Package ‘kdensity’

July 11, 2019

Type Package

Title Kernel Density Estimation with Parametric Starts and Asymmetric Kernels

Version 1.0.1

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Description Handles univariate non-parametric density estimation with parametric starts and asymmetric kernels in a simple and flexible way. Kernel density estimation with parametric starts involves fitting a parametric density to the data before making a correction with kernel density estimation, see Hjort & Glad (1995) <doi:10.1214/aos/1176324627>. Asymmetric kernels make kernel density estimation more efficient on bounded intervals such as (0, 1) and the positive half-line. Supported asymmetric kernels are the gamma kernel of Chen (2000) <doi:10.1023/A:1004165218295>, the beta kernel of Chen (1999) <doi:10.1016/S0167-9473(99)00010-9>, and the copula kernel of Jones & Henderson (2007) <doi:10.1093/biomet/asm068>. User-supplied kernels, parametric starts, and bandwidths are supported.

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Encoding UTF-8

LazyData true

Suggests extraDistr, SkewHyperbolic, testthat, covr, EQL, knitr, rmarkdown

Imports assertthat

RoxygenNote 6.1.1

VignetteBuilder knitr

NeedsCompilation no

Repository CRAN

Date/Publication 2019-07-11 13:43:21 UTC

R topics documented:

bandwidths *Bandwidth Selectors*

Description

The available options for bandwidth selectors, passed as the bw argument to kdensity.

Arguments

x	The input data.
kernel_str	A string specifying the kernel, e.g. "gaussian."
start_str	A string specifying the parametric start, e.g. "normal".
support	The domain of definition for the kernel. (-Inf, Inf) for symmetric kernels.

Details

The bandwidth functions are not exported. They are members of the environment bw_environments, and can be accessed by kdensity:::bw_environments.

Bandwidth selectors

"nrd0", "nrd", "bcv", "SJ": Bandwidth selectors from stats. They are documented in bandwidth stats:bandwidth. "nrd0" is the standard bandwidth selector for symmetric kernels with constant parametric starts.

"ucv": Unbiased cross validation. The standard option for asymmetric kernels.

"RHE": Selector for parametric starts with a symmetric kernel, based on a reference rule with Hermite polynomials. Described in Hjort & Glad (1995). The default method in kdensity when a parametric start is supplied and the kernel is symmetric.

"JH": Selector for the Gaussian copula kernel, based on normal reference rule. Described in Jones & Henderson. The default method when the gcopula kernel is used in kdensity.

Structure

The bandwidth selector is a function of four arguments: The data x, a kernel string kernel, a start string start, and a support vector support. To obtain the functions associated with these strings, use get_kernel and get_start. The function should return a double.

References

Jones, M. C., and D. A. Henderson. "Kernel-type density estimation on the unit interval." Biometrika 94.4 (2007): 977-984. Hjort, Nils Lid, and Ingrid K. Glad. "Nonparametric density estimation with a parametric start." The Annals of Statistics (1995): 882-904.

See Also

`kdensity`, `bandwidth` for the bandwidth selectors of `density`. In addition, `kernels`; `parametric_starts`

Examples

```
## Not a serious bandwidth function.
silly_width = function(x, kernel = NULL, start = NULL, support = NULL) {
  rexp(1)
}
kdensity(mtcars$mpg, start = "gumbel", bw = silly_width)
```

`kdensity` *Parametrically guided kernel density estimation*

Description

`kdensity` computes a parametrically guided kernel density estimate for univariate data. It supports asymmetric kernels and parametric starts through the `kernel` and `start` arguments.

Usage

```
kdensity(x, bw = NULL, adjust = 1, kernel = NULL, start = NULL,
  support = NULL, na.rm = FALSE, normalized = TRUE,
  tolerance = 0.01)
```

Arguments

`x` Numeric vector containing the data.

`bw` A bandwidth function. Can be either a string, a custom-made function, or a double. The supported bandwidth functions are documented in `bandwidths`.

`adjust` An adjustment constant, so that `h = adjust*bw*sd`, where `sd` varies with the chosen kernel.

`kernel` The kernel function. Can be chosen from the list of built-in kernels or be custom-made. See `kernels` for details.

`start` Parametric start. Can be chosen from the list of built-in parametric starts or be custom-made. See `parametric_starts` for details.

