Package ‘nprotreg’

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Title Nonparametric Rotations for Sphere-Sphere Regression

Version 1.0.1

Description Fits sphere-sphere regression models by estimating locally weighted rotations. Simulation of sphere-sphere data according to non-rigid rotation models. Provides methods for bias reduction applying iterative procedures within a Newton-Raphson learning scheme. Cross-validation is exploited to select smoothing parameters. See Marco Di Marzio, Agnese Panzera & Charles C. Taylor (2018) <doi:10.1080/01621459.2017.1421542>.

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R topics documented:

`convert_cartesian_to_spherical` *Converts Cartesian to Spherical Coordinates.*

Description

The Cartesian coordinates of points on a 3-dimensional sphere with unit radius and center at the origin are converted to the equivalent longitude and latitude coordinates, measured in radians.

Usage

```
convert_cartesian_to_spherical(cartesian_coords)
```

Arguments

`cartesian_coords`
A matrix whose rows contain the Cartesian coordinates of the specified points.

Value

A matrix of rows containing the longitude and latitude of specific points on a 3-dimensional sphere.

See Also

http://mathworld.wolfram.com/SphericalCoordinates.html.

Other Conversion functions: `convert_spherical_to_cartesian()`

Examples

```
library(nprotreg)

# Define the Cartesian coordinates of the North and South Poles.

north_pole <- cbind(0, 0, 1)
south_pole <- cbind(0, 0, -1)
cartesian_coords <- rbind(north_pole, south_pole)

# Get the corresponding Spherical coordinates.

spherical_coords <- convert_cartesian_to_spherical(cartesian_coords)
```

convert_spherical_to_cartesian
Converts Spherical to Cartesian Coordinates.

Description

The longitude and latitude coordinates of points on a 3-dimensional sphere with unit radius and center at the origin are converted to the equivalent Cartesian coordinates.

Usage

```
convert_spherical_to_cartesian(spherical_coords)
```

Arguments

`spherical_coords`
A matrix of rows containing the longitude and latitude, measured in radians, of specific points on a 3-dimensional sphere.

Value

A matrix whose rows contain the Cartesian coordinates of the specified points.

See Also

http://mathworld.wolfram.com/SphericalCoordinates.html.

Other Conversion functions: `convert_cartesian_to_spherical()`

Examples

```
library(nprotreg)

# Define the Spherical coordinates of the North and South Poles.

north_pole <- cbind(0, pi / 2)
south_pole <- cbind(0, - pi / 2)
spherical_coords <- rbind(north_pole, south_pole)

# Get the corresponding Cartesian coordinates.

cartesian_coords <- convert_spherical_to_cartesian(spherical_coords)
```

cross_validate_concentration

Cross-validates The Concentration Parameter In A 3D Spherical Regression.

Description

Returns a cross-validated value for the concentration parameter in a 3D regression, relating specific explanatory points to response ones, given a weighting scheme for the observed data set. This function supports the method for sphere-sphere regression proposed by Di Marzio et al. (2018).

Usage

```
cross_validate_concentration(
  concentration_upper_bound = 10,
  explanatory_points,
  response_points,
  weights_generator = weight_explanatory_points,
  number_of_expansion_terms = 1,
  number_of_iterations = 1,
  allow_reflections = FALSE
)
```

Arguments

`concentration_upper_bound`
: A scalar numeric value representing the upper end-point of the interval to be searched for the required minimizer. Defaults to `10`.

`explanatory_points`
: An m-by-3 matrix whose rows contain the Cartesian coordinates of the explanatory points used to calculate the regression estimators.

`response_points`
: An m-by-3 matrix whose rows contain the Cartesian coordinates of the response points corresponding to the explanatory points.

`weights_generator`
: A function that, given a matrix of n evaluation points, returns an m-by-n matrix whose j-th column contains the weights assigned to the explanatory points while analyzing the j-th evaluation point. Defaults to `weight_explanatory_points`.

