

Package ‘pch’

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Type Package

Title Piecewise Constant Hazards Models for Censored and Truncated Data

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Description Using piecewise constant hazards models is a very flexible approach for the analysis of survival data. The time line is divided into sub-intervals; for each interval, a different hazard is estimated using Poisson regression.

Depends survival

Imports stats

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Description

This function estimates piecewise exponential models on right-censored, left-truncated data. The effect of covariates, and not just the baseline hazard, varies across intervals. Moreover, a special handling of zero-risk regions is implemented. Differently from the `phreg` function available in the `eha` package, this function is mainly intended to be used as a nonparametric maximum likelihood estimator.

Usage

```
pchreg(formula, breaks, data, weights, splinex = NULL)
```

Arguments

<code>formula</code>	an object of class “ <code>formula</code> ”: a symbolic description of the regression model. The response must be a <code>Surv</code> object as returned by <code>Surv</code> (see ‘Details’).
<code>breaks</code>	either a numeric vector of two or more unique cut points or a single number (greater than or equal to 1) giving the number of intervals into which the time variable is to be cut. If missing, the number of intervals is set to $\max(5, \min(50, \text{ceiling}(n1/q/5)))$, where $n1$ is the number of events, and q is the number of predictors.
<code>data</code>	an optional data frame containing the variables in the model.
<code>weights</code>	an optional vector of weights to be used in the fitting process.
<code>splinex</code>	either <code>NULL</code> , or an object created with <code>splinex</code> (see ‘Details’).

Details

The left side of `formula` must be of the form `Surv(time, event)` if the data are right-censored, and `Surv(time0, time, event)` if the data are right-censored and left-truncated ($time0 < time$). Using `Surv(time)` is also allowed and indicates that the data are neither censored nor truncated. Note that the response variable (and thus the `breaks`) can be negative.

To fit the model, the time interval is first divided in sub-intervals as defined by `breaks`. When the location of `breaks` is not specified, the empirical quantiles of `time[event == 1]` are used as cut points. If there is a probability mass, this may result in two or more `breaks` being equal: in this case, an interval that only includes the mass point is created automatically.

A different constant hazard (exponential) model is then fitted in each sub-interval, using Poisson regression to model the log-hazard as a linear function of covariates. Within each interval, the risk of the event may be zero at some covariate values. For each covariate x , the algorithm will try to identify a threshold c such that all events (in any given interval) occur when $x < c$ ($x > c$). A zero risk will be automatically fitted above (below) the threshold, using an offset of -100 on the log-hazard.

This type of model can be utilized to obtain a nonparametric maximum likelihood estimator of a conditional distribution, achieving the flexibility of nonparametric estimators while keeping the

model parametric in practice. Users unfamiliar with this approach are recommended reading Geman and Hwang (1982) for an overview, and the paper by Akerberg, Chen and Hahn (2012) describing how this approach can be applied to simplify inference in two-step semiparametric models.

The number of parameters is equal to the number of intervals, multiplied by the number of covariates. Both quantities are usually supposed to increase with the sample size. The special function `splines` is a handy tool that facilitates implementing the linear predictor. When `splines` is not NULL, each column of the original design matrix (as defined by `formula`) is automatically replaced with the corresponding spline basis. See the documentation of `splines` for details.

Value

An object of class “pch”, which is a list with the following items:

<code>call</code>	the matched call.
<code>beta</code>	a matrix of regression coefficients. Rows correspond to covariates, while columns correspond to different time intervals.
<code>breaks</code>	the used cut points, with attributes 'h' indicating the length of each interval, and 'k' denoting the number of intervals.
<code>covar</code>	the estimated asymptotic covariance matrix.
<code>logLik</code>	the value of the maximized log-likelihood, with attribute “df” indicating the number of free model parameters.
<code>lambda</code>	the fitted hazard values in each interval.
<code>Lambda</code>	the fitted cumulative hazard values at the end of each interval.
<code>mf</code>	the model frame used.
<code>x</code>	the model matrix.
<code>conv.status</code>	a code indicating the convergence status. It takes value 0 if the algorithm has converged successfully; 1 if convergence has not been achieved; and 2 if, although convergence has been achieved, more than 1% of observations have an associated survival numerically equal to zero, indicating that the solution is not well-behaved or the model is misspecified.

The accessor functions `summary`, `coef`, `predict`, `nobs`, `logLik`, `AIC`, `BIC` can be used to extract information from the fitted model. This function is mainly intended for prediction and simulation: see `predict.pch`.

