vsn**, authored by Wolfgang Huber, available under LGPL.

The output style of the iht print function is strongly inspired on the summary of the ic.test function from the **ic.infer** package.

## Author(s)

Leonard Vanbrabant and Yves Rosseel - Ghent University

#### References

Groemping, U. (2010). Inference With Linear Equality And Inequality Constraints Using R: The Package ic.infer. *Journal of Statistical Software*, Forthcoming.

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Silvapulle, M. (1992a). Robust tests of inequality constraints and one-sided hypotheses in the linear model. *Biometrika*, **79**, 621–630.

Silvapulle, M. (1992b). Robust wald-type tests of one-sided hypotheses in the linear model. *Journal of the American Statistical Association*, **87**, 156–161.

Silvapulle, M. (1996). Robust bounded influence tests against one-sided hypotheses in general parametric models. *Statistics & probability letters*, **31**, 45–50.

Silvapulle, M.J. and Sen, P.K. (2005). Constrained Statistical Inference. Wiley, New York

Vanbrabant, L. and Kuiper, R. (n.d.). Giving the complement a compliment: Evaluating a theorybased hypothesis against its complement using the GORIC.

#### See Also

See also restriktor, iht, packages boot, goric, ic.infer, mvtnorm, and quadprog.

## Examples

AngerManagement Reduction of aggression levels Dataset (4 treatment groups)

## Description

The anger management dataset consists of reduction of aggression levels between week 1 (intake) and week 8 (end of training) from four different treatment groups (No-exercises, Physical-exercises, Behavioral-exercises, combination of physical and behavioral exercises).

#### Usage

data(AngerManagement)

## Burns

## Format

A data frame of 40 observations of 4 treatment variables and covariate age.

Anger reduction in aggression levels

Group No, Physical, Behavioral, Both

Age persons' age

## References

Hoijtink, H. Informative Hypotheses: Theory and Practice for Behavioral and Social Scientists Boca Raton, FL: Taylor & Francis, 2012.

## Examples

head(AngerManagement)

Burns

*Relation between the response variable PTSS and gender, age, TBSA, guilt and anger.* 

## Description

Simulated dataset based on the original model parameters. The original data are based on two cohort studies in children from 0 to 4 and 8 to 18 years old with burns and their mother.

#### Usage

data(Burns)

#### Format

A data frame of 278 observations of 4 variables.

PTSS post-traumatic stress symptoms

gender gender

age age in years

TBSA estimated percentage total body surface area affected by second and third degree burns

guilt parental guilt feelings in relation to the burn event

anger parental anger feelings in relation to the burn event

#### References

Bakker A, Van der Heijden PG, Van Son MJ, Van Loey NE. Course of traumatic stress reactions in couples after a burn event to their young child. Health Psychology 2013; 10(32):1076-1083, doi:10.1037/a0033983.

Egberts MR, van de Schoot R, Boekelaar A, Hendrickx H, Geenen R, NEE V. Child and adolescent internalizing and externalizing problems 12 months postburn: the potential role of preburn functioning, parental posttraumatic stress, and informant bias. Child \& Adolescent Psychiatry 2016; 25:791-803.

## Examples

head(Burns)

conTest

*function for informative hypothesis testing (iht)* 

## Description

conTest tests linear equality and/or inequality restricted hypotheses for linear models.

## Usage

#### Arguments

object	an object of class lm or rlm. In this case, the constraint syntax needs to be specified OR
	an object of class restriktor. The constraints are inherited from the fitted restriktor object and do not to be specified again.
constraints	there are two ways to constrain parameters. First, the constraint syntax consists of one or more text-based descriptions, where the syntax can be specified as a literal string enclosed by single quotes. Only the names of coef(model) can be used as names. See details restriktor for more information.
	Second, the constraint syntax consists of a matrix $R$ (or a vector in case of one constraint) and defines the left-hand side of the constraint $R\theta \ge rhs$ , where each row represents one constraint. The number of columns needs to correspond to the number of parameters estimated ( $\theta$ ) by model. The rows should be linear independent, otherwise the function gives an error. For more information about constructing the matrix $R$ and $rhs$ see the details in the restriktor function.
type	hypothesis test type "A", "B", "C", "global", or "summary" (default). See details for more information.
test	test statistic; for information about the null-distribution see details.

	<ul> <li>for object of class lm; if "F" (default), the F-bar statistic (Silvapulle, 1996) is computed. If "LRT", a likelihood ratio test statistic (Silvapulle and Sen, 2005, chp 3.) is computed. If "score", a global score test statistic (Silvapulle and Silvapulle, 1995) is computed. Note that, in case of equality constraints only, the usual unconstrained F-, Wald-, LR- and score-test statistic is computed.</li> <li>for object of class rlm; if "F" (default), a robust likelihood ratio type test statistic (Silvapulle, 1992a) is computed. If "Wald", a robust Wald test statistic (Silvapulle, 1992b) is computed. If "score", a global score test statistic (Silvapulle, 1992b) is computed. If "score", a global score test statistic (Silvapulle, 1992b) is computed. If "score", a global score test statistic (Silvapulle, and Silvapulle, 1995) is computed. Note that, in case of equality constraints only, unconstrained robust F-, Wald-, score-test statistics are computed.</li> </ul>
	• for object of class glm; if "F" (default), the F-bar statistic (Silvapulle, 1996) is computed. If "LRT", a likelihood ratio test statistic (Silvapulle and Sen, 2005, chp 4.) is computed. If "score", a global score test statistic (Silvapulle and Silvapulle, 1995) is computed. Note that, in case of equality constraints only, the usual unconstrained F-, Wald-, LR- and score-test statistic is computed.
rhs	vector on the right-hand side of the constraints; $R\theta \ge rhs$ . The length of this vector equals the number of rows of the constraints matrix $R$ and consists of zeros by default. Note: only used if constraints input is a matrix or vector.
neq	integer (default = 0) treating the number of constraints rows as equality con- straints instead of inequality constraints. For example, if $neq = 2$ , this means that the first two rows of the constraints matrix $R$ are treated as equality con- straints. Note: only used if constraints input is a matrix or vector.
	futher options for the conTest and/or restriktor function. See details for more information.

#### Details

The following hypothesis tests are available:

- Type A: Test H0: all constraints with equalities ("=") active against HA: at least one inequality restriction (">") strictly true.
- Type B: Test H0: all constraints with inequalities (">") (including some equalities ("=")) active against HA: at least one restriction false (some equality constraints may be maintained).
- Type C: Test H0: at least one restriction false ("<") against HA: all constraints strikty true (">"). This test is based on the intersection-union principle (Silvapulle and Sen, 2005, chp 5.3). Note that, this test only makes sense in case of no equality constraints.
- Type global: equal to Type A but H0 contains additional equality constraints. This test is analogue to the global F-test in lm, where all coefficients but the intercept equal 0.

The null-distribution of hypothesis test Type C is based on a t-distribution (one-sided). Its power can be poor in case of many inequalty constraints. Its main role is to prevent wrong conclusions from significant results from hypothesis test Type A.

The exact finite sample distributions of the non-robust F-, score- and LR-test statistics based on restricted OLS estimates and normally distributed errors, are a mixture of F-distributions under the

null hypothesis (Wolak, 1987). For the robust tests, we found that the results based on these mixtures of F-distributions approximate the tail probabilities better than their asymptotic distributions.

Note that, in case of equality constraints only, the null-distribution of the (non-)robust F-test statistics are based on an F-distribution. The (non-)robust Wald- and (non-)robust score-test statistics are based on chi-square distributions.

If object is of class lm or rlm, the conTest function internally calls the restriktor function. Arguments for the restriktor function can be passed on via the .... Additional arguments for the conTest function can also passed on via the .... See for example conTestF for all available arguments.

## Value

An object of class conTest, for which a print is available. More specifically, it is a list with the following items:

CON	a list with useful information about the constraints.
Amat	constraints matrix.
bvec	vector of right-hand side elements.
meq	number of equality constraints.
meq.alt	same as input neq.alt.
iact	number of active constraints.
type	same as input.
test	same as input.
Ts	test-statistic value.
df.residual	the residual degrees of freedom.
pvalue	tail probability for Ts.
b.eqrestr	equality restricted regression coefficients. Only available for type = "A" and type = "global", else b.eqrestr = NULL.
b.unrestr	unrestricted regression coefficients.
b.restr	restricted regression coefficients.
b.restr.alt	restricted regression coefficients under HA if some equality constraints are main- tained.
Sigma	variance-covariance matrix of unrestricted model.
R2.org	unrestricted R-squared.
R2.reduced	restricted R-squared.
boot	same as input.
model.org	original model.

## Author(s)

Leonard Vanbrabant and Yves Rosseel

#### conTest

## References

Robertson, T., Wright, F.T. and Dykstra, R.L. (1988). Order Restricted Statistical Inference New York: Wiley.

Shapiro, A. (1988). Towards a unified theory of inequality-constrained testing in multivariate analysis. *International Statistical Review* **56**, 49–62.

Silvapulle, M. (1992a). Robust tests of inequality constraints and one-sided hypotheses in the linear model. *Biometrika*, **79**, 621–630.

Silvapulle, M. (1992b). Robust Wald-Type Tests of One-Sided Hypotheses in the Linear Model. *Journal of the American Statistical Association*, **87**, 156–161.

Silvapulle, M. and Silvapulle, P. (1995). A score test against one-sided alternatives. *American statistical association*, **90**, 342–349.

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Silvapulle, M. (1996) Robust bounded influence tests against one-sided hypotheses in general parametric models. *Statistics & probability letters*, **31**, 45–50.

Silvapulle, M.J. and Sen, P.K. (2005). Constrained Statistical Inference. Wiley, New York

Wolak, F. (1987). An exact test for multiple inequality and equality constraints in the linear regression model. *Journal of the American statistical association*, **82**, 782–793.

## See Also

quadprog, conTest

## Examples

```
## example 1:
# the data consist of ages (in months) at which an
# infant starts to walk alone.
# prepare data
DATA1 <- subset(ZelazoKolb1972, Group != "Control")</pre>
# fit unrestricted linear model
fit1.lm <- lm(Age ~ -1 + Group, data = DATA1)</pre>
# the variable names can be used to impose constraints on
# the corresponding regression parameters.
coef(fit1.lm)
# constraint syntax: assuming that the walking
# exercises would not have a negative effect of increasing the
# mean age at which a child starts to walk.
myConstraints1 <- ' GroupActive < GroupPassive;</pre>
                     GroupPassive < GroupNo '
conTest(fit1.lm, myConstraints1)
```

```
# another way is to first fit the restricted model
fit.restr1 <- restriktor(fit1.lm, constraints = myConstraints1)</pre>
conTest(fit.restr1)
## Not run:
  # Or in matrix notation.
  Amat1 <- rbind(c(-1, 0, 1),
                 c( 0, 1, -1))
  myRhs1 <- rep(0L, nrow(Amat1))</pre>
  myNeq1 <- 0
  conTest(fit1.lm, constraints = Amat1,
          rhs = myRhs1, neq = myNeq1)
## End(Not run)
## Artificial examples ##
# generate data
n <- 10
means <- c(1,2,1,3)
nm <- length(means)</pre>
group <- as.factor(rep(1:nm, each = n))</pre>
y <- rnorm(n * nm, rep(means, each = n))</pre>
DATA2 <- data.frame(y, group)</pre>
# fit unrestricted linear model
fit2.lm <- lm(y \sim -1 + group, data = DATA2)
coef(fit2.lm)
## example 2: increasing means
myConstraints2 <- ' group1 < group2</pre>
                    group2 < group3
                    group3 < group4 '
# compute F-test for hypothesis test Type A and compute the tail
# probability based on the parametric bootstrap. We only generate 9
# bootstrap samples in this example; in practice you may wish to
# use a much higher number.
conTest(fit2.lm, constraints = myConstraints2, type = "A",
        boot = "parametric", R = 9)
# or fit restricted linear model
fit2.con <- restriktor(fit2.lm, constraints = myConstraints2)</pre>
conTest(fit2.con)
## Not run:
  # increasing means in matrix notation.
  Amat2 <- rbind(c(-1, 1, 0, 0),
```

```
c( 0,-1, 1, 0),
                 c( 0, 0,-1, 1))
  myRhs2 <- rep(0L, nrow(Amat2))</pre>
  myNeq2 <- 0
  conTest(fit2.con, constraints = Amat2, rhs = myRhs2, neq = myNeq2,
          type = "A", boot = "parametric", R = 9)
## End(Not run)
## example 3: equality constraints only.
myConstraints3 <- ' group1 == group2</pre>
                    group2 == group3
                    group3 == group4 '
conTest(fit2.lm, constraints = myConstraints3)
# or
fit3.con <- restriktor(fit2.lm, constraints = myConstraints3)</pre>
conTest(fit3.con)
## example 4:
# combination of equality and inequality constraints.
myConstraints4 <- ' group1 == group2</pre>
                    group3 < group4 '
conTest(fit2.lm, constraints = myConstraints4, type = "B", neq.alt = 1)
# fit resticted model and compute model-based bootstrapped
# standard errors. We only generate 9 bootstrap samples in this
# example; in practice you may wish to use a much higher number.
# Note that, a warning message may be thrown because the number of
# bootstrap samples is too low.
fit4.con <- restriktor(fit2.lm, constraints = myConstraints4,</pre>
                       se = "boot.model.based", B = 9)
conTest(fit4.con, type = "B", neq.alt = 1)
## example 5:
# restriktor can also be used to define effects using the := operator
# and impose constraints on them. For example, is the
# average effect (AVE) larger than zero?
# generate data
n <- 30
b0 <- 10; b1 = 0.5; b2 = 1; b3 = 1.5
X <- c(rep(c(0), n/2), rep(c(1), n/2))
set.seed(90)
Z <- rnorm(n, 16, 5)
y <- b0 + b1*X + b2*Z + b3*X*Z + rnorm(n, 0, sd = 10)
DATA3 = data.frame(cbind(y, X, Z))
# fit linear model with interaction
```

