# Package 'fExoticOptions'

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Description Provides a collection of functions to evaluate barrier options, Asian options, binary options, currency translated options, lookback options, multiple asset options and multiple exercise options.
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# **R** topics documented:

AsianOptions	2
BarrierOptions	4
BinaryOptions	10
CurrencyTranslatedOptions	15
LookbackOptions	
MultipleAssetsOptions	22
MultipleExercisesOptions	27
	33

Index

AsianOptions

# Description

This is a collection of functions to valuate asian options. Asian options are path-dependent options, with payoffs that depend on the average price of the underlying asset or the average exercise price. There are two categories or types of Asian options: average rate options (also known as average price options) and average strike options. The payoffs depend on the average price of the underlying asset over a predetermined time period. An average is less volatile than the underlying asset, therefore making Asian options less expensive than standard European options. Asian options are commonly used in currency and commodity markets. Asian options are of interest in markets with thinly traded assets. Due to the little effect it will have on the option's value, options based on an average, such as Asian options, have a reduced incentive to manipulate the underlying price at expiration.

The functions are:

GeometricAverageRateOption	Geometric Average Rate Option,
TurnbullWakemanAsianApproxOption	Turnbull and Wakeman's Approximation,
LevyAsianApproxOption	Levy's Approximation.

## Usage

```
GeometricAverageRateOption(TypeFlag, S, X, Time, r, b, sigma,
        title = NULL, description = NULL)
TurnbullWakemanAsianApproxOption(TypeFlag, S, SA, X, Time, time,
        tau, r, b, sigma, title = NULL, description = NULL)
LevyAsianApproxOption(TypeFlag, S, SA, X, Time, time, r, b,
        sigma, title = NULL, description = NULL)
```

b	the annualized cost-of-carry rate, a numeric value; e.g. 0.1 means 10% pa.
description	a character string which allows for a brief description.
r	a numeric value, the annualized rate of interest; e.g. 0.25 means 25% pa.
S, SA	the asset price, a numeric value.
sigma	a numeric value, the annualized volatility of the underlying security; e.g. $0.3$ means 30% volatility pa.
tau	[TurnWakeAsianApprox*] - is the time to the beginning of the average period.
time, Time	a numeric value, the time to maturity measured in years; e.g. 0.5 means 6 months.
title	a character string which allows for a project title.

### AsianOptions

TypeFlag	a character string either "c" for a call option or a "p" for a put option.
Х	the exercise price, a numeric value.

#### Details

The Geometric average is the nth root of the product of the n sample points. The Arithmetic average is the sum of the stock values divided by the number of sampling points. Although Geometric Asian options are not commonly used in practice, they are often used as a good initial guess for the price of arithmetic Asian options. This technique is used to improve the convergence rate of the Monte Carlo model when pricing arithmetic Asian options.

Two cases are considered, the geometric and the arithmetic average-rate option. For the latter one can choose between three different kinds of approximations: Turnbull and Wakeman's approximations, Levy's approximation and Curran's approximation. [Haug's Book, Chapter 2.12]

#### Value

The option price, a numeric value.

#### Note

The functions implement the algorithms to valuate plain vanilla options as described in Chapter 2.12 of Haug's Book (1997).

# Author(s)

Diethelm Wuertz for the Rmetrics R-port.

# References

Haug E.G. (1997); *The complete Guide to Option Pricing Formulas*, Chapter 2.12, McGraw-Hill, New York.

### Examples

## Examples from Chapter 2.12 in E.G. Haug's Option Guide (1997)

```
## Geometric Average Rate Option:
GeometricAverageRateOption(TypeFlag = "p", S = 80, X = 85,
Time = 0.25, r = 0.05, b = 0.08, sigma = 0.20)
```

```
## Turnbull Wakeman Approximation:
```

TurnbullWakemanAsianApproxOption(TypeFlag = "p", S = 90, SA = 88, X = 95, Time = 0.50, time = 0.25, tau = 0.0, r = 0.07, b = 0.02, sigma = 0.25)

```
## Levy Asian Approximation:
    LevyAsianApproxOption(TypeFlag = "c", S = 100, SA = 100, X = 105,
    Time = 0.75, time = 0.50, r = 0.10, b = 0.05, sigma = 0.15)
```

BarrierOptions

#### Description

A collection and description of functions to valuate barrier options. Barrier options are pathdependent options, with payoffs that depend on the price of the underlying asset at expiration and whether or not the asset price crosses a barrier during the life of the option. There are two categories or types of Barrier options: "knock-in" and "knock-out". "Knock-in" or "in" options are paid for up front, but you do not receive the option until the asset price crosses the barrier. "Knock-out" or "out" options come into existence on the issue date but becomes worthless if the asset price hits the barrier before the expiration date. If the option is a knock-in (knock-out), a predetermined cash rebate may be paid at expiration if the option has not been knocked in (knocked-out) during its lifetime. The barrier monitoring frequency specifies how often the price is checked for a breach of the barrier. All of the analytical models have a flag to change the monitoring frequency where the default frequency is continuous.

The functions are:

StandardBarrierOption	Standard Barrier Option,
DoubleBarrierOption	Double Barrier Option,
PTSingleAssetBarrierOption	Partial Time Single Asset Barrier Option,
TwoAssetBarrierOption	Two Asset Barrier Option,
PTTwoAssetBarrierOption	Partial Time Two Asset Barrier Option,
LookBarrierOption	Lookback Barrier Option,
DiscreteBarrierOption	Discrete Barrier Option
SoftBarrierOption	Soft Barrier Option.

#### Usage

```
StandardBarrierOption(TypeFlag, S, X, H, K, Time, r, b, sigma,
    title = NULL, description = NULL)
DoubleBarrierOption(TypeFlag, S, X, L, U, Time, r, b, sigma,
    delta1, delta2, title = NULL, description = NULL)
PTSingleAssetBarrierOption(TypeFlag, S, X, H, time1, Time2, r,
    b, sigma, title = NULL, description = NULL)
TwoAssetBarrierOption(TypeFlag, S1, S2, X, H, Time, r, b1, b2,
    sigma1, sigma2, rho, title = NULL, description = NULL)
PTTwoAssetBarrierOption(TypeFlag, S1, S2, X, H, time1, Time2, r,
    b1, b2, sigma1, sigma2, rho, title = NULL, description = NULL)
LookBarrierOption(TypeFlag, S, X, H, time1, Time2, r, b, sigma,
    title = NULL, description = NULL)
DiscreteBarrierOption(S, H, sigma, dt, title = NULL,
    description = NULL)
SoftBarrierOption(TypeFlag, S, X, L, U, Time, r, b, sigma,
    title = NULL, description = NULL)
```

# **BarrierOptions**

b	the annualized cost-of-carry rate, a numeric value; e.g. 0.1 means 10% pa.
b1, b2	[PTTwoAssetBarrier*] - the annualized cost-of-carry rate for the first and second asset, a numeric value.
delta1, delta2	[DoubleBarrier*] - numeric values which determine the curvature of the lower L and upper U bounds. The case of delta1=delta2=0 corresponds to two flat boundaries, delta1<0 <delta2 a="" as<br="" boundary="" corresponds="" exponentially="" growing="" lower="" to="">time elapses, while the upper boundary will be exponentially decaying, delta1&gt;0&gt;delta2 correponds to a convex downward lower boundary and a con- vex upward upper boundary.</delta2>
description	a character string which allows for a brief description.
dt	[DiscreteBarrier*] - time between monitoring instants, a numeric value.
Н	[StandardBarrier*] - the barrier value, a numeric value.
К	[StandardBarrier*] - for an "In"-Barrier a prespecified cash rebate which is paid out at option expira- tion if the option has not been knocked in during its lifetime, for an "Out"-Barrier a prespecified cash rebate which is paid out at option ex- piration if the option has not been knocked out before its lifetime, a numerical value.
L, U	[DoubleBarrier*] - the lower and upper boundary to be touched, numerical values.
r	the annualized rate of interest, a numeric value; e.g. 0.25 means 25% pa.
rho	[TwoAssetBarrier*] - the correlation of the volatility between the first and second asset, a numeric value.
S	the asset price, a numeric value.
S1, S2	[PTTwoAssetBarrier*] - the price of the first and second asset, a numeric value.
sigma	the annualized volatility of the underlying security, a numeric value; e.g. 0.3 means 30% volatility pa.
sigma1, sigma2	[PTTwoAssetBarrier*] - the annualized volatility of the first and second underlying security, numeric values.
Time	the time to maturity measured in years, a numeric value; e.g. 0.5 means 6 months.
time1, Time2	[PTSingleAssetBarrier*][PTTwoAssetBarrier*] - so called type "A" options (see the TypeFlag argument) will have the location of the monitoring period starting at the options starting date and ending at an arbitrary time time1 before expiration time Time2. Partial-time-end-barrier op- tions will have the location of the monitoring period starting at an arbitrary time

