Package 'missSBM'

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Type Package

Title Handling Missing Data in Stochastic Block Models

Version 0.2.1

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Description

When a network is partially observed (here, NAs in the adjacency matrix rather than 1 or 0 due to missing information between node pairs), it is possible to account for the underlying process that generates those NAs. 'missSBM' adjusts the popular stochastic block model from network data sampled under various missing data conditions, as described in Tabouy, Barbillon and Chi-quet (2019) <doi:10.1080/01621459.2018.1562934>.

URL https://jchiquet.github.io/missSBM

BugReports https://github.com/jchiquet/missSBM/issues

License GPL-3

Encoding UTF-8

LazyData true

RoxygenNote 6.1.1

Depends R (>= 3.4.0)

Imports Rcpp, methods, ape, igraph, nloptr, ggplot2, corrplot, R6, magrittr

LinkingTo Rcpp, RcppArmadillo

Collate 'RcppExports.R' 'SBM-Class.R' 'SBM_fit-Class.R' 'SBM_fit_covariates-Class.R' 'SBM_fit_nocovariate-Class.R' 'SBM_sampler-Class.R' 'er_network.R' 'estimate.R' 'frenchblog2007.R' 'missSBM-package.R' 'utils_missSBM.R' 'networkSampling-Class.R' 'networkSampling_fit-Class.R' 'missSBM_fit-Class.R' 'missSBM_collection-Class.R' 'missSBM_fit-Class.R' 'missSBM_collection-Class.R' 'networkSampler-Class.R' 'prepare_data.R' 'sample.R' 'sampledNetwork-Class.R' 'simulate.R' 'utils-pipe.R' 'utils_initialization.R' 'war.R'

Suggests aricode, blockmodels, testthat, covr, knitr, rmarkdown,

VignetteBuilder knitr

NeedsCompilation yes

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dyadSampler

Virtual class for all dyad-centered samplers

Description

Virtual class for all dyad-centered samplers

Usage

dyadSampler

Format

An object of class R6ClassGenerator of length 24.

er_network

Description

A dataset containing the weighted PPI network centred around the ESR1 (ER) protein

Usage

er_network

Format

A sparse symmetric matrix with 741 rows and 741 columns ESR1

Source

https://string-db.org/

Examples

data("er_network")
class(er_network)

estimate

Estimation of SBMs with missing data

Description

Variational inference from sampled network data on a collection of Stochastic Block Models indexed by block number.

Usage

```
estimate(sampledNet, vBlocks, sampling, clusterInit = "hierarchical",
  useCovariates = TRUE, control = list())
```

Arguments

| sampledNet | An object with class sampledNetwork, typically obtained with the function prepare_data (real-word data) or sample (simulation). |
|------------|--|
| vBlocks | The vector of number of blocks considered in the collection |
| sampling | The sampling design for the modelling of missing data: MAR designs ("dyad", "node") and NMAR designs ("double-standard", "block-dyad", "block-node", "degree") |

estimate

| clusterInit | Initial method for clustering: either a character in "hierarchical", "spectral" or |
|---------------|---|
| | "kmeans", or a list with length(vBlocks) vectors, each with size ncol(adjacencyMatrix), |
| | providing a user-defined clustering. Default is "hierarchical". |
| useCovariates | logicial. If covariates are present in sampledNet, should they be used for the |
| | inference or of the network sampling design, or just for the SBM inference? |
| | default is TRUE. |
| control | a list of parameters controlling the variational EM algorithm. See details. |
| | |

Details

The list of parameters control essentially tunes the optimization process and the variational EM algorithm, with the following parameters

- "threshold"stop when an optimization step changes the objective function by less than threshold. Default is 1e-4.
- "maxIter"V-EM algorithm stops when the number of iteration exceeds maxIter. Default is 200
- "fixPointIter"number of fix-point iterations in the Variational E step. Default is 5.
- "cores" integer for number of cores used. Default is 1.
- "trace" integer for verbosity (0, 1, 2). Default is 1. Useless when cores > 1

The different sampling designs are split into two families in which we find dyad-centered and node-centered samplings. See <doi:10.1080/01621459.2018.1562934> for complete description.

