# Package 'markovchain'

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Type Package

Title Easy Handling Discrete Time Markov Chains

Version 0.8.5 Date 2020-05-21

Maintainer Giorgio Alfredo Spedicato <spedicato\_giorgio@yahoo.it>

**Description** Functions and S4 methods to create and manage discrete time Markov chains more easily. In addition functions to perform statistical (fitting and drawing random variates) and probabilistic (analysis of their structural proprieties) analysis are provided.

License GPL-2

**Depends** R (>= 3.6.0), methods

**Imports** igraph, Matrix, matlab, expm, stats4, parallel, Rcpp (>= 1.0.2), RcppParallel, utils, stats, grDevices

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VignetteBuilder utils, knitr

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Author Giorgio Alfredo Spedicato [aut, cre] (<a href="https://orcid.org/0000-0002-0315-8888">https://orcid.org/0000-0002-0315-8888</a>), Tae Seung Kang [aut],

Sai Bhargav Yalamanchi [aut],

2 R topics documented:

Mildenberger Thoralf [ctb] (<a href="https://orcid.org/0000-0001-7242-1873">https://orcid.org/0000-0001-7242-1873</a>), Deepak Yadav [aut], Ignacio Cordón [aut] (<a href="https://orcid.org/0000-0002-3152-0231">https://orcid.org/0000-0002-3152-0231</a>), Vandit Jain [ctb], Toni Giorgino [ctb] (<a href="https://orcid.org/0000-0001-6449-0596">https://orcid.org/0000-0001-6449-0596</a>), Richèl J.C. Bilderbeek [ctb] (<a href="https://orcid.org/0000-0003-3666-7205">https://orcid.org/0000-0003-3666-7205</a>), Shreyash Maheshwari [ctb]

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# Description

The package contains classes and method to create and manage (plot, print, export for example) discrete time Markov chains (DTMC). In addition it provide functions to perform statistical (fitting and drawing random variates) and probabilistic (analysis of DTMC proprieties) analysis

#### **Details**

Package: markovchain
Type: Package
Version: 0.8.2
Date: 2020-01-5
License: GPL-2

Depends: R (>= 3.6.0), methods, expm, matlab, igraph, Matrix

### Author(s)

Giorgio Alfredo Spedicato Maintainer: Giorgio Alfredo Spedicato <spedicato\_giorgio@yahoo.it>

#### References

Discrete-Time Markov Models, Bremaud, Springer 1999

### **Examples**

```
# create some markov chains
statesNames=c("a","b")
mcA<-new("markovchain", transitionMatrix=matrix(c(0.7,0.3,0.1,0.9),byrow=TRUE,</pre>
           nrow=2, dimnames=list(statesNames, statesNames)))
statesNames=c("a","b","c")
mcB<-new("markovchain", states=statesNames, transitionMatrix=</pre>
           \texttt{matrix}(\texttt{c}(\texttt{0.2}, \texttt{0.5}, \texttt{0.3}, \texttt{0,1}, \texttt{0,0.1}, \texttt{0.8}, \texttt{0.1}), \texttt{ nrow=3,}
           byrow=TRUE, dimnames=list(statesNames, statesNames)))
statesNames=c("a","b","c","d")
\texttt{matrice} < -\texttt{matrix} (\texttt{c}(0.25, 0.75, 0, 0, 0.4, 0.6, 0, 0, 0, 0, 1, 0.9, 0, 0, 0.7, 0.3), \ \texttt{nrow=4}, \ \texttt{byrow=TRUE})
mcC<-new("markovchain", states=statesNames, transitionMatrix=matrice)</pre>
mcD<-new("markovchain", transitionMatrix=matrix(c(0,1,0,1), nrow=2,byrow=TRUE))
#operations with S4 methods
mcA<sup>2</sup>
steadyStates(mcB)
absorbingStates(mcB)
markovchainSequence(n=20, markovchain=mcC, include=TRUE)
```

absorptionProbabilities

Absorption probabilities

# **Description**

Computes the absorption probability from each transient state to each recurrent one (i.e. the (i, j) entry or (j, i), in a stochastic matrix by columns, represents the probability that the first not transient state we can go from the transient state i is j (and therefore we are going to be absorbed in the communicating recurrent class of j)

# Usage

```
absorptionProbabilities(object)
```

blanden 5

### **Arguments**

object

the markovchain object

#### Value

A named vector with the expected number of steps to go from a transient state to any of the recurrent ones

### Author(s)

Ignacio Cordón

### References

C. M. Grinstead and J. L. Snell. Introduction to Probability. American Mathematical Soc., 2012.

### **Examples**

blanden

Mobility between income quartiles

### **Description**

This table show mobility between income quartiles for father and sons for the 1970 cohort born

# Usage

data(blanden)

#### **Format**

An object of class table with 4 rows and 4 columns.

### **Details**

The rows represent fathers' income quartile when the son is aged 16, whilst the columns represent sons' income quartiles when he is aged 30 (in 2000).

### Source

Personal reworking

6 committorAB

### References

Jo Blanden, Paul Gregg and Stephen Machin, Intergenerational Mobility in Europe and North America, Center for Economic Performances (2005)

### **Examples**

```
data(blanden)
mobilityMc<-as(blanden, "markovchain")</pre>
```

committorAB

Calculates committor of a markovchain object with respect to set A, B

# **Description**

Returns the probability of hitting states rom set A before set B with different initial states

### Usage

```
committorAB(object,A,B,p)
```

### **Arguments**

```
object a markovchain class object

A a set of states

B a set of states

p initial state (default value : 1)
```

### **Details**

The function solves a system of linear equations to calculate probaility that the process hits a state from set A before any state from set B

### Value

Return a vector of probabilities in case initial state is not provided else returns a number

conditional Distribution 7

conditionalDistribution

conditionalDistribution of a Markov Chain

# Description

It extracts the conditional distribution of the subsequent state, given current state.

# Usage

```
conditionalDistribution(object, state)
```

# Arguments

object A markovchain object.

state Subsequent state.

### Value

A named probability vector

#### Author(s)

Giorgio Spedicato, Deepak Yadav

#### References

A First Course in Probability (8th Edition), Sheldon Ross, Prentice Hall 2010

### See Also

markovchain

createSequenceMatrix

craigsendi

CD4 cells counts on HIV Infects between zero and six month

# Description

This is the table shown in Craig and Sendi paper showing zero and six month CD4 cells count in six brakets

### Usage

```
data(craigsendi)
```

#### **Format**

```
The format is: table [1:3, 1:3] 682 154 19 33 64 19 25 47 43 - attr(*, "dimnames")=List of 2 ..$: chr [1:3] "0-49" "50-74" "75-UP" ..$: chr [1:3] "0-49" "50-74" "75-UP"
```

### **Details**

Rows represent counts at the beginning, cols represent counts after six months.

### **Source**

Estimation of the transition matrix of a discrete time Markov chain, Bruce A. Craig and Peter P. Sendi, Health Economics 11, 2002.