`number_of_expansion_terms`
: The number of terms to be included in the expansion of the matrix exponential applied while approximating a local rotation matrix. Must be `1` or `2`. Defaults to `1`.

`number_of_iterations`
: The number of rotation fitting steps to be executed. At each step, the points estimated during the previous step are exploited as the current explanatory points. Defaults to `1`.

allow_reflections
: A logical scalar value. If set to TRUE signals that reflections are allowed. Defaults to FALSE. It is ignored if number_of_expansion_terms is 2.

Details

Function weights_generator must be prototyped as having the following three arguments:

evaluation_points a matrix whose n rows are the Cartesian coordinates of given evaluation points.

explanatory_points a matrix whose m rows are the Cartesian coordinates of given explanatory points.

concentration A non negative scalar whose reciprocal value is proportional to the bandwidth applied while estimating a spherical regression model.

It is also expected that weights_generator will return a non NULL numerical m-by-n matrix whose j-th column contains the weights assigned to the explanatory points while analyzing the j-th evaluation point.

Value

A list having two components, concentration, a scalar, numeric value representing the cross-validated concentration for the specified 3D regression, and objective, the value of the cross-validating objective function at argument concentration.

References

Marco Di Marzio, Agnese Panzera & Charles C. Taylor (2018) Nonparametric rotations for sphere-sphere regression, Journal of the American Statistical Association, DOI: 10.1080/01621459.2017.1421542

See Also

Other Regression functions: fit_regression(), get_equally_spaced_points(), get_skew_symmetric_matrix(), simulate_regression(), simulate_rigid_regression(), weight_explanatory_points()

Examples

```
library(nprotreg)

# Define a matrix of explanatory points.

number_of_explanatory_points <- 50

explanatory_points <- get_equally_spaced_points(
  number_of_explanatory_points)

# Define a matrix of response points by simulation.

local_rotation_composer <- function(point) {
  independent_components <- (1 / 2) *
    c(exp(2.0 * point[3]), - exp(2.0 * point[2]), exp(2.0 * point[1]))
```

```
}

local_error_sampler <- function(point) {
  rnorm(3)
}

response_points <- simulate_regression(explanatory_points,
                                       local_rotation_composer,
                                       local_error_sampler)

# Define an upper bound for concentration.

concentration_upper_bound <- 1

# Use default weights generator.

weights_generator <- weight_explanatory_points

# Cross-validate concentration parameter.

cv_info <- cross_validate_concentration(
  concentration_upper_bound,
  explanatory_points,
  response_points,
  weights_generator,
  number_of_expansion_terms = 1,
  number_of_iterations = 2,
  allow_reflections = FALSE
)

# Get the cross-validated concentration value.

cat("cross-validated concentration value: \n")
print(cv_info$concentration)
```

---

expm *Computes the Exponential of a 3D Skew Symmetric Matrix.*

Description

The exponential of a skew-symmetric matrix is computed by means of the Rodrigues' formula.

Usage

```
expm(skew_symmetric_matrix)
```

Arguments

`skew_symmetric_matrix`
A 3-by-3 skew-symmetric matrix.

Value

A 3-by-3 rotation matrix representing the exponential of the specified skew-symmetric matrix.

`fit_regression` *Fits a 3D Spherical Regression.*

Description

Returns 3D spherical points obtained by locally rotating the specified evaluation points, given an approximated model for local rotations and a weighting scheme for the observed data set. This function implements the method for sphere-sphere regression proposed by Di Marzio et al. (2018).

Usage

```
fit_regression(
  evaluation_points,
  explanatory_points,
  response_points,
  concentration,
  weights_generator = weight_explanatory_points,
  number_of_expansion_terms = 1,
  number_of_iterations = 1,
  allow_reflections = FALSE
)
```

Arguments

`evaluation_points`
: An n-by-3 matrix whose rows contain the Cartesian coordinates of the points at which the regression will be estimated.