- Missing at Random (MAR)
 - "dyad": parameter = p and

$$p = P(Dyad(i, j)issampled)$$

– "node": parameter = p and

$$p = P(Nodeiissampled)$$

- "covar-dyad": parameter = beta in R^M and

P(Dyad(i, j)issampled) = logistic(parameter'covarArray(i, j,))

- "covar-node": parameter = nu in R^M and

P(Nodeiissampled) = logistic(parameter'covarMatrix(i,))

- Not Missing At Random (NMAR)
 - "double-standard": parameter = (p0,p1) and

p0 = P(Dyad(i, j)issampled|thedyadisequal to 0) =

, p1 = P(Dyad(i,j) is sampled | the dyad is equal to 1)

- "block-node": parameter = c(p(1),...,p(Q)) and

p(q) = P(Nodeiissampled|nodeiisinclusterq)

- "block-dyad": parameter = c(p(1,1),...,p(Q,Q)) and

p(q, l) = P(Edge(i, j)issampled|nodeiisinclusterqandnodejisinclusterl)

- "degree": parameter = c(a,b) and

logit(a + b * Degree(i)) = P(Nodeiissampled | Degree(i))

frenchblog2007

Value

Returns an R6 object with class missSBM_collection.

See Also

sample, simulate, missSBM_collection and missSBM_fit.

Examples

```
## SBM parameters
directed <- FALSE
N <- 300 # number of nodes
Q <- 3 # number of clusters
alpha <- rep(1,Q)/Q
                      # mixture parameter
pi <- diag(.45,Q) + .05 # connectivity matrix</pre>
## simulate a SBM without covariates
sbm <- missSBM::simulate(N, alpha, pi, directed)</pre>
## Sample network data
samplingParameters <- .5 # the sampling rate</pre>
sampling <- "dyad" # the sampling design</pre>
sampledNet <- missSBM::sample(sbm$adjacencyMatrix, sampling, samplingParameters)</pre>
## Inference :
vBlocks <- 1:5 # number of classes
collection <- missSBM::estimate(sampledNet, vBlocks, sampling)</pre>
collection$ICL
coef(collection$bestModel$fittedSBM, "connectivity")
myModel <- collection$bestModel</pre>
plot(myModel, "monitoring")
coef(myModel, "sampling")
coef(myModel, "connectivity")
```

head(predict(myModel))
head(fitted(myModel))

frenchblog2007

Political Blogosphere network prior to 2007 French presidential election

Description

French Political Blogosphere network dataset consists of a single day snapshot of over 200 political blogs automatically extracted the 14 October 2006 and manually classified by the "Observatoire Presidentielle" project. Originally part of the 'mixer' package

Usage

frenchblog2007

Format

An igraph object with 196 nodes. The vertex attribute "party" provides a possible clustering of the nodes.

Source

https://www.linkfluence.com/

Examples

```
data(frenchblog2007)
igraph::V(frenchblog2007)$party
igraph::plot.igraph(frenchblog2007,
   vertex.color = factor(igraph::V(frenchblog2007)$party),
   vertex.label = NA
  )
```

| mis | sSBM |
|-----|--------|
| | 000011 |

Adjusting Stochastic Block Models under various missing data conditions

Description

The missSBM package provides five functions:

- simulatea function to define and draw network data according to a stochastic block model
- samplea function to sample an existing network according to a variety of sampling designs
- estimatea function to perform inference of SBM from network data with missing entries under various sampling designs.
- prepare_dataa function to format real-world network data (adjacency matrix and covariates) to perform the estimation under missing data condition
- smootha function to smooth an existing collection of SBM estimation, to avoid being trapped in local maxima.