#### References

see source

# **Examples**

```
data(craigsendi)
csMc<-as(craigsendi, "markovchain")
steadyStates(csMc)</pre>
```

createSequenceMatrix Function to fit a discrete Markov chain

### **Description**

Given a sequence of states arising from a stationary state, it fits the underlying Markov chain distribution using either MLE (also using a Laplacian smoother), bootstrap or by MAP (Bayesian) inference.

createSequenceMatrix 9

### Usage

```
createSequenceMatrix(
  stringchar,
  toRowProbs = FALSE,
  sanitize = FALSE,
  possibleStates = character()
)
markovchainFit(
  data,
  method = "mle",
  byrow = TRUE,
  nboot = 10L,
  laplacian = 0,
  name = "",
  parallel = FALSE,
  confidencelevel = 0.95,
  confint = TRUE,
  hyperparam = matrix(),
  sanitize = FALSE,
  possibleStates = character()
)
```

# Arguments

stringchar It can be a n x n matrix or a character vector or a list toRowProbs converts a sequence matrix into a probability matrix put 1 in all rows having rowSum equal to zero sanitize possibleStates Possible states which are not present in the given sequence It can be a character vector or a n x n matrix or a n x n data frame or a list data Method used to estimate the Markov chain. Either "mle", "map", "bootstrap" or method "laplace" it tells whether the output Markov chain should show the transition probabilities byrow by row. Number of bootstrap replicates in case "bootstrap" is used. nboot laplacian Laplacian smoothing parameter, default zero. It is only used when "laplace" method is chosen. Optional character for name slot. name parallel Use parallel processing when performing Boostrap estimates. confidencelevel

 $\alpha$ 

level for conficence intervals width. Used only when method equal to "mle".

createSequenceMatrix

confint a boolean to decide whether to compute Confidence Interval or not.

hyperparam Hyperparameter matrix for the a priori distribution. If none is provided, default

value of 1 is assigned to each parameter. This must be of size  $k \times k$  where k is the number of states in the chain and the values should typically be non-negative

integers.

#### **Details**

Disabling confint would lower the computation time on large datasets. If data or stringchar contain NAs, the related NA containing transitions will be ignored.

#### Value

A list containing an estimate, log-likelihood, and, when "bootstrap" method is used, a matrix of standards deviations and the bootstrap samples. When the "mle", "bootstrap" or "map" method is used, the lower and upper confidence bounds are returned along with the standard error. The "map" method also returns the expected value of the parameters with respect to the posterior distribution.

#### Note

This function has been rewritten in Rcpp. Bootstrap algorithm has been defined "heuristically". In addition, parallel facility is not complete, involving only a part of the bootstrap process. When data is either a data. frame or a matrix object, only MLE fit is currently available.

### Author(s)

Giorgio Spedicato, Tae Seung Kang, Sai Bhargav Yalamanchi

#### References

A First Course in Probability (8th Edition), Sheldon Ross, Prentice Hall 2010

Inferring Markov Chains: Bayesian Estimation, Model Comparison, Entropy Rate, and Out-of-Class Modeling, Christopher C. Strelioff, James P. Crutchfield, Alfred Hubler, Santa Fe Institute

Yalamanchi SB, Spedicato GA (2015). Bayesian Inference of First Order Markov Chains. R package version 0.2.5

### See Also

markovchainSequence, markovchainListFit

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```
na.sequenceMatr <- createSequenceMatrix(na.sequence, sanitize = FALSE)
mcFitMLE <- markovchainFit(data = na.sequence)

# data can be a list of character vectors
sequences <- list(x = c("a", "b", "a"), y = c("b", "a", "b", "a", "c"))
mcFitMap <- markovchainFit(sequences, method = "map")
mcFitMle <- markovchainFit(sequences, method = "mle")</pre>
```

ctmc-class

Continuous time Markov Chains class

### **Description**

The S4 class that describes ctmc (continuous time Markov chain) objects.

### **Arguments**

states Name of the states. Must be the same of colnames and rownames of the gener-

ator matrix

byrow TRUE or FALSE. Indicates whether the given matrix is stochastic by rows or by

columns

generator Square generator matrix

name Optional character name of the Markov chain

#### Methods

```
dim signature(x = "ctmc"): method to get the size
initialize signature(.Object = "ctmc"): initialize method
states signature(object = "ctmc"): states method.
steadyStates signature(object = "ctmc"): method to get the steady state vector.
plot signature(x = "ctmc", y = "missing"): plot method for ctmc objects
```

#### Note

- 1. ctmc classes are written using S4 classes
- 2. Validation method is used to assess whether either columns or rows totals to zero. Rounding is used up to 5th decimal. If state names are not properly defined for a generator matrix, coercing to ctmc object leads to overriding states name with artificial "s1", "s2", ... sequence

#### References

Introduction to Stochastic Processes with Applications in the Biosciences (2013), David F. Anderson, University of Wisconsin at Madison. Sai Bhargav Yalamanchi, Giorgio Spedicato

### See Also

generatorToTransitionMatrix,rctmc

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### **Examples**

ctmcFit

Function to fit a CTMC

### Description

This function fits the underlying CTMC give the state transition data and the transition times using the maximum likelihood method (MLE)

### Usage

```
ctmcFit(data, byrow = TRUE, name = "", confidencelevel = 0.95)
```

### **Arguments**

states. The second is a numeric vector denoting the corresponding transition

times.

byrow Determines if the output transition probabilities of the underlying embedded

DTMC are by row.

name Optional name for the CTMC.

confidencelevel

Confidence level for the confidence interval construnction.

## **Details**

Note that in data, there must exist an element wise corresponding between the two elements of the list and that data[[2]][1] is always 0.

#### Value

It returns a list containing the CTMC object and the confidence intervals.

### Author(s)

Sai Bhargav Yalamanchi

expectedRewards 13

### References

Continuous Time Markov Chains (vignette), Sai Bhargav Yalamanchi, Giorgio Alfredo Spedicato 2015

#### See Also

rctmc

### **Examples**

```
data <- list(c("a", "b", "c", "a", "b", "a", "c", "b", "c"), c(0, 0.8, 2.1, 2.4, 4, 5, 5.9, 8.2, 9)) ctmcFit(data)
```

expectedRewards

Expected Rewards for a markovchain

### **Description**

Given a markovchain object and reward values for every state, function calculates expected reward value after n steps.

### Usage

```
expectedRewards(markovchain,n,rewards)
```

## **Arguments**

markovchain the markovchain-class object n no of steps of the process

rewards vector depicting rewards coressponding to states

### **Details**

the function uses a dynamic programming approach to solve a recursive equation described in reference.

#### Value

returns a vector of expected rewards for different initial states

### Author(s)

Vandit Jain

### References

Stochastic Processes: Theory for Applications, Robert G. Gallager, Cambridge University Press

### **Examples**

```
\label{transMatr} $$\operatorname{transMatr} < \operatorname{matrix}(c(0.99,0.01,0.01,0.99),\operatorname{nrow}=2,\operatorname{byrow}=TRUE)$$ simpleMc<-new("markovchain", states=c("a","b"),$$$ transitionMatrix=transMatr)$$ expectedRewards(simpleMc,1,c(0,1))$
```

expected Rewards Before Hitting A

Expected first passage Rewards for a set of states in a markovchain

### **Description**

Given a markovchain object and reward values for every state, function calculates expected reward value for a set A of states after n steps.

### Usage

```
expectedRewardsBeforeHittingA(markovchain, A, state, rewards, n)
```

### **Arguments**

markovchain the markovchain-class object

A set of states for first passage expected reward

state initial state

rewards vector depicting rewards coressponding to states

n no of steps of the process

#### Details

The function returns the value of expected first passage rewards given rewards coressponding to every state, an initial state and number of steps.

# Value

returns a expected reward (numerical value) as described above

# Author(s)

Sai Bhargav Yalamanchi, Vandit Jain

ExpectedTime 15

ExpectedTime	Returns expected hitting time from state i to state j

# Description

Returns expected hitting time from state i to state j

# Usage

```
ExpectedTime(C,i,j,useRCpp)
```

### **Arguments**

С	A CTMC S4 object
i	Initial state i
j	Final state j
useRCpp	logical whether to use Rcpp

#### **Details**

According to the theorem, holding times for all states except j should be greater than 0.

### Value

A numerical value that returns expected hitting times from i to j

### Author(s)

Vandit Jain

### References

Markovchains, J. R. Norris, Cambridge University Press

```
states <- c("a","b","c","d") byRow <- TRUE gen <- matrix(data = c(-1, 1/2, 1/2, 0, 1/4, -1/2, 0, 1/4, 1/6, 0, -1/3, 1/6, 0, 0, 0), nrow = 4,byrow = byRow, dimnames = list(states,states)) ctmc <- new("ctmc",states = states, byrow = byRow, generator = gen, name = "testctmc") ExpectedTime(ctmc,1,4,TRUE)
```

16 firstPassage

firstPassage

First passage across states

### **Description**

This function compute the first passage probability in states

### Usage