	time1 before expiration time Time2, and ending at expiration time. [LookBarrier*] - the lookbarrier option's barrier monitoring period starts at the options starting
1	date and ends at an arbitrary time time1 before expiration time Time2.
title	a character string which allows for a project title.
TypeFlag	usually a character string either "c" for a call option or a "p" for a put option.
	[StandardBarrier*] - here "cdi" denotes a down-and-in call, "cui" denotes an up-and-in call, "cdo" denotes a down-and-out call, and "cuo" denotes an up-and-out call. Similarily, the type flags for the corresponding puts are "pdi", "pui", "pdo", and "puo".
	[DoubleBarrier*] -
	here "co" denotes an up-and-out-down-and-out call, "ci" denotes an up-and-in-down-and-in call, "po" denotes an up-and-out-down-and-out put, and "pi" denotes an up-and-in-down-and-in call.
	[PTSingleAssetBarrier*] - here "cdoA" denotes a down-and-out call of type "A",
	"cuoA" denotes a uown-and-out call of type "A",
	"pdoA" denotes a down-and-out put of type "A",
	"puoA" denotes an up-and-out put of type "A",
	"coB1" denotes an out-call of type "B1",
	"poB1" denotes an out-call of type "B1", "cdoB2" denotes a down-and-out call of type "B2",
	"cuoB2" denotes an up-and-out call of type "B2".
	Note, a partial-time-start-barrier option is called a type "A" option, a partial- time-end-out-call is a called a type "B" option. There are two types of "B" options: "B1" is defined such that only a barrier hit or crossed causes the op- tion to be knocked out, and a "B2" is defined such that a down-and-out-call is knocked out as soon as the underlying price is below the barrier.
	[TwoAssetBarrier*][PTTwoAssetBarrier*] -
	here "cuo" denotes an up-and-out call,
	"cui" denotes an up-and-out can,
	"cdo" denotes a down-and-out call,
	"cdi" denotes a down-and-in call,
	"puo" denotes an up-and-out put,
	"pui" denotes an up-and-in put,
	"pdo" denotes a down-and-out put,

	"pdi" denotes a down-and-in put.
	[LookBarrier*][SoftBarrier*] -
	here "cuo" denotes an up-and-out call,
	"cui" denotes an up-and-in call,
	"pdo" denotes a down-and-out put,
	"pdi" denotes a down-and-in put.
Х	the exercise price, a numeric value.

#### Details

#### Single [Standard] Barrier Options:

There are four types of single barrier options. The type flag "cdi" denotes a down-and-in call, "cui" denotes an up-and-in call, "cdo" denotes a down-and-out call, and "cuo" denotes an upand-out call. Similarily, the type flags for the corresponding puts are cdi, cui, cdo, and cuo. A down-and-in option comes into existence and knocked-in only if the asset price falls to the barrier level. An up-and- in option comes into existence and knocked-in only if the asset price rises to the barrier level. A down-and-out option comes into existence and knocked-out only if the asset price falls to the barrier level. An up-and-in option comes into existence and knocked-out only if the asset price rises to the barrier level. European single barrier options can be priced analytically using a model introduced by Reiner and Rubinstein (1991). A trinomial lattice is used for the numerical calculation of an American or European style single barrier options. [Haug's Book, Chapter 2.10.1]

#### **Double Barrier Options:**

A double barrier option is either knocked in or knocked out if the asset price touches the lower or upper barrier during its lifetime. The type flag "co" denotes an up-and-out-down-and-out call, "ci" denotes an up-and-in-down-and-in call, "po" denotes an up-and-out-down-and-out put, and "pi" denotes an up-and-in-down-and-in call. Once a barrier is crossed, the option comes into existence if it is a knock-in barrier or becomes worthless if it is a knocked out barrier. Double barrier options can be priced analytically using a model introduced by Ikeda and Kunitomo (1992). [Haug's Book, Chapter 2.10.2]

#### **Partial-Time Barrier Options:**

For single asset partial-time barrier options, the monitoring period for a barrier crossing is confined to only a fraction of the option's lifetime. There are two types of partial-time barrier options: partial-time-start and partial-time-end. Partial-time-start barrier options have the monitoring period start at time zero and end at an arbitrary date before expiration. Partial-time-end barrier options have the monitoring period start at an arbitrary date before expiration and end at expiration. Partialtime-end barrier options are then broken down again into two categories: B1 and B2. Type B1 is defined such that only a barrier hit or crossed causes the option to be knocked out. There is no difference between up and down options. Type B2 options are defined such that a down-and-out call is knocked out as soon as the underlying price is below the barrier. Similarly, an up-and-out call is knocked out as soon as the underlying price is above the barrier. Partial-time barrier options can be priced analytically using a model introduced by Heynen and Kat (1994). [Haug's Book, Chapter 2.10.3]

# **Two-Asset Barrier Options:**

The underlying asset, Asset 1, determines how much the option is in or out-of-the-money. The other asset, Asset 2, is the trigger asset that is linked to barrier hits. Two-asset barrier options can be priced analytically using a model introduced by Heynen and Kat (1994). [Haug's Book, Chapter 2.10.4]

#### **Lookback Barrier Options:**

A look-barrier option is the combination of a forward starting fixed strike Lookback option and a partial time barrier option. The option's barrier monitoring period starts at time zero and ends at an arbitrary date before expiration. If the barrier is not triggered during this period, the fixed strike Lookback option will be kick off at the end of the barrier tenor. Lookback barrier options can be priced analytically using a model introduced by Bermin (1996). [Haug's Book, Chapter 2.10.6]

## **Partial-Time-Two-Asset Options:**

Partial-time two-asset barrier options are similar to standard two-asset barrier options, except that the barrier hits are monitored only for a fraction of the option's lifetime. The option is knocked in or knocked out is Asset 2 hits the barrier during the monitoring period. The payoff depends on Asset 1 and the strike price. Partial-time two-asset barrier options can be priced analytically using a model introduced by Bermin (1996).

[Haug's Book, Chapter 2.10.5]

# Soft Barrier Options:

A soft-barrier option is similar to a standard barrier option, except that the barrier is no longer a single level. Rather, it is a soft range between a lower level and an upper level. Soft-barrier options are knocked in or knocked out proportionally. Introduced by Hart and Ross (1994), the valuation formula can be used to price soft-down-and-in call and soft-up-and-in put options. The value of the related "out" option can be determined by subtracting the "in" option value from the value of a standard plain option. Soft-barrier options can be priced analytically using a model introduced by Hart and Ross (1994).

[Haug's Book, Chapter 2.10.8]

# Value

The option price, a numeric value.

# Note

The functions implement the algorithms to valuate plain vanilla options as described in Chapter 2.10 of Haug's Book (1997).

#### **BarrierOptions**

#### Author(s)

Diethelm Wuertz for the Rmetrics R-port.

#### References

Haug E.G. (1997); *The complete Guide to Option Pricing Formulas*, Chapter 2.10, McGraw-Hill, New York.

## Examples

```
## Examples from Chapter 2.10 in E.G. Haug's Option Guide (1997)
## Standard Barrier Option [2.10.1]:
   # down-and-out Barrier Call
  StandardBarrierOption(TypeFlag = "cdo", S = 100, X = 90,
    H = 95, K = 3, Time = 0.5, r = 0.08, b = 0.04, sigma = 0.25)
## Double Barrier Option [2.10.2]:
  DoubleBarrierOption(TypeFlag = "co", S = 100, X = 100, L = 50,
    U = 150, Time = 0.25, r = 0.10, b = 0.10, sigma = 0.15,
     delta1 = -0.1, delta2 = 0.1)
## Partial Time Single-Asset Barrier Option [2.10.3]:
  PTSingleAssetBarrierOption(TypeFlag = "coB1", S = 95, X = 110,
     H = 100, time1 = 0.5, Time2 = 1, r = 0.20, b = 0.20,
     sigma = 0.25)
## Two Asset Barrier Option [2.10.4]:
   TwoAssetBarrierOption(TypeFlag = "puo", S1 = 100, S2 = 100,
    X = 110, H = 105, Time = 0.5, r = 0.08, b1 = 0.08, b2 = 0.08,
     sigma1 = 0.2, sigma2 = 0.2, rho = -0.5)
## PT Two Asset Barrier Option [2.10.5]:
  PTTwoAssetBarrierOption(TypeFlag = "pdo", S1 = 100, S2 = 100,
    X = 100, H = 85, time1 = 0.5, Time2 = 1, r = 0.1, b1 = 0.1,
     b2 = 0.1, sigma1 = 0.25, sigma2 = 0.30, rho = -0.5)
## Look Barrier Option [2.10.6]:
  LookBarrierOption(TypeFlag = "cuo", S = 100, X = 100, H = 130,
     time1 = 0.25, Time2 = 1, r = 0.1, b = 0.1, sigma = 0.15)
  LookBarrierOption(TypeFlag = "cuo", S = 100, X = 100, H = 110,
     time1 = 1, Time2 = 1, r = 0.1, b = 0.1, sigma = 0.30)
## Discrete Barrier Option [2.10.7]:
  DiscreteBarrierOption(S = 100, H = 105, sigma = 0.25, dt = 0.1)
## Soft Barrier Option [2.10.8]:
  SoftBarrierOption(TypeFlag = "cdo", S = 100, X = 100, L = 70,
    U = 95, Time = 0.5, r = 0.1, b = 0.05, sigma = 0.20)
```

**BinaryOptions** 

# Description

A collection and description of functions to valuate binary options. Binary options, also known as digital options, have discontinuous payoffs. They can be used as building blocks to develop options with more complicated payoffs. For example, a regular European call option is equivalent to a long position in an asset-or-nothing call and a short position in a cash-or-nothing call, where the both options have the same strike price and the cash payoff of the cash-or-nothing option equals the strike price. Unlike standard European style options, the payout for binary options does not depend on how much it is in-the-money but rather whether or not it is on the money. The option's payoff is fixed at the options inception and is based on the price of the underlying asset on the expiration date. Binary options may also incorporate barriers, as is the case with binary-barrier options.

