# Package 'networkTomography'

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Type Package

Title Tools for network tomography

Version 0.3

Author Alexander W Blocker, Paul Koullick, Edoardo Airoldi

Maintainer Alexander W Blocker <ablocker@gmail.com>

**Description** networkTomography implements the methods developed and evaluated in Blocker and Airoldi (2011) and Airoldi and Blocker (2012). These include the authors' own dynamic multilevel model with calibration based upon a Gaussian state-space model in addition to implementations of the methods of Tebaldi & West (1998; Poisson-Gamma model with MCMC sampling), Zhang et al. (2002; tomogravity), Cao et al. (2000; Gaussian model with mean-variance relation), and Vardi (1996; method of moments). Data from the 1router network of Cao et al. (2000), the Abilene network of Fang et al. (2007), and the CMU network of Blocker and Airoldi (2011) are included for testing and reproducibility.

## License LGPL-2

LazyLoad yes

URL https://github.com/awblocker/networkTomography

**Depends** R (>= 2.10.0),

Imports coda (>= 0.11-3), igraph (>= 0.5), KFAS (>= 1.0), limSolve (>= 1.4), plyr, Rglpk (>= 0.3),

NeedsCompilation yes

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Abilene data from Fang et al. (2007)

# Description

Data from the 12 node Abilene network from Fang et al. (2007). Both the OD flows and the topology correspond to the actual network. This is the X1 dataset from the given paper.

# Usage

abilene

#### Objects

The list abilene, which contains several objects:

- A, the routing matrix for this network (truncated for full row rank)
- X, a matrix of origin-destination flows formatted for analysis
- Y, a matrix of link loads formatted for analysis
- A. full, the routing matrix for this network without truncatation for full row rank)
- Y.full, a matrix of link loads corresponding to codeA.full

In this data, we have A %\*% t(X) == t(Y) and A.full %\*% t(X) == t(Y.full)

# Variables

The list abilene contains the following:

- The routing matrix A. The columns of this matrix correspond to individual OD flows (the columns of X), and its rows correspond to individual link loads (the columns of Y).
- The OD matrix X. Columns correspond to individual OD flows, and the rows correspond to observations.
- The link load matrix Y. Columns of the Y matrix correspond to individual link loads, and the rows correspond to observations.
- The routing matrix A.full. This is the complete routing matrix before reduction for full row-rank.
- The link load matrix Y.full, corresponding to A.full.

#### References

J. Fang, Y. Vardi, and C.-H. Zhang. An iterative tomogravity algorithm for the estimation of network traffic. In R. Liu, W. Strawderman, and C.-H. Zhang, editors, Complex Datasets and Inverse Problems: Tomography, Networks and Beyond, volume 54 of Lecture Notes-Monograph Series. IMS, 2007.

agg

Function to aggregate results from matrix to matrix

#### Description

Defaults to mean, SD, limits, and given quantiles. Used to limit memory consumption from MCMC runs.

ach

#### Usage

agg(mat, q = c(0.05, 0.16, 0.5, 0.84, 0.95))

#### Arguments

mat	input numeric matrix to summarize
q	quantiles of mat's columns to provide in summary matrix

# Value

matrix with each row corresponding to a summary measure and each column corresponding to a column of mat

#### Examples

mat <- matrix(rnorm(5e3), ncol=5)
agg(mat)</pre>

bayesianDynamicFilter Function for inference with multilevel state-space model

# Description

Particle filtering with sample-resample-move algorithm for multilevel state-space model of Blocker & Airoldi (2011). This has log-normal autoregressive dynamics on OD intensities, log-normal emission distributions, and truncated normal observation densities. This can return full (all particles) output, but it is typically better to aggregate results as you go to reduce memory consumption. It can also run forward or backward filtering for smoothing. These results are combined via a separate function for smoothing; however, this procedure typically performs poorly due to differences between the distributions of particles from forward and reverse filtering.

## Usage

```
bayesianDynamicFilter(Y, A, prior, lambda0, sigma0, phi0, rho = 0.1,
tau = 2, m = 1000, verbose = FALSE, Xdraws = 5 * m, Xburnin = m,
Movedraws = 10, nThresh = 10, aggregate = FALSE, backward = FALSE,
tStart = 1)
```

#### Arguments

Y	matrix (n x l) of observed link loads over time, one observation per row
A	routing matrix (l x k) for network; must be of full row rank
prior	list containing priors for lambda and phi; must have
	• mu, a matrix (n x k) containing the prior means for the log-change in e
	lambda at each time

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	• sigma, a matrix (n x k) containing the prior standard deviations for the log- change in each lambda at each time
	• a list phi, containing the numeric prior df and a numeric vector scale of length n
lambda0	numeric vector (length k) of time 0 prior means for OD flows
sigma0	numeric vector (length k) of time 0 prior standard deviations for OD flows
phi0	numeric starting value for phi at time 0
rho	numeric fixed autoregressive parameter for dynamics on lambda; see reference for details
tau	numeric fixed power parameter for variance structure on truncated normal noise; see reference for details
m	integer number of particles to use
verbose	logical activates verbose diagnostic output
Xdraws	integer number of draws to perform for xsample RDA
Xburnin	integer number of burnin draws to discard for xsample proposals RDA in addition to baseline number of draws
Movedraws	integer number of iterations to run for each move step
nThresh	numeric effective number of independent particles below which redraw will be performed
aggregate	logical to activate aggregation of MCMC results; highly
backward	logical to activate reverse filtering (for smoothing
tStart	integer time index to begin iterations from

list containing:

- xList
- lambdaList
- phiList
- y
- rho
- prior
- n
- 1
- k
- A
- A\_qr
- A1
- A1\_inv
- A2

bell.labs

- nEff
- tStart
- backward
- aggregate

## References

A.W. Blocker and E.M. Airoldi. Deconvolution of mixing time series on a graph. Proceedings of the Twenty-Seventh Conference Annual Conference on Uncertainty in Artificial Intelligence (UAI-11) 51-60, 2011.