conTest-methods *Methods for conTest* 

#### Description

Print function for objects of class conTest.

#### Usage

## S3 method for class 'conTest'
print(x, digits = max(3, getOption("digits") - 2), ...)

## Arguments

х	an object of class conTest.
digits	the number of significant digits to use when printing.
	no additional arguments for now.

## Examples

```
# unrestricted linear model for ages (in months) at which an
# infant starts to walk alone.
# prepare data
DATA <- subset(ZelazoKolb1972, Group != "Control")
# fit unrestricted linear model
fit.lm <- lm(Age ~ -1 + Group, data = DATA)
# restricted linear model with restrictions that the walking
# exercises would not have a negative effect of increasing the
# mean age at which a child starts to walk.
fit.con <- restriktor(fit.lm, constraints = "GroupActive < GroupPassive;
GroupPassive < GroupNo")</pre>
```

## conTestC

conTest(fit.con)

conTestC

one-sided t-test for iht

## Description

conTestC tests linear inequality restricted hypotheses for (robust) linear models by a one-sided ttest. This method is based on the union-intersection principle. It is called by the conTest function if all restrictions are equalities. For more information see details.

#### Usage

## S3 method for class 'restriktor'
conTestC(object, ...)

#### Arguments

object	an object of class restriktor.
	no additional arguments for now.

#### Details

Hypothesis test Type C:

• Test H0: at least one restriction false ("<") against HA: all constraints strikty true (">"). This test is based on the intersection-union principle. Note that, this test only makes sense in case of no equality constraints.

The null-distribution of hypothesis test Type C is based on a t-distribution (one-sided). Its power can be poor in case of many inequality constraints. Its main role is to prevent wrong conclusions from significant results from hypothesis test Type A.

## Value

An object of class conTest, for which a print is available. More specifically, it is a list with the following items:

CON	a list with useful information about the constraints.
Amat	constraints matrix.
bvec	vector of right-hand side elements.
meq	number of equality constraints.
test	same as input.
Ts	test-statistic value.

#### conTestC

df.residual	the residual degrees of freedom.
pvalue	tail probability for Ts.
b.unrestr	unrestricted regression coefficients.
b.restr	restricted regression coefficients.
Sigma	variance-covariance matrix of unrestricted model.
R2.org	unrestricted R-squared.
R2.reduced	restricted R-squared.
boot	"no", not used (yet).
model.org	original model.

## Author(s)

Leonard Vanbrabant and Yves Rosseel

## References

Silvapulle, M.J. and Sen, P.K. (2005, chapter 5.). *Constrained Statistical Inference*. Wiley, New York

## See Also

quadprog, conTest

## Examples

```
## example 1:
# the data consist of ages (in months) at which an
# infant starts to walk alone.
# prepare data
DATA1 <- subset(ZelazoKolb1972, Group != "Control")</pre>
# fit unrestricted linear model
fit1.lm <- lm(Age ~ -1 + Group, data = DATA1)</pre>
# the variable names can be used to impose constraints on
# the corresponding regression parameters.
coef(fit1.lm)
# constraint syntax: assuming that the walking
# exercises would not have a negative effect of increasing the
# mean age at which a child starts to walk.
myConstraints1 <- ' GroupActive < GroupPassive;</pre>
                    GroupPassive < GroupNo '
conTest(fit1.lm, myConstraints1, type = "C")
# another way is to first fit the restricted model
```

## conTestF

conTestF

F-bar test for iht

#### Description

conTestF tests linear equality and/or inequality restricted hypotheses for linear models by F-tests. It can be used directly and is called by the conTest function if test = "F".

#### Usage

```
## S3 method for class 'conLM'
conTestF(object, type = "A", neq.alt = 0,
            boot = "no", R = 9999, p.distr = rnorm,
            parallel = "no", ncpus = 1L, cl = NULL, seed = 1234,
            verbose = FALSE, control = NULL, ...)
## S3 method for class 'conRLM'
conTestF(object, type = "A", neq.alt = 0,
            boot = "no", R = 9999, p.distr = rnorm,
            parallel = "no", ncpus = 1L, cl = NULL, seed = 1234,
            verbose = FALSE, control = NULL, ...)
## S3 method for class 'conGLM'
conTestF(object, type = "A", neq.alt = 0,
            boot = "no", R = 9999, p.distr = rnorm,
            parallel = "no", ncpus = 1L, cl = NULL, seed = 1234,
            verbose = FALSE, control = NULL, ...)
## S3 method for class 'conGLM'
conTestF(object, type = "A", neq.alt = 0,
            boot = "no", R = 9999, p.distr = rnorm,
            parallel = "no", ncpus = 1L, cl = NULL, seed = 1234,
            verbose = FALSE, control = NULL, ...)
```

## Arguments

0	
object	an object of class conLM, conRLM or conGLM.
type	hypothesis test type "A", "B", "C", "global", or "summary" (default). See details for more information.
neq.alt	integer: number of equality constraints that are maintained under the alternative hypothesis (for hypothesis test type "B"), see example 3.
boot	the null-distribution of these test-statistics (except under type "C") takes the form of a mixture of F-distributions. The tail probabilities can be computed di- rectly via bootstrapping; if "parametric", the p-value is computed based on the parametric bootstrap. By default, samples are drawn from a normal distribution with mean zero and varance one. See p.distr for other distributional options. If "model.based", a model-based bootstrap method is used. Instead of com- puting the p-value via simulation, the p-value can also be computed using the chi-bar-square weights. If "no", the p-value is computed based on the weights obtained via simulation (mix.weights = "boot") or using the multivariate nor- mal distribution function (mix.weights = "pmvnorm"). Note that, these weights are already available in the restriktor objected and do not need to be estimated again. However, there are two exception for objects of class conRLM, namely for computing the p-value for the robust test = "Wald" and the robust "score". In these cases the weights need to be recalculated.
R	integer; number of bootstrap draws for boot. The default value is set to 9999.
p.distr	random generation distribution for the parametric bootstrap. For all available distributions see ?distributions. For example, if rnorm, samples are drawn from the normal distribution (default) with mean zero and variance one. If rt, samples are drawn from a t-distribution. If rchisq, samples are drawn from a chi-square distribution. The distributional parameters will be passed in via
parallel	the type of parallel operation to be used (if any). If missing, the default is set "no".
ncpus	integer: number of processes to be used in parallel operation: typically one would chose this to the number of available CPUs.
cl	an optional parallel or snow cluster for use if parallel = "snow". If not supplied, a cluster on the local machine is created for the duration of the conTest call.
seed	seed value. The default value is set to 1234.
verbose	logical; if TRUE, information is shown at each bootstrap draw.
control	a list of control arguments:
	<ul> <li>absval tolerance criterion for convergence (default = sqrt(.Machine\$double.eps)). Only used for model of class lm.</li> <li>maxit the maximum number of iterations for the optimizer (default = 10000). Only used for model of class mlm (not yet supported).</li> <li>tol numerical tolerance value. Estimates smaller than tol are set to 0.</li> </ul>
	additional arguments to be passed to the p.distr function.

## conTestF

## Details

The following hypothesis tests are available:

- Type A: Test H0: all constraints with equalities ("=") active against HA: at least one inequality restriction (">") strictly true.
- Type B: Test H0: all constraints with inequalities (">") (including some equalities ("=")) active against HA: at least one restriction false (some equality constraints may be maintained).
- Type C: Test H0: at least one restriction false ("<") against HA: all constraints strikty true (">"). This test is based on the intersection-union principle (Silvapulle and Sen, 2005, chp 5.3). Note that, this test only makes sense in case of no equality constraints.
- Type global: equal to Type A but H0 contains additional equality constraints. This test is analogue to the global F-test in lm, where all coefficients but the intercept equal 0.

The null-distribution of hypothesis test Type C is based on a t-distribution (one-sided). Its power can be poor in case of many inequalty constraints. Its main role is to prevent wrong conclusions from significant results from hypothesis test Type A.

The exact finite sample distributions of the non-robust F-, score- and LR-test statistics based on restricted OLS estimates and normally distributed errors, are a mixture of F-distributions under the null hypothesis (Wolak, 1987). In agreement with Silvapulle (1992), we found that the results based on these mixtures of F-distributions approximate the tail probabilities of the robust tests better than their asymptotic distributions. Therefore, all p-values for hypothesis test Type "A", "B" and "global" are computed based on mixtures of F-distributions.

Note that, in case of equality constraints only, the null-distribution of the (robust) F-test statistics is based on an F-distribution. The (robust) Wald- and (robust) score-test statistics are based on chi-square distributions.

#### Value

An object of class conTest, for which a print is available. More specifically, it is a list with the following items:

CON	a list with useful information about the constraints.
Amat	constraints matrix.
bvec	vector of right-hand side elements.
meq	number of equality constraints.
meq.alt	same as input neq.alt.
iact	number of active constraints.
type	same as input.
test	same as input.
Ts	test-statistic value.
df.residual	the residual degrees of freedom.
pvalue	tail probability for Ts.
b.eqrestr	equality restricted regression coefficients. Only available for type = "A" and type = "global", else b.eqrestr = NULL.

b.unrestr	unrestricted regression coefficients.
b.restr	restricted regression coefficients.
b.restr.alt	restricted regression coefficients under HA if some equality constraints are main- tained. Only available for type = "B" else b.restr.alt = NULL.
Sigma	variance-covariance matrix of unrestricted model.
R2.org	unrestricted R-squared, not available for objects of class conGLM.
R2.reduced	restricted R-squared, not available for objects of class conGLM.
boot	same as input.
model.org	original model.

#### Author(s)

Leonard Vanbrabant and Yves Rosseel

## References

Kudo, A. (1963) A multivariate analogue of the one-sided test. *Biometrika*, 50, 403–418.

Silvapulle, M. (1992a). Robust tests of inequality constraints and one-sided hypotheses in the linear model. *Biometrika*, **79**, 621–630.

Silvapulle, M. (1996) On an F-type statistic for testing one-sided hypotheses and computation of chi-bar-squared weights. *Statistics & probability letters*, **28**, 137–141.

Silvapulle, M.J. and Sen, P.K. (2005). Constrained Statistical Inference. Wiley, New York

Wolak, F. (1987). An exact test for multiple inequality and equality constraints in the linear regression model. *Journal of the American statistical association*, **82**, 782–793.

