# Package 'ilc'

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ilc-package

*Generalised Lee-Carter models using iterative fitting algorithms* 

#### Description

The package implements a specialised iterative regression method for the analysis of age-period mortality based on a class of generalised Lee-Carter type modelling structure. Within the modelling framework of Renshaw and Haberman (2006), we use a Newton-Raphson iterative process to generate parameter estimates based on Poisson (or Gaussian) likelihood. In addition, we develop and implement here a stratified Lee-Carter model.

## Details

The package contains methods for the analysis of a class of six different types of log-linear models in the GLM framework with Poisson or Gaussian errors that includes the basic LC model too. Also, this package includes tools for the fitting and analysis of the stratified LC model using an additional covariate (other than age and period). There are also made available some general diagnostic tools to analyse the data and the graduation results.

#### Author(s)

Zoltan Butt, Steven Haberman and Han Lin Shang

Maintainer: Zoltan Butt <Z.Butt@city.ac.uk>

## coef.elca

#### References

Lee, R. and Carter, L. (1992), "Modelling and forecasting U.S. mortalit", *Journal of the American Statistical Association* **87**, 659-671.

Lee, L. (2000), "The Lee-Carter method for forecasting mortality, with various extensions and applications", *North American Actuarial Journal* **4**, 80-93.

Renshaw, A. E. and Haberman, S. (2003a), "Lee-Carter mortality forecasting: a parallel generalised linear modelling approach for England and Wales mortality projections", *Journal of the Royal Statistical Society, Series C*, **52**(1), 119-137.

Renshaw, A. E. and Haberman, S. (2003b), "Lee-Carter mortality forecasting with age specific enhancement", *Insurance: Mathematics and Economics*, **33**, 255-272.

Renshaw, A. E. and Haberman, S. (2006), "A cohort-based extension to the Lee-Carter model for mortality reduction factors", *Insurance: Mathematics and Economics*, **38**, 556-570.

Renshaw, A. E. and Haberman, S. (2008), "On simulation-based approaches to risk measurement in mortality with specific reference to Poisson Lee-Carter modelling", *Insurance: Mathematics and Economics*, **42**(2), 797-816.

Renshaw, A. E. and Haberman, S. (2009), "On age-period-cohort parametric mortality rate projections", *Insurance: Mathematics and Economics*, **45**(2), 255-270.

coef.elca

Extract extended Lee-Carter coefficients from an object of class elca

#### Description

Extract extended Lee-Carter coefficients from an object of class elca

#### Usage

```
## S3 method for class 'elca'
coef(object, ...)
```

#### Arguments

object	Data object of class elca
	Other arguments

## Author(s)

Zoltan Butt, Steven Haberman and Han Lin Shang

#### See Also

print.elca

coef.lca

## Description

Extract Lee-Carter coefficients from an object of class lca

## Usage

```
## S3 method for class 'lca'
coef(object, ...)
```

#### Arguments

object	Data object of class lca
	Other arguments

## Author(s)

Zoltan Butt, Steven Haberman and Han Lin Shang

#### See Also

coef.elca

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Extract Lee-Carter coefficients from an object of class rh

## Description

Extract Lee-Carter coefficients from an object of class rh.

## Usage

## S3 method for class 'rh'
coef(object, ...)

## Arguments

object	Data object of class rh
	Other arguments

## Author(s)

Zoltan Butt, Steven Haberman and Han Lin Shang

## dd.cmi.pens

## See Also

print.rh

dd.cmi.pens

Male mortality data of UK pensioners provided by Continuous Mortality Investigation UK

## Description

The dataset is made up by age- and time-specific mortality (hazard) rates and population (exposure) for male pensioners in the UK. Specifically, it covers individual ages between 50 - 108 and calendar years between 1983 - 2003.

#### Usage

data("dd.cmi.pens")

#### Format

Object of class demogdata

## Source

Continuous Mortality Investigation (http://www.actuaries.org.uk/research-and-resources/pages/continuous-mortality-investigation)

#### See Also

demogdata

## Examples

```
# print data summary:
dd.cmi.pens
#Mortality data for CMI
# Series: male
# Years: 1983 - 2003
# Ages: 50 - 108
```

dd.rfp

Artificial (stratified) mortality experience (with Poisson error) for testing the SLC regression

## Description

It transforms a base age-period (2-dimensional) experience of mortality rates into artificially 'stratified' (3-dimensional) mortality rates by overlaying an extra random effect (i.e. other than age and period). Thus, it augments the log of the 2-dimensional mortality rates by an additive effect (with any number of levels) having Poisson distribution with means specified in the rfp argument of the function. It also randomises the base central exposures by a similar additive effect having a normal distribution with mean 0 and a constant age-specific standard deviation, which is calculated arbitrarily as the square root of the age-specific standard errors of the observed exposures. The latter adjustment is applied in order to further randomize the artificially created data. The purpose of the artificial data is to test the Stratified Lee-Carter regression method.

#### Usage

dd.rfp(ddata, rfp)

#### Arguments

ddata	mortality data object of class demogdata
rfp	vector of the means of the artificial additive effect, whereas the length of this
	argument determines the number of levels of the extra factor.

#### Details

Consider a cross-classified mortality experience observed over age (x) and period (t) made up of  $k \times n$  data cells. This function will augment the observed data with Poisson distributed random 'noise' (reduction factors) corresponding to an extra covariate (g) having l levels with means specified in the rfp vector. That is, it creates an artificial data object made up by  $k \times n \times l$  data cells containing the number of deaths corresponding to each subgroup of the stratified mortality experience.