Details

These function leads to the manipulation of a variety of R object, with their respective fields and methods. They are all automatically generated by the top-level functions itemized above, so that the user should generally to use their constructor or internal methods directly. The user should only have a basic understanding of the fields of each object to manipulate the output in R. The main objects are the following:

- sampledNetworkan object to store sampled network data (i.e. with missing dyads)
- SBM_sampler an object to define a SBM to sample from
- SBM_fitan object to define and store an SBM fit
- networkSampleran object to define a network sampling to sample from

- networkSamplingan object to define and store a network sampling fit
- missSBM_fitan object that put together an SBM fit and and network sampling fit the main point of the missSBM package !
- missSBM_collectionan object to store a collection of missSBM_fit, ordered by number of block

Author(s)

Timothée Tabouy, Pierre Barbillon, Julien Chiquet

References

Timothée Tabouy, Pierre Barbillon & Julien Chiquet (2019) "Variational Inference for Stochastic Block Models from Sampled Data", Journal of the American Statistical Association, <doi:10.1080/01621459.2018.1562934>

missSBM_collection An object to represent a collection of missSBM_fit

Description

This R6 class stores a collection of missSBM_fit. Comes with basic printing methods an field access.

Usage

missSBM_collection

Format

An object of class R6ClassGenerator of length 24.

Fields

models a list of models

ICL the vector of Integrated Classification Criterion (ICL) associated to the models in the collection (the smaller, the better)

bestModel the best model according to the ICL

optimizationStatus a data.frame summarizing the optimization process for all models

See Also

The function estimate, which produces an instance of this class. The function smooth can be used to smooth the ICL on a collection of model, as post-treatment.

missSBM_fit

Description

This class is designed to adjust a Stochastic Block Model on a network with missing entries.

Usage

missSBM_fit

Format

An object of class R6ClassGenerator of length 24.

networkSampler

Definition of R6 Class 'networkSampling_sampler'

Description

This class is use to define a sampling model for a network. Inherits from 'networkSampling'. Owns a rSampling method which takes an adjacency matrix as an input and send back an object with class sampledNetwork.

Usage

networkSampler

Format

An object of class R6ClassGenerator of length 24.

See Also

sampledNetwork

networkSampling

Description

this virtual class is the mother of all subtypes of networkSampling (either sampler or fit) It is used to define a sampling model for a network. It has a rSampling method which takes an adjacency matrix as an input and send back an object with class sampledNetwork.

Usage

networkSampling

Format

An object of class R6ClassGenerator of length 24.

networkSamplingDyads_fit

Virtual class used to define a family of networkSamplingDyads_fit

Description

Virtual class used to define a family of networkSamplingDyads_fit

Usage

```
networkSamplingDyads_fit
```

Format

An object of class R6ClassGenerator of length 24.

networkSamplingNodes_fit

Virtual class used to define a family of networkSamplingNodes_fit

Description

Virtual class used to define a family of networkSamplingNodes_fit

Usage

networkSamplingNodes_fit

Format

An object of class R6ClassGenerator of length 24.

prepare_data

Prepare network data for estimation with missing data

Description

This function puts together the adjacency matrix of a network and an optional list of covariates into a single sampledNetwork object, ready to use for inference with the estimate function of the missSBM package.

Usage

```
prepare_data(adjacencyMatrix, covariates = NULL,
    similarity = missSBM::::l1_similarity)
```

Arguments

adjacencyMatrix

| | The adjacency matrix of the network (NAs allowed) |
|------------|---|
| covariates | An optional list with M entries (the M covariates). If the covariates are node- centred, each entry of covariates must be a size-N vector; if the covariates are dyad-centred, each entry of covariates must be N x N matrix. |
| similarity | An optional R x R -> R function to compute similarities between node covariates. Default is $11_similarity$, that is, $-abs(x-y)$. Only relevant when the covariates is a list of size-N vectors. |

Value

Returns an R6 object with class sampledNetwork.

sample

See Also

estimate and sampledNetwork.

