Computational details of demographic functions

C. Dutang and G.A. Spedicato

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Survival probabilities

For notation details, see [@bowers1997actuarial].

Using the well-known relation ${}_{s+u}p_y = {}_up_{y+s} \times {}_sp_y$, we compute ${}_tp_x$ as for all $x, t \in \mathbb{R}_+$

$$ {}_tp_x = \frac{{}_{t+\epsilon_x}p_{\lfloor x \rfloor}}{{}_{\epsilon_x}p_{\lfloor x \rfloor}} = \frac{{}_{\lfloor u \rfloor}p_{\lfloor x \rfloor} \times {}_{\epsilon_u}p_{\lfloor x \rfloor + \lfloor u \rfloor}}{{}_{\epsilon_x}p_{\lfloor x \rfloor}} $$

with $\epsilon_x = x - \lfloor x \rfloor$, $u = t + \epsilon_x$ and $\epsilon_u = u - \lfloor u \rfloor$.

Then we estimate ${}_np_m$ as for all $n, m \in \mathbb{N}$

$$ {}_np_m = \frac{l_{n+m}}{l_m}. $$

We interpolate fractional age probabilities by three classical assumptions for all $y \in [0, 1), m \in \mathbb{N}$

$$ {}_yp_m = \begin{cases} 1 - y(1 - p_m) & \text{if uniform distribution} \\ (p_m)^y & \text{if constant force} \\ \frac{p_m}{1-(1-y)(1-p_m)} & \text{if hyperbolic distribution} \end{cases} $$

Examples of non-integer times

```
library(lifecontingencies)
data("soa08Act")
pXt <- Vectorize(lifecontingencies:::pxtold, "x")
pxT <- Vectorize(lifecontingencies:::pxtold, "t")
pxtvect <- pxt

z <- 1:6/3
#non integer time
cbind(t=z, pxtvect(soa08Act, x=100, t=z, fractional = "lin"), pxT(object=soa08Act, x=100, t=z, fractiona
```

```
##              t
## [1,] 0.3333333 0.8639604 0.8639604
## [2,] 0.6666667 0.7279208 0.7279208
## [3,] 1.0000000 0.5918812 0.5918812
## [4,] 1.3333333 0.5056079 0.5056079
## [5,] 1.6666667 0.4193345 0.4193345
## [6,] 2.0000000 0.3330612 0.3330612
```

```
cbind(t=z, pxtvect(soa08Act, x=100, t=z, fractional = "hyp"), pxT(object=soa08Act, x=100, t=z, fractiona
```

```
##              t
## [1,] 0.3333333 0.8131121 0.8131121
## [2,] 0.6666667 0.6850791 0.6850791
## [3,] 1.0000000 0.5918812 0.5918812
```

```
## [4,] 1.3333333 0.4701083 0.4701083
## [5,] 1.6666667 0.3898924 0.3898924
## [6,] 2.0000000 0.3330612 0.3330612
```

```
cbind(t=z, pxtvect(soa08Act, x=100, t=z, fractional = "exp"), pxT(object=soa08Act, x=100, t=z, fractiona
```

```
##              t
## [1,] 0.3333333 0.8396111 0.8396111
## [2,] 0.6666667 0.7049468 0.7049468
## [3,] 1.0000000 0.5918812 0.5918812
## [4,] 1.3333333 0.4886498 0.4886498
## [5,] 1.6666667 0.4034232 0.4034232
## [6,] 2.0000000 0.3330612 0.3330612
```

Examples of non-integer ages

```
x <- 50+0:6/6
#non-integer age
cbind(x=x, pxtvect(soa08Act, x=x, t=1, fractional = "lin"), pXt(object=soa08Act, x=x, t=1, fractional =
```

```
##             x
## [1,] 50.00000 0.9940801 0.9940801
## [2,] 50.16667 0.9939968 0.9939968
## [3,] 50.33333 0.9939134 0.9939134
## [4,] 50.50000 0.9938298 0.9938298
## [5,] 50.66667 0.9937460 0.9937460
## [6,] 50.83333 0.9936620 0.9936620
## [7,] 51.00000 0.9935779 0.9935779
```

```
cbind(x=x, pxtvect(soa08Act, x=x, t=1, fractional = "hyp"), pXt(object=soa08Act, x=x, t=1, fractional =
```

```
##             x
## [1,] 50.00000 0.9940801 0.9940801
## [2,] 50.16667 0.9939960 0.9939960
## [3,] 50.33333 0.9939120 0.9939120
## [4,] 50.50000 0.9938282 0.9938282
## [5,] 50.66667 0.9937446 0.9937446
## [6,] 50.83333 0.9936612 0.9936612
## [7,] 51.00000 0.9935779 0.9935779
```

```
cbind(x=x, pxtvect(soa08Act, x=x, t=1, fractional = "exp"), pXt(object=soa08Act, x=x, t=1, fractional =
```

```
##             x
## [1,] 50.00000 0.9940801 0.9940801
## [2,] 50.16667 0.9939964 0.9939964
## [3,] 50.33333 0.9939127 0.9939127
## [4,] 50.50000 0.9938290 0.9938290
## [5,] 50.66667 0.9937453 0.9937453
## [6,] 50.83333 0.9936616 0.9936616
## [7,] 51.00000 0.9935779 0.9935779
```

Examples of large ages

```
x <- 135:145
#high-age
cbind(x=x, pxtvect(soa08Act, x=x, t=1), pXt(object=soa08Act, x=x, t=1))
```

```
##         x
##  [1,] 135 1.932519e-06 1.932519e-06
##  [2,] 136 5.431077e-07 5.431077e-07
##  [3,] 137 1.350422e-07 1.350422e-07
##  [4,] 138 2.935883e-08 2.935883e-08
##  [5,] 139 5.508989e-09 5.508989e-09
##  [6,] 140 0.000000e+00 0.000000e+00
##  [7,] 141 0.000000e+00 0.000000e+00
##  [8,] 142 0.000000e+00 0.000000e+00
##  [9,] 143 0.000000e+00 0.000000e+00
## [10,] 144 0.000000e+00 0.000000e+00
## [11,] 145 0.000000e+00 0.000000e+00
```