#### Value

Multidimensional mortality data object of rhdata class with the following components:

age	vector of ages
year	vector of years
covariates	names of covariates
deaths	3-dimensional array of death counts
рор	3-dimensional array of exposure
mu	3-dimensional array of force of mortality
label	data label
name	data name

elca.rh

#### Author(s)

Z. Butt and S. Haberman and H. L. Shang

#### See Also

elca.rh

## Examples

```
# vector of means of the additional effect (other than age and period):
rfp <- c(0.5, 1.2, -0.7, 2.5)
# create artificial stratified mortality experience of rhdata class:
rfp.cmi <- dd.rfp(dd.cmi.pens, rfp)
# print stratified rhdata data summary:
rfp.cmi
# plot the base level experience in the stratified rhdata
# a. central exposures:
matplot(rfp.cmi$age, rfp.cmi$pop[,,1], type='1', xlab='Age', ylab='Ec', main='Base Level')
# b. deaths:
matplot(rfp.cmi$age, rfp.cmi$deaths[,,1], type='1', xlab='Age', ylab='D', main='Base Level')
# c. log mortality rates:
matplot(rfp.cmi$age, log(rfp.cmi$mu[,,1]), type='1', xlab='Age', ylab='log(mu)', main='Base Level')
```

elca.rh

*Extended (Stratified) Lee-Carter model (with a single extra parameter)* 

#### Description

A purpose-built regression routine to fit the extended Lee-Carter model with an extra additive effect of an observable factor (other than age and period) on the log mortality mortality rates.

#### Usage

```
elca.rh(dat, year = dat$year, age = dat$age, dec.conv = 6,
error = c("poisson", "gaussian"),
restype = c("logrates", "rates", "deaths", "deviance"),
scale = F, interpolate = F, verbose = T, spar = NULL, ax.fix = NULL)
```

dat	rhdata class multidimensional mortality data object
year	vector of years to be included in the regression (all available years by default)
age	vector of ages to be included in the regression (all available ages by default)
dec.conv	number of decimal places used to achieve convergence. The lower the value the faster the convergence of the fitting algorithm. Note: very high values could over fit the parameters.

error	type of error structure of the model choice (Poisson distribution of the errors by default)
restype	types of residuals, which also controls the type of the fitted value. Thus, in the cases of logrates and rates the function returns as fitted values the log and untransformed mortality rates, respectively. Likewise, the choices of deaths and deviance correspond to the fitted number of deaths
scale	logical, if TRUE, re-scale the interaction parameters so that the $k_t$ has drift parameter equal to 1 (see also <code>lca</code> )
interpolate	logical, if TRUE, replace before regression all zero or missing values in the mortality rates of dat argument by interpolation across calendar years (see also smooth.demogdata)
verbose	logical, it controls the amount of process information
spar	numerical smoothing spline parameter in the interval (0,1] (with a recommended value of 0.6). If it is not NULL, the interaction effects (i.e. $\beta_x^{(0,1)}$ ) are smoothed out after the initial regression. Consequently, the period and/or cohort effects are adjusted (smoothed out) accordingly.
ax.fix	vector of constant age effect to be used in the model (e.g. the fitted values of a standard LC regression to the experience of a large population). If NULL the base ax values are estimated from dat

## Details

This function models the number of deaths for a group within a generalised Lee-Carter framework with a Poisson or Gaussian error structure. The methodology quantifies the differences in the mortality experience of population subgroups differentiated by an additional measurable covariate (other than age and period). Additional covariate, for instance, could be related to geographical, socio-economic or race differences.

## Value

An object of class elca with the following components:

lca	list of fitted lca model objects by the level of the extra factor
age	vector of fitted ages
year	vector of fitted years
ag	parameter estimates of the effects of the extra factor
ах	parameter estimates (or ax.fix) of (mean) age-specific mortality rates across the entire fitting period
bx	parameter estimates of age-specific interaction effect between age and period
kt	parameter estimates of year-specific period trend of mortality rates
adjust	type of error structure used in fitting (e.g. "poisson" or "gaussian")
label	data label
call	copy of the R call to the model
conv.iter	number of iterations used to reach convergence

## extract.deaths

mdev	mean deviance of total and base lack of fit (see also lca)
model	string expression of the fitted model
df	degree of freedom of the fitted GLM model

## Author(s)

Z. Butt and S. Haberman and H. L. Shang

#### References

Li, N. and Lee, R. D. (2005), 'Cohort mortality forecasts for a group of populations: an extension of the Lee-Carter method', Demography, 42(3), 575-594. Renshaw and Haberman (2006), 'A cohort-based extension to the Lee-Carter model for mortality reduction factors.', Insurance: Mathematics and Economics, 38, 556-570.

## See Also

dd.rfp,link{rhdata}

#### Examples

```
rfp <- c(0.5, 1.2, -0.7, 2.5)
rfp.cmi <- dd.rfp(dd.cmi.pens, rfp)
mod6e <- elca.rh(rfp.cmi, age=50:100, interp=TRUE, dec=3, verb=TRUE)
# display model summary and diagnostics:
mod6e; coef(mod6e)</pre>
```

#### Description

This function calculates and outputs the corresponding (observed) number of deaths from a demogdata type mortality data for a choice of ages and calendar years.