#### See Also

Other bayesianDynamicModel: buildPrior; move\_step

bell.labs

Bell Labs 1 router data from Cao et al. (2000)

# Description

Data from 4-node network with star topology collected from Bell Labs; used in Cao et al. (2000).

#### Usage

bell.labs

# Objects

The list bell.labs, which contains several objects:

- A, the routing matrix for this network (truncated for full row rank)
- df, a data.frame with all data
- X, a matrix of origin-destination flows formatted for analysis
- Y, a matrix of link loads formatted for analysis
- tvec, a vector of times

In this data, we have A %\*% t(X) == t(Y).

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#### buildPrior

#### Variables

The list bell.labs contains the following:

- The routing matrix A. The columns of this matrix correspond to individual OD flows (the columns of X), and its rows correspond to individual link loads (the columns of Y).
- The data.frame df, containing
  - value, level of traffic recorded
  - nme, name of flow or load
  - method, whether flow was directly observered or inferred (all observed)
  - time, time of observation
  - od, flag for origin-destination vs. link loads
  - orig, origin of flow or load
  - dest, destination of flow or load
  - node, node involved in flow or load
- The OD matrix X. Columns correspond to individual OD flows, and the rows correspond to observations.
- The link load matrix Y. Columns of the Y matrix correspond to individual link loads, and the rows correspond to observations.
- The vector tvec, containing the time in decimal hours since midnight for each observation.

#### References

J. Cao, D. Davis, S. Van Der Viel, and B. Yu. Time-varying network tomography: router link data. Journal of the American Statistical Association, 95:1063-75, 2000.

buildPrior

Construct prior from calibration model estimates

## Description

Builds prior from appropriately structured output of the calibration model from Blocker & Airoldi (2011). Handles all formatting so result can be fed directly to bayesianDynamicFilter.

#### Usage

```
buildPrior(xHat, varHat, phiHat, Y, A, rho = 0.9, phiPriorDf = ncol(A)/2,
backward = FALSE, lambdaMin = 1, ipfp.maxit = 1e+06, ipfp.tol = 1e-06)
```

# Arguments

xHat	matrix (n x k) of estimates for OD flows from calibration model, one time point per row
varHat	matrix (n x k) of estimated variances for OD flows from calibration, one time point per row
phiHat	numeric vector (length n) of estimates for phi from calibration model
Υ	matrix (n x l) of observed link loads, one time point per row
A	routing matrix (l x k) for network; must be of full row rank
phiPriorDf	numeric prior convolution parameter for independent inverse-gamma priors on $phi_t$
rho	numeric fixed autoregressive parameter for dynamics on lambda; see reference for details
backward	logical to activate construction of reversed prior (for smoothing applications)
lambdaMin	numeric value at which to floor estimated OD flows for prior construction
ipfp.maxit	integer maximum number of iterations for IPFP
ipfp.tol	numeric tolerance for convergence of IPFP iterations

# Value

list containing priors for lambda and phi, consisting of:

- mu, a matrix (n x k) containing the prior means for the log-change in each lambda at each time
- sigma, a matrix (n x k) containing the prior standard deviations for the log-change in each lambda at each time
- a list phi, containing the numeric prior df and a numeric vector scale of length n

#### References

A.W. Blocker and E.M. Airoldi. Deconvolution of mixing time series on a graph. Proceedings of the Twenty-Seventh Conference Annual Conference on Uncertainty in Artificial Intelligence (UAI-11) 51-60, 2011.

# See Also

Other bayesianDynamicModel: bayesianDynamicFilter; move\_step

buildRoutingMat

Build routing matrices for linked star topologies; that is, a set of startopology networks with links between a subset of routers

#### Description

Build routing matrices for linked star topologies; that is, a set of star-topology networks with links between a subset of routers

#### Usage

buildRoutingMat(nVec, Cmat)

#### Arguments

nVec	integer vector containing number of nodes in each sub-network (length m)
Cmat	matrix (m x m) containing a one for each linked sub-network; only upper trian-
	gular part is used

# Value

routing matrix of dimension at least 2\*sum(nVec) x sum(nVec^2)

# See Also

buildStarMat, which this function depends upon

## Examples

```
nVec <- c(3, 3, 3)
Cmat <- diag(3)
Cmat[1,2] <- Cmat[2,3] <- 1
buildRoutingMat(nVec, Cmat)</pre>
```

buildRoutingMatrix Build routing matrix from table of link relationships

#### Description

Constructs routing matrix from link relationships. Determines routes using (weighted) shortest-path calculation (mirroring OSPF). Currently handles tied paths arbitrarily; will incorporate fractions for tie resolution in next version. Can optionally include aggregate source and destination flows for each node; this can make a major difference for some topologies. Tomogravity methods typically make use of such information, which most routers collect. Note that resulting routing matrix need not be of full row rank.

# Usage

```
buildRoutingMatrix(nodes, src, dest, weights = NULL, agg = FALSE,
    sep = "_", aggChar = "*", verbose = 0)
```

# Arguments

nodes	vector (lenght n) of node identifiers
src	vector (length m) of sources, one per link, matched with dest
dest	vector (length n) of destination identifiers, one per link, matched with src
weights	numeric vector (length m) of weights for each link; used in shortest-path routing calculations (roughly OSPF)
agg	logical for whether to include aggregate source and destination flows for each node
sep	character separator between node id's for link and OD names
aggChar	character to indicate aggregate flows; should be distinct from sep
verbose	integer level of verbosity; 0 is silent, >=1 are increasing levels of reporting

# Value

List consisting of routing matrix A (dense) of dimensions m x n and iGraph object for network topo

buildStarMat Build routing matrix for star network topology	
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# Description

Build routing matrix for star network topology

# Usage

```
buildStarMat(n)
```

# Arguments

n integer number of nodes in the network

# Value

matrix of dimension 2n x n^2 that transforms OD flows to link loads

# Examples

buildStarMat(3)

calcN

# Description

Compute total traffic from a particular time.

