

Package ‘pwrFDR’

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Title FDR Power

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Imports methods, stats

Description This is a package for calculating power and sample size in multiple testing situations using the Benjamini-Hochberg (BH) false discovery rate (FDR) procedure. The package computes power and sample size in one of either two ways, using the average power or the lambda-power. See Izmirlian, G. (2018) <arXiv:1801.03989>.

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cCDF . SoM

*Complementary CDF for the True Positive Fraction***Description**

Computes the complementary CDF for the True Positive Fraction, S_m/M_m , via approximation based upon the asymptotic distribution.

Usage

```
cCDF.SoM(lambda, x = NULL, groups, effect.size, n.sample, r.1, FDR, N.tests,
          control)
```

Arguments

lambda	Argument of the complementary CDF. Result will be $\Pr(S_m/M_m > \text{lambda})$
x	In the abbreviated call sequence, the user only needs to specify lambda, N. tests, and x, an object of class pwr returned by the function pwrFDR.
groups	The number of experimental groups to compare. Default value is 2.
effect.size	The effect size (mean over standard deviation) for test statistics having non-zero means. Assumed to be a constant (in magnitude) over non-zero mean test statistics.
n.sample	The number of experimental replicates.
r.1	The proportion of all test statistics that are distributed under HA.
FDR	The false discovery rate.
N.tests	The number of simultaneous hypothesis tests.
control	Optionally, a list with components with the following components: 'groups', used when distop=3 (F-dist), specifying number of groups. 'version', used only in the 'JL' method, choice 0 gives the 'JL' version as published, whereas choice 1 replaces the FDR with $r.0 * \text{FDR}$ resulting in the infinite simultaneous tests limiting average power, which is the 'Iz' version, but this is redundant because you can specify the 'Iz' method to use this option. 'tol' is a convergence criterion used in iterative methods which is set to $1e-8$ by default 'max.iter' is an iteration limit, set to 1000 by default 'distop', specifying the distribution family of the central and non-centrally located sub-populations. =1 gives normal (2 groups) =2 gives t- (2 groups) and =3 gives F- (2+ groups) 'CS', correlation structure, for use only with 'method="simulation"' which will simulate m simultaneous tests with correlations 'rho' in blocks of size 'n.WC'. Specify as list $\text{CS} = \text{list}(\text{rho}=0.80, \text{n.WC}=50)$ for example

Details

The complementary CDF for the True Positive Fraction, S_m/M_m , is approximated using its asymptotic distribution. Since

$$m^{0.5} (S_m/M_m - \text{average.power}) \xrightarrow{D} N(0, \text{sigma}^2)$$

then

$$P(S_m/M_m > \text{lambda})$$

$$\sim 1 - \Phi(m^{0.5} (\text{lambda} - \text{average.power})/\text{sigma})$$

The approximation is reasonable as long as $m \text{ sigma}^2$ is large enough. A formula for the asymptotic variance is given in the cited manuscript. There is a user level function, `var.rtm.SoM`, which computes the asymptotic variance.

NOTE: the capabilities of this function are available in the function, `pwrFDR` by requesting the `lambda` power by specifying the argument, `lambda`

Value

An object of class "vzv" which is a list having components

<code>cCDF . SoM</code>	The result
<code>average . power</code>	The average power at the supplied arguments
<code>c . g</code>	The per test threshold that is equivalent to the BH-FDR, on the test statistic scale
<code>gamma</code>	The limiting proportion of tests that were called significant
<code>objective</code>	Result of optimization producing the average power, should be close to zero.
<code>err . III</code>	The probability mass on side of the oppositely signed alternative in two sided tests
<code>sigma . rtm . SoM</code>	The square root of the asymptotic variance of the root-m scaled true positive fraction, $m^{0.5} * S_m/M_m$
<code>call</code>	The call which produced the result

Author(s)

Grant Izmirlian <izmirlian at nih dot gov>

References

Izmirlian G. (2017) Average Power and λ -power in Multiple Testing Scenarios when the Benjamini-Hochberg False Discovery Rate Procedure is Used. arXiv:1801.03989

See Also

[pwrFDR cCDF . ToJ](#)

Examples

```
## Example 1: Explicit call
ccdf <- cCDF.SoM(lambda=0.8201087, effect.size=0.79, n.sample=46, r.1=2000/54675,
                FDR=0.15, N.tests=1000)

ccdf

## Example 2: Abbreviated call using result of pwrFDR
rslt.avgp <- pwrFDR(effect.size=0.79, n.sample=46, r.1=2000/54675, FDR=0.15)
ccdf <- cCDF.SoM(lambda=0.8201087, x=rslt.avgp, N.tests=1000)
ccdf
```

cCDF.ToJ

*Complementary CDF for the False Discovery Fraction***Description**

Computes the complementary CDF for the False Discovery Fraction, T_m/J_m , via approximation based upon the asymptotic distribution.

Usage

```
cCDF.ToJ(lambda, x=NULL, groups, effect.size, n.sample, r.1, FDR, N.tests,
         control)
```