```
firstPassage(object, state, n)
```

### **Arguments**

object A markovchain object

state Initial state

n Number of rows on which compute the distribution

### **Details**

Based on Feres' Matlab listings

### Value

A matrix of size 1:n x number of states showing the probability of the first time of passage in states to be exactly the number in the row.

### Author(s)

Giorgio Spedicato

### References

Renaldo Feres, Notes for Math 450 Matlab listings for Markov chains

# See Also

```
conditionalDistribution
```

```
simpleMc <- new("markovchain", states = c("a", "b"), \\ transitionMatrix = matrix(c(0.4, 0.6, .3, .7), \\ nrow = 2, byrow = TRUE)) \\ firstPassage(simpleMc, "b", 20)
```

firstPassageMultiple 17

firstPassageMultiple function to calculate first passage probabilities

### **Description**

The function calculates first passage probability for a subset of states given an initial state.

### Usage

```
firstPassageMultiple(object, state, set, n)
```

# Arguments

```
object a markovchain-class object
state intital state of the process (charactervector)
set set of states A, first passage of which is to be calculated
n Number of rows on which compute the distribution
```

#### Value

A vector of size n showing the first time proabilities

### Author(s)

Vandit Jain

### References

Renaldo Feres, Notes for Math 450 Matlab listings for Markov chains; MIT OCW, course - 6.262, Discrete Stochastic Processes, course-notes, chap -05

### See Also

```
firstPassage
```

18 fitHigherOrder

fitHigherOrder

Functions to fit a higher order Markov chain

### **Description**

Given a sequence of states arising from a stationary state, it fits the underlying Markov chain distribution with higher order.

# Usage

```
fitHigherOrder(sequence, order = 2)
seq2freqProb(sequence)
seq2matHigh(sequence, order)
```

### **Arguments**

sequence A character list.

order Markov chain order

### Value

A list containing lambda, Q, and X.

#### Note

This function is written in Rcpp.

### Author(s)

Giorgio Spedicato, Tae Seung Kang

### References

Ching, W. K., Huang, X., Ng, M. K., & Siu, T. K. (2013). Higher-order markov chains. In Markov Chains (pp. 141-176). Springer US.

Ching, W. K., Ng, M. K., & Fung, E. S. (2008). Higher-order multivariate Markov chains and their applications. Linear Algebra and its Applications, 428(2), 492-507.

fitHighOrderMultivarMC

Function to fit Higher Order Multivariate Markov chain

## **Description**

Given a matrix of categorical sequences it fits Higher Order Multivariate Markov chain.

### Usage

```
fitHighOrderMultivarMC(seqMat, order = 2, Norm = 2)
```

### **Arguments**

seqMat a matrix or a data frame where each column is a categorical sequence

order Multivariate Markov chain order. Default is 2.

Norm Norm to be used. Default is 2.

#### Value

an homme object

### Author(s)

Giorgio Spedicato, Deepak Yadav

#### References

W.-K. Ching et al. / Linear Algebra and its Applications

20 freq2Generator

_	~~	
tren	ı2Gene	rator

Returns a generator matrix corresponding to frequency matrix

### **Description**

The function provides interface to calculate generator matrix corresponding to a frequency matrix and time taken

## Usage

```
freq2Generator(P, t = 1, method = "QO", logmethod = "Eigen")
```

### **Arguments**

P relative frequency matrix

t (default value = 1)

method one among "QO"(Quasi optimaisation), "WA"(weighted adjustment), "DA"(diagonal

adjustment)

logmethod method for computation of matrx algorithm (by default : Eigen)

#### Value

returns a generator matix with same dimnames

### References

E. Kreinin and M. Sidelnikova: Regularization Algorithms for Transition Matrices. Algo Research Quarterly 4(1):23-40, 2001

```
sample <- matrix(c(150,2,1,1,1,200,2,1,2,1,175,1,1,1,1,150),nrow = 4,byrow = TRUE)
sample_rel = rbind((sample/rowSums(sample))[1:dim(sample)[1]-1,],c(rep(0,dim(sample)[1]-1),1))
freq2Generator(sample_rel,1)

data(tm_abs)
tm_rel=rbind((tm_abs/rowSums(tm_abs))[1:7,],c(rep(0,7),1))
## Derive quasi optimization generator matrix estimate
freq2Generator(tm_rel,1)</pre>
```

generatorToTransitionMatrix

Function to obtain the transition matrix from the generator

# Description

The transition matrix of the embedded DTMC is inferred from the CTMC's generator

### Usage

```
generatorToTransitionMatrix(gen, byrow = TRUE)
```

### **Arguments**

gen The generator matrix

byrow Flag to determine if rows (columns) sum to 0

### Value

Returns the transition matrix.

### Author(s)

Sai Bhargav Yalamanchi

#### References

Introduction to Stochastic Processes with Applications in the Biosciences (2013), David F. Anderson, University of Wisconsin at Madison

# See Also

```
rctmc,ctmc-class
```

22 hittingProbabilities

HigherOrderMarkovChain-class

Higher order Markov Chains class

# **Description**

The S4 class that describes HigherOrderMarkovChain objects.

hittingProbabilities Hitting probabilities for markovchain

### **Description**

Given a markovchain object, this function calculates the probability of ever arriving from state i to j

### Usage

```
hittingProbabilities(object)
```

### **Arguments**

object

the markovchain-class object

# Value

a matrix of hitting probabilities

### Author(s)

Ignacio Cordón

# References

R. Vélez, T. Prieto, Procesos Estocásticos, Librería UNED, 2013

```
M <- matlab::zeros(5, 5)
M[1,1] <- M[5,5] <- 1
M[2,1] <- M[2,3] <- 1/2
M[3,2] <- M[3,4] <- 1/2
M[4,2] <- M[4,5] <- 1/2
mc <- new("markovchain", transitionMatrix = M)
hittingProbabilities(mc)</pre>
```

holson 23

holson

Holson data set

### **Description**

A data set containing 1000 life histories trajectories and a categorical status (1,2,3) observed on eleven evenly spaced steps.

# Usage

data(holson)

### **Format**

A data frame with 1000 observations on the following 12 variables.

id unique id

time1 observed status at i-th time

time2 observed status at i-th time

time3 observed status at i-th time

time4 observed status at i-th time

time5 observed status at i-th time

time6 observed status at i-th time

time7 observed status at i-th time

time8 observed status at i-th time

time9 observed status at i-th time

time10 observed status at i-th time

time11 observed status at i-th time

# **Details**

The example can be used to fit a markovchain or a markovchainList object.

#### **Source**

Private communications

# References

Private communications

# Examples

data(holson)

head(holson)

24 ictmc-class

hommc-class

An S4 class for representing High Order Multivariate Markovchain (HOMMC)

### **Description**

An S4 class for representing High Order Multivariate Markovchain (HOMMC)

# Slots

order an integer equal to order of Multivariate Markovchain states a vector of states present in the HOMMC model P array of transition matrices

Lambda a vector which stores the weightage of each transition matrices in P byrow if FALSE each column sum of transition matrix is 1 else row sum = 1 name a name given to hommc

#### Author(s)

Giorgio Spedicato, Deepak Yadav

### **Examples**

ictmc-class

An S4 class for representing Imprecise Continuous Time Markovchains

## **Description**

An S4 class for representing Imprecise Continuous Time Markovchains

### **Slots**

```
states a vector of states present in the ICTMC model

Q matrix representing the generator demonstrated in the form of variables
range a matrix that stores values of range of variables
name given to ICTMC
```

```
imprecise Probability at T
```

Calculating full conditional probability using lower rate transition matrix

### **Description**

This function calculates full conditional probability at given time s using lower rate transition matrix

### Usage