#### Usage

calcN(yt, A1)

# Arguments

yt	length-m numeric vectors of observed aggregate flows at a particular time
A1	m x m matrix containing the full-rank portion of the network's routing matrix,
	as supplied by decomposeA

# Examples

```
data(bell.labs)
A.decomp <- decomposeA(bell.labs$A)
total.traffic <- calcN(yt=bell.labs$Y[1,], A1=A.decomp$A1)
total.traffic == sum(bell.labs$X[1,])</pre>
```

calibration_ssm	Estimation for the linear SSM calibration model of Blocker & Airoldi
	(2011)

#### Description

Maximum likelihood estimation of the parameters of the calibration model from Blocker & Airoldi (2011) via direct numerical maximization of the marginal log-likelihood. This relies upon efficient Kalman smoothing to evaluate the marginal likelihood, which is provided here by the KFAS package.

# Usage

```
calibration_ssm(tme, y, A, Ft, Rt, lambda0, phihat0, tau = 2, w = 11,
initScale = 1/(1 - diag(Ft)^2), nugget = sqrt(.Machine$double.eps),
verbose = FALSE, logTrans = TRUE, method = "L-BFGS-B",
optimArgs = list())
```

#### Arguments

tme	integer time at which to center moving window for estimation
У	matrix (n x m) of observed link loads from all times (not just the window used for estimation; one observation per row
A	routing matrix (m x k) for network; should be full row rank
Ft	matrix (k x k) containing fixed autoregressive parameters for state evolution equation; upper-left block of overall matrix for expanded state
Rt	covariance matrix for observation equation; typically small and fixed
lambda0	matrix (n x k) of initial estimates for lambda (e.g. obtained via IPFP)
phihat0	numeric vector (length n) of initial estimates for phi
tau	numeric power parameter for mean-variance relationship
W	number of observations to use for rolling-window estimation; handles boundary cases cleanly
initScale	numeric inflation factor for time-zero state covariance; defaults to steady-state variance setting
nugget	small positive value to add to diagonal of state evolution covariance matrix to ensure numerical stability
verbose	logical to select verbose output from algorithm
logTrans	logical whether to log-transform parameters for optimization. If FALSE, sets method to "L-BFGS-B".
method	optimization method to use (in optim calls)
optimArgs	list of arguments to append to control argument for optim. Can include all arguments except for fnscale, which is automatically set

#### Value

list containing lambdahat, a numeric vector (length k) containing the MLE for lambda; phihat, the MLE for phi; xhat, the smoothed estimates of the OD flows for the window used as a k x w matrix; and varhat, a k x w matrix containing the diagonal of the estimated covariance for each OD flow in the window

# References

A.W. Blocker and E.M. Airoldi. Deconvolution of mixing time series on a graph. Proceedings of the Twenty-Seventh Conference Annual Conference on Uncertainty in Artificial Intelligence (UAI-11) 51-60, 2011.

# See Also

Other calibrationModel: llCalibration; mle\_filter

# сти

# Examples

cmu

CMU data from Blocker & Airoldi (2011)

## Description

Data from the 12 node CMU network used in Blocker & Airoldi (2011). The OD flows are actual, observed traffic from a CMU network. The topology does not, however, correspond to the original network due to security considerations.

#### Usage

cmu

## Objects

The list cmu, which contains several objects:

- A, the routing matrix for this network (truncated for full row rank)
- X, a matrix of origin-destination flows formatted for analysis
- Y, a matrix of link loads formatted for analysis
- A. full, the routing matrix for this network without truncatation for full row rank)
- Y. full, a matrix of link loads corresponding to codeA.full

In this data, we have A %\*% t(X) == t(Y) and A.full %\*% t(X) == t(Y.full)

# Variables

The list cmu contains the following:

- The routing matrix A. The columns of this matrix correspond to individual OD flows (the columns of X), and its rows correspond to individual link loads (the columns of Y).
- The OD matrix X. Columns correspond to individual OD flows, and the rows correspond to
  observations.

- The link load matrix Y. Columns of the Y matrix correspond to individual link loads, and the rows correspond to observations.
- The routing matrix A.full. This is the complete routing matrix before reduction for full row-rank.
- The link load matrix Y.full, corresponding to A.full.

# References

A.W. Blocker and E.M. Airoldi. Deconvolution of mixing time series on a graph. Proceedings of the Twenty-Seventh Conference Annual Conference on Uncertainty in Artificial Intelligence (UAI-11) 51-60, 2011.

decomposeA	Compute pivoted decomposition of routing matrix A into full-rank and
	remainder, as in Cao et al. 2000, via the QR decomposition.

## Description

Compute pivoted decomposition of routing matrix A into full-rank and remainder, as in Cao et al. 2000, via the QR decomposition.

#### Usage

decomposeA(A)

## Arguments

A routing matrix of dimension m x k

# Value

list containing two matrices: A1 (m x m), a full-rank subset of the columns of A, and A2 (m x k - m), the remaining columns

	• •	
diag	ind	
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Make vector of 1-dimensional diagonal indices for square matrix

# Description

Compute vector of indices for efficient access to diagonal of a square matrix

#### Usage

diag\_ind(n)

# diag\_mat

#### Arguments

n

integer dimension of (square) matrix

# Value

integer vector of length n with indices (unidimensional) of square matrix

#### See Also

diag\_mat

# Examples

ind <- diag\_ind(5)
diag\_mat(seq(5))[ind]</pre>

diag\_mat

# Make diagonal matrix from vector

# Description

Build matrix with supplied vector on diagonal; this is much faster than diag due to the use of matrix instead of array

# Usage

diag\_mat(x)

# Arguments

x numeric vector for diagonal

## Value

matrix of size length(x) x length(x) with x along diagonal

# See Also

diag\_ind

# Examples

diag\_mat(seq(5))

dobj.dxt.tomogravity Analytic gradient of objective function of Zhang et al. 2003

# Description

Requires bounded optimization to maintain positive OD flows, and only those flows that are not deterministically zero should be included in the estimation.