Arguments

lambda	Argument of the complementary CDF. Result will be $\Pr(S_m/M_m > \lambda)$
x	In the abbreviated call sequence, the user only needs to specify lambda, N.tests, and x, an object of class pwr returned by the function pwrFDR.
groups	The number of experimental groups to compare. Default value is 2.
effect.size	The effect size (mean over standard deviation) for test statistics having non-zero means. Assumed to be a constant (in magnitude) over non-zero mean test statistics.
n.sample	The number of experimental replicates.
r.1	The proportion of all test statistics that are distributed under H_A .
FDR	The false discovery rate.
N.tests	The number of simultaneous hypothesis tests.
control	Optionally, a list with components with the following components: 'groups', used when <code>distop=3</code> (F-dist), specifying number of groups. 'version', used only in the 'JL' method, choice 0 gives the 'JL' version as published, whereas choice 1 replaces the FDR with $r.0 * \text{FDR}$ resulting in the infinite simultaneous tests limiting average power, which is the 'Iz' version, but this is redundant because you can specify the 'Iz' method to use this option. 'tol' is a convergence criterion used in iterative methods which is set to $1e-8$ by default 'max.iter' is an

iteration limit, set to 1000 by default 'distop', specifying the distribution family of the central and non-centrally located sub-populations. =1 gives normal (2 groups) =2 gives t- (2 groups) and =3 gives F- (2+ groups) 'CS', correlation structure, for use only with 'method="simulation"' which will simulate m simultaneous tests with correlations 'rho' in blocks of size 'n.WC'. Specify as list CS = list(rho=0.80, n.WC=50) for example

Details

The complementary CDF for the False Discovery Fraction, T_m/J_m , is approximated using its asymptotic distribution. Since

$$m^{0.5} (T_m/J_m - (1 - r.1) FDR) \xrightarrow{D} N(0, \tau^2)$$

then

$$P(T_m/J_m > \lambda)$$

$$\sim 1 - \Phi(m^{0.5} (\lambda - (1 - r.1) FDR) / \tau)$$

The approximation is reasonable as long as $m \tau^2$ is large enough A formula for the asymptotic variance is given in the cited manuscript. There is a user level function, var.rtm.ToJ, which computes the asymptotic variance.

Value

An object of class "vvv" which is a list having components

cCDF.ToJ	The result
average.power	The average power at the supplied arguments
c.g	The per test threshold that is equivalent to the BH-FDR, on the test statistic scale
gamma	The limiting proportion of tests that were called significant
objective	Result of optimization producing the average power, should be close to zero.
err.III	The probability mass on side of the oppositely signed alternative in two sided tests
sigma.rtm.ToJ	The square root of the asymptotic variance of the root-m scaled false discovery fraction, $m^{0.5} * T_m/J_m$
call	The call which produced the result

Author(s)

Grant Izmirlian <izmirlian at nih dot gov>

References

Izmirlian G. (2017) Average Power and λ -power in Multiple Testing Scenarios when the Benjamini-Hochberg False Discovery Rate Procedure is Used. arXiv:1801.03989

See Also

[controlFDF cCDF.SoM](#)

Examples

```
## Example 1: Explicit call
ccdf <- cCDF.ToJ(lambda=(1-2000/54675)*0.15, effect.size=0.79, n.sample=46, r.1=2000/54675,
                FDR=0.15, N.tests=1000)

ccdf

## Example 2: Abbreviated call using result of pwrFDR
rslt.avgp <- pwrFDR(effect.size=0.79, n.sample=46, r.1=2000/54675, FDR=0.15)
ccdf <- cCDF.ToJ(lambda=(1-2000/54675)*0.15, x=rslt.avgp, N.tests=1000)
ccdf
```

CDF.Pval

*CDF of pooled (H0 and HA) population p-values***Description**

Computes the CDF of the pooled population p-values under the mixture model, e.g. the p-values are i.i.d. with CDF a mixture between a uniform (CDF in the null distributed population) and a concave function (CDF in the non-null distributed population).

Usage

```
CDF.Pval(u, groups, r.1, effect.size, n.sample, control)
```

Arguments

u	Argument of the CDF. Result will be $\Pr(P_i \leq u)$
groups	The number of experimental groups to compare. Default value is 2.
r.1	The proportion of all test statistics that are distributed under HA.
effect.size	The effect size (mean over standard deviation) for test statistics having non-zero means. Assumed to be a constant (in magnitude) over non-zero mean test statistics.
n.sample	The number of experimental replicates.
control	Optionally, a list with components with the following components: 'groups', used when <code>distop=3</code> (F-dist), specifying number of groups. 'version', used only in the 'JL' method, choice 0 gives the 'JL' version as published, whereas choice 1 replaces the FDR with $r.0 \cdot \text{FDR}$ resulting in the infinite simultaneous tests limiting average power, which is the 'Iz' version, but this is redundant because you can specify the 'Iz' method to use this option. 'tol' is a convergence criterion used in iterative methods which is set to $1e-8$ by default 'max.iter' is an iteration limit, set to 1000 by default 'distop', specifying the distribution family of the central and non-centrally located sub-populations. =1 gives normal (2 groups) =2 gives t- (2 groups) and =3 gives F- (2+ groups) 'CS', correlation structure, for use only with 'method="simulation"' which will simulate m simultaneous tests with correlations 'rho' in blocks of size 'n.WC'. Specify as list <code>CS = list(rho=0.80, n.WC=50)</code> for example

Details

Computes the CDF of the pooled population p-values under the mixture model, e.g. the p-values are i.i.d. with CDF a mixture between a uniform (CDF in the null distributed population) and a concave function (CDF in the non-null distributed population). If F_{c0} is the cCDF of a test statistic under H_0 and F_{cA} is the cCDF of a test statistic under H_A then the CDF of the P-values is

$$G(u) = (1-r)u + r F_{cA}(F_{c0}^{-1}(u))$$

The limiting positive call fraction, $\lim_m J_m/m = \text{gamma}$ (a.s.) is the solution to the equation

$$G(\text{gamma} f) = \text{gamma}$$

where f is the FDR

Value

A list with components

call	The call which produced the result
u	The argument that was passed to the function
CDF.Pval	The value of the CDF

Author(s)

Grant Izmirlian <izmirlian at nih dot gov>

References

Genovese, C. and L. Wasserman. (2004) A stochastic process approach to false discovery control. *Annals of Statistics*. 32 (3), 1035-1061.