#### See Also

quadprog, conTest

## Examples

```
## example 1:
# the data consist of ages (in months) at which an
# infant starts to walk alone.
# prepare data
DATA1 <- subset(ZelazoKolb1972, Group != "Control")
# fit unrestricted linear model
fit1.lm <- lm(Age ~ -1 + Group, data = DATA1)
# the variable names can be used to impose constraints on
# the corresponding regression parameters.
coef(fit1.lm)
# constraint syntax: assuming that the walking
```

<sup>#</sup> exercises would not have a negative effect of increasing the

## conTestF

```
# mean age at which a child starts to walk.
myConstraints1 <- ' GroupActive < GroupPassive;</pre>
                    GroupPassive < GroupNo '</pre>
conTest(fit1.lm, myConstraints1)
# another way is to first fit the restricted model
fit.restr1 <- restriktor(fit1.lm, constraints = myConstraints1)</pre>
conTest(fit.restr1)
## Not run:
  # Or in matrix notation.
  Amat1 <- rbind(c(-1, 0, 1),
                 c( 0, 1, -1))
  myRhs1 <- rep(0L, nrow(Amat1))</pre>
  myNeq1 <- 0
  conTest(fit1.lm, constraints = Amat1,
          rhs = myRhs1, neq = myNeq1)
## End(Not run)
## Artificial examples ##
# generate data
n <- 10
means <- c(1,2,1,3)
nm <- length(means)</pre>
group <- as.factor(rep(1:nm, each = n))</pre>
y <- rnorm(n * nm, rep(means, each = n))</pre>
DATA2 <- data.frame(y, group)</pre>
# fit unrestricted linear model
fit2.lm <- lm(y \sim -1 + group, data = DATA2)
coef(fit2.lm)
## example 2: increasing means
myConstraints2 <- ' group1 < group2</pre>
                    group2 < group3</pre>
                    group3 < group4 '
# compute F-test for hypothesis test Type A and compute the tail
# probability based on the parametric bootstrap. We only generate 9
# bootstrap samples in this example; in practice you may wish to
# use a much higher number.
conTest(fit2.lm, constraints = myConstraints2, type = "A",
        boot = "parametric", R = 9)
```

# or fit restricted linear model

```
fit2.con <- restriktor(fit2.lm, constraints = myConstraints2)</pre>
conTest(fit2.con)
## Not run:
  # increasing means in matrix notation.
  Amat2 <- rbind(c(-1, 1, 0, 0),
                 c( 0,-1, 1, 0),
                 c( 0, 0,-1, 1))
  myRhs2 <- rep(0L, nrow(Amat2))</pre>
  myNeq2 <- 0
  conTest(fit2.con, constraints = Amat2, rhs = myRhs2, neq = myNeq2,
          type = "A", boot = "parametric", R = 9)
## End(Not run)
## example 3:
# combination of equality and inequality constraints.
myConstraints3 <- ' group1 == group2</pre>
                    group3 < group4 '
conTest(fit2.lm, constraints = myConstraints3, type = "B", neq.alt = 1)
# fit resticted model and compute model-based bootstrapped
# standard errors. We only generate 9 bootstrap samples in this
# example; in practice you may wish to use a much higher number.
# Note that, a warning message may be thrown because the number of
# bootstrap samples is too low.
fit3.con <- restriktor(fit2.lm, constraints = myConstraints3,</pre>
                       se = "boot.model.based", B = 9)
conTest(fit3.con, type = "B", neq.alt = 1)
## example 4:
# restriktor can also be used to define effects using the := operator
# and impose constraints on them. For example, is the
# average effect (AVE) larger than zero?
# generate data
n <- 30
b0 < -10; b1 = 0.5; b2 = 1; b3 = 1.5
X <- c(rep(c(0), n/2), rep(c(1), n/2))
set.seed(90)
Z <- rnorm(n, 16, 5)
y <- b0 + b1*X + b2*Z + b3*X*Z + rnorm(n, 0, sd = 10)
DATA3 = data.frame(cbind(y, X, Z))
# fit linear model with interaction
fit4.lm <- lm(y ~ X*Z, data = DATA3)</pre>
# constraint syntax
myConstraints4 <- ' AVE := X + 16.86137*X.Z;</pre>
                    AVE > 0 '
```

conTest(fit4.lm, constraints = myConstraints4)

```
conTestLRT
```

Likelihood-ratio-bar test for iht

## Description

conTestLRT tests linear equality and/or inequality restricted hypotheses for linear models by LR-tests. It can be used directly and is called by the conTest function if test = "LRT".

## Usage

```
## S3 method for class 'conLM'
conTestLRT(object, type = "A", neq.alt = 0,
            boot = "no", R = 9999, p.distr = rnorm,
            parallel = "no", ncpus = 1L, cl = NULL, seed = 1234,
            verbose = FALSE, control = NULL, ...)
## S3 method for class 'conGLM'
conTestLRT(object, type = "A", neq.alt = 0,
            boot = "no", R = 9999, p.distr = rnorm,
            parallel = "no", ncpus = 1L, cl = NULL, seed = 1234,
            verbose = FALSE, control = NULL, ...)
## S3 method for class 'conMLM'
conTestLRT(object, type = "A", neq.alt = 0,
            boot = "no", R = 9999, p.distr = rnorm,
            parallel = "no", ncpus = 1L, cl = NULL, seed = 1234,
            verbose = FALSE, control = NULL, ...)
## S3 method for class 'conMLM'
conTestLRT(object, type = "A", neq.alt = 0,
            boot = "no", R = 9999, p.distr = rnorm,
            parallel = "no", ncpus = 1L, cl = NULL, seed = 1234,
            verbose = FALSE, control = NULL, seed = 1234,
            verbose = FALSE, control = NULL, seed = 1234,
            verbose = FALSE, control = NULL, seed = 1234,
            verbose = FALSE, control = NULL, seed = 1234,
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            verbose = FALSE, control = NULL, seed = 1234,
            verbose = FALSE, control = NULL, seed = 1234,
            verbose = FALSE, control = NULL, seed = 1234,
            verbose = FALSE, control = NULL,
            verbose = FALSE,
            verbose = FALSE,
            verbose = FALSE,
```

## Arguments

object	an object of class conLM, conMLM or conGLM.
type	hypothesis test type "A", "B", "C", "global", or "summary" (default). See details for more information.
neq.alt	integer: number of equality constraints that are maintained under the alternative hypothesis (for hypothesis test type "B"), see example 3.
boot	the null-distribution of these test-statistics (except under type "C", see details) takes the form of a mixture of F-distributions. The tail probabilities can be computed directly via bootstrapping; if "parametric", the p-value is computed

	based on the parametric bootstrap. By default, samples are drawn from a normal distribution with mean zero and varance one. See p.distr for other distributional options. If "model.based", a model-based bootstrap method is used. Instead of computing the p-value via simulation, the p-value can also be computed using the chi-bar-square weights. If "no", the p-value is computed based on the weights obtained via simulation (mix.weights = "boot") or using the multivariate normal distribution function (mix.weights = "pmvnorm"). Note that, these weights are already available in the restriktor objected and do not need to be estimated again. However, there are two exception for objects of class conRLM, namely for computing the p-value for the robust test = "Wald" and the robust "score". In these cases the weights need to be recalculated.
R	integer; number of bootstrap draws for boot. The default value is set to 9999.
p.distr	random generation distribution for the parametric bootstrap. For all available distributions see ?distributions. For example, if rnorm, samples are drawn from the normal distribution (default) with mean zero and variance one. If rt, samples are drawn from a t-distribution. If rchisq, samples are drawn from a chi-square distribution. The random generation distributional parameters will be passed in via
parallel	the type of parallel operation to be used (if any). If missing, the default is set "no".
ncpus	integer: number of processes to be used in parallel operation: typically one would chose this to the number of available CPUs.
cl	an optional parallel or snow cluster for use if parallel = "snow". If not supplied, a cluster on the local machine is created for the duration of the conTest call.
seed	seed value. The default value is set to 1234.
verbose	logical; if TRUE, information is shown at each bootstrap draw.
control	a list of control arguments:
	<ul> <li>absval tolerance criterion for convergence (default = sqrt(.Machine\$double.eps)). Only used for model of class lm.</li> <li>maxit the maximum number of iterations for the optimizer (default = 10000). Only used for model of class mlm (not yet supported).</li> <li>tol numerical tolerance value. Estimates smaller than tol are set to 0. additional arguments to be passed to the p.distr function.</li> </ul>
•••	additional arguments to be passed to the pluisti function.

## Details

The following hypothesis tests are available:

- Type A: Test H0: all constraints with equalities ("=") active against HA: at least one inequality restriction (">") strictly true.
- Type B: Test H0: all constraints with inequalities (">") (including some equalities ("=")) active against HA: at least one restriction false (some equality constraints may be maintained).
- Type C: Test H0: at least one restriction false ("<") against HA: all constraints strikty true (">"). This test is based on the intersection-union principle (Silvapulle and Sen, 2005, chp 5.3). Note that, this test only makes sense in case of no equality constraints.

## conTestLRT

• Type global: equal to Type A but H0 contains additional equality constraints. This test is analogue to the global F-test in lm, where all coefficients but the intercept equal 0.

The null-distribution of hypothesis test Type C is based on a t-distribution (one-sided). Its power can be poor in case of many inequalty constraints. Its main role is to prevent wrong conclusions from significant results from hypothesis test Type A.

The exact finite sample distributions of the non-robust F-, score- and LR-test statistics based on restricted OLS estimates and normally distributed errors, are a mixture of F-distributions under the null hypothesis (Wolak, 1987). In agreement with Silvapulle (1992), we found that the results based on these mixtures of F-distributions approximate the tail probabilities of the robust tests better than their asymptotic distributions. Therefore, all p-values for hypothesis test Type "A", "B" and "global" are computed based on mixtures of F-distributions.

## Value

An object of class conTest, for which a print is available. More specifically, it is a list with the following items:

a list with useful information about the constraints.
constraints matrix.
vector of right-hand side elements.
number of equality constraints.
same as input neq.alt.
number of active constraints.
same as input.
same as input.
test-statistic value.
the residual degrees of freedom.
tail probability for Ts.
equality restricted regression coefficients. Only available for type = "A" and type = "global", else b.eqrestr = NULL.
unrestricted regression coefficients.
restricted regression coefficients.
restricted regression coefficients under HA if some equality constraints are main- tained. Only available for type = "B" else b_restr_alt = NULL.
variance-covariance matrix of unrestricted model.
unrestricted R-squared, not available for objects of class conGLM.
restricted R-squared, not available for objects of class conGLM.
same as input.
original model.