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References

- Akerberg, D., Chen, X., and Hahn, J. (2012). A Practical Asymptotic Variance Estimator for Two-Step Semiparametric Estimators. *The Review of Economics and Statistics*, 94(2), 481-498.
- Friedman, M. (1982). Piecewise Exponential Models for Survival Data with Covariates. *The Annals of Statistics*, 10(1), pp. 101-113.
- Geman, S., and Hwang, C.R. (1982). Nonparametric Maximum Likelihood Estimation by the Method of Sieves. *The Annals of Statistics*, 10(2), 401-414.

See Also

[predict.pch](#), [splineX](#)

Examples

```
# using simulated data

n <- 1000
x <- runif(n)
time <- rnorm(n, 1 + x, 1 + x)
cens <- rnorm(n,2,2)
y <- pmin(time,cens) # censored variable
d <- (time <= cens) # indicator of the event

model <- pchreg(Surv(y,d) ~ x, breaks = 20)

# see the documentation of predict.pch
```

predict.pch

Prediction from Fitted Piecewise Constant Hazards Models

Description

This function returns predictions for an object of class “pch”, usually the result of a call to [pchreg](#).

Usage

```
## S3 method for class 'pch'
predict(object, type = c("distr", "quantile", "sim"),
        newdata, p, sim.method = c("quantile", "sample"), ...)
```

Arguments

object	a “pch” object.
type	a character string (just the first letter can be used) indicating the type of prediction. See ‘Details’.
newdata	optional data frame in which to look for variables with which to predict. It must include all the covariates that enter the model and, if type = ‘distr’, also the time variable. If omitted, the original data are used.
p	vector of quantiles, to be specified if type = “quantile”.
sim.method	a character string (just the first letter can be used) indicating the simulation method if type = “sim”.
...	for future methods.

Details

If `type = "distr"` (the default), this function returns a data frame with columns (`haz`, `Haz`, `Surv`, `f`) containing the fitted values of the hazard function, the cumulative hazard, the survival function, and the probability density function, respectively.

If `type = "quantile"`, a data frame with the fitted quantiles (corresponding to the supplied values of `p`) is returned.

If `type = "sim"`, new data are simulated from the fitted model. Two methods are available: with `sim.method = "quantile"`, data are simulated by applying the estimated quantile function to a vector of random uniform numbers; if `sim.method = "sample"`, the quantile function is only used to identify the time interval, and the data are resampled from the observed values in the interval. The second method only works properly if there is a large number of breaks. However, it is less sensitive to model misspecification and facilitates sampling from distributions with a probability mass or non compact support.

If the data are censored, some high quantiles may not be estimated: beyond the last observable quantile, all types of predictions (including `type = "sim"` with `sim.method = "sample"`) are computed assuming that the hazard remains constant after the last interval.

Predictions are computed at `newdata`, if supplied. In the current implementation, `newdata` must include all the variables that are needed for the prediction. Note that if `type = "distr"`, new values of the response variable are also required.

Value

If `type = "distr"`, a 4-columns data frame with columns (`haz`, `Haz`, `Surv`, `f`). If `type = "quantile"`, a named data frame with a column for each value of `p`. If `type = "sim"`, a vector of simulated data.

The presence of NA values will always cause the prediction to be NA.

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See Also

[pchreg](#)