Izmirlian G. (2017) Average Power and λ -power in Multiple Testing Scenarios when the Benjamini-Hochberg False Discovery Rate Procedure is Used. arXiv:1801.03989

See Also

[CDF.Pval.HA](#)

Examples

```
## First calculate an average power for a given set of parameters
rslt.avgp <- pwrFDR(effect.size=0.79, n.sample=46, r.1=2000/54675, FDR=0.15)

## Now verify that G( gamma f ) = gamma

gamma <- rslt.avgp$gamma
f <- rslt.avgp$call$FDR

G.gma.f <- CDF.Pval(u=gamma*f, r.1=2000/54675, effect.size=0.79, n.sample=46)

c(G.of.gamma.f=G.gma.f$CDF.Pval, gamma=gamma)
```

CDF.Pval.HA

*CDF of p-values for test statistics distributed under HA.***Description**

Computes the CDF of p-values for test statistics distributed under HA.

Usage

```
CDF.Pval.HA(u, groups = 2, r.1, effect.size, n.sample, control)
```

Arguments

u	Argument of the CDF. Result will be $\Pr(P_i \leq u)$
groups	The number of experimental groups to compare. Default value is 2.
r.1	The proportion of all test statistics that are distributed under HA.
effect.size	The effect size (mean over standard deviation) for test statistics having non-zero means. Assumed to be a constant (in magnitude) over non-zero mean test statistics.
n.sample	The number of experimental replicates.
control	Optionally, a list with components with the following components: 'groups', used when <code>distop=3</code> (F-dist), specifying number of groups. 'version', used only in the 'JL' method, choice 0 gives the 'JL' version as published, whereas choice 1 replaces the FDR with $r.0 * \text{FDR}$ resulting in the infinite simultaneous tests limiting average power, which is the 'Iz' version, but this is redundant because you can specify the 'Iz' method to use this option. 'tol' is a convergence criterion used in iterative methods which is set to $1e-8$ by default 'max.iter' is an iteration limit, set to 1000 by default 'distop', specifying the distribution family of the central and non-centrally located sub-populations. =1 gives normal (2 groups) =2 gives t- (2 groups) and =3 gives F- (2+ groups) 'CS', correlation structure, for use only with 'method="simulation"' which will simulate m simultaneous tests with correlations 'rho' in blocks of size 'n.WC'. Specify as list <code>CS = list(rho=0.80, n.WC=50)</code> for example

Details

Computes the CDF of p-values for test statistics distributed under HA. If F_{c_0} is the cCDF of a test statistic under H_0 and F_{c_A} is the cCDF of a test statistic under HA then the CDF of a P-value for a test statistic distributed under HA is

$$G_A(u) = F_{c_A}(F_{c_0}^{-1}(u))$$

The limiting true positive fraction is the infinite simultaneous tests average power,

$$\lim_m S_m/M_m = \text{average.power (a.s.)},$$

which is used to approximate the average power for finite 'm', is G_1 at gamma f:

$$G_1(\text{gamma f}) = \text{average.pwer}$$

where f is the FDR and $\text{gamma} = \lim_m J_m/m$ (a.s.) is the limiting positive call fraction.

Value

A list with components

call The call which produced the result
 u The argument that was passed to the function
 CDF.Pval.HA The value of the CDF

Author(s)

Grant Izmirlian <izmirlian at nih dot gov>

References

Genovese, C. and L. Wasserman. (2004) A stochastic process approach to false discovery control. *Annals of Statistics*. 32 (3), 1035-1061.

Izmirlian G. (2017) Average Power and λ -power in Multiple Testing Scenarios when the Benjamini-Hochberg False Discovery Rate Procedure is Used. arXiv:1801.03989

See Also

[CDF.Pval](#)

Examples

```
## First calculate an average power for a given set of parameters
rslt.avgp <- pwrFDR(effect.size=0.79, n.sample=46, r.1=2000/54675, FDR=0.15)

## Now verify that G_A( gamma f ) = average.power

gamma <- rslt.avgp$gamma
f <- rslt.avgp$call$FDR

GA.gma.f <- CDF.Pval.HA(u=gamma*f, r.1=2000/54675, effect.size=0.79, n.sample=46)

c(G.A.of.gamma.f=GA.gma.f$CDF.Pval.HA, average.power=rslt.avgp$average.power)
```

controlFDF

Calculates a reduced FDR required to control the FDF

Description

Calculates a reduced FDR required to bound the the false discovery rate in probability.

Usage

```
controlFDF(groups=2, FDR, r.1, N.tests, effect.size, n.sample,
            use.prob=c("f*", "f", "user"), prob=NULL)
```

Arguments

groups	The number of experimental groups to compare. Default value is 2.
FDR	The false discovery rate.
r.1	The proportion of simultaneous tests that are non-centrally located
N.tests	The number of simultaneous hypothesis tests.
effect.size	The effect size (mean over standard deviation) for test statistics having non-zero means. Assumed to be a constant (in magnitude) over non-zero mean test statistics.
n.sample	The number of experimental replicates.
use.prob	This sets the value of the probability that the FDF exceeds lambda*. Set this to the character string 'f*' (default) and the routine uses the value of f*. Set this to 'f' and the routine uses the value of the FDR. Set this to 'user' and then you can specify any desired probability in the argument 'prob'.
prob	The desired probability that the FDF exceeds lambda* that is user specified when the argument 'use.prob' is set to 'user'.

Details

Calculates a reduced FDR required to bound the the false discovery rate in probability...e.g. finds f* so that when the BH-FDR procedure at FDR, f*, is used, we ensure that

$$\Pr(T/J > (1-r) f) < (1-r) f^*$$

where 'f' is the original false discovery rate and 'r' is the proportion of non-null distributed test statistics.