## Author(s)

Leonard Vanbrabant and Yves Rosseel

## References

Silvapulle, M.J. and Sen, P.K. (2005). Constrained Statistical Inference. Wiley, New York

#### See Also

quadprog, conTest

## Examples

```
## example 1:
# the data consist of ages (in months) at which an
# infant starts to walk alone.
# prepare data
DATA1 <- subset(ZelazoKolb1972, Group != "Control")</pre>
# fit unrestricted linear model
fit1_lm <- lm(Age ~ -1 + Group, data = DATA1)</pre>
# the variable names can be used to impose constraints on
# the corresponding regression parameters.
coef(fit1_lm)
# constraint syntax: assuming that the walking
# exercises would not have a negative effect of increasing the
# mean age at which a child starts to walk.
myConstraints1 <- ' GroupActive < GroupPassive;</pre>
                   GroupPassive < GroupNo '
conTest(fit1_lm, myConstraints1, test = "LRT")
# another way is to first fit the restricted model
fit_restr1 <- restriktor(fit1_lm, constraints = myConstraints1)</pre>
conTest(fit_restr1, test = "LRT")
## Not run:
 # Or in matrix notation.
 Amat1 <- rbind(c(-1, 0, 1),
                c( 0, 1, -1))
 myRhs1 <- rep(0L, nrow(Amat1))</pre>
 myNeq1 <- 0
 conTest(fit1_lm, constraints = Amat1, test = "LRT",
         rhs = myRhs1, neq = myNeq1)
## End(Not run)
## Artificial examples ##
```

## conTestLRT

```
# generate data
n <- 10
means <- c(1,2,1,3)
nm <- length(means)</pre>
group <- as.factor(rep(1:nm, each = n))</pre>
y <- rnorm(n * nm, rep(means, each = n))</pre>
DATA2 <- data.frame(y, group)</pre>
# fit unrestricted linear model
fit2_lm <- lm(y \sim -1 + group, data = DATA2)
coef(fit2_lm)
## example 2: increasing means
myConstraints2 <- ' group1 < group2</pre>
                     group2 < group3
                     group3 < group4 '
# compute F-test for hypothesis test Type A and compute the tail
# probability based on the parametric bootstrap. We only generate 9
# bootstrap samples in this example; in practice you may wish to
# use a much higher number.
conTest(fit2_lm, constraints = myConstraints2, type = "A", test = "LRT",
        boot = "parametric", R = 9)
# or fit restricted linear model
fit2_con <- restriktor(fit2_lm, constraints = myConstraints2)</pre>
conTest(fit2_con, test = "LRT")
## Not run:
  # increasing means in matrix notation.
  Amat2 <- rbind(c(-1, 1, 0, 0),
                 c( 0,-1, 1, 0),
                 c( 0, 0,-1, 1))
  myRhs2 <- rep(0L, nrow(Amat2))</pre>
  myNeq2 <- 0
  conTest(fit2_con, constraints = Amat2, rhs = myRhs2, neq = myNeq2,
          type = "A", test = "LRT", boot = "parametric", R = 9)
## End(Not run)
## example 3:
# combination of equality and inequality constraints.
myConstraints3 <- ' group1 == group2</pre>
                    group3 < group4 '
conTest(fit2_lm, constraints = myConstraints3, type = "B",
        test = "LRT", neq.alt = 1)
# fit resticted model and compute model-based bootstrapped
# standard errors. We only generate 9 bootstrap samples in this
```

```
# example; in practice you may wish to use a much higher number.
# Note that, a warning message may be thrown because the number of
# bootstrap samples is too low.
fit3_con <- restriktor(fit2_lm, constraints = myConstraints3,</pre>
                       se = "boot.model.based", B = 9)
conTest(fit3_con, type = "B", test = "LRT", neq.alt = 1)
## example 4:
# restriktor can also be used to define effects using the := operator
# and impose constraints on them. For example, is the
# average effect (AVE) larger than zero?
# generate data
n <- 30
b0 <- 10; b1 = 0.5; b2 = 1; b3 = 1.5
X <- c(rep(c(0), n/2), rep(c(1), n/2))
set.seed(90)
Z <- rnorm(n, 16, 5)
y <- b0 + b1*X + b2*Z + b3*X*Z + rnorm(n, 0, sd = 10)
DATA3 = data.frame(cbind(y, X, Z))
# fit linear model with interaction
fit4_lm <- lm(y ~ X*Z, data = DATA3)</pre>
# constraint syntax
myConstraints4 <- ' AVE := X + 16.86137*X.Z;</pre>
                    AVE > 0 '
conTest(fit4_lm, constraints = myConstraints4, test = "LRT")
# or
fit4_con <- restriktor(fit4_lm, constraints = ' AVE := X + 16.86137*X.Z;</pre>
                                                 AVE > 0 ')
conTest(fit4_con, test = "LRT")
```

conTestScore Score-bar test for iht

#### Description

conTestScore tests linear equality and/or inequality restricted hypotheses for (robust) linear models by score-tests. It can be used directly and is called by the conTest function if test = "score".

## Usage

```
verbose = FALSE, control = NULL, ...)
## S3 method for class 'conRLM'
conTestScore(object, type = "A", neq.alt = 0,
            boot = "no", R = 9999, p.distr = rnorm,
            parallel = "no", ncpus = 1L, cl = NULL, seed = 1234,
            verbose = FALSE, control = NULL, ...)
## S3 method for class 'conGLM'
conTestScore(object, type = "A", neq.alt = 0,
            boot = "no", R = 9999, p.distr = rnorm,
            parallel = "no", ncpus = 1L, cl = NULL, seed = 1234,
            verbose = FALSE, control = NULL, ...)
```

## Arguments

-	
object	an object of class conLM, conRLM or conGLM.
type	hypothesis test type "A", "B", "C", "global", or "summary" (default). See details for more information.
neq.alt	integer: number of equality constraints that are maintained under the alternative hypothesis (for hypothesis test type "B"), see example 3.
boot	the null-distribution of these test-statistics (except under type "C", see details) takes the form of a mixture of F-distributions. The tail probabilities can be computed directly via bootstrapping; if "parametric", the p-value is computed based on the parametric bootstrap. By default, samples are drawn from a normal distribution with mean zero and varance one. See p.distr for other distributional options. If "model.based", a model-based bootstrap method is used. Instead of computing the p-value via simulation, the p-value can also be computed using the chi-bar-square weights. If "no", the p-value is computed based on the weights obtained via simulation (mix.weights = "boot") or using the multivariate normal distribution function (mix.weights = "pmvnorm"). Note that, these weights are already available in the restriktor objected and do not need to be estimated again. However, there are two exception for objects of class conRLM, namely for computing the p-value for the robust test = "Wald" and the robust "score". In these cases the weights need to be recalculated.
R	integer; number of bootstrap draws for boot. The default value is set to 9999.
p.distr	random generation distribution for the parametric bootstrap. For all available distributions see ?distributions. For example, if rnorm, samples are drawn from the normal distribution (default) with mean zero and variance one. If rt, samples are drawn from a t-distribution. If rchisq, samples are drawn from a chi-square distribution. The random generation distributional parameters will be passed in via
parallel	the type of parallel operation to be used (if any). If missing, the default is set "no".
ncpus	integer: number of processes to be used in parallel operation: typically one would chose this to the number of available CPUs.

cl	an optional parallel or snow cluster for use if parallel = "snow". If not supplied, a cluster on the local machine is created for the duration of the conTest call.
seed	seed value. The default value is set to 1234.
verbose	logical; if TRUE, information is shown at each bootstrap draw.
control	a list of control arguments:
	<ul> <li>absval tolerance criterion for convergence (default = sqrt(.Machine\$double.eps)).</li> <li>Only used for model of class lm.</li> </ul>
	• maxit the maximum number of iterations for the optimizer (default = 10000). Only used for model of class mlm (not yet supported).
	• tol numerical tolerance value. Estimates smaller than tol are set to 0.
	additional arguments to be passed to the p.distr function.

#### Details

The following hypothesis tests are available:

- Type A: Test H0: all constraints with equalities ("=") active against HA: at least one inequality restriction (">") strictly true.
- Type B: Test H0: all constraints with inequalities (">") (including some equalities ("=")) active against HA: at least one restriction false (some equality constraints may be maintained).
- Type C: Test H0: at least one restriction false ("<") against HA: all constraints strikty true (">"). This test is based on the intersection-union principle (Silvapulle and Sen, 2005, chp 5.3). Note that, this test only makes sense in case of no equality constraints.
- Type global: equal to Type A but H0 contains additional equality constraints. This test is analogue to the global F-test in lm, where all coefficients but the intercept equal 0.

The null-distribution of hypothesis test Type C is based on a t-distribution (one-sided). Its power can be poor in case of many inequality constraints. Its main role is to prevent wrong conclusions from significant results from hypothesis test Type A.

The exact finite sample distributions of the non-robust F-, score- and LR-test statistics based on restricted OLS estimates and normally distributed errors, are a mixture of F-distributions under the null hypothesis (Wolak, 1987). In agreement with Silvapulle (1992), we found that the results based on these mixtures of F-distributions approximate the tail probabilities of the robust tests better than their asymptotic distributions. Therefore, all p-values for hypothesis test Type "A", "B" and "global" are computed based on mixtures of F-distributions.

#### Value

An object of class conTest, for which a print is available. More specifically, it is a list with the following items:

CON	a list with useful information about the constraints.
Amat	constraints matrix.
bvec	vector of right-hand side elements.
meq	number of equality constraints.
meq.alt	same as input neq.alt.

## conTestScore

iact	number of active constraints.
type	same as input.
test	same as input.
Ts	test-statistic value.
df.residual	the residual degrees of freedom.
pvalue	tail probability for Ts.
b.eqrestr	equality restricted regression coefficients. Only available for type = "A" and type = "global", else b.eqrestr = NULL.
b.unrestr	unrestricted regression coefficients.
b.restr	restricted regression coefficients.
b.restr.alt	restricted regression coefficients under HA if some equality constraints are main- tained. Only available for type = "B" else b.restr.alt = NULL.
Sigma	variance-covariance matrix of unrestricted model.
R2.org	unrestricted R-squared, not available for objects of class conGLM.
R2.reduced	restricted R-squared, not available for objects of class conGLM.
boot	same as input.
model.org	original model.

## Author(s)

Leonard Vanbrabant and Yves Rosseel

## References

Silvapulle, M. and Silvapulle, P. (1995). A score test against one-sided alternatives. *American statistical association*, **90**, 342–349.

Silvapulle, M. (1996) Robust bounded influence tests against one-sided hypotheses in general parametric models. *Statistics & probability letters*, **31**, 45–50.

Silvapulle, M.J. and Sen, P.K. (2005). Constrained Statistical Inference. Wiley, New York

## See Also

quadprog, conTest

## Examples

```
## example 1:
# the data consist of ages (in months) at which an
# infant starts to walk alone.
# prepare data
DATA1 <- subset(ZelazoKolb1972, Group != "Control")
# fit unrestricted linear model
fit1.lm <- lm(Age ~ -1 + Group, data = DATA1)</pre>
```

```
# the variable names can be used to impose constraints on
# the corresponding regression parameters.
coef(fit1.lm)
# constraint syntax: assuming that the walking
# exercises would not have a negative effect of increasing the
# mean age at which a child starts to walk.
myConstraints1 <- ' GroupActive < GroupPassive;</pre>
                    GroupPassive < GroupNo '</pre>
conTest(fit1.lm, myConstraints1, test = "score")
# another way is to first fit the restricted model
fit.restr1 <- restriktor(fit1.lm, constraints = myConstraints1)</pre>
conTest(fit.restr1, test = "score")
## Not run:
  # Or in matrix notation.
  Amat1 <- rbind(c(-1, 0, 1),
                 c( 0, 1, -1))
  myRhs1 <- rep(0L, nrow(Amat1))</pre>
  myNeq1 <- 0
  conTest(fit1.lm, constraints = Amat1, test = "score",
          rhs = myRhs1, neq = myNeq1)
## End(Not run)
## Artificial examples ##
# generate data
n <- 10
means <- c(1,2,1,3)
nm <- length(means)</pre>
group <- as.factor(rep(1:nm, each = n))</pre>
y <- rnorm(n * nm, rep(means, each = n))</pre>
DATA2 <- data.frame(y, group)</pre>
# fit unrestricted linear model
fit2.lm <- lm(y \sim -1 + group, data = DATA2)
coef(fit2.lm)
## example 2: increasing means
myConstraints2 <- ' group1 < group2</pre>
                    group2 < group3
                    group3 < group4 '
```

# compute F-test for hypothesis test Type A and compute the tail
# probability based on the parametric bootstrap. We only generate 9

```
# bootstrap samples in this example; in practice you may wish to
# use a much higher number.
conTest(fit2.lm, constraints = myConstraints2, type = "A", test = "score",
        boot = "parametric", R = 9)
# or fit restricted linear model
fit2.con <- restriktor(fit2.lm, constraints = myConstraints2)</pre>
conTest(fit2.con, test = "score")
## Not run:
  # increasing means in matrix notation.
  Amat2 <- rbind(c(-1, 1, 0, 0),
                 c( 0,-1, 1, 0),
                 c( 0, 0,-1, 1))
  myRhs2 <- rep(0L, nrow(Amat2))</pre>
  myNeq2 <- 0
  conTest(fit2.con, constraints = Amat2, rhs = myRhs2, neq = myNeq2,
          type = "A", test = "score", boot = "parametric", R = 9)
## End(Not run)
## example 3:
# combination of equality and inequality constraints.
myConstraints3 <- ' group1 == group2</pre>
                    group3 < group4 '
conTest(fit2.lm, constraints = myConstraints3, type = "B", test = "score", neq.alt = 1)
# fit resticted model and compute model-based bootstrapped
# standard errors. We only generate 9 bootstrap samples in this
# example; in practice you may wish to use a much higher number.
# Note that, a warning message may be thrown because the number of
# bootstrap samples is too low.
fit3.con <- restriktor(fit2.lm, constraints = myConstraints3,</pre>
                       se = "boot.model.based", B = 9)
conTest(fit3.con, type = "B", test = "score", neq.alt = 1)
## example 4:
# restriktor can also be used to define effects using the := operator
# and impose constraints on them. For example, is the
# average effect (AVE) larger than zero?
# generate data
n <- 30
b0 < -10; b1 = 0.5; b2 = 1; b3 = 1.5
X <- c(rep(c(0), n/2), rep(c(1), n/2))
set.seed(90)
Z <- rnorm(n, 16, 5)
y <- b0 + b1*X + b2*Z + b3*X*Z + rnorm(n, 0, sd = 10)
DATA3 = data.frame(cbind(y, X, Z))
```

conTestWald

Wald-bar test for robust iht

## Description

conTestWald tests linear equality and/or inequality restricted hypotheses for linear models by Wald-tests. It can be used directly and is called by the conTest function if test = "Wald".