#### Usage

```
extract.deaths(data, ages = data$age, years = data$year, combine.upper = T,
fill.method = NULL, series = names(data$rate)[1])
```

data	mortality data object of demogdata class
ages	vector of ages to extract
years	vector of years to extract
combine.upper	logical, if TRUE, ages above max(ages) will be grouped together

fill.method	string value indicating the method to be used for correcting missing or 0 tran- sition rates before estimating the number of deaths (e.g. one of "perks", "inter- polate" or "mspline"). By default is set to NULL, which corresponds to no data correction.
series	target series name (e.g. 'males') or index number (e.g. 1) of the data object to be extracted

#### Value

A 'fictive' demogdata class object in which the (mortality) rate component is replaced by the extracted number of deaths.

#### Note

When estimating the number of deaths (as the product between mortality rates and exposures), some assumptions will need to be made for the cases where the mortality rates are missing (NA). That is, it is not possible to estimate the number of deaths where the exposure (population) is zero because for those cases the corresponding hazard rate (mu) will most likely be NA in the dataset (unless mu was estimated by other means, like a moving average or smoothing, etc.). However, it is reasonable to assume that zero exposures correspond to no observed deaths, which is implemented here. Further, when a fill.method is specified, then the zero and the missing mortality rates are corrected before calculating the number of deaths.

#### Author(s)

Z. Butt and S. Haberman and H. L. Shang

#### See Also

insp.dd, extract.ages, extract.years

#### Examples

```
# 'observed' number of deaths (i.e. no data correction)
extract.deaths(dd.cmi.pens, ages=55:100)
# number of deaths with corrections using Perks mortality model
tmp.d <- extract.deaths(dd.cmi.pens, ages=55:100, fill='perks')</pre>
# Note: to further improve the plot the user can change the vertical
# axis label and/or main title (amongst other plotting parameters).
plot(tmp.d, transf=FALSE, ylab='Number of Deaths')
                                                      # change ylab
plot_dd(tmp.d, transf=FALSE, ylab='Number of Deaths', lpar=list(x.int=-0.2, y.int=0.9, cex=0.85))
plot_dd(tmp.d, y=1995:2003, transf=FALSE, lty=1:5, ylab='Number of Deaths',
  main=paste(tmp.d$lab, ": male (1995-2003)", sep='')) # change main title
```

fill.rhdata

## Description

Interpolate/Smooth an object of class rhdata

## Usage

```
fill.rhdata(data, method = c("mspline", "interpolate", "perks"), ...)
```

## Arguments

data	Data object of class rh
method	Method of interpolation or smoothing
	Other arguments

## Value

An object does not have missing values

#### Author(s)

Zoltan Butt, Steven Haberman and Han Lin Shang

fitted_plots	Miscellaneous plotting functions for lca and lca.rh type regression
	objects

## Description

A diagnostic plot with two graphical regions showing the fitted log rates by the given ages and calendar years.

## Usage

```
fitted_plots(lca.obj, file = paste("fit", deparse(substitute(lca.obj)),
    "ps", sep = "."), view = T, labs = T, col)
```

#### Arguments

lca.obj	an object of class lca.rh
file	an optional string value indicating the output postscript file name (i.e. with extension .ps). By default, it concatenates "fit." and the model object name (with extension ".ps"). If it is set to NULL, the plot will be sent instead to the active graphical window
view	logical, if TRUE (and file argument is not NULL) then Ghostview will be launched with the created .ps file $% \left( \frac{1}{2}\right) =0$
labs	logical, if TRUE, it adds age/years text labels to fitted curves
col	color palette to be used in the plot

## Value

Diagnostic plots of fitted curves by age and by calendar year of a Lee-Carter model object.

## Author(s)

Z. Butt and S. Haberman and H. L. Shang

## See Also

residual\_plots,lca.rh

## Examples

```
mod6 <- lca.rh(dd.cmi.pens, mod='lc', max=110)
# send fitted plots with legends to 'fit.mod6.ps' file:
fitted_plots(mod6)
# send fitted plots without legends to active graphics window
fitted_plots(mod6, file=NULL, labs=FALSE)</pre>
```

flc.plot	Miscellaneous plotting functions for lca and lca.rh type regression
	objects. Plot of forecasted Lee-Carter model based on a fitted model
	object

## Description

Plots the forecasted period effect and life expectancy of the fitted Lee-Carter model in a single graphical window.

## Usage

flc.plot(lca.obj, at = 65, ...)

## fle.plot

#### Arguments

lca.obj	an object of class lca
at	target age at which to calculate life expectancy
	additional arguments to forecast function

## Details

It makes use of a univariate ARIMA process (i.e. random walk with drift) in order to extrapolate the period effect  $k_t$ , which is illustrated by the fitted calendar years together with the corresponding forecasted life expectancy at the specified target age.

## Value

Plot

#### Author(s)

Z. Butt and S. Haberman and H. L. Shang

## See Also

residual\_plots, fitted\_plots, fle.plot, forecast, life.expectancy

## Examples

mod6 <- lca.rh(dd.cmi.pens, mod='lc', interpolate=TRUE)
flc.plot(mod6, at=60, h=30, level=90)</pre>

fle.plot	Miscellaneous plotting functions for lca and lca.rh type regression
	objects. Plot of forecasted life expectancy based on a fitted Lee-Carter
	model object

## Description

Compute the historical and forecasted life expectancy from a fitted LC object and plot them in a single (overlay) figure.