`explanatory_points`
: An m-by-3 matrix whose rows contain the Cartesian coordinates of the explanatory points used to calculate the regression estimators.

`response_points`
: An m-by-3 matrix whose rows contain the Cartesian coordinates of the response points corresponding to the explanatory points.

`concentration`
: A non negative scalar whose reciprocal value is proportional to the bandwidth applied while estimating a spherical regression model.

`weights_generator`
: A function that, given a matrix of n evaluation points, returns an m-by-n matrix whose j-th column contains the weights assigned to the explanatory points while analyzing the j-th evaluation point. Defaults to `weight_explanatory_points`.

`number_of_expansion_terms`
: The number of terms to be included in the expansion of the matrix exponential applied while approximating a local rotation matrix. Must be `1` or `2`. Defaults to `1`.

`number_of_iterations`
: The number of rotation fitting steps to be executed. At each step, the points estimated during the previous step are exploited as the current explanatory points. Defaults to `1`.

`allow_reflections`
: A logical scalar value. If set to `TRUE` signals that reflections are allowed. Defaults to `FALSE`. It is ignored if `number_of_expansion_terms` is `2`.

Details

Function `weights_generator` must be prototyped as having the following three arguments:

- `evaluation_points` a matrix whose n rows are the Cartesian coordinates of given evaluation points.
- `explanatory_points` a matrix whose m rows are the Cartesian coordinates of given explanatory points.
- `concentration` A non negative scalar whose reciprocal value is proportional to the bandwidth applied while estimating a spherical regression model.

It is also expected that `weights_generator` will return a non `NULL` numerical m-by-n matrix whose j-th column contains the weights assigned to the explanatory points while analyzing the j-th evaluation point.

Value

A `number_of_iterations`-length vector of lists, with the `s`-th list having two components, `fitted_response_points`, an n-by-3 matrix whose rows contain the Cartesian coordinates of the fitted points at iteration `s`, and `explanatory_points`, an m-by-3 matrix whose rows contain the Cartesian coordinates of the points exploited as explanatory at iteration `s`.

References

Marco Di Marzio, Agnese Panzera & Charles C. Taylor (2018) Nonparametric rotations for sphere-sphere regression, Journal of the American Statistical Association, DOI: 10.1080/01621459.2017.1421542

See Also

Other Regression functions: `cross_validate_concentration()`, `get_equally_spaced_points()`, `get_skew_symmetric_matrix()`, `simulate_regression()`, `simulate_rigid_regression()`, `weight_explanatory_po`