Examples

```
data(war)
adj_beligerent <- war$beligerent %>% igraph::as_adj(sparse = FALSE)
sampledNet_war_nocov <- prepare_data(adj_beligerent)
sampledNet_war_withcov <- prepare_data(adj_beligerent, list(military_power = war$beligerent$power))</pre>
```

sample

Sampling of network data

Description

This function samples observations in an adjacency matrix according to a given sampling design. The final results is an adjacency matrix with the dimension as the input, yet with additional NAs.

Usage

```
sample(adjacencyMatrix, sampling, parameters, clusters = NULL,
    covariates = NULL, similarity = l1_similarity, intercept = 0)
```

Arguments

adjacencyMatrix

| aajaceneynaerii | |
|-----------------|--|
| | The N x N adjacency matrix of the network to sample. If adjacencyMatrix is symmetric, we assume an undirected network with no loop; otherwise the network is assumed directed. |
| sampling | The sampling design used to sample the adjacency matrix, see details |
| parameters | The sampling parameters adapted to each sampling |
| clusters | An optional clustering membership vector of the nodes, only necessary for block samplings |
| covariates | A list with M entries (the M covariates). If the covariates are node-centred, each entry of covariates must be a size-N vector; if the covariates are dyad-centred, each entry of covariates must be N x N matrix. |
| similarity | An optional function to compute similarities between node covariates. Default is l1_similarity, that is, -abs(x-y). Only relevant when the covariates are node-centered (i.e. covariates is a list of size-N vectors). |
| intercept | An optional intercept term to be added in case of the presence of covariates. Default is 0. |

Details

The different sampling designs are split into two families in which we find dyad-centered and node-centered samplings. See <doi:10.1080/01621459.2018.1562934> for complete description.

- Missing at Random (MAR)
 - "dyad": parameter = p

$$p = P(Dyad(i, j)issampled)$$

- "node": parameter = p and

$$p = P(Nodeiissampled)$$

- "covar-dyad": parameter = beta in R^M and

P(Dyad(i, j)issampled) = logistic(parameter'covarArray(i, j,))

- "covar-node": parameter = nu in R^M and

$$P(Nodeiissampled) = logistic(parameter'covarMatrix(i,))$$

- Not Missing At Random (NMAR)
 - "double-standard": parameter = (p0,p1) and

p0 = P(Dyad(i, j)issampled|thedyadisequal to 0) =

, p1 = P(Dyad(i,j) is sampled | the dyad is equal to 1)

- "block-node": parameter = c(p(1),...,p(Q)) and

p(q) = P(Nodeiissampled|nodeiisinclusterq)

- "block-dyad": parameter = c(p(1,1),...,p(Q,Q)) and

p(q,l) = P(Edge(i,j) is sampled | node iis incluster q and node j is incluster l)

- "degree": parameter = c(a,b) and

logit(a + b * Degree(i)) = P(Nodeiissampled | Degree(i))

Value

an object with class sampledNetwork containing all the useful information about the sampling. Can then feed the estimate function.

See Also

The class sampledNetwork

sampledNetwork

Examples

```
## SBM parameters
directed <- FALSE
N <- 300 # number of nodes
0 <- 3 # number of clusters
alpha <- rep(1,Q)/Q
                      # mixture parameter
pi <- diag(.45,Q) + .05 # connectivity matrix</pre>
## simulate a SBM without covariates
sbm <- missSBM::simulate(N, alpha, pi, directed)</pre>
## Sample network data
# some sampling design and their associated parameters
sampling_parameters <- list(</pre>
   "dyad" = .3,
   "node" = .3,
   "double-standard" = c(0.4, 0.8),
   "block-node" = c(.3, .8, .5),
   "block-dyad" = pi,
   "degree" = c(.01, .01)
 )
sampled_networks <- list()</pre>
for (sampling in names(sampling_parameters)) {
  sampled_networks[[sampling]] <-</pre>
     missSBM::sample(
       adjacencyMatrix = sbm$adjacencyMatrix,
       sampling
                   = sampling,
                     = sampling_parameters[[sampling]],
       parameters
       cluster
                     = sbm$memberships
     )
}
## SS0000 long, but fancy
old_par <- par(mfrow = c(2,3))
for (sampling in names(sampling_parameters)) {
  plot(sampled_networks[[sampling]],
    clustering = sbm$memberships, main = paste(sampling, "sampling"))
}
par(old_par)
```

sampledNetwork An R6 Class to represent sampled network data

Description

The function sample and prepare_data produces an instance of an object with class sampledNetwork.