# Usage

dobj.dxt.tomogravity(xt, yt, A, srcDstInd, lambda)

# Arguments

xt	length-k numeric vector of point-to-point flows
yt	length-m numeric vector of observed aggregate flows
A	m x k routing matrix, $yt = A xt$
srcDstInd	list of source and destination flow indices corresponding to each point-to-point flow, as produced by getSrcDstIndices
lambda	regularization parameter for mutual information prior. Note that this is scaled by the squared total traffic in the objective function before scaling the mutual information prior.

#### Value

numeric vector of length k containing gradient of objective function with respect to xt

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Check for deterministically-known OD flows at single time

# Description

Uses xranges from limSolve to find deterministically-known OD flows

# Usage

getActive(y, A)

У	numeric vector of link loads, dimension m
A	routing matrix of dimension m x k

# getSrcDstIndices

# Value

logical vector of length k; TRUE for unknown OD flows, FALSE for known

# Examples

```
data(bell.labs)
getActive(bell.labs$Y[1,], bell.labs$A)
```

getSrcDstIndices Find indices of source and destination for each point-to-point flow

## Description

This works only for routing matrices that include all aggregate source and destination flows. It is often easier to build these indices manually via string processing or during the construction of the routing matrix.

#### Usage

```
getSrcDstIndices(A)
```

# Arguments

A

routing matrix of dimension m x k. This should be the reduced-rank version including all aggregate source and destination flows.

# Value

list consisting of two component, src and dst, which are integer vectors of length k containing the index (in y = A x) of the source and destination flows that each point-to-point flow is part of.

# Examples

```
data(cmu)
src.dst.ind <- getSrcDstIndices(cmu$A.full)</pre>
```

grad\_iid

# Description

Computes gradient of Q-function with respect to log(c(lambda,phi)) for EM algorithm from Cao et al. (2000) for their locally IID model.

#### Usage

grad\_iid(logtheta, c, M, rdiag, epsilon)

# Arguments

logtheta	numeric vector (length k+1) of log(lambda) (1:k) and log(phi) (last entry)
с	power parameter in model of Cao et al. (2000)
М	matrix $(n \ x \ k)$ of conditional expectations for OD flows, one time per row
rdiag	numeric vector (length k) containing diagonal of conditional covariance matrix R
epsilon	numeric nugget to add to diagonal of covariance for numerical stability

## Value

numeric vector of same length as logtheta containing calculated gradient

#### References

J. Cao, D. Davis, S. Van Der Viel, and B. Yu. Time-varying network tomography: router link data. Journal of the American Statistical Association, 95:1063-75, 2000.

# See Also

Other CaoEtAl: Q\_iid; Q\_smoothed; R\_estep; grad\_smoothed; locally\_iid\_EM; m\_estep; phi\_init; smoothed\_EM

# Description

Computes gradient of Q-function with respect to log(c(lambda,phi)) for EM algorithm from Cao et al. (2000) for their smoothed model.

# Usage

```
grad_smoothed(logtheta, c, M, rdiag, eta0, sigma0, V, eps.lambda, eps.phi)
```

## Arguments

logtheta	numeric vector (length k+1) of log(lambda) (1:k) and log(phi) (last entry)
С	power parameter in model of Cao et al. (2000)
Μ	matrix $(n \ x \ k)$ of conditional expectations for OD flows, one time per row
rdiag	numeric vector (length k) containing diagonal of conditional covariance matrix R
eta0	numeric vector (length k+1) containing value for log(c(lambda, phi)) from pre- vious time (or initial value)
sigma0	covariance matrix (k+1 x k+1) of log(c(lambda, phi)) from previous time (or initial value)
V	evolution covariance matrix (k+1 x k+1) for $\log(c(\text{lambda, phi}))$ (random walk)
eps.lambda	numeric small positive value to add to lambda for numerical stability; typically $0$
eps.phi	numeric small positive value to add to phi for numerical stability; typically 0

#### Value

numeric vector of same length as logtheta containing calculated gradient

## References

J. Cao, D. Davis, S. Van Der Viel, and B. Yu. Time-varying network tomography: router link data. Journal of the American Statistical Association, 95:1063-75, 2000.

# See Also

Other CaoEtAl: Q\_iid; Q\_smoothed; R\_estep; grad\_iid; locally\_iid\_EM; m\_estep; phi\_init; smoothed\_EM

gravity

# Description

Run tomogravity estimation on complete time series of aggregate flows

## Usage

gravity(Y, srcDstInd)

# Arguments

Y	n x m matrix contain one vector of observed aggregate flows per row
srcDstInd	list of source and destination flow indices corresponding to each point-to-point
	flow, as produced by getSrcDstIndices

# Value

Xhat, a n x k matrix containing a vector of estimated point-to-point flows (for each time point) per row

#### See Also

Other gravity: gravity.fit

# Examples

```
data(cmu)
srcDstInd <- getSrcDstIndices(cmu$A.full)
estimate <- gravity(Y=cmu$Y[1:3,], srcDstInd=srcDstInd)</pre>
```

gravity.fit	Gravity estimation for a single time point
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# Description

Gravity estimation for a single time point

# Usage

gravity.fit(yt, srcDstInd)

# ipfp

## Arguments

yt	length-m numeric vector of observed aggregate flows at time t
srcDstInd	list of source and destination flow indices corresponding to each point-to-point flow as produced by getSrcDstIndices
	now, as produced by getth epstimatees

# Value

xhat, a numeric vector of length k providing gravity estimates of the point-to-point flows of interest

## See Also

Other gravity: gravity

# Examples

```
data(cmu)
srcDstInd <- getSrcDstIndices(cmu$A.full)
estimate <- gravity.fit(yt=cmu$Y.full[1,], srcDstInd=srcDstInd)</pre>
```

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Function to run basic IPFP (iterative proportional fitting procedure)

## Description

Use IPFP starting from x0 to produce vector x s.t. Ax = y within tolerance. Need to ensure that x0 >= 0.