Examples

```
library(nprotreg)

# Create 100 equally spaced design points on the sphere.

number_of_explanatory_points <- 100

explanatory_points <- get_equally_spaced_points(
  number_of_explanatory_points
)
```

```
# Define the regression model, where the rotation for a given "point"
# is obtained from the exponential of a skew-symmetric matrix with the
# following components.

local_rotation_composer <- function(point) {
  independent_components <- (1 / 8) *
    c(exp(2.0 * point[3]), - exp(2.0 * point[2]), exp(2.0 * point[1]))
}

# Define an error term given by a small rotation, similarly defined
# from a skew-symmetric matrix with random entries.

local_error_sampler <- function(point) {
  rnorm(3, sd = .01)
}

# Generate the matrix of responses, using the regression model
# and the error model.

response_points <- simulate_regression(
  explanatory_points,
  local_rotation_composer,
  local_error_sampler
)

# Create some "test data" for which the response will be predicted.

evaluation_points <- rbind(
  cbind(.5, 0, .8660254),
  cbind(-.5, 0, .8660254),
  cbind(1, 0, 0),
  cbind(0, 1, 0),
  cbind(-1, 0, 0),
  cbind(0, -1, 0),
  cbind(.5, 0, -.8660254),
  cbind(-.5, 0, -.8660254)
)

# Define a weight function for nonparametric fit.

weights_generator <- weight_explanatory_points

# Set the concentration parameter.

concentration <- 5

# Or obtain this by cross-validation: see
# the `cross_validate_concentration` function.

# Fit regression.

fitted_model <- fit_regression(
```

```
  evaluation_points,
  explanatory_points,
  response_points,
  concentration,
  weights_generator,
  number_of_expansion_terms = 1,
  number_of_iterations = 2
)

# Extract the point corresponding to the
# second evaluation point fitted at
# the first iteration.

cat("Point fitted at iteration 1 corresponding to the second evaluation point: \n")
cat(fitted_model[[1]]$fitted_response_points[2, ], "\n")

## Not run:
# Create some plots to view the results.

# 3D plot.

library(rgl)

plot3d(
  explanatory_points,
  type = "n",
  xlab = "x",
  ylab = "y",
  zlab = "z",
  box = TRUE,
  axes = TRUE
)
spheres3d(0, 0, 0, radius = 1, lit = FALSE, color = "white")
spheres3d(0, 0, 0, radius = 1.01, lit = FALSE, color = "black", front = "culled")
text3d(c(0, 0, 1), text = "N", adj = 0)

ll <- 10
vv1 <- (ll - (0:(ll))) / ll
vv2 <- 1 - vv1
plot3d(explanatory_points, add = TRUE, col = 2)
for (i in 1:dim(explanatory_points)[1]) {
  m <- outer(vv1, explanatory_points[i,], "*") +
    outer(vv2, response_points[i,], "*")
  m <- m / sqrt(apply(m ^ 2, 1, sum))
  lines3d(m, col = 3)
}

plot3d(evaluation_points, add = TRUE, col = 4)

for (i in 1:dim(evaluation_points)[1]) {
  m <- outer(vv1, evaluation_points[i,], "*") +
    outer(vv2, fitted_model[[1]]$fitted_response_points[i,], "*")
  m <- m / sqrt(apply(m ^ 2, 1, sum))
```

```
  lines3d(m, col = 1)
}

# 2D plot.

explanatory_spherical_coords <- convert_cartesian_to_spherical(explanatory_points)
response_spherical_coords <- convert_cartesian_to_spherical(response_points)

plot(
  x = explanatory_spherical_coords[, 1],
  y = explanatory_spherical_coords[, 2],
  pch = 20,
  cex = .7,
  col = 2,
  xlab = "longitude",
  ylab = "latitude"
)

for (i in 1:dim(explanatory_spherical_coords)[1]) {
  column <- 1
  if ((explanatory_spherical_coords[i, 1] - response_spherical_coords[i, 1]) ^ 2 +
      (explanatory_spherical_coords[i, 2] - response_spherical_coords[i, 2]) ^ 2 > 4)
        column <- "grey"
  lines(
    c(explanatory_spherical_coords[i, 1], response_spherical_coords[i, 1]),
    c(explanatory_spherical_coords[i, 2], response_spherical_coords[i, 2]),
    col = column
  )
}

evaluation_spherical_coords <- convert_cartesian_to_spherical(
  evaluation_points
)

fitted_response_spherical_coords <- convert_cartesian_to_spherical(
  fitted_model[[1]]$fitted_response_points
)

points(
  x = evaluation_spherical_coords[, 1],
  y = evaluation_spherical_coords[, 2],
  pch = 20,
  cex = .7,
  col = 4
)

for (i in 1:dim(evaluation_spherical_coords)[1]) {
  column <- 3
  if ((evaluation_spherical_coords[i, 1] - fitted_response_spherical_coords[i, 1]) ^ 2 +
     (evaluation_spherical_coords[i, 2] - fitted_response_spherical_coords[i, 2]) ^ 2 > 4)
        column <- "grey"
  lines(
    c(evaluation_spherical_coords[i, 1], fitted_response_spherical_coords[i, 1]),
```

```
    c(evaluation_spherical_coords[i, 2], fitted_response_spherical_coords[i, 2]),
    col = column
  )
}


## End(Not run)
```

---

get_equally_spaced_points

Generates Equally Spaced Points On A 3D Sphere.

