Exposure rating, destruction rate models and the mbbefd package

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1 Introduction and general notation

Exposure rating is a tool for insurance pricing that allocates premium to bands of damage ratios or severity of losses. First ideas were published in (Salzmann 1963). It is often used to price non-proportional reinsurance contracts, such as excess of loss (XL) reinsurance.

Exposure rating uses the loss experience of a similar portfolio of policies to estimate the expected losses of the portfolio to be covered. The method is frequently used as a benchmark when there is no sufficient credible claims history from the client.

In this vignette, we first present the general notation and concepts of loss modelling. Secondly, we focus on destruction models implemented in the mbbefd package. One popular model uses the MBBEFD distribution introduced by (Bernegger 1997). Finally, we provide an example of pricing of XL contract.

1.1 Loss Distribution

First lets assume we have perfect information, i.e. we know the loss distribution for a certain risk.

To keep it simple I assume the loss distribution is log-normal with a mean (M) of 65 and a coefficient of variation (CV) of 30%. The corresponding log-normal parameters μ and σ can be derived via:

$$\sigma^2 = \log(1 + CV^2)$$
$$\mu = \log(M) - \sigma^2/2$$

The following chart shows the corresponding probability density curve $f(x)$, cumulative distribution function $F(x)$ and survival function $S(x) = 1 - F(x)$.
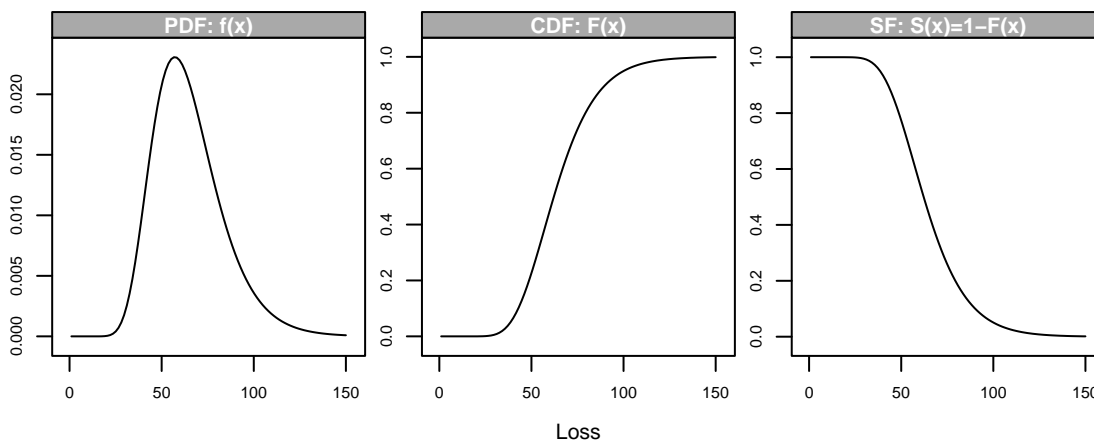
In the insurance context the survival function is often called *exceedance probability function*, as it describes the probability of exceeding a certain loss.

1.2 Expected Loss

Let's define X as the random variable that describes the ground up loss. The expected loss (in plain English the probability weighted sum of the losses) is then:

$$E[X] = \int_{-\infty}^{\infty} x\, f(x) dx$$

Assuming losses are not negative ($P(X < 0) = 0$) this simplifies to:

$$E[X] = \int_{0}^{\infty} x\, f(x) dx$$

1.3 Limited Expected Loss

Let's further assume that losses are limited by α in my contract. Then the limited expected value (LEV), written as $E[X \wedge \alpha]$, is given as

$$E[X \wedge \alpha] = \int_{0}^{\alpha} x\, f(x) dx + \int_{\alpha}^{\infty} \alpha\, f(x) dx$$

To evaluate this sum of integrals I recall that $F(x) = \int f(x) dx$ and $F(\infty) = 1$. Hence:

$$\int_{\alpha}^{\infty} \alpha\, f(x) dx = \alpha \int_{\alpha}^{\infty} f(x) dx = \alpha - \alpha F(\alpha)$$
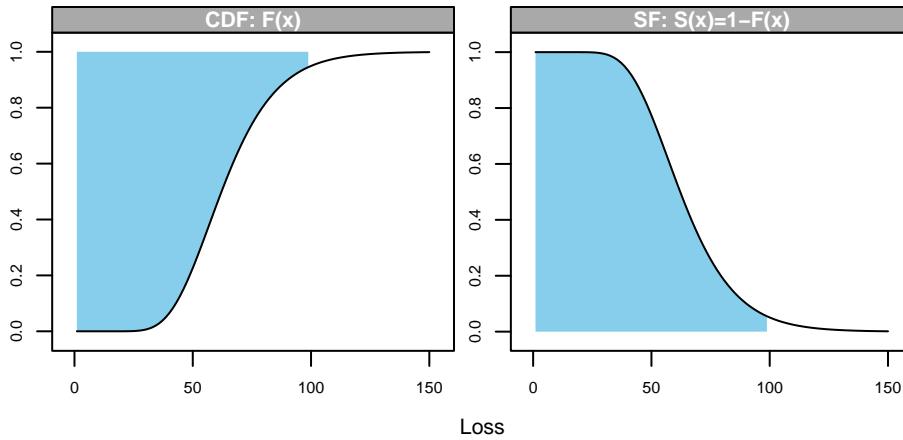
The integration by parts theorem $\int u(x) v'(x)\, dx = u(x) v(x) - \int v(x)\, u'(x) dx$ helps me with the first part of the integral:

$$\int_{0}^{\alpha} x\, f(x) dx = \alpha F(\alpha) - \int_{0}^{\alpha} F(x) dx$$
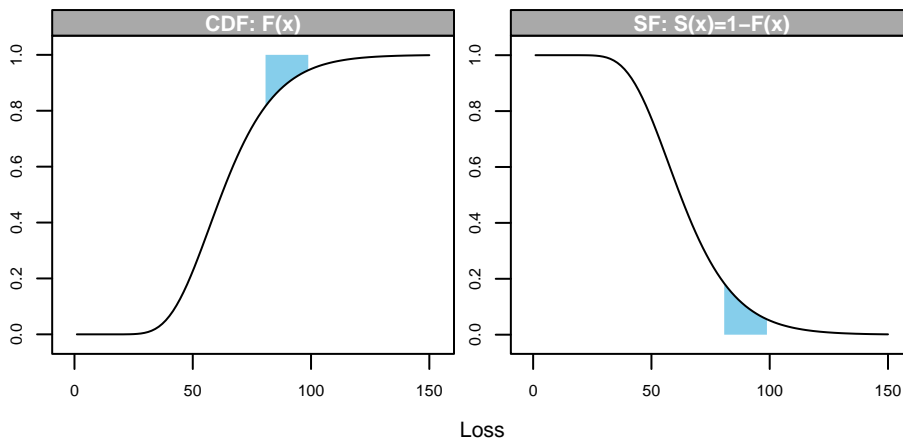
Therefore:

$$E[X \wedge \alpha] = \alpha - \int_{0}^{\alpha} F(x) dx = \int_{0}^{\alpha} 1 - F(x) dx = \int_{0}^{\alpha} S(x) dx$$

The integral describes the area above the CDF up to α, or the area under the survival function up to α. In the example below the limit α was set to 100.

1.4 Loss Cost of a Layer

Suppose I want to insure the layer of claims between 80 and 100.

The expected loss cost would be the difference between the limited expected values of $E[X \wedge 100] - E[X \wedge 80]$.

```
S <- function(x){ 1 - plnorm(x, mu, sigma) }
(lyr <- integrate(S, 80, 100)$value)
```

```
## [1] 2.158244
```

1.5 Increased Limit Factor

On the other hand the ratio of the original loss cost with a limit of 100 to a limit of 80 is called an increased limit factor (ILF):

```
(ILF <- integrate(S, 0, 100)$value / integrate(S, 0, 80)$value)
```

```
## [1] 1.03481
```

Therefore we would expect the LEV to increase by 3.5% as the limit increases from 80 to 100.

Note ILFs are often used for pricing casualty business.

1.6 Exposure curves

From the loss distribution we could read of the expected loss for any layer. However, often we will not have that level of information about the risk.

Instead, we will have to infer information from others risks that share similar loss characteristics as a function of the overall exposure, assuming that the relative loss size distribution is independent of the individual risk characteristic.

Hence, we will require a view on the expected full value loss cost for the overall exposure.

1.6.1 Normalising loss experience using TIV and MPL

To make risks more comparable we look at ratio of losses to the underlying exposure, where the exposure is given as the sum insured (SI), or better the total insured value (TIV), or perhaps as the maximum probable loss (MPL).

Other definitions and measures are also popular, such as possible maximum loss (PML), estimate maximum loss (EML), or maximum foreseeable loss (MFL).

While the TIV and SI are straightforward to understand, the other metrics can be little more challenging to assess.

Suppose we insure a big production plant with of several buildings against fire. It is unlikely that a fire will destroy the whole facility, instead it is believed that the distance between the building will ensure that fires can be contained in a local area. Hence the MPL might be only the highest value of any of those buildings.

Shifting the metric from loss amounts to damage ratio or destruction rates (loss as a % of the exposure metric) allows us to compare and benchmarks risks.

1.6.2 Analysing deductibles

Most insurance policies are written with a deductible, so that the insured will cover the first \$X of the losses.

Thus, the reinsurer needs to understand, by how much the claims burden is reduced for a given deductible.

From the previous section we have learned how to calculate the limited expected value, which is the loss cost to the insured.

