# Package 'dad'

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	<b>iption</b> The data consist of a set of variables measured on several groups of individuals. To each group is associated an estimated probability density function. The package provides tools to create or manage such data and functional methods (principal component analysis, multidimensional scaling, cluster analysis, discriminant analysis) for such probability densities.
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# **Description**

The three-way data consists of a set of variables measured on several groups of individuals. To each group is associated an estimated probability density function. The package provides functional methods (principal component analysis, multidimensional scaling, cluster analysis, discriminant analysis...) for such probability densities.

## **Details**

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To cite dad, use citation("dad").

The main functions applying to the probability densities are:

- fpcad: functional principal component analysis,
- fpcat: functional principal component analysis applied to data indexed according to time,
- fmdsd: multidimensional scaling,
- fhclustd: hierarchical clustering,
- fdiscd.misclass: functional discriminant analysis in order to compute the misclassification ratio with the one-leave-out method,
- fdiscd.predict: discriminant analysis in order to predict the class (synonymous with cluster, not to be confused with the class attribute of an R object) of each probability density whose class is unknown.
- mdsdd: multidimensional scaling of discrete probability distributions,
- discdd.misclass: functional discriminant analysis of discrete probability distributions, in order to compute the misclassification ratio with the one-leave-out method,
- discdd.predict: discriminant analysis of discrete probability distributions, in order to predict the class of each probability distribution whose class is unknown,

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The above functions are completed by:

• A print() method for objects of class fpcad, fmdsd, fdiscd.misclass, fdiscd.predict or mdsdd, in order to display the results of the corresponding function,

- A plot() method for objects of class fpcad, fmdsd, fhclustd or mdsdd, in order to display some useful graphics attached to the corresponding function,
- A generic function interpret that applies to objects of class fpcad fmdsd or mdsdd, helps the user to interpret the scores returned by the corresponding function, in terms of moments (fpcad or fmdsd) or in terms of marginal probability distributions (mdsdd).

We also introduce classes of objects and tools in order to handle collections of data frames:

 folder creates an object of class folder, that is a list of data frames which have in common the same columns.

The following functions apply to a folder and compute some statistics on the columns of its elements: mean.folder, var.folder, cor.folder, skewness.folder or kurtosis.folder.

- folderh creates an object of class folderh, that is a list of data frames with a hierarchic relation between each pair of consecutive data frames.
- foldert creates an object of class foldert, that is a list of data frames indexed according to time, concerning the same individuals and variables or not.
- read.mtg creates an object of class foldermtg from an MTG (Multiscale Tree Graph) file containing plant architecture data.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard with the contributions from Gilles Hunault, Julie Bourbeillon and Besnik Pumo

## References

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Yousfi, S., Boumaza, R., Aissani, D., Adjabi, S. (2014). Optimal bandwith matrices in functional principal component analysis of density functions. Journal of Statistical Computation and Simulation, 85 (11), 2315-2330.

appendtofolderh	Adds a data frame to a folderh.

## **Description**

Creates an object of class folderh by appending a data frame to an object of class folderh. The appended data frame will be the first or last element of the returned folderh.

# Usage

```
appendtofolderh(fh, df, key, after = FALSE)
```

# **Arguments**

fh	object of class	folderh.

df data frame to be appended to fh.

key character string. The key defining the relation 1toN between df and the first (if

after = FALSE, the default value) or last (if after = TRUE) data frame of fh.

after logical. If FALSE (default), the data frame df is related to the first data frame of

fh, and is appended as the first element of the returned folderh. If TRUE, df is related to the last data frame of fh and becomes the last element of the returned

folderh.

#### Value

Returns an object of class folderh, that is a list of n+1 data frames where n is the number of data frames of fh. The value of the attribute  $\operatorname{attr}(,\text{"keys"})$  is  $\operatorname{c(key,attr(fh,"keys"))}$  if  $\operatorname{after} = \operatorname{FALSE}$ ,  $\operatorname{c(attr(fh,"keys"),key)}$  otherwise.

## Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### See Also

folderh.

8 as.data.frame.folder

```
as.data.frame.folder Folder to data frame
```

# **Description**

Builds a data frame from an object of class folder.

# Usage

```
## S3 method for class 'folder'
as.data.frame(x, row.names = NULL, optional = FALSE, ..., group.name = "group")
```

## **Arguments**

x object of class folder that is a list of data frames with the same column names. row.names, optional

for consistency with as.data.frame. as.data.frame.folder does not take them into account.

... further arguments passed to or from other methods.

group.name the name of the grouping variable. It is the name of the last column of the returned data frame.

## **Details**

The data frame is simply obtained by row binding the data frames of the folder and adding a factor (as last column). The name of this column is given by group.name argument. The levels of this factor are the names of the elements of the folder.

#### Value

```
as.data.frame.folder returns a data frame.
```

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### See Also

folder: object of class folder. as.folder.data.frame: build an object of class folder from a data frame.

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## **Examples**

```
data(iris)
iris.fold <- as.folder(iris, "Species")
print(iris.fold)
iris.df <- as.data.frame(iris.fold)
print(iris.df)</pre>
```

as.data.frame.folderh Hierarchic folder to data frame

#### **Description**

Builds a data frame from a folderh.

#### Usage

## **Arguments**

## Value

as.data.frame.folderh returns a data frame whose row names are those of x[[elt]] (that is x[[j]]). The data frame contains the values of x[[elt]] and the corresponding values of the data frames x[[k]], these correspondances being defined by the keys of the hierarchic folder.

The column names of the returned data frame are organized in three parts.

- 1. The first part consists in the key names keys[k],..., keys[j-1].
- 2. The second part consists in the values of x[[j]].
- 3. The third part consists in the values of x[[k]] except the key keys[k].

See the examples to view these details.

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## Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### See Also

```
folder, folderh, as.folder.folderh.
```

```
# First example: rose flowers
data(roseflowers)
flg <- roseflowers$variety</pre>
flx <- roseflowers$flower</pre>
flfh <- folderh(flg, "rose", flx)</pre>
print(flfh)
fldf <- as.data.frame(flfh)</pre>
print(fldf)
# Second example: castles
data(castles.dated)
cag <- castles.dated$periods</pre>
cax <- castles.dated$stones</pre>
cafh <- folderh(cag, "castle", cax)</pre>
print(cafh)
cadf <- as.data.frame(cafh)</pre>
print(summary(cadf))
# Third example: leaves (example of a folderh with more than two data frames)
data(roseleaves)
lvr <- roseleaves$rose</pre>
lvs <- roseleaves$stem</pre>
lvl <- roseleaves$leaf</pre>
lvll <- roseleaves$leaflet</pre>
lfh <- folderh(lvr, "rose", lvs, "stem", lvl, "leaf", lvll)</pre>
lf1 <- as.data.frame(lfh, elt = "lvs", key = "rose")</pre>
print(lf1)
lf2 <- as.data.frame(lfh, elt = "lvl", key = "rose")</pre>
print(lf2)
lf3 <- as.data.frame(lfh, elt = "lvll", key = "rose")</pre>
print(lf3)
lf4 <- as.data.frame(lfh, elt = "lvll", key = "stem")</pre>
print(lf4)
```

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```
as.data.frame.foldert foldert to data frame
```

# **Description**

Builds a data frame from an object of class foldert.

## Usage

```
## S3 method for class 'foldert'
as.data.frame(x, row.names = NULL, optional = FALSE, ..., group.name = "time")
```

## Arguments

Х

object of class foldert with the same row names. An object of class foldert is a list of data frames with the same column names, each of them corresponding to a time of observation.

row.names, optional

for consistency with as.data.frame. as.data.frame.foldert does not take them into account.

... further arguments passed to or from other methods.

group.name

the name of the grouping variable. It is the name of the last column of the returned data frame.

As the observations are indexed by time, the default value is group.name = "time".

# **Details**

```
as.data.frame.foldert uses as.data.frame.folder.
```

# Value

```
as.data.frame.foldert returns a data frame.
```

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

## See Also

foldert: object of class foldert. as.foldert.data.frame: build an object of class foldert from a data frame. as.foldert.array: build an object of class foldert from a 3d-array.

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## **Examples**

```
data(floribundity)
ftflor <- foldert(floribundity, cols.select = "union", rows.select = "union")
print(ftflor)
dfflor <- as.data.frame(ftflor)
summary(dfflor)</pre>
```

as.folder

Coerce to a folder

## **Description**

Coerces a data frame or an object of class "folderh" to an object of class "folder".

# Usage

```
as.folder(x, ...)
```

# **Arguments**

x an object of class data. frame or folderh.

- data.frame: see as.folder.data.frame
- folderh: see as.folder.folderh

. . . further arguments passed to or from other methods.

## Value

an object of class folder.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### See Also

folder: objects of class folder. as.data.frame.folder: build a data frame from an object of class folder. as.folder.data.frame: build an object of class folder from a data frame. as.folder.folderh: build an object of class folder from an object of class folderh.

as.folder.data.frame

```
as.folder.data.frame Data frame to folder
```

# **Description**

Builds an object of class folder from a data frame.

## Usage

```
## S3 method for class 'data.frame'
as.folder(x, groups = tail(colnames(x), 1), ...)
```

## **Arguments**

x data frame.

groups string. The name of the column of x containing the grouping variable. x[,groups]

must be a factor, otherwise, there is an error.

If omitted, the last column of x is used as grouping variable.

... further arguments passed to or from other methods.

## Value

as.folder.data.frame returns an object of class folder that is a list of data frames with the same column names.

Each element of the folder contains the data corresponding to one level of x[,groups].

## Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

## See Also

folder: objects of class folder. as.data.frame.folder: build a data frame from an object of class folder. as.folder.folderh: build an object of class folder from an object of class folderh.

```
# First example: iris (Fisher)
data(iris)
iris.fold <- as.folder(iris, "Species")
print(iris.fold)

# Second example: roses
data(roses)
roses.fold <- as.folder(roses, "rose")
print(roses.fold)</pre>
```

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as.folder.folderh Hierarchic folder to folder

## Description

Creates an object of class folder, that is a list of data frames with the same column names, from a folderh

#### Usage

```
## S3 method for class 'folderh'
as.folder(x, elt = names(x)[2], key = attr(x, "keys")[1], ...)
```

#### **Arguments**

X	object of class folderh containing N (N>1) data frames: $x[[1]],, x[[N]],$ related by (N-1) keys: $keys[1],, keys[N-1].$
elt	string. The name of one element of $x$ , that is data frame, say the $j$ -th, whose rows are distributed among the data frames of the returned folder. See details.
key	string. The name of an element of attr(x, "keys"), that is the key, say the k-th with k <j, are="" data="" details.<="" factor="" folder.="" frames="" is="" levels="" names="" of="" returned="" see="" td="" the="" which="" whose=""></j,>
	further arguments passed to or from other methods.

#### Value

as.folder.folderh returns an object of class folder, a list of data frames with the same columns. These data frames contain the values of x[[elt]] (or x[[j]]) and the corresponding values of the data frames x[[j-1]], ... x[[k]], these correspondances being defined by the keys of the hierarchic folder. The names of these data frames are given by the levels of the key attr(x, "keys")[k]).

The rows of the data frame x[[elt]] (or x[[j]]) are distributed among the data frames of the returned folder accordingly to the levels of the key attr(x, "keys")[k]. So the row names of the 1-th data frame of the returned folder consist in the rows of x[[j]] corresponding to the 1-th level of the key attr(x, "keys")[k].

The column names of the data frames of the returned folder are the union of the column names of the data frames x[[k]],...,x[[j]] and are organized in two parts.

- The first part consists in the columns of x[[k]] except the column corresponding to the key attr(x, "keys")[k].
- 2. For each i=k+1,...,j the column names of the data frame x[[i]] are reorganized so that the key attr(x,"keys")[i] is its first column. The columns of the reorganized data frames x[[k+1]],..., x[[j]] are concatenated. The result forms the second part.

## Notice that if:

• the folderh has two data frames df1 and df2, where the factor corresponding to the key has T levels, and one column of df2, say df2[, "Fa"], is a factor with levels "a1", ..., "ap"

as.folder.folderh

ullet and the folder returned by as.folder includes T data frames dat1, ..., datT,

then each of dat1, ..., datT has a column named "Fa" which is a factor with the same levels "a1", ..., "ap" as df2[, "Fa"].

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

## See Also

folder, folderh. as.folder.folderh to build an object of class folder from an object of class folderh. as.data.frame.folder to build a data frame from an object of class folder. as.data.frame.folderh to build a data frame from an object of class folderh.

```
# First example: flowers
data(roseflowers)
flg <- roseflowers$variety
flx <- roseflowers$flower</pre>
flfh <- folderh(flg, "rose", flx)</pre>
print(flfh)
flf <- as.folder(flfh)</pre>
print(flf)
# Second example: castles
data(castles.dated)
cag <- castles.dated$periods</pre>
cax <- castles.dated$stones</pre>
cafh <- folderh(cag, "castle", cax)</pre>
print(cafh)
caf <- as.folder(cafh)</pre>
print(caf)
# Third example: leaves (example of a folderh of more than two data frames)
data(roseleaves)
lvr <- roseleaves$rose</pre>
lvs <- roseleaves$stem</pre>
lvl <- roseleaves$leaf</pre>
lvll <- roseleaves$leaflet</pre>
lfh <- folderh(lvr, "rose", lvs, "stem", lvl, "leaf", lvll)</pre>
lf1 <- as.folder(lfh, elt = "lvs", key = "rose")</pre>
print(lf1)
lf2 <- as.folder(lfh, elt = "lvl", key = "rose")</pre>
```

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```
print(1f2)

lf3 <- as.folder(lfh, elt = "lvll", key = "rose")
print(1f3)

lf4 <- as.folder(lfh, elt = "lvll", key = "stem")
print(lf4)</pre>
```

as.folderh

Coerce to a folderh

# **Description**

Coerces an object to an object of class folderh.

# Usage

```
as.folderh(x, classes)
```

# **Arguments**

x an object to be coerced to an object of class folderh. In the current version, it is an object of class "foldermtg" (see as.folderh.foldermtg).

classes argument useful for as.folderh.foldermtg.

## Value

an object of class folderh.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### See Also

as. folderh. foldermtg: build an object of class folderh from an object of class foldermtg.

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as.folderh.foldermtg Build a hierarchic folder from an object of class foldermtg

## **Description**

Creates an object of class folderh from an object of class foldermtg.

# Usage

```
## S3 method for class 'foldermtg'
as.folderh(x, classes)
```

# **Arguments**

x object of class foldermtg.

classes character vector. Codes of the vertex classes in the returned folderh. These

codes are the names of the elements (data frames) of x containing the features

on the vertices corresponding to the codes.

These codes must be distinct, and the corresponding classes must have distinct

scales (see foldermtg). Otherwise, there is an error.

These codes, except the one with the highest scale, are the keys of the returned

folderh.

#### **Details**

This function uses folderh.

#### Value

An object of class folderh. Its elements are the data frames of x containing the features on vertices. Hence, each data frame matches with a class of vertex, and a scale. These data frames are in increasing order of the scale.

A column (factor) is added to the first data frame, containing the identifier of the vertex. Two columns are added to the second data frame:

- 1. the first one is a factor which gives, for each vertex, the name of the vertex of the first data frame which is its "parent",
- 2. and the second one is also a factor and contains the vertex's identifier.

And so on for the third and following data frames, if relevant.

The column containing the vertex identifiers is redundant with the row names; anyway, it is necessary for folderh.

The key of the relationship between the two first data frame is given by the first column of each of these data frames. If there are more than two data frames, the key of the relationship between the n-th and (n+1)-th data frames (n>1) is given by the second column of the n\$th data frame and the first column of the (n+1)-th data frame.

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#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### See Also

read.mtg: reads a MTG file and creates an object of class "foldermtg". folderh: object of class folderh.

## **Examples**

```
mtgfile <- system.file("extdata/plant1.mtg", package = "dad")
x <- read.mtg(mtgfile)

# folderh containing the plant ("P") and the stems ("A")
as.folderh(x, classes = c("P", "A"))

# folderh containing the plant ("P"), axes ("A") and phytomers ("M")
as.folderh(x, classes = c("P", "A", "M"))

# folderh containing the plant ("P") and the phytomers ("M")
as.folderh(x, classes = c("P", "M"))

# folderh containing the axes and phytomers
fhPM <- as.folderh(x, classes = c("A", "M"))
# coerce this folderh into a folder, and compute statistics on this folder
fPM <- as.folder(fhPM)
mean(fPM)</pre>
```

as.foldert

Coerce to a foldert

# Description

Coerces a data frame or array to an object of class foldert.

## Usage

```
as.foldert(x, ...)
```

## **Arguments**

```
x an object of class data. frame or array.
```

- data.frame: see as.foldert.data.frame
- array: see as.foldert.array

arguments passed to as.foldert.data.frame or as.foldert.array, further arguments passed to or from other methods.

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## Value

```
an object of class foldert.
```

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

as.foldert.array

Data frame to foldert

## **Description**

Builds an object of class foldert from a 3d-array.

#### Usage

```
## S3 method for class 'array'
as.foldert(x, ind = 1, var = 2, time = 3, ...)
```

## **Arguments**

```
    x a 3d-array.
    ind, var, time three distinct integers among 1, 2 and 3.
    ind gives the dimension of the observations, var gives the dimension of the variables and ind gives the dimension of the times.
    ... further arguments passed to or from other methods.
```

# Value

an object ft of class foldert that is a list of data frames, each of them corresponding to a time of observation; these data frames have the same column names.

They necessarily have the same row names (attr(ft, "same.rows")=TRUE). The "times" attribute of ft: attr(ft, "times") is a numeric vector, an ordered factor or an object of class Date, and contains the values of the dimension of x given by time argument.

## Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

## See Also

```
foldert: objects of class foldert.
as.foldert.data.frame: build an object of class foldert from a data frame.
```

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## **Examples**

as.foldert.data.frame Data frame to foldert

## **Description**

Builds an object of class foldert from a data frame.

# Usage

```
## S3 method for class 'data.frame'
as.foldert(x, method = 1, ind = 1, timecol = 2, nvar = NULL, same.rows = TRUE, ...)
```

# Arguments

Χ.

data frame.

method

1 or 2. Indicates the layout of the data frame x and, therefore, the method used to extract the data and build the foldert.

- If method = 1, there is a column containing the identifiers of the measured objects and a column containing the times. The other columns contain the observations.
- If method = 2, there is a column containing the identifiers of the measured objects, and the observations are organized as follows:
  - the observations corresponding to the 1st time are on columns timecol
     : (timecol + nvar -1)
  - the observations corresponding to the 2nd time are on columns (timecol + nvar): (timecol + 2 \* nvar -1)
  - and so on.

ind

string or numeric. The name of the column of x containing the indentifiers of the measured objects, or the number of this column.

timecol

string or numeric.

• If method = 1, timecol is the name or the number of the column of x containing the times of observation, or the number of this column. x[,timecol] must be of class "numeric", "ordered", "Date", "POSIX1t" or "POSIXct", otherwise, there is an error.

as.foldert.data.frame 21

• If method=2, timecol is the name or the number of the first column corresponding to the first observation. If there are duplicated column names and several columns are named by timecol, the first one is considered.

nvar integer. If method=2, indicates the number of variables observed at each time.

Omitted if method=1.

same.rows logical. If TRUE (default), the elements of the returned foldert are data frames

with the same row names.

Necessarily TRUE if method = 2.

... further arguments passed to or from other methods.

#### Value

an object ft of class foldert, that is a list of data frames organised according to time; these data frames have the same column names.