Description

Generates points approximately equally spaced on a 3D sphere.

Usage

```
get_equally_spaced_points(number_of_points)
```

Arguments

`number_of_points`
A scalar, positive integer representing the number of points to get.

Value

A `number_of_points`-by-3 matrix whose rows contain the Cartesian coordinates of the equally spaced points.

See Also

Other Regression functions: `cross_validate_concentration()`, `fit_regression()`, `get_skew_symmetric_matrix()`, `simulate_regression()`, `simulate_rigid_regression()`, `weight_explanatory_points()`

Examples

```
library(nprotreg)

# Define the number of points to get.

number_of_points <- 5

# Get the Cartesian coordinates of the equally spaced points.

equally_spaced_points <- get_equally_spaced_points(number_of_points)
```

`get_skew_symmetric_matrix` *Gets a 3-by-3 Skew Symmetric Matrix.*

Description

Returns the 3-by-3 skew symmetric matrix having the specified independent components.

Usage

```
get_skew_symmetric_matrix(independent_components)
```

Arguments

`independent_components`
A vector containing the independent components of the matrix to get.

Details

Given a vector of components, say $[x, y, z]$, this function will return matrix

$$\begin{matrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{matrix}$$

Value

The 3-by-3 skew symmetric matrix corresponding to the specified independent components.

See Also

https://en.wikipedia.org/wiki/Skew-symmetric_matrix.

Other Regression functions: `cross_validate_concentration()`, `fit_regression()`, `get_equally_spaced_points()`, `simulate_regression()`, `simulate_rigid_regression()`, `weight_explanatory_points()`

Examples

```
library(nprotreg)

# Define a vector of independent components.

independent_components <- cbind(1, 2, 3)

# Get the corresponding 3-by-3 skew symmetric matrix.

m <- get_skew_symmetric_matrix(independent_components)
```

`logm` *Computes the Logarithm of a 3D Rotation Matrix.*

Description

Computes the Logarithm of a 3D Rotation Matrix.

Usage

```
logm(rotation_matrix)
```

Arguments

`rotation_matrix`
A 3-by-3 rotation matrix.

Value

A 3-by-3 skew-symmetric matrix representing the logarithm of the specified rotation matrix.

`nprotreg` *nprotreg: Nonparametric Rotations for Sphere-Sphere Regression.*

Description

The nprotreg package provides several categories of functions.

Regression functions

Regression functions provide support for simulating and fitting 3-dimensional spherical regression models.

- `cross_validate_concentration`
- `fit_regression`
- `get_equally_spaced_points`
- `get_skew_symmetric_matrix`
- `simulate_regression`
- `simulate_rigid_regression`
- `weight_explanatory_points`

Conversion functions

Conversion functions transform coordinates of points on a 3-dimensional sphere with unit radius and center at the origin.

- `convert_cartesian_to_spherical`
- `convert_spherical_to_cartesian`

`simulate_regression` *Simulates a 3D Spherical Regression.*

Description

Returns the response points corresponding to the specified explanatory points, given a model for local rotations and an error term sampler.

Usage

```
simulate_regression(
  explanatory_points,
  local_rotation_composer,
  local_error_sampler
)
```

Arguments

`explanatory_points`
: An *m*-by-3 matrix whose rows contain the Cartesian coordinates of the points at which the regression will be simulated.

`local_rotation_composer`
: A function that returns a 3-length numeric vector representing the independent components of a skew symmetric matrix local to an explanatory point, given its Cartesian coordinates.

`local_error_sampler`
: A function that returns a 3-length numeric vector representing a sampled error term local to an explanatory point, given its Cartesian coordinates.