The exposure curve (or also called *first loss curve* or *original loss cure*) is defined as the proportion of the LEV for a given deductible d compared to the overall expected claims cost $E[X]$:

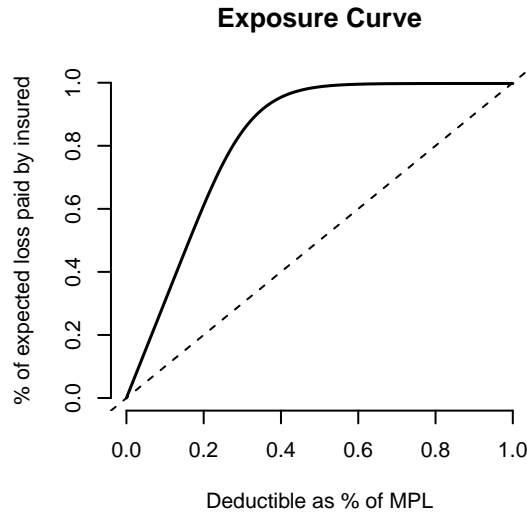$$G(d) = \frac{E[X \wedge d]}{E[X]}$$

For our example, using a log-normal distribution and a MPL of 200 we get:

```
MPL <- 200
ExpectedLoss <- 65
Deductible <- seq(0, MPL, 1)
G <- sapply(Deductible, function(x){
  LEVx <- integrate(S, 0, x)$value
  LEVx/ExpectedLoss
})
plot(Deductible/MPL, G,
     t="l", bty="n", lwd=1.5,
     main="Exposure Curve",
```

```
    xlab="Deductible as % of MPL",
    ylab="% of expected loss paid by insured",
    xlim=c(0,1), ylim=c(0,1))
abline(a=0, b=1, lty=2)
```

Exposure Curve

The steepness of the curve is related to the severity of the loss distribution. The closer the curve is to the diagonal the greater the proportion of large loss.

If all losses were total losses then the exposure curve would be identical with the diagonal.

Different perils and exposures will have different exposure curves. Well known exposure curves are those used by Swiss Re and Lloyd's.

2 Destruction rate models

2.1 Classic distributions

As already mentioned, the exposure curve function of X is defined as the ratio of the limited expected value and the expectation. Since the loss X is a positive random variable, the exposure curve is

$$G_X(d) = \frac{\int_0^d (1 - F_X(x))dx}{\int_0^1 (1 - F_X(x))dx}.$$

Note that the exposure curve is a concave function for $d \in]0, 1[$.

There is a direct link between the distribution function and the exposure curve. Since

$$F_X(x) = \left(1 - \frac{G'_X(x)}{G'_X(0)}\right) \mathbb{1}_{[0,1[}(x) + \mathbb{1}_{[1,+\infty[}(x),$$

defining the exposure curve or the distribution function is equivalent. The exposure curve is also a concave increasing function, see e.g.(Antal 2003).

2.1.1 Uniform distribution

The most trivial example of exposure curve is obtained for the uniform distribution on I. We consider $F_X(x) = x$ leading to

$$G_X(d) = d(2 - d).$$

2.1.2 Beta distribution

A more interesting example is obtained for the Beta distribution on I (e.g.(Johnson, Kotz, and Balakrishnan 1994)) for which the density is $f_X(x) = x^{a-1}(1-x)^{b-1}/\beta(a, b)$ for $x \in]0, 1[$ and $a, b > 0$ where $\beta(., .)$ denotes the beta function, see e.g.(Olver et al. 2010). The distribution function is obtained in terms of the incomplete beta ratio function $F_X(x) = \beta(x; a, b)/\beta(a, b) = I(x; a, b)$. We get

$$G_X(d)\beta(a, b; x) - \frac{x^a(1-x)^b}{a\beta(a, b)} + x(1 - \beta(a, b; x))\frac{a+b}{a},$$

where $\beta(.; ., .)$ denotes the incomplete beta function.

2.2 One-inflated distributions

2.2.1 Characterizations

Let us consider a continuous distribution function F_0 of a random variable X_0. The corresponding distribution function of the one-inflated random variable X_1 is

$$F_1(x) = (1 - p_1)F_0(x) + p_1 \mathbb{1}_{[1,+\infty[}(x).$$

There is no density but an improper density $(1 - p)F_0'(x)$ and a probability mass p_1 at $x = 1$.

2.2.2 The one-inflated beta distribution

We consider the one-inflated beta distribution. Using Section 2.1.2, we obtain the following distribution function

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ I(x; a, b)(1 - p_1) & \text{if } 0 \le x < 1 \\ 1 & \text{if } x \ge 1 \end{cases}$$

where $I(x; a, b)$ denotes the incomplete beta ratio function. This leads to a non-null probability at $x = 1$, $P(X = 1) = p_1$. The improper density function is

$$\tilde{f}_X(x) = (1 - p_1)\frac{x^{a-1}(1-x)^{b-1}}{\beta(a, b)}.$$

The expectation is

$$E(X) = p_1 + (1 - p_1)\frac{a}{a+b}.$$

The exposure curve is

$$G_X(d) = \frac{(1 - p_1)\left(1 - I(d; a, b)\frac{b}{a+b} - \frac{d^a(1-d)^b}{(a+b)\beta(a,b)}\right) + p_1 d}{p_1 + (1 - p_1)\frac{a}{a+b}}.$$

2.3 The MBBEFD distribution, first parametrization

We denote this first parametrization by MBBEFD(a, b). We define the parameter domain $\mathcal{D}_{a,b}$ as

$$\mathcal{D}_{a,b} = \{(a, b) \in \mathbb{R}^2, a + 1 > 0, a(1 - b) > 0, b > 0\} \cup \{(a, b), a = +\infty, b < 1\}. \tag{1}$$

Let us note that this domain includes two particular sets $\mathcal{D}_{a,1} = \{(a, 1), a + 1 > 0\}$ and $\mathcal{D}_{0,b} = \{(0, b), b > 0\}$.