Usage

sampledNetwork

Format

An object of class R6ClassGenerator of length 24.

Details

All fields of this class are only accessible for reading. This class comes with a basic plot, summary and print methods

Fields

samplingRate percentage of observed dyads
nNodes number of nodes
nDyads number of dyads
is_directed direction
adjacencyMatrix adjacency matrix (with NA)
covarMatrix the matrix of covariates (if applicable)
covarArray the array of covariates (if applicable)
dyads list of potential dyads in the network
missingDyads array indices of missing dyads
observedDyads array indices of observed dyads
samplingMatrix matrix of observed and non-observed edges
observedNodes vector of observed and non-observed nodes
NAs boolean for NA entries in the adjacencyMatrix

Examples

)

```
summary(sampled_network)
print(sampled_network)
plot(sampled_network, clustering = sbm$memberships)
```

SBM_fit

R6 Class definition of an SBM-fit

Description

This class is designed to adjust a Stochastic Block Model on a fully observed network. The doVEM method performs inference via Variational EM.

Usage

SBM_fit

Format

An object of class R6ClassGenerator of length 24.

Details

This class is virtual: inference is effective only for instance of one of the two child classes 'SBM_fit_nocovariate' and 'SBM_fit_covariates'

| SBM_sampler | An R6 Class to represent a sampler for a SBM |
|-------------|--|
| | |

Description

The function simulate produces an instance of an object with class SBM_sampler.

Usage

SBM_sampler

Format

An object of class R6ClassGenerator of length 24.

Details

All fields of this class are only accessible for reading. This class comes with a set of methods, some of them being useful for the user (see examples)

- R6 methods:\$rBlocks(), \$rAdjancencyMatrix()
- S3 methodsprint(), plot()

Fields

nNodes The number of nodes

nBlocks The number of blocks

nCovariates The number of covariates

nDyads The number of possible dyad in the network (depends on the direction)

direction A character indicating if the network is directed or undirected

hasCovariates a boolean indicating if the model has covariates

mixtureParam the vector of mixture parameters

connectParam the matrix of connectivity: inter/intra probabilities of connection when the network does not have covariates, or a logit scaled version of it.

covarParam the vector of parameters associated with the covariates

covarArray the array of covariates

See Also

The function simulate.

Examples

```
## SBM parameters
directed <- FALSE
N <- 300 # number of nodes
Q <- 3 # number of clusters
alpha <- rep(1,Q)/Q # mixture parameter
pi <- diag(.45,Q) + .05 # connectivity matrix
gamma <- log(pi/(1-pi)) # logit transform fo the model with covariates
M <- 2 # two Gaussian covariates
covarMatrix <- matrix(rnorm(N*M,mean = 0, sd = 1), N, M)
covarParam <- rnorm(M, -1, 1)</pre>
```

draw a SBM without covariates through simulateSBM sbm <- missSBM::simulate(N, alpha, pi, directed)</pre>

```
## equivalent construction from the SBM_sampler class itslef
sbm_s <- SBM_sampler$new(directed, N, alpha, pi)
sbm_s$rBlocks() # draw some blocks
sbm_s$rAdjMatrix() # draw some edges
```

```
coef(sbm_s, "mixture")
coef(sbm_s, "connectivity")
summary(sbm_s)
```

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simulate

Description

Generates a realization (blocks and adjacency matrix) of a Stochastic Block model