Value

f.star	The reduced FDR required to bound the FDF in probability
obj	Result of optimization yielding 'f.star' should be close to 0
L.star	The bound on the FDF, should be (1-r) f. See above.
P.star	The probability that the FDF is greater than L.star. See above.
average.power	Resulting average power.
c.g	The BH-FDR threshold on the scale of the test statistics.
gamma	The proportion of all 'm' tests declared significant.
objective	Result of optimization yielding the 'average.power'.
err.III	Mass on the wrong side of the threshold.
sigma.rtm.SOM	Asymptotic variance of the true positive fraction.

Author(s)

Grant Izmirlian <izmirlian at nih dot gov>

References

Izmirlian G. (2017) Average Power and λ -power in Multiple Testing Scenarios when the Benjamini-Hochberg False Discovery Rate Procedure is Used. arXiv:1801.03989

See Also

[pwrFDR](#)

Examples

```
## at FDR=0.15 and other parameters, it takes n.sample=46 replicates for
## average power > 80%
pwr.46.15 <- pwrFDR(FDR=0.15, r.1=0.03, N.tests=1000, effect.size=0.79, n.sample=46)

## when there are 'only' N.tests=1000 simultaneous tests, the distribution of the
## false discovery fraction, FDF, is not so highly spiked at the FDR=0.15
## You need to set the FDR down to FDR=0.0657 to ensure that Pr( T/J > 0.145 ) < 0.0657
fstr <- controlFDF(FDR=0.15, r.1=0.03, N.tests=1000, effect.size=0.8, n.sample=46)

## at all the above settings, with FDR=0.0657 at an n.sample of 46, we only have 69%
## average power.
pwr.46.0657 <- pwrFDR(FDR=0.065747, r.1=0.03, N.tests=1000, effect.size=0.79, n.sample=46)

## it'll cost 7 more replicates to get the average power up over 80%.
pwr.53.0657 <- pwrFDR(FDR=0.065747, r.1=0.03, N.tests=1000, effect.size=0.8, n.sample=53)

## it costs only 8.75% more to get it right!
```

detail

The detail extraction function for simulated power objects

Description

Objects created by the `pwrFDR` function with option `method=="simulation"` are returned with an attribute named `detail`. This is its extractor function

Usage

```
detail(obj)
```

Arguments

`obj` An object created by the `pwrFDR` function with option `method=="simulation"`.

Value

A list with components

reps	A data frame of <code>n.sim</code> rows containing the results of the simulations as columns: <code>M1</code> , the \forall of non-null distributed statistics, <code>J</code> , the \forall of statistical tests rejected under the BH-FDR procedure, and <code>S</code> , the \forall of true positives.
CCDF	A data frame of dimension <code>n.T</code> , the number of test statistics with non-zero mean, by 2. The first column is <code>s</code> , taking values 1 to <code>n.T</code> , and the second column is the monte carlo estimated complementary CDF for the distribution of the number of true positives, <code>S</code>
X	A single simulation replicate of the <code>m</code> raw test statistics, included as a sanity check

Author(s)

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dists *The Distribution family object*

Description

The `pwrFDR` package currently incorporates 3 distribution types, normal, t and F. The first two of these are strictly for statistics formed from two group comparison while the third is for statistics formed from the omnibus test of any difference among an arbitrary number of groups ≥ 2 . The structure is general and user expandable. One must specify the density, CDF and quantile function for a given distribution and its parameters under the null and under the alternative. These parameters must be expressions to be evaluated inside the kernel of the power program, functions of the arguments `n.sample`, `groups` and `effect.size`. This is not used directly by the user at all unless she (he) wants to add a distribution type.

Format

A data frame with 3 observations on the following 6 variables.

`pars0` a list vector having components `'c(nd, p1, p2, ...)'` where `'nd'` is the distribution number starting with 0, and `p1, p2, ...`, are parameters of the distribution, which are functions of `'n.sample'`, `'groups'` and `'effect.size'` as mentioned above. These must be expressed as a call e.g. `as.call(expression(c, nd, p1, p2, ...))` etc. `'pars0'` are the parameters under the null.

`pars1` a list vector. See directly above. Parameters under the alternative.

`minv` a list vector with components given the values `-Inf` or `0`, which will be used to decide if the two sided corrections are used or not.

`ddist` a list vector with components set to functions, each one computing the probability density function corresponding to the particular distribution. A function of arguments `'x'` and `'par'`. See details below.

`pdist` a list vector with components set to the functions, each one computing the cumulative distribution function corresponding to the particular distribution. A function of arguments 'x' and 'par'. See details below.

`qdist` a list vector with components set to the functions, each one computing the quantile function (inverse cumulative distribution function) corresponding to the particular distribution. A function of arguments 'x' and 'par'. See details below.

Details

The instance shipped with the current version of the package was created using the following commands. Read this and you'll figure out how to add a distribution type yourself.

```
"dists" <-
as.data.frame(rbind(
### Normal with 2 groups ###
c(pars0=as.call(expression(c,0,ncp=0,sd=1)),
pars1=as.call(expression(c,0,ncp=(n.sample/2)^0.5*effect.size, sd=1)),
minv=-Inf,
ddist=function(x, par) dnorm(x, mean=par[2], sd=par[3]),
pdist=function(x, par) pnorm(x, mean=par[2], sd=par[3]),
qdist=function(x, par) qnorm(x, mean=par[2], sd=par[3])),

### t with 2 groups ###
c(pars0=as.call(expression(c,1,ncp=0, ndf=2*n.sample - 2)),
pars1=as.call(expression(c,1,ncp=(n.sample/2)^0.5*effect.size, ndf=2*n.sample - 2)),
minv=-Inf,
ddist=function(x, par) dt(x, ncp=par[2], df=par[3]),
pdist=function(x, par) pt(x, ncp=par[2], df=par[3]),
qdist=function(x, par) qt(x, ncp=par[2], df=par[3])),

### F with 'groups' groups, effect.size=theta*c(0, 0.5, 0.5, ..., 0.5, 1) ###
c(pars0=as.call(expression(c,2,ncp=0, ndf1=groups-1, ndf2=groups*(n.sample-1))),
pars1=as.call(expression(c,2,ncp=n.sample*effect.size^2/2, ndf1=groups-1, ndf2=groups*(n.sample-1))),
minv=0,
ddist=function(x, par) df(x, ncp=par[2], df1=par[3], df2=par[4]),
pdist=function(x, par) pf(x, ncp=par[2], df1=par[3], df2=par[4]),
qdist=function(x, par) qf(x, ncp=par[2], df1=par[3], df2=par[4]))))
```

Source

This isn't 'data' data, its a kind of a 'family' object.