## Usage

fle.plot(lca.obj, at = 65, ...)

lca.obj	an object of class lca
at	target age at which to calculate life expectancy
	additional arguments to forecast function

#### Details

It makes use of the life.expectancy and forecast functions from the demography and forecast packages, respectively, in order to compute life expectancy at the specified target age.

#### Value

Plot

#### Author(s)

Z. Butt and S. Haberman and H. L. Shang

## See Also

residual\_plots, fitted\_plots, flc.plot, forecast, life.expectancy

## Examples

```
mod6 <- lca.rh(dd.cmi.pens, mod='lc', interpolate=TRUE)
fle.plot(mod6, at=60, h=30, level=90)</pre>
```

insp.dd

Miscellaneous utility functions for demogdata type mortality data

#### Description

This function can extract any subset of the source data, such as the mortality (hazard) rates and population (exposure), by a given vector of ages and calendar years. Similarly, the function can output the observed number of deaths by age and calendar years, based on the source data sets included in the demogdata object.

#### Usage

```
insp.dd(data, what = c("rate", "pop", "deaths"), ages = data$age,
years = data$year, series = names(data$rate)[1])
```

## Arguments

data	demogdata mortality data object
what	specifies the required type of data matrix to be extracted
ages	vector of ages to inspect in the data
years	vector of years to inspect in the data
series	target series name (e.g. 'males') or index number (e.g. 1) of the data object to be extracted

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## lca.dev.res

#### Details

A subset of mortality rates and population (exposures) can be directly inspected (i.e. extracted) from the corresponding component data matrices in the source demogdata object by using this function. To inspect the observed number of deaths by age and calendar years the function calls extract.deaths without data corrections.

#### Value

Returns a subset data matrix of the source data.

#### Author(s)

Zoltan Butt, Steven Haberman and Han Lin Shang

## See Also

extract.deaths, extract.ages, extract.years

#### Examples

```
# inspect mortality (hazard) rates:
insp.dd(dd.cmi.pens, 'rate', age=50:80, year=1985:1990)
# inspect exposure (population) values:
insp.dd(dd.cmi.pens, 'pop', age=50:80, year=1985:1990)
# inspect 'estimated' number of deaths
insp.dd(dd.cmi.pens, 'deaths', age=50:80, year=1985:1990)
```

lca.dev.res	
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Miscellaneous utility functions for lca and lca.rh type regression objects. Deviance residuals of the Lee-Carter model

#### Description

A simple utility function to replace the original residuals (e.g. logrates, rates, deaths) of a LC fit with deviance residuals without the need to re-estimate the regression parameters. We note that the estimation of the parameters can be particularly slow in the case of the APC model.

## Usage

```
lca.dev.res(lca.obj, pop, clip = 0)
```

lca.obj	an object of class lca
рор	matrix of population data corresponding to the fitted mortality rates
clip	number of years to clip from start and end of cohort years

## Details

The Lee-Carter regression object contains the type of residuals specified in the original function call, which might need to be changed for further analysis, but without actually re-running the entire iterative estimation process.

#### Value

An identical regression object as lca.obj containing the corresponding deviance residuals

## Author(s)

Zoltan Butt, Steven Haberman and Han Lin Shang

## See Also

lca.rh

## Examples

```
# original model object with 'logrates' residuals
mod6 <- lca.rh(dd.cmi.pens, mod="lc", error="gauss", max=110, interpolate=TRUE)
# adjusted model object with 'deviance' residuals:
dev6 <- lca.dev.res(mod6, insp.dd(dd.cmi.pens, "pop"))</pre>
```

lca.rh

A class of generalised Lee-Carter models

#### Description

A purpose-built regression routine to fit any of the six variants of the class of Lee-Carter model structures using an iterative Newton-Raphson fitting method.

## Usage

```
lca.rh(dat, year = dat$year, age = dat$age, series = 1, max.age = 100,
  dec.conv = 6, clip = 3, error = c("poisson", "gaussian"),
  model = c("m", "h0", "h1", "h2", "ac", "lc"),
    restype = c("logrates", "rates", "deaths", "deviance"), scale = F,
    interpolate = F, verbose = T, spar = NULL)
```

dat	source data object of demogdata class
year	vector of years to be included in the regression (all available years by default)
age	vector of ages to be included in the regression (all available ages by default)
series	numerical index corresponding to the target series to be used from the source data

max.age	highest age to be used in the regression
dec.conv	number of decimal places used to achieve convergence. The lower the value the faster the convergence of the fitting algorithm.
clip	number of marginal birth cohorts to exclude from the regression (i.e., give 0 weights). It is only applicable to the first 5 models (see below)
error	type of error structure of the model choice (Poisson distribution of the errors by default)
model	a character (see usage) or a numeric value (1-6) to specify the model choice
restype	types of residuals, which also controls the type of the fitted value. Thus, in the cases of logrates and rates the function returns as fitted values the log and untransformed mortality rates, respectively. Likewise, the choices of deaths and deviance correspond to the fitted number of deaths
scale	logical, if TRUE, re-scale the interaction parameters so that the $k_t$ has drift parameter equal to 1 (see also <code>lca</code> )
interpolate	logical, if TRUE, replace before regression all zero or missing values in the mortality rates of dat argument by interpolation across calendar years (see also smooth.demogdata)
verbose	logical, if TRUE, the program prints out the updated deviance values along with the starting and final parameter estimates
spar	numerical smoothing spline parameter in the interval (0,1] (with a recommended value of 0.6). If it is not NULL, the interaction effects (i.e. $\beta_x^{(0,1)}$ ) are smoothed out after the initial regression. Consequently, the period and/or cohort effects are adjusted (smoothed out) accordingly.