## Usage

## Arguments

object	an object of class conRLM.
type	hypothesis test type "A", "B", "C", "global", or "summary" (default). See details for more information.
neq.alt	integer: number of equality constraints that are maintained under the alternative hypothesis (for hypothesis test type "B"), see example 3.
boot	the null-distribution of these test-statistics (except under type "C", see details) takes the form of a mixture of F-distributions. The tail probabilities can be computed directly via bootstrapping; if "parametric", the p-value is computed based on the parametric bootstrap. By default, samples are drawn from a normal distribution with mean zero and varance one. See p.distr for other distributional options. If "model.based", a model-based bootstrap method is used. Instead of computing the p-value via simulation, the p-value can also be computed using the chi-bar-square weights. If "no", the p-value is computed based on

	the weights obtained via simulation (mix.weights = "boot") or using the mul- tivariate normal distribution function (mix.weights = "pmvnorm"). Note that, these weights are already available in the restriktor objected and do not need to be estimated again. However, there are two exception for objects of class conRLM, namely for computing the p-value for the robust test = "Wald" and the robust "score". In these cases the weights need to be recalculated.
R	integer; number of bootstrap draws for boot. The default value is set to 9999.
p.distr	random generation distribution for the parametric bootstrap. For all available distributions see ?distributions. For example, if rnorm, samples are drawn from the normal distribution (default) with mean zero and variance one. If rt, samples are drawn from a t-distribution. If rchisq, samples are drawn from a chi-square distribution. The random generation distributional parameters will be passed in via
parallel	the type of parallel operation to be used (if any). If missing, the default is set "no".
ncpus	integer: number of processes to be used in parallel operation: typically one would chose this to the number of available CPUs.
cl	an optional parallel or snow cluster for use if parallel = "snow". If not supplied, a cluster on the local machine is created for the duration of the conTest call.
seed	seed value. The default value is set to 1234.
verbose	logical; if TRUE, information is shown at each bootstrap draw.
control	a list of control arguments:
	<ul> <li>absval tolerance criterion for convergence (default = sqrt(.Machine\$double.eps)).</li> <li>Only used for model of class lm.</li> </ul>
	<ul> <li>maxit the maximum number of iterations for the optimizer (default = 10000).</li> <li>Only used for model of class mlm (not yet supported).</li> </ul>
	• tol numerical tolerance value. Estimates smaller than tol are set to 0.
	additional arguments to be passed to the p.distr function.

## Details

The following hypothesis tests are available:

- Type A: Test H0: all constraints with equalities ("=") active against HA: at least one inequality restriction (">") strictly true.
- Type B: Test H0: all constraints with inequalities (">") (including some equalities ("=")) active against HA: at least one restriction false (some equality constraints may be maintained).
- Type C: Test H0: at least one restriction false ("<") against HA: all constraints strikty true (">"). This test is based on the intersection-union principle (Silvapulle and Sen, 2005, chp 5.3). Note that, this test only makes sense in case of no equality constraints.
- Type global: equal to Type A but H0 contains additional equality constraints. This test is analogue to the global F-test in lm, where all coefficients but the intercept equal 0.

The null-distribution of hypothesis test Type C is based on a t-distribution (one-sided). Its power can be poor in case of many inequalty constraints. Its main role is to prevent wrong conclusions from significant results from hypothesis test Type A.

The exact finite sample distributions of the non-robust F-, score- and LR-test statistics based on restricted OLS estimates and normally distributed errors, are a mixture of F-distributions under the null hypothesis (Wolak, 1987). In agreement with Silvapulle (1992), we found that the results based on these mixtures of F-distributions approximate the tail probabilities of the robust tests better than their asymptotic distributions. Therefore, all p-values for hypothesis test Type "A", "B" and "global" are computed based on mixtures of F-distributions.

#### Value

An object of class conTest, for which a print is available. More specifically, it is a list with the following items:

CON	a list with useful information about the constraints.
Amat	constraints matrix.
bvec	vector of right-hand side elements.
meq	number of equality constraints.
meq.alt	same as input neq.alt.
iact	number of active constraints.
type	same as input.
test	same as input.
Ts	test-statistic value.
df.residual	the residual degrees of freedom.
pvalue	tail probability for Ts.
b.eqrestr	equality restricted regression coefficients. Only available for type = "A" and type = "global", else b.eqrestr = NULL.
b.unrestr	unrestricted regression coefficients.
b.restr	restricted regression coefficients.
b.restr.alt	restricted regression coefficients under HA if some equality constraints are main- tained. Only available for type = "B" else b.restr.alt = NULL.
Sigma	variance-covariance matrix of unrestricted model.
R2.org	unrestricted R-squared, not available for objects of class conGLM.
R2.reduced	restricted R-squared, not available for objects of class conGLM.
boot	same as input.
model.org	original model.

## Author(s)

Leonard Vanbrabant and Yves Rosseel

#### References

Silvapulle, M. (1992b). Robust Wald-Type Tests of One-Sided Hypotheses in the Linear Model. *Journal of the American Statistical Association*, **87**, 156–161.

Silvapulle, M. (1996) Robust bounded influence tests against one-sided hypotheses in general parametric models. *Statistics & probability letters*, **31**, 45–50.

Silvapulle, M.J. and Sen, P.K. (2005). Constrained Statistical Inference. Wiley, New York

## conTestWald

## See Also

quadprog, conTest

## Examples

```
library(MASS)
## example 1:
# the data consist of ages (in months) at which an
# infant starts to walk alone.
# prepare data
DATA1 <- subset(ZelazoKolb1972, Group != "Control")</pre>
# fit unrestricted robust linear model
fit1.rlm <- rlm(Age ~ -1 + Group, data = DATA1, method = "MM")</pre>
# the variable names can be used to impose constraints on
# the corresponding regression parameters.
coef(fit1.rlm)
# constraint syntax: assuming that the walking
# exercises would not have a negative effect of increasing the
# mean age at which a child starts to walk.
myConstraints1 <- ' GroupActive < GroupPassive;</pre>
                   GroupPassive < GroupNo '</pre>
conTest(fit1.rlm, myConstraints1, test = "Wald")
# another way is to first fit the restricted model
fit.restr1 <- restriktor(fit1.rlm, constraints = myConstraints1)</pre>
conTest(fit.restr1, test = "Wald")
## Not run:
  # Or in matrix notation.
  Amat1 <- rbind(c(-1, 0, 1),
                 c( 0, 1, -1))
  myRhs1 <- rep(0L, nrow(Amat1))</pre>
  myNeq1 <- 0
  conTest(fit1.rlm, constraints = Amat1, test = "Wald",
         rhs = myRhs1, neq = myNeq1)
## End(Not run)
## Artificial examples ##
# generate data
n <- 30
means <- c(1,2,1,3)
```

```
group <- as.factor(rep(1:nm, each = n))</pre>
y <- rnorm(n * nm, rep(means, each = n))</pre>
DATA2 <- data.frame(y, group)</pre>
# fit unrestricted robust linear model
fit2.rlm <- rlm(y ~ -1 + group, data = DATA2, method = "MM")</pre>
coef(fit2.rlm)
## example 2: increasing means
myConstraints2 <- ' group1 < group2</pre>
                     group2 < group3
                     group3 < group4 '
# compute Wald-test for hypothesis test Type A and compute the tail
# probability based on the parametric bootstrap. We only generate 9
# bootstrap samples in this example; in practice you may wish to
# use a much higher number.
conTest(fit2.rlm, constraints = myConstraints2, type = "A",
        test = "Wald", boot = "parametric", R = 9)
# or fit restricted robust linear model
fit2.con <- restriktor(fit2.rlm, constraints = myConstraints2)</pre>
conTest(fit2.con, test = "Wald")
## Not run:
  # increasing means in matrix notation.
  Amat2 <- rbind(c(-1, 1, 0, 0),
                 c( 0,-1, 1, 0),
                 c( 0, 0,-1, 1))
  myRhs2 <- rep(0L, nrow(Amat2))</pre>
  myNeq2 <- 0
  conTest(fit2.con, constraints = Amat2, rhs = myRhs2, neq = myNeq2,
          type = "A", test = "Wald", boot = "parametric", R = 9)
## End(Not run)
## example 3:
# combination of equality and inequality constraints.
myConstraints3 <- ' group1 == group2</pre>
                    group3 < group4 '
conTest(fit2.rlm, constraints = myConstraints3, type = "B", test = "Wald", neq.alt = 1)
# fit robust resticted model and compute model-based bootstrapped
# standard errors. We only generate 9 bootstrap samples in this
# example; in practice you may wish to use a much higher number.
# Note that, a warning message may be thrown because the number of
# bootstrap samples is too low.
fit3.con <- restriktor(fit2.rlm, constraints = myConstraints3,</pre>
```

36

nm <- length(means)</pre>

```
se = "boot.model.based", B = 9)
conTest(fit3.con, type = "B", test = "Wald", neq.alt = 1)
## example 4:
# restriktor can also be used to define effects using the := operator
# and impose constraints on them. For example, is the
# average effect (AVE) larger than zero?
# generate data
n <- 30
b0 <- 10; b1 = 0.5; b2 = 1; b3 = 1.5
X <- c(rep(c(0), n/2), rep(c(1), n/2))
set.seed(90)
Z <- rnorm(n, 16, 5)
y <- b0 + b1*X + b2*Z + b3*X*Z + rnorm(n, 0, sd = 10)
DATA3 = data.frame(cbind(y, X, Z))
# fit linear model with interaction
fit3.rlm <- rlm(y ~ X*Z, data = DATA3, method = "MM")</pre>
# constraint syntax
myConstraints4 <- ' AVE := X + 16.86137*X.Z;</pre>
                    AVE > 0 '
conTest(fit3.rlm, constraints = myConstraints4, test = "Wald")
# or
fit3.con <- restriktor(fit3.rlm, constraints = ' AVE := X + 16.86137*X.Z;</pre>
                                                  AVE > 0 ')
conTest(fit3.con, test = "Wald")
```

conTest\_ceq

Tests for iht with equality constraints only

#### Description

conTest\_ceq tests linear equality restricted hypotheses for (robust) linear models by F-, Wald-, and score-tests. It can be used directly and is called by the conTest function if all restrictions are equalities.