The functions are:

GapOption	Gap Option,
CashOrNothingOption	Cash Or Nothing Option,
TwoAssetCashOrNothingOption	Two Asset Cash Or Nothing Option,
AssetOrNothingOption	Asset Or Nothing Option,
SuperShareOption	Super Share Option,
BinaryBarrierOption	Binary Barrier Option.

# Usage

```
GapOption(TypeFlag, S, X1, X2, Time, r, b, sigma, title = NULL,
    description = NULL)
CashOrNothingOption(TypeFlag, S, X, K, Time, r, b, sigma,
    title = NULL, description = NULL)
TwoAssetCashOrNothingOption(TypeFlag, S1, S2, X1, X2, K, Time, r,
    b1, b2, sigma1, sigma2, rho, title = NULL, description = NULL)
AssetOrNothingOption(TypeFlag, S, X, Time, r, b, sigma,
    title = NULL, description = NULL)
SuperShareOption(S, XL, XH, Time, r, b, sigma, title = NULL,
    description = NULL)
BinaryBarrierOption(TypeFlag, S, X, H, K, Time, r, b, sigma,
    eta, phi, title = NULL, description = NULL)
```

b	the annualized cost-of-carry rate, a numeric value; e.g. 0.1 means 10% pa.
b1, b2	[TwoAssetCashOrNothing*] - the annualized cost-of-carry rate for the first and second asset, a numeric value.
description	a character string which allows for a brief description.

eta, phi

# [BinaryBarrier\*] -

eta, phi	[BinaryBarrier*] -
	a set of parameters to price 28 different types of Binary Barrier options:
	01: eta=+1, phi=NA, [S>H] down-and-in cash-at-hit-or-nothing,
	02: eta=-1, phi=NA, [S <h] cash-at-hit-or-nothing,<="" th="" up-and-in=""></h]>
	03: eta=+1, phi=NA, [S>H] down-and-in asset-at-hit-or-nothing,
	04: eta=-1, phi=NA, [S <h] asset-at-hit-or-nothing,<="" th="" up-and-in=""></h]>
	05: eta=+1, phi=-1, [S>H] down-and-in cash-at-expiry-or-nothing,
	06: eta=-1, phi=+1, [S <h] cash-at-expiry-or-nothing,<="" th="" up-and-in=""></h]>
	07: eta=+1, phi=-1, [S>H] down-and-in asset-at-expiry-or-nothing,
	08: eta=-1, phi=+1, [S <h] asset-at-expiry-or-nothing,<="" th="" up-and-in=""></h]>
	09: eta=+1, phi=+1, [S>H] down-and-out cash-or-nothing,
	10: eta=-1, phi=-1, [S <h] cash-or-nothing,<="" th="" up-and-out=""></h]>
	11: eta=+1, phi=+1, [S>H] down-and-out asset-or-nothing,
	12: eta=-1, phi=-1, [S <h] asset-or-nothing,<="" th="" up-and-out=""></h]>
	13: eta=+1, phi=+1, [S>H] down-and-in cash-or-nothing call,
	14: eta=-1, phi=+1, [S <h] call,<="" cash-or-nothing="" th="" up-and-in=""></h]>
	15: eta=+1, phi=+1, [S>H] down-and-in asset-or-nothing call,
	16: eta=-1, phi=+1, [S <h] asset-or-nothing="" call,<="" th="" up-and-in=""></h]>
	17: eta=+1, phi=-1, [S>H] down-and-in cash-or-nothing put,
	18: eta=-1, phi=-1, [S <h] cash-or-nothing="" put,<="" th="" up-and-in=""></h]>
	19: eta=+1, phi=-1, [S>H] down-and-in asset-or-nothing put,
	20: eta=-1, phi=-1, [S <h] asset-or-nothing="" put,<="" th="" up-and-in=""></h]>
	21: eta=+1, phi=+1, [S>H] down-and-out cash-or-nothing call,
	22: eta=-1, phi=+1, [S <h] call,<="" cash-or-nothing="" th="" up-and-out=""></h]>
	23: eta=+1, phi=+1, [S>H] down-and-out asset-or-nothing call,
	24: eta=-1, phi=-1, [S <h] asset-or-nothing="" call,<="" th="" up-and-out=""></h]>
	25: eta=+1, phi=-1, [S>H] down-and-out cash-or-nothing put,
	26: eta=-1, phi=-1, [S <h] cash-or-nothing="" put,<="" th="" up-and-out=""></h]>
	27: eta=+1, phi=-1, [S>H] down-and-out asset-or-nothing put,
	28: eta=-1, phi=-1, [S <h] asset-or-nothing="" put.<="" th="" up-and-out=""></h]>
Н	[BinaryBarrier*] -
	the barrier value, a numeric value.
К	[CashOrNothing*] -
	the cash amount at expiry if the option is in the money, a numerical value.
	[TwoAssetCashOrNothing*] -
	for the cash-or-nothing call the cash amount at expiry if asset S1 is above the
	strike X1 and asset S2 is above strike X2 at expiration,
	for the cash-or-nothing put the cash amount at expiry if asset S1 is below the
	strike X1 and asset S2 is below strike X2 at expiration,
	for the cash-or-nothing up-down the cash amount at expiry if asset S1 is above
	the strike X1 and asset S2 is below strike X2 at expiration,
	for the cash-or-nothing down-up the cash amount at expiry if asset S1 is below
	the strike X1 and asset S2 is above strike X2 at expiration.
	[BinaryBarrier*] -
	the prespecified cash amount, a numeric value.
r	the annualized rate of interest, a numeric value; e.g. 0.25 means 25% pa.
rho	[TwoAssetCashOrNothing*] -
	the correlation of the volatility between the first and second asset, a numeric
	· · · · · · · · · · · · · · · · · · ·

	value.
S	the asset price, a numeric value.
S1, S2	[TwoAssetCashOrNothing*] - the price of the first and second asset, a numeric value.
sigma	the annualized volatility of the underlying security, a numeric value; e.g. 0.3 means 30% volatility pa.
sigma1, sigma2	[TwoAssetCashOrNothing*] - the annualized volatility of the first and second underlying security, numeric values.
Time	the time to maturity measured in years, a numeric value; e.g. 0.5 means 6 months.
title	a character string which allows for a project title.
TypeFlag	a character string either "c" for a call option or a "p" for a put option. [TwoAssetCashOrNothing*] - a character string either "c" for a call option, or a "p" for a put option, or a "ud" for an up-down option, or a "du" for a down-up option. [BinaryBarrier*] - an integer between 1 and 28, selecting one of the 28 types, for a definition lookup the arguments eta and phi.
Х	the exercise price, a numeric value.
X1, X2	[GapOption][TwoAssetCashOrNothing*] - the first and the second exercise price, a numeric value.
XL, XH	[SuperShare*] - the lower and upper boundary strike, a numeric value.

#### Details

# **Gap Options:**

The payoff on a gap option depends on the usual factors of a plain option, but is also affected by a "gap" amount of exercise prices, which may be positive or negative. Note, that a gap call (put) option is equivalent to being long (short) an asset-or-nothing call (put) and short (long) a cash-or-nothing call (put). The option price is calculated analytically according to Reiner and Rubinstein (1991).

[Haug's Book, Chapter 2.11.1]

# **Cash-or-Nothing Options:**

For this option a predetermined amount is paid at expiration if the asset is above for a call or below for a put some strike level. The amount independent of the path taken. These options require no payment of an exercise price. The exercise price determines whether or not the option returns a payoff. The value of a cash-or-nothing call (put) option is the present value of the fixed cash payoff multiplied by the probability that the terminal price will be greater than (less than) the exercise price. The option price is calculated analytically according to Reiner and Rubinstein (1991). [Haug's Book, Chapter 2.11.2]

#### **BinaryOptions**

#### **Two-Asset-Cash-Or-Nothing Options:**

These options are building blocks for constructing more complex exotic options. There are four types of two-asset cash-or-nothing options, the first two situations are: A two-asset-cash-or-nothing call pays out a fixed cash amount if the price of the first asset is above (below) the strike price of the first asset and the price of the second asset is also above (below) the strike price of the second asset at expiration. The other two situations arise under the following conditions: A two-asset cash-or-nothing down-up pays out a fixed cash amount if the price of the first asset is is below (above) the strike price of the first asset and the price of the second asset is above (below) the strike price of the second asset as amount if the price of the first asset is below (above) the strike price of the first asset and the price of the second asset is above (below) the strike price of the second asset at expiration. The option price is calculated analytically according to Heynen and Kat (1996).