#### Usage

```
ipfp(y, A, x0, tol = .Machine$double.eps, maxit = 1000, verbose = FALSE,
full = FALSE)
```

#### Arguments

У	numeric constraint vector (length nrow)
A	constraint matrix (nrow x ncol)
x0	numeric initial vector (length ncol)
tol	numeric tolerance for IPFP; defaults to <code>.Machine\$double.eps</code>
maxit	integer maximum number of iterations for IPFP; defaults to 1e3
verbose	logical parameter to select verbose output from C function
full	logical parameter to select full return (with diagnostic info)

# Value

if not full, vector of length ncol containing solution obtained by IPFP. If full, list containing solution (as x), number of iterations (as iter), and norm of Ax - y (as errNorm)

## Examples

```
A <- buildStarMat(3)
x <- rgamma(ncol(A), 10, 1/100)
y <- A %*% x
x0 <- x * rgamma(length(x), 10, 10)
ans <- ipfp(y, A, x0, full=TRUE)
print(ans)
print(x)</pre>
```

11Calibration Evaluate marginal log-likelihood for calibration SSM

# Description

Evaluates marginal log-likelihood for calibration SSM of Blocker & Airoldi (2011) using Kalman filtering. This is very fast and numerically stable, using the univariate Kalman filtering and smoothing functions of KFAS with Fortran implementations.

#### Usage

```
llCalibration(theta, Ft, yt, Zt, Rt, k = ncol(Ft), tau = 2,
initScale = 1/(1 - diag(Ft)<sup>2</sup>), nugget = sqrt(.Machine$double.eps))
```

# Arguments

theta	numeric vector (length k+1) of parameters. theta[-1] = log(lambda), and theta[1] = log(phi)
Ft	evolution matrix (k x k) for OD flows; include fixed
yt	matrix (k x n) of observed link loads, one observation per column
Zt	observation matrix for system; should be routing matrix A
Rt	covariance matrix for observation equation; typically small and fixed
k	integer number of OD flows to infer
tau	numeric power parameter for mean-variance relationship
initScale	numeric inflation factor for time-zero state covariance; defaults to steady-state variance setting
nugget	small positive value to add to diagonal of state evolution covariance matrix to ensure numerical stability

## Value

numeric marginal log-likelihood obtained via Kalman smoothing

# References

A.W. Blocker and E.M. Airoldi. Deconvolution of mixing time series on a graph. Proceedings of the Twenty-Seventh Conference Annual Conference on Uncertainty in Artificial Intelligence (UAI-11) 51-60, 2011.

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# locally\_iid\_EM

## See Also

Other calibrationModel: calibration\_ssm; mle\_filter

locally_iid_EM	Run EM algorithm to obtain MLE for locally IID model of Cao et al.
	(2000)

#### Description

Runs EM algorithm to compute MLE for the locally IID model of Cao et al. (2000). Uses numerical optimization of Q-function for each M-step with analytic computation of its gradient.

# Usage

```
locally_iid_EM(Y, A, lambda0, phi0 = NULL, c = 2, maxiter = 1000,
tol = 1e-06, epsilon = 0.01, method = "L-BFGS-B", checkActive = FALSE)
```

# Arguments

Y	matrix (h x k) of observations in local window; columns correspond to OD flows, and rows are individual observations	
A	routing matrix (m x k) for network being analyzed	
lambda0	initial vector of values (length k) for lambda; ipfp is a good way to obtain this	
phi0	initial value for covariance scale phi; initializes automatically using phi_initi if NULL, but you can likely do better	
С	power parameter in model of Cao et al. (2000)	
maxiter	maximum number of EM iterations to run	
tol	tolerance (in relative change in Q function value) for stopping EM iterations	
epsilon	numeric nugget to add to diagonal of covariance for numerical stability	
method	optimization method to use (in optim calls)	
checkActive	logical check for deterministically known OD flows	

#### Value

list with 3 elements: lambda, the estimated value of lambda; phi, the estimated value of phi; and iter, the number of iterations run

# References

J. Cao, D. Davis, S. Van Der Viel, and B. Yu. Time-varying network tomography: router link data. Journal of the American Statistical Association, 95:1063-75, 2000.

#### See Also

Other CaoEtAl: Q\_iid; Q\_smoothed; R\_estep; grad\_iid; grad\_smoothed; m\_estep; phi\_init; smoothed\_EM

mle\_filter

#### Description

Run Kalman filtering and smoothing at calculated MLE for parameters of calibration SSM. This is used to obtain point and covariance estimates for the actual OD flows X following estimation of other parameters.

# Usage

```
mle_filter(mle, Ft, yt, Zt, Rt, k = ncol(Ft), tau = 2, initScale = 1/(1 -
diag(Ft)^2), nugget = sqrt(.Machine$double.eps))
```

#### Arguments

mle	numeric vector (length k+1) of parameters. theta[-1] = log(lambda), and theta[1] = log(phi)
Ft	evolution matrix (k x k) for OD flows; include fixed
yt	matrix (k x n) of observed link loads, one observation per column
Zt	observation matrix for system; should be routing matrix A
Rt	covariance matrix for observation equation; typically small and fixed
k	integer number of OD flows to infer
tau	numeric power parameter for mean-variance relationship
initScale	numeric inflation factor for time-zero state covariance; defaults to steady-state variance setting
nugget	small positive value to add to diagonal of state evolution covariance matrix to ensure numerical stability

#### Value

numeric marginal log-likelihood obtained via Kalman smoothing list containing result of Kalman smoothing; see SSModel and KFS for details

#### References

A.W. Blocker and E.M. Airoldi. Deconvolution of mixing time series on a graph. Proceedings of the Twenty-Seventh Conference Annual Conference on Uncertainty in Artificial Intelligence (UAI-11) 51-60, 2011.