If method = 1, they can have the same row names (attr(ft, "same.rows") = TRUE) or not (attr(ft, "same.rows") = FALSE). The time attribute attr(ft, "times") has the same class as x[, timecol] (numeric vector, ordered factor or object of class "Date", "POSIX1t" or "POSIXct") and contains the values of x[, timecol].

If method = 2, they necessarily have the same row names: attr(ft, "same.rows") = TRUE and attr(ft, "times") is 1:length(ft).

The rownames of each data frame are the identifiers of the individuals, as given by x[,ind].

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

## See Also

```
foldert: objects of class foldert.
as.data.frame.foldert: build a data frame from an object of class foldert.
as.foldert.array: build an object of class foldert from a 3d-array.
```

22 association measures

```
ft1 <- as.foldert(x, method = 1, ind = "ind", timecol = "t", same.rows = TRUE)
print(ft1)
ft2 <- as.foldert(x, method = 1, ind = "ind", timecol = "t", same.rows = FALSE)
print(ft2)
data(castles.dated)
periods <- castles.dated$periods
stones <- castles.dated$stones</pre>
stones$stone <- rownames(stones)</pre>
castledf <- merge(periods, stones, by = "castle")</pre>
castledf$period <- as.numeric(castledf$period)</pre>
castledf$stone <- as.factor(paste(as.character(castledf$castle),</pre>
                              as.character(castledf$stone), sep = "_"))
castfoldt1 <- as.foldert(castledf, method = 1, ind = "stone", timecol = "period",</pre>
                           same.rows = FALSE)
summary(castfoldt1)
# Second example: method = 2
times <- as.Date(c("2017-03-01", "2017-04-01", "2017-05-01"))
y1 \leftarrow data.frame(z1=rep(0,6), z2=rep(0,6))
y2 \leftarrow data.frame(z1=rnorm(6,1,1), z2=rnorm(6,3,2))
y3 <- data.frame(z1=rnorm(6,3,2), z2=rnorm(6,6,3))
y \leftarrow cbind(ind = 1:6, y1, y2, y3)
ft3 <- as.foldert(y, method = 2, ind = "ind", timecol = 2, nvar = 2)</pre>
print(ft3)
```

#### **Description**

Computes pairwise association measures (Cramer's V, Pearson's contingency coefficient, phi, Tschuprow's T) between the categorical variables of a data frame, using functions of the package DescTools (see Assocs).

# Usage

```
cramer.data.frame(x, check = TRUE)
pearson.data.frame(x, check = TRUE)
phi.data.frame(x, check = TRUE)
tschuprow.data.frame(x, check = TRUE)
```

## Arguments

x a data frame (can also be a tibble). Its columns should be factors.

check logical. If TRUE (default) the function checks if each column of x is a factor, and

there is a warning if it is not.

#### Value

A square matrix whose elements are the pairwise association measures.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

# **Examples**

```
data(roses)
xr = roses[,c("Sha", "Den", "Sym", "rose")]
xr$Sha = cut(xr$Sha, breaks = c(0, 5, 7, 10))
xr$Den = cut(xr$Den, breaks = c(0, 4, 6, 10))
xr$Sym = cut(xr$Sym, breaks = c(0, 6, 8, 10))
cramer.data.frame(xr)
pearson.data.frame(xr)
phi.data.frame(xr)
tschuprow.data.frame(xr)
```

association measures for folder

Association measures between categorical variables of the data frames of a folder

# Description

Computes the pairwise association measures (Cramer's V, Pearson's contingency coefficient, phi, Tschuprow's T) between the categorical variables of an object of class folder. The computation is carried out using the functions cramer.data.frame, tschuprow.data.frame, pearson.data.frame or phi.data.frame. These functions are built from corresponding functions of the package DescTools (see Assocs)

## Usage

```
cramer.folder(xf)
tschuprow.folder(xf)
pearson.folder(xf)
phi.folder(xf)
```

#### **Arguments**

xf

an object of class folder that is a list of data frames with the same column names. Its columns should be factors, otherwise there is a warning.

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## Value

A list the length of which is equal to the number of data frames of the folder. Each element of the list is a square matrice giving the pairwise association measures of the variables of the corresponding data frame.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

## **Examples**

```
data(roses)
xr = roses[,c("Sha", "Den", "Sym", "rose")]
xr$Sha = cut(xr$Sha, breaks = c(0, 5, 7, 10))
xr$Den = cut(xr$Den, breaks = c(0, 4, 6, 10))
xr$Sym = cut(xr$Sym, breaks = c(0, 6, 8, 10))
xfolder = as.folder(xr, groups = "rose")
cramer.folder(xfolder)
pearson.folder(xfolder)
phi.folder(xfolder)
tschuprow.folder(xfolder)
```

bandwidth.parameter

Parameter of the normal reference rule

## **Description**

Computation of the parameter of the normal reference rule in order to estimate the (matrix) bandwidth.

# Usage

```
bandwidth.parameter(p, n)
```

## **Arguments**

p sample dimension.

n sample size.

#### **Details**

The parameter is equal to:

$$h = (\frac{4}{n(p+2)})^{\frac{1}{p+4}}$$

It is based on the minimisation of the asymptotic mean integrated square error in density estimation when using the Gaussian kernel method (Wand and Jones, 1995).

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#### Value

Returns the value required by the functions fpcad, fmdsd, fdiscd.misclass and fdiscd.predict when their argument windowh is set to NULL.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

Boumaza, R., Yousfi, S., Demotes-Mainard, S. (2015). Interpreting the principal component analysis of multivariate density functions. Communications in Statistics - Theory and Methods, 44 (16), 3321-3339.

Wand, M. P., Jones, M. C. (1995). Kernel Smoothing. Boca Raton, FL: Chapman and Hall.

# **Examples**

```
# Sample size :
n <- 20
# Number of variables :
p <- 3
bandwidth.parameter(p, n)</pre>
```

castles

Alsacian castles by year of building

# Description

The data were collected by J.M. Rudrauf on Alsacian castles whose building year is known (even approximatively). On each castle, he measured 4 structural parameters on a sample of building stones.

These data are about the same castles as in castles. dated data set.

# Usage

```
data(castles)
```

#### **Format**

castles is a list of 46 data frames. Each of these data frames matches with one year (between 1136 and 1510) and contains measures on one or several castles which have been built since that year.

Each data frame has 5 to 101 rows (stones) and 5 columns: height, width, edging, boss (numeric) and castle (factor).

26 castles.dated

#### **Source**

Rudrauf, J.M., Boumaza, R. (2001). Contribution a l'etude de l'architecture medievale: les caracteristiques des pierres a bossage des chateaux forts alsaciens. Centre de Recherches Archeologiques Medievales de Saverne, 5, 5-38.

## **Examples**

```
data(castles)
foldert(castles)
```

castles.dated

Dated Alsacian castles

# **Description**

The data were collected by J.M. Rudrauf on Alsacian castles whose building period is known (even approximately). On each castle, he measured 4 structural parameters on a sample of building stones.

#### Usage

```
data(castles.dated)
```

#### **Format**

castles.dated is a list of two data frames:

- castles.dated\$stones: this first data frame has 1262 cases (rows) and 5 variables (columns) that are named height, width, edging, boss (numeric) and castle (factor).
- castles.dated\$periods: this second data frame has 68 cases and 2 variables named castle and period; the column castle corresponds to the levels of the factor castle of the first data frame; the column period is a factor with 6 levels indicating the approximative building period. Thus this factor defines 6 classes of castles.

#### Source

Rudrauf, J.M., Boumaza, R. (2001). Contribution a l'etude de l'architecture medievale: les caracteristiques des pierres a bossage des chateaux forts alsaciens. Centre de Recherches Archeologiques Medievales de Saverne, 5, 5-38.

```
data(castles.dated)
summary(castles.dated$stones)
summary(castles.dated$periods)
```

castles.nondated 27

castles.nondated

Non dated Alsacian castles

## **Description**

The data were collected by J.M. Rudrauf on Alsacian castles whose building period is unknown. On each castle, he measured 4 structural parameters on a sample of building stones.

# Usage

```
data(castles.nondated)
```

#### **Format**

castles. nondated is a list of two data frames:

- castles.nondated\$stones: this first data frame has 1280 cases (rows) and 5 variables (columns) that are named height, width, edging, boss (numeric) and castle (factor).
- castles.nondated\$periods: this second data frame has 67 cases and 2 variables named castle and period; the column castle corresponds to the levels of the factor castle of the first data frame; the column period is a factor indicating NA as the building period is unknown.

Notice that the data frames corresponding to the castles whose building period is known are those in castles.dated.

#### Source

Rudrauf, J.M., Boumaza, R. (2001). Contribution a l'etude de l'architecture medievale: les caracteristiques des pierres a bossage des chateaux forts alsaciens. Centre de Recherches Archeologiques Medievales de Saverne, 5, 5-38.

```
data(castles.nondated)
summary(castles.nondated$stones)
summary(castles.nondated$periods)
```

28 cor.folder

cor.folder	Correlation matrices of a folder of data sets
cor. Foraci	Correlation matrices of a folder of data sets

# **Description**

Computes the correlation matrices of the elements of an object of class folder.

## Usage

```
cor.folder(x, use = "everything", method = "pearson")
```

# **Arguments**

х	an object of class folder that is a list of data frames with the same column names.
use	an optional character string giving a method for computing covariances in the presence of missing values. This must be (an abbreviation of) one of the strings "everything", "all.obs", "complete.obs", "na.or.complete", or "pairwise.complete.obs" (see var).
method	a character string indicating which correlation coefficient (or covariance) is to be computed. One of "pearson" (default), "kendall", or "spearman": can be abbreviated.

# **Details**

It uses cor to compute the variance matrix of the numeric columns of each element of the folder. If some columns of the data frames are not numeric, there is a warning, and the variances are computed on the numeric columns only.

## Value

A list whose elements are the correlation matrices of the elements of the folder.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

## See Also

folder to create an object is of class folder. mean.folder, var.folder, skewness.folder, kurtosis.folder for other statistics for folder objects.

cut.data.frame

## **Examples**

```
# First example: iris (Fisher)
data(iris)
iris.fold <- as.folder(iris, "Species")
iris.cor <- cor.folder(iris.fold)
print(iris.cor)

# Second example: roses
data(roses)
roses.fold <- as.folder(roses, "rose")
roses.cor <- cor.folder(roses.fold)
print(roses.cor)</pre>
```

cut.data.frame

Change numeric variables into factors

## **Description**

This function changes numerical columns of a data frame x into factors. For each of these columns, its range is divided into intervals and the values of this column is recoded according to which interval they fall.

For that, cut is applied to each column of x.

# Usage

```
## S3 method for class 'data.frame'
cut(x, breaks, labels = NULL, include.lowest = FALSE, right = TRUE, dig.lab = 3L,
    ordered_result = FALSE, cutcol = NULL, ...)
```

## **Arguments**

x data frame (can also be a tibble).

breaks list or numeric.

- If breaks is a list, its length is equal to the number of columns in the data frame. It can be:
  - a list of numeric vectors. The  $j^{th}$  element corresponds to the column x[,j], and is a vector of two or more unique cut points
  - or a list of single numbers (each greater or equal to 2). breaks[[j]] element gives the number of intervals into which th  $j^{th}$  variable of the folder is to be cut. The elements breaks[[j]] corresponding to nonnumeric columns must be NULL; if not, there is a warning.
- If breaks is a numeric vector, it gives the number of intervals into which every column x[,j] is to be cut (see cut).

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list of character vectors. If given, its length is equal to the number of columns of labels x. labels[[j]] gives the labels for the intervals of the  $j^{th}$  columns of the data frame. By default, the labels are constructed using "(a,b]" interval notation. If labels = FALSE, simple integer codes are returned instead of a factor. See cut. include.lowest logical, indicating if, for each column x[,j], an x[i,j] equal to the lowest (or highest, for right = FALSE) 'breaks' value should be included (see cut). right logical, indicating if the intervals should be closed on the right (and open on the left) or vice versa (see cut). dig.lab integer or integer vector, which is used when labels are not given. It determines the number of digits used in formatting the break numbers. • If it is a single value, it gives the number of digits for all variables of the folder (see cut). • If it is a list of integers, its length is equal to the number of variables, and the  $j^{th}$  element gives the number of digits for the  $j^{th}$  variable of the folder. ordered\_result logical: should the results be ordered factors? (see cut) cutcol numeric vector: indices of the columns to be converted into factors. These columns must all be numeric. Otherwise, there is a warning.

#### Value

A data frame with the same column and row names as x.

If cutcol is given, each numeric column x[,j] whose number is contained in cutcol is replaced by a factor. The other columns are unmodified.

further arguments passed to or from other methods.

If any column x[, j] whose number is in cutcol is not numeric, it is unmodified.

If cutcol is omitted, every numerical columns are replaced by factors.

## Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

```
data("roses")
x <- roses[roses$rose %in% c("A", "B"), c("Sha", "Sym", "Den", "rose")]
cut(x, breaks = 3)
cut(x, breaks = 5)
cut(x, breaks = c(0, 4, 6, 10))
cut(x, breaks = list(c(0, 6, 8, 10), c(0, 5, 7, 10), c(0, 6, 7, 10)))
cut(x, breaks = list(c(0, 6, 8, 10), c(0, 5, 7, 10)), cutcol = 1:2)</pre>
```

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cut.folder	In a folder: change numeric variables into factors	
------------	--	--

# Description

This function applies to a folder. For each elements (data frames) of this folder, it changes its numerical columns into factors, using cut.data.frame.

# Usage

```
## S3 method for class 'folder'
cut(x, breaks, labels = NULL, include.lowest = FALSE, right = TRUE, dig.lab = 3L,
    ordered_result = FALSE, cutcol = NULL, ...)
```

# **Arguments**

x	an object of class folder.
breaks	list or numeric, defining the intervals into which the variables of each element of the folder is to be cut. See cut.folder.
labels	list of character vectors. If not omitted, it gives the labels for the intervals of each column of the elements of $x$ . See $cut.folder$ .
include.lowest	logical, indicating if a value equal to the lowest (or highest, for right = FALSE) 'breaks' value should be included (see cut.folder).
right	logical, indicating if the intervals should be closed on the right (and open on the left) or vice versa (see cut.folder).
dig.lab	integer or integer vector, which is used when labels are not given. It determines the number of digits used in formatting the break numbers. See cut.folder.
ordered_result	logical: should the results be ordered factors? (see cut.folder)
cutcol	numeric vector: indices of the columns of the elements of $x$ to be converted into factors. These columns must all be numeric. Otherwise, there is a warning. See $cut.folder$ .
	further arguments passed to or from other methods.

# Value

An object of class folder with the same length and names as x. Its elements (data frames) have the same column and row names as the elements of x.

For more details, see cut.data.frame

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

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## **Examples**

```
data("roses")

x <- as.folder(roses[, c("Sha", "Den", "Sym", "rose")], groups = "rose")
summary(x)

x3 <- cut(x, breaks = 3)
summary(x3)

x7 <- cut(x, breaks = 7)
summary(x7)</pre>
```

ddchisqsym

Distance between probability distributions of discrete variables given samples

# **Description**

Symmetrized chi-squared distance between two multivariate (q > 1) or univariate (q = 1) discrete probability distributions, estimated from samples.

## Usage

```
ddchisqsym(x1, x2)
```

## **Arguments**

x1, x2 vectors or data frames of q columns (can also be a tibble).

If they are data frames and have not the same column names, there is a warning.

#### **Details**

Let  $p_1$  and  $p_2$  denote the estimated probability distributions of the discrete samples  $x_1$  and  $x_2$ . The symmetrized chi-squared distance between the discrete probability distributions of the samples are computed using the ddchisqsympar function.

## Value

The distance between the two probability distributions.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

## References

Deza, M.M. and Deza E. (2013). Encyclopedia of distances. Springer.

ddchisqsympar 33

## See Also

ddchisqsympar: chi-squared distance between two discrete distributions, given the probabilities on their common support.

Other distances: ddhellinger, ddjeffreys, ddjensen, ddlp.

# **Examples**

ddchisqsympar

Distance between discrete probability distributions given the probabilities on their common support

#### **Description**

Symmetrized chi-squared distance between two discrete probability distributions on the same support (which can be a Cartesian product of q sets), given the probabilities of the states (which are q-tuples) of the support.

# Usage

```
ddchisqsympar(p1, p2)
```

#### **Arguments**

p1 array (or table) the dimension of which is q. The first probability distribution on the support.

p2 array (or table) the dimension of which is q. The second probability distribution on the support.

## **Details**

The chi-squared distance between two discrete distributions  $p_1$  and  $p_2$  is given by:

$$\sum_{x} (p_1(x) - p_2(x))^2 / p_2(x)$$

Then the symmetrized chi-squared distance is given by the formula:

$$||p_1 - p_2|| = \sum_{x} (p_1(x) - p_2(x))^2 / (p_1(x) + p_2(x))$$

34 ddhellinger

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

#### References

Deza, M.M. and Deza E. (2013). Encyclopedia of distances. Springer.

#### See Also

ddchisqsym: chi-squared distance between two estimated discrete distributions, given samples.

Other distances: ddhellingerpar, ddjeffreyspar, ddjensenpar, ddlppar.

## **Examples**

ddhellinger

Distance between probability distributions of discrete variables given samples

# **Description**

Hellinger (or Matusita) distance between two multivariate (q > 1) or univariate (q = 1) discrete probability distributions, estimated from samples.

# Usage

```
ddhellinger(x1, x2)
```

## **Arguments**

x1, x2 data frames of q columns or vectors (can also be tibbles).

If they are data frames and have not the same column names, there is a warning.

ddhellingerpar 35

## **Details**

Let  $p_1$  and  $p_2$  denote the estimated probability distributions of the discrete samples  $x_1$  and  $x_2$ . The Matusita distance between the discrete probability distributions of the samples are computed using the ddhellingerpar function.

## Value

The distance between the two probability distributions.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

#### References

Deza, M.M. and Deza E. (2013). Encyclopedia of distances. Springer.

#### See Also

ddhellingerpar: Hellinger metric (Matusita distance) between two discrete distributions, given the on their common support probabilities.

Other distances: ddchisqsym, ddjeffreys, ddjensen, ddlp.

# **Examples**

ddhellingerpar

Distance between discrete probability distributions given the probabilities on their common support

## **Description**

Hellinger (or Matusita) distance between two discrete probability distributions on the same support (which can be a Cartesian product of q sets), given the probabilities of the states (which are q-tuples) of the support.

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## Usage

```
ddhellingerpar(p1, p2)
```

# Arguments

p1 array (or table) the dimension of which is q. The first probability distribution on the support.
 p2 array (or table) the dimension of which is q. The second probability distribution

on the support.

#### **Details**

The Hellinger distance between two discrete distributions  $p_1$  and  $p_2$  is given by:  $\sqrt{\sum_x (\sqrt{p_1(x)} - \sqrt{p_2(x)})^2}$ Notice that some authors divide this expression by  $\sqrt{2}$ .

## Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

#### References

Deza, M.M. and Deza E. (2013). Encyclopedia of distances. Springer.

# See Also

ddhellinger: Hellinger distance between two estimated discrete distributions, given samples.

Other distances: ddchisqsympar, ddjeffreyspar, ddjensenpar, ddlppar.

ddjeffreys 37

3 3	Divergence between probability distributions of discrete variables given samples

# Description

Jeffreys divergence (symmetrized Kullback-Leibler divergence) between two multivariate (q > 1) or univariate (q = 1) discrete probability distributions, estimated from samples.

### Usage

```
ddjeffreys(x1, x2)
```

# Arguments

x1, x2 vectors or data frames of q columns (can also be a tibble).