#### Usage

# Arguments

object	an object of class conLM, conRLM or conGLM.
test	test statistic; for information about the null-distribution see details.
	<ul> <li>for object of class lm and glm; if "F" (default), the classical F-statistic is computed. If "Wald", the classical Wald-statistic is computed. If "score", the classical score test statistic is computed.</li> <li>for object of class rlm; if "F" (default), a robust likelihood ratio type test statistic (Silvapulle, 1992a) is computed. If "Wald", a robust Wald test statistic (Silvapulle, 1992b) is computed. If "score", a score test statistic (Silvapulle, 1996) is computed.</li> </ul>
boot	if "parametric", the p-value is computed based on the parametric bootstrap. See p.distr for available distributions. If "model.based", a model-based bootstrap method is used. Model-based bootstrapping is not supported for the conGLM object yet.
R	integer; number of bootstrap draws for boot. The default value is set to 9999.
p.distr	the p.distr function is specified by this function. For all available distributions see ?distributions. For example, if rnorm, samples are drawn from the normal distribution (default) with mean zero and variance one. If rt, samples are drawn from a t-distribution. If rchisq, samples are drawn from a chi-square distribution. The distributional parameters will be passed in via
parallel	the type of parallel operation to be used (if any). If missing, the default is set "no".
ncpus	integer: number of processes to be used in parallel operation: typically one would chose this to the number of available CPUs.
cl	an optional parallel or snow cluster for use if parallel = "snow". If not supplied, a cluster on the local machine is created for the duration of the conTest call.
seed	seed value. The default value is set to 1234.
verbose	logical; if TRUE, information is shown at each bootstrap draw.
	additional arguments to be passed to the p.distr function.

# Value

An object of class conTest, for which a print is available. More specifically, it is a list with the following items:

CON a list with useful information about the constraints.

#### conTest\_ceq

Amat	constraints matrix.
bvec	vector of right-hand side elements.
meq	number of equality constraints.
test	same as input.
Ts	test-statistic value.
df.residual	the residual degrees of freedom.
pvalue	tail probability for Ts.
b_unrestr	unrestricted regression coefficients.
b_restr	restricted regression coefficients.
R2_org	unrestricted R-squared.
R2_reduced	restricted R-squared.

## Author(s)

Leonard Vanbrabant and Yves Rosseel

## References

Silvapulle, M. (1992a). Robust tests of inequality constraints and one-sided hypotheses in the linear model. *Biometrika*, **79**, 621–630.

Silvapulle, M. (1996) Robust bounded influence tests against one-sided hypotheses in general parametric models. *Statistics & probability letters*, **31**, 45–50.

Silvapulle, M. (1992b). Robust Wald-Type Tests of One-Sided Hypotheses in the Linear Model. *Journal of the American Statistical Association*, **87**, 156–161.

Silvapulle, M. (1996) Robust bounded influence tests against one-sided hypotheses in general parametric models. *Statistics & probability letters*, **31**, 45–50.

## See Also

quadprog, conTest

## Examples

```
## example 1:
# the data consist of ages (in months) at which an
# infant starts to walk alone.
# prepare data
DATA1 <- subset(ZelazoKolb1972, Group != "Control")
# fit unrestricted linear model
fit1.lm <- lm(Age ~ -1 + Group, data = DATA1)
# the variable names can be used to impose constraints on
# the corresponding regression parameters.
coef(fit1.lm)
```

```
# constraint syntax: assuming that the walking
# exercises would not have a negative effect of increasing the
# mean age at which a child starts to walk.
myConstraints1 <- ' GroupActive == GroupPassive;</pre>
                    GroupPassive == GroupNo '
conTest(fit1.lm, myConstraints1)
# another way is to first fit the restricted model
fit_restr1 <- restriktor(fit1.lm, constraints = myConstraints1)</pre>
conTest(fit_restr1)
## Not run:
  # Or in matrix notation.
  Amat1 <- rbind(c(-1, 0, 1),
                 c( 0, 1, -1))
  myRhs1 <- rep(0L, nrow(Amat1))</pre>
  myNeq1 < -2
  conTest(fit1.lm, constraints = Amat1,
          rhs = myRhs1, neq = myNeq1)
## End(Not run)
```

conTest\_summary function for computing all available hypothesis tests

## Description

conTest\_summary computes all available hypothesis tests and returns and object of class conTest for which a print function is available. The conTest\_summary can be used directly and is called by the conTest function if type = "summary".

## Usage

```
## S3 method for class 'restriktor'
conTest_summary(object, test = "F", ...)
```

#### Arguments

object	an object of class restriktor.
test	test statistic; for information about the null-distribution see details.

• for object of class lm; if "F" (default), the classical F-statistic is computed. If "Wald", the classical Wald-statistic is computed. If "score", the classical score test statistic is computed.
• for object of class rlm; if "F" (default), a robust likelihood ratio type test statistic (Silvapulle, 1992a) is computed. If "Wald", a robust Wald test statistic (Silvapulle, 1992b) is computed. If "score", a score test statistic (Silvapulle, 1996) is computed.
 the same arguments as passed to the conTest function, except for type, of course.

## Value

An object of class conTest, for which a print is available. More specifically, it is a list with the following items:

CON	a list with useful information about the constraints.
Amat	constraints matrix.
bvec	vector of right-hand side elements.
meq	number of equality constraints.
meq.alt	same as input neq.alt.
iact	number of active constraints.
type	same as input.
test	same as input.
Ts	test-statistic value.
df.residual	the residual degrees of freedom.
pvalue	tail probability for Ts.
b.eqrestr	equality restricted regression coefficients. Only available for type = "A" and type = "global", else b.eqrestr = NULL.
b.unrestr	unrestricted regression coefficients.
b.restr	restricted regression coefficients.
b.restr.alt	restricted regression coefficients under HA if some equality constraints are main- tained.
Sigma	variance-covariance matrix of unrestricted model.
R2.org	unrestricted R-squared.
R2.reduced	restricted R-squared.
boot	same as input.
model.org	original model.

# Author(s)

Leonard Vanbrabant and Yves Rosseel

#### References

Shapiro, A. (1988). Towards a unified theory of inequality-constrained testing in multivariate analysis. *International Statistical Review* **56**, 49–62.

Silvapulle, M. (1992a). Robust tests of inequality constraints and one-sided hypotheses in the linear model. *Biometrika*, **79**, 621–630.

Silvapulle, M. (1992b). Robust Wald-Type Tests of One-Sided Hypotheses in the Linear Model. *Journal of the American Statistical Association*, **87**, 156–161.

Silvapulle, M. and Silvapulle, P. (1995). A score test against one-sided alternatives. *American statistical association*, **90**, 342–349.

Silvapulle, M. (1996) On an F-type statistic for testing one-sided hypotheses and computation of chi-bar-squared weights. *Statistics & probability letters*, **28**, 137–141.

Silvapulle, M. (1996) Robust bounded influence tests against one-sided hypotheses in general parametric models. *Statistics & probability letters*, **31**, 45–50.

Silvapulle, M.J. and Sen, P.K. (2005). Constrained Statistical Inference. Wiley, New York

Wolak, F. (1987). An exact test for multiple inequality and equality constraints in the linear regression model. *Journal of the American statistical association*, **82**, 782–793.

## See Also

quadprog, conTest

#### Examples

```
## example 1:
# the data consist of ages (in months) at which an
# infant starts to walk alone.
# prepare data
DATA1 <- subset(ZelazoKolb1972, Group != "Control")</pre>
# fit unrestricted linear model
fit1.lm <- lm(Age ~ -1 + Group, data = DATA1)</pre>
# the variable names can be used to impose constraints on
# the corresponding regression parameters.
coef(fit1.lm)
# constraint syntax: assuming that the walking
# exercises would not have a negative effect of increasing the
# mean age at which a child starts to walk.
myConstraints1 <- ' GroupActive < GroupPassive;</pre>
                     GroupPassive < GroupNo '</pre>
conTest(fit1.lm, myConstraints1)
# another way is to first fit the restricted model
fit.restr1 <- restriktor(fit1.lm, constraints = myConstraints1)</pre>
```

con\_weights\_boot

function for computing the chi-bar-square weights based on Monte Carlo simulation.

## Description

The null-distribution of the test statistics under inequality constraints takes the form of mixtures of F-distributions. This function computes these mixing weights (a.k.a chi-bar-square weights and level probabilities). It can be used directly and is called by the conTest function.

## Usage

#### Arguments

VCOV	variance-covariance matrix of the data for which the weights are to be calculated.
Amat	constraints matrix $R$ (or a vector in case of one constraint) and defines the left- hand side of the constraint $R\theta \ge rhs$ , where each row represents one constraint. The number of columns needs to correspond to the number of parameters esti- mated ( $\theta$ ). The rows should be linear independent, otherwise the function gives an error. For more information about constructing the matrix $R$ and $rhs$ see restriktor.
meq	integer (default = 0) treating the number of constraints rows as equality con- straints instead of inequality constraints. For example, if $meq = 2$ , this means that the first two rows of the constraints matrix $R$ are treated as equality con- straints.
R	integer; number of bootstrap draws for mix.bootstrap. The default value is set to 99999.

parallel	the type of parallel operation to be used (if any). If missing, the default is set "no".
ncpus	integer: number of processes to be used in parallel operation: typically one would chose this to the number of available CPUs.
cl	an optional parallel or snow cluster for use if parallel = "snow". If not supplied, a cluster on the local machine is created for the duration of the conTest call.
seed	seed value.
verbose	logical; if TRUE, information is shown at each bootstrap draw.
	no additional arguments for now.

## Value

The function returns a vector with the mixing weights

# Author(s)

Leonard Vanbrabant and Yves Rosseel

## References

Silvapulle, M.J. and Sen, P.K. (2005, p.79). Constrained Statistical Inference. Wiley, New York.

## Examples

```
W <- matrix(c(1,0.5,0.5,1),2,2)
Amat <- rbind(c(0,1))
meq <- 0L
# we only generate 99 bootstrap samples in this
# example; in practice you may wish to use a much higher number.
wt.bar <- con_weights_boot(W, Amat, meq, R = 99)
wt.bar</pre>
```

Exam

Relation between exam scores and study hours, anxiety scores and average point scores.

## Description

The data provide information about students' exam scores, average point score, the amount of study hours and anxiety scores.

#### Usage

data(Exam)

goric

## Format

A data frame of 20 observations of 4 variables.

Scores exam scores

Hours study hours

Anxiety anxiety scores

APS average point score

## References

The original source of these data is http://staff.bath.ac.uk/pssiw/stats2/examrevision.sav.

## Examples

head(Exam)

goric	Generalized Order-Restrikted Information Criterion (Approximation)
	Weights

## Description

The goric function computes GORIC(A) weights, which are comparable to the Akaike weights.

## Usage

```
goric(object, ...)
## Default S3 method:
goric(object, ..., comparison = c("unconstrained", "complement", "none"),
    VCOV = NULL, sample.nobs = NULL, type = "goric", bound = NULL, debug = FALSE)
## S3 method for class 'con_goric'
print(x, digits = max(3, getOption("digits") - 4), ...)
## S3 method for class 'con_goric'
summary(object, brief = TRUE, digits = max(3, getOption("digits") - 4), ...)
## S3 method for class 'con_goric'
summary(object, brief = TRUE, digits = max(3, getOption("digits") - 4), ...)
```

#### Arguments

object	<ul> <li>an object containing the outcome of a statistical analysis. Currently, the following objects can be processed:</li> <li>a fitted object of class restriktor.</li> <li>a fitted unconstrained object of class lm, rlm or glm.</li> <li>a numeric vector containing the unconstrained estimates resulting from any statistical analysis.</li> </ul>
х	an object of class goric.
	this depends on the class of the object. If object is of class restriktor, further objects of class restriktor can be passed. If object is of class lm, rlm or glm, the constraints can be passed. If object is of class numeric, the constraints can be passed. See details for more information.
comparison	if "unconstrained" (default) the unconstrained model is included in the set of models. If "complement" then the restricted object is compared against its com- plement. Note that the complement can only be computed for one model/hypothesis at a time (for now). If "none" the model is only compared against the models provided by the user.
VCOV	variance-coviance matrix. Only needed if object is of class numeric and type = "gorica".
sample.nobs	not used for now.
type	if "goric" (default), the generalized order-restricted information criterion value is computed. If "gorica" the log-likihood is computed using the multivariate normal distribution function.
bound	not used yet.
digits	the number of significant digits to use when printing.
debug	if TRUE, debugging information is printed out.
brief	if FALSE, an extended overview is printed.

## Details

The GORIC(A) values themselves are not interpretable and the interest lie in their differences. The GORIC(A) weights reflect the support of each hypothesis in the set. To compare two hypotheses (and not one to the whole set), one can examine the ratio of the two corresponding GORIC(A) weights. To avoid selecting a weakly supported hypothesis as the best one, the unconstrained hypothesis is usually included as safeguard.