2.3.1 Characterization by the exposure curve

The MBBEFD distribution is defined by the following exposure curve for $(a, b) \in \mathcal{D}_{a,b}$

$$\forall x \in I, \ G_X(x) = \begin{cases} \frac{\ln(\frac{a+b^x}{a+1})}{\ln(\frac{a+b}{a+1})} & \text{if } a(1 - b) > 0 \\ \frac{1-b^x}{1-b} & \text{if } a = +\infty \text{ and } b < 1 \\ x & \text{if } a = 0 \text{ or } b = 1. \end{cases} \tag{2}$$

The two special cases of $a(1 - b) = 0$ correspond to $\mathcal{D}_{a,1}$ and $\mathcal{D}_{0,b}$. Note that the denominator is a normalizing constant to ensure the term belongs to $[0, 1]$.

2.3.2 Distribution, density and quantile functions

Differentiating G_X, we obtain the following distribution function for $(a, b) \in \mathcal{D}_{a,b}$

$$\forall x \in I, \ F_X(x) = \begin{cases} \left(1 - \frac{(a+1)b^x}{a+b^x}\right)\mathbb{1}_{[0,1[}(x) + \mathbb{1}_{[1,+\infty[}(x) & \text{if } a(1 - b) > 0 \\ (1 - b^x)\mathbb{1}_{[0,1[}(x) + \mathbb{1}_{[1,+\infty[}(x) & \text{if } a = +\infty \text{ and } b < 1 \\ \mathbb{1}_{[1,+\infty[}(x) & \text{if } a = 0 \text{ or } b = 1. \end{cases} \tag{3}$$

Note that the MBBEFD distribution is a mixed-type distribution with mass probability at $x = 1$

$$P(X = 1) = \frac{(a + 1)b}{a + b} = p_{a,b}, \tag{4}$$

which equals to 1 when $a(1 - b) = 0$. In other words, for $\mathcal{D}_{a,1}$ and $\mathcal{D}_{0,b}$, X has a Dirac distribution at $x = 1$. When $a = +\infty$, the total loss probability is $P(X = 1) = b$.

For $a(1 - b) > 0$, the improper density function is

$$\tilde{f}_X(x) \begin{cases} -\frac{a(a+1)b^x \ln(b)}{(a+b^x)^2}\mathbb{1}_{[0,1[}(x) & \text{if } a(1 - b) > 0 \\ -\ln(b)b^x\mathbb{1}_{[0,1[}(x) & \text{if } a = +\infty \text{ and } b < 1 \\ 0 & \text{if } a = 0 \text{ or } b = 1. \end{cases} \tag{5}$$

The quantile function is

$$\forall p \in [0, 1], \ q_X(p) = \begin{cases} \frac{\ln\left(\frac{(1-p)a}{a+p}\right)}{\ln(b)}\mathbb{1}_{[0,1-p_{a,b}[}(p) + \mathbb{1}_{[1-p_{a,b},1]}(p) & \text{if } a(1 - b) > 0 \\ \frac{\ln(1-p)}{\ln(b)}\mathbb{1}_{[0,1-b[}(p) + \mathbb{1}_{[1-b,1]}(p) & \text{if } a = +\infty \text{ and } b < 1 \\ \mathbb{1}_{]0,1]}(p) & \text{if } a = 0 \text{ or } b = 1. \end{cases} \tag{6}$$

2.3.3 Moments

Using the definition of the exposure curve, we have $E(X) = 1/G'_X(0)$. The expectation for MBBEFD(a, b) is

$$E(X) = \frac{\ln(\frac{a+b}{a+1})}{\ln(b)}(a + 1).$$

When $a = 0$ or $b = 1$, the expectation is simply $E(X) = 1$.

2.4 The MBBEFD distribution, second parametrization

2.4.1 Parameter domain

For fitting purposes and for verifying parameter constraints, (Bernegger 1997) proposed a second parametrization $\text{MBBEFD}(g, b)$. Using the following parameter $g = 1/p_{a,b}$, it is possible to reformulate the $\text{MBBEFD}(a, b)$. That is

$$g = \frac{a+b}{(a+1)b} \Leftrightarrow a = \frac{(g-1)b}{1-gb}.$$

So $g \geq 1$ guarantees that $\frac{a+b}{(a+1)b} \in [0,1]$, in addition to $b > 0$. The special case $g = 1$ leading to a Dirac distribution at $x = 1$ corresponds to $a(1 - b) = 0$ in the previous parametrization. The parameter domain is

$$\widetilde{\mathcal{D}}_{g,b} = \left\{ (g, b) \in \mathbb{R}^2, b > 0, g \geq 1 \right\}.$$

2.4.2 Characterization by the exposure curve

The exposure curve is defined for $x \in [0, 1]$ as

$$G_X(x) = \begin{cases} \frac{\ln(\frac{(g-1)b}{1-b} + \frac{1-gb}{1-b} b^x)}{\ln(gb)} & \text{if } g > 1, b \neq 1, b \neq 1/g \\ \frac{\ln(1+(g-1)x)}{\ln(g)} & \text{if } g > 1, b = 1 \\ \frac{1-b^x}{1-b} & \text{if } g > 1, bg = 1 \\ x & \text{if } g = 1 \text{ or } b = 0 \end{cases} \tag{7}$$

Note that the case $g > 1, bg = 1$ implies $g = 1/b, b < 1$.