Izmirlan G. (2017) Average Power and λ -power in Multiple Testing Scenarios when the Benjamini-Hochberg False Discovery Rate Procedure is Used. arXiv:1801.03989

gentempfilenm *Generate a tempfile name*

Description

Generates a tempfile name with an optional user specified prefix and suffix Result is a character string

Usage

```
gentempfilenm(prfx = "temp", sfx = ".txt")
```

Arguments

prfx prefix for the file name, e.g. "temp"
sfx suffix (file extension) for the file name, e.g. ".txt"

Value

a character string containing the randomly generated name of the tempfile.

Author(s)

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logit *Computes the logit transform*

Description

Computes the logit transform for objects of type numeric and objects of class "pwr".

Usage

```
logit(mu)
```

Arguments

mu A real number on the interval [0, 1]

Value

A numeric equal to the logit of mu, a real number.

Author(s)

Grant Izmirlian <izmirlian at mail.nih.gov>

logitInv	<i>Computes the inverse logit transform</i>
----------	---

Description

Computes the inverse logit transform for objects of type numeric and objects of class "pwr".

Usage

```
logitInv(eta)
```

Arguments

eta	Any real number
-----	-----------------

Value

A numeric equal to the logit inverse of mu, a real number on the interval [0, 1]

Author(s)

Grant Izmirlian <izmirlian at mail.nih.gov>

paste	<i>The paste operator</i>
-------	---------------------------

Description

A binary operator shortcut for paste(x,y)

Usage

```
x %,% y
```

Arguments

x	a character string
y	a character string

Value

The concatenated character string

Author(s)

Grant Izmirlian <izmirlian@nih.gov>

Examples

```
library(pwrFDR)
"var" %,% (1:10)
```

pwrFDR *Power in the BH-FDR procedure.*

Description

Calculates average power or λ -power in the Benjamini-Hochberg FDR procedure, or sample size required for given average- or λ - power.

Usage

```
pwrFDR(groups = 2, effect.size, n.sample, r.1, FDR, use.LFD = FALSE,
        N.tests, average.power, L.power, lambda, method = c("approximate",
        "simulation", "JL", "Iz"), control = list(version = 0,
        tol = 1e-08, max.iter = 1000, distopt = 1, CS = list(NULL),
        verb = FALSE), n.sim = 1000, temp.file)
```

Arguments

groups	The number of experimental groups to compare. Default value is 2.
effect.size	The effect size (mean over standard deviation) for test statistics having non-zero means. Assumed to be a constant (in magnitude) over non-zero mean test statistics.
n.sample	The number of experimental replicates. Required for calculation of power
r.1	The proportion of simultaneous tests that are non-centrally located
FDR	The false discovery rate.
use.LFD	Under the default value, 'FALSE', the procedure is controlled under the specified FDR. Specifying "use.LFD=TRUE" will instead first find an FDR, FDR.star, so that the probability that the FDF exceeds FDR.star is less than or equal to the originally specified FDR. Power (either average- or lambda-) or sample size under the average- or lambda- power is then calculated when the procedure is controlled at FDR.star. This is recommended when the distribution of the FDF is wide enough to matter but still well approximated by its asymptotic distribution, e.g. between several hundred and several thousand simultaneous tests. The argument 'N.tests' is required when "use.LFD=TRUE".
N.tests	The number of simultaneous hypothesis tests.
average.power	The desired average power. Sample size calculation requires specification of either 'average.power' or 'L.power'.
L.power	The desired λ -power (see details for explanation). Sample size calculation requires specification of either 'average.power' or 'L.power'.

lambda	The λ -power threshold, required when calculating the λ -power (see details for explanation) or when calculating the sample size required for λ -power.
method	Specify the method whereby the average power is calculated. The selection 'approximate' uses the approximation technique of Izmirlian (2016). To use direct simulation, specify 'simulation'. Specification of 'JL' uses the method of Jung (2005) and Liu (2007). Specification of 'Iz' uses the form which was shown (Izmirlian) to be the large number of tests limit of the average power. Valid selections are 'Iz' (default), 'JL', and any substring of 'approximate' or 'simulation'.
control	Optionally, a list with components with the following components: 'groups', used when distop=3 (F-dist), specifying number of groups. 'version', used only in the 'JL' method, choice 0 gives the 'JL' version as published, whereas choice 1 replaces the FDR with $r.0 * \text{FDR}$ resulting in the infinite simultaneous tests limiting average power, which is the 'Iz' version, but this is redundant because you can specify the 'Iz' method to use this option. 'tol' is a convergence criterion used in iterative methods which is set to $1e-8$ by default 'max.iter' is an iteration limit, set to 1000 by default 'distop', specifying the distribution family of the central and non-centrally located sub-populations. =1 gives normal (2 groups) =2 gives t- (2 groups) and =3 gives F- (2+ groups) 'CS', correlation structure, for use only with 'method="simulation"' which will simulate m simultaneous tests with correlations 'rho' in blocks of size 'n.WC'. Specify as list $\text{CS} = \text{list}(\text{rho}=0.80, \text{n.WC}=50)$ for example
n.sim	If 'simulation' method is chosen you may specify number of simulations. Default is 1000.
temp.file	If 'simulation' method is chosen you may specify a tempfile where the current simulation replicate is updated. Very usefull for batch runs. You can use the included utility 'gentempfilenm'

Details

Power for the BH-FDR procedure on 'm' simultaneous tests, at a given FDR, f , is computed under the following model. A-priori, a proportion, 'r.1', of the 'm' test statistics are given a non-centrally located distribution, the remaining proportion are given a centrally distribution. Each of the non-centrally located test statistics is given the same location parameter. Suppose that a total of 'M' are chosen to have a non-central distribution, that a total of 'J' of all 'm' statistics are declared significant, and that of these, 'S' come from the non-centrally located population. This results in the following table.