```
impreciseProbabilityatT(C,i,t,s,error,useRCpp)
```

### **Arguments**

С	a ictmc class object
i	initial state at time t
t	initial time t. Default value = $0$
S	final time
error	error rate. Default value = $0.001$
useRCpp	logical whether to use RCpp implementation; by default TRUE

#### Author(s)

Vandit Jain

### References

Imprecise Continuous-Time Markov Chains, Thomas Krak et al., 2016

```
states <- c("n","y")
Q <- matrix(c(-1,1,1,-1),nrow = 2,byrow = TRUE,dimnames = list(states,states))
range <- matrix(c(1/52,3/52,1/2,2),nrow = 2,byrow = 2)
name <- "testictmc"
ictmc <- new("ictmc",states = states,Q = Q,range = range,name = name)
impreciseProbabilityatT(ictmc,2,0,1,10^-3,TRUE)</pre>
```

26 inferHyperparam

inferHyperparam	Function to infer the hyperparameters for Bayesian inference from an a priori matrix or a data set
Im er nyper par alli	

### **Description**

Since the Bayesian inference approach implemented in the package is based on conjugate priors, hyperparameters must be provided to model the prior probability distribution of the chain parameters. The hyperparameters are inferred from a given a priori matrix under the assumption that the matrix provided corresponds to the mean (expected) values of the chain parameters. A scaling factor vector must be provided too. Alternatively, the hyperparameters can be inferred from a data set.

### Usage

```
inferHyperparam(transMatr = matrix(), scale = numeric(), data = character())
```

### **Arguments**

transMatr A valid transition matrix, with dimension names.

scale A vector of scaling factors, each element corresponds to the row names of the

provided transition matrix transMatr, in the same order.

data A data set from which the hyperparameters are inferred.

### **Details**

transMatr and scale need not be provided if data is provided.

### Value

Returns the hyperparameter matrix in a list.

#### Note

The hyperparameter matrix returned is such that the row and column names are sorted alphanumerically, and the elements in the matrix are correspondingly permuted.

### Author(s)

Sai Bhargav Yalamanchi, Giorgio Spedicato

#### References

Yalamanchi SB, Spedicato GA (2015). Bayesian Inference of First Order Markov Chains. R package version 0.2.5

is.accessible 27

### See Also

```
markovchainFit, predictiveDistribution
```

### **Examples**

is.accessible

Verify if a state j is reachable from state i.

### **Description**

This function verifies if a state is reachable from another, i.e., if there exists a path that leads to state j leaving from state i with positive probability

### Usage

```
is.accessible(object, from, to)
```

### **Arguments**

object A markovchain object.

from The name of state "i" (beginning state).

to The name of state "j" (ending state).

#### **Details**

It wraps an internal function named reachabilityMatrix.

### Value

A boolean value.

### Author(s)

Giorgio Spedicato, Ignacio Cordón

28 is.CTMCirreducible

### References

James Montgomery, University of Madison

### See Also

```
is.irreducible
```

### **Examples**

is.CTMCirreducible

Check if CTMC is irreducible

### **Description**

This function verifies whether a CTMC object is irreducible

# Usage

```
is.CTMCirreducible(ctmc)
```

### **Arguments**

ctmc

a ctmc-class object

### Value

a boolean value as described above.

### Author(s)

Vandit Jain

#### References

Continuous-Time Markov Chains, Karl Sigman, Columbia University

is.irreducible 29

### **Examples**

is.irreducible

Function to check if a Markov chain is irreducible (i.e. ergodic)

# Description

This function verifies whether a markovchain object transition matrix is composed by only one communicating class.

### Usage

```
is.irreducible(object)
```

# Arguments

object

A markovchain object

### Details

It is based on .communicatingClasses internal function.

### Value

A boolean values.

### Author(s)

Giorgio Spedicato

### References

Feres, Matlab listings for Markov Chains.

### See Also

summary

is.regular

### **Examples**

is.regular

Check if a DTMC is regular

# Description

Function to check wether a DTCM is regular

### Usage

```
is.regular(object)
```

# Arguments

object

a markovchain object

### **Details**

A Markov chain is regular if some of the powers of its matrix has all elements strictly positive

# Value

A boolean value

# Author(s)

Ignacio Cordón

## References

Matrix Analysis. Roger A.Horn, Charles R.Johnson. 2nd edition. Corollary 8.5.8, Theorem 8.5.9

### See Also

```
is.irreducible
```

is.TimeReversible 31

### **Examples**

is.TimeReversible

checks if ctmc object is time reversible

### **Description**

The function returns checks if provided function is time reversible

### Usage

```
is.TimeReversible(ctmc)
```

### **Arguments**

ctmc

a ctmc-class object

#### Value

Returns a boolean value stating whether ctmc object is time reversible a boolean value as described above

### Author(s)

Vandit Jain

### References

INTRODUCTION TO STOCHASTIC PROCESSES WITH R, ROBERT P. DOBROW, Wiley

32 markovchain-class

kullback

Example from Kullback and Kupperman Tests for Contingency Tables

### **Description**

A list of two matrices representing raw transitions between two states

# Usage

```
data(kullback)
```

#### **Format**

A list containing two 6x6 non - negative integer matrices

markovchain-class

Markov Chain class

# **Description**

The S4 class that describes markovchain objects.

### **Arguments**

states Name of the states. Must be the same of colnames and rownames of the transi-

tion matrix

byrow TRUE or FALSE indicating whether the supplied matrix is either stochastic by

rows or by columns

transitionMatrix

Square transition matrix

name Optional character name of the Markov chain

### Creation of objects

Objects can be created by calls of the form new("markovchain", states, byrow, transitionMatrix,...).

#### Methods

- \* signature(e1 = "markovchain", e2 = "markovchain"): multiply two markovchain objects
- \* signature(e1 = "markovchain", e2 = "matrix"): markovchain by matrix multiplication
- \* signature(e1 = "markovchain",e2 = "numeric"): markovchain by numeric vector multiplication
- \* signature(e1 = "matrix", e2 = "markovchain"): matrix by markov chain

markovchain-class 33

```
* signature(e1 = "numeric",e2 = "markovchain"): numeric vector by markovchain multipli-
[ signature(x = "markovchain", i = "ANY", j = "ANY", drop = "ANY"): ...
^ signature(e1 = "markovchain",e2 = "numeric"): power of a markovchain object
== signature(e1 = "markovchain",e2 = "markovchain"): equality of two markovchain object
!= signature(e1 = "markovchain",e2 = "markovchain"): non-equality of two markovchain
     object
absorbingStates signature(object = "markovchain"): method to get absorbing states
canonicForm signature(object = "markovchain"): return a markovchain object into canonic
     form
coerce signature(from = "markovchain", to = "data.frame"): coerce method from markovchain
     to data.frame
conditionalDistribution signature(object = "markovchain"): returns the conditional proba-
    bility of subsequent states given a state
coerce signature(from = "data.frame",to = "markovchain"): coerce method from data.frame
     to markovchain
coerce signature(from = "table", to = "markovchain"): coerce method from table to markovchain
coerce signature(from = "msm", to = "markovchain"): coerce method from msm to markovchain
coerce signature(from = "msm.est", to = "markovchain"): coerce method from msm.est (but
    only from a Probability Matrix) to markovchain
coerce signature(from = "etm", to = "markovchain"): coerce method from etm to markovchain
coerce signature(from = "sparseMatrix", to = "markovchain"): coerce method from sparseMatrix
     to markovchain
coerce signature(from = "markovchain", to = "igraph"): coercing to igraph objects
coerce signature(from = "markovchain", to = "matrix"): coercing to matrix objects
coerce signature(from = "markovchain",to = "sparseMatrix"): coercing to sparseMatrix
    objects
coerce signature(from = "matrix", to = "markovchain"): coercing to markovchain objects
     from matrix one
dim signature(x = "markovchain"): method to get the size
names signature(x = "markovchain"): method to get the names of states
names<- signature(x = "markovchain", value = "character"): method to set the names of
     states
initialize signature(.Object = "markovchain"): initialize method
plot signature(x = "markovchain", y = "missing"): plot method for markovchain objects
predict signature(object = "markovchain"): predict method
print signature(x = "markovchain"): print method.
show signature(object = "markovchain"): show method.
sort signature(x = "markovchain", decreasing=FALSE): sorting the transition matrix.
states signature(object = "markovchain"): returns the names of states (as names.
```

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```
steadyStates signature(object = "markovchain"): method to get the steady vector.
summary signature(object = "markovchain"): method to summarize structure of the markov chain
transientStates signature(object = "markovchain"): method to get the transient states.
t signature(x = "markovchain"): transpose matrix
transitionProbability signature(object = "markovchain"): transition probability
```

#### Note

- 1. markovchain object are backed by S4 Classes.
- 2. Validation method is used to assess whether either columns or rows totals to one. Rounding is used up to .Machine\$double.eps \* 100. If state names are not properly defined for a probability matrix, coercing to markovhcain object leads to overriding states name with artificial "s1", "s2", ... sequence. In addition, operator overloading has been applied for +, \*,' ==,! = operators.