2.4.3 Distribution, density and quantile functions

The resulting distribution function is

$$F_X(x) = \begin{cases} \left(1 - \frac{1-b}{(g-1)b^{1-x}+1-gb}\right) \mathbb{1}_{[0,1[}(x) + \mathbb{1}_{[1,+\infty[}(x) & \text{if } g > 1, b \neq 1, b \neq 1/g \\ \left(1 - \frac{1}{1+(g-1)x}\right) \mathbb{1}_{[0,1[}(x) + \mathbb{1}_{[1,+\infty[}(x) & \text{if } g > 1, b = 1 \\ (1 - b^x) \mathbb{1}_{[0,1[}(x) + \mathbb{1}_{[1,+\infty[}(x) & \text{if } g > 1, bg = 1 \\ \mathbb{1}_{[1,+\infty[}(x) & \text{if } g = 1 \text{ or } b = 0 \end{cases} \tag{8}$$

As in the previous parametrization, there is a non-null probability at $x = 1$,

$$P(X = 1) = 1/g. \tag{9}$$

The improper density function is for $x \in]0, 1[$

$$\tilde{f}_X(x) = \begin{cases} -\frac{(1-b)\ln(b)b^{1-x}}{((g-1)b^{1-x}+1-gb)^2} & \text{if } g > 1, b \neq 1, b \neq 1/g \\ \frac{g-1}{(1+(g-1)x)^2} & \text{if } g > 1, b = 1 \\ -\ln(b)b^x & \text{if } g > 1, bg = 1 \\ 0 & \text{if } g = 1 \text{ or } b = 0 \end{cases} \tag{10}$$

The quantile function is

$$\forall p \in [0,1], \; q_X(p) = \begin{cases} \left(1 - \frac{\ln\left(\frac{gb-1}{g-1} + \frac{1-b}{(1-p)(g-1)}\right)}{\ln(b)}\right) \mathbb{1}_{[0,1-1/g[}(p) + \mathbb{1}_{[1-1/g,1]}(p) & \text{if } g > 1, b \neq 1, b \neq 1/g \\ \frac{p}{(1-p)(g-1)} \mathbb{1}_{[0,1-1/g[}(p) + \mathbb{1}_{[1-1/g,1]}(p) & \text{if } g > 1, b = 1 \\ \frac{\ln(1-p)}{\ln(b)} \mathbb{1}_{[0,1-1/g[}(p) + \mathbb{1}_{[1-1/g,1]}(p) & \text{if } g > 1, bg = 1 \\ \mathbb{1}_{]0,1]}(p) & \text{if } g = 1 \text{ or } b = 0 \end{cases} \tag{11}$$

2.4.4 Moments

Let us compute the first two moments.

$$E(X) = 1/G_X'(0) = \begin{cases} \frac{\ln(gb)(1-b)}{\ln(b)(1-gb)} & \text{if } g > 1, b \neq 1, b \neq 1/g \\ \frac{\ln(g)}{g-1} & \text{if } g > 1, b = 1 \\ \frac{b-1}{\ln(b)} & \text{if } g > 1, bg = 1 \\ 1 & \text{if } g = 1 \text{ or } b = 0 \end{cases}$$

3 Fitting methods and pricing examples

3.1 Fitting methods: MLE and TLMME

We consider two fitting methods, namely the maximum likelihood estimation (MLE) and the total-loss-moment matching estimation. Both methods are implemented in the fitDR function of the mbbefd package.

As its name suggest, the maximum likelihood function consists in maximizing the likelihood function defined as

$$\mathcal{L}(\theta, x_1, \ldots, x_n) = \prod_{i=1}^{n} f_X(x_i)$$

where f_X denotes the density of the loss random variable X and θ is the parameter vector. When X is a continuous non-inflated distribution, f_X is the usual density function (obtained by differentiating the distribution function) when X is a mixed-type distribution with a non-null probability $P(X = 1)$, f_X is the product of $P(X = x)\mathbb{1}_{x=1}$ and $F_X'(x)\mathbb{1}_{x\neq 1}$.

For example when considering the beta distribution, the likelihood function is

$$\mathcal{L}((a,b), x_1, \ldots, x_n) = \prod_{i=1}^{n} \frac{x_i^{a-1}(1-x_i)^{b-1}}{\beta(a,b)}.$$

The total-loss-moment matching estimation consists in matching the total loss and the first moments of a distribution. That is we solve the system

$$\begin{cases} P(X = 1; \theta) = tl_n \\ E(X; \theta) = \bar{x}_n \\ \vdots \\ E(X^k; \theta) = m_{n,k} \end{cases} \quad \text{where} \quad \begin{cases} tl_n = \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}_{x_i=1}, \\ \bar{x}_n = \frac{1}{n}\sum_{i=1}^{n} x_i, \\ m_{n,k} = \frac{1}{n}\sum_{i=1}^{n} (x_i)^k. \end{cases}$$

The number of equation $k + 1$ equals the number of parameters. This methods is relevant only for one-inflated distributions as for classic continuous distributions such as the beta distribution we know that $P(X = 1) = 0$.