1.		rej H0	acc H0	row Total
2.	H0 is FALSE	S	M-S	M
3.	H0 is TRUE	J-S	(m-M)-(J-S)	m-M.1
4.	col Total	J	m-J	m

By default, when 'n.sample' is specified, the function computes the the average power,

AVERAGE POWER: $E[S/M]$

This is approximated using the infinite tests limit.

When 'n.sample' and ' λ ' are both specified, the function computes the λ -power, which is the probability:

$$\lambda\text{-POWER: } P\{S/M > \lambda\}$$

that the true positive fraction S/M exceeds the threshold, ' λ '. This is done using an asymptotic approximation. If the user desires instead the sample size required for given average- or λ - power, then leave the argument 'n.sample' unspecified and specify instead either the 'average.power' or the ' λ ', the threshold for the λ -power. By default, the model uses the assumption that test statistics are distributed as t-distributed as in the two group comparison. If the user specifies a value of 'groups' larger than 2 then the model assumes the test statistics are F-distributed as in the omnibus F-test for any difference. The above approximations are done under the default method. If the user wants to compare with simulated answers to obtain simulation estimates of the 'exact' values, one need only specify 'method="sim"'.

Value

An object of class "pwr" with with components including:

call	The call which produced the result
average.power	Resulting average power.
L.power	When ' λ ' is specified, the λ -power is also computed
L.eq	The λ at which the λ -power and average-power are equal.
n.sample	If 'n.sample' is missing from the argument list, then the sample size required for the specified average- or λ - power.
FDR.star	If "use.LFD=TRUE" was specified, the FDR at which the probability that the FDF exceeds FDR.star is less than or equal to the originally specified FDR.
c.g	The BH-FDR threshold on the scale of the test statistics.
gamma	The proportion of all 'm' tests declared significant.
objective	Result of optimization yielding the 'average.power'.
err.III	Mass on the wrong side of the threshold.
sigma.rtm.Som	Asymptotic variance of the true positive fraction.

Author(s)

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References

- Izmirlian G. (2017) Average Power and λ -power in Multiple Testing Scenarios when the Benjamini-Hochberg False Discovery Rate Procedure is Used. arXiv:1801.03989
- Jung S-H. (2005) Sample size for FDR-control in microarray data analysis. *Bioinformatics*; 21:3097-3104.
- Liu P. and Hwang J-T. G. (2007) Quick calculation for sample size while controlling false discovery rate with application to microarray analysis. *Bioinformatics*; 23:739-746.

See Also[controlFDF](#)**Examples**

```
## Example 1a: average power
rslt.avgp <- pwrFDR(effect.size=0.79, n.sample=46, r.1=2000/54675, FDR=0.15)
rslt.avgp

## Example 1b: lambda-power
rslt.lpwr <- pwrFDR(effect.size=0.79, n.sample=46, r.1=2000/54675,
                  FDR=0.15, lambda=0.80, N.tests=54675)
rslt.lpwr

## Example 1c: sample size required for given average power
rslt.ss.avgp <- pwrFDR(effect.size=0.79, average.power=0.82,
                    r.1=2000/54675, FDR=0.15)
rslt.ss.avgp

## Example 1d: sample size required for given lambda-power
rslt.ss.lpwr <- pwrFDR(effect.size=0.79, L.power=0.82, lambda=0.80,
                    r.1=2000/54675, FDR=0.15, N.tests=54675)
rslt.ss.lpwr

## Example 1e: simulation
rslt.sim <- update(rslt.avgp, method="sim", n.sim=500, N.tests=1000)
rslt.sim

## Example 2: methods for adding, subtracting, multiplying, dividing, exp, log,
## logit and inverse logit
rslt.avgp - rslt.sim
logit(rslt.avgp)      ## etc

## Example 3: Compare the asymptotic distribution of S/M with kernel
## density estimate from simulated data
pdf <- with(detail(rslt.sim)$reps, density(S/M1))

med <- with(detail(rslt.sim)$reps, median(S/M1))
avg <- rslt.sim$average.power
sd <- rslt.sim$v.SoM.emp^0.5

rng.x <- range(pdf$x)
rng.y <- range(c(pdf$y, dnorm(pdf$x, mean=avg, sd=sd)))

plot(rng.x, rng.y, xlab="u", ylab="PDF for S/M", type="n")
with(pdf, lines(x, y))
lines(rep(rslt.sim$average.power, 2), rng.y, lty=2)
lines(pdf$x, dnorm(pdf$x, mean=avg, sd=sd), lty=3)
```

var.J.o.rtm	<i>Calculates asymptotic variance of significant call fraction, J_m/m in the BH-FDR procedure on $m=N$ tests simultaneous tests.</i>
-------------	--

Description

A function which computes the asymptotic variance of the proportion of significant calls, J_N/N .