If they are data frames and have not the same column names, there is a warning.

### **Details**

Let  $p_1$  and  $p_2$  denote the estimated probability distributions of the discrete samples  $x_1$  and  $x_2$ . The Jeffreys divergence between the discrete probability distributions of the samples are computed using the ddjeffreyspar function.

## Value

The divergence between the two probability distributions.

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

## References

Deza, M.M. and Deza E. (2013). Encyclopedia of distances. Springer.

### See Also

ddjeffreyspar: Jeffreys distances between two discrete distributions, given the probabilities on their common support.

Other distances: ddchisqsym, ddhellinger, ddjensen, ddlp.

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## **Examples**

ddjeffreyspar

Distance between discrete probability distributions given the probabilities on their common support

## **Description**

Jeffreys divergence (symmetrized Kullback-Leibler divergence) between two discrete probability distributions on the same support (which can be a Cartesian product of q sets), given the probabilities of the states (which are q-tuples) of the support.

### Usage

```
ddjeffreyspar(p1, p2)
```

## Arguments

p1 array (or table) the dimension of which is q. The first probability distribution on the support.

p2 array (or table) the dimension of which is q. The second probability distribution on the support.

### **Details**

The Jeffreys divergence  $||p_1 - p_2||$  between two discrete distributions  $p_1$  and  $p_2$  is given by the formula:

$$||p_1 - p_2|| = \sum_{x} (p_1(x) - p_2(x)) log(p_1(x)/p_2(x))$$

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

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### References

Deza, M.M. and Deza E. (2013). Encyclopedia of distances. Springer.

#### See Also

ddjeffreys: Jeffreys distance between two estimated discrete distributions, given samples. Other distances: ddchisqsympar, ddhellingerpar, ddjensenpar, ddlppar.

# **Examples**

ddjensen

Divergence between probability distributions of discrete variables given samples

## Description

Jensen-Shannon divergence between two multivariate (q > 1) or univariate (q = 1) discrete probability distributions, estimated from samples.

## Usage

```
ddjensen(x1, x2)
```

# **Arguments**

x1, x2 vectors or data frames of q columns (can also be tibbles).

If they are data frames and have not the same column names, there is a warning.

### **Details**

Let  $p_1$  and  $p_2$  denote the estimated probability distributions of the discrete samples  $x_1$  and  $x_2$ . The Jensen-Shannon divergence between the discrete probability distributions of the samples are computed using the ddjensenpar function.

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### Value

The distance between the two probability distributions.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

#### References

Deza, M.M. and Deza E. (2013). Encyclopedia of distances. Springer.

#### See Also

ddjensenpar: Jeffreys distances between two discrete distributions, given the probabilities on their common support.

Other distances: ddchisqsym, ddhellinger, ddjeffreys, ddlp.

## **Examples**

ddjensenpar

Divergence between discrete probability distributions given the probabilities on their common support

# Description

Jensen-Shannon divergence between two discrete probability distributions on the same support (which can be a Cartesian product of q sets), given the probabilities of the states (which are q-tuples) of the support.

### Usage

```
ddjensenpar(p1, p2)
```

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# **Arguments**

p1	array (or table) the dimension of which is $q$ . The first probability distribution on
	the support.

p2 array (or table) the dimension of which is q. The second probability distribution on the support.

### **Details**

The Jensen-Shannon divergence  $||p_1 - p_2||$  between two discrete distributions  $p_1$  and  $p_2$  is given by the formula:

$$||p_1 - p_2|| = \sum_{x} (p_1(x)\log(2p_1(x)/(p_1(x) + p_2(x)))) + (p_2(x)\log(2p_2(x)/(p_1(x) + p_2(x))))$$

## Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

#### References

Deza, M.M. and Deza E. (2013). Encyclopedia of distances. Springer.

### See Also

ddjensen: Jensen-Shannon distance between two estimated discrete distributions, given samples.

Other distances: ddchisqsympar, ddhellingerpar, ddjeffreyspar, ddlppar.

# **Examples**

42 ddlp

ddlp	Distance between probability distributions of discrete variables given samples

### **Description**

 $L^p$  distance between two multivariate (q > 1) or univariate (q = 1) discrete probability distributions, estimated from samples.

### Usage

```
ddlp(x1, x2, p = 1)
```

## Arguments

x1, x2 vectors or data frames of q columns (can also be tibbles).

If they are data frames and have not the same column names, there is a warning.

p integer. Parameter of the distance.

### **Details**

Let  $p_1$  and  $p_2$  denote the estimated probability distributions of the discrete samples  $x_1$  and  $x_2$ . The  $L^p$  distance between the discrete probability distributions of the samples are computed using the ddlppar function.

### Value

The distance between the two discrete probability distributions.

## Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

### References

Deza, M.M. and Deza E. (2013). Encyclopedia of distances. Springer.

## See Also

ddlppar:  $L^p$  distance between two discrete distributions, given the probabilities on their common support.

Other distances: ddchisqsym, ddhellinger, ddjeffreys, ddjensen.

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### **Examples**

ddlppar

Distance between discrete probability distributions given the probabilities on their common support

# Description

 $L^p$  distance between two discrete probability distributions on the same support (which can be a Cartesian product of q sets), given the probabilities of the states (which are q-tuples) of the support.

### Usage

```
ddlppar(p1, p2, p = 1)
```

## **Arguments**

p1	array (or table) the dimension of which is $q$ . The first probability distribution on the support.
p2	array (or table) the dimension of which is $q$ . The second probability distribution on the support.
р	integer. Parameter of the distance.

### **Details**

The  $L^p$  distance  $||p_1 - p_2||$  between two discrete distributions  $p_1$  and  $p_2$  is given by the formula:

$$||p_1 - p_2||^p = \sum_{x} |p_1(x) - p_2(x)|^p$$

If p = 1, it is the variational distance.

If p = 2, it is the Patrick-Fisher distance.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

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### References

Deza, M.M. and Deza E. (2013). Encyclopedia of distances. Springer.

#### See Also

ddlp:  $L^p$  distance between two estimated discrete distributions, given samples. Other distances: ddchisqsympar, ddhellingerpar, ddjeffreyspar, ddjensenpar.

### **Examples**

departments

French departments and regions

# Description

Departments and regions of metropolitan France.

### Usage

```
data(departments)
```

### Format

departments is a data frame with 96 rows and 4 columns (factors):

```
coded: departments: numbers
named: departments: names
coder: regions: ISO code
namer: region: names
```

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

discdd.misclass 45

#### Source

INSEE. Code officiel g\'eographique au 1er janvier 2018.

### **Examples**

```
data(departments)
print(departments)
```

discdd.misclass

Misclassification ratio in functional discriminant analysis of discrete probability distributions.

## **Description**

Computes the one-leave-out misclassification ratio of the rule assigning T groups of individuals, one group after another, to the class of groups (among K classes of groups) which achieves the minimum of the distances or divergences between the probability distribution associated to the group to assign and the K probability distributions associated to the K classes.

## Usage

### **Arguments**

xf

object of class folderh with two data frames or list of arrays (or tables).

- If it is a folderh:
  - The first data frame has at least two columns. One column contains the names of the T groups (all the names must be different). An other column is a factor with K levels partitionning the T groups into K classes.
  - The second one has (q+1) columns. The first q columns are factors (otherwise, they are coerced into factors). The last column is a factor with T levels defining T groups. Each group, say t, consists of  $n_t$  individuals.
- If it is a list of arrays or tables, the  $t^{th}$  element  $(t=1,\ldots,T)$  is the table of the joint distribution (absolute or relative frequencies) of the  $t^{th}$  group. These arrays have the same shape: Each array (or table) xf[[i]] has:
  - the same dimension(s). If q=1 (univariate),  $\dim(xf[[i]])$  is an integer. If q>1 (multivariate),  $\dim(xf[[i]])$  is an integer vector of length q.
  - the same dimension names dimnames(xf[[i]]) (is non NULL). These dimnames are the names of the variables.

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class.var

string (if xf is an object of class "folderh") or data.frame with two columns (if xf is a list of arrays).

- If xf is of class "folder", class.var is the name of the class variable.
- If xf is a list of arrays or a list of tables, class.var is a data.frame with at least two columns named "group" and "class". The "group" column contains the names of the T groups (all the names must be different). The "class" column is a factor with K levels partitioning the T groups into K classes.

distance

The distance or dissimilarity used to compute the distance matrix between the densities. It can be:

- "11" (default) the  $L^p$  distance with p=1
- "12" the  $L^p$  distance with p=2
- "chisqsym" the symmetric Chi-squared distance
- "hellinger" the Hellinger metric (Matusita distance)
- "jeffreys" the Jeffreys distance (symmetrised Kullback-Leibler divergence)
- "jensen" the Jensen-Shannon distance
- "lp" the  $L^p$  distance with p given by the argument p of the function.

crit

р

1 or 2. In order to select the densities associated to the classes. See Details.

integer. Optional. When distance = "lp" ( $L^p$  distance with p>2), p is the parameter of the distance.

#### **Details**

• If xf is an object of class "folderh" containing the data:

The T probability distributions  $f_t$  corresponding to the T groups of individuals are estimated by frequency distributions within each group.

To the class k consisting of  $T_k$  groups is associated the probability distribution  $g_k$ , knowing that when using the one-leave-out method, we do not include the group to assign in its class k. The crit argument selects the estimation method of the  $g_k$ 's.

- crit=1 The probability distribution  $g_k$  is estimated using the whole data of this class, that is the rows of x corresponding to the  $T_k$  groups of the class k.
  - The estimation of the  $g_k$ 's uses the same method as the estimation of the  $f_t$ 's.
- crit=2 The  $T_k$  probability distributions  $f_t$  are estimated using the corresponding data from xf. Then they are averaged to obtain an estimation of the density  $g_k$ , that is  $g_k = \frac{1}{T_k} \sum f_t$ .
- If xf is a list of arrays (or list of tables):

The  $t^{th}$  array is the joint frequency distribution of the  $t^{th}$  group. The frequencies can be absolute or relative.

To the class k consisting of  $T_k$  groups is associated the probability distribution  $g_k$ , knowing that when using the one-leave-out method, we do not include the group to assign in its class k. The crit argument selects the estimation method of the  $g_k$ 's.

- crit=1  $g_k = \frac{1}{\sum n_t} \sum n_t f_t$ , where  $n_t$  is the total of xf[[t]]. Notice that when xf[[t]] contains relative frequencies, its total is 1. That is equivalent to crit=2.
- crit=2  $g_k = \frac{1}{T_k} \sum f_t$ .

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#### Value

Returns an object of class discdd.misclass, that is a list including:

classification

data frame with 4 columns:

- factor giving the group name. The column name is the same as that of the column (q + 1) of x,
- the prior class of the group if it is available, or NA if not,
- alloc: the class allocation computed by the discriminant analysis method,
- misclassed: boolean. TRUE if the group is misclassed, FALSE if it is well-classed, NA if the prior class of the group is unknown.

```
confusion.mat confusion matrix, misalloc.per.class the misclassification ratio per class, misclassed the misclassification ratio, distances matrix with T rows and K columns, of the distances (d_{tk}): d_{tk} is the distance between the group t and the class t, proximities matrix of the proximity indices (in percents) between the groups and the classes. The proximity between the group t and the class t is: (1/d_{tk})/\sum_{l=1}^{l=K}(1/d_{tl}).
```

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

Rudrauf, J.M., Boumaza, R. (2001). Contribution à l'étude de l'architecture médiévale: les caractéristiques des pierres à bossage des châteaux forts alsaciens, Centre de Recherches Archéologiques médiévales de Saverne, 5, 5-38.

## **Examples**

```
# Example 1 with a folderh obtained by converting numeric variables
data("castles.dated")
stones <- castles.dated$stones
periods <- castles.dated$periods
stones$height <- cut(stones$height, breaks = c(19, 27, 40, 71), include.lowest = TRUE)
stones$width <- cut(stones$width, breaks = c(24, 45, 62, 144), include.lowest = TRUE)
stones$edging <- cut(stones$edging, breaks = c(0, 3, 4, 8), include.lowest = TRUE)
stones$boss <- cut(stones$boss, breaks = c(0, 6, 9, 20), include.lowest = TRUE)

castlefh <- folderh(periods, "castle", stones)

# Default: dist="l1", crit=1
discdd.misclass(castlefh, "period")

# Hellinger distance, crit=2
discdd.misclass(castlefh, "period", distance = "hellinger", crit = 2)</pre>
```

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```
# Example 2 with a list of 96 arrays
data("dspgd2015")
data("departments")
classes <- departments[, c("coded", "namer")]
names(classes) <- c("group", "class")

# Default: dist="11", crit=1
discdd.misclass(dspgd2015, classes)

# Hellinger distance, crit=2
discdd.misclass(dspgd2015, classes, distance = "hellinger", crit = 2)</pre>
```

discdd.predict

Predicting the class of a group of individuals with discriminant analysis of probability distributions.

# Description

Assigns several groups of individuals, one group after another, to the class of groups (among K classes of groups) which achieves the minimum of the distances or divergences between the probability distribution associated to the group to assign and the K probability distributions associated to the K classes.

### Usage

## Arguments

xf

object of class folderh with two data frames or list of arrays (or tables).

- If it is a folderh:
  - The first data.frame has at least two columns. One column contains the names of the T groups (all the names must be different). An other column is a factor with K levels partitionning the T groups into K classes.
  - The second one has (q+1) columns. The first q columns are factors (otherwise, they are coerced into factors). The last column is a factor with T levels defining T groups. Each group, say t, consists of  $n_t$  individuals.
- If it is a list of arrays or tables, the  $t^{th}$  element  $(t=1,\ldots,T)$  is the table of the joint distribution (absolute or relative frequencies) of the  $t^{th}$  group. These arrays have the same shape: Each array (or table) xf[[i]] has:

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- the same dimension(s). If q = 1 (univariate), dim(xf[[i]]) is an integer. If q > 1 (multivariate), dim(xf[[i]]) is an integer vector of length q.
- the same dimension names dimnames(xf[[i]]) (is non NULL). These dimnames are the names of the variables.

class.var

string (if xf is an object of class "folderh") or data.frame with two columns (if xf is a list of arrays).

- If xf is of class "folder", class.var is the name of the class variable.
- If xf is a list of arrays or a list of tables, class.var is a data.frame with at least two columns named "group" and "class". The "group" column contains the names of the T groups (all the names must be different). The "class" column is a factor with K levels partitioning the T groups into K

distance

The distance or dissimilarity used to compute the distance matrix between the densities. It can be:

- "11" (default) the  $L^p$  distance with p=1
- "12" the  $L^p$  distance with p=2
- "chisqsym" the symmetric Chi-squared distance
- "hellinger" the Hellinger metric (Matusita distance)
- "jeffreys" the Jeffreys distance (symmetrised Kullback-Leibler divergence)
- "jensen" the Jensen-Shannon distance
- "lp" the  $L^p$  distance with p given by the argument p of the function.

crit

1 or 2. In order to select the densities associated to the classes. See Details.

misclass.ratio logical (default FALSE). If TRUE, the confusion matrix and misclassification ratio are computed on the groups whose prior class is known. In order to compute the misclassification ratio by the one-leave-out method, use the discdd.misclass function.

p

integer. Optional. When distance = "lp" ( $L^p$  distance with p > 2), p is the parameter of the distance.

### **Details**

• If xf is an object of class "folderh" containing the data:

The T probability distributions  $f_t$  corresponding to the T groups of individuals are estimated by frequency distributions within each group.

To the class k consisting of  $T_k$  groups is associated the probability distribution  $g_k$ . The crit argument selects the estimation method of the  $g_k$ 's.

- crit=1 The probability distribution  $g_k$  is estimated using the whole data of this class, that is the rows of x corresponding to the  $T_k$  groups of the class k. The estimation of the  $g_k$ 's uses the same method as the estimation of the  $f_t$ 's.
- crit=2 The  $T_k$  probability distributions  $f_t$  are estimated using the corresponding data from xf. Then they are averaged to obtain an estimation of the density  $g_k$ , that is  $g_k =$  $\frac{1}{T_h} \sum f_t$ .

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• If xf is a list of arrays (or list of tables):

The  $t^{th}$  array is the joint frequency distribution of the  $t^{th}$  group. The frequencies can be absolute or relative.

To the class k consisting of  $T_k$  groups is associated the probability distribution  $g_k$ . The crit argument selects the estimation method of the  $g_k$ 's.

- crit=1  $g_k = \frac{1}{\sum n_t} \sum n_t f_t$ , where  $n_t$  is the total of xf[[t]]. Notice that when xf[[t]] contains relative frequencies, its total is 1. That is equivalent to crit=2.

– crit=2 
$$g_k = \frac{1}{T_k} \sum f_t.$$

#### Value

Returns an object of class discdd.predict, that is a list including:

prediction data frame with 3 columns:

- factor giving the group name. The column name is the same as that of the column (q + 1) of x,
- class.known: the prior class of the group if it is available, or NA if not,
- class.predict: the class allocation predicted by the discriminant analysis method. If misclass.ratio = TRUE, the class allocations are computed for all groups. Otherwise (default), they are computed only for the groups whose class is unknown.

distances

matrix with T rows and K columns, of the distances  $(d_{tk})$ :  $d_{tk}$  is the distance between the group t and the class k, computed with the measure given by argument.

proximities

matrix of the proximities (in percents). The proximity of a group t to the class

k is computed as so:  $(1/d_{tk})/\sum_{l=1}^{l=K}(1/d_{tl})$ . the confusion matrix (if misclass.ratio = TRUE)

confusion.mat misclassed

the misclassification ratio (if misclass.ratio = TRUE)

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### References

Rudrauf, J.M., Boumaza, R. (2001). Contribution à l'étude de l'architecture médiévale: les caractéristiques des pierres à bossage des châteaux forts alsaciens, Centre de Recherches Archéologiques médiévales de Saverne, 5, 5-38.

## **Examples**

```
data(castles.dated)
data(castles.nondated)
stones <- rbind(castles.dated$stones, castles.nondated$stones)
periods <- rbind(castles.dated$periods, castles.nondated$periods)
stones$height <- cut(stones$height, breaks = c(19, 27, 40, 71), include.lowest = TRUE)</pre>
```

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```
stones$width <- cut(stones$width, breaks = c(24, 45, 62, 144), include.lowest = TRUE)
stones$edging <- cut(stones$edging, breaks = c(0, 3, 4, 8), include.lowest = TRUE)
stones$boss <- cut(stones$boss, breaks = c(0, 6, 9, 20), include.lowest = TRUE)

castlesfh <- folderh(periods, "castle", stones)

# Default: dist="11", crit=1
discdd.predict(castlesfh, "period")

# With the calculation of the confusion matrix and misclassification ratio discdd.predict(castlesfh, "period", misclass.ratio = TRUE)

# Hellinger distance
discdd.predict(castlesfh, "period", distance = "hellinger")

# crit=2
discdd.predict(castlesfh, "period", crit = 2)</pre>
```

distl2d

 $L^2$  distance between probability densities

## **Description**

 $L^2$  distance between two multivariate (p > 1) or univariate (dimension: p = 1) probability densities, estimated from samples.

## Usage

```
distl2d(x1, x2, method = "gaussiand", check = FALSE, varw1 = NULL, varw2 = NULL)
```

### **Arguments**

x1, x2

the samples from the probability densities (see 12d.

method

string. It can be:

- "gaussiand" if the densities are considered to be Gaussian.
- "kern" if they are estimated using the Gaussian kernel method.

check

logical. When TRUE (the default is FALSE) the function checks if the covariance matrices (if method = "gaussiand") or smoothing bandwidth matrices (if method = "kern") are not degenerate, before computing the inner product.