In case of one order-constrained hypothesis, say H1, the complement Hc can be computed as competing hypothesis. The complement is defined as Hc = not H1.

If the object(s) is of class restriktor the constraints are automatically extracted. Otherwise, the constraint syntax can be parsed via the .... If the object is an unconstrained model of class lm, rlm or glm, then the constraints can be specified in two ways, see restriktor. Note that if the constraints are written in matrix notation, then the constraints for each model/hypothesis is put in a named list. For example, h1 <-list(constraints = "x1 > 0", rhs = 0, neq = 0). The rhs and neq are not required if they are equal to 0. If type = "gorica", then the object might be a (named) numeric vector. The constraints can again be specified in two ways, see restriktor. For examples, see below.

goric

## Value

The function returns a dataframe with the log-likelihood, penalty term, GORIC(A) values and the GORIC(A) weights. Furthermore, a dataframe is returned with the relative GORIC(A) weights.

## Author(s)

Leonard Vanbrabant and Rebecca Kuiper

#### References

Kuiper, R.M., Hoijtink, H., and Silvapulle, M.J. (2011). An Akaike-type information criterion for model selection under inequality constraints. *Biometrika*, **98**, 2, 495–501.

Vanbrabant, L. and Kuiper, R. (n.d.). Giving the complement a compliment: Evaluating a theorybased hypothesis against its complement using the GORIC.

#### Examples

est <- coef(fit.lm)</pre>

```
library(MASS)
## lm
## unrestricted linear model for ages (in months) at which an
## infant starts to walk alone.
# prepare data
DATA <- subset(ZelazoKolb1972, Group != "Control")</pre>
# fit unrestrikted linear model
fit1.lm <- lm(Age ~ Group, data = DATA)</pre>
# some artificial restrictions
fit1.con <- restriktor(fit1.lm, constraints = "GroupPassive > 0; GroupPassive < GroupNo")</pre>
fit2.con <- restriktor(fit1.lm, constraints = "GroupPassive > 0; GroupPassive > GroupNo")
fit3.con <- restriktor(fit1.lm, constraints = "GroupPassive == 0; GroupPassive < GroupNo")</pre>
fit4.con <- restriktor(fit1.lm) # unrestricted model</pre>
goric(fit1.con, fit2.con, fit3.con, fit4.con)
# fit1.con versus the complement
goric(fit1.con, comparison = "complement")
## GORICA
# generate data
n <- 10
x1 <- rnorm(n)
x2 <- rnorm(n)
y < -1 + x1 + x2 + rnorm(n)
# fit unconstrained linear model
fit.lm <- lm(y \sim x1 + x2)
# extract unconstrained estimates
```

```
# unconstrained variance-covariance matrix
VCOV <- vcov(fit.lm)</pre>
## constraint syntax (character)
h1 <- "x1 > 0"
h2 <- "x1 > 0; x2 > 0"
# use fitted unconstrained linear model
out <- goric(fit.lm, h1, h2, type = "gorica")</pre>
# use unconstrained estimates
out <- goric(est, VCOV = VCOV, h1, h2, type = "gorica")</pre>
## constraint syntax (matrix notation)
h1 \le list(constraints = c(0,1,0))
h2 <- list(constraints = rbind(c(0,1,0), c(0,0,1)))
out <- goric(fit.lm, h1, h2, type = "gorica")</pre>
out <- goric(est, VCOV = VCOV, h1, h2, type = "gorica")</pre>
## mlm
# generate data
n <- 30
mu <- c(1,2,3,4)
Sigma <- matrix(5,4,4)</pre>
  diag(Sigma) <- c(10,10,10,10)
# 4 Y's.
Y <- mvrnorm(n, mu, Sigma)
# fit unrestricted multivariate linear model
fit2.mlm <- lm(Y \sim 1)
# constraints
myConstraints2 <- rbind(c(-1,1,0,0), c(0,-1,1,0), c(0,0,-1,1))</pre>
# fit restricted multivariate linear model
fit5.con <- restriktor(fit2.mlm, constraints = myConstraints2)</pre>
```

Hurricanes

The Hurricanes Dataset

#### Description

The data provide information on the effect of El Nino (Cold, Neutral, Warm) on the number of hurricanes from 1950 to 1995.

#### Usage

data(Hurricanes)

#### restriktor

## Format

A data frame of 46 observations of 3 variables.

Year

Hurricanes Number of Hurricanes

ElNino 1=Cold, 2=Neutral, 3=Warm

## References

The original source of these data is the National Hurricane Center in Australia. The dataset was extracted from the table on page 5 in Silvapulle and Sen (2005).

#### Examples

head(Hurricanes)

restriktor

Estimating linear regression models with (in)equality restrictions

## Description

Function restriktor estimates the parameters of an univariate and a multivariate linear model (1m), a robust estimation of the linear model (rlm) and a generalized linear model (glm) subject to linear equality and linear inequality restrictions. It is a convenience function. The real work horses are the conLM, conMLM, conRLM and the conGLM functions.

#### Usage

```
restriktor(object, constraints = NULL, ...)
## S3 method for class 'lm'
conLM(object, constraints = NULL, se = "standard",
     B = 999, rhs = NULL, neq = 0L, mix.weights = "pmvnorm",
     mix.bootstrap = 99999L, parallel = "no", ncpus = 1L,
     cl = NULL, seed = NULL, control = list(),
     verbose = FALSE, debug = FALSE, ...)
## S3 method for class 'rlm'
conRLM(object, constraints = NULL, se = "standard",
       B = 999, rhs = NULL, neq = 0L, mix.weights = "pmvnorm",
      mix.bootstrap = 99999L, parallel = "no", ncpus = 1L,
       cl = NULL, seed = NULL, control = list(),
       verbose = FALSE, debug = FALSE, ...)
## S3 method for class 'glm'
conGLM(object, constraints = NULL, se = "standard",
      B = 999, rhs = NULL, neq = 0L, mix.weights = "pmvnorm",
```

```
mix.bootstrap = 99999L, parallel = "no", ncpus = 1L,
cl = NULL, seed = NULL, control = list(),
verbose = FALSE, debug = FALSE, ...)
## S3 method for class 'mlm'
conMLM(object, constraints = NULL, se = "none",
B = 999, rhs = NULL, neq = 0L, mix.weights = "pmvnorm",
mix.bootstrap = 99999L, parallel = "no", ncpus = 1L,
cl = NULL, seed = NULL, control = list(),
verbose = FALSE, debug = FALSE, ...)
```

#### Arguments

object	a fitted linear model object of class "lm", "mlm", "rlm" or "glm". For class "rlm" only the loss function bisquare is supported for now, otherwise the function gives an error.
constraints	there are two ways to constrain parameters. First, the constraint syntax consists of one or more text-based descriptions, where the syntax can be specified as a literal string enclosed by single quotes. Only the names of coef(model) can be used as names. See details for more information. Note that objects of class "mlm" do not (yet) support this method.
	Second, the constraint syntax consists of a matrix $R$ (or a vector in case of one constraint) and defines the left-hand side of the constraint $R\theta \ge rhs$ , where each row represents one constraint. The number of columns needs to correspond to the number of parameters estimated ( $\theta$ ) by model. The rows should be linear independent, otherwise the function gives an error. For more information about

se if "standard" (default), conventional standard errors are computed based on inverting the observed augmented information matrix. If "const", homoskedastic standard errors are computed. If "HC0" or just "HC", heteroskedastic robust standard errors are computed (a.k.a Huber White). The options "HC1", "HC2", "HC3", "HC4", "HC4m", and "HC5" are refinements of "HC0". For more details about heteroskedastic robust standard errors see the **sandwich** package. If "boot.standard", bootstrapped standard errors are computed using standard bootstrapping. If "boot.model.based" or "boot.residual", bootstrapped standard errors are computed. Note that for objects of class "mlm" no standard errors are available (yet).

constructing the matrix R and rhs see details.

- B integer; number of bootstrap draws for se. The default value is set to 999. Parallel support is available.
- rhs vector on the right-hand side of the constraints;  $R\theta \ge rhs$ . The length of this vector equals the number of rows of the constraints matrix R and consists of zeros by default. Note: only used if constraints input is a matrix or vector.
- neq integer (default = 0) treating the number of constraints rows as equality constraints instead of inequality constraints. For example, if neq = 2, this means that the first two rows of the constraints matrix R are treated as equality constraints. Note: only used if constraints input is a matrix or vector.

## restriktor

m	ix.weights	if "pmvnorm" (default), the chi-bar-square weights are computed based on the multivariate normal distribution function with additional Monte Carlo steps. If "boot", the chi-bar-square weights are computed using parametric bootstrap- ping. If "none", no chi-bar-square weights are computed. The weights are nec- essary in the restriktor.summary function for computing the GORIC. More- over, the weights are re-used in the iht function for computing the p-value for the test-statistic, unless the p-value is computed directly via bootstrapping.
m	ix.bootstrap	integer; number of bootstrap draws for mix.weights = "boot". The default value is set to 99999. Parallel support is available.
р	arallel	the type of parallel operation to be used (if any). If missing, the default is set "no".
n	cpus	integer: number of processes to be used in parallel operation: typically one would chose this to the number of available CPUs.
с	1	an optional parallel or snow cluster for use if parallel = "snow". If not supplied, a cluster on the local machine is created for the duration of the restriktor call.
S	eed	seed value.
с	ontrol	a list of control arguments:
		<ul> <li>absval tolerance criterion for convergence (default = sqrt(.Machine\$double.eps)).</li> <li>maxit the maximum number of iterations for the optimizer (default = 10000).</li> <li>tol numerical tolerance value. Estimates smaller than tol are set to 0.</li> </ul>
v	erbose	logical; if TRUE, information is shown at each bootstrap draw.
d	ebug	if TRUE, debugging information about the constraints is printed out.
	•••	no additional arguments for now.

## Details

The constraint syntax can be specified in two ways. First as a literal string enclosed by single quotes as shown below:

```
myConstraints <- '
# 1. inequality constraints
    x1 > 0
    x1 < x2
! 2. equality constraints
    x3 == x4; x4 == x5 '</pre>
```

The variable names x1 to x5 refer to the corresponding regression coefficient. Thus, constraints are impose on regression coefficients and not on the data.

Second, the above constraints syntax can also be written in matrix/vector notation as:

(The first column refers to the intercept, the remaining five columns refer to the regression coefficients x1 to x5.)

```
myConstraints <-
    rbind(c(0, 0, 0, -1, 1, 0), #equality constraint x3 == x4
        c(0, 0, 0, 0, -1, 1), #equality constraint x4 == x5
        c(0, 1, 0, 0, 0, 0), #inequality constraint x1 > rhs
        c(0, -1, 1, 0, 0, 0)) #inequality constraint x1 < x2</pre>
```

# the length of rhs is equal to the number of myConstraints rows. myRhs <- c(0,0,0,0)

```
# the first two rows should be considered as equality constraints myNeq <- 2 \,
```

Blank lines and comments can be used in between the constraints, and constraints can be split over multiple lines. Both the hashtag (#) and the exclamation (!) characters can be used to start a comment. Multiple constraints can be placed on a single line if they are separated by a semicolon (;).

There can be three types of text-based descriptions in the constraints syntax:

- Equality constraints: The "==" operator can be used to define equality constraints (e.g., x1 == 1 or x1 == x2).
- 2. Inequality constraints: The "<" or ">" operator can be used to define inequality constraints (e.g., x1 > 1 or x1 < x2).
- 3. Newly defined parameters: The ":=" operator can be used to define new parameters, which take on values that are an arbitrary function of the original model parameters. The function must be specified in terms of the parameter names in coef(model) (e.g., new := x1 + 2\*x2). By default, the standard errors for these defined parameters are computed by using the so-called Delta method.