`support` The support of the data. Must be compatible with the supplied `x` and the supplied `start` and `kernel`. Is used to find the normalization constant, see `normalized`.

`na.rm` Logical; if `TRUE`, `NA`s will be removed from `x`.

`normalized` Logical; if `TRUE`, the density is normalized.

`tolerance` Numeric; the relative error to tolerate in normalization.

Details

The default values for bw, kernel, start, and support are interdependent, and are chosen to make sense. E.g., the default value for support when start = beta is c(0, 1).

The start argument defaults to uniform, which corresponds to ordinary kernel density estimation. The typical default value for kernel is gaussian.

If normalized is FALSE and start != "uniform", the resulting density will not integrate to 1 in general.

Value

kdensity returns an S3 function object of class "kdensity". This is a callable function with the following elements, accessible by '$':

bw_str, bw, adjust, h The bandwidth function, the resulting bandwidth, the adjust argument, and the adjusted bandwidth.

kernel, start, support Name of the kernel, name of the parametric start, and the support of the density.

data.name, n, range, has.na Name of the data, number of observations, the range of the data, and whether the data x contained NA values.

call The call to kdensity.

estimates Named numeric vector containing the parameter estimates from the parametric start.

References

Hjort, Nils Lid, and Ingrid K. Glad. "Nonparametric density estimation with a parametric start." The Annals of Statistics (1995): 882-904.

Jones, M. C., and D. A. Henderson. "Miscellanea kernel-type density estimation on the unit interval." Biometrika 94.4 (2007): 977-984.

Chen, Song Xi. "Probability density function estimation using gamma kernels." Annals of the Institute of Statistical Mathematics 52.3 (2000): 471-480.

Silverman, Bernard W. Density estimation for statistics and data analysis. Vol. 26. CRC press, 1986.

See Also

The stats package function density.

Examples

```
## Use gamma kernels to model positive data, the concentration of
## theophylline

concentration = Theoph$conc + 0.001
plot(kdensity(concentration, start = "gamma", kernel = "gamma", adjust = 1/3),
     ylim = c(0, 0.15), lwd = 2, main = "Concentration of theophylline")
lines(kdensity(concentration, start = "gamma", kernel = "gaussian"),
      lty = 2, col = "grey", lwd = 2)
```

```
lines(kdensity(concentration, start = "gaussian", kernel = "gaussian"),
      lty = 3, col = "blue", lwd = 2)
lines(kdensity(concentration, start = "gaussian", kernel = "gamma", adjust = 1/3),
      lty = 4, col = "red", lwd = 2)
rug(concentration)

## Using a density and and estimator from another package.

skew_hyperbolic = list(
  density   = SkewHyperbolic::dskewhyp,
  estimator = function(x) SkewHyperbolic::skewhypFit(x, printOut = FALSE)$param,
  support   = c(-Inf, Inf)
)

kde = kdensity(diff(LakeHuron), start = skew_hyperbolic)
plot(kde, lwd = 2, col = "blue",
     main = "Annual differences in water level (ft) of Lake Huron, 1875 - 1972")
lines(kde, plot_start = TRUE, lty = 2, lwd = 2) # Plots the skew hyperbolic density.
rug(diff(LakeHuron))

kde$estimates # Also: coef(kde)
# Displays the parameter estimates:
#        mu     delta      beta        nu
# -1.140713  3.301112  2.551657 26.462469
```

kernels *Kernel functions*

Description

Kernel functions are an important part of kdensity. This document lists the available built-in functions and the structure of them. Any kernel in the list can be used in kdensity by using kernel = "kernel" for the intended kernel.

Details

Be careful combining kernels with compact support with parametric starts, as the normalizing integral typically fails to converge. Use gaussian instead.

Symmetric kernels

gaussian, normal: The Gaussian kernel. The default argument when starts is supported on R. epanechnikov, rectangular (uniform), triangular, biweight, cosine, optcosine: Standard symmetric kernels, also used in density. tricube, triweight: Standard symmetric kernels. Not supported by density. laplace: Uses the Laplace density, also known as the double exponential density.

Asymmetric kernels

gamma, gamma_biased: The gamma kernel of Chen (2000). For use on the positive half-line. gamma is the recommended biased-corrected kernel. gcopula: The Gaussian copula kernel of Jones & Henderson (2007). For use on the unit interval. beta, beta_biased: The beta kernel of Chen (1999). For use on the unit interval. beta is the recommended bias-corrected kernel.