Details

Let E be the *m*-by-3 matrix of explanatory points. This function will return an *m*-by-*3* matrix whose i-th row is obtained by transposition of the following expression:

$$exp(\Phi(\epsilon(x)))exp(\Phi(s(x)))x$$

where x is the transpose of the i-th row of E. Terms $\epsilon(x)$ and $s(x)$ are obtained by evaluating at x functions `local_error_sampler` and `local_rotation_composer`, respectively, while matrix $\Phi(c)$, for a 3-length numeric vector c, is the skew symmetric matrix having its independent components represented by the entries of c (for a thorough discussion, see function `get_skew_symmetric_matrix`).

Functions `local_error_sampler` and `local_rotation_composer` must be prototyped as having one argument, `point`, representing the Cartesian coordinates of a point on a 3D sphere, and returning a non `NULL` numerical object having length equal to `3`.

Value

An *m*-by-*3* matrix whose rows contain the Cartesian coordinates of the response points corresponding to the explanatory points.

See Also

Other Regression functions: cross_validate_concentration(), fit_regression(), get_equally_spaced_points(), get_skew_symmetric_matrix(), simulate_rigid_regression(), weight_explanatory_points()

Examples

```
library(nprotreg)

# Define a matrix of explanatory points.

explanatory_points <- rbind(
  cbind(.5, 0, .8660254),
  cbind(-.5, 0, .8660254),
  cbind(1, 0, 0),
  cbind(0, 1, 0),
  cbind(-1, 0, 0),
  cbind(0, -1, 0),
  cbind(.5, 0, -.8660254),
  cbind(-.5, 0, -.8660254)
)

# Define a local rotation composer.

local_rotation_composer <- function(point) {
  independent_components <- (1 / 2) *
    c(exp(2.0 * point[3]), - exp(2.0 * point[2]), exp(2.0 * point[1]))
}

# Define a local error sampler.

local_error_sampler <- function(point) {
  rnorm(3)
}

# Get the corresponding 8-by-3 matrix of response points.
# Rows corresponds to explanatory points,
# columns to Cartesian coordinates.

response_points <- simulate_regression(explanatory_points,
                                       local_rotation_composer,
                                       local_error_sampler)

# Get the response point corresponding to the second
# explanatory point.

cat("Response point corresponding to the second explanatory point: \n")
cat(response_points[2, ], "\n")
```

`simulate_rigid_regression` *Simulates a Rigid 3D Spherical Regression.*

Description

Returns the response points corresponding to the specified explanatory points, given a rigid rotation model and an error term sampler.

Usage

```
simulate_rigid_regression(
  explanatory_points,
  rotation_matrix,
  local_error_sampler
)
```

Arguments

`explanatory_points`
: An *m*-by-3 matrix whose rows contain the Cartesian coordinates of the points at which the regression will be simulated.

`rotation_matrix`
: A 3-by-3 rotation matrix.

`local_error_sampler`
: A function that returns a 3-length numeric vector representing a sampled error term local to an explanatory point, given its Cartesian coordinates.

Details

Let E be the *m*-by-3 matrix of explanatory points. This function will return an *m*-by-*3* matrix whose i-th row is obtained by transposition of the following expression:

$$exp(\Phi(\epsilon(x)))Rx$$

where x is the transpose of the i-th row of E and R is `rotation_matrix`. Term $\epsilon(x)$ is obtained by evaluating at x function `local_error_sampler`, while matrix $\Phi(c)$, for a 3-length numeric vector c, is the skew symmetric matrix having its independent components represented by the entries of c (for a thorough discussion, see function `get_skew_symmetric_matrix`).

Function `local_error_sampler` must be prototyped as having one argument, `point`, representing the Cartesian coordinates of a point on a 3D sphere, and returning a non `NULL` numerical object having length equal to `3`.

Value

An *m*-by-*3* matrix whose rows contain the Cartesian coordinates of the response points corresponding to the explanatory points.