[Haug's Book, Chapter 2.11.3]

#### **Asset-Or-Nothing Options:**

In this option a predetermined asset value is paid if the asset is, at expiration, above for a call or below for a put some strike level, independent of the path taken. For a call (put) the terminal price is greater than (less than) the exercise price, the call (put) expires worthless. The exercise price is never paid. Instead, the value of the asset relative to the exercise price determines whether or not the option returns a payoff. The value of an asset-or-nothing call (put) option is the present value of the asset multiplied by the probability that the terminal price will be greater than (less than) the exercise price. The option price is calculated analytically according to Cox and Rubinstein (1985). [Haug's Book, Chapter 2.11.4]

#### **Supershare Options:**

These options represents a contingent claim on a fraction of the underlying portfolio. The contingency is that the value of the portfolio must lie between a lower and an upper bound at expiration. If the value lies within these boundaries, the supershare is worth a proportion of the assets underlying the portfolio, else the supershare expires worthless. A supershare has a payoff that is basically like a spread of two asset-or-nothing calls, in which the owner of a supershare purchases an asset-ornothing call with an strike price of the lower strike and sells an asset-or-nothing call with an strike price of the upper strike. The option price is calculated analytically according to Hakansson (1976). [Haug's Book, Chapter 2.11.5]

#### **Binary Barrier Options:**

These options combine characteristics of both binary and barrier options. They are path dependent with a discontinuous payoff. Similar to barrier options, the payoff depends on whether or not the asset price crosses a predetermined barrier. There are 28 different types of binary barrier options, which can be divided into two main categories: Cash-or-nothing and Asset-or-nothing barrier options. Cash-or-nothing barrier options pay out a predetermined cash amount or nothing, depending on whether the asset price has hit the barrier. Asset-or-nothing barrier options pay out the value of the asset or nothing, depending on whether the asset price has crossed the barrier. The barrier monitoring frequency can be adjusted to account for discrete monitoring using an approximation developed by Broadie, Glasserman, and Kou (1995). Binary-barrier options can be priced analytically using a model introduced by Reiner and Rubinstein (1991). [Haug's Book, Chapter 2.11.6]

# Value

The option price, a numeric value.

#### Note

The functions implement the algorithms to valuate plain vanilla options as described in Chapter 2.11 of Haug's Book (1997).

# Author(s)

Diethelm Wuertz for the Rmetrics R-port.

#### References

Cox J.C., Rubinstein M. (1985); Innovations in Option Markets, Prentice-Hall, New Jersey.

Hakkansson N.H. (1976); *The Purchasing Power Fund: A New Kind of Financial Intermediary*, Financial Analysts Journal 32, 49–59.

Haug E.G. (1997); *The complete Guide to Option Pricing Formulas*, Chapter 2.11, McGraw-Hill, New York.

Heinen R.C., Kat H.M. (1996); Brick by Brick, Risk Magazine 9, 6.

Reiner E., Rubinstein M. (1991); Unscrambling the Binary Code; Risk Magazine 4, 9.

## Examples

```
## Examples from Chapter 2.11 in E.G. Haug's Option Guide (1997)
```

```
## Gap Option [2.11.1]:
GapOption(TypeFlag = "c", S = 50, X1 = 50, X2 = 57, Time = 0.5,
r = 0.09, b = 0.09, sigma = 0.20)
```

```
## Cash Or Nothing Option [2.11.2]:
CashOrNothingOption(TypeFlag = "p", S = 100, X = 80, K = 10,
Time = 9/12, r = 0.06, b = 0, sigma = 0.35)
```

```
## Two Asset Cash Or Nothing Option [2.11.3]:
  # Type 1 - call:
  TwoAssetCashOrNothingOption(TypeFlag = "c", S1 = 100, S2 = 100,
    X1 = 110, X2 = 90, K = 10, Time = 0.5, r = 0.10, b1 = 0.05,
    b2 = 0.06, sigma1 = 0.20, sigma2 = 0.25, rho = 0.5)
  # Type 2 - put:
  TwoAssetCashOrNothingOption(TypeFlag = "p", S1 = 100, S2 = 100,
    X1 = 110, X2 = 90, K = 10, Time = 0.5, r = 0.10, b1 = 0.05,
    b2 = 0.06, sigmal = 0.20, sigma2 = 0.25, rho = -0.5)
  # Type 3 - down-up:
  TwoAssetCashOrNothingOption(TypeFlag = "ud", S1 = 100, S2 = 100,
    X1 = 110, X2 = 90, K = 10, Time = 1, r = 0.10, b1 = 0.05,
    b2 = 0.06, sigma1 = 0.20, sigma2 = 0.25, rho = 0)
  # Type 4 - up-down:
  TwoAssetCashOrNothingOption(TypeFlag = "du", S1 = 100, S2 = 100,
    X1 = 110, X2 = 90, K = 10, Time = 1, r = 0.10, b1 = 0.05,
```

```
b2 = 0.06, sigma1 = 0.20, sigma2 = 0.25, rho = 0)
## Asset Or Nothing Option [2.11.4]:
AssetOrNothingOption(TypeFlag = "p", S = 70, X = 65, Time = 0.5,
r = 0.07, b = 0.07 - 0.05, sigma = 0.27)
## Super Share Option [2.11.5]:
SuperShareOption(S = 100, XL = 90, XH = 110, Time = 0.25, r = 0.10,
b = 0, sigma = 0.20)
## Binary Barrier Option [2.11.6]:
BinaryBarrierOption(TypeFlag = "6", S = 95, X=102, H = 100,
K = 15, Time = 0.5, r = 0.1, b = 0.1, sigma = 0.20)
BinaryBarrierOption(TypeFlag = "12", S = 95, X = 98, H = 100,
K = 15, Time = 0.5, r = 0.1, b = 0.1, sigma = 0.20)
```

CurrencyTranslatedOptions

#### Valuation of Currency Translated Options

#### Description

This is a collection of functions to valuate currency translated options. Currency translated options are options on foreign assets where the payoff is exchanged into domestic currency at expiration. For example, a US investor is interested in buying an option that is linked to the Nikkei index that is priced in yen. There are two types or risks, changing prices and exchange rates, to consider when valuing currency-translated options.

The functions are:

FEInDomesticFXOption	Foreign Exchange In Domestic Currency Option,
QuantoOption	Quanto Option,
EquityLinkedFXOption	Equity Linked FX Option,
TakeoverFXOption	Takeover FX Option.

# Usage

```
FEInDomesticFXOption(TypeFlag, S, E, X, Time, r, q, sigmaS,
    sigmaE, rho, title = NULL, description = NULL)
QuantoOption(TypeFlag, S, Ep, X, Time, r, rf, q, sigmaS,
    sigmaE, rho, title = NULL, description = NULL)
EquityLinkedFXOption(TypeFlag, E, S, X, Time, r, rf, q,
    sigmaS, sigmaE,rho, title = NULL, description = NULL)
TakeoverFXOption(V, B, E, X, Time, r, rf, sigmaV, sigmaE,
    rho, title = NULL, description = NULL)
```

В	[TakeoverFX*] - the value of the foreign firm in the foreign currency at the option expiration, a numeric value.
description	a character string which allows for a brief description.
E	[FEInDomesticFX*] - the spot exchange rate specified in units of the domestic currency per unit of the foreign currency, a numeric value. [TakeoverFX*] - the currency price quoted in units of the domestic currency per unit of the foreign currency.
Ep	[Quanto*] - the predetermined exchange rate specified in units of domestic currency per unit of foreign currency.
q	[FEInDomesticFX*][EquityLinkedFX*] - the instantaneous proportional dividend payout rate of the underlying asset, a numerical value.
r	[FEInDomesticFX*][TakeoverFX*] - the domestic interest rate, a numeric value. E.g. 0.25 means 25% p.a.
rf	[TakeoverFX*] - the foreign interest rate, a numeric value.
rho	[TakeoverFX*] - the correlation between annualized volatility of the currency price quoted in units of the domestic currency per unit of the foreign currency and the annual- ized volatility of the value of the foreign firm, a numeric value.
S	[FEInDomesticFX*][EquityLinkedFX*] - the underlying asset price in foreign currency, a numeric value.
sigmaE	[TakeoverFX*] - the annualized volatility of the currency price quoted in units of the domestic currency per unit of the foreign currency, a numeric value; e.g. 0.3 means 30% volatility pa.
sigmaS	[Quanto*] - the annualized volatility of the underlying asset, a numeric value; e.g. 0.3 means 30% volatility pa.
sigmaV	[TakeoverFX*] - the annualized volatility of the value of the foreign firm, a numeric value; e.g. 0.3 means 30% volatility pa.
Time	the time to maturity, a numeric value.
title	a character string which allows for a project title.
TypeFlag	a character string either "c" for a call option or a "p" for a put option.
٧	[TakeoverFX*] - the value of the foreign firm in the foreign currency, a numeric value.

Х

[FEInDomesticFX\*] the strike (delivery) price in domestic currency, a numeric value. [TakeoverFX\*] the strike price quoted in units of the domestic currency per unit of the foreign currency.

#### Details

## **Equity Linked Foreign Exchange Options:**

An equity-linked foreign-exchange option is an option on the foreign exchange rate and is linked to the forward price of a stock or equity index. This option can be priced analytically using a model introduced by Reiner (1992).

#### **Quanto Options:**

A fixed exchange-rate foreign-equity option (Quanto) is denominated in another currency than that of the underlying equity exposure. The face value of the currency protection expands or contracts to cover changes in the foreign currency value of the underlying asset. Quanto options can be priced analytically using a model published by Dravid, Richardson, and Sun (1993).

#### **Foreign Equity Options:**

A foreign equity option is an option on a foreign asset where the strike price is specified in either domestic or foreign currency and the payoff at expiration is valued in domestic currency. Foreign equity options can be priced analytically using a model introduced by Reiner (1992).