The fitDR function of the mbbefd package returns an object of class "DR" inheriting from the "fitdist" class for which generic functions are available print, plot, summary, gofstat. There is also a bootstrap procedure implemented in the bootDR function.

3.2 Examples

Let us fit some distributions with a simulated dataset based on a mixture of two distributions (regular beta and mbbefd).

```r
library(mbbefd)
set.seed(123456)
x <- c(rbeta(50, 3, 1/2), rmbbefd(50, 1/2, 1/10))
f1 <- fitDR(x, "mbbefd", method="mle")
summary(f1)
```

```
## Fitting of the distribution ' mbbefd ' by maximum likelihood
## Parameters :
##      estimate Std. Error
## a 0.035909278         NA
## b 0.009872438         NA
## Loglikelihood:  -39.69145   AIC:  83.3829   BIC:  88.59324
## Correlation matrix:
## [1] NA
```
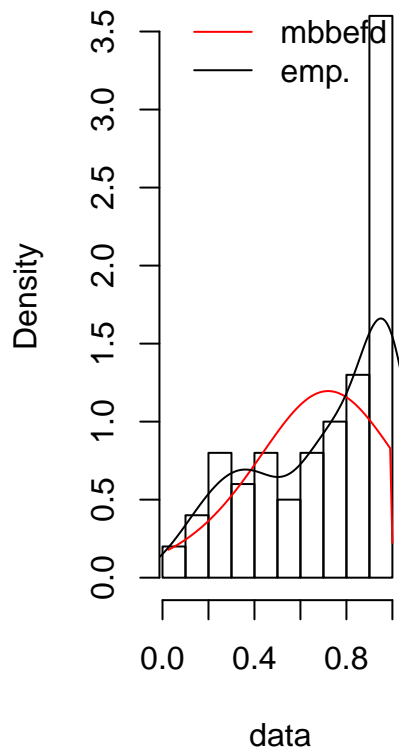
```r
b1 <- bootDR(f1, niter=20)
summary(b1)
```

```
## Parametric bootstrap medians and 95% percentile CI
##        Median         2.5%       97.5%
## a 0.03603436 0.012784198 0.06813441
## b 0.00930872 0.002569712 0.02273344
```
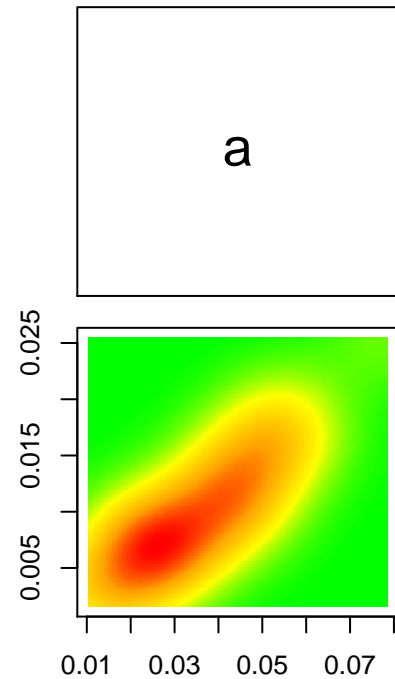
```r
par(mfrow=c(1, 2))
denscomp(f1, demp=TRUE)
plot(b1, enhance=TRUE)
```

The graph correspond respectively to the histogram (and empirical density) against the fitted density and a two-dimensional level curve and scatter plot for bootstrap parameters.

As in the fitdistrplus package, we can compare multiple fits on the same graph or with the same statistics. We just have to pass a list of "DR" objects.

```
f2 <- fitDR(x, "oibeta", method="mle")
f3 <- fitDR(x, "oiunif", method="mle")
gofstat(list(f1, f2, f3))
```

```
## Goodness-of-fit statistics
##                                1-mle-mbbefd 2-mle-oibeta 3-mle-oiunif
## Kolmogorov-Smirnov statistic      0.1400000    0.1400000    0.2154661
## Cramer-von Mises statistic        0.2715687    0.1414871    1.7856663
## Anderson-Darling statistic              Inf          Inf          Inf
##
## Goodness-of-fit criteria
##                                1-mle-mbbefd 2-mle-oibeta 3-mle-oiunif
## Akaike's Information Criterion     83.38290     41.84037     82.99270
## Bayesian Information Criterion     88.59324     49.65588     85.59787
```

```
par(mfrow=c(1, 2))
cdfcomp(list(f1, f2, f3), leg=c("mbbefd", "oibeta", "oiunif"))
denscomp(list(f1, f2, f3), leg=c("mbbefd", "oibeta", "oiunif"), ylim=c(0,4), xleg="topleft")
```