Notice that if p=1, it checks if the variances or smoothing parameters are not zero.

\_

varw1, varw2 the band

the bandwidths when the densities are estimated by the kernel method (see 12d.

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### **Details**

The function dist12d computes the distance between  $f_1$  and  $f_2$  from the formula

$$||f_1 - f_2||^2 = \langle f_1, f_1 \rangle + \langle f_2, f_2 \rangle - 2 \langle f_1, f_2 \rangle$$

For some information about the method used to compute the  $L^2$  inner product or about the arguments, see 12d.

#### Value

The  $L^2$  distance between the two densities.

Be careful! If check = FALSE and one smoothing bandwidth matrix is degenerate, the result returned can not be considered.

## Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### See Also

matdist12d in order to compute pairwise distances between several densities.

## **Examples**

```
require(MASS)
m1 <- c(0,0)
v1 <- matrix(c(1,0,0,1),ncol = 2)
m2 <- c(0,1)
v2 <- matrix(c(4,1,1,9),ncol = 2)
x1 <- mvrnorm(n = 3,mu = m1,Sigma = v1)
x2 <- mvrnorm(n = 5, mu = m2, Sigma = v2)
distl2d(x1, x2, method = "gaussiand")
distl2d(x1, x2, method = "kern")
distl2d(x1, x2, method = "kern", varw1 = v1, varw2 = v2)</pre>
```

distl2dnorm

 $L^2$  distance between  $L^2$ -normed probability densities

## Description

 $L^2$  distance between two multivariate (p > 1) or univariate (dimension: p = 1)  $L^2$ -normed probability densities, estimated from samples, where a  $L^2$ -normed probability density is the original probability density function divided by its  $L^2$ -norm.

### Usage

```
distl2dnorm(x1, x2, method = "gaussiand", check = FALSE, varw1 = NULL, varw2 = NULL)
```

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### **Arguments**

x1, x2 the samples from the probability densities (see 12d. method string. It can be:

- "gaussiand" if the densities are considered to be Gaussian.
- "kern" if they are estimated using the Gaussian kernel method.

check logical. When TRUE (the default is FALSE) the function checks if the covari-

ance matrices (if method = "gaussiand") or smoothing bandwidth matrices (if method = "kern") are not degenerate, before computing the inner product.

Notice that if p=1, it checks if the variances or smoothing parameters are not zero.

zero.

varw1, varw2 the bandwidths when the densities are estimated by the kernel method (see 12d.

#### **Details**

Given densities  $f_1$  and  $f_2$ , the function dist12dnormpar computes the distance between the  $L^2$ -normed densities  $f_1/||f_1||$  and  $|f_2|||f_2||$ :

$$2-2 < f_1, f_2 > /(||f_1||||f_2||)$$

For some information about the method used to compute the  $L^2$  inner product or about the arguments, see 12d.

#### Value

The  $L^2$  distance between the two  $L^2$ -normed densities.

Be careful! If check = FALSE and one smoothing bandwidth matrix is degenerate, the result returned can not be considered.

## Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

## See Also

dist12d for the distance between two probability densities.

matdistl2dnorm in order to compute pairwise distances between several  $L^2$ -normed densities.

#### **Examples**

```
require(MASS)
m1 <- c(0,0)
v1 <- matrix(c(1,0,0,1),ncol = 2)
m2 <- c(0,1)
v2 <- matrix(c(4,1,1,9),ncol = 2)
x1 <- mvrnorm(n = 3,mu = m1,Sigma = v1)
x2 <- mvrnorm(n = 5, mu = m2, Sigma = v2)
distl2dnorm(x1, x2, method = "gaussiand")
distl2dnorm(x1, x2, method = "kern")
distl2dnorm(x1, x2, method = "kern", varw1 = v1, varw2 = v2)</pre>
```

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distl2dnormpar	$L^2$ distance between $L^2$ -normed Gaussian densities given their parameters
	rameters

## Description

 $L^2$  distance between two multivariate (p>1) or univariate (dimension: p=1)  $L^2$ -normed Gaussian densities, given their parameters (mean vectors and covariance matrices if the densities are multivariate, or means and variances if univariate) where a  $L^2$ -normed probability density is the original probability density function divided by its  $L^2$ -norm.

## Usage

```
distl2dnormpar(mean1, var1, mean2, var2, check = FALSE)
```

# **Arguments**

mean1, mean2 means of the probability densities.

var1, var2 variances (p = 1) or covariance matrices (p > 1) of the probability densities.

check logical. When TRUE (the default is FALSE) the function checks if the covariance

matrices are not degenerate, before computing the inner product. If the variables are univariate, it checks if the variances are not zero.

### **Details**

Given densities  $f_1$  and  $f_2$ , the function dist12dnormpar computes the distance between the  $L^2$ -normed densities  $f_1/||f_1||$  and  $f_2/||f_2||$ :

$$2-2 < f_1, f_2 > /(||f_1||||f_2||)$$

For some information about the method used to compute the  $L^2$  inner product or about the arguments, see 12dpar; the norm ||f|| of the multivariate Gaussian density f is equal to  $(4\pi)^{-p/4}det(var)^{-1/4}$ .

### Value

The  $L^2$  distance between the two  $L^2$ -normed Gaussian densities.

Be careful! If check = FALSE and one variance matrix is degenerated (or one variance is zero if the densities are univariate), the result returned must not be considered.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

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### See Also

dist12dpar for the distance between two probability densities.

matdist12d in order to compute pairwise distances between several densities.

## **Examples**

```
u1 <- c(1,1,1);

v1 <- matrix(c(4,0,0,0,16,0,0,0,25),ncol = 3);

u2 <- c(0,1,0);

v2 <- matrix(c(1,0,0,0,1,0,0,0,1),ncol = 3);

distl2dnormpar(u1,v1,u2,v2)
```

distl2dpar

 $L^2$  distance between Gaussian densities given their parameters

# **Description**

 $L^2$  distance between two multivariate (p > 1) or univariate (dimension: p = 1) Gaussian densities, given their parameters (mean vectors and covariance matrices if the densities are multivariate, or means and variances if univariate).

## Usage

```
distl2dpar(mean1, var1, mean2, var2, check = FALSE)
```

# **Arguments**

mean1, mean2 means of the probability densities.

var1, var2 variances (p = 1) or covariance matrices (p > 1) of the probability densities.

check logical. When TRUE (the default is FALSE) the function checks if the covariance

matrices are not degenerate, before computing the inner product.

If the variables are univariate, it checks if the variances are not zero.

## **Details**

The function dist12dpar computes the distance between two densities, say  $f_1$  and  $f_2$ , from the formula:

$$||f_1 - f_2||^2 = \langle f_1, f_1 \rangle + \langle f_2, f_2 \rangle - 2 \langle f_1, f_2 \rangle$$

.

For some information about the method used to compute the  $L^2$  inner product or about the arguments, see 12dpar.

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#### Value

The  $L^2$  distance between the two densities.

Be careful! If check = FALSE and one variance matrix is degenerated (or one variance is zero if the densities are univariate), the result returned must not be considered.

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### See Also

matdist12d in order to compute pairwise distances between several densities.

### **Examples**

```
u1 <- c(1,1,1);

v1 <- matrix(c(4,0,0,0,16,0,0,0,25),ncol = 3);

u2 <- c(0,1,0);

v2 <- matrix(c(1,0,0,0,1,0,0,0,1),ncol = 3);

distl2dpar(u1,v1,u2,v2)
```

dspg

Diploma x Socio professional group

### **Description**

Contingency tables of the counts of Diploma x Socio professional group of France

### Usage

```
data(dspg)
```

### **Format**

dspg is a list of 7 arrays (each one corresponding to a year: 1968, 1975, 1982, 1990, 1999, 2010, 2015) of 4 rows (each one corresponding to a level of diploma) and 6 columns (each one corresponding to a socio professional group).

```
• csp: Socio professional group
```

• diplome: Diploma

• agri: farmer (agriculteur)

• arti: craftsperson (artisan)

• cadr: senior manager (cadre sup\'erieur)

• pint: middle manager (profession interm\'ediaire)

• empl: employee (employ\'e)

• ouvr: worker (ouvrier)

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bepc: brevetcap: NVQ (cap)bac: baccalaureate

• sup: higher education (sup\'erieur)

## Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

### Source

INSEE. Population active de 25 \'a 54 ans ayant un emploi et ch\\^omeurs par cat\'egorie socioprofessionnelle et dipl\\^ome par commune et d\'epartement (1968 \'a 2015).

# **Examples**

```
data(dspg)
names(dspg)
print(dspg[[1]])
```

dspgd2015

Diploma x Socio professional group by departement in 2015

## **Description**

Contingency tables of the counts of Diploma x Socio professional group by metroplitan France departement in year 2015.

## Usage

```
data(dspgd2015)
```

#### **Format**

dspgd2015 is a list of 96 arrays (each one corresponding to a department, designated by its official geographical code) of 4 rows (each one corresponding to a level of diploma) and 6 columns (each one corresponding to a socio professional group).

• csp: Socio professional group

• diplome: Diploma

• agri: farmer (agriculteur)

• arti: craftsperson (artisan)

• cadr: senior manager (cadre sup\'erieur)

• pint: middle manager (profession interm\'ediaire)

• empl: employee (employ\'e)

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• ouvr: worker (ouvrier)

bepc: brevetcap: NVQ (cap)bac: baccalaureate

• sup: higher education (sup\'erieur)

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

#### Source

www.insee.fr/fr/statistiques/1893185 Population active de 25 \'a 54 ans ayant un emploi et ch\^omeurs par cat\'egorie socioprofessionnelle et dipl\^ome par commune et d\'epartement (1968 \'a 2015).

## **Examples**

```
data(dspgd2015)
names(dspgd2015)
print(dspgd2015[[1]])
```

dstatis.inter

Dual STATIS method (interstructure stage)

### Description

Performs the first stage (interstructure) of the dual STATIS method in order to describe a data folder, consisting of T groups of individuals on which are observed p variables. It returns an object of class dstatis.

# Usage

### **Arguments**

xf object of class folder. Its elements are data frames with p numeric columns. If

there are non numeric columns, there is an error. The  $t^{th}$  element (t = 1, ..., T)

matches with the  $t^{th}$  group.

normed logical. If TRUE (default), the scalar products are normed.

centered logical. If TRUE (default), the scalar products are centered.

data.scaled logical. If TRUE, the data of each group are centered and scaled. The analysis

is then performed on the correlation matrices. If FALSE (default), the analysis is

performed on the covariance matrices.

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numeric. Number of returned principal scores (default nb. factors = 3). nb.factors nb.values numerical. Number of returned eigenvalues (default nb. values = 10). string. If provided, the subtitle for the graphs. sub.title logical. If TRUE (default), the barplot of the eigenvalues is plotted. plot.eigen plot.score logical. If TRUE, the graphs of principal scores are plotted. A new graphic device is opened for each pair of principal scores defined by nscore argument. numeric vector. If plot.score = TRUE, the numbers of the principal scores nscore which are plotted. By default it is equal to nscore = 1:3. Its components cannot be greater than nb. factors. string. Name of the grouping variable. Default: groupname = "group". group.name filename string. Name of the file in which the results are saved. By default (filename = NULL) the results are not saved.

#### **Details**

The covariance matrices (if data.scale is FALSE) or correlation matrices (if TRUE) per group are computed. The matrix W of the scalar products between these covariance matrices is then computed.

To perform the STATIS method, see the function DSTATIS of the multigroup package.

#### Value

Returns an object of class dstatis, that is a list including:

inertia data frame of the eigenvalues and percentages of inertia.

 $contributions \quad data \ frame \ of \ the \ contributions \ to \ the \ first \ nb. \ factors \ principal \ components.$ 

qualities data frame of the qualities on the first nb. factors principal factors.

scores data frame of the first nb. factors scores of the spectral decomposition of W.

norm vector of the  $L^2$  norms of the densities.

means list of the means.

variances list of the covariance matrices.
correlations list of the correlation matrices.
skewness list of the skewness coefficients.
kurtosis list of the kurtosis coefficients.

## Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

## References

Lavit, C., Escoufier, Y., Sabatier, R., Traissac, P. (1994). The ACT (STATIS method). Computational Statistics & Data Analysis, 18 (1994), 97-119.

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## See Also

```
print.dstatis, plot.dstatis, interpret.dstatis. DSTATIS
```

## **Examples**

```
data(roses)
rosesf <- as.folder(roses[,c("Sha","Den","Sym","rose")])

# Dual STATIS on the covariance matrices
result1 <- dstatis.inter(rosesf, data.scaled = FALSE, group.name = "rose")
print(result1)
plot(result1)

# Dual STATIS on the correlation matrices
result2 <- dstatis.inter(rosesf, data.scaled = FALSE, group.name = "rose")
print(result2)
plot(result2)</pre>
```

fdiscd.misclass

Misclassification ratio in functional discriminant analysis of probability densities.

### **Description**

Computes the one-leave-out misclassification ratio of the rule assigning T groups of individuals, one group after another, to the class of groups (among K classes of groups) which achieves the minimum of the distances or divergences between the density function associated to the group to assign and the K density functions associated to the K classes.

# Usage

#### **Arguments**

xf

object of class folderh with two data frames:

- The first one has at least two columns. One column contains the names of the T groups (all the names must be different). An other column is a factor with K levels partitionning the T groups into K classes.
- The second one has (p+1) columns. The first p columns are numeric (otherwise, there is an error). The last column is a factor with T levels defining T groups. Each group, say t, consists of  $n_t$  individuals.

class.var

string. The name of the class variable.

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distance

The distance or dissimilarity used to compute the distance matrix between the densities. It can be:

- "jeffreys" (default) the Jeffreys measure (symmetrised Kullback-Leibler divergence),
- "hellinger" the Hellinger (Matusita) distance,
- "wasserstein" the Wasserstein distance,
- "12" the  $L^2$  distance,
- "12norm" (only available when crit = 1) the densities are normed and the  $L^2$  distance between these normed densities is used;

If gaussiand = FALSE, the densities are estimated by the Gaussian kernel method and the distance is "12" or "12norm".

1, 2 or 3. In order to select the densities associated to the classes. See Details. crit

> If distance is "hellinger", "jeffreys" or "wasserstein", crit is necessarily 1 (see Details).

logical. If TRUE (default), the probability densities are supposed Gaussian. If gaussiand

FALSE, densities are estimated using the Gaussian kernel method.

If distance is "hellinger", "jeffreys" or "wasserstein", gaussiand is

necessarily TRUE.

strictly positive numeric value. If windowh = NULL (default), the bandwidths are

computed using the bandwidth.parameter function.

Omitted when distance is "hellinger", "jeffreys" or "wasserstein" (see Details).

### **Details**

The T probability densities  $f_t$  corresponding to the T groups of individuals are either parametrically estimated (gaussiand = TRUE) or estimated using the Gaussian kernel method (gaussiand = FALSE). In the latter case, the windowh argument provides the list of the bandwidths to be used. Notice that in the multivariate case (p>1), the bandwidths are positive-definite matrices.

The argument windowh is a numerical value, the matrix bandwidth is of the form hS, where Sis either the square root of the covariance matrix (p>1) or the standard deviation of the estimated density.

If windowh = NULL (default), h in the above formula is computed using the bandwidth.parameter

To the class k consisting of  $T_k$  groups is associated the density denoted  $g_k$ . The crit argument selects the estimation method of the K densities  $g_k$ .

- 1. The density  $g_k$  is estimated using the whole data of this class, that is the rows of x corresponding to the  $T_k$  groups of the class k.
  - The estimation of the densities  $g_k$  uses the same method as the estimation of the  $f_t$ .
- 2. The  $T_k$  densities  $f_t$  are estimated using the corresponding data from x. Then they are averaged to obtain an estimation of the density  $g_k$ , that is  $g_k = \frac{1}{T_k} \sum f_t$ .
- 3. Each previous density  $f_t$  is weighted by  $n_t$  (the number of rows of x corresponding to  $f_t$ ). Then they are averaged, that is  $g_k = \frac{1}{\sum n_t} \sum n_t f_t$ .

windowh

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The last two methods are only available for the  $L^2$ -distance. If the divergences between densities are computed using the Hellinger or Wasserstein distance or the Jeffreys measure, only the first of these methods is available.

The distance or dissimilarity between the estimated densities is either the  $L^2$  distance, the Hellinger distance, the Jeffreys measure (symmetrised Kullback-Leibler divergence) or the Wasserstein distance.

- If it is the L^2 distance (distance="12" or distance="12norm"), the densities can be either parametrically estimated or estimated using the Gaussian kernel.
- If it is the Hellinger distance (distance="hellinger"), the Jeffreys measure (distance="jeffreys") or the Wasserstein distance (distance="wasserstein"), the densities are considered Gaussian and necessarily parametrically estimated.

#### Value

Returns an object of class fdiscd.misclass, that is a list including:

classification

data frame with 4 columns:

- factor giving the group name. The column name is the same as that of the column (p + 1) of x,
- the prior class of the group if it is available, or NA if not,
- alloc: the class allocation computed by the discriminant analysis method,
- misclassed: boolean. TRUE if the group is misclassed, FALSE if it is well-classed, NA if the prior class of the group is unknown.

confusion.mat confusion matrix,

misalloc.per.class

the misclassification ratio per class,

misclassed the misclassification ratio,

distances matrix with T rows and K columns, of the distances  $(d_{tk})$ :  $d_{tk}$  is the distance

between the group t and the class k, computed with the measure given by argu-

ment distance ( $L^2$ -distance, Hellinger distance or Jeffreys measure),

proximities matrix of the proximity indices (in percents) between the groups and the classes.

The proximity of the group t to the class k is computed as so:  $(1/d_{tk})/\sum_{l=1}^{l=K} (1/d_{tl})$ .

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### References

Boumaza, R. (2004). Discriminant analysis with independently repeated multivariate measurements: an  $L^2$  approach. Computational Statistics & Data Analysis, 47, 823-843.

Rudrauf, J.M., Boumaza, R. (2001). Contribution? l'?tude de l'architecture m?di?vale: les caract?ristiques des pierres? bossage des ch?teaux forts alsaciens. Centre de Recherches Arch?ologiques M?di?vales de Saverne, 5, 5-38.

fdiscd.predict 63

## **Examples**

```
data(castles.dated)
castles.stones <- castles.dated$stones
castles.periods <- castles.dated$periods
castlesfh <- folderh(castles.periods, "castle", castles.stones)
result <- fdiscd.misclass(castlesfh, "period")
print(result)</pre>
```

fdiscd.predict

Predicting the class of a group of individuals with discriminant analysis of probability densities.

## **Description**

Assigns several groups of individuals, one group after another, to the class of groups (among K classes of groups) which achieves the minimum of the distances or divergences between the density function associated to the group to assign and the K density functions associated to the K classes.

### Usage

### **Arguments**

xf

object of class folderh with two data frames:

- The first one has at least two columns. One column contains the names of the *T* groups (all the names must be different). An other column is a factor with *K* levels partitionning the T groups into K classes..
- The second one has (p+1) columns. The first p columns are numeric (otherwise, there is an error). The last column is a factor with T levels defining T groups. Each group, say t, consists of  $n_t$  individuals.

Notice that for the versions earlier than 2.0, fdiscd.predict applied to two data frames.

class.var

string. The name of the class variable.

distance

The distance or divergence used to compute the distance matrix between the densities. It can be:

- "jeffreys" (default) the Jeffreys measure (symmetrised Kullback-Leibler divergence),
- "hellinger" the Hellinger (Matusita) distance,
- "wasserstein" the Wasserstein distance,
- "12" the  $L^2$  distance,
- "12norm" the densities are normed and the L<sup>2</sup> distance between these normed densities is used;

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If gaussiand = FALSE, the densities are estimated by the Gaussian kernel method and the distance is "12" or "12norm".

crit 1, 2 or 3. In order to select the densities associated to the classes. See Details.