#### **Takeover Foreign Exchange Options:**

A takeover foreign exchange call option gives the buyer the right purchase a specified number of units of foreign currency at a strike price if the corporate takeover is successful. This option can be priced analytically using a model introduced by Schnabel and Wei (1994). [Haug's Book, Chapter 2.13.4]

# Value

The option price, a numeric value.

# Note

The functions implement the algorithms to valuate plain vanilla options as described in Chapter 1 of Haug's Book (1997).

# Author(s)

Diethelm Wuertz for the Rmetrics R-port.

### References

Haug E.G. (1997); *The Complete Guide to Option Pricing Formulas*, Chapter 2.13, McGraw-Hill, New York.

# Examples

```
## Examples from Chapter 2.13 in E.G. Haug's Option Guide (1997)
## Foreign Equity Options Struck in Domestic Currency [2.13.1]:
  FEInDomesticFXOption(TypeFlag = "c", S = 100, E = 1.5,
    X = 160, Time = 0.5, r = 0.08, q = 0.05, sigmaS = 0.20,
    sigmaE = 0.12, rho = 0.45)
## Fixed Exchange-Rate Foreign-Equity Option [2.13.2]:
  QuantoOption(TypeFlag = "c", S = 100, Ep = 1.5, X = 105,
    Time = 0.5, r = 0.08, rf = 0.05, q = 0.04, sigmaS= 0.2,
    sigmaE = 0.10, rho = 0.30)
## Equity Linked Foreign Exchange Option [2.13.3]:
  EquityLinkedFXOption(TypeFlag = "p", E = 1.5, S = 100,
    X = 1.52, Time = 0.25, r = 0.08, rf = 0.05, q = 0.04,
    sigmaS = 0.20, sigmaE = 0.12, rho = -0.40)
## Takeover Foreign-Exchange Option [2.13.4]:
  TakeoverFXOption(V = 100, B = 100, E = 1.5, X = 1.55, Time = 1,
    r = 0.08, rf = 0.06, sigmaV = 0.20, sigmaE = 0.25, rho = 0.1)
```

LookbackOptions

Valuation of Lookback Options

#### Description

A collection and description of functions to valuate lookback options. The payoff from a pathdependent lookback call (put) depends on the exercise price being set to the minimum (maximum) asset price achieved during the life of the option. Thus, a lookback call (put) allows the purchaser to buy (sell) the asset at its minimum (maximum) price.

The functions are:

FloatingStrikeLookbackOption	Floating Strike Lookback Option,
FixedStrikeLookbackOption	Fixed Strike Lookback Option,
PTFloatingStrikeLookbackOption	PT Floating Strike Lookback Option,
PTFixedStrikeLookbackOption	PT Fixed Strike Lookback Option,
ExtremeSpreadOption	Extreme Spread Options.

# LookbackOptions

# Usage

```
FloatingStrikeLookbackOption(TypeFlag, S, SMinOrMax, Time, r,
    b, sigma, title = NULL, description = NULL)
FixedStrikeLookbackOption(TypeFlag, S, SMinOrMax, X, Time, r,
    b, sigma, title = NULL, description = NULL)
PTFloatingStrikeLookbackOption(TypeFlag, S, SMinOrMax, time1,
    Time2, r, b, sigma, lambda, title = NULL, description = NULL)
PTFixedStrikeLookbackOption(TypeFlag, S, X, time1, Time2, r, b,
    sigma, title = NULL, description = NULL)
ExtremeSpreadOption(TypeFlag, S, SMin, SMax, time1, Time2, r, b,
    sigma, title = NULL, description = NULL)
```

b	the annualized cost-of-carry rate, a numeric value; e.g. 0.1 means 10% pa.
description	a character string which allows for a brief description.
lambda	The lambda factor enables the creation of so-called "fractional" lookback op- tions where the strike is fixed at some percentage or below the extremum, i.e. lambda is greater than 1 for calls, and between 0 and 1 for puts.
r	the annualized rate of interest, a numeric value; e.g. 0.25 means 25% pa.
S	the asset price, a numeric value.
sigma	the annualized volatility of the underlying security, a numeric value; e.g. $0.3$ means 30% volatility pa.
SMax, SMin	[ExtremeSpread*] - the maximum (minimum) value of the underlying asset. Note, the payoff at maturity of the extreme spread call (put) equals the positive part of the difference between the maximum (minimum) value of the underlying asset, SMax, of the second (first) period and the maximum (minimum) of the underlying asset of the first (second) period. Likewise, reverse conditions are valid for the reverse extreme spread option.
SMinOrMax	the lowest price observed of the underlying in the case of the coll, or the highest price in the case of the put. A numeric value.
Time	the time to maturity measured in years, a numeric value; e.g. 0.5 means 6 months.
time1, Time2	<pre>[PTFloatingStrikeLookback*] - the time to the end of the lookback period time1, and the time to expiry Time2 where time1<time2, -="" [extremespread*]="" [ptfixedstrikelookback*]="" and="" at="" ending="" expiry="" lookback="" maturity="" of="" one="" option.<="" other="" period="" periods,="" pre="" predetermined="" starting="" starts,="" the="" time="" time1="" time1,="" time2="" time2,="" to="" today="" two="" where=""></time2,></pre>
title	a character string which allows for a project title.

TypeFlag	usually a character string either "c" for a call option or a "p" for a put option
	except for
	[ExtremeSpread*] -
	a character string either,
	"c" for the extreme call,
	"p" for the extreme put,
	"cr" for the reverse extreme call,
	"pr" for the revers extreme put.
Х	the exercise price, a numeric value.

# Details

# **Floating Strike Lookback Options:**

The lookback call (put) option gives the holder the right to buy (sell) an asset at its lowest (highest) price observed during the life of the option. This observed price is applied as the strike price. The payout for a call option is essentially the asset price minus the minimum spot price observed during the life of the option. The payout for a put option is essentially the maximum spot price observed during the life of the option minus the asset price. Therefore, a floating strike lookback option is always in the money and should always be exercised. Floating strike options can be priced analytically using a model introduced by Goldman, Sosin, and Gatto (1979). Monte Carlo simulation is used for the numerical calculation of a European style floating strike options. [Haug's Book, Chapter 2.9.1]

# **Fixed Strike Lookback Options:**

For a fixed strike lookback option, the strike price is known in advance. The call option payoff is given by the difference between the maximum observed price of the underlying asset during the life of the option and the fixed strike price. The put option payoff is given by the difference between the fixed strike price and the minimum observed price of the underlying asset during the life of the option. A fixed strike lookback call (put) option payoff is equal to that of a standard plain call (put) option when the final asset price is the maximum (minimum) observed value during the options life. Fixed strike lookback options can be priced analytically using a model introduced by Conze and Viswanathan (1991).

[Haug's Book, Chapter 2.9.2]

# **Partial-Time Floating Strike Options:**

For a partial-time floating strike lookback option, the lookback period starts at time zero and ends at an arbitrary date before expiration. Except for the partial lookback period, the option is similar to a floating strike lookback option. The partial-time floating strike lookback option is cheaper than a similar standard floating strike lookback option. Partial-time floating strike lookback options can be priced analytically using a model introduced by Heynen and Kat (1994). [Haug's Book, Chapter 2.9.3]

#### **Partial-Time Fixed Strike Options:**

For a partial-time fixed strike lookback option, the lookback period starts at a predetermined date

#### LookbackOptions

after the initialization date of the option. The partial-time fixed strike lookback call option payoff is given by the difference between the maximum observed price of the underlying asset during the lookback period and the fixed strike price. The partial-time fixed strike lookback put option payoff is given by the difference between the fixed strike price and the minimum observed price of the underlying asset during the lookback period. The partial-time fixed strike lookback option is cheaper than a similar standard fixed strike lookback option. Partial-time fixed strike lookback options can be priced analytically using a model introduced by Heynen and Kat (1994). [Haug's Book, Chapter 2.9.4]

#### **Extreme Spread Options:**

The time to maturity of an extreme spread option is divided into two periods: one period starting at time zero and ending at some arbitrary date, and another starting at that arbitrary date and ending at the expiration date. A payoff at maturity of an extreme spread call (put) option equals the positive part of the difference between the maximum (minimum) value of the underlying asset of the second (first) period and the maximum (minimum) value of the underlying asset of the first (second) period.[1] The payoff at expiration of a reverse extreme spread call (put) option equals the positive part of the difference between the minimum (maximum) of the underlying asset of the second (first) period and the minimum (maximum) value of the underlying asset of the first (second) period. Extreme spread options can be priced analytically using a model introduced by Bermin (1996).

[Haug's Book, Chapter 2.9.5]

## Value

The option price, a numeric value.

#### Note

The functions implement the algorithms to valuate plain vanilla options as described in Chapter 2.9 of Haug's Book (1997).

## Author(s)

Diethelm Wuertz for the Rmetrics R-port.

## References

Bermin H.P. (1996b); *Exotic Lookback Options: The case of Extreme Spread Options*, Department of Economics, Lund University, Sweden.

Conze A., Viswanathan R. (1991); Path Dependent Options: The Case of Lookback Options, Journal of Finance 46, 1893–1907.

Goldmann B.M., Sosin H.B., Gatto M.A. (1993); *Path Dependent Options: Buy at the Low, Sell at the High*, Journal of Finance 34, 1111.