Empirical and theoretical CDFs   **Histogram and theoretical densities**
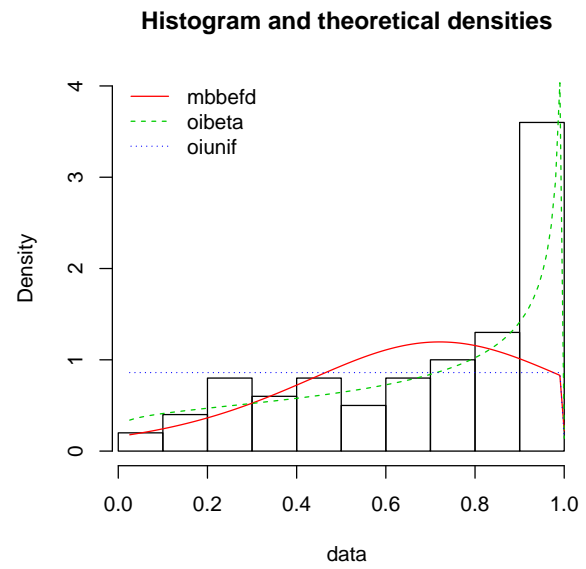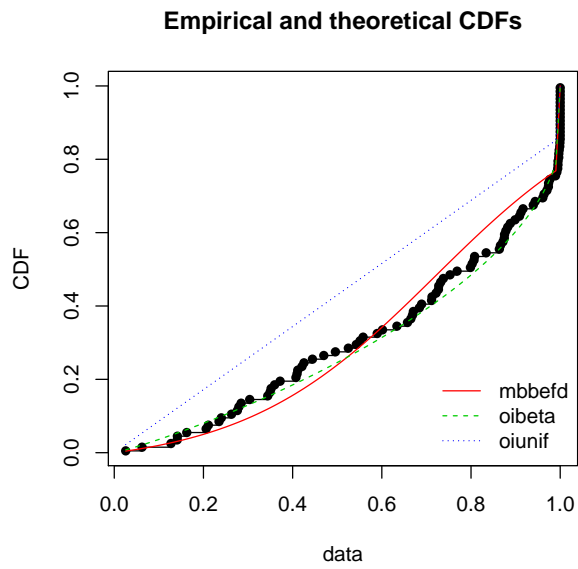
Of course, we can also compare the different fitted exposure curves using the function `eccomp`. That is

```r
eccomp(list(f1, f2, f3), leg=c("mbbefd", "oibeta", "oiunif"), do.points=FALSE)
```


Emp. and theo. exposure curve(s)
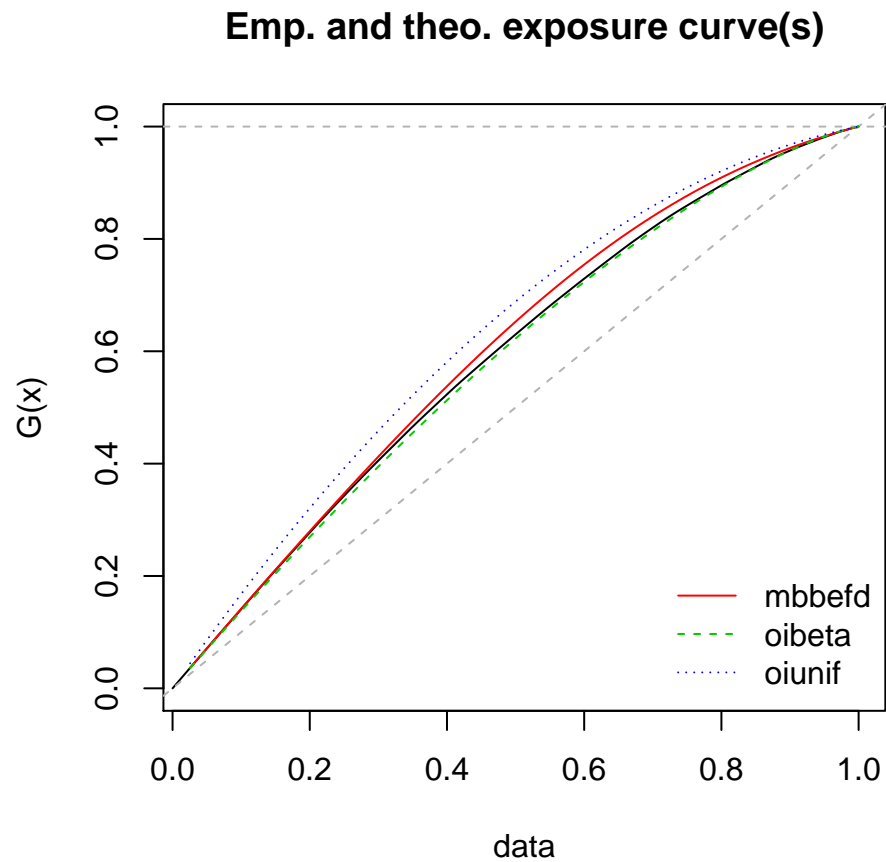
3.3 Computation of rate on line (example)

The following example data is taken from the earlier Swiss Re technical paper above. It is the aim to find the risk premium for a per risk WXL cover (fire only) with the limits

$$\text{CHF } 3,500,000 \text{ xs CHF } 1,500,000$$

The total gross net premium income (GNPI) is CHF $95,975,000$ in 2004, with an expected loss ratio of 55%. Therefore we have a view on the mean expected loss cost.