## Details

Implements the modelling approach proposed in Renshaw and Haberman (2006), which extends the basic Lee-Carter model within the GLM framework. The function makes use of tailored iterative Newton-Raphson fitting algorithms to estimate the graduation parameters of the six variants within this class of extended Lee-Carter models.

## Value

A Lee-Carter type fitted object with the following components:

label	data label
age	vector of fitted ages
year	vector of fitted fitted years
<series></series>	matrix of observed (source) mortality rates used for fitting. It is named the same way as the chosen series
ах	parameter estimates of (mean) age-specific mortality rates across the entire fit- ting period
bx	parameter estimates of age-specific interaction effect between age and period
kt	parameter estimates of year-specific period trend of mortality rates

df	degree of freedom of the fitted GLM model
residuals	residuals of the fitted model in the form of a functional time series object
fitted	fitted values of the fitted model in the form of a functional time series object
varprop	percent of variance
У	source mortality data in the form of a functional time series object
mdev	mean deviance of total and base lack of fit (see also lca)
model	string expression of the fitted model
adjust	type of error structure (e.g. "poisson" or "gaussian")
call	copy of the R call to the model
conv.iter	number of iterations used to reach convergence
bx0	parameter estimates of age-specific interaction effect between age and cohort (only applies to the age-period-cohort model)
itx	parameter estimates of year-specific cohort trend of mortality rates (only applies to the age-period-cohort model)

#### Author(s)

Zoltan Butt, Steven Haberman and Han Lin Shang

#### References

Renshaw, A. E. and Haberman, S. (2003a), "Lee-Carter mortality forecasting: a parallel generalised linear modelling approach for England and Wales mortality projections", *Journal of the Royal Statistical Society, Series C*, **52**(1), 119-137.

Renshaw, A. E. and Haberman, S. (2003b), "Lee-Carter mortality forecasting with age specific enhancement", *Insurance: Mathematics and Economics*, **33**, 255-272.

Renshaw, A. E. and Haberman, S. (2006), "A cohort-based extension to the Lee-Carter model for mortality reduction factors", *Insurance: Mathematics and Economics*, **38**, 556-570.

Renshaw, A. E. and Haberman, S. (2008), "On simulation-based approaches to risk measurement in mortality with specific reference to Poisson Lee-Carter modelling", *Insurance: Mathematics and Economics*, **42**(2), 797-816.

Renshaw, A. E. and Haberman, S. (2009), "On age-period-cohort parametric mortality rate projections", *Insurance: Mathematics and Economics*, **45**(2), 255-270.

#### See Also

dd.rfp,elca.rh,lca

## Examples

```
# standard LC model with Gaussian errors (corresponding to SVD graduation):
# correct 0 or missing mortality rates before graduation
mod6g <- lca.rh(dd.cmi.pens, mod='lc', error='gauss', max=110, interpolate=TRUE)
# AP LC model with Poisson errors
mod6p <- lca.rh(dd.cmi.pens, mod='lc', error='pois', interpolate=TRUE)</pre>
```

#### matflc.plot

```
# Model Summary, Coefficients and Plotting:
mod6p; coef(mod6p); plot(mod6p)
# Comparison with standard fitting method
# Standard LC model (with Gaussian errors) - SVD fit (demography package)
modlc <- lca(dd.cmi.pens, interp=TRUE, adjust='none')
# Gaussian (SVD) - Gaussian (iterative)
round(modlc$ax-mod6g$ax, 4)
round(modlc$bx-mod6g$bx, 4)
round(modlc$bx-mod6g$bx, 4)
# ------ #
# APC LC model fitted to restricted age range with 'deviance' residuals
# the remaining 0/NA values reestimated:
# WARNING: for proper fit recommend dec=6, but it can lead to slow convergence!
mod1 <- lca.rh(dd.cmi.pens, age=60:100, mod='m', interpolate=TRUE, res='dev', dec=1)</pre>
```

```
matflc.plot
```

Miscellaneous plotting functions for lca and lca.rh type regression objects. Plot of forecasted Lee-Carter models based on a series of fitted model objects

#### Description

Comparison plots of the forecasted period effect and life expectancy of a series of fitted Lee-Carter models

#### Usage

```
matflc.plot(lca.obj, lca.base, at = 65, label = NULL, ...)
```

#### Arguments

lca.obj	a list of fitted model objects of class lca (such as returned by elca.rh function)
lca.base	base fitted model object of class lca to be used in comparison
at	target age at which to calculate life expectancy
label	a data label
	additional arguments to forecast function

#### Details

The function makes use of a univariate ARIMA process (i.e. random walk with drift) in order to extrapolate the period effects  $k_t$  of the model objects in lca.obj, which is illustrated by the calendar years together with the corresponding forecasted life expectancy for a given age.