Structure

A kernel is a list containing two mandatory elements and one optional element. The mandatory element 'kernel' is the kernel function. It takes arguments y, x, h, where x is the data supplied to kdensity and y is the point of evaluation. h is the bandwidth. Internally, the kernel function is evaluated as 1/h*kernel(y, x, h). It should be vectorized in x, but vectorization in y is not needed.

The second mandatory element is support, stating the domain of definition for the kernel. This is used to distinguish kernels on the unit interval / positive half-line from kernels on R.

sd is used for symmetric kernels, and states the standard error of the kernel. This is used to make kernels comparable to the Gaussian kernel when calculating bandwidths.

References

Chen, Song Xi. "Probability density function estimation using gamma kernels." Annals of the Institute of Statistical Mathematics 52.3 (2000): 471-480. Jones, M. C., and D. A. Henderson. "Kernel-type density estimation on the unit interval." Biometrika 94.4 (2007): 977-984. Chen, Song Xi. "Beta kernel estimators for density functions." Computational Statistics & Data Analysis 31.2 (1999): 131-145.

See Also

kdensity; parametric_starts; bandwidths.

Examples

```
gaussian = list(
  kernel  = function(y, x, h) dnorm((y-x)/h),
  sd = 1,
  support = c(-Inf, Inf)
)

gcopula = list(
  kernel  = function(y, x, h) {
    rho = 1 - h^2
    inside = rho^2*(qnorm(y)^2 + qnorm(x)^2)-2*rho*qnorm(y)*qnorm(x)
    exp(-inside/(2*(1-rho^2)))
  },
  support = c(0, 1)
)
```

parametric_starts *Parametric starts*

Description

A parametric start is a density function with an associated estimator which is used as a starting point in kdensity. Several parametric starts are implemented, all with maximum likelihood estimation. Custom-made parametric starts are possible, see the Structure section.

Built-in starts

uniform, constant: Selecting the uniform start makes kdensity act like an ordinary kernel density estimator. The default value for any choice of kernel or support. gaussian, normal: The normal distribution. A natural choice for densities on the real line $(-\infty, \infty)$. laplace, gumbel: Distributions on $(-\infty, \infty)$. exponential, gamma, lognormal, inverse_gaussian, weibull: Densities supported on the positive real line $(0, \infty)$. beta, kumaraswamy: The beta and Kumaraswamy distributions, supported on the unit interval $[0, 1]$. pareto: The Pareto distribution, supported on $[1, \infty)$. Has heavy tails.

Structure

The parametric start contains three elements: The density function, an estimation function, and the support of the density. The parameters of the density function must partially match the parameters of the estimator function. The estimator function takes one argument, a numeric vector, which is passed from kdensity.

See Also

kdensity; kernels; bandwidths

Examples

```
start_exponential = list(
 density = dexp,
 estimator = function(data) {
   c(rate = 1/mean(data))
 },
 support = c(0, Inf)
)

start_inverse_gaussian = list(
 density = extraDistr::dwald,
 estimator = function(data) {
  c(mu = mean(data),
    lambda = mean(1/data - 1/mean(data)))
  },
 support = c(0, Inf)
)
```

`plot.kdensity` *Plot Method for Kernel Density Estimation*

Description

The `plot` method for `kdensity` objects.

Usage

```
## S3 method for class 'kdensity'
plot(x, range = NULL, plot_start = FALSE,
  zero_line = TRUE, ...)
```

Arguments

`x` a `kdensity` object.

`range` range of x values.

`plot_start` logical; if `TRUE`, plots the parametric start instead of the kernel density estimate.

`zero_line` logical; if `TRUE`, add a base line at `y = 0`.

`...` further plotting parameters.

Value

None.

See Also

`kdensity`

Examples

```
## Using the data set "precip" to eye-ball the similarity between
## a kernel fit, a parametric fit, and a kernel with parametric start fit.
kde_gamma = kdensity(precip, kernel = "gaussian", start = "gamma")
kde = kdensity(precip, kernel = "gaussian", start = "uniform")

plot(kde_gamma, main = "Annual Precipitation in US Cities")
lines(kde_gamma, plot_start = TRUE, lty = 2)
lines(kde, lty = 3)
rug(precip)
```

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