Haug E.G. (1997); The Complete Guide to Option Pricing Formulas, McGraw-Hill, New York.

Heynen R.C., Kat H.M. (1994); Selective Memory, Risk Magazine 7, 1994.

# Examples

```
## Examples from Chapter 2.9 in E.G. Haug's Option Guide (1997)
## Floating Strike Lookback Option [2.9.1]:
  FloatingStrikeLookbackOption(TypeFlag = "c", S = 120,
     SMinOrMax = 100, Time = 0.5, r = 0.10, b = 0.10-0.06,
     sigma = 0.30)
## Fixed Strike Lookback Option [2.9.2]:
  FixedStrikeLookbackOption(TypeFlag = "c", S = 100,
     SMinOrMax = 100, X = 105, Time = 0.5, r = 0.10, b = 0.10,
     sigma = 0.30)
## Partial Time Floating Strike Lookback Option [2.9.3]:
  PTFloatingStrikeLookbackOption(TypeFlag = "p", S = 90,
     SMinOrMax = 90, time1 = 0.5, Time2 = 1, r = 0.06, b = 0.06,
     sigma = 0.20, lambda = 1)
## Partial Time Fixed Strike Lookback Option [2.9.4]:
  PTFixedStrikeLookbackOption(TypeFlag = "c", S = 100, X = 90,
     time1 = 0.5, Time2 = 1, r = 0.06, b = 0.06, sigma = 0.20)
## Extreme Spread Option [2.9.5]:
  ExtremeSpreadOption(TypeFlag = "c", S = 100, SMin = NA,
     SMax = 110, time1 = 0.5, Time2 = 1, r = 0.1, b = 0.1,
     sigma = 0.30)
  ExtremeSpreadOption(TypeFlag = "cr", S = 100, SMin = 90,
     SMax = NA, time1 = 0.5, Time2 = 1, r = 0.1, b = 0.1,
     sigma = 0.30)
```

MultipleAssetsOptions Valuation of Mutiple Assets Options

# Description

A collection and deswcription of functions to valuate multiple asset options. Multiple asset options, as the name implies, are options whose payoff is based on two (or more) assets. The two assets are associated with one another through their correlation coefficient.

The functions are:

Two Asset Correlation Option,
Exchange-One-Asset-For-Another,
European or American Option,
Exchange Option on an Exchange Option,
Option on the Min/Max of 2 Risky Assets,
Spread Option Approximation.

# Usage

```
TwoAssetCorrelationOption(TypeFlag, S1, S2, X1, X2, Time, r,
b1, b2, sigma1, sigma2, rho, title = NULL, description = NULL)
EuropeanExchangeOption(S1, S2, Q1, Q2, Time, r, b1, b2,
sigma1, sigma2, rho, title = NULL, description = NULL)
AmericanExchangeOption(S1, S2, Q1, Q2, Time, r, b1, b2, sigma1,
sigma2, rho, title = NULL, description = NULL)
ExchangeOnExchangeOption(TypeFlag, S1, S2, Q, time1, Time2, r,
b1, b2, sigma1, sigma2, rho, title = NULL, description = NULL)
TwoRiskyAssetsOption(TypeFlag, S1, S2, X, Time, r, b1,
b2, sigma1, sigma2, rho, title = NULL, description = NULL)
SpreadApproxOption(TypeFlag, S1, S2, X, Time, r, sigma1,
sigma2, rho, title = NULL, description = NULL)
```

b1, b2	the annualized cost-of-carry rate for the first and second asset, a numeric value; e.g. 0.1 means 10% pa.
description	a character string which allows for a brief description.
Q, Q1, Q2	additionally, quantity of the first and second asset.
r	the annualized rate of interest, a numeric value; e.g. 0.25 means 25% p.a.
rho	the correlation coefficient between the returns on the two assets.
S1, S2	the first and second asset price, numeric values.
sigma1, sigma2	the annualized volatility of the first and second underlying security, a numeric value; e.g. 0.3 means 30% volatility p.a.
Time	the time to maturity measured in years, a numeric value; e.g. 0.5 means 6 months.
time1, Time2	the time to maturity measured in years, a numeric value; e.g. 0.5 means 6 months.
title	a character string which allows for a project title.
TypeFlag	usually a character string either "c" for a call option or a "p" for a put option, except for
	[ExchangeOnExchange*] - a character string either,
	"1" denotes: option to exchange Q*S2 for the option to exchange S2 for S1,
	"2" denotes option to exchange the option to exchange S2 for S1, in return for $Q*S2$ ,
	"3" denotes: option to exchange Q*S2 for the option to exchange S1 for S2,
	"4" denotes option to exchange the option to exchange S1 for S2, in return for $0$ *S2;
	[TwoRiskyAssets*] - a character string either,
	"cmin" denotes: call on the minimum,
	"cmax" denotes: call on the maximum, "pmin" denotes: call on the minimum,
	"pmax" denotes: call on the maximum of two risky assets.
х	the exercise price, a numeric value.
X1, X2	the first and second exercise price, numeric values.
*	1 /

# Details

#### **Two-Asset Correlation Options:**

A two asset correlation options have two underlying assets and two strike prices. A two asset correlation call option on two assets S1 and S2 with a strike prices X1 and X2 has a payoff of max(S2-X2,0) if S1>X1 and 0 otherwise, and a put option has a payoff of max(X2-S2,0) if S1<X1 and 0 otherwise. Two asset correlation options can be priced analytically using a model introduced by Zhang (1995).

[Haug's Book, Chapter 2.8.1]

#### **Exchange-One-Asset-For-Another Options:**

The exchange option gives the holder the right to exchange one asset for another. The payoff for this option is the difference between the prices of the two assets at expiration. The analytical calculation of European exchange option is based on a modified Black Scholes formula originally introduced by Margrabe (1978). A binomial lattice is used for the numerical calculation of an American or European style exchange option.

[Haug's Book, Chapter 2.8.2]

# **Exchange-On-Exchange Options:**

Exchange options on exchange options can be found embedded in many sequential exchange opportunities [1]. As an example, a bond holder converting into a stock, later exchanging the shares received for stocks of an acquiring firm. This complex option can be priced analytically using a model introduced by Carr (1988).

[Haug's Book, Chapter 2.8.3]

#### **Portfolio Options:**

A portfolio option is an American (or European) style option on the maximum of the sum of the prices of two assets and a fixed strike price. A portfolio call option on two assets S1 and S2 with a strike price X has a payoff of max((S1+S2)-X,0) and a put option has a payoff of max((X-(S1+S2),0)). A binomial lattice is used for the numerical calculation of an American or European style portfolio options.

#### **Rainbow Options:**

A rainbow option is an American (or European) style option on the maximum (or minimum) of two underlying assets. These types of rainbow options are generally referred to as two-color rainbow options. There are four general types of two-color rainbow options: maximum or best of two risky assets, the minimum or worst of two risky assets, the better of two risky assets, and the worse of two risky assets. A maximum rainbow call option on two assets S1 and S2 with a strike price X has a payoff of max(max(S1,S2)-X,0) and a put option has a payoff of max(X-max(S1,S2),0). A minimum rainbow call option on two assets S1 and S2 with a strike parameter to a very Small number (1e-8) to calculate better and worse rainbow option types. The analytical calculation of European rainbow option is based on Rubinstein's (1991) model. A binomial lattice is used for

24

#### **MultipleAssetsOptions**

the numerical calculation of an American or European style rainbow options.

#### **Spread Options:**

A spread option is a standard option on the difference of the values of two assets. Spread options a related to exchange options. If the strike price is set to zero, a spread option is equivalent to an exchange option. A spread call option on two assets S1 and S2 with a strike price X has a payoff of max(S1-S2-X,0) and a put option has a payoff of max(X-S1+S2,0). The analytical calculation of European spread option is based on Gauss-Legendre integration and the Black-Scholes model. A binomial lattice is used for the numerical calculation of an American or European style spread options.

[Haug's Book, Chapter 2.8.5]

# **Dual Strike Options:**

A dual strike option is an American (European) option whose payoff involves receiving the best payoff of two standard American (European) style plain options. These options have two underlying assets and two strike prices. The payoff of a dual strike call option is the maximum of asset one minus strike one or asset two minus strike two. The payoff of a dual strike put option is the maximum of strike call option is the maximum of asset one or strike two minus asset two. The payoff of a reverse dual strike call option is the maximum of asset one minus strike one or strike two. The payoff of a reverse dual strike put option is the maximum of strike one or asset two. The payoff of a reverse dual strike put option is the maximum of strike one or asset two. The payoff of a reverse dual strike put option is the maximum of strike one minus asset one or asset two minus asset one or asset two minus strike two. A binomial lattice is used for the numerical calculation of an American or European style dual strike and reverse dual strike options.

# Value

The option price, a numeric value.

#### Note

The functions implement the algorithms to valuate plain vanilla options as described in Chapter 2.8 of Haug's Book (1997).

# Author(s)

Diethelm Wuertz for the Rmetrics R-port.

#### References

Black F. (1976); The Pricing of Commodity Contracs, Journal of Financial Economics 3, 167–179.

Boyle P.P., Evnine J., Gibbs S. (1989); *Numerical Evaluation of Multivariate Contingent Claims*, Review of Financial Studies 2, 241–250.

Boyle P.P., Tse Y.K. (1990); An Algorithm for Computing Values of Options on the Maximum or Minimum of Several Assets, Journal of Financial and Quantitative Analysis 25, 215–227.