The risk profile given by the cedant is from the year 2002. However, instead of re-indexing the historical data to 2004, we back-index the data to 2002 with a factor of $457/550 = 0.83$:

$$\text{CHF } 2,908,182 \text{ xs CHF } 1,246,364$$

The data presented below is from the client, with pre-selected exposure curves, in this case the parameter c of the MBBEFD curve.

The policies have been grouped into different exposure bands. The mean MPL is simply the average of the lower and upper band.

Max MPL '000	Mean MPL Gross Loss '000	Gross Premium '000	Exposure Curve Parameter c
150	75	33,434	1.5
250	200	14,568	1.5
400	325	6,324	1.5
600	500	4,584	2
800	700	3,341	2
1,000	900	1,405	2
1,250	1,125	1,169	3
1,500	1,375	683	3
1,750	1,625	613	3
2,000	1,875	554	3
2,500	2,250	700	4
3,000	2,750	552	4
4,000	3,500	1,194	4
5,500	4,750	1,490	4
9,000	7,250	4,177	4
12,500	10,750	3,527	4
18,000	15,250	3,249	4
24,000	21,000	2,712	4
36,000	30,000	2,588	4
48,000	42,000	1,988	4
72,000	60,000	657	4
90,000	81,000	1,918	4

In order to use the exposure curves we need the expected loss and the deductible as % of the MPL.

We have to consider three scenarios:

3.3.1 Losses below deductible

Losses with a maximum MPL below the deductible are not relevant and hence no reinsurance premium needs to be calculated.

3.3.2 Losses above deductible, but below limit

As an example I use the band that has a maximum MPL of CHF 4m, with a mean MPL gross loss of CH 3.5m and gross premium of CH 1.194m.

The client would retain 1,246/3,500=35.6% of the maximum MPL. Using our exposure curve we can read of that this would reduce our expected claims burden by 79.5%.

Therefore the reinsurance premium for this band is 1,194k · 20.5% =244.8k.

3.3.3 Losses above limit

Here I select the band with a maximum MPL of CHF 90m and a mean gross MPL of CHF 81m. Because of the limit of CHF 4154.5 only that amount matters. From the exposure curve we can read of again the average proportion of loss paid by the cedant 0.8%.

To calculate the premium we first have to derive the proportion of premium that is attached to the risk below the limit: 4,155k/8,100k · 1,918k=98.4k.

Multiplying the premium with the reinsurer's share of the loss gives us a premium of CHF 23.4k.

3.3.4 Rate on Line

For the overall portfolio we can calculate the rate on line, which is the loss cost of the layer compared to the total loss cost, using the expected loss ratio of 55%:

```r
XL_ROL <- function(Deductible, Limit, MaxMPL,
                   MeanMPL, GrossPremium, C, ULR){
  DedPerMPL <- Deductible / ifelse(MaxMPL < Deductible, Deductible,
                                   ifelse(MaxMPL<Limit, MaxMPL, Limit))
  LossShare <- 1 - apply(cbind(DedPerMPL, C), 1,
                         function(x){
                           ecMBBEFD(x[1],
                                    b=swissRe(x[2])['b'],
                                    g=swissRe(x[2])['g'])
                         })
  NetPremium <- GrossPremium * ifelse(MaxMPL < Limit, 1,
                                      Limit/MaxMPL)
  XL_Premium <- NetPremium * LossShare
  # Rate on Line
  sum(XL_Premium)/sum(NetPremium)*ULR
}

rol <- XL_ROL(Deductible = D,
              Limit = L,
              MaxMPL = ClientData$MaxMPL,
              MeanMPL = ClientData$MeanMPLGrossLoss,
              GrossPremium = ClientData$GrossPremium,
              C=ClientData$ExposureCurve, ULR=0.55)
```

Applying the function to our data we get a rate on line of 1.55%. Differences to the Swiss Re paper are a result of rounding errors.

References

Antal, P. 2003. "Quantitative Methods in Reinsurance." Swiss Re.

Bernegger, S. 1997. "The Swiss Re Exposure Curves and the MBBEFD Distribution Class." *ASTIN Bulletin* 27 (1): 99–111. http://www.casact.net/library/astin/vol27no1/99.pdf.

Johnson, N., S. Kotz, and N. Balakrishnan. 1994. *Continuous Univariate Distributions.* Vol. 2. Wiley Interscience.

Olver, F. W. J., D. W. Lozier, R. F. Boisvert, and C. W. Clark, eds. 2010. *NIST Handbook of Mathematical Functions.* Cambridge University Press. http://dlmf.nist.gov/.

Salzmann, Ruth E. 1963. "Rating by Layer of Insurance." *PCAS* L: 15–26. https://www.casact.org/pubs/proceed/proceed63/63015.pdf.