If distance is "hellinger", "jeffreys" or "wasserstein", crit is neces-

sarily 1 (see Details).

gaussiand logical. If TRUE (default), the probability densities are supposed Gaussian. If

FALSE, densities are estimated using the Gaussian kernel method.

If distance is "hellinger", "jeffreys" or "wasserstein", gaussiand is

necessarily TRUE.

windowh strictly positive number. If windowh = NULL (default), the bandwidths are com-

puted using the bandwidth.parameter function.

Omitted when distance is "hellinger", "jeffreys" or "wasserstein" (see

Details).

misclass.ratio logical (default FALSE). If TRUE, the confusion matrix and misclassification ratio

are computed on the groups whose prior class is known. In order to compute the misclassification ratio by the one-leave-out method, use the fdiscd.misclass

function.

#### **Details**

To the group t is associated the density denoted  $f_t$ . To the class k consisting of  $T_k$  groups is associated the density denoted  $g_k$ . The crit argument selects the estimation method of the K densities  $g_k$ .

- 1. The density  $g_k$  is estimated using the whole data of this class, that is the rows of x corresponding to the  $T_k$  groups of the class k.
- 2. The  $T_k$  densities  $f_t$  are estimated using the corresponding data from x. Then they are averaged to obtain an estimation of the density  $g_k$ , that is  $g_k = (1/T_k) \sum f_t$ .
- 3. Each previous density  $f_t$  is weighted by  $n_t$  (the number of rows of x corresponding to  $f_t$ ). Then they are averaged, that is  $g_k = (1/\sum n_t) \sum n_t f_t$ .

The last two methods are available only for the  $L^2$ -distance. If the divergences between densities are computed using the Hellinger or Wasserstein distance or the Jeffreys measure, only the first of these methods is available.

## Value

Returns an object of class fdiscd.predict, that is a list including:

prediction data frame with 3 columns:

- factor giving the group name. The column name is the same as that of the column (p+1) of x,
- class.known: the prior class of the group if it is available, or NA if not,
- class.predict: the class allocation predicted by the discriminant analysis method. If misclass.ratio = TRUE, the class allocations are computed for all groups. Otherwise (default), they are computed only for the groups whose class is unknown.

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distances	matrix with $T$ rows and $K$ columns, of the distances $(d_{tk})$ : $d_{tk}$ is the distance between the group $t$ and the class $k$ , computed with the measure given by argument distance ( $L^2$ -distance, Hellinger distance or Jeffreys measure),
proximities	matrix of the proximities (in percents). The proximity of a group $t$ to the class $k$ is computed as so: $(1/d_{tk})/\sum_{l=1}^{l=K} (1/d_{tl})$ .
confusion.mat	the confusion matrix (if misclass.ratio = TRUE)
misclassed	the misclassification ratio (if misclass.ratio = TRUE)

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

Boumaza, R. (2004). Discriminant analysis with independently repeated multivariate measurements: an  $L^2$  approach. Computational Statistics & Data Analysis, 47, 823-843.

Rudrauf, J.M., Boumaza, R. (2001). Contribution? l'?tude de l'architecture m?di?vale: les caract?ristiques des pierres? bossage des ch?teaux forts alsaciens. Centre de Recherches Arch?ologiques M?di?vales de Saverne, 5, 5-38.

# **Examples**

```
data(castles.dated)
data(castles.nondated)
castles.stones <- rbind(castles.dated$stones, castles.nondated$stones)</pre>
castles.periods <- rbind(castles.dated$periods, castles.nondated$periods)</pre>
castlesfh <- folderh(castles.periods, "castle", castles.stones)</pre>
# With the L^2-distance
# - crit=1
resultl2.1 <- fdiscd.predict(castlesfh, "period", distance="12", crit=1)</pre>
print(result12.1)
# - crit=2
## Not run:
resultl2.2 <- fdiscd.predict(castlesfh, "period", distance="12", crit=2)</pre>
print(result12.2)
## End(Not run)
# - crit=3
result12.3 <- fdiscd.predict(castlesfh, "period", distance="12", crit=3)</pre>
print(result12.3)
# With the Hellinger distance
resulthelling <- fdiscd.predict(castlesfh, "period", distance="hellinger")</pre>
print(resulthelling)
# With the Jeffreys measure
```

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```
resultjeff <- fdiscd.predict(castlesfh, "period", distance="jeffreys")
print(resultjeff)</pre>
```

fhclustd

Hierarchic cluster analysis of probability densities

### **Description**

Performs functional hierarchic cluster analysis of probability densities. It returns an object of class fhclustd. It applies hclust to the distance matrix between the T densities.

## Usage

### **Arguments**

xf

object of class folder. Its elements are data frames with p numeric columns. If there are non numeric columns, there is an error. The  $t^{th}$  element  $(t=1,\ldots,T)$  matches with the  $t^{th}$  group.

gaussiand

logical. If TRUE (default), the probability densities are supposed Gaussian. If FALSE, densities are estimated using the Gaussian kernel method.

If distance is "hellinger", "jeffreys" or "wasserstein", gaussiand is necessarily TRUE (see Details).

distance

The distance or divergence used to compute the distance matrix between the densities. It can be:

- "jeffreys" (default) the Jeffreys measure (symmetrised Kullback-Leibler divergence),
- "hellinger" the Hellinger (Matusita) distance,
- "wasserstein" the Wasserstein distance,
- "12" the  $L^2$  distance,
- "12norm" the densities are normed and the  $L^2$  distance between these normed densities is used;

If gaussiand = FALSE, the densities are estimated by the Gaussian kernel method and the distance can be "12" (default) or "12norm".

windowh

either a list of T bandwidths (one per density associated to a group), or a strictly positive number. If windowh = NULL (default), the bandwidths are automatically computed. See Details.

Omitted when distance is "hellinger", "jeffreys" or "wasserstein" (see Details).

data.centered

logical. If TRUE (default is FALSE), the data of each group are centered.

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data.scaled logical. If TRUE (default is FALSE), the data of each group are centered (even if

data.centered = FALSE) and scaled.

common.variance

logical. If TRUE (default is FALSE), a common covariance matrix (or correlation matrix if data.scaled = TRUE), computed on the whole data, is used. If FALSE

(default), a covariance (or correlation) matrix per group is used.

sub. title string. If provided, the subtitle for the graphs.

filename string. Name of the file in which the results are saved. By default (filename =

NULL) the results are not saved.

method.hclust the agglomeration method to be used for the clustering. See the method argu-

ment of the hclust function.

group.name string. Name of the grouping variable. Default: group.name = "group".

### **Details**

In order to compute the distances/dissimilarities between the groups, the T probability densities  $f_t$  corresponding to the T groups of individuals are either parametrically estimated (gaussiand = TRUE) or estimated using the Gaussian kernel method (gaussiand = FALSE). In the latter case, the windowh argument provides the list of the bandwidths to be used. Notice that in the multivariate case (p>1), the bandwidths are positive-definite matrices. The distances between the T groups of individuals are given by the  $L^2$ -distances between the T probability densities  $f_t$  corresponding to these groups. The hclust function is then applied to the distance matrix to perform the hierarchical clustering on the T groups.

If windowh is a numerical value, the matrix bandwidth is of the form hS, where S is either the square root of the covariance matrix (p>1) or the standard deviation of the estimated density.

If windowh = NULL (default), h in the above formula is computed using the bandwidth.parameter function.

The distance or dissimilarity between the estimated densities is either the  $L^2$  distance, the Hellinger distance, the Jeffreys measure (symmetrised Kullback-Leibler divergence) or the Wasserstein distance.

- If it is the L^2 distance (distance="12" or distance="12norm"), the densities can be either parametrically estimated or estimated using the Gaussian kernel.
- If it is the Hellinger distance (distance="hellinger"), the Jeffreys measure (distance="jeffreys") or the Wasserstein distance (distance="wasserstein"), the densities are considered Gaussian and necessarily parametrically estimated.

### Value

Returns an object of class fhclustd, that is a list including:

distances matrix of the  $L^2$ -distances between the estimated densities.

clust an object of class hclust.

## Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

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### See Also

fdiscd.predict, fdiscd.misclass

# **Examples**

```
data(castles.dated)
stones <- castles.dated$stones</pre>
periods <- castles.dated$periods
periods123 <- periods[periods$period %in% 1:3, "castle"]</pre>
stones123 <- stones[stones$castle %in% periods123, ]</pre>
stones123$castle <- as.factor(as.character(stones123$castle))</pre>
yf <- as.folder(stones123)</pre>
# Jeffreys measure (default):
resultjef <- fhclustd(yf)</pre>
print(resultjef)
print(resultjef, dist.print = TRUE)
plot(resultjef)
plot(resultjef, hang = -1)
# Use cutree (stats package) to get the partition
cutree(resultjef$clust, k = 1:4)
cutree(resultjef$clust, k = 5)
cutree(resultjef$clust, h = 0.041)
# Hellinger distance:
resulthel <- fhclustd(yf, distance = "hellinger")</pre>
print(resulthel)
print(resulthel, dist.print = TRUE)
plot(resulthel)
plot(resulthel, hang = -1)
# Use cutree (stats package) to get the partition
cutree(resulthel$clust, k = 1:4)
cutree(resulthel\$clust, k = 5)
cutree(resulthel$clust, h = 0.041)
## Not run:
# L2-distance:
xf <- as.folder(stones)</pre>
result <- fhclustd(xf, distance = "12")
print(result)
print(result, dist.print = TRUE)
plot(result)
plot(result, hang = -1)
```

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```
# Use cutree (stats package) to get the partition
cutree(result$clust, k = 1:5)
cutree(result$clust, k = 5)
cutree(result$clust, h = 0.18)
## End(Not run)
periods123 <- periods[periods$period %in% 1:3, "castle"]</pre>
stones123 <- stones[stones$castle %in% periods123, ]</pre>
stones123$castle <- as.factor(as.character(stones123$castle))</pre>
yf <- as.folder(stones123)</pre>
result123 <- fhclustd(yf, distance = "12")
print(result123)
print(result123, dist.print = TRUE)
plot(result123)
plot(result123, hang = -1)
# Use cutree (stats package) to get the partition
cutree(result123$clust, k = 1:4)
cutree(result123$clust, k = 5)
cutree(result123$clust, h = 0.041)
```

floribundity

Rose flowering

### **Description**

These data are collected on eight rosebushes from four varieties, during summer 2010 in Angers, France. They give measures of the flowering.

# Usage

```
data("floribundity")
```

### **Format**

floribundity is a list of 16 data frames, each corresponding to an observation date. Each one of these data frames has 3 or 4 columns:

- rose: the number of the rosebush, that is an identifier.
- variety: factor. The variety of the rosebush.
- area (when available): numeric. The ratio of flowering area to the whole plant area, measured on the photograph of the rosebush.
- nflowers (when available): integer. The number of flowers on the rosebush.

The row names of these data frames are the rose identifiers.

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### **Examples**

```
data(floribundity)
foldt <- foldert(floribundity, times = as.Date(names(floribundity)), rows.select = "union")
summary(foldt)</pre>
```

fmdsd

Multidimensional scaling of probability densities

### **Description**

Applies the multidimensional scaling (MDS) method to probability densities in order to describe a data folder, consisting of T groups of individuals on which are observed p variables. It returns an object of class fmdsd. It applies cmdscale to the distance matrix between the T densities.

### Usage

### Arguments

xf

object of class folder. Its elements are data frames with p numeric columns. If there are non numeric columns, there is an error. The  $t^{th}$  element  $(t=1,\ldots,T)$  matches with the  $t^{th}$  group.

Notice that for the versions earlier than 2.0, fmdsd applied to a data frame.

gaussiand

logical. If TRUE (default), the probability densities are supposed Gaussian. If FALSE, densities are estimated using the Gaussian kernel method.

distance

The distance or divergence used to compute the distance matrix between the densities

If gaussiand = TRUE, the densities are parametrically estimated and the distance can be:

- "jeffreys" (default) the Jeffreys measure (symmetrised Kullback-Leibler divergence),
- "hellinger" the Hellinger (Matusita) distance,
- "wasserstein" the Wasserstein distance.
- "12" the  $L^2$  distance,
- "12norm" the densities are normed and the L<sup>2</sup> distance between these normed densities is used;

If gaussiand = FALSE, the densities are estimated by the Gaussian kernel method and the distance can be "12" (default) or "12norm".

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windowh either a list of T bandwidths (one per density associated to a group), or a strictly

positive number. If windowh = NULL (default), the bandwidths are automatically

computed. See Details.

Omitted when distance is "hellinger", "jeffreys" or "wasserstein" (see

Details).

data.centered logical. If TRUE (default is FALSE), the data of each group are centered.

data.scaled logical. If TRUE (default is FALSE), the data of each group are centered (even if

data.centered = FALSE) and scaled.

common.variance

logical. If TRUE (default is FALSE), a common covariance matrix (or correlation

matrix if data.scaled = TRUE), computed on the whole data, is used. If FALSE

(default), a covariance (or correlation) matrix per group is used.

add logical indicating if an additive constant should be computed and added to the

non diagonal dissimilarities such that the modified dissimilarities are Euclidean

(default TRUE; see add argument of cmdscale).

nb. factors numeric. Number of returned principal coordinates (default nb. factors = 3).

Warning: The plot.fmdsd and interpret.fmdsd functions cannot take into

account more than nb. factors principal factors.

nb.values numeric. Number of returned eigenvalues (default nb.values = 10).

sub. title string. Subtitle for the graphs (default NULL).

plot.eigen logical. If TRUE (default), the barplot of the eigenvalues is plotted.

plot.score logical. If TRUE, the graphs of new coordinates are plotted. A new graphic device

is opened for each pair of coordinates defined by nscore argument.

nscore numeric vector. If plot.score = TRUE, the numbers of the principal coordinates

which are plotted. By default it is equal to nscore = 1:3. Its components cannot

be greater than nb. factors.

filename string. Name of the file in which the results are saved. By default (filename =

NULL) they are not saved.

group.name string. Name of the grouping variable. Default: groupname = "group".

# **Details**

In order to compute the distances/dissimilarities between the groups, the T probability densities  $f_t$  corresponding to the T groups of individuals are either parametrically estimated (gaussiand = TRUE) or estimated using the Gaussian kernel method (gaussiand = FALSE). In the latter case, the windowh argument provides the list of the bandwidths to be used. Notice that in the multivariate case (p>1), the bandwidths are positive-definite matrices.

If windowh is a numerical value, the matrix bandwidth is of the form hS, where S is either the square root of the covariance matrix (p>1) or the standard deviation of the estimated density.

If windowh = NULL (default), h in the above formula is computed using the bandwidth.parameter function.

The distance or dissimilarity between the estimated densities is either the  $L^2$  distance, the Hellinger distance, the Jeffreys measure (symmetrised Kullback-Leibler divergence) or the Wasserstein distance.

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• If it is the L^2 distance (distance="12" or distance="12norm"), the densities can be either parametrically estimated or estimated using the Gaussian kernel.

• If it is the Hellinger distance (distance="hellinger"), the Jeffreys measure (distance="jeffreys") or the Wasserstein distance (distance="wasserstein"), the densities are considered Gaussian and necessarily parametrically estimated.

#### Value

Returns an object of class fmdsd, i.e. a list including:

inertia data frame of the eigenvalues and percentages of inertia.
scores data frame of the nb. factors first principal coordinates.

means list of the means.

variances list of the covariance matrices.
correlations list of the correlation matrices.
skewness list of the skewness coefficients.
kurtosis list of the kurtosis coefficients.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

Boumaza, R., Yousfi, S., Demotes-Mainard, S. (2015). Interpreting the principal component analysis of multivariate density functions. Communications in Statistics - Theory and Methods, 44 (16), 3321-3339.

Delicado, P. (2011). Dimensionality reduction when data are density functions. Computational Statistics & Data Analysis, 55, 401-420.

Yousfi, S., Boumaza, R., Aissani, D., Adjabi, S. (2014). Optimal bandwith matrices in functional principal component analysis of density function. Journal of Statistical Computation and Simulation, 85 (11), 2315-2330.

Cox, T.F., Cox, M.A.A. (2001). Multimensional Scaling, second ed. Chapman & Hall/CRC.

# See Also

fpcad print.fmdsd, plot.fmdsd, interpret.fmdsd, bandwidth.parameter

## **Examples**

```
data(roses)
rosesf <- as.folder(roses[,c("Sha","Den","Sym","rose")])
# MDS on Gaussian densities (on sensory data)
# using the Jeffreys measure (default):
resultjeff <- fmdsd(rosesf, distance = "jeffreys")</pre>
```

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```
print(resultjeff)
plot(resultjeff)
# using the Hellinger distance:
resulthellin <- fmdsd(rosesf, distance = "hellinger")</pre>
print(resulthellin)
plot(resulthellin)
# using the Wasserstein distance:
resultwass <- fmdsd(rosesf, distance = "wasserstein")</pre>
print(resultwass)
plot(resultwass)
# Gaussian case, using the L2-distance:
resultl2 <- fmdsd(rosesf, distance = "12")</pre>
print(result12)
plot(result12)
# Gaussian case, using the L2-distance between normed densities:
resultl2norm <- fmdsd(rosesf, distance = "l2norm")</pre>
print(resultl2norm)
plot(resultl2norm)
## Not run:
# Non Gaussian case, using the L2-distance,
# the densities are estimated using the Gaussian kernel method:
result <- fmdsd(rosesf, distance = "12", gaussiand = FALSE, group.name = "rose")
print(result)
plot(result)
## End(Not run)
```

folder

Folder of data sets

### **Description**

Creates an object of class "folder" (called folder below), that is a list of data frames with the same column names. Thus, these data sets are on the same variables. They can be on the same individuals or not.

### Usage

```
folder(x1, x2 = NULL, ..., cols.select = "intersect", rows.select = "")
```

# Arguments

x1

data frame (can also be a tibble) or list of data frames.

• If x1 is a data frame, x2 must be provided.

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• If x1 is a list of data frames, its elements are the datasets of the folder. In this case, there is no x2 argument.

x2 data frame. Must be provided if x1 is a data frame.

.. optional. One or several data frames. When x1 and x2 are data frames, these are the other data frames.

cols.select

string. Gives the method used to choose the column names of the data frames of the folder. This argument can be:

"intersect" (default) the column names of the data frames in the folder are the intersection of the column names of all the data frames given as arguments.

"union" the column names of the data frames in the folder are the union of the column names of all the data frames given as arguments. When necessary, the rows of the returned data frames are completed by NA. If cols.select is a character vector, it gives the column names selected in the data frames given as arguments. The corresponding columns constitute the columns of the elements of the returned folder. Notice that when a column name is not present in all data frames (given as arguments), the data are completed by NA.

rows.select

string. Gives the method used to choose the row names of the data frames of the folder. This argument can be:

"" (default) the data frames of the folder have the same rows as those which were passed as arguments.

"intersect" the row names of the data frames in the folder are the intersection of the row names of all the data frames given as arguments.

"union" the row names of the data frames in the folder are the union of the row names of all the data frames given as arguments. When necessary, the columns of the data frames returned are completed by NA.

### Details

The class folder has a logical attributes attr(, "same.rows").

The data frames in the returned folder all have the same column names. That means that the same variables are observed in every data sets.

If the rows. select argument is "union" or "intersect", the elements of the returned folder have the same rows. That means that the same individuals are present in every data sets. This allows to consider the evolution of each individual among time.

If rows.select is "", every rows of this folder are different, and the row names are made unique by adding the name of the data frame to the row names. In this case, The individuals of the data sets are assumed to be all different. Or, at least, the user does not mind if they are the same or not.

#### Value

Returns an object of class "folder", that is a list of data frames.

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

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### See Also

is. folder to test if an object is of class folder. folderh to build a folder of several data frames with a hierarchic relation between each pair of consecutive data frames.

### **Examples**