#### Author(s)

Giorgio Spedicato

#### References

A First Course in Probability (8th Edition), Sheldon Ross, Prentice Hall 2010

#### See Also

markovchainSequence,markovchainFit

```
#show markovchain definition
showClass("markovchain")
#create a simple Markov chain
transMatr<-matrix(c(0.4,0.6,.3,.7),nrow=2,byrow=TRUE)</pre>
simpleMc<-new("markovchain", states=c("a","b"),</pre>
            transitionMatrix=transMatr,
            name="simpleMc")
#power
simpleMc^4
#some methods
steadyStates(simpleMc)
absorbingStates(simpleMc)
simpleMc[2,1]
t(simpleMc)
is.irreducible(simpleMc)
#conditional distributions
conditionalDistribution(simpleMc, "b")
#example for predict method
mcFit<-markovchainFit(data=sequence)</pre>
```

markovchainList-class 35

```
predict(mcFit$estimate, newdata="b",n.ahead=3)
#direct conversion
myMc<-as(transMatr, "markovchain")

#example of summary
summary(simpleMc)
## Not run: plot(simpleMc)</pre>
```

markovchainList-class Non homogeneus discrete time Markov Chains class

### Description

A class to handle non homogeneous discrete Markov chains

#### **Arguments**

```
markovchains Object of class "list": a list of markovchains

name Object of class "character": optional name of the class
```

### **Objects from the Class**

A markovchainlist is a list of markovchain objects. They can be used to model non homogeneous discrete time Markov Chains, when transition probabilities (and possible states) change by time.

### Methods

```
[[ signature(x = "markovchainList"): extract the i-th markovchain
dim signature(x = "markovchainList"): number of markovchain underlying the matrix
predict signature(object = "markovchainList"): predict from a markovchainList
print signature(x = "markovchainList"): prints the list of markovchains
show signature(object = "markovchainList"): same as print
```

### Note

The class consists in a list of markovchain objects. It is aimed at working with non homogeneous Markov chains.

### Author(s)

Giorgio Spedicato

### References

A First Course in Probability (8th Edition), Sheldon Ross, Prentice Hall 2010

36 markovchainListFit

### See Also

markovchain

#### **Examples**

```
showClass("markovchainList")
#define a markovchainList
statesNames=c("a","b")
mcA<-new("markovchain", name="MCA",</pre>
         transitionMatrix=matrix(c(0.7,0.3,0.1,0.9),
                           byrow=TRUE, nrow=2,
                           dimnames=list(statesNames, statesNames))
        )
mcB<-new("markovchain", states=c("a","b","c"), name="MCB",</pre>
         transitionMatrix=matrix(c(0.2,0.5,0.3,0,1,0,0.1,0.8,0.1),
         nrow=3, byrow=TRUE))
mcC<-new("markovchain", states=c("a","b","c","d"), name="MCC",</pre>
         transitionMatrix=matrix(c(0.25,0.75,0,0,0.4,0.6,
                                     0,0,0,0,0.1,0.9,0,0,0.7,0.3),
                                  nrow=4, byrow=TRUE)
)
mcList<-new("markovchainList",markovchains=list(mcA, mcB, mcC),</pre>
           name="Non - homogeneous Markov Chain")
```

markovchainListFit

markovchainListFit

### **Description**

Given a data frame or a matrix (rows are observations, by cols the temporal sequence), it fits a non-homogeneous discrete time markov chain process (storing row). In particular a markovchainList of size = ncol - 1 is obtained estimating transitions from the n samples given by consecutive column pairs.

### Usage

```
markovchainListFit(data, byrow = TRUE, laplacian = 0, name)
```

# Arguments

data Either a matrix or a data.frame or a list object.

byrow Indicates whether distinc stochastic processes trajectiories are shown in distinct

rows.

laplacian Laplacian correction (default 0).

name Optional name.

markovchainSequence 37

#### **Details**

If data contains NAs then the transitions containing NA will be ignored.

#### Value

A list containing two slots: estimate (the estimate) name

#### **Examples**

```
# using holson dataset
data(holson)
# fitting a single markovchain
singleMc <- markovchainFit(data = holson[,2:12])
# fitting a markovchainList
mclistFit <- markovchainListFit(data = holson[, 2:12], name = "holsonMcList")</pre>
```

 ${\tt markovchainSequence}$ 

Function to generate a sequence of states from homogeneous Markov chains.

## **Description**

Provided any markovchain object, it returns a sequence of states coming from the underlying stationary distribution.

#### Usage

```
markovchainSequence(
   n,
   markovchain,
   t0 = sample(markovchain@states, 1),
   include.t0 = FALSE,
   useRCpp = TRUE
)
```

# **Arguments**

n Sample size
markovchain markovchain object
t0 The initial state
include.t0 Specify if the initial state shall be used
useRCpp Boolean. Should RCpp fast implementation being used? Default is yes.

#### **Details**

A sequence of size n is sampled.

#### Value

A Character Vector

#### Author(s)

Giorgio Spedicato

#### References

A First Course in Probability (8th Edition), Sheldon Ross, Prentice Hall 2010

#### See Also

```
markovchainFit
```

#### **Examples**

```
# define the markovchain object
statesNames <- c("a", "b", "c")
mcB <- new("markovchain", states = statesNames,
    transitionMatrix = matrix(c(0.2, 0.5, 0.3, 0, 0.2, 0.8, 0.1, 0.8, 0.1),
    nrow = 3, byrow = TRUE, dimnames = list(statesNames, statesNames)))
# show the sequence
outs <- markovchainSequence(n = 100, markovchain = mcB, t0 = "a")</pre>
```

 ${\tt meanAbsorptionTime}$ 

Mean absorption time

# Description

Computes the expected number of steps to go from any of the transient states to any of the recurrent states. The Markov chain should have at least one transient state for this method to work

# Usage

```
meanAbsorptionTime(object)
```

# Arguments

object

the markovchain object

#### Value

A named vector with the expected number of steps to go from a transient state to any of the recurrent ones

meanFirstPassageTime

#### Author(s)

Ignacio Cordón

#### References

C. M. Grinstead and J. L. Snell. Introduction to Probability. American Mathematical Soc., 2012.

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#### **Examples**

meanFirstPassageTime Mean First Passage Time for irreducible Markov chains

#### **Description**

Given an irreducible (ergodic) markovchain object, this function calculates the expected number of steps to reach other states

#### Usage

```
meanFirstPassageTime(object, destination)
```

# **Arguments**

object the markovchain object

destination a character vector representing the states respect to which we want to compute

the mean first passage time. Empty by default

#### **Details**

For an ergodic Markov chain it computes:

- If destination is empty, the average first time (in steps) that takes the Markov chain to go from initial state i to j. (i, j) represents that value in case the Markov chain is given row-wise, (j, i) in case it is given col-wise.
- If destination is not empty, the average time it takes us from the remaining states to reach the states in destination