#### Value

Plot

## Author(s)

Z. Butt and S. Haberman and H. L. Shang

## See Also

matfle.plot, flc.plot, elca.rh

## Examples

```
rfp.cmi <- dd.rfp(dd.cmi.pens, c(0.5, 1.2, -0.7, 2.5))
mod6e <- elca.rh(rfp.cmi, age=50:70, interpolate=TRUE, dec=3)
# plot with original (fitted) base values
matflc.plot(mod6e$lca, label='RFP CMI')
# use a standard LC model fitting as base values
mod6 <- lca.rh(dd.cmi.pens, mod='lc', error='gauss', max.age = 70, interpolate=TRUE)
matflc.plot(mod6e$lca, mod6, label='RFP CMI')</pre>
```

matfle.plot	Miscellaneous plotting functions for lca and lca.rh type regression
	objects. Plot of forecasted life expectancy based on a series of fitted
	Lee-Carter model objects

## Description

Compute the historical and forecasted life expectancy of a series of fitted Lee-Carter models and plot them in one comparative figure

## Usage

```
matfle.plot(lca.obj, lca.base, at = 65, label = NULL, ...)
```

## Arguments

lca.obj	a list of fitted model objects of class lca (such as returned by elca.rh function)
lca.base	base fitted model object of class lca to be used in comparison
at	target age at which to calculate life expectancy
label	a data label
	additional arguments to forecast function

#### Details

It makes use of the life.expectancy and forecast functions from the demography and forecast packages, respectively, in order to compute life expectancy at the specified target age for each of the model objects in lca.obj.

## plot.elca

## Value

Plot

## Author(s)

Z. Butt and S. Haberman and H. L. Shang

## See Also

matflc.plot, fle.plot, elca.rh

#### Examples

```
rfp.cmi <- dd.rfp(dd.cmi.pens, c(0.5, 1.2, -0.7, 2.5))
mod6e <- elca.rh(rfp.cmi, age=50:100, interpolate=TRUE, dec=3)
# plot with original (fitted) base values
matfle.plot(mod6e$lca, label='RFP CMI')
# use a standard LC model fitting as base values
mod6 <- lca.rh(dd.cmi.pens, mod='lc', error='gauss', interpolate=TRUE)
matfle.plot(mod6e$lca, mod6, label='RFP CMI')</pre>
```

```
plot.elca
```

Plot an object of class elca

#### Description

Plot an object of class elca

## Usage

## S3 method for class 'elca'
plot(x, ...)

#### Arguments

Х	An object of class elca
	Other arguments

## Value

A plot

## Author(s)

Zoltan Butt, Steven Haberman and Han Lin Shang

## See Also

print.elca

plot.rh

## Description

Plot an object of class rh

## Usage

## S3 method for class 'rh'
plot(x, ...)

## Arguments

х	An object of class rh
	Other arguments

## Value

A plot

## Author(s)

Zoltan Butt, Steven Haberman and Han Lin Shang

## See Also

print.rh

plot_coh_pars	Miscellaneous plotting functions for lca.rh type regression objects.
	Plot of the cohort effects of the generalised Lee-Carter model

## Description

This function plots the age- and time-specific patterns of the cohort effects (only) obtained from the fitting of a generalised Lee-Carter model.

## Usage

plot\_coh\_pars(lca.obj)

## Arguments

lca.obj an object of class lca.rh (containing a generalised LC model with a cohort effect)

## plot\_dd

## Value

A plot with two graphical regions showing the age- and time-specific cohort parameters (i.e.  $beta_x^{(0)}$  and  $iota_t$ ).

## Author(s)

Z. Butt and S. Haberman and H. L. Shang

## References

Renshaw, A. E. and Haberman, S. (2006), "A cohort-based extension to the Lee-Carter model for mortality reduction factors", *Insurance: Mathematics and Economics*, **38**, 556-570.

R. D. Lee and L. Carter (1992) "Modeling and forecasting U.S. mortality", Journal of the American Statistical Association, 87(419), 659-671.

## See Also

plot.per.pars,lca.rh

#### Examples

mod1 <- lca.rh(dd.cmi.pens, age=60:100, mod='m', interpolate=TRUE, res='dev', dec=1)
plot\_coh\_pars(mod1)</pre>

plot_dd	Miscellaneous plotting functions for demogdata type mortality data.
	Versatile plotting tool with an optional legend.

## Description

Plot an object of class demogdata

## Usage

plot\_dd(dd.obj, year = dd.obj\$year, col = rainbow(length(year), start = 0.1), lpos = "UL", lpar = list(), ppar = T, ...)

dd.obj	a mortality type data object of class demogdata (or number of deaths type data object returned by extract.deaths)
year	vector of years to be included in the plotting (all available years by default)
col	color palette to be used in the plot (by default, it uses a sequence of <i>rainbow</i> colors)

#### plot\_per\_pars

lpos	a text identifier (one of "UR", "LR", "UL", "LL", "UC", "LC", "CL", "CR"; whereas the abbreviation is made up by U/L/C=Upper/Lower/Center, L/R/C=Left/Right/Center) or a list containing the coordinates (e.g. x and y) of the upper left corner of the legend/object.
lpar	list of additional arguments to be passed on to legend (other than legend, title, col or text.col)
ppar	logical, if FALSE, ignores in the legend the plotting arguments lty, lwd and pch (i.e. in case they are given in) and hence it creates a legend containing only text
	additional plotting arguments that are passed on to both plot and legend func- tions (see par)

## Value

Plot of mortality rates or number of deaths.