Usage

Arguments

| nNodes | The number of nodes |
|--------------|---|
| mixtureParam | The mixture parameters |
| connectParam | The connectivity matrix (inter/intra clusters probabilities. provided on a logit scale for a model with covariates) |
| directed | Boolean variable to indicate whether the network is directed or not. Default to FALSE. |
| covariates | A list with M entries (the M covariates). Each entry of the list must be an N x N matrix. |
| covarParam | An optional vector of parameters associated with the covariates, with size M |

Value

an object with class SBM_sampler

See Also

The class SBM_sampler

Examples

```
## SBM parameters
directed <- FALSE
N <- 300 # number of nodes
Q <- 3 # number of clusters
M <- 2 # two Gaussian covariates
alpha <- rep(1, Q)/Q # mixture parameters
pi <- diag(.45, Q) + .05 # connectivity matrix
eta <- rnorm(M, -1, 1) # covariate parametes
gamma <- log(pi/(1-pi)) # logit transform of pi for the model with covariates
X <- replicate(M, matrix(rnorm(N * N ,mean = 0, sd = 1), N, N), simplify = FALSE)
## draw a SBM without covariates
```

sbm <- missSBM::simulate(N, alpha, pi, directed)</pre>

smooth

```
coef(sbm, "connectivity")
## draw a SBM model with node-centred covariates
sbm_cov <- missSBM::simulate(N, alpha, gamma, directed, X, eta)
coef(sbm_cov, "covariates")
old_param <- par(mfrow = c(1,2))
plot(sbm)
plot(sbm_prov)
par(old_param)</pre>
```

smooth

Smooth the path ICL in a collection of missSBM_fit models

Description

Apply a split and/or merge strategy of the clustering in a path of models in a collection of SBM ordered by number of block. The goal is to find better initialization. This results in a "smoothing" of the ICL, that should be close to concave.

Usage

```
smooth(Robject, type = c("forward", "backward", "both"),
    control = list())
```

Arguments

| Robject | an object with class missSBM_collection, i.e. an output from estimate |
|---------|---|
| type | character indicating what kind of ICL smoothing should be use among "for- |
| | ward", "backward" or "both". Default is "forward". |
| control | a list controlling the variational EM algorithm. See details. |

Details

The list of parameters control controls the optimization process and the variational EM algorithm, with the following entries

- "iterates" integer for the number of iteration of smoothing. Default is 1.
- "threshold"stop when an optimization step changes the objective function by less than threshold. Default is 1e-4.
- "maxIter"V-EM algorithm stops when the number of iteration exceeds maxIter. Default is 200
- "fixPointIter"number of fix-point iteration for the Variational E step. Default is 5.
- "cores" integer for number of cores used. Default is 1.
- "trace"integer for verbosity. Useless when cores > 1

Value

an invisible missSBM_collection, in which the ICL has been smoothed

```
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```

War data set

Description

This dataset contains two networks where the nodes are countries and an edge in network "beligerent" means that the two countries have been at least once at war between years 1816 to 2007 while an edge in network "alliance" means that the two countries have had a formal alliance between years 1816 to 2012. The network 'beligerent' have less nodes since countries which have not been at war are not considered.

Usage

war

Format

A list with 2 two igraph objects, alliance and beligerent. Each graph have three attributes: 'name' (the country name), 'power' (a score related to military power: the higher, the better) and 'trade' (a score related to the trade effort between pairs of countries).

Source

networks were extracted from <http://www.correlatesofwar.org/>

References

Sarkees, Meredith Reid and Frank Wayman (2010). Resort to War: 1816 - 2007. Washington DC: CQ Press.

Gibler, Douglas M. 2009. International military alliances, 1648-2008. CQ Press

Examples

```
data(war)
class(war$beligerent)
igraph::gorder(war$alliance)
igraph::gorder(war$beligerent)
igraph::edges(war$alliance)
igraph::get.graph.attribute(war$alliance)
```

war

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