Variable names of interaction effects in objects of class lm, rlm and glm contain a semi-colon (:) between the variables. To impose constraints on parameters of interaction effects, the semi-colon must be replaced by a dot (.) (e.g., x3:x4 becomes x3.x4). In addition, the intercept variable names is shown as "(Intercept)". To impose restrictions on the intercept both parentheses must be replaced by a dot ".Intercept." (e.g., Intercept. > 10). Note: in most practical situations we do not impose restrictions on the intercept because we do not have prior knowledge about the intercept. Moreover, the sign of the intercept can be changed arbitrarily by shifting the response variable *y*.

Each element can be modified using arithmetic operators. For example, if  $x^2$  is expected to be twice as large as  $x^1$ , then " $2*x^2 = x^1$ ".

If constraints = NULL, the unrestricted model is fitted.

## Value

An object of class restriktor, for which a print and a summary method are available. More specifically, it is a list with the following items:

CON	a list with useful information about the restrictions.
call	the matched call.
timing	how much time several tasks take.

# restriktor

parTable	a parameter table with information about the observed variables in the model and the imposed restrictions.
b.unrestr	unrestricted regression coefficients.
b.restr	restricted regression coefficients.
residuals	restricted residuals.
wresid	a working residual, weighted for "inv.var" weights only (rlm only)
fitted	restricted fitted mean values.
weights	(only for weighted fits) the specified weights.
wgt	the weights used in the IWLS process (rlm only).
scale	the robust scale estimate used (rlm only).
stddev	a scale estimate used for the standard errors.
R2.org	unrestricted R-squared.
R2.reduced	restricted R-squared.
df.residual	the residual degrees of freedom
s2.unrestr	mean squared error of unrestricted model.
s2.restr	mean squared error of restricted model.
loglik	restricted log-likelihood.
Sigma	variance-covariance matrix of unrestricted model.
constraints	matrix with restrictions.
rhs	vector of right-hand side elements.
neq	number of equality restrictions.
wt.bar	chi-bar-square mixing weights or a.k.a. level probabilities.
iact	active restrictions.
converged	did the IWLS converge (rlm only)?
iter	number of iteration needed for convergence (rlm only).
bootout	object of class boot. Only available if bootstrapped standard errors are requested, else bootout = NULL.
control	list with control options.
model.org	original model.
se	as input. This information is needed in the summary function.
information	observed information matrix with the inverted information matrix and the aug- mented information matrix as attributes.

# Author(s)

Leonard Vanbrabant and Yves Rosseel

#### References

Schoenberg, R. (1997). Constrained Maximum Likelihood. *Computational Economics*, **10**, 251–266.

Shapiro, A. (1988). Towards a unified theory of inequality-constrained testing in multivariate analysis. *International Statistical Review* **56**, 49–62.

Silvapulle, M.J. and Sen, P.K. (2005). Constrained Statistical Inference. Wiley, New York

#### See Also

iht, goric

## Examples

```
## lm
## unrestricted linear model for ages (in months) at which an
## infant starts to walk alone.
# prepare data
DATA1 <- subset(ZelazoKolb1972, Group != "Control")</pre>
# fit unrestricted linear model
fit1.lm <- lm(Age ~ -1 + Group, data = DATA1)</pre>
# the variable names can be used to impose restrictions on
# the corresponding regression parameters.
coef(fit1.lm)
# restricted linear model with restrictions that the walking
# exercises would not have a negative effect of increasing the
# mean age at which a child starts to walk.
fit1.con <- restriktor(fit1.lm, constraints = ' GroupActive < GroupPassive;</pre>
                                                GroupPassive < GroupNo ')</pre>
summary(fit1.con)
## Not run:
# Or in matrix notation.
myConstraints1 <- rbind(c(-1, 1, 0),</pre>
                        c( 0,-1, 1))
myRhs1 <- rep(0L, nrow(R1))</pre>
myNeq1 <- 0
fit1.con <- restriktor(fit1.lm, constraints = myConstraints1,</pre>
                       rhs = myRhs1, neq = myNeq1)
summary(fit1.con)
## End(Not run)
## Artificial examples ##
library(MASS)
```

## restriktor

```
## mlm
# generate data
n <- 30
mu <- rep(0, 4)
Sigma <- matrix(5,4,4)</pre>
  diag(Sigma) <- c(10,10,10,10)
# 4 Y's.
Y <- mvrnorm(n, mu, Sigma)
# fit unrestricted multivariate linear model
fit.mlm <- lm(Y \sim 1)
# constraints
myConstraints2 <- diag(0,4)</pre>
  diag(myConstraints2) <- 1</pre>
# fit restricted multivariate linear model
fit2.con <- restriktor(fit.mlm, constraints = myConstraints2)</pre>
summary(fit2.con)
## rlm
# generate data
n <- 10
means <- c(1,2,1,3)
nm <- length(means)</pre>
group <- as.factor(rep(1:nm, each = n))</pre>
y <- rnorm(n * nm, rep(means, each = n))</pre>
DATA2 <- data.frame(y, group)</pre>
# fit unrestricted robust linear model
fit3.rlm <- rlm(y ~ -1 + group, data = DATA2, method = "MM")</pre>
coef(fit3.rlm)
## increasing means
myConstraints3 <- ' group1 < group2</pre>
                     group2 < group3
                      group3 < group4 '
# fit restricted robust linear model and compute
# Huber-White (robust) standard errors.
fit3.con <- restriktor(fit3.rlm, constraints = myConstraints2,</pre>
                         se = "HC0")
summary(fit3.con)
## Not run:
## increasing means in matrix notation.
myConstraints3 <- rbind(c(-1, 1, 0, 0),</pre>
                          c( 0,-1, 1, 0),
                          c( 0, 0,-1, 1))
myRhs3 <- rep(0L, nrow(myConstraints3))</pre>
```

```
myNeq2 <- 0
fit3.con <- restriktor(fit3.rlm, constraints = myConstraints3,</pre>
                       rhs = myRhs2, neq = myNeq2, se = "HC0")
summary(fit3.con)
## End(Not run)
## equality restrictions only.
myConstraints4 <- ' group1 == group2</pre>
                    group2 == group3
                    group3 == group4 '
fit4.con <- restriktor(fit3.rlm, constraints = myConstraints4)</pre>
summary(fit4.con)
## combination of equality and inequality restrictions.
myConstraints5 <- ' group1 == group2</pre>
                    group3 < group4 '
# fit restricted model and compute model-based bootstrapped
# standard errors. We only generate 9 bootstrap samples in this
# example; in practice you may wish to use a much higher number.
fit5.con <- restriktor(fit3.rlm, constraints = myConstraints4,</pre>
                       se = "boot.model.based", B = 9)
# an error is probably thrown, due to a too low number of bootstrap draws.
summary(fit5.con)
# restriktor can also be used to define effects using the := operator
# and impose restrictions on them. For example, compute the average
# effect (AVE) and impose the restriction AVE > 0.
# generate data
n <- 30
b0 <- 10; b1 = 0.5; b2 = 1; b3 = 1.5
X <- c(rep(c(0), n/2), rep(c(1), n/2))
set.seed(90)
Z <- rnorm(n, 16, 5)
y <- b0 + b1*X + b2*Z + b3*X*Z + rnorm(n, 0, sd = 10)
DATA3 = data.frame(cbind(y, X, Z))
# fit linear model with interaction
fit6.lm <- lm(y ~ X*Z, data = DATA3)</pre>
fit6.con <- restriktor(fit6.lm, constraints = ' AVE := X + 16.86137*X.Z;</pre>
                                                  AVE > 0 ')
summary(fit6.con)
```

restriktor-methods Methods for restriktor

## restriktor-methods

## Description

restricted estimation and confidence intervals for (robust) linear (in)equality restricted hypotheses.

## Usage

```
## S3 method for class 'restriktor'
print(x, digits = max(3, getOption("digits") - 2), ...)
## S3 method for class 'restriktor'
summary(object, bootCIs = TRUE,
            bty = "perc", level = 0.95, goric = "goric", ...)
## S3 method for class 'summary.restriktor'
print(x, digits = max(3, getOption("digits") - 2),
            signif.stars = getOption("show.signif.stars"), ...)
## S3 method for class 'restriktor'
coef(object, ...)
## S3 method for class 'restriktor'
model.matrix(object, ...)
## S3 method for class 'restriktor'
## S4 method for class 'restrikt
```

## Arguments

object	an object of class restriktor.
x	an object of class restriktor.
bootCIs	if TRUE (default), nonparametric bootstrap confidence intervals are generated. Only available if object contains bootout object.
bty	a character string representing the type of interval required. The value should be any of the values "norm", "basic", "perc", "bca". The value "stud" is not supported. For more details see boot.ci.
level	the confidence level of the interval (default = $0.95$ ).
goric	if "goric" (default), the generalized order-restricted information criterion value is computed. If "gorica" the log-likihood is computed using the multivariate normal distribution function. If "goricc" or "goricca", a small sample ver- sion of the "goric" or "gorica" is computed.
digits	the number of significant digits to use when printing.
signif.stars	If TRUE, "significance stars are printed for each coefficient.
	no additional arguments for now.

## Details

The function print returns the restricted coefficients. The output from the print.summary.conLM function provides information that is comparable with the output from print.summary.lm. Additional information is provided about the unrestricted and restricted R-square and by default the output of the GORIC. If bootstrapped standard errors are requested (e.g., option se = "boot.model.based" in the restriktor function and bootCI = TRUE in the summary function) standard errors and confidence intervals are provided.

## Value

The function summary computes and returns a list of summary statistics of the fitted unrestricted and restricted (robust) linear model given in object, plus

se.type	type of standard error computed, equal to input se in the restriktor function.
residuals	the weighted residuals.
coefficients	a p x 4 matrix with columns for the estimated coefficient, its standard error, t- statistic and corresponding p-value. If bootCIs = TRUE and the bootout object is available in the object, bootstrapped standard errors and confidence intervals are produced.
rdf	residual degrees of freedom.
R2.org	unrestricted R-squared.
R2.reduced	restricted R-squared.
goric	goric value and attributed its penalty term and log-likelihood.

## Examples

```
# unrestricted linear model for ages (in months) at which an
# infant starts to walk alone.
# prepare data
DATA <- subset(ZelazoKolb1972, Group != "Control")
# fit unrestricted linear model
fit.lm <- lm(Age ~ -1 + Group, data = DATA)
# restricted linear model with restrictions that the walking
# exercises would not have a negative effect of increasing the
# mean age at which a child starts to walk.
fit.con <- restriktor(fit.lm, constraints = ' GroupActive < GroupPassive;
GroupPassive < GroupNo ')</pre>
```

summary(fit.con)

ZelazoKolb1972 "Walking" in the newborn (4 treatment groups)

## Description

The Zelazo, Zelazo and Kolb (1972) dataset consists of ages (in months) at which an infant starts to walk alone from four different treatment groups (Active-exercise, Passive-exercise, 8 week Control, No-exercise).

## Usage

```
data(ZelazoKolb1972)
```

# Format

A data frame of 23 observations of 4 treatment variables.

Age Age in months

Group Active-exercise, Passive-exercise, 8-week Control group, No-exercise

#### References

Zelazo, P.R., Zelazo, N.A., and Kolb, S. (1972). "Walking in the Newborn". Science, New Series, 176, 314-315

## Examples

head(ZelazoKolb1972)

# Index

```
AngerManagement, 4
boot.ci, 57
Burns, 5
coef.con_goric(goric), 45
coef.restriktor(restriktor-methods), 56
con_weights_boot, 43
conGLM.glm(restriktor), 49
conLM.lm(restriktor), 49
conMLM.mlm(restriktor), 49
conRLM.rlm(restriktor), 49
conTest, 6, 9, 14, 18, 24, 29, 35, 39, 41, 42
conTest-methods, 12
conTest_ceq, 37
conTest_summary, 40
conTestC, 13
conTestF, 8, 15
conTestLRT, 21
conTestScore, 26
conTestWald, 32
Exam, 44
goric, 45, 54
Hurricanes, 48
iht, 4, 51, 54
iht (conTest), 6
iht-methods (conTest-methods), 12
logLik.restriktor (restriktor-methods),
        56
model.matrix.restriktor
        (restriktor-methods), 56
print.con_goric(goric), 45
print.conTest (conTest-methods), 12
print.restriktor(restriktor-methods),
        56
```