#### Author(s)

Zoltan Butt, Steven Haberman and Han Lin Shang

#### See Also

extract.deaths, plot.demogdata, legend, par, rainbow

## Examples

```
# plot log mortality rates with repositioned legend
plot_dd(dd.cmi.pens, xlim=c(40, 110), lpar=list(x.intersp=-0.2, y.intersp=0.9, cex=0.85))
# plot (untransformed) mortality rates with repositioned legend
plot_dd(dd.cmi.pens, age=60:95, lpar=list(x.intersp=-0.2, y.intersp=0.9, cex=0.85), transf=FALSE)
# plot a small subset of log mortality rates (calendar years: 1985 - 1995)
# and add a line with the overall mean rates
plot_dd(dd.cmi.pens, lpos=list(x=0.85,y=0.55), year=1985:1995,
lpar=list(x.intersp=-0.1, y.intersp=0.95, cex=0.9))
lines(mean(dd.cmi.pens),lwd=2, lty=3, col='red')
# legend(coord('LC'), legend='mean rate', lwd=2, lty=3, col='red', text.col='red')
# plot number of (extracted) deaths:
tmp.d <- extract.deaths(dd.cmi.pens, ages=55:100, y=1995:2003)
plot_dd(tmp.d, transf=FALSE, lty=1:5, ylab='Number of Deaths',
main=paste(tmp.d$lab, ": male (1995-2003)", sep=''))
```

plot\_per\_pars

Miscellaneous plotting functions for lca and lca.rh type regression objects. Plot of the period effects of the (generalised) Lee-Carter model

## print.elca

#### Description

This function plots the age- and time-specific patterns of the period effects (only) obtained from the fitting of a generalised Lee-Carter model.

## Usage

```
plot_per_pars(lca.obj)
```

## Arguments

lca.obj an object of class lca

## Value

A plot with two graphical regions showing the age- and time-specific period parameters (i.e.  $beta_x^{(1)}$  and  $kappa_t$ ).

## Author(s)

Z. Butt and S. Haberman and H. L. Shang

## References

Renshaw, A. E. and Haberman, S. (2006), "A cohort-based extension to the Lee-Carter model for mortality reduction factors", *Insurance: Mathematics and Economics*, **38**, 556-570.

R. D. Lee and L. Carter (1992) "Modeling and forecasting U.S. mortality", Journal of the American Statistical Association, 87(419), 659-671.

## See Also

plot\_coh\_pars, lca.rh

## Examples

mod6 <- lca.rh(dd.cmi.pens, mod='lc', interpolate=TRUE)
plot\_per\_pars(mod6)</pre>

print.elca

Print function for class elca

## Description

Print an object of class elca

## Usage

## S3 method for class 'elca'
print(x, ...)

## print.rh

## Arguments

х	Data object of class elca
	Other arguments

## Author(s)

Zoltan Butt, Steven Haberman and Han Lin Shang

## See Also

print.rh

print.rh

## Print function for class rh

## Description

Print an object of class rhdata

## Usage

## S3 method for class 'rh'
print(x,...)

## Arguments

х	Data object of class rh
	Other arguments

## Author(s)

Zoltan Butt, Steven Haberman and Han Lin Shang

## See Also

print.rhdata

print.rhdata

## Description

Print an object of class rhdata

## Usage

## S3 method for class 'rhdata'
print(x, ...)

## Arguments

Х	Data object of class rhdata
	Other arguments

## Author(s)

Zoltan Butt, Steven Haberman and Han Lin Shang

## See Also

print.rh

residual_plots	Miscellaneous plotting functions for lca and lca.rh type regression
	objects.

## Description

A diagnostic plot with three graphical regions showing the residual values by the given ages, calendar years and (cohort) years of birth.

#### Usage

```
residual_plots(lca.obj, file = paste("res", deparse(substitute(lca.obj)),
"ps", sep = "."), view = T)
```

lca.obj	an object of class 1ca
file	an optional string value indicating the output postscript file name (i.e. with extension .ps). By default, it concatenates "res." and the model object name (with extension ".ps"). If it is set to NULL, the plot will be sent instead to the active graphical window.
view	logical, if TRUE (and file argument is not NULL) then Ghostview will be launched with the created .ps file

#### Value

Diagnostic plot of the residual values of a Lee-Carter model object.

## Author(s)

Zoltan Butt, Steven Haberman and Han Lin Shang

## See Also

fitted\_plots, lca.rh

#### Examples

```
mod6d <- lca.rh(dd.cmi.pens, mod='lc', restype='dev', interpolate=TRUE)
# send fitted plots with legends to 'res.mod6.ps' file:
residual_plots(mod6d)
# send fitted plots to active graphics window
residual_plots(mod6d, file=NULL)</pre>
```

rhdata

Data formatting utility for the extended (Stratified) LC model function

#### Description

It creates rhdata class object suitable for fitting the extended SLC model using elca.rh iterative fitting method. Basically, it transforms a two-dimensional survival data into three-dimensional arrays of population (exposure) and mortality rates dependent on age, calendar time and additional covariate(s).

#### Usage

```
rhdata(dat, covar, xbreaks = 60:96, xlabels = 60:95,
ybreaks = mdy.date(1, 1, 1999:2008), ylabels = 1999:2007,
name = NULL, label = NULL)
```

dat	data.frame containing individual observations of survival data along with val- ues of additional covariate(s). Thus, the data set needs to contain the following named columns of individual survival records: - 'event' = binary value corre- sponding to the survival event (1 - fail/death or 0 - survive); - 'dob' = Julian date corresponding to the date of birth (or origin) of the survival time; - 'dev' = Julian date corresponding to date of event (or end) of the survival time. In ad- dition, there should he at least one surve column corresponding to characterize
	dition, there should be at least one extra column corresponding to observations related to any additional covariate(s) (e.g. socio-economic factors).
covar	(partial) covariate name(s) or position number(s) in the dat data set. The covari- ate(s) must be of class 'factor'.