```
# First example
x1 \leftarrow data.frame(x = rnorm(10), y = 1:10)
x2 \leftarrow data.frame(x = rnorm(10), z = runif(10, 1, 10))
f1 \leftarrow folder(x1, x2)
print(f1)
f2 <- folder(x1, x2, cols.select = "union")</pre>
print(f2)
#Second example
data(iris)
iris.set <- iris[iris$Species == "setosa", 1:4]</pre>
iris.ver <- iris[iris$Species == "versicolor", 1:4]</pre>
iris.vir <- iris[iris$Species == "virginica", 1:4]</pre>
irisf1 <- folder(iris.set, iris.ver, iris.vir)</pre>
print(irisf1)
listofdf <- list(df1 = iris.set,df2 = iris.ver,df3 = iris.vir)</pre>
irisf2 <- folder(listofdf,x2 = NULL)</pre>
print(irisf2)
```

folderh

Hierarchic folder of n data frames related in pairs by (n-1) keys

### **Description**

Creates an object of class folderh, that is a list of n > 1 data frames whose rows are related by (n-1) keys, each key defining a relation "1 to N" between the two adjacent data frames passed as arguments of the function.

#### Usage

```
folderh(df1, key1, df2, ..., na.rm = TRUE)
```

### **Arguments**

df1	data frame (can also be a tibble) with at least two columns. It contains a factor (whose name is given by key1 argument) whose levels are taken exactly once.
key1	character string. The name of the factor of the data frames df1 and df2 which contains the key of the relations "1 to $N$ " between the two datasets.
df2	data frame (or tibble) with at least two columns. It contains a factor column (named by keys argument) with the same levels as df1[,key1] (see Details).

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optional. One or several supplementary character strings and data frames, ordered as follows: key2,df3,.... The argument key2 indicates the key defining the relation "1 to N" between the data frames df2 and df3, and so on.

logical. If TRUE, the rows of each data frame for which the key is NA are removed.

#### **Details**

The object of class folderh is a list of  $n \geq 2$  data frames.

- If no optional arguments are given via ..., that is n=2, the two data frames of the list have a column named by the attribute  $\operatorname{attr}(, \text{"keys"})$  (argument key1), which is a factor with the same levels. Each one of these levels occur exactly once in the first data frame of the list.
- If some supplementary data frames and supplementary strings key2,df3, ... are given as optional arguments, n is the number of data frames given as arguments. Then, the attribute attr(,"keys") is a vector of n-1 character strings. For  $i=1,\ldots,N-1$ , its i-th element is the name of a column of the i-th and (i+1)-th data frames of the folderh, which are factors with the same levels. Each one of these levels occur exactly once in the i-th data frame.

If there are more than two data frames, folderh computes a folderh with the two last data frames, and then uses the function appendtofolderh to append each one of the other data frames to the folderh.

#### Value

Returns an object of class folderh. Its elements are the data frames passed as arguments, and the attribute attr(, "keys") contains the character arguments.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### See Also

is.folderh to test if an object is of class folderh. folder for a folder of data frames with no hierarchic relation between them. as.folder.folderh (or as.data.frame.folderh) to build an object of class folder (or a data frame) from an object of class folderh,

```
# First example: rose flowers
data(roseflowers)
df1 <- roseflowers$variety
df2 <- roseflowers$flower
fh1 <- folderh(df1, "rose", df2)
print(fh1)

# Second example
data(roseleaves)
roses <- roseleaves$rose
stems <- roseleaves$stem
leaves <- roseleaves$leaf</pre>
```

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```
leaflets <- roseleaves$leaflet
fh2 <- folderh(roses, "rose", stems, "stem", leaves, "leaf", leaflets)
print(fh2)</pre>
```

foldermtg

foldermtg

### **Description**

An object of S3 class "foldermtg" is built and returned by the function read.mtg.

### Value

An object of this S3 class is a list of at least 5 data frames (see the Value section in read.mtg): classes, description, features, topology, coordinates...

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### References

```
Cokelaer, T. and Pradal, C. (2010). MTG user guide
```

#### See Also

```
read.mtg print.foldermtg mtgorder
```

```
mtgfile1 <- system.file("extdata/plant1.mtg", package = "dad")
x1 <- read.mtg(mtgfile1)
print(x1)

mtgfile2 <- system.file("extdata/plant2.mtg", package = "dad")
x2 <- read.mtg(mtgfile2)
print(x2)</pre>
```

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foldert

Folder of data sets among time

### **Description**

Creates an object of class "foldert" (called foldert below), that is a list of data frames, each of them corresponding to a time of observation. These data sets are on the same variables. They can be on the same individuals or not.

### Usage

```
foldert(x1, x2 = NULL, ..., times = NULL, cols.select = "intersect", rows.select = "")
```

### **Arguments**

x1 data frame (can also be a tibble) or list of data frames.

- If x1 is a data frame, x2 must be provided.
- If x1 is a list of data frames, its elements are the datasets of the folder. In this case, there is no x2 argument.

x2 data frame. Must be provided if x1 is a data frame. Omitted if x1 is a list of data frames.

optional. One or several data frames when x1 is a data frame. These supplementary data frames are added to the list of data frames constituting the returned foldert.

Vector of the "times" of observations. It can be either numeric, or an ordered factor or an object of class "Date", "POSIXlt" or "POSIXct". If omitted, it is 1:N where N is the number of data frame arguments (if x1 is a data frame) or the length of x1 (if it is a list).

So there is an order relationship between these times.

string or character vector. Gives the method used to choose the column names of the data frames of the foldert. This argument can be:

"intersect" (default) the column names of the data frames in the foldert are the intersection of the column names of all the data frames given as arguments.

"union" the column names of the data frames in the foldert are the union of the column names of all the data frames given as arguments. When necessary, the rows of the returned data frames are completed by NA.

If cols.select is a character vector, it gives the column names selected in the data frames given as arguments. The corresponding columns constitute the columns of the elements of the returned foldert. Notice that when a column name is not present in all data frames (given as arguments), the data are completed by NA.

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• • •

times

cols.select

foldert 79

rows.select

string. Gives the method used to choose the row names of the data frames of the foldert. This argument can be:

"" (default) the data frames of the foldert have the same rows as those which were passed as arguments.

"intersect" the row names of the data frames in the foldert are the intersection of the row names of all the data frames given as arguments.

"union" the row names of the data frames in the foldert are the union of the row names of all the data frames given as arguments. When necessary, the columns of the data frames returned are completed by NA.

#### **Details**

The class "foldert" has an attribute attr(, "times") (the times argument, when provided) and a logical attributes attr(, "same.rows").

The data frames in the returned foldert all have the same column names. That means that the same variables are observed in every data sets.

If the rows. select argument is "union" or "intersect", the elements of the returned foldert have the same rows. That means that the same individuals are present in every data sets. This allows to consider the evolution of each individual among time.

If rows.select is "", every rows of this foldert are different, and the row names are made unique by adding the name of the data frame to the row names. In this case, The individuals of the data sets are assumed to be all different. Or, at least, the user does not mind if they are the same or not.

#### Value

Returns an object of class "foldert", that is a list of data frames. The elements of this list are ordered according to time.

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### See Also

is.foldert to test if an object is of class foldert. as.foldert.data.frame: build an object of class foldert from a data frame. as.foldert.array: build an object of class foldert from a 3d-array.

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```
row.names = c(paste0("i", c(1, 2, 4, 6))),
                stringsAsFactors = TRUE)
z \leftarrow data.frame(xyz = c("A", "B", "C"),
                z1 = rnorm(3),
                row.names = c("i1", "i2", "i5"),
                stringsAsFactors = TRUE)
# Columns selected by the user
ftc. <- foldert(x, y, z, cols.select = c("xyz", "x1", "y1", "z1"))
print(ftc.)
# cols.select = "union": all the variables (columns) of each data frame are kept
ftcun <- foldert(x, y, z, cols.select = "union")</pre>
print(ftcun)
# cols.select = "intersect": only variables common to all data frames
ftcint <- foldert(x, y, z, cols.select = "intersect")</pre>
print(ftcint)
# rows.select = "": the rows of the data frames are unchanged
# and the rownames are made unique
ftr. <- foldert(x, y, z, rows.select = "")</pre>
print(ftr.)
# rows.select = "union": all the individuals (rows) of each data frame are kept
ftrun <- foldert(x, y, z, rows.select = "union")</pre>
print(ftrun)
# rows.select = "intersect": only individuals common to all data frames
ftrint <- foldert(x, y, z, rows.select = "intersect")</pre>
print(ftrint)
# Define the times (times argument)
ftimes <- foldert(x, y, z, times = as.Date(c("2018-03-01", "2018-04-01", "2018-05-01")))
print(ftimes)
```

fpcad

Functional PCA of probability densities

### Description

Performs functional principal component analysis of probability densities in order to describe a data folder, consisting of T groups of individuals on which are observed p variables. It returns an object of class fpcad.

#### Usage

```
fpcad(xf, gaussiand = TRUE, windowh = NULL, normed = TRUE,
    centered = TRUE, data.centered = FALSE, data.scaled = FALSE,
    common.variance = FALSE, nb.factors = 3, nb.values = 10, sub.title = "",
```

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```
plot.eigen = TRUE, plot.score = FALSE, nscore = 1:3, group.name = "group",
filename = NULL)
```

# Arguments

xf	object of class folder. Its elements are data frames with $p$ numeric columns. If there are non numeric columns, there is an error. The $t^{th}$ element $(t=1,\ldots,T)$ matches with the $t^{th}$ group.  Notice that for the versions earlier than 2.0, fpcad applied to a data frame.
gaussiand	logical. If TRUE (default), the probability densities are supposed Gaussian. If FALSE, densities are estimated using the Gaussian kernel method.
windowh	either a list of $T$ bandwidths (one per density associated to a group), or a strictly positive number. If windowh = NULL (default), the bandwidths are automatically computed. See Details.
normed	logical. If TRUE (default), the densities are normed before computing the distances.
centered	logical. If TRUE (default), the densities are centered.
data.centered	logical. If TRUE (default is FALSE), the data of each group are centered.
data.scaled	logical. If TRUE (default is FALSE), the data of each group are centered (even if data.centered = FALSE) and scaled.
common.varianc	
	logical. If TRUE (default is FALSE), a common covariance matrix (or correlation matrix if data.scaled = TRUE), computed on the whole data, is used. If FALSE (default), a covariance (or correlation) matrix per group is used.
nb.factors	numeric. Number of returned principal scores (default nb.factors = 3).
	Warning: The plot.fpcad and interpret.fpcad functions cannot take into account more than nb.factors principal factors.
nb.values	numerical. Number of returned eigenvalues (default nb.values = 10).
sub.title	string. If provided, the subtitle for the graphs.
plot.eigen	logical. If TRUE (default), the barplot of the eigenvalues is plotted.
plot.score	logical. If TRUE, the graphs of principal scores are plotted. A new graphic device is opened for each pair of principal scores defined by nscore argument.
nscore	numeric vector. If plot.score = TRUE, the numbers of the principal scores which are plotted. By default it is equal to nscore = 1:3. Its components cannot be greater than nb.factors.
group.name	string. Name of the grouping variable. Default: groupname = "group".
filename	string. Name of the file in which the results are saved. By default (filename = NULL) the results are not saved.

# **Details**

The T probability densities  $f_t$  corresponding to the T groups of individuals are either parametrically estimated (gaussiand = TRUE) or estimated using the Gaussian kernel method (gaussiand =

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FALSE). In the latter case, the windowh argument provides the list of the bandwidths to use. Notice that in the multivariate case (p>1) the bandwidths are positive-definite matrices.

If windowh is a numerical value, the matrix bandwidth is of the form hS, where S is either the square root of the covariance matrix (p>1) or the standard deviation of the estimated density.

If windowh = NULL (default), h in the above formula is computed using the bandwidth.parameter function.

#### Value

Returns an object of class fpcad, that is a list including:

inertia data frame of the eigenvalues and percentages of inertia.

contributions data frame of the contributions to the first nb. factors principal components.

qualities data frame of the qualities on the first nb. factors principal factors.

scores data frame of the first nb. factors principal scores.

norm vector of the  $L^2$  norms of the densities.

means list of the means.

variances list of the covariance matrices.
correlations list of the correlation matrices.
skewness list of the skewness coefficients.
kurtosis list of the kurtosis coefficients.

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

Boumaza, R. (1998). Analyse en composantes principales de distributions gaussiennes multidimensionnelles. Revue de Statistique Appliqu?e, XLVI (2), 5-20.

Boumaza, R., Yousfi, S., Demotes-Mainard, S. (2015). Interpreting the principal component analysis of multivariate density functions. Communications in Statistics - Theory and Methods, 44 (16), 3321-3339.

Delicado, P. (2011). Dimensionality reduction when data are density functions. Computational Statistics & Data Analysis, 55, 401-420.

Yousfi, S., Boumaza, R., Aissani, D., Adjabi, S. (2014). Optimal bandwith matrices in functional principal component analysis of density functions. Journal of Statistical Computation and Simulation, 85 (11), 2315-2330.

#### See Also

print.fpcad, plot.fpcad, interpret.fpcad, bandwidth.parameter

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### **Examples**

```
# Case of a normed non-centred PCA of Gaussian densities (on 3 architectural
# characteristics of roses: shape (Sha), foliage density (Den) and symmetry (Sym))
rosesf <- as.folder(roses[,c("Sha","Den","Sym","rose")])</pre>
result3 <- fpcad(rosesf, group.name = "rose")</pre>
print(result3)
plot(result3)
# Flower colors of the roses
scores <- result3$scores</pre>
scores <- data.frame(scores, color = scores$rose, stringsAsFactors = TRUE)</pre>
colours <- scores$rose</pre>
colours <- factor(c(A = "yellow", B = "yellow", C = "pink", D = "yellow", E = "red",</pre>
                   F = "yellow", G = "pink", H = "pink", I = "yellow", J = "yellow"))
levels(scores$color) <- c(A = "yellow", B = "yellow", C = "pink", D = "yellow", E = "red",</pre>
                        F = "yellow", G = "pink", H = "pink", I = "yellow", J = "yellow")
# Scores according to the first two principal components, per color
plot(result3, nscore = 1:2, color = colours)
```

fpcat

Functional PCA of probability densities among time

### Description

Performs functional principal component analysis of probability densities in order to describe a data "foldert", consisting of individuals on which are observed p variables on T times. It returns an object of class fpcat.

#### Usage

```
fpcat(xf, gaussiand = TRUE, windowh = NULL, normed=TRUE,
    centered=TRUE, data.centered = FALSE, data.scaled = FALSE,
    common.variance = FALSE, nb.factors = 3, nb.values = 10, sub.title = "",
    plot.eigen = TRUE, plot.score = FALSE, nscore = 1:3, group.name="time",
    filename = NULL)
```

#### **Arguments**

xf

object of class foldert that is a list of data frames with the same column names, each of them corresponding to a time of observation. Its elements are data frames with p numeric columns. If there are non numeric columns, there is an error. The  $t^{th}$  element  $(t=1,\ldots,T)$  matches with the  $t^{th}$  time of observation.

All other arguments are the same as for fpcad, except group.name.

gaussiand

logical. If TRUE (default), the probability densities are supposed Gaussian. If FALSE, densities are estimated using the Gaussian kernel method (as fpcad).

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windowh either a list of T bandwidths (one per density associated to a group), or a strictly

positive number. If windowh = NULL (default), the bandwidths are automatically

computed (as fpcad). See Details.

normed logical. If TRUE (default), the densities are normed before computing the dis-

tances (as fpcad).

centered logical. If TRUE (default), the densities are centered (as fpcad).

data.centered logical. If TRUE (default is FALSE), the data of each group are centered (as

fpcad).

data.scaled logical. If TRUE (default is FALSE), the data of each group are centered (even if

data.centered = FALSE) and scaled (as fpcad).

common.variance

logical. If TRUE (default is FALSE), a common covariance matrix (or correlation matrix if data.scaled = TRUE), computed on the whole data, is used. If FALSE (default), a covariance (or correlation) matrix per group is used (as fpcad).

nb.factors numeric. Number of returned principal scores (default nb.factors = 3) (as

fpcad).

Warning: The plot.fpcad and interpret.fpcad functions cannot take into

account more than nb. factors principal factors (as fpcad).

nb. values numerical. Number of returned eigenvalues (default nb. values = 10) (as fpcad).

sub. title string. Subtitle for the graphs (default NULL) (as fpcad).

plot.eigen logical. If TRUE (default), the barplot of the eigenvalues is plotted (as fpcad).

plot.score logical. If TRUE, the graphs of principal scores are plotted. A new graphic

device is opened for each pair of principal scores defined by nscore argument

(as fpcad).

nscore numeric vector. If plot.score = TRUE, the numbers of the principal scores

which are plotted. By default it is equal to nscore = 1:3. Its components cannot

be greater than nb. factors (as fpcad).

group.name name of the grouping variable, that is the observation times. Default: "time".

filename string. Name of the file in which the results are saved. By default (filename =

NULL) the results are not saved (as fpcad).

## **Details**

The T probability densities  $f_t$  corresponding to the T times of observation are either parametrically estimated or estimated using the Gaussian kernel method (see fpcad for the use of the arguments indicating the method used to estimate these densities).

### Value

Returns an object of class fpcat, that is a list including:

times vector of the times of observation.

inertia data frame of the eigenvalues and percentages of inertia.

contributions data frame of the contributions to the first nb. factors principal components.

fpcat 85

qualities data frame of the qualities on the first nb. factors principal factors.

scores data frame of the first nb. factors principal scores.

norm vector of the  $L^2$  norms of the densities.

means list of the means.

variances list of the covariance matrices.
correlations list of the correlation matrices.
skewness list of the skewness coefficients.
kurtosis list of the kurtosis coefficients.

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### References

Boumaza, R. (1998). Analyse en composantes principales de distributions gaussiennes multidimensionnelles. Revue de Statistique Appliqu?e, XLVI (2), 5-20.

Boumaza, R., Yousfi, S., Demotes-Mainard, S. (2015). Interpreting the principal component analysis of multivariate density functions. Communications in Statistics - Theory and Methods, 44 (16), 3321-3339.

Delicado, P. (2011). Dimensionality reduction when data are density functions. Computational Statistics & Data Analysis, 55, 401-420.

Yousfi, S., Boumaza, R., Aissani, D., Adjabi, S. (2014). Optimal bandwith matrices in functional principal component analysis of density functions. Journal of Statistical Computation and Simulation, 85 (11), 2315-2330.

#### See Also

print.fpcat, plot.fpcat, bandwidth.parameter

```
times <- as.Date(c("2017-03-01", "2017-04-01", "2017-05-01", "2017-06-01"))
x1 <- data.frame(z1=rnorm(6,1,5), z2=rnorm(6,3,3))
x2 <- data.frame(z1=rnorm(6,4,6), z2=rnorm(6,5,2))
x3 <- data.frame(z1=rnorm(6,7,2), z2=rnorm(6,8,4))
x4 <- data.frame(z1=rnorm(6,9,3), z2=rnorm(6,10,2))
ft <- foldert(x1, x2, x3, x4, times = times, rows.select="intersect")
print(ft)
result <- fpcat(ft)
print(result)
plot(result)</pre>
```

86 getcol.folder

getcol.folder

Select columns in all elements of a folder

### **Description**

Select columns in all data frames of a folder.