Usage

```
var.J.o.rtm(x, groups, effect.size, n.sample, r.1, FDR, N.tests, control)
```

Arguments

x	Calls to this function can be made either specifying the single argument, x, which is an object of class "pwr" returned from the function, pwrFDR or by a full call specification, by specifying all the arguments required by the function pwrFDR (see below)
groups	The number of experimental groups to compare. Default value is 2.
effect.size	The effect size (mean over standard deviation) for test statistics having non-zero means. Assumed to be a constant (in magnitude) over non-zero mean test statistics.
n.sample	The number of experimental replicates. Required for calculation of average power or L.power.
r.1	The proportion of simultaneous tests that are non-centrally located
FDR	the false discovery rate.
N.tests	The number of simultaneous tests.
control	Optionally, a list with components with the following components: 'groups', used when distop=3 (F-dist), specifying number of groups. 'version', used only in the 'JL' method, choice 0 gives the 'JL' version as published, whereas choice 1 replaces the FDR with $r.0 * FDR$ resulting in the infinite simultaneous tests limiting average power, which is the 'Iz' version, but this is redundant because you can specify the 'Iz' method to use this option. 'tol' is a convergence criterion used in iterative methods which is set to $1e-8$ by default 'max.iter' is an iteration limit, set to 1000 by default 'distop', specifying the distribution family of the central and non-centrally located sub-populations. =1 gives normal (2 groups) =2 gives t- (2 groups) and =3 gives F- (2+ groups) 'CS', correlation structure, for use only with 'method="simulation"' which will simulate m simultaneous tests with correlations 'rho' in blocks of size 'n.WC'. Specify as list CS = list(rho=0.80, n.WC=50) for example

Details

The proportion of significant calls, J_m/m , i.e., the proportion of all hypothesis tests declared significant by the Benjamini-Hochberg procedure is shown in the cited publication to be root- m consistent and asymptotically normal i.e.

$$m^{1/2} (J_m/m - \gamma) \xrightarrow{D} N(0, \tau^2)$$

which is to say that the distribution of J_m/m is spiked about its mean, γ , and the width dies off as $1/m^{1/2}$. This is of interest in its own right, and is also needed as an intermediate in the calculation of the asymptotic variance of the empirical average power, S_m/M_m . The normal approximation for this latter quantity can be used to power multiple testing experiments on a lower quantile of the empirical mean rather than on its expected value. The width of this distribution is non-negligible for as many as 200 simultaneous tests, where the average power could be 80% but the lower quantile of the empirical average power could be as low as 50%.

Value

Returns a value of class `vvv`, containing components

<code>var.J.o.rtm</code>	The computed asymptotic variance
<code>power</code>	The average power
<code>gamma</code>	The expected proportion of significant calls
<code>c.g</code>	The 'q-value', which is the value of the criterion on the scale of the statistic (t of given number of $2n - 2$ degrees of freedom) which can be used as a per test criterion resulting in the equivalent Benjamini-Hochberg procedure
<code>call</code>	The call which produced the result

Author(s)

Grant Izmirlian <izmirli@nih.gov>

References

Izmirlian G. (2017) Average Power and λ -power in Multiple Testing Scenarios when the Benjamini-Hochberg False Discovery Rate Procedure is Used. arXiv:1801.03989

See Also

[var.rtm.SOM](#) [var.rtm.ToJ](#)

Examples

```
## call using result of pwrFDR
rslt.Iz <- pwrFDR(effect.size=0.79, n.sample=46, r.1=2000/54675, FDR=0.15)
vJ <- var.J.o.rtm(rslt.Iz)

## call via argument list specification
vJ <- var.J.o.rtm(effect.size=0.79, n.sample=46, r.1=2000/54675, FDR=0.15)
```

var.rtm.SoM

Calculates asymptotic variance of true positive fraction, S_m/M_m .

Description

A function which computes the asymptotic variance of the true positive fraction, S_m/M_m in the BH-FDR procedure on $m=N$.tests simultaneous tests.

Usage

```
var.rtm.SoM(x, groups, effect.size, n.sample, r.1, FDR, N.tests, control)
```

Arguments

x	Calls to this function can be made either specifying the single argument, x, which is an object of class "pwr" returned from the function, pwrFDR or by a full call specification, by specifying all the arguments required by the function pwrFDR (see below)
groups	The number of experimental groups to compare. Default value is 2.
effect.size	The effect size (mean over standard deviation) for test statistics having non-zero means. Assumed to be a constant (in magnitude) over non-zero mean test statistics.
n.sample	The number of experimental replicates. Required for calculation of power
r.1	The proportion of simultaneous tests that are non-centrally located
FDR	The false discovery rate.
N.tests	Number of simultaneous tests.
control	Optionally, a list with components with the following components: 'groups', used when distop=3 (F-dist), specifying number of groups. 'version', used only in the 'JL' method, choice 0 gives the 'JL' version as published, whereas choice 1 replaces the FDR with $r.0 * FDR$ resulting in the infinite simultaneous tests limiting average power, which is the 'Iz' version, but this is redundant because you can specify the 'Iz' method to use this option. 'tol' is a convergence criterion used in iterative methods which is set to $1e-8$ by default 'max.iter' is an iteration limit, set to 1000 by default 'distop', specifying the distribution family of the central and non-centrally located sub-populations. =1 gives normal (2 groups) =2 gives t- (2 groups) and =3 gives F- (2+ groups) 'CS', correlation structure, for use only with 'method="simulation"' which will simulate m simultaneous tests with correlations 'rho' in blocks of size 'n.WC'. Specify as list CS = list(rho=0.80, n.WC=50) for example

Details

The true positive fraction, S_m/M_m , i.e., is the proportion of all non-centrally located statistical tests that are declared significant by the Benjamini-Hochberg procedure. It is shown, in the cited publication, to be root-m consistent and asymptotically normal i.e.