## See Also

Other calibrationModel: calibration\_ssm; llCalibration

move\_step

Move step of sample-resample-move algorithm for multilevel statespace model

# Description

Function to execute single MCMC-based move step for bayesianDynamicFilter. This can use two types of stopping rules: number of iterations or number of accepted moves for the X particles. The former is used by default, but the latter adapts better to low acceptance rates (sometimes with substantial computational cost). Most updates in this algorithm are Metropolis-Hastings with customized proposals.

## Usage

```
move_step(y, X, tme, lambda, phi, lambdatm1, phitm1, prior, A, A1_inv, A2, rho,
tau, m = ncol(X), l = nrow(A1_inv), k = length(lambda), ndraws = 10,
minAccepts = 0, verbose = FALSE)
```

У	numeric vector (length l) of observed link loads		
Х	matrix (m x k) of particles for OD flows, one particle per row, in pivoted order		
tme	integer time index currently used in estimation		
lambda	matrix (m x k) of particles for OD intensities, one particle per row, in pivoted order		
phi	numeric vector (length m) of particles for phi		
lambdatm1	lambda matrix (m x k) of particles for OD intensities from previous time, one particle per row, in pivoted order		
phitm1	numeric vector (length m) of particles for phi from previous time		
prior	list containing priors for hyperparameters; see ${\tt bayesianDynamicFilter}$ for details		
A	routing matrix (l x k) for network		
A1_inv	inverse of full-rank portion of routing matrix (l x l)		
A2	remainder of routing matrix (l x k-l)		
rho	numeric fixed autoregressive parameter for dynamics on lambda; see reference for details		
tau	numeric fixed power parameter for variance structure on truncated normal noise; see reference for details		
m	integer number of particles		
1	integer number of observed link loads		
k	integer number of OD flows to infer		
ndraws	integer number of draws to perform (can be overriden by minAccepts)		

m\_estep

minAccepts	integer minimum number of acceptances before results are returned; activates
	alternative stopping rule if >= 1
verbose	logical activates verbose diagnostic output

# Value

list containing updated values of X, lambda, and phi

#### References

A.W. Blocker and E.M. Airoldi. Deconvolution of mixing time series on a graph. Proceedings of the Twenty-Seventh Conference Annual Conference on Uncertainty in Artificial Intelligence (UAI-11) 51-60, 2011.

#### See Also

Other bayesianDynamicModel: bayesianDynamicFilter; buildPrior

m_estep	Compute conditional expectations for EM algorithms of Cao et al.
	(2000)

#### Description

Computes conditional expectation of OD flows for E-step of EM algorithm from Cao et al. (2000) for their locally IID model.

# Usage

m\_estep(yt, lambda, phi, A, c, epsilon)

# Arguments

yt	numeric vector (length m) of link loads from single time
lambda	numeric vector (length k) of mean OD flows from last M-step
phi	numeric scalar scale for covariance matrix of xt
A	routing matrix (m x k) for network being analyzed
с	power parameter in model of Cao et al. (2000)
epsilon	numeric nugget to add to diagonal of covariance for numerical stability

# Value

numeric vector of same size as lambda with conditional expectations of x

# obj.tomogravity

## References

J. Cao, D. Davis, S. Van Der Viel, and B. Yu. Time-varying network tomography: router link data. Journal of the American Statistical Association, 95:1063-75, 2000.

# See Also

Other CaoEtAl: Q\_iid; Q\_smoothed; R\_estep; grad\_iid; grad\_smoothed; locally\_iid\_EM; phi\_init; smoothed\_EM

obj.tomogravity Objective function of Zhang et al. 2003

# Description

Requires bounded optimization to maintain positive OD flows, and only those flows that are not deterministically zero should be included in the estimation.

#### Usage

obj.tomogravity(xt, yt, A, srcDstInd, lambda)

## Arguments

xt	length-k numeric vector of point-to-point flows	
yt	length-m numeric vector of observed aggregate flows	
A	m x k routing matrix, $yt = A xt$	
srcDstInd	list of source and destination flow indices corresponding to each point-to-point flow, as produced by getSrcDstIndices	
lambda	regularization parameter for mutual information prior. Note that this is scaled by the squared total traffic in the objective function before scaling the mututal information prior.	

#### Value

numeric value of objective function to minimize in tomogravity estimation

phi\_init

## Description

Uses a crude estimator to get a starting point for phi in the model of Cao et al. (2000).

#### Usage

phi\_init(Y, A, lambda0, c)

#### Arguments

Υ	matrix (n x k) of observed link loads over time
A	routing matrix (m x k)
lambda0	numeric vector (length k) of initial guesses for lambda
с	power parameter in model of Cao et al. (2000)

## Value

numeric starting value for phi

# References

J. Cao, D. Davis, S. Van Der Viel, and B. Yu. Time-varying network tomography: router link data. Journal of the American Statistical Association, 95:1063-75, 2000.

# See Also

Other CaoEtAl: Q\_iid; Q\_smoothed; R\_estep; grad\_iid; grad\_smoothed; locally\_iid\_EM; m\_estep; smoothed\_EM

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Q function for locally IID EM algorithm of Cao et al. (2000)

# Description

Computes the Q function (expected log-likelihood) for the EM algorithm of Cao et al. (2000) for their locally IID model.