Examples

```
# using simulated data

##### EXAMPLE 1 - Continuous distribution #####

n <- 1000
x <- runif(n)
time <- rnorm(n, 1 + x, 1 + x)
cens <- rnorm(n,2,2)
y <- pmin(time,cens) # censored variable
d <- (time <= cens) # indicator of the event
model <- pchreg(Surv(y,d) ~ x, breaks = 20)
```

```

# predicting hazard, cumulative hazard, survival, density

pred <- predict(model, type = "distr")
plot(pred$Surv, 1 - pnorm(y, 1 + x, 1 + x)); abline(0,1)
# true vs fitted survival

# predicting quartiles

predQ <- predict(model, type = "quantile", p = c(0.25,0.5,0.75))
plot(x,time)
points(x, qnorm(0.5, 1 + x, 1 + x), col = "red") # true median
points(x, predQ$p0.5, col = "green")           # fitted median

# simulating new data

tsim1 <- predict(model, type = "sim", sim.method = "quantile")
tsim2 <- predict(model, type = "sim", sim.method = "sample")

qt <- quantile(time, (1:9)/10) # deciles of t
q1 <- quantile(tsim1, (1:9)/10) # deciles of tsim1
q2 <- quantile(tsim2, (1:9)/10) # deciles of tsim2

par(mfrow = c(1,2))
plot(qt,q1, main = "sim.method = 'quantile'"); abline(0,1)
plot(qt,q2, main = "sim.method = 'sample'"); abline(0,1)

# prediction with newdata

predict(model, type = "distr", newdata = data.frame(y = 0, x = 0.5)) # need y!
predict(model, type = "quantile", p = 0.5, newdata = data.frame(x = 0.5))
predict(model, type = "sim", sim.method = "sample", newdata = data.frame(x = c(0,1)))

##### EXAMPLE 2 - non-compact support #####
# to simulate, sim.method = "sample" is recommended #####

n <- 1000
t <- c(rnorm(n,-5), rnorm(n,5))
model <- pchreg(Surv(t) ~ 1, breaks = 30)

tsim1 <- predict(model, type = "sim", sim.method = "quantile")
tsim2 <- predict(model, type = "sim", sim.method = "sample")

par(mfrow = c(1,3))
hist(t, main = "true distribution")
hist(tsim1, main = "sim.method = 'quantile'") # the empty spaces are 'filled'
hist(tsim2, main = "sim.method = 'sample'") # perfect!

```

splineX *Including Interactions and Splines in Piecewise Constant Hazard Regression*

Description

This function can be used within a call to `pchreg` to automatically include spline functions in the linear predictor of the model.

Usage

```
splineX(method = c("ns", "bs"), df = 2, degree = 2, v = 0.98, ...)
```

Arguments

<code>method</code>	a character string indicating whether natural splines (<code>ns</code>) or B-splines (<code>bs</code>) should be used. Default is “ <code>ns</code> ”.
<code>df</code>	the degrees of freedom of the spline basis.
<code>degree</code>	the degree of the polynomial (only for <code>bs</code>).
<code>v</code>	a value between 0 and 1 determining how many principal components of the design matrix must be used in model fitting (see “Details”).
<code>...</code>	for future arguments.

Details

The piecewise constant hazard model implemented by `pchreg` can be used as a nonparametric maximum likelihood estimator, in which the number of parameters is allowed to increase with the sample size in order to achieve any desired flexibility. Modeling the effect of covariates is as important as setting a sufficiently large number of breaks.

By letting `splineX = splineX(...)`, each column of the original design matrix is automatically replaced by the corresponding spline basis, defined by `method`, `df`, and `degree`.

This modeling approach has the drawback of generating a potentially large design matrix: to reduce its dimension, you can choose $v < 1$. In this case, only the principal components explaining at least a proportion v of the variance are used to fit the model (see “Examples”).

Value

The function returns its arguments, to be passed to an internal function `build.splineX` that actually computes the design matrix.

Note

A multidimensional spline can be created by including a term like, for example `ns(x1, df)*ns(x2, df)`. This is not supported by `splineX`, as it may generate a very large design matrix.

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See Also[pchreg](#)**Examples**

```
require(splines)
n <- 1000
x1 <- runif(n,-2,2)
x2 <- runif(n,-2,2)
t <- rexp(n, exp(-abs(x1 - x2)))

# a simple model
model1 <- pchreg(Surv(t) ~ x1 + x2)

# using splinex: the same as ~ ns(x1, df = 2) + ns(x2, df = 2)
model2 <- pchreg(Surv(t) ~ x1 + x2, splinex = splinex("ns", v = 1))

# include interaction: ~ ns(x1, df = 2) + ns(x2, df = 2) + ns(x1*x2, df = 2)
model3 <- pchreg(Surv(t) ~ x1 * x2, splinex = splinex("ns", v = 1))

# the same as model 3, only keep the PCs explaining at least 95 percent of the variance
model4 <- pchreg(Surv(t) ~ x1 * x2, splinex = splinex("ns", v = 0.95))

# true CDF vs fitted

trueF <- pexp(t, exp(-abs(x1 - x2)))
par(mfrow = c(2,2))
plot(trueF, 1 - predict(model1)$Surv); abline(0,1, col = "red") # does not fit
plot(trueF, 1 - predict(model2)$Surv); abline(0,1, col = "red") # neither
plot(trueF, 1 - predict(model3)$Surv); abline(0,1, col = "red") # great!
plot(trueF, 1 - predict(model4)$Surv); abline(0,1, col = "red") # almost as good
```


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