$m^{1/2} (S_m/M_m - \text{average.power}) \xrightarrow{D} N(0, \sigma^2)$

which is to say that the distribution of S_m/M_m is spiked about its mean, the average power, and the width dies off as $1/m^{1/2}$. This normal approximation is of interest in its own right and is also useful for powering multiple testing experiments on a more conservative operating characteristic than the average power, $E[S_m/M_m]$. For example, we can power the experiment on the lower 10th percentile of the distribution of S_m/M_m , which is approximated as

$\text{average.power} + \text{qnorm}(0.10) * (\text{vS}/\text{N.tests})^{0.5}$

While the width of this distribution is negligible for micro-array studies, e.g. when $\text{N.tests}=54675$, it is non-negligible for as many as 200 simultaneous tests, where the average power could be 80% but the lower quantile of the empirical average power could be as low as 50%.

Value

Returns a value of class `vvv`, containing components

<code>var.rtm.SoM</code>	The computed asymptotic variance
<code>power</code>	The average power
<code>gamma</code>	The expected proportion of significant calls
<code>c.g</code>	The 'q-value', which is the value of the criterion on the scale of the statistic (t of given number of 2n - 2 degrees of freedom) which can be used as a per test criterion resulting in the equivalent Benjamini-Hochberg procedure
<code>call</code>	The call which produced the result

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References

Izmirlian G. (2017) Average Power and λ -power in Multiple Testing Scenarios when the Benjamini-Hochberg False Discovery Rate Procedure is Used. arXiv:1801.03989

See Also

[var.J.o.rtm](#) [var.rtm.ToJ](#)

Examples

```
## call using result of pwrFDR
rslt.Iz <- pwrFDR(effect.size=0.79, n.sample=46, r.1=2000/54675, FDR=0.15)
vS <- var.rtm.SoM(rslt.Iz)

## call via argument list specification
vS <- var.rtm.SoM(effect.size=0.79, n.sample=46, r.1=2000/54675, FDR=0.15)
```

var.rtm.ToJ *Calculates asymptotic variance of false discovery fraction, T_N/J_N .*

Description

A function which computes the asymptotic variance of the false discovery fraction, T_N/J_N .

Usage

```
var.rtm.ToJ(x, groups, effect.size, n.sample, r.1, FDR, N.tests, control)
```

Arguments

x	Calls to this function can be made either specifying the single argument, x, which is an object of class "pwr" returned from the function, pwrFDR or by a full call specification, by specifying all the arguments required by the function pwrFDR (see below)
groups	The number of experimental groups to compare. Default value is 2.
effect.size	The effect size (mean over standard deviation) for test statistics having non-zero means. Assumed to be a constant (in magnitude) over non-zero mean test statistics.
n.sample	The number of experimental replicates. Required for calculation of power
r.1	The proportion of simultaneous tests that are non-centrally located
FDR	The false discovery rate.
N.tests	Number of simultaneous tests.
control	Optionally, a list with components with the following components: 'groups', used when distop=3 (F-dist), specifying number of groups. 'version', used only in the 'JL' method, choice 0 gives the 'JL' version as published, whereas choice 1 replaces the FDR with $r.0 * FDR$ resulting in the infinite simultaneous tests limiting average power, which is the 'Iz' version, but this is redundant because you can specify the 'Iz' method to use this option. 'tol' is a convergence criterion used in iterative methods which is set to $1e-8$ by default 'max.iter' is an iteration limit, set to 1000 by default 'distop', specifying the distribution family of the central and non-centrally located sub-populations. =1 gives normal (2 groups) =2 gives t- (2 groups) and =3 gives F- (2+ groups) 'CS', correlation structure, for use only with 'method="simulation"' which will simulate m simultaneous tests with correlations 'rho' in blocks of size 'n.WC'. Specify as list CS = list(rho=0.80, n.WC=50) for example

Details

The false discovery fraction (FDF), T_m/J_m , i.e., is the proportion of all centrally located statistical tests that are declared significant by the Benjamini-Hochberg procedure. It is shown, in the cited publication, to be root-m consistent and asymptotically normal i.e.

$$m^{1/2} (T_m/J_m - (1-r.1) f) \xrightarrow{D} N(0, \sigma^2)$$

which is to say that the distribution of T_m/J_m is spiked about its mean, the false discovery rate, and the width dies off as $1/m^{1/2}$. This normal approximation is of interest in its own right, allowing tighter control over the false discovery fraction that afforded by controlling its mean, the false discovery rate. One can determine the FDR required to bound the FDF by a given value with a probability no less than a given value.

Value

Returns a value of class `vvv`, containing components

<code>var.J.o.rtm</code>	The computed asymptotic variance
<code>power</code>	The average power
<code>gamma</code>	The expected proportion of significant calls
<code>c.g</code>	The 'q-value', which is the value of the criterion on the scale of the statistic (t of given number of $2n - 2$ degrees of freedom) which can be used as a per test criterion resulting in the equivalent Benjamini-Hochberg procedure
<code>call</code>	The call which produced the result

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References

Izmirlian G. (2017) Average Power and λ -power in Multiple Testing Scenarios when the Benjamini-Hochberg False Discovery Rate Procedure is Used. arXiv:1801.03989

See Also

[var.J.o.rtm](#) [var.rtm.SOM](#)

Examples

```
## call using result of pwrFDR
rslt.Iz <- pwrFDR(effect.size=0.79, n.sample=46, r.1=2000/54675, FDR=0.15)
vJ <- var.J.o.rtm(rslt.Iz)

## call via argument list specification
vJ <- var.J.o.rtm(effect.size=0.79, n.sample=46, r.1=2000/54675, FDR=0.15)
```

%over%

Division operator with divide by zero clobbering

Description

$x \text{ %over% } y = x/y$ when $y \neq 0$, equals 0 when $y = 0$.

Usage

$x \text{ %over% } y$

Arguments

x, y Numeric or complex vectors or objects that can be coerced to such.

Value

x/y when $y \neq 0$, otherwise 0.

Author(s)

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