See Also

Other Regression functions: cross_validate_concentration(), fit_regression(), get_equally_spaced_points(), get_skew_symmetric_matrix(), simulate_regression(), weight_explanatory_points()

Examples

```
library(nprotreg)

# Define a matrix of explanatory points.

explanatory_points <- rbind(
  cbind(.5, 0, .8660254),
  cbind(-.5, 0, .8660254),
  cbind(1, 0, 0),
  cbind(0, 1, 0),
  cbind(-1, 0, 0),
  cbind(0, -1, 0),
  cbind(.5, 0, -.8660254),
  cbind(-.5, 0, -.8660254)
)

# Define a rotation matrix.

rotation_matrix <- rbind(
    cbind(-0.69492055764131177575, 0.71352099052778772403, 0.08929285886191218324),
    cbind(-0.19200697279199935297, -0.30378504433947051133, 0.93319235382364695841),
    cbind(0.69297816774177023458, 0.63134969938371787723, 0.34810747783026463331)
)

# Define a local error sampler.

local_error_sampler <- function(point) {
  rnorm(3)
}

# Get the corresponding 8-by-3 matrix of response points.
# Rows corresponds to explanatory points,
# columns to Cartesian coordinates.

response_points <- simulate_rigid_regression(explanatory_points,
                                             rotation_matrix,
                                             local_error_sampler)

# Get the response point corresponding to the second
# explanatory point.

cat("Response point corresponding to the second explanatory point: \n")
cat(response_points[2, ], "\n")
```

`weight_explanatory_points` *Weights the Specified Explanatory Points in a 3D Spherical Regression.*

Description

Returns the weights assigned to the specified explanatory points for each evaluation point under study, given a concentration parameter.

Usage

```
weight_explanatory_points(evaluation_points, explanatory_points, concentration)
```

Arguments

`evaluation_points`
: An n-by-3 matrix whose rows contain the Cartesian coordinates of the points on which the regression will be estimated.

`explanatory_points`
: An m-by-3 matrix whose rows contain the Cartesian coordinates of the explanatory points used to calculate the regression estimators.

`concentration`
: A non negative scalar whose reciprocal value is proportional to the bandwidth applied while estimating a spherical regression model.

Details

Let X be the m-by-3 matrix of explanatory points, and E the n-by-3 matrix of evaluation points, and κ the concentration parameter. This function will return an m-by-n matrix whose (i, j) entry is defined as follows:

$$exp(\kappa(s(i, j) - 1))$$

where $s(i, j)$ is the scalar product of the i-th row of X and the j-th row of E.

Value

An m-by-n matrix whose j-th column contains the weights assigned to the explanatory points while analyzing the j-th evaluation point.

See Also

Other Regression functions: `cross_validate_concentration()`, `fit_regression()`, `get_equally_spaced_points()`, `get_skew_symmetric_matrix()`, `simulate_regression()`, `simulate_rigid_regression()`

Examples

```
library(nprotreg)

# Define a matrix of evaluation points.

north_pole <- cbind(0, 0, 1)
south_pole <- cbind(0, 0, -1)
evaluation_points <- rbind(north_pole, south_pole)

# Define a matrix of explanatory points

explanatory_points <- rbind(
  cbind(.5, 0, .8660254),
  cbind(-.5, 0, .8660254),
  cbind(1, 0, 0),
  cbind(0, 1, 0),
  cbind(-1, 0, 0),
  cbind(0, -1, 0),
  cbind(.5, 0, -.8660254),
  cbind(-.5, 0, -.8660254)
)

# Define a value for the concentration parameter.

concentration <- 1.0

# Get the corresponding 8-by-2 matrix of weights.
# Columns corresponds to evaluation points,
# rows to explanatory ones.

weights <- weight_explanatory_points(evaluation_points,
                                     explanatory_points,
                                     concentration)

# Get the weights assigned to the explanatory points
# while analyzing the second evaluation point.

cat("Weights assigned while analyzing the second evaluation point: \n")
cat(weights[, 2], "\n")
```

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