Carr P.P. (1988) *The Valuation of Sequential Exchange Opportunities*, Journal of Finance 43, 1235–1256.

Haug E.G. (1997); The Complete Guide to Option Pricing Formulas, McGraw-Hill, New York.

Johnson H. (1987) *Options on the Maximum or the Minimum of Several Assets*, Journal of Financial and Quantitative Analysis 22, 277–283.

Kirk E. (1995); *Correlation in the Energy Markets*, in: Managing Energy Price Risk, Risk Publications and Enron, London, pp. 71–78.

Margrabe W. (1998); *The Value of an Option to Exchange one Asset for Another*, Journal of Finance 33, 177–186.

Rich D.R, Chance D.M. (1993); An Alternative Approach to the Pricing of Options on Multiple Assets, Journal of Financial Engineering 2, 271–285.

Rubinstein M. (1991) Somewhere over the Rainbow, Risk Magazine 4, 10.

Stulz R.M. (1982); *Options on the Minimum or Maximum of Two Risky Assets*, Journal of Financial Economics 10, 161–185.

Zhang P.G. (1995); Correlation Digital Options Journal of Financial Engineering 3, 5.

# Examples

```
## Examples from Chapter 2.8 in E.G. Haug's Option Guide (1997)
## Two Asset Correlation Options [2.8.1]:
  TwoAssetCorrelationOption(TypeFlag = "c", S1 = 52, S2 = 65,
    X1 = 50, X2 = 70, Time = 0.5, r = 0.10, b1 = 0.10, b2 = 0.10,
    sigma1 = 0.2, sigma2 = 0.3, rho = 0.75)
## European Exchange Options [2.8.2]:
  EuropeanExchangeOption(S1 = 22, S2 = 0.20, O1 = 1, O2 = 1,
    Time = 0.1, r = 0.1, b1 = 0.04, b2 = 0.06, sigma1 = 0.2,
    sigma2 = 0.25, rho = -0.5)
## American Exchange Options [2.8.2]:
  AmericanExchangeOption(S1 = 22, S2 = 0.20, Q1 = 1, Q2 = 1,
    Time = 0.1, r = 0.1, b1 = 0.04, b2 = 0.06, sigma1 = 0.2,
    sigma2 = 0.25, rho = -0.5)
## Exchange Options On Exchange Options [2.8.3]:
  for (flag in 1:4) print(
  ExchangeOnExchangeOption(TypeFlag = as.character(flag),
    S1 = 105, S2 = 100, Q = 0.1, time1 = 0.75, Time2 = 1.0, r = 0.1,
    b1 = 0.10, b2 = 0.10, sigma1 = 0.20, sigma2 = 0.25, rho = -0.5))
## Two Risky Assets Options [2.8.4]:
  TwoRiskyAssetsOption(TypeFlag = "cmax", S1 = 100, S2 = 105,
    X = 98, Time = 0.5, r = 0.05, b1 = -0.01, b2 = -0.04,
    sigma1 = 0.11, sigma2 = 0.16, rho = 0.63)
  TwoRiskyAssetsOption(TypeFlag = "pmax", S1 = 100, S2 = 105,
    X = 98, Time = 0.5, r = 0.05, b1 = -0.01, b2 = -0.04,
    sigma1 = 0.11, sigma2 = 0.16, rho = 0.63)
## Spread-Option Approximation [2.8.5]:
  SpreadApproxOption(TypeFlag = "c", S1 = 28, S2 = 20, X = 7,
```

```
Time = 0.25, r = 0.05, sigma1 = 0.29, sigma2 = 0.36, rho = 0.42)
```

MultipleExercisesOptions

Valuation of Mutiple Exercises Options

## Description

A collection and description of functions to valuate multiple exercise options. Multiple exercises options, as the name implies, are options whose payoff is based on multiple exercise dates.

The functions are:

ExecutiveStockOption	Executive Stock Option,
ForwardStartOption	Forward Start Option,
RatchetOption	Ratchet Option,
TimeSwitchOption	Time Switch Option,
SimpleChooserOption	Simple Chooser Option,
ComplexChooserOption	Complex Chooser Option,
OptionOnOption	Option On Option,
WriterExtendibleOption	Writer Extendible Option,
HolderExtendibleOption	Holder Extendible Option.

# Usage

```
ExecutiveStockOption(TypeFlag, S, X, Time, r, b, sigma, lambda,
    title = NULL, description = NULL)
ForwardStartOption(TypeFlag, S, alpha, time1, Time2, r, b, sigma,
    title = NULL, description = NULL)
RatchetOption(TypeFlag, S, alpha, time1, Time2, r, b, sigma,
    title = NULL, description = NULL)
TimeSwitchOption(TypeFlag, S, X, Time, r, b, sigma, A, m, dt,
    title = NULL, description = NULL)
SimpleChooserOption(S, X, time1, Time2, r, b, sigma,
    title = NULL, description = NULL)
ComplexChooserOption(S, Xc, Xp, Time, Timec, Timep, r, b, sigma,
    doprint = FALSE, title = NULL, description = NULL)
OptionOnOption(TypeFlag, S, X1, X2, time1, Time2, r, b, sigma,
    doprint = FALSE, title = NULL, description = NULL)
WriterExtendibleOption(TypeFlag, S, X1, X2, time1, Time2, r, b, sigma,
    title = NULL, description = NULL)
HolderExtendibleOption(TypeFlag, S, X1, X2, time1, Time2, r, b, sigma, A,
    title = NULL, description = NULL)
```

A	[HolderExtendible*] - defined by the amount A*dt the investor receives at maturity time Time for each time interval deltat the corresponding asset price has exceeded the exercise price X, in the case of a call option, or the corresponding asset price has been below the exercise price X, in the case of a put option. A numeric value.
alpha	[Ratchet*] - the exercise price is alpha times the asset price S after the known time time. alpha is a numeric value. If alpha is less than unity, the call (put) will start 100*(1-alpha) percent in the money (out-of-the-money); if alpha is unity, the option will start at the money; and if alpha is larger than unity, the call (put) will start 100*(alpha-1) percentage out of the money (in-the-money).
b	the annualized cost-of-carry rate, a numeric value; e.g. 0.1 means 10% pa.
description	a character string which allows for a brief description.
doprint	a logical. Should the critical value I be printed? By defaut FALSE.
dt	the time interval; a numeric value.
lambda	the jump rate pa.
m	defined by the number of time units where the option has already fulfilled the thresold condition. This applies to cases, for which some of the option's total lifetime has already passed. An integer value.
r	the annualized rate of interest, a numeric value; e.g. 0.25 means 25% pa.
S	the asset price, a numeric value.
sigma	the annualized volatility of the underlying security, a numeric value; e.g. 0.3 means 30% volatility pa.
Time	the time to maturity measured in years, a numeric value; e.g. 0.5 means 6 months.
Timec, Timep	[ComplexChooser*] - decision time measured in years, e.g. 0.5 means 6 months. Timec, is the time to maturity of the call option, Timep, is the time to maturity of the put option, both also measured in years. Numeric values.
time1, Time2	the time to maturity, Time1, measured in years, e.g. 0.5 means 6 months, and the elapsed time in the future, Time2. In detail, the forward start option with time to maturity Time1 starts at-the-money or proportinally in-the-money or out-of-the-money after this elapsed time Time2 in the future.
title	a character string which allows for a project title.
TypeFlag	usually a character string either "c" for a call option or a "p" for a put option; [OptionOnOption] - a character string either "cc" for a call-on-call option, or "cp" for a call-on-put
	option, or "pc" for a put-on-call option, or "pp" for a put-on-put option.
Х	the exercise price, a numeric value.
Хс, Хр	[ComplexChooser*] - the exercise price of the call option, Xc, and the exercise price of the put option, Xp, numeric values.
X1, X2	the exercise price of the underlying option, $X1$ , and the exercise price of the option on the option, $X2$ , numeric values.

#### Details

#### **Executive Stock Options:**

Executive stock options are usually at-the-money options that are issued to motivate employees to act in the best interest of the company. They cannot be sold and often last as long as 10 or 15 years. The executive model takes into account that an employee often looses their options when they leave the company before expiration. The value of an executive option equals the standard Black-Scholes model multiplied by the probability that the employee will stay with the firm until the option expires. Executive stock options can be priced analytically using a model published by Jennergren and Naslund (1993).

[Haug's Book, Chapter 2.1]

# **Forward Start Options:**

A forward start option is an option which is paid for today, but will start at some determined time in the future known as the issue date. The option usually starts at-the-money or proportionally in or out-of-the-money at a future date. The strike is set to a positive constant a times the asset price S at a future date. If a is less than one, the call (put) will start 1 - a percent in-the-money (out-ofthe-money); if a is one, the option will start at-the-money; and if a is larger than one, the call (put) will start a - 1 percent out-of-the-money (in-the-money).[1] Forward start options can be priced analytically using a model published by Rubinstein (1990). [Haug's Book, Chapter 2.2]

# Ratchet [Compound] Options:

A compound option is an option on an option. Therefore, when one option is exercised, the underlying security is another option. There are four types of possible compound options: a call on a call, a call on a put, a put on a call, and a put on a put. The owner of a compound option has until the expiration date of the compound option to determine whether to exercise the compound option. If exercised, the owner will receive the underlying option with its own exercise price and time until expiration. If the underlying option is exercised, the owner will receive the underlying security. European compound options can be priced analytically using a model published by Rubinstein (1991). A binomial lattice is used for the numerical calculation of an American or European style exchange option. A ratchet option is also called sometimes a "moving strike option" or "cliquet option". [Haug's Book, Chapter 2.3]

#### **Time-Switch Options:**

For a discrete time-switch call (put) option, the holder receives an amount ADt at expiration for each time interval, Dt, the corresponding asset price has been above (below) the strike price. If some of the option's total lifetime has passed, it is required to add a fixed amount to the pricing formula. Discrete time-switch options can be priced analytically using a model published by Pechtl (1995).