### Usage

```
getcol.folder(object, name)
```

### **Arguments**

object of class folder that is a list of data frames with the same column names.

name character vector. The names of the columns to be selected in each data frame of

the folder.

#### Value

A folder with the same number of elements as object. Its  $k^{th}$  element is a data frame, and its columns are the columns of object[[k]] given by name.

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### See Also

```
folder: object of class folder.

rmcol.folder: remove columns in all elements of a folder.

getrow.folder: select rows in all elements of a folder.

rmrow.folder: remove rows in all elements of a folder.
```

```
data(iris)
iris.fold <- as.folder(iris, "Species")
getcol.folder(iris.fold, "Sepal.Length")
getcol.folder(iris.fold, c("Petal.Length", "Petal.Width"))</pre>
```

getcol.foldert 87

getcol	tοl	dert

Select columns in all elements of a foldert

### **Description**

Select columns in all data frames of a foldert.

### Usage

```
getcol.foldert(object, name)
```

### Arguments

object of class foldert that is a list of data frames with the same column names,

each of them corresponding to a time of observation.

name character vector. The names of the columns to be selected in each data frame of

the foldert.

#### Value

A foldert with the same number of elements as object. Its  $k^{th}$  element is a data frame, and its columns are the columns of object[[k]] given by name.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### See Also

```
foldert: object of class foldert.

rmcol.foldert: remove columns in all elements of a foldert.

getrow.foldert: select rows in all elements of a foldert.

rmrow.foldert: remove rows in all elements of a foldert.
```

```
data(floribundity)
ft0 <- foldert(floribundity, cols.select = "union")
getcol.foldert(ft0, c("rose", "variety"))</pre>
```

88 getrow.folder

getrow.folder

Select rows in all elements of a folder

### **Description**

Select rows in all data frames of a folder.

# Usage

```
getrow.folder(object, name)
```

## **Arguments**

object of class folder that is a list of data frames with the same column names.

name character vector. The names of the rows to be selected in each data frame of the

folder.

#### Value

A folder with the same number of elements as object. Its  $k^{th}$  element is a data frame, and its rows are the rows of object[[k]] given by name.

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### See Also

```
folder: object of class folder.

rmrow.folder: remove rows in all elements of a folder.

getcol.folder: select rows in all elements of a folder.

rmcol.folder: remove rows in all elements of a folder.
```

```
data(iris)
iris.fold <- as.folder(iris, "Species")
getrow.folder(iris.fold, c(1:5, 51:55, 101:105))</pre>
```

getrow.foldert 89

getrow.	fo1	dert
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Select rows in all elements of a foldert

### **Description**

Select rows in all data frames of a foldert.

### Usage

```
getrow.foldert(object, name)
```

### **Arguments**

object of class foldert that is a list of data frames with the same column names,

each of them corresponding to a time of observation.

name character vector. The names of the rows to be selected in each data frame of the

foldert.

#### Value

A foldert with the same number of elements as object. Its  $k^{th}$  element is a data frame, and its rows are the rows of object[[k]] given by name.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### See Also

```
foldert: object of class foldert.

rmrow.foldert: remove rows in all elements of a foldert.

getcol.foldert: select columns in all elements of a foldert.

rmcol.foldert: remove columns in all elements of a foldert.
```

```
data(floribundity)
ft0 <- foldert(floribundity, cols.select = "union", rows.select = "union")
getrow.foldert(ft0, c("16", "51"))</pre>
```

90 hclustdd

hclustdd

Hierarchic cluster analysis of discrete probability distributions

### **Description**

Performs functional hierarchic cluster analysis of discrete probability distributions. It returns an object of class helustdd. It applies helust to the distance matrix between the T distributions.

### Usage

### Arguments

xf

object of class folder, or list of arrays (or tables).

If it is a folder, its elements are data frames with q numeric columns. The  $t^{th}$  element (t=1,...,T) matches with the  $t^{th}$  group.

If it is a list of arrays (or tables), the  $t^{th}$  element  $(t=1,\ldots,T)$  is the table of the joint distribution (absolute or relative frequencies) of the  $t^{th}$  group. These arrays have the same shape, and their elements are non-negative numbers (if they are not, there is an error).

distance

The distance or divergence used to compute the distance matrix between the discrete distributions (see Details). It can be:

- "11" (default) the  $L^p$  distance with p=1
- "12" the  $L^p$  distance with p=2
- "chisqsym" the symmetric Chi-squared distance
- "hellinger" the Hellinger metric (Matusita distance)
- "jeffreys" the Jeffreys distance (symmetrised Kullback-Leibler divergence)
- "jensen" the Jensen-Shannon distance
- "lp" the  $L^p$  distance with p given by the argument p of the function.

sub.title

string. If provided, the subtitle for the graphs.

filename

string. Name of the file in which the results are saved. By default (filename = NULL) the results are not saved.

method.hclust

the agglomeration method to be used for the clustering. See the method argument of the hclust function.

group.name

string. Name of the grouping variable. Default: group.name = "group".

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#### **Details**

In order to compute the distances/dissimilarities between the groups, the T probability distributions  $f_t$  corresponding to the T groups of individuals are estimated from observations. Then the distances/dissimilarities between the estimated distributions are computed, using the distance or divergence defined by the distance argument:

If the distance is "11", "12" or "1p", the distances are computed by the function matddlppar. Otherwise, it can be computed by matddchisqsympar ("chisqsym"), matddhellingerpar ("hellinger"), matddjeffreyspar ("jeffreys") or matddjensenpar ("jensen").

### Value

Returns an object of class helustdd, that is a list including:

```
distances matrix of the L^2-distances between the estimated densities. clust an object of class helust.
```

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### See Also

hclustdd

```
# Example 1 with a folder (10 groups) of 3 factors
# obtained by converting numeric variables
data(roses)
xr = roses[,c("Sha", "Den", "Sym", "rose")]
xr$Sha = cut(xr$Sha, breaks = c(0, 5, 7, 10))
xr$Den = cut(xr$Den, breaks = c(0, 4, 6, 10))
xr$Sym = cut(xr$Sym, breaks = c(0, 6, 8, 10))
xf = as.folder(xr, groups = "rose")
a = hclustdd(xf)
print(a)
print(a, dist.print = TRUE)
plot(a)
plot(a, hang = -1)
# Example 2 with a list of 7 arrays
data(dspg)
x1 = dspg
hclustdd(x1)
```

92 hellinger

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Hellinger distance between Gaussian densities

### **Description**

Hellinger distance between two multivariate (p > 1) or univariate (p = 1) Gaussian densities (see Details).

### Usage

```
hellinger(x1, x2, check = FALSE)
```

### Arguments

x1	a matrix or data frame of $n_1$ rows (observations) and $p$ columns (variables) (can also be a tibble) or a vector of length $n_1$ .
x2	matrix or data frame (or tibble) of $n_2$ rows and $p$ columns or vector of length $n_2$ .
check	logical. When TRUE (the default is FALSE) the function checks if the covariance matrices are not degenerate (multivariate case) or if the variances are not zero (univariate case).

### **Details**

The Hellinger distance between the two Gaussian densities is computed by using the hellingerpar function and the density parameters estimated from samples.

# Value

Returns the *Hellinger* distance between the two probability densities.

Be careful! If check = FALSE and one smoothing bandwidth matrix is degenerate, the result returned can not be considered.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

McLachlan, G.J. (1992). Discriminant analysis and statistical pattern recognition. John Wiley & Sons, New York .

#### See Also

hellingerpar: Hellinger distance between Gaussian densities, given their parameters.

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### **Examples**

```
require(MASS)
m1 <- c(0,0)
v1 <- matrix(c(1,0,0,1),ncol = 2)
m2 <- c(0,1)
v2 <- matrix(c(4,1,1,9),ncol = 2)
x1 <- mvrnorm(n = 3,mu = m1,Sigma = v1)
x2 <- mvrnorm(n = 5, mu = m2, Sigma = v2)
hellinger(x1, x2)</pre>
```

hellingerpar

Hellinger distance between Gaussian densities given their parameters

### **Description**

Hellinger distance between two multivariate (p > 1) or univariate (p = 1) Gaussian densities given their parameters (mean vectors and covariance matrices if the densities are multivariate, or means and variances if univariate) (see Details).

### Usage

```
hellingerpar(mean1, var1, mean2, var2, check = FALSE)
```

#### **Arguments**

mean1	p-length numeric vector: the mean of the first Gaussian density.
var1	$p \times p$ symmetric numeric matrix $(p > 1)$ or numeric $(p = 1)$ : the covariance matrix $(p > 1)$ or the variance $(p = 1)$ of the first Gaussian density.
mean2	p-length numeric vector: the mean of the second Gaussian density.
var2	$p \times p$ symmetric numeric matrix $(p > 1)$ or numeric $(p = 1)$ : the covariance matrix $(p > 1)$ or the variance $(p = 1)$ of the second Gaussian density.
check	logical. When TRUE (the default is FALSE) the function checks if the covariance matrices are not degenerate (multivariate case) or if the variances are not zero (univariate case).

### **Details**

The mean vectors (m1 and m2) and variance matrices (v1 and v2) given as arguments (mean1, mean2, var1 and var2) are used to compute the Hellinger distance between the two Gaussian densities, equal to:

$$(2(1-2^{p/2}det(v1v2)^{1/4}det(v1+v2)^{-1/2}exp((-1/4)t(m1-m2)(v1+v2)^{-1}(m1-m2))))^{1/2}$$

If p=1 the means and variances are numbers, the formula is the same ignoring the following operators: t (transpose of a matrix or vector) and det (determinant of a square matrix).

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### Value

The Hellinger distance between two Gaussian densities.

Be careful! If check = FALSE and one covariance matrix is degenerated (multivariate case) or one variance is zero (univariate case), the result returned must not be considered.

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

McLachlan, G.J. (1992). Discriminant analysis and statistical pattern recognition. John Wiley & Sons, New York .

### See Also

hellinger: Hellinger distance between Gaussian densities estimated from samples.

### **Examples**

```
m1 <- c(1,1)
v1 <- matrix(c(4,1,1,9),ncol = 2)
m2 <- c(0,1)
v2 <- matrix(c(1,0,0,1),ncol = 2)
hellingerpar(m1,v1,m2,v2)
```

interpret

Scores of fmdsd, dstatis, fpcad, or fpcat vs. moments, or scores of mdsdd vs. marginal distributions or association measures

### **Description**

This generic function provides a tool for the interpretation of the results of fmdsd, dstatis, fpcad, fpcat or mdsdd function.

#### Usage

```
interpret(x, nscore = 1:3, ...)
```

### **Arguments**

Х

 $object\ of\ class\ {\tt fmdsd},\ {\tt dstatis},\ {\tt fpcad},\ {\tt fpcat}\ or\ {\tt mdsdd}.$ 

```
• fmdsd: see interpret.fmdsd
```

• dstatis: see interpret.dstatis

• fpcad: see interpret.fpcad

• fpcat: see interpret.fpcat

• mdsdd: see interpret.mdsdd

interpret.dstatis 95

nscore numeric vector. Selects the columns of the data frame x\$scores to be inter-

preted.

Warning: Its components cannot be greater than the nb. factors argument in

the call of the fpcad or fpcat function.

... Arguments to be passed to the methods, such as moment (for interpret.fmdsd,

 $interpret. \, dstatis, \, interpret. \, fpcad \, \, and \, \, interpret. \, fpcat), \, or \, \, mma \, \, (for \, \, interpret. \, fpcad), \, or \, \, mma \, \, (for \, \, interpret. \, fpcad), \, or \, \, mma \, \, (for \, \, interpret. \, fpcad), \, or \, \, mma \, \, (for \, \, interpret. \, fpcad), \, or \, \, mma \, \, (for \, \, interpret. \, fpcad), \, or \, \, mma \, \, (for \, \, interpret. \, fpcad), \, or \, \, mma \, \, (for \, \, interpret. \, fpcad), \, or \, \, mma \, \, (for \, \, interpret. \, fpcad), \, or \, \, mma \, \, (for \, \, interpret. \, fpcad), \, or \, \, mma \, \, (for \, \, interpret. \, fpcad), \, or \, \, mma \, \, (for \, \, interpret. \, fpcad), \, or \, \, mma \, \, (for \, \, interpret. \, fpcad), \, or \, \, mma \, \, (for \, \, interpret. \, fpcad), \, or \, \, mma \, \, (for \, \, interpret. \, fpcad), \, or \, \, mma \, \, (for \, \, interpret. \, fpcad), \, or \, \, mma \, \, (for \, \, interpret. \, fpcad), \, or \, \, mma \, \, (for \, \, interpret. \, fpcad), \, or \, \, interpret. \, or \, \, (for \, \, interpret. \, fpcad), \, or \, \, interpret. \, or \, \, (for \, \, interpret. \, fpcad), \, or \, \, (for \, \, interpret. \, fpcad)$ 

interpret.mdsdd).

#### Value

Returns a list including:

pearson matrix of Pearson correlations between selected scores and moments, probabil-

ities or associations.

spearman matrix of Spearman correlations between selected scores and moments, proba-

bilities or associations.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

#### References

Boumaza, R., Yousfi, S., Demotes-Mainard, S. (2015). Interpreting the principal component analysis of multivariate density functions. Communications in Statistics - Theory and Methods, 44 (16), 3321-3339.

#### See Also

interpret.fmdsd; interpret.dstatis; interpret.fpcad; interpret.fpcat; interpret.mdsdd.

interpret.dstatis

Scores of the dstatis function vs. moments of the densities

### **Description**

Applies to an object of class dstatis, plots the principal scores vs. the moments of the densities (means, standard deviations, variances, correlations, skewness and kurtosis coefficients), and computes the correlations between these scores and moments.

### Usage

96 interpret.dstatis

#### **Arguments**

x object of class dstatis (returned by the dstatis.inter function).

nscore numeric vector. Selects the columns of the data frame x\$scores to be inter-

preted.

Warning: Its components cannot be greater than the nb.factors argument in

the call of the dstatis.inter function.

moment characters string. Selects the moments to cross with scores:

• "mean" (means)

• "sd" (standard deviations)

• "cov" (covariances)

• "cor" (correlation coefficients)

• "skewness" (skewness coefficients)

• "kurtosis" (kurtosis coefficients)

• "all" (for univariate densities only. It simultaneously considers means, standard deviations, variances and skewness and kurtosis coefficients)

... Arguments to be passed to methods.

#### **Details**

A new graphics device is opened for each score column. A device can contain up to 9 graphs. If there are too many (more than 36) graphs for each score, a multipage PDF file is created in the current working directory, and the graphs are displayed in it.

The number of principal scores to be interpreted cannot be greater than nb.factors of the data frame x\$scores returned by the function dstatis.inter.

### Value

Returns a list including:

pearson matrix of Pearson correlations between selected scores and moments.

spearman matrix of Spearman correlations between selected scores and moments.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

Lavit, C., Escoufier, Y., Sabatier, R., Traissac, P. (1994). The ACT (STATIS method). Computational Statistics & Data Analysis, 18 (1994), 97-119.

#### See Also

dstatis.inter; plot.dstatis.

interpret.fmdsd 97

### **Examples**

```
data(roses)
rosesf <- as.folder(roses[,c("Sha","Den","Sym","rose")])
# Dual STATIS on the covariance matrices
## Not run:
result <- dstatis.inter(rosesf, group.name = "rose")
interpret(result)
interpret(result, moment = "var")
interpret(result, moment = "cor")
interpret(result, nscore = 1:2)
## End(Not run)</pre>
```

interpret.fmdsd

Scores of the fmdsd function vs. moments of the densities

### **Description**

Applies to an object of class fmdsd, plots the scores vs. the moments of the densities (means, standard deviations, variances, correlations, skewness and kurtosis coefficients), and computes the correlations between these scores and moments.

### Usage

### **Arguments**

Х

object of class fmdsd (returned by the fmdsd function).

nscore

numeric vector. Selects the columns of the data frame x\$scores consisting of the score vectors.

Warning: Its components cannot be greater than the nb.factors argument in the call of the fmdsd function.

moment

character string. Selects the moments to cross with scores:

- "mean" (means, which is the default value)
- "sd" (standard deviations)
- "cov" (covariances)
- "cor" (correlation coefficients)
- "skewness" (skewness coefficients)
- "kurtosis" (kurtosis coefficients)
- "all" (for univariate densities only. It simultaneously considers means, standard deviations, variances and skewness and kurtosis coefficients)

.. Arguments to be passed to methods.

98 interpret.fmdsd

### **Details**

A new graphics device is opened for each score column. A device can contain up to 9 graphs. If there are too many (more than 36) graphs for each score, a multipage PDF file is created in the current working directory, and the graphs are displayed in it.

The number of principal scores to be interpreted cannot be greater than nb.factors of the data frame x\$scores returned by the function fmdsd.

### Value

Returns a list including:

pearson matrix of Pearson correlations between selected scores and moments.

spearman matrix of Spearman correlations between selected scores and moments.

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

Boumaza, R., Yousfi, S., Demotes-Mainard, S. (2015). Interpreting the principal component analysis of multivariate density functions. Communications in Statistics - Theory and Methods, 44 (16), 3321-3339.

Delicado, P. (2011). Dimensionality reduction when data are density functions. Computational Statistics & Data Analysis, 55, 401-420.

### See Also

fmdsd; plot.fmdsd.

```
data(roses)
x <- roses[,c("Sha","Den","Sym","rose")]
rosesfold <- as.folder(x)
result1 <- fmdsd(rosesfold)
interpret(result1)
## Not run:
interpret(result1, moment = "var")
## End(Not run)
interpret(result1, nscore = 1:2)</pre>
```

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interpret.fpcad

Scores of the fpcad function vs. moments of the densities

### **Description**

Applies to an object of class fpcad, plots the principal scores vs. the moments of the densities (means, standard deviations, variances, correlations, skewness and kurtosis coefficients), and computes the correlations between these scores and moments.

### Usage

### **Arguments**

x object of class fpcad (returned by the fpcad function).

nscore numeric vector. Selects the columns of the data frame x\$scores to be inter-

preted.

Warning: Its components cannot be greater than the nb. factors argument in

the call of the fpcad function.

moment characters string. Selects the moments to cross with scores:

• "mean" (means)

• "sd" (standard deviations)

• "cov" (covariances)

• "cor" (correlation coefficients)

• "skewness" (skewness coefficients)

• "kurtosis" (kurtosis coefficients)

• "all" (for univariate densities only. It simultaneously considers means, standard deviations, variances and skewness and kurtosis coefficients)

. . . Arguments to be passed to methods.

#### **Details**

A new graphics device is opened for each score column. A device can contain up to 9 graphs. If there are too many (more than 36) graphs for each score, a multipage PDF file is created in the current working directory, and the graphs are displayed in it.

The number of principal scores to be interpreted cannot be greater than nb.factors of the data frame x\$scores returned by the function fpcad.

#### Value

Returns a list including:

pearson matrix of Pearson correlations between selected scores and moments.

spearman matrix of Spearman correlations between selected scores and moments.

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#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

Boumaza, R., Yousfi, S., Demotes-Mainard, S. (2015). Interpreting the principal component analysis of multivariate density functions. Communications in Statistics - Theory and Methods, 44 (16), 3321-3339.

#### See Also

fpcad; plot.fpcad.