## rhdata

xbreaks	a sequence of age break points (including the starting and closing values) to be used for sub-grouping the input data set dat in order to calculate age-specific exposures and mortality rates. By default, it is set to $60:96$ that corresponds to integer ages between $60 - 95$ .
xlabels	a sequence of age labels to be used for the sequence defined in xbreaks.
ybreaks	a sequence of year break points (as Julian calendar dates) to be used for sub- grouping the input data set dat in order to calculate year-specific exposures and mortality rates. By default, it is set to mdy.date(1, 1, 1999:2008) that corresponds to whole years between 1st of January of years 1999 - 2008.
ylabels	a sequence of year labels to be used for the sequence defined in ybreaks.
name	name of subset data series (e.g. male, female or total)
label	label (name) of overall data source (e.g. CMI)

#### Details

While the rhdata function can sub-group the input data by more than one additional covariates (possibly useful for other preliminary analysis), the fitting method implemented in elc.rh can only handle a single additional covariate. Also, currently, there are no generic methods to plot or to extract parts of the rhdata class object, but there are a few illustrations provided below how these might be carried out.

#### Value

List object defined as class rhdata made up by the following components:

year	vector of year labels
age	vector of age labels
covariates	vector of levels of the additional covariate
deaths	3-dimensional array of number of deaths (by age-year-covariate)
рор	3-dimensional array of population (exposure) (by age-year-covariate)
mu	3-dimensional array of central mortality rates (by age-year-covariate)
label	label (name) of overall data source
name	name of subset data series

#### Author(s)

Z. Butt and S. Haberman and H. L. Shang

#### References

Renshaw, A. E. and Haberman, S. (2003a), "Lee-Carter mortality forecasting: a parallel generalised linear modelling approach for England and Wales mortality projections", *Journal of the Royal Statistical Society, Series C*, **52**(1), 119-137.

Renshaw, A. E. and Haberman, S. (2003b), "Lee-Carter mortality forecasting with age specific enhancement", *Insurance: Mathematics and Economics*, **33**, 255-272.

Renshaw, A. E. and Haberman, S. (2006), "A cohort-based extension to the Lee-Carter model for mortality reduction factors", *Insurance: Mathematics and Economics*, **38**, 556-570.

Renshaw, A. E. and Haberman, S. (2008), "On simulation-based approaches to risk measurement in mortality with specific reference to Poisson Lee-Carter modelling", *Insurance: Mathematics and Economics*, **42**(2), 797-816.

Renshaw, A. E. and Haberman, S. (2009), "On age-period-cohort parametric mortality rate projections", *Insurance: Mathematics and Economics*, **45**(2), 255-270.

#### See Also

30

elca.rh, dd.rfp, demogdata, mdy.date

#### Examples

```
# See data set 'tab' provided in the ilc package
# names(tab)
# [1] "refno" "dob" "dev"
                             "event" "cov1" "cov2"
# Get multidimensional survival data:
mdat <- rhdata(tab, covar='cov2', xbreaks=60:96, xlabels=60:95,</pre>
  ybreaks=mdy.date(1,1,2000:2006), ylabels=2000:2005, name='M', label='CMI')
# Warning: although rhdata() can sort by more than a single parameter, for ex.
   covar=c('cov1', 'cov2'), the SLC fitting only works at the moment with
    a single extra covariate.
#
# print data summary:
mdat
#Multidimensional Mortality data for: MDat [M]
#Across covariates:
          years: 2000 - 2005
#
#
          ages: 60 - 95
#
          cov2: 0, 1, 2, 3
# Graphical illustrations of mdat data levels (by the additional factor):
# plot of exposures:
matplot(mdat$age, mdat$pop[,,1], type='l', xlab='Age', ylab='Ec', main='Base Level')
matplot(mdat$age, mdat$pop[,,2], type='l', xlab='Age', ylab='Ec', main='Level 1')
# plot of deaths:
matplot(mdat$age, mdat$deaths[,,1], type='l', xlab='Age', ylab='D', main='Base Level')
matplot(mdat$age, mdat$deaths[,,2], type='l', xlab='Age', ylab='D', main='Level 1')
# plot of log mortality rates:
matplot(mdat$age, log(mdat$mu[,,1]), type='l', xlab='Age', ylab='log(mu)', main='Base Level')
matplot(mdat$age, log(mdat$mu[,,2]), type='l', xlab='Age', ylab='log(mu)', main='Level 1')
```

tab

Sample survival data with additional effects (other than age and time).

#### Description

An artificial sample of individual observations of survival times along with two additional effects (fictive covariates).

tab

#### Usage

tab

## Format

Object of class data.frame

#### Details

The data set contains the following named columns of fictive individual survival records: - 'refno' = unique reference numbers; - 'dob' = Julian date corresponding to the date of birth (or origin) of the survival time; - 'dev' = Julian date corresponding to date of event (or end) of the survival time. - 'event' = binary value corresponding to the survival event (1 - fail/death or 0 - survive); - 'cov1' = first additional covariate with 13 levels; - 'cov1' = second additional covariate with 4 levels;

## Source

NA

#### See Also

rhdata

## Examples

```
# print out the first 10 observations:
tab[1:10,]
# sub-group by a single additional covariate and merge ages above 95:
mdat <- rhdata(tab, covar=c('cov2'), xbreaks=c(40:95, 105), xlabels=c(40:94, '95>'),
ybreaks=mdy.date(1,1,2000:2007), ylabels=2000:2006, name='M', label='CMI')
mdat
```

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