[Haug's Book, Chapter 2.4]

#### **Simple Chooser Options:**

A chooser option allows the holder to determine at some date, after the trade date, whether the option becomes a plain vanilla call or put. Chooser options are also called "as you like it" options. Chooser options are useful for hedging a future event that might not occur. Due to their increased flexibility, chooser options are more expensive than plain vanilla options. It is assumed at the options expiration date that a holder of the chooser option will choose the more valuable of the put or call option. The less valuable option that was not chosen will become worthless. Chooser options can be priced analytically using a model introduced by Rubinstein (1991). [Haug's Book, Chapter 2.5.1]

#### **Complex Chooser Options:**

A complex chooser option allows the holder to determine at some date, after the trade date, whether the option is to be a standard call chooser model, a complex chooser option will be more expensive than a plain vanilla option. Complex chooser options can be priced analytically using a model introduced by Rubinstein (1991).

[Haug's Book, Chapter 2.5.2]

# **Option On Options:**

This derivative prices options on options. An option on an option is more expensive to purchase than the underlying option itself, as the purchaser has received a price guarantee and effectively extended the life of the option. These options provide the benefit of a guaranteed price for the option at a date in the future. Options on Options can be prices as published by Geske (1977). His model was later extended and discussed by Geske (1979), Hodges and Selby (1987), and Rubinstein (1991). [Haug's Book, Chapter 2.6]

# Writer [Holder] Extendible Options:

Writer extendible options can be found embedded in various financial contracts. For example, corporate warrants often give the issuing firm the right to extend the life of the warrants. These options can be exercised at their initial maturity, but are extended to a new maturity if they are out-of-the-money at initial maturity. Discrete time-switch options can be priced analytically using a model published by Longstaff (1995).

[Haug's Book, Chapter 2.6]

# Value

The option valuation programs return an object of class "fOPTION" with the following slots:

@call	the function call.
@parameters	a list with the input parameters.
@price	a numeric value with the value of the option.
@title	a character string with the name of the test.
@description	a character string with a brief description of the test.

#### Note

Options on options are also known as compound options or as mother-and-daughter options.

#### Author(s)

Diethelm Wuertz for the Rmetrics R-port.

#### References

Geske R. (1977); *The Valuation of Corporate Liabilities as Compound Options*, Journal of Financial and Quantitative Analysis, 541–552.

Geske R. (1979); The Valuation of Compound Options, Journal of Financial Economics 7, 63-81.

Haug E.G. (1997); *The complete Guide to Option Pricing Formulas*, Chapter 2.8.1, McGraw-Hill, New York.

Hodges S.D., Selby J.P. (1987); On the Evaluation of Compound Options; Management Science 33, 347–355.

Jennergren L.P., Naslund B. (1993); A Comment on Valuation of Executive Stock Options and the FASB Proposal, The Accounting Review 68, 179, 1993.

Longstaff F.A. (1990); *Pricing Options with Extendible Maturities: Analysis and Applications*, Journal of Finance 45, 474–491.

Pechtl A. (1990); Classified Information, Risk Magazine 8.

Rubinstein, M. (1990); Pay Now, Choose Later, Risk Magazine 3.

Rubinstein M. (1991); Options for the Undecide, Risk Magazine 4.

Rubinstein M. (1991); Double Trouble; Risk Magazine 5.

# Examples

```
## Examples from Chapter 2.1 - 2.7 in E.G. Haug's Option Guide (1997)
```

```
## ExecutiveStockOption [2.1]:
    ExecutiveStockOption(TypeFlag = "c", S = 60, X = 64, Time = 2,
    r = 0.07, b = 0.07-0.03, sigma = 0.38, lambda = 0.15)
```

```
## ForwardStartOption [2.2]:
    ForwardStartOption(TypeFlag = "c", S = 60, alpha = 1.1,
        time1 = 1, Time2 = 1/4, r = 0.08, b = 0.08-0.04, sigma = 0.30)
```

## Ratchet Option [2.3]: RatchetOption(TypeFlag = "c", S = 60, alpha = 1.1, time1 = c(1.00, 0.75), Time2 = c(0.75, 0.50), r = 0.08, b = 0.04, sigma = 0.30)

## Time Switch Option [2.4]: TimeSwitchOption(TypeFlag = "c", S = 100, X = 110, Time = 1, r = 0.06, b = 0.06, sigma = 0.26, A = 5, m = 0, dt = 1/365)

- ## Simple Chooser Option [2.5.1]: SimpleChooserOption(S = 50, X = 50, time1 = 1/4, Time2 = 1/2, r = 0.08, b = 0.08, sigma = 0.25)
- ## Complex Chooser Option [2.5.2]: ComplexChooserOption(S = 50, Xc = 55, Xp = 48, Time = 0.25, Timec = 0.50, Timep = 0.5833, r = 0.10, b = 0.1-0.05,

```
sigma = 0.35, doprint = TRUE)
## Option On Option [2.6]:
    OptionOnOption(TypeFlag = "pc", S = 500, X1 = 520, X2 = 50,
        time1 = 1/2, Time2 = 1/4, r = 0.08, b = 0.08-0.03, sigma = 0.35)
## Holder Extendible Option [2.7.1]:
    HolderExtendibleOption(TypeFlag = "c", S = 100, X1 = 100,
        X2 = 105, time1 = 0.50, Time2 = 0.75, r = 0.08, b = 0.08,
        sigma = 0.25, A = 1)
## Writer Extendible Option [2.7.2]:
    WriterExtendibleOption(TypeFlag = "c", S = 80, X1 = 90, X2 = 82,
        time1 = 0.50, Time2 = 0.75, r = 0.10, b = 0.10, sigma = 0.30)
```

# Index

\*Topic math AsianOptions, 2 BarrierOptions, 4 BinaryOptions, 10 CurrencyTranslatedOptions, 15 LookbackOptions, 18 MultipleAssetsOptions, 22 MultipleExercisesOptions, 27 AmericanExchangeOption (MultipleAssetsOptions), 22 AsianOptions, 2 AssetOrNothingOption (BinaryOptions), 10 BarrierOptions, 4 BinaryBarrierOption (BinaryOptions), 10 BinaryOptions, 10 CashOrNothingOption (BinaryOptions), 10 ComplexChooserOption (MultipleExercisesOptions), 27 CurrencyTranslatedOptions, 15 DiscreteBarrierOption (BarrierOptions), 4 DoubleBarrierOption (BarrierOptions), 4 EquityLinkedFXOption (CurrencyTranslatedOptions), 15 EuropeanExchangeOption (MultipleAssetsOptions), 22 ExchangeOnExchangeOption (MultipleAssetsOptions), 22 ExecutiveStockOption (MultipleExercisesOptions), 27 ExtremeSpreadOption (LookbackOptions), 18 FEInDomesticFXOption

CurrencyTranslatedOptions), 15

FixedStrikeLookbackOption
 (LookbackOptions), 18
FloatingStrikeLookbackOption
 (LookbackOptions), 18
ForwardStartOption
 (MultipleExercisesOptions), 27

GapOption (BinaryOptions), 10 GeometricAverageRateOption (AsianOptions), 2

HolderExtendibleOption (MultipleExercisesOptions), 27

LevyAsianApproxOption (AsianOptions), 2 LookbackOptions, 18 LookBarrierOption (BarrierOptions), 4

MultipleAssetsOptions, 22 MultipleExercisesOptions, 27

OptionOnOption (MultipleExercisesOptions), 27

PTFixedStrikeLookbackOption (LookbackOptions), 18 PTFloatingStrikeLookbackOption (LookbackOptions), 18 PTSingleAssetBarrierOption (BarrierOptions), 4 PTTwoAssetBarrierOption (BarrierOptions), 4

QuantoOption (CurrencyTranslatedOptions), 15

RatchetOption (MultipleExercisesOptions), 27

SimpleChooserOption
 (MultipleExercisesOptions), 27

INDEX

SoftBarrierOption (BarrierOptions), 4 SpreadApproxOption (MultipleAssetsOptions), 22 StandardBarrierOption (BarrierOptions), 4 SuperShareOption (BinaryOptions), 10 TakeoverFXOption (CurrencyTranslatedOptions), 15 TimeSwitchOption (MultipleExercisesOptions), 27 TurnbullWakemanAsianApproxOption (AsianOptions), 2 TwoAssetBarrierOption (BarrierOptions), 4 TwoAssetCashOrNothingOption (BinaryOptions), 10 TwoAssetCorrelationOption (MultipleAssetsOptions), 22 TwoRiskyAssetsOption (MultipleAssetsOptions), 22

```
WriterExtendibleOption
    (MultipleExercisesOptions), 27
```

34