#### Usage

Q\_iid(logtheta, c, M, rdiag, epsilon)

# Q\_smoothed

#### Arguments

logtheta	numeric vector (length k+1) of log(lambda) (1:k) and log(phi) (last entry)
С	power parameter in model of Cao et al. (2000)
М	matrix (n x k) of conditional expectations for OD flows, one time per row
rdiag	numeric vector (length k) containing diagonal of conditional covariance matrix R
epsilon	numeric nugget to add to diagonal of covariance for numerical stability

#### Value

numeric value of Q function; not vectorized in any way

# References

J. Cao, D. Davis, S. Van Der Viel, and B. Yu. Time-varying network tomography: router link data. Journal of the American Statistical Association, 95:1063-75, 2000.

# See Also

Other CaoEtAl: Q\_smoothed; R\_estep; grad\_iid; grad\_smoothed; locally\_iid\_EM; m\_estep; phi\_init; smoothed\_EM

Q\_smoothed

*Q* function for smoothed EM algorithm of Cao et al. (2000)

# Description

Computes the Q function (expected log-likelihood) for the EM algorithm of Cao et al. (2000) for their smoothed model.

### Usage

Q\_smoothed(logtheta, c, M, rdiag, eta0, sigma0, V, eps.lambda, eps.phi)

logtheta	numeric vector (length k+1) of log(lambda) (1:k) and log(phi) (last entry)
С	power parameter in model of Cao et al. (2000)
М	matrix (n x k) of conditional expectations for OD flows, one time per row
rdiag	numeric vector (length k) containing diagonal of conditional covariance matrix R
eta0	numeric vector (length k+1) containing value for log(c(lambda, phi)) from pre- vious time (or initial value)
sigma0	covariance matrix (k+1 x k+1) of log(c(lambda, phi)) from previous time (or initial value)

# R\_estep

V	evolution covariance matrix (k+1 x k+1) for log(c(lambda, phi)) (random walk)
eps.lambda	numeric small positive value to add to lambda for numerical stability; typically $0$
eps.phi	numeric small positive value to add to phi for numerical stability; typically 0

# Value

numeric value of Q function; not vectorized in any way

## References

J. Cao, D. Davis, S. Van Der Viel, and B. Yu. Time-varying network tomography: router link data. Journal of the American Statistical Association, 95:1063-75, 2000.

# See Also

Other CaoEtAl: Q\_iid; R\_estep; grad\_iid; grad\_smoothed; locally\_iid\_EM; m\_estep; phi\_init; smoothed\_EM

R_estep	Compute conditional covariance matrix for EM algorithms of Cao et
	al. (2000)

#### Description

Computes conditional covariance of OD flows for E-step of EM algorithm from Cao et al. (2000) for their locally IID model.

# Usage

R\_estep(lambda, phi, A, c, epsilon)

# Arguments

lambda	numeric vector (length k) of mean OD flows from last M-step
phi	numeric scalar scale for covariance matrix of xt
A	routing matrix (m x k) for network being analyzed
с	power parameter in model of Cao et al. (2000)
epsilon	numeric nugget to add to diagonal of covariance for numerical stability

# Value

conditional covariance matrix (k x k) of OD flows given parameters

# smoothed\_EM

#### References

J. Cao, D. Davis, S. Van Der Viel, and B. Yu. Time-varying network tomography: router link data. Journal of the American Statistical Association, 95:1063-75, 2000.

# See Also

Other CaoEtAl: Q\_iid; Q\_smoothed; grad\_iid; grad\_smoothed; locally\_iid\_EM; m\_estep; phi\_init; smoothed\_EM

smoothed_EM	Run EM algorithm to obtain MLE (single time) for smoothed model of
	<i>Cao et al. (2000)</i>

# Description

Runs EM algorithm to compute MLE for the smoothed model of Cao et al. (2000). Uses numerical optimization of Q-function for each M-step with analytic computation of its gradient. This performs estimation for a single time point using output from the previous one.

# Usage

smoothed\_EM(Y, A, eta0, sigma0, V, c = 2, maxiter = 1000, tol = 1e-06, eps.lambda = 0, eps.phi = 0, method = "L-BFGS-B")

Y	matrix (h x k) of observations in local window; columns correspond to OD flows, and rows are individual observations
A	routing matrix (m x k) for network being analyzed
eta0	numeric vector (length k+1) containing value for $\log(c(\text{lambda, phi}))$ from previous time (or initial value)
sigma0	covariance matrix (k+1 x k+1) of log(c(lambda, phi)) from previous time (or initial value)
V	evolution covariance matrix (k+1 x k+1) for $\log(c(\text{lambda, phi}))$ (random walk)
с	power parameter in model of Cao et al. (2000)
maxiter	maximum number of EM iterations to run
tol	tolerance (in relative change in Q function value) for stopping EM iterations
eps.lambda	numeric small positive value to add to lambda for numerical stability; typically $\boldsymbol{0}$
eps.phi	numeric small positive value to add to phi for numerical stability; typically 0
method	optimization method to use (in optim calls)

list with 5 elements: lambda, the estimated value of lambda; phi, the estimated value of phi; iter, the number of iterations run; etat, log(c(lambda, phi)); and sigmat, the inverse of the Q functions Hessian at its mode

## References

J. Cao, D. Davis, S. Van Der Viel, and B. Yu. Time-varying network tomography: router link data. Journal of the American Statistical Association, 95:1063-75, 2000.

# See Also

Other CaoEtAl: Q\_iid; Q\_smoothed; R\_estep; grad\_iid; grad\_smoothed; locally\_iid\_EM; m\_estep; phi\_init

strphour

Convert time string to decimal hour

# Description

Convert time string to decimal hour

# Usage

strphour(x, fmt = "(%m/%d/%y %H:%M:%S)")

# Arguments

х	input character vector of times
fmt	input character format for times

#### Value

numeric vector of decimal times in hours

# Examples

strphour("31/08/87 12:53:29")

thin

# Description

Returns a vector of indices with a given spacing for thinning MCMC results

# Usage

thin(m, interval = 10)

# Arguments

m	integer length of results
interval	thinning interval

# Value

integer vector of indices for thinning

tomogravity

*Run tomogravity estimation on complete time series of aggregate flows* 

# Description

The aggregate flows Y and their corresponding routing matrix A must include all aggregate source and destination flows.