#### Value

a Matrix of the same size with the average first passage times if destination is empty, a vector if destination is not

40 meanNumVisits

#### Author(s)

Toni Giorgino, Ignacio Cordón

#### References

C. M. Grinstead and J. L. Snell. Introduction to Probability. American Mathematical Soc., 2012.

# **Examples**

meanNumVisits

Mean num of visits for markovchain, starting at each state

## **Description**

Given a markovchain object, this function calculates a matrix where the element (i, j) represents the expect number of visits to the state j if the chain starts at i (in a Markov chain by columns it would be the element (j, i) instead)

## Usage

```
meanNumVisits(object)
```

# Arguments

object

the markovchain-class object

#### Value

a matrix with the expect number of visits to each state

#### Author(s)

Ignacio Cordón

meanRecurrenceTime 41

#### References

R. Vélez, T. Prieto, Procesos Estocásticos, Librería UNED, 2013

#### **Examples**

```
M <- matlab::zeros(5, 5)
M[1,1] <- M[5,5] <- 1
M[2,1] <- M[2,3] <- 1/2
M[3,2] <- M[3,4] <- 1/2
M[4,2] <- M[4,5] <- 1/2
mc <- new("markovchain", transitionMatrix = M)
meanNumVisits(mc)</pre>
```

meanRecurrenceTime

Mean recurrence time

# **Description**

Computes the expected time to return to a recurrent state in case the Markov chain starts there

#### Usage

```
meanRecurrenceTime(object)
```

#### Arguments

object

the markovchain object

#### Value

For a Markov chain it outputs is a named vector with the expected time to first return to a state when the chain starts there. States present in the vector are only the recurrent ones. If the matrix is ergodic (i.e. irreducible), then all states are present in the output and order is the same as states order for the Markov chain

#### Author(s)

Ignacio Cordón

#### References

C. M. Grinstead and J. L. Snell. Introduction to Probability. American Mathematical Soc., 2012.

#### **Examples**

```
 \begin{tabular}{ll} $m < - matrix(1 / 10 * c(6,3,1, & 2,3,5, & & \\ & & 2,3,5, & & \\ & & 4,1,5), \; ncol = 3, \; byrow = TRUE) \\ $mc < - new("markovchain", states = c("s","c","r"), \; transitionMatrix = m)$ $meanRecurrenceTime(mc)$ \\ \end{tabular}
```

multinomialConfidenceIntervals

A function to compute multinomial confidence intervals of DTMC

## **Description**

Return estimated transition matrix assuming a Multinomial Distribution

## Usage

```
multinomialConfidenceIntervals(
  transitionMatrix,
  countsTransitionMatrix,
  confidencelevel = 0.95
)
```

#### **Arguments**

transitionMatrix

An estimated transition matrix.

counts Transition Matrix

Empirical (conts) transition matrix, on which the transitionMatrix was performed.

confidencelevel

confidence interval level.

#### Value

Two matrices containing the confidence intervals.

#### References

Constructing two-sided simultaneous confidence intervals for multinomial proportions for small counts in a large number of cells. Journal of Statistical Software 5(6) (2000)

#### See Also

markovchainFit

name 43

#### **Examples**

name

Method to retrieve name of markovchain object

# **Description**

This method returns the name of a markovchain object

# Usage

```
name(object)
## S4 method for signature 'markovchain'
name(object)
```

## **Arguments**

object

A markovchain object

# Author(s)

Giorgio Spedicato, Deepak Yadav

name<-

Method to set name of markovchain object

#### **Description**

This method modifies the existing name of markovchain object

## Usage

```
name(object) <- value
## S4 replacement method for signature 'markovchain'
name(object) <- value</pre>
```

## **Arguments**

object A markovchain object

value New name of markovchain object

## Author(s)

Giorgio Spedicato, Deepak Yadav

## **Examples**

names, markovchain-method

Returns the states for a Markov chain object

## **Description**

Returns the states for a Markov chain object

#### Usage

```
## S4 method for signature 'markovchain'
names(x)
```

noofVisitsDist 45

#### **Arguments**

x object we want to return states for

# Description

This function would return a joint pdf of the number of visits to the various states of the DTMC during the first N steps.

#### Usage

```
noofVisitsDist(markovchain,N,state)
```

## **Arguments**

markovchain a markovchain-class object

N no of steps state the initial state

#### **Details**

This function would return a joint pdf of the number of visits to the various states of the DTMC during the first N steps.

#### Value

a numeric vector depicting the above described probability density function.

# Author(s)

Vandit Jain

46 period

period

Various function to perform structural analysis of DTMC

# **Description**

These functions return absorbing and transient states of the markovchain objects.

# Usage

```
period(object)

communicatingClasses(object)

recurrentClasses(object)

transientClasses(object)

transientStates(object)

recurrentStates(object)

absorbingStates(object)

canonicForm(object)
```

#### **Arguments**

object

A markovchain object.

#### Value

period returns a integer number corresponding to the periodicity of the Markov chain (if it is irreducible)

absorbingStates returns a character vector with the names of the absorbing states in the Markov chain

communicatingClasses returns a list in which each slot contains the names of the states that are in that communicating class

recurrentClasses analogously to communicatingClasses, but with recurrent classes transientClasses analogously to communicatingClasses, but with transient classes transientStates returns a character vector with all the transient states for the Markov chain recurrentStates returns a character vector with all the recurrent states for the Markov chain canonicForm returns the Markov chain reordered by a permutation of states so that we have blocks submatrices for each of the recurrent classes and a collection of rows in the end for the transient states

period 47

#### Author(s)

Giorgio Alfredo Spedicato, Ignacio Cordón

#### References

Feres, Matlab listing for markov chain.

#### See Also

markovchain

```
statesNames <- c("a", "b", "c")</pre>
mc <- new("markovchain", states = statesNames, transitionMatrix =</pre>
          matrix(c(0.2, 0.5, 0.3,
                    0, 1, 0,
                    0.1, 0.8, 0.1), nrow = 3, byrow = TRUE,
                  dimnames = list(statesNames, statesNames))
         )
communicatingClasses(mc)
recurrentClasses(mc)
recurrentClasses(mc)
absorbingStates(mc)
transientStates(mc)
recurrentStates(mc)
canonicForm(mc)
# periodicity analysis
A \leftarrow matrix(c(0, 1, 0, 0, 0.5, 0, 0.5, 0, 0.5, 0, 0.5, 0, 0.5, 0, 1, 0),
            nrow = 4, ncol = 4, byrow = TRUE)
mcA <- new("markovchain", states = c("a", "b", "c", "d"),</pre>
          transitionMatrix = A,
          name = ^{"}A")
is.irreducible(mcA) #true
period(mcA) #2
# periodicity analysis
B \leftarrow matrix(c(0, 0, 1/2, 1/4, 1/4, 0, 0,
                    0, 0, 1/3, 0, 2/3, 0, 0,
                    0, 0, 0, 0, 0, 1/3, 2/3,
                    0, 0, 0, 0, 0, 1/2, 1/2,
                    0, 0, 0, 0, 0, 3/4, 1/4,
                    1/2, 1/2, 0, 0, 0, 0, 0,
                    1/4, 3/4, 0, 0, 0, 0, 0), byrow = TRUE, ncol = 7)
mcB <- new("markovchain", transitionMatrix = B)</pre>
period(mcB)
```

48 predictiveDistribution

pred	i	<u>_</u>	١Н	_	m	m	_
$DI \vdash CI$	- 1		ΙП	( )	ш	ш	

Simulate a higher order multivariate markovchain

# Description

This function provides a prediction of states for a higher order multivariate markovchain object

#### Usage

```
predictHommc(hommc,t,init)
```

# **Arguments**

homme a homme-class object
t no of iterations to predict

init matrix of previous states size of which depends on hommc

#### **Details**

The user is required to provide a matrix of giving n previous coressponding every categorical sequence. Dimensions of the init are s X n, where s is number of categorical sequences and n is order of the home.

#### Value

The function returns a matrix of size s X t displaying t predicted states in each row coressponding to every categorical sequence.

# Author(s)

Vandit Jain

predictiveDistribution

predictiveDistribution

# Description

The function computes the probability of observing a new data set, given a data set

# Usage

```
predictiveDistribution(stringchar, newData, hyperparam = matrix())
```

predictiveDistribution 49

#### Arguments

stringchar This is the data using which the Bayesian inference is performed.

newData This is the data whose predictive probability is computed.

hyperparam This determines the shape of the prior distribution of the parameters. If none is

provided, default value of 1 is assigned to each parameter. This must be of size kxk where k is the number of states in the chain and the values should typically

be non-negative integers.

#### **Details**

The underlying method is Bayesian inference. The probability is computed by averaging the likelihood of the new data with respect to the posterior. Since the method assumes conjugate priors, the result can be represented in a closed form (see the vignette for more details), which is what is returned.

#### Value

The log of the probability is returned.

#### Author(s)

Sai Bhargav Yalamanchi

#### References

Inferring Markov Chains: Bayesian Estimation, Model Comparison, Entropy Rate, and Out-of-Class Modeling, Christopher C. Strelioff, James P. Crutchfield, Alfred Hubler, Santa Fe Institute

Yalamanchi SB, Spedicato GA (2015). Bayesian Inference of First Order Markov Chains. R package version 0.2.5

# See Also

markovchainFit

priorDistribution

preproglucacon

Preprogluccacon DNA protein bases sequences

## **Description**

Sequence of bases for preproglucacon DNA protein

# Usage

```
data(preproglucacon)
```

#### **Format**

A data frame with 1572 observations on the following 2 variables.

V1 a numeric vector, showing original coding

preproglucacon a character vector, showing initial of DNA bases (Adenine, Cytosine, Guanine, Thymine)

#### Source

Avery Henderson

#### References

Averuy Henderson, Fitting markov chain models on discrete time series such as DNA sequences

## **Examples**

```
data(preproglucacon)
preproglucaconMc<-markovchainFit(data=preproglucacon$preproglucacon)</pre>
```

 ${\tt priorDistribution}$ 

priorDistribution

## **Description**

Function to evaluate the prior probability of a transition matrix. It is based on conjugate priors and therefore a Dirichlet distribution is used to model the transitions of each state.

# Usage

```
priorDistribution(transMatr, hyperparam = matrix())
```

priorDistribution 51

#### **Arguments**

transMatr The transition matrix whose probability is the parameter of interest.

hyperparam The hyperparam matrix (optional). If not provided, a default value of 1 is as-

sumed for each and therefore the resulting probability distribution is uniform.

#### **Details**

The states (dimnames) of the transition matrix and the hyperparam may be in any order.

#### Value

The log of the probabilities for each state is returned in a numeric vector. Each number in the vector represents the probability (log) of having a probability transition vector as specified in corresponding the row of the transition matrix.

#### Note

This function can be used in conjunction with inferHyperparam. For example, if the user has a prior data set and a prior transition matrix, he can infer the hyperparameters using inferHyperparam and then compute the probability of their prior matrix using the inferred hyperparameters with priorDistribution.

#### Author(s)

Sai Bhargav Yalamanchi, Giorgio Spedicato

#### References

Yalamanchi SB, Spedicato GA (2015). Bayesian Inference of First Order Markov Chains. R package version 0.2.5

#### See Also

predictiveDistribution, inferHyperparam

```
\begin{split} \text{priorDistribution}(\text{matrix}(c(0.5,\ 0.5,\ 0.5,\ 0.5),\\ \text{nrow} &= 2,\\ \text{dimnames} &= \text{list}(c("a",\ "b"),\ c("a",\ "b"))),\\ \text{matrix}(c(2,\ 2,\ 2,\ 2),\\ \text{nrow} &= 2,\\ \text{dimnames} &= \text{list}(c("a",\ "b"),\ c("a",\ "b")))) \end{split}
```

52 probabilityatT

prob	٦ah	٠i ٦	i 1	-1/2	+ T

Calculating probability from a ctmc object

# Description

This function returns the probability of every state at time t under different conditions

#### Usage

```
probabilityatT(C,t,x0,useRCpp)
```

# Arguments

С	A CTMC S4 object
t	final time t
x0	initial state
useRCpp	logical whether to use RCpp implementation

#### **Details**

The initial state is not mandatory, In case it is not provided, function returns a matrix of transition function at time t else it returns vector of probabilities of transition to different states if initial state was x0

# Value

returns a vector or a matrix in case x0 is provided or not respectively.

#### Author(s)

Vandit Jain

## References

INTRODUCTION TO STOCHASTIC PROCESSES WITH R, ROBERT P. DOBROW, Wiley

```
states <- c("a","b","c","d") 
byRow <- TRUE 
gen <- matrix(data = c(-1, 1/2, 1/2, 0, 1/4, -1/2, 0, 1/4, 1/6, 0, -1/3, 1/6, 0, 0, 0), 
nrow = 4,byrow = byRow, dimnames = list(states,states)) 
ctmc <- new("ctmc",states = states, byrow = byRow, generator = gen, name = "testctmc") 
probabilityatT(ctmc,1,useRCpp = TRUE)
```

rain 53

rain

Alofi island daily rainfall

# **Description**

Rainfall measured in Alofi Island

# Usage

```
data(rain)
```

#### **Format**

A data frame with 1096 observations on the following 2 variables.

V1 a numeric vector, showing original coding rain a character vector, showing daily rainfall millilitres brackets

#### **Source**

Avery Henderson

#### References

Avery Henderson, Fitting markov chain models on discrete time series such as DNA sequences

# **Examples**

```
data(rain)
rainMc<-markovchainFit(data=rain$rain)</pre>
```

rctmc

rctmc

# **Description**

The function generates random CTMC transitions as per the provided generator matrix.

# Usage

```
rctmc(n, ctmc, initDist = numeric(), T = 0, include.T0 = TRUE,
  out.type = "list")
```

54 rctmc

#### **Arguments**

n The number of samples to generate.

ctmc The CTMC S4 object.

initDist The initial distribution of states.

T The time up to which the simulation runs (all transitions after time T are not

returned).

include. T0 Flag to determine if start state is to be included.

out.type "list" or "df"

#### **Details**

In order to use the T0 argument, set n to Inf.

#### Value

Based on out.type, a list or a data frame is returned. The returned list has two elements - a character vector (states) and a numeric vector (indicating time of transitions). The data frame is similarly structured.

#### Author(s)

Sai Bhargav Yalamanchi

#### References

Introduction to Stochastic Processes with Applications in the Biosciences (2013), David F. Anderson, University of Wisconsin at Madison

# See Also

generatorToTransitionMatrix,ctmc-class

rmarkovchain 55

rmarkovchain	Function to generate a sequence of states from homogeneous or non- homogeneous Markov chains.

# Description

Provided any markovchain or markovchainList objects, it returns a sequence of states coming from the underlying stationary distribution.

# Usage

```
rmarkovchain(
    n,
    object,
    what = "data.frame",
    useRCpp = TRUE,
    parallel = FALSE,
    num.cores = NULL,
    ...
)
```

# **Arguments**

n	Sample size
object	Either a markovchain or a markovchainList object
what	It specifies whether either a data.frame or a matrix (each rows represent a simulation) or a list is returned.
useRCpp	Boolean. Should RCpp fast implementation being used? Default is yes.
parallel	Boolean. Should parallel implementation being used? Default is yes.
num.cores	Number of Cores to be used
	additional parameters passed to the internal sampler

#### **Details**

When a homogeneous process is assumed (markovchain object) a sequence is sampled of size n. When a non - homogeneous process is assumed, n samples are taken but the process is assumed to last from the begin to the end of the non-homogeneous markov process.