## Usage

```
tomogravity(Y, A, lambda, lower = 0, normalize = FALSE,
  .progress = "none", control = list())
```

Y	n x m matrix contain one vector of observed aggregate flows per row. This should include all observed aggegrate flows with none removed due to redundancy.
A	m x k routing matrix. This need not be of full row rank and must include all source and destination flows.
lambda	Regularization parameter for mutual information prior. Note that this is scaled by the squared total traffic in the objective function before scaling the mutual information prior.
lower	Component-wise lower bound for xt in L-BFGS-B optimization.

normalize	If TRUE, xt and yt are scaled by N. Typically used in conjunction with calcN to normalize traffic to proportions, easing the tuning of lambda.
.progress	name of the progress bar to use, see $\verb"create_progress_bar"$ in plyr documentation
control	List of control information for optim.

A list containing three elements:

- resultList, a list containing the output from running tomogravity. fit on each timepoint
- changeFromInit, a vector of length n containing the relative L\_1 change between the initial (IPFP) point-to-point flow estimates and the final tomogravity estimates
- Xhat, a n x k matrix containing a vector of estimated point-to-point flows (for each time point) per row

# See Also

Other tomogravity: tomogravity.fit

## Examples

tomogravity.fit Tomogravity estimation for a single time point using L-BFGS-B

# Description

Tomogravity estimation for a single time point using L-BFGS-B

#### Usage

```
tomogravity.fit(yt, A, srcDstInd, lambda, N = 1, normalize = FALSE,
lower = 0, control = list())
```

yt	length-m numeric vector of observed aggregate flows at time t
A	m x k routing matrix
srcDstInd	list of source and destination flow indices corresponding to each point-to-point flow, as produced by getSrcDstIndices
lambda	regularization parameter for mutual information prior. Note that this is scaled by the squared total traffic in the objective function before scaling the mututal information prior.

Ν	total traffic for normalization. Unused if normalized is FALSE.
normalize	If TRUE, xt and yt are scaled by N. Typically used in conjunction with calcN to normalize traffic to proportions, easing the tuning of lambda.
lower	Component-wise lower bound for xt in L-BFGS-B optimization.
control	List of control information for optim.

A list as returned by optim, with element par containing the estimated point-to-point flows and elementer gr containing the analytic gradient evaluated at the estimate.

#### See Also

Other tomogravity: tomogravity

# Examples

twMCMC

Function to run MCMC sampling for model of Tebaldi & West (1998)

# Description

Runs MCMC sampling for the gamma-Poisson model presented in Tebaldi & West (1998). The algorithm used is a modification of that presented in the original paper. It uses a joint proposal for  $(x_k, lambda_k)$  to greatly accelerate convergence.

### Usage

```
twMCMC(Y, A, prior, ndraws = 120000, burnin = 20000, verbose = 0)
```

Y	numeric vector of observed link loads at a single time (length k)
A	routing matrix of dimension (k x n); needs to be full row rank
prior	parameters for conjugate gamma prior (convolution and rate)
ndraws	integer number of draws for sampler to produce (excluding burn-in)
burnin	integer number of additional draws to discard as burnin
verbose	integer level of verbosity; levels > 1 have no effect currently

list consisting of matrix of draws for X XDraws, matrix of draws for X lambdaDraws, and vector of acceptances per OD flow accepts

#### References

C. Tebaldi and M. West. Bayesian inference on network traffic using link count data. Journal of the American Statistical Association, 93(442):557-573, 1998.

#### Examples

vardi.algorithm Run algorithm of Vardi (1996) given B and S matrices

#### Description

Runs moment-matching algorithm of Vardi (1996) until convergence

#### Usage

```
vardi.algorithm(A, Y, lambda, tol = 0.001)
```

#### Arguments

A	routing matrix (m x k)
Υ	matrix of link loads over time (m x n, one column per time)
lambda	numeric vector of starting values for OD flows (length k)
tol	numeric tolerance for halting iterations

# Value

numeric vector of length k with estimated OD flows

# References

Y. Vardi. Network tomography: estimating source-destination traffic intensities from link data. Journal of the American Statistical Association, 91:365-377, 1996.

# vardi.compute.BS

#### See Also

Other vardi: vardi.compute.BS; vardi.iteration

vardi.compute.BS Compute B and S matrices in algorithm of Vardi (1996)

## Description

Function to compute B and S matrices for moment equations of Vardi's method (1996). It's not particularly efficient, but it works.

## Usage

vardi.compute.BS(A, Y)

#### Arguments

A	routing matrix (m x k)
Y	matrix of link loads over time (m x n, one column per time)

#### Value

list containing two entries for the B and S matrices, respectively

#### References

Y. Vardi. Network tomography: estimating source-destination traffic intensities from link data. Journal of the American Statistical Association, 91:365-377, 1996.

#### See Also

Other vardi: vardi.algorithm; vardi.iteration

vardi.iteration Execute single iteration for algorithm of Vardi (1996)

#### Description

Function to compute B and S matrices for moment equations of Vardi's method (1996). It's not particularly efficient, but it works.

#### Usage

vardi.iteration(A, yBar, lambda, B, S)

# Arguments

A	routing matrix (m x k)
yBar	numeric vector of mean link loads (length m)
lambda	value of lambda from last iteration
В	B matrix computed by vardi.compute.BS
S	S matrix computed by vardi.compute.BS

# Value

numeric vector of length k with updated lambda

# References

Y. Vardi. Network tomography: estimating source-destination traffic intensities from link data. Journal of the American Statistical Association, 91:365-377, 1996.

# See Also

Other vardi: vardi.algorithm; vardi.compute.BS

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