# Value

Character Vector, data.frame, list or matrix

# Note

Check the type of input

56 sales

#### Author(s)

Giorgio Spedicato

#### References

A First Course in Probability (8th Edition), Sheldon Ross, Prentice Hall 2010

#### See Also

markovchainFit, markovchainSequence

#### **Examples**

```
# define the markovchain object
statesNames <- c("a", "b", "c")
mcB <- new("markovchain", states = statesNames,</pre>
   transitionMatrix = matrix(c(0.2, 0.5, 0.3, 0, 0.2, 0.8, 0.1, 0.8, 0.1),
   nrow = 3, byrow = TRUE, dimnames = list(statesNames, statesNames)))
# show the sequence
outs <- rmarkovchain(n = 100, object = mcB, what = "list")
#define markovchainList object
statesNames <- c("a", "b", "c")
mcA <- new("markovchain", states = statesNames, transitionMatrix =</pre>
   matrix(c(0.2, 0.5, 0.3, 0, 0.2, 0.8, 0.1, 0.8, 0.1), nrow = 3,
   byrow = TRUE, dimnames = list(statesNames, statesNames)))
mcB <- new("markovchain", states = statesNames, transitionMatrix =</pre>
   matrix(c(0.2, 0.5, 0.3, 0, 0.2, 0.8, 0.1, 0.8, 0.1), nrow = 3,
   byrow = TRUE, dimnames = list(statesNames, statesNames)))
mcC <- new("markovchain", states = statesNames, transitionMatrix =</pre>
   matrix(c(0.2, 0.5, 0.3, 0, 0.2, 0.8, 0.1, 0.8, 0.1), nrow = 3,
   byrow = TRUE, dimnames = list(statesNames, statesNames)))
mclist <- new("markovchainList", markovchains = list(mcA, mcB, mcC))</pre>
# show the list of sequence
rmarkovchain(100, mclist, "list")
```

sales

Sales Demand Sequences

# **Description**

Sales demand sequences of five products (A, B, C, D, E). Each row corresponds to a sequence. First row corresponds to Sequence A, Second row to Sequence B and so on.

show,hommc-method 57

# Usage

```
data("sales")
```

#### **Format**

An object of class matrix (inherits from array) with 269 rows and 5 columns.

#### **Details**

The example can be used to fit High order multivariate markov chain.

# **Examples**

```
data("sales")
# fitHighOrderMultivarMC(seqMat = sales, order = 2, Norm = 2)
```

show, hommc-method

Function to display the details of hommc object

# Description

This is a convenience function to display the slots of homme object in proper format

## Usage

```
## S4 method for signature 'hommc'
show(object)
```

## **Arguments**

object

An object of class homme

states

Defined states of a transition matrix

# Description

This method returns the states of a transition matrix.

# Usage

```
states(object)
## S4 method for signature 'markovchain'
states(object)
```

58 steadyStates

# **Arguments**

object

A discrete markovchain object

#### Value

The character vector corresponding to states slot.

#### Author(s)

Giorgio Spedicato

#### References

A First Course in Probability (8th Edition), Sheldon Ross, Prentice Hall 2010

#### See Also

markovchain

## **Examples**

steadyStates

Stationary states of a markovchain object

## **Description**

This method returns the stationary vector in matricial form of a markovchain object.

# Usage

```
steadyStates(object)
```

## **Arguments**

object

A discrete markovchain object

# Value

A matrix corresponding to the stationary states

tm\_abs 59

## Note

The steady states are identified starting from which eigenvectors correspond to identity eigenvalues and then normalizing them to sum up to unity. When negative values are found in the matrix, the eigenvalues extraction is performed on the recurrent classes submatrix.

## Author(s)

Giorgio Spedicato

## References

A First Course in Probability (8th Edition), Sheldon Ross, Prentice Hall 2010

#### See Also

markovchain

## **Examples**

tm\_abs

Single Year Corporate Credit Rating Transititions

# Description

Matrix of Standard and Poor's Global Corporate Rating Transition Frequencies 2000 (NR Removed)

# Usage

```
data(tm_abs)
```

#### **Format**

```
The format is: num [1:8, 1:8] 17 2 0 0 0 0 0 1 455 ... - attr(*, "dimnames")=List of 2 ..$ : chr [1:8] "AAA" "AA" "A" "BBB" ... ... $ : chr [1:8] "AAA" "AA" "ABBB" ...
```

#### References

European Securities and Markets Authority, 2016 https://cerep.esma.europa.eu/cerep-web/statistics/transitionMatrice.xhtml

60 transition2Generator

# **Examples**

```
data(tm_abs)
```

transition2Generator Return the generator matrix for a corresponding transition matrix

# Description

Calculate the generator matrix for a corresponding transition matrix

## Usage

```
transition2Generator(P, t = 1, method = "logarithm")
```

## **Arguments**

Р transition matrix between time 0 and t

t time of observation

"logarithm" returns the Matrix logarithm of the transition matrix method

#### Value

A matrix that represent the generator of P

# See Also

rctmc

```
mymatr \leftarrow matrix(c(.4, .6, .1, .9), nrow = 2, byrow = TRUE)
Q <- transition2Generator(P = mymatr)</pre>
expm::expm(Q)
```

transitionProbability 61

transitionProbability Function to get the transition probabilities from initial to subsequent states.

## **Description**

This is a convenience function to get transition probabilities.

# Usage

```
transitionProbability(object, t0, t1)
## S4 method for signature 'markovchain'
transitionProbability(object, t0, t1)
```

## **Arguments**

object A markovchain object.

t0 Initial state.

t1 Subsequent state.

#### Value

Numeric Vector

#### Author(s)

Giorgio Spedicato

#### References

A First Course in Probability (8th Edition), Sheldon Ross, Prentice Hall 2010

#### See Also

markovchain

verifyMarkovProperty Various functions to perform statistical inference of DTMC

#### **Description**

These functions verify the Markov property, assess the order and stationarity of the Markov chain.

This function tests whether an empirical transition matrix is statistically compatible with a theoretical one. It is a chi-square based test

Verifies that the s elements in the input list belongs to the same DTMC

#### Usage

```
verifyMarkovProperty(sequence, verbose = TRUE)
assessOrder(sequence, verbose = TRUE)
assessStationarity(sequence, nblocks, verbose = TRUE)
verifyEmpiricalToTheoretical(data, object, verbose = TRUE)
verifyHomogeneity(inputList, verbose = TRUE)
```

# **Arguments**

sequence An empirical sequence.

verbose Should test results be printed out?

nblocks Number of blocks.

data matrix, character or list to be converted in a raw transition matrix

object a markovchain object

inputList A list of items that can coerced to transition matrices

#### Value

Verification result

a list with following slots: statistic (the chi - square statistic), dof (degrees of freedom), and corresponding p-value

a list of transition matrices?

#### Author(s)

Tae Seung Kang, Giorgio Alfredo Spedicato

#### References

Anderson and Goodman.

## See Also

markovchain

```
sequence <- c("a", "b", "a", "a", "a", "a", "b", "a", "b", 
"a", "b", "a", "a", "b", "b", "b", "a")
mcFit <- markovchainFit(data = sequence, byrow = FALSE)</pre>
verifyMarkovProperty(sequence)
assessOrder(sequence)
assessStationarity(sequence, 1)
#Example taken from Kullback Kupperman Tests for Contingency Tables and Markov Chains
0,0,0,0,0,2,2,1,1,1,1,1,2,1,2,0,0,0,1,2,2,2,0,0,0,1,1)
mc=matrix(c(5/8,1/4,1/8,1/4,1/2,1/4,1/4,3/8,3/8),byrow=TRUE, nrow=3)
rownames(mc)<-colnames(mc)<-0:2; theoreticalMc<-as(mc, "markovchain")</pre>
verifyEmpiricalToTheoretical(data=sequence,object=theoreticalMc)
data(kullback)
verifyHomogeneity(inputList=kullback,verbose=TRUE)
```

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