### **Examples**

```
data(roses)
rosefold <- as.folder(roses[,c("Sha","Den","Sym","rose")])
result1 <- fpcad(rosefold)
interpret(result1)
## Not run:
interpret(result1, moment = "var")
## End(Not run)
interpret(result1, moment = "cor")
interpret(result1, nscore = 1:2)</pre>
```

interpret.fpcat

Scores of the fpcat function vs. moments of the densities

### Description

This function applies to an object of class fpcat and does the same as for an object of class fpcad: it plots the principal scores vs. the moments of the densities (means, standard deviations, variances, correlations, skewness and kurtosis coefficients), and computes the correlations between these scores and moments.

In the next version of the package, this function will be updated by some more graphics.

# Usage

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#### **Arguments**

x object of class fpcat (returned by the fpcat function).

nscore numeric vector. Selects the columns of the data frame x\$scores to be inter-

preted.

Warning: Its components cannot be greater than the nb.factors argument in

the call of the fpcat function.

moment characters string. Selects the moments to cross with scores:

• "mean" (means)

• "sd" (standard deviations)

• "cov" (covariances)

• "cor" (correlation coefficients)

• "skewness" (skewness coefficients)

• "kurtosis" (kurtosis coefficients)

• "all" (for univariate densities only. It simultaneously considers means, standard deviations, variances and skewness and kurtosis coefficients)

... Arguments to be passed to methods.

#### **Details**

A new graphics device is opened for each score column. A device can contain up to 9 graphs. If there are too many (more than 36) graphs for each score, a multipage PDF file is created in the current working directory, and the graphs are displayed in it.

The number of principal scores to be interpreted cannot be greater than nb.factors of the data frame x\$scores returned by the function fpcat.

### Value

Returns a list including:

pearson matrix of Pearson correlations between selected scores and moments.

spearman matrix of Spearman correlations between selected scores and moments.

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

Boumaza, R., Yousfi, S., Demotes-Mainard, S. (2015). Interpreting the principal component analysis of multivariate density functions. Communications in Statistics - Theory and Methods, 44 (16), 3321-3339.

#### See Also

fpcat; plot.fpcat.

interpret.mdsdd

### **Examples**

```
# Alsacian castles with their building year
data(castles)
castyear <- foldert(lapply(castles, "[", 1:4))
fpcayear <- fpcat(castyear, group.name = "year")
interpret(fpcayear)
## Not run:
interpret(fpcayear, moment="var")
## End(Not run)</pre>
```

interpret.mdsdd

Scores of the mdsdd function vs. marginal probability distributions or association measures

# Description

Applies to an object of class "mdsdd", plots the scores vs. the marginal probability distributions or pairwise association measures of the discrete variables, and computes the correlations between these scores and probabilities or association measures (see Details).

### Usage

```
## S3 method for class 'mdsdd'
interpret(x, nscore = 1:3, mma = c("marg1", "marg2", "assoc"), ...)
```

#### **Arguments**

x object of class mdsdd (returned by the mdsdd function).

nscore numeric vector. Selects the columns of the data frame x\$scores consisting of

the score vectors.

Warning: Its components cannot be greater than the nb.factors argument in

the call of the mdsdd function.

mma character. Indicates which measures will be considered:

• "marg1": the probability distribution of each variable.

• "marg2": the joint probability distribution of each pair of variables.

• "assoc": the pairwise association measures of the variables.

... Arguments to be passed to methods.

#### **Details**

A new graphics device is opened for each score column. A device can contain up to 9 graphs. If there are too many (more than 36) graphs for each score, a multipage PDF file is created in the current working directory, and the graphs are displayed in it.

The number of principal scores to be interpreted cannot be greater than nb.factors of the data frame x\$scores returned by the function mdsdd.

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### Value

Returns a list including:

pearson matrix of Pearson correlations between selected scores and probabilities or as-

sociation measures.

spearman matrix of Spearman correlations between selected scores and probabilities or

association measures.

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

#### See Also

```
mdsdd; plot.mdsdd.
```

```
# INSEE (France): Diploma x Socio professional group, seven years.
data(dspg)
xlista = dspg
a <- mdsdd(xlista)
interpret(a)
# Example 3 with a list of 96 arrays (departments)
## Not run:
data(dspgd2015)
xd = dspgd2015
res = mdsdd(xd, group.name = "coded")
interpret(res)
plot(res, fontsize.points = 0.7)
# Each department is represented by its name
data(departments)
coor = merge(res$scores, departments, by = "coded")
dev.new()
plot(coor$PC.1, coor$PC.2, type ="n")
text(coor$PC.1, coor$PC.2, coor$named, cex = 0.5)
# Each department is represented by its region
dev.new()
plot(coor$PC.1, coor$PC.2, type ="n")
text(coor$PC.1, coor$PC.2, coor$coder, cex = 0.7)
## End(Not run)
```

is.discdd.predict

is.discdd.misclass

Class discdd.misclass

### **Description**

Tests if its argument is an object of class discdd.misclass (see Details of the function discdd.misclass).

# Usage

```
is.discdd.misclass(x)
```

### **Arguments**

Х

object to be tested.

### Value

TRUE if its argument is of class discdd.misclass, and FALSE otherwise.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### See Also

discdd.misclass.

is.discdd.predict

Class discdd.predict

### **Description**

Tests if its argument is an object of class discdd.predict (see Details of the function discdd.predict).

### Usage

```
is.discdd.predict(x)
```

### **Arguments**

Х

object to be tested.

### Value

TRUE if its argument is of class discdd.predict, and FALSE otherwise.

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### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### See Also

discdd.predict.

is.dstatis

Class dstatis

# Description

Tests if its argument is an object of class dstatis (see Details of the function dstatis.inter).

### Usage

```
is.dstatis(x)
```

# Arguments

Х

object to be tested.

### Value

TRUE if its argument is of class dstatis, and FALSE otherwise.

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### See Also

dstatis.inter.

is.fdiscd.predict

is.fdiscd.misclass

Class fdiscd.misclass

### **Description**

Tests if its argument is an object of class fdiscd.misclass (see Details of the function fdiscd.misclass).

### Usage

```
is.fdiscd.misclass(x)
```

### **Arguments**

Х

object to be tested.

#### Value

TRUE if its argument is of class fdiscd.misclass, and FALSE otherwise.

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### See Also

fdiscd.misclass.

is.fdiscd.predict

Class fdiscd.predict

### **Description**

Tests if its argument is an object of class fdiscd.predict (see Details of the function fdiscd.predict)...

# Usage

```
is.fdiscd.predict(x)
```

# **Arguments**

Х

object to be tested.

## Value

TRUE if its argument is of class fdiscd.predict, and FALSE otherwise.

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### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### See Also

fdiscd.predict.

is.fhclustd

Class fhclustd

# Description

Tests if its argument is an object of class fhclustd (see Details of the function fhclustd).

### Usage

is.fhclustd(x)

# Arguments

x

object to be tested.

### Value

TRUE if its argument is of class fhclustd, and FALSE otherwise.

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### See Also

fhclustd.

is.folder

is.fmdsd

 ${\it Class}\ {\sf fmdsd}$ 

# Description

Tests if its argument is an object of class fmdsd (see Details of the function fmdsd).

### Usage

```
is.fmdsd(x)
```

### **Arguments**

Χ

object to be tested.

### Value

TRUE if its argument is of class fmdsd, and FALSE otherwise.

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### See Also

fmdsd.

is.folder

Class folder

# Description

Tests if its argument is an object of class folder (see folder).

### Usage

```
is.folder(x)
```

# Arguments

Х

object to be tested.

### Value

TRUE if its argument is of class folder, and FALSE otherwise.

is.folderh

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### See Also

folder to create an object of class folder.

is.folderh

Class folderh

# Description

Tests if its argument is an object of class folderh (see folderh).

# Usage

is.folderh(x)

# Arguments

Х

object to be tested.

### Value

TRUE if its argument is of class folderh, and FALSE otherwise.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

# See Also

folderh to create an object of class folderh.

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is.foldermtg

 ${\it Class}$  foldermtg

# Description

Tests if its argument is an object of class foldermtg (see read.mtg).

## Usage

```
is.foldermtg(x)
```

## **Arguments**

Х

object to be tested.

### Value

TRUE if its argument is of class foldermtg, and FALSE otherwise.

## Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### See Also

read.mtg to read a MTG file and create an object of class foldermtg.

is.foldert

Class foldert

## **Description**

Tests if its argument is an object of class foldert (see foldert).

# Usage

```
is.foldert(x)
```

## Arguments

Χ

object to be tested.

### Value

TRUE if its argument is of class foldert, and FALSE otherwise.

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## Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

## See Also

foldert to create an object of class foldert.

is.fpcad

Class fpcad

# Description

Tests if its argument is an object of class fpcad (see Details of the function fpcad).

# Usage

```
is.fpcad(x)
```

# Arguments

Χ

object to be tested.

### Value

TRUE if its argument is of class fpcad, and FALSE otherwise.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

# See Also

fpcad.

jeffreys jeffreys

is.mdsdd

Class mdsdd

### **Description**

Tests if its argument is an object of class mdsdd (see Details of the function mdsdd).

## Usage

```
is.mdsdd(x)
```

### **Arguments**

Х

object to be tested.

## Value

TRUE if its argument is of class mdsdd, and FALSE otherwise.

## Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### See Also

mdsdd.

jeffreys

Jeffreys measure between Gaussian densities

### **Description**

Jeffreys measure (or symmetrised Kullback-Leibler divergence) between two multivariate (p>1) or univariate (p=1) Gaussian densities given samples (see Details).

## Usage

```
jeffreys(x1, x2, check = FALSE)
```

### **Arguments**

x1	a matrix or data frame of $n_1$ rows (observations) and $p$ columns (variables) (can
	also be a tibble) or a vector of length $n_1$ .

x2 matrix or data frame (or tibble) of  $n_2$  rows and p columns or vector of length

 $n_2$ .

check logical. When TRUE (the default is FALSE) the function checks if the covariance

matrices are not degenerate (multivariate case) or if the variances are not zero

(univariate case).

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#### **Details**

The Jeffreys measure between the two Gaussian densities is computed by using the jeffreyspar function and the density parameters estimated from samples.

### Value

Returns the Jeffreys measure between the two probability densities.

Be careful! If check = FALSE and one smoothing bandwidth matrix is degenerate, the result returned must not be considered.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

Thabane, L., Safiul Haq, M. (1999). On Bayesian selection of the best population using the Kullback-Leibler divergence measure. Statistica Neerlandica, 53(3): 342-360.

## See Also

jeffreyspar: Jeffreys measure between Gaussian densities, given their parameters.

### **Examples**

```
require(MASS)
m1 <- c(0,0)
v1 <- matrix(c(1,0,0,1),ncol = 2)
m2 <- c(0,1)
v2 <- matrix(c(4,1,1,9),ncol = 2)
x1 <- mvrnorm(n = 3,mu = m1,Sigma = v1)
x2 <- mvrnorm(n = 5, mu = m2, Sigma = v2)
jeffreys(x1, x2)</pre>
```

jeffreyspar

Jeffreys measure between Gaussian densities given their parameters

## Description

Jeffreys measure (or symmetrised Kullback-Leibler divergence) between two multivariate (p > 1) or univariate (p = 1) Gaussian densities, given their parameters (mean vectors and covariance matrices if they are multivariate, means and variances if univariate) (see Details).

```
jeffreyspar(mean1, var1, mean2, var2, check = FALSE)
```

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### **Arguments**

mean1	<i>p</i> -length numeric vector: the mean of the first Gaussian density.
var1	$p \times p$ symmetric numeric matrix $(p > 1)$ or numeric $(p = 1)$ : the covariance matrix $(p > 1)$ or the variance $(p = 1)$ of the first Gaussian density.
mean2	p-length numeric vector: the mean of the second Gaussian density.
var2	$p \times p$ symmetric numeric matrix $(p > 1)$ or numeric $(p = 1)$ : the covariance matrix $(p > 1)$ or the variance $(p = 1)$ of the second Gaussian density.
check	logical. When TRUE (the default is FALSE) the function checks if the covariance matrices are not degenerate (multivariate case) or if the variances are not zero (univariate case).

#### **Details**

Let m1 and m2 the mean vectors, v1 and v2 the covariance matrices, the Jeffreys' measure of the two Gaussian densities is equal to:

$$(1/2)t(m1-m2)(v1^{-1}+v2^{-1})(m1-m2)-(1/2)tr((v1-v2)(v1^{-1}-v2^{-1}))$$

.

If p=1 the means and variances are numbers, the formula is the same ignoring the following operators: t (transpose of a matrix or vector) and tr (trace of a square matrix).

#### Value

The Jeffreys measure between two Gaussian densities.

Be careful! If check = FALSE and one covariance matrix is degenerated (multivariate case) or one variance is zero (univariate case), the result returned must not be considered.

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### References

McLachlan, G.J. (1992). Discriminant analysis and statistical pattern recognition. John Wiley & Sons, New York .

Thabane, L., Safiul Haq, M. (1999). On Bayesian selection of the best population using the Kullback-Leibler divergence measure. Statistica Neerlandica, 53(3): 342-360.

#### See Also

jeffreys: Jeffreys measure of two parametrically estimated Gaussian densities, given samples.

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### **Examples**

```
m1 <- c(1,1)
v1 <- matrix(c(4,1,1,9),ncol = 2)
m2 <- c(0,1)
v2 <- matrix(c(1,0,0,1),ncol = 2)
jeffreyspar(m1,v1,m2,v2)
```

kurtosis.folder

Kurtosis coefficients of a folder of data sets

## **Description**

Computes the kurtosis coefficient by column of the elements of an object of class folder.

### Usage

```
kurtosis.folder(x, na.rm = FALSE, type = 3)
```

# Arguments

```
x an object of class folder.

na.rm logical. Should missing values be omitted from the calculations? (see kurtosis)

type an integer between 1 and 3 (see kurtosis).
```

### **Details**

It uses kurtosis to compute the mean by numeric column of each element of the folder. If some columns of the data frames are not numeric, there is a warning, and the means are computed on the numeric columns only.

#### Value

A list whose elements are the kurtosis coefficients by column of the elements of the folder.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### See Also

folder to create an object is of class folder. mean. folder, var. folder, cor. folder, skewness. folder for other statistics for folder objects.

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### **Examples**

```
# First example: iris (Fisher)
data(iris)
iris.fold <- as.folder(iris, "Species")</pre>
iris.kurtosis <- kurtosis.folder(iris.fold)</pre>
print(iris.kurtosis)
# Second example: roses
data(roses)
roses.fold <- as.folder(roses, "rose")</pre>
roses.kurtosis <- kurtosis.folder(roses.fold)</pre>
print(roses.kurtosis)
```

12d

 $L^2$  inner product of probability densities

## **Description**

 $L^2$  inner product of two multivariate (p > 1) or univariate (p = 1) probability densities, estimated from samples.

#### **Usage**

```
12d(x1, x2, method = "gaussiand", check = FALSE, varw1 = NULL, varw2 = NULL)
```

## **Arguments**

х1 a matrix or data frame of  $n_1$  rows (observations) and p columns (variables) (can

also be a tibble) or a vector of length  $n_1$ .

matrix or data frame (or tibble) of  $n_2$  rows and p columns or vector of length x2

 $n_2$ .

method string. It can be:

• "gaussiand" if the densities are considered to be Gaussian.

• "kern" if they are estimated using the Gaussian kernel method.

logical. When TRUE (the default is FALSE) the function checks if the covaricheck

ance matrices (if method = "gaussiand") or smoothing bandwidth matrices (if method = "kern") are not degenerate, before computing the inner product.

Notice that if p = 1, it checks if the variances or smoothing parameters are not zero.

varw1, varw2

 $p \times p$  symmetric matrices: the smoothing bandwidths for the estimation of the probability densities. If they are omitted, the smoothing bandwidths are computed using the normal reference rule matrix bandwidth (see details).

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#### **Details**

• If method = "gaussiand", the mean vectors and the variance matrices (v1 and v2) of the two samples are computed, and they are used to compute the inner product using the 12dpar function.

• If method = "kern", the densities of both samples are estimated using the Gaussian kernel method. These estimations are then used to compute the inner product. if varw1 and varw2 arguments are omitted, the smoothing bandwidths are computed using the normal reference rule matrix bandwidth:

$$h_1 v_1^{1/2}$$

where

$$h_1 = (4/(n_1(p+2)))^{1/(p+4)}$$

for the first density. Idem for the second density after making the necessary changes.

#### Value

The  $L^2$  inner product of the two probability densities.

Be careful! If check = FALSE and one smoothing bandwidth matrix is degenerate, the result returned can not be considered.

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### References

Boumaza, R., Yousfi, S., Demotes-Mainard, S. (2015). Interpreting the principal component analysis of multivariate density functions. Communications in Statistics - Theory and Methods, 44 (16), 3321-3339.

Wand, M., Jones, M. (1995). Kernel smoothing. Chapman and Hall/CRC, London.

Yousfi, S., Boumaza R., Aissani, D., Adjabi, S. (2014). Optimal bandwith matrices in functional principal component analysis of density functions. Journal of Statistical Computational and Simulation, 85 (11), 2315-2330.

### See Also

12dpar for Gaussian densities whose parameters are given.

## **Examples**

```
require(MASS)
m1 <- c(0,0)
v1 <- matrix(c(1,0,0,1),ncol = 2)
m2 <- c(0,1)
v2 <- matrix(c(4,1,1,9),ncol = 2)
x1 <- mvrnorm(n = 3,mu = m1,Sigma = v1)
x2 <- mvrnorm(n = 5, mu = m2, Sigma = v2)
l2d(x1, x2, method = "gaussiand")
l2d(x1, x2, method = "kern")</pre>
```

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```
12d(x1, x2, method = "kern", varw1 = v1, varw2 = v2)
```

12dpar

 $L^2$  inner product of Gaussian densities given their parameters

## **Description**

 $L^2$  inner product of multivariate (p > 1) or univariate (p = 1) Gaussian densities, given their parameters (mean vectors and covariance matrices if the densities are multivariate, or means and variances if univariate).

#### Usage

```
12dpar(mean1, var1, mean2, var2, check = FALSE)
```

## **Arguments**

mean1	p-length numeric vector: the mean of the first Gaussian density.
var1	$p \times p$ symmetric numeric matrix $(p > 1)$ or numeric $(p = 1)$ : the covariance matrix $(p > 1)$ or the variance $(p = 1)$ of the first Gaussian density.
mean2	p-length numeric vector: the mean of the second Gaussian density.
var2	$p \times p$ symmetric numeric matrix $(p > 1)$ or numeric $(p = 1)$ : the covariance matrix $(p > 1)$ or the variance $(p = 1)$ of the second Gaussian density.
check	logical. When TRUE (the default is FALSE) the function checks if the covariance matrices are not degenerate (multivariate case) or if the variances are not zero (univariate case).

#### **Details**

Computes the inner product of two Gaussian densities, equal to:

$$(2\pi)^{-p/2} det(var1 + var2)^{-1/2} exp(-(1/2)t(mean1 - mean2)(var1 + var2)^{-1}(mean1 - mean2))$$

If p = 1 the means and variances are numbers, the formula is the same ignoring the following operators: t (transpose of a matrix or vector) and det (determinant of a square matrix).

### Value

The  $L^2$  inner product between two Gaussian densities.

Be careful! If check = FALSE and one covariance matrix is degenerated (multivariate case) or one variance is zero (univariate case), the result returned must not be considered.

# Author(s)

matddchisqsym 119

### References

M. Wand and M. Jones (1995). Kernel Smoothing. Chapman and Hall, London.

#### See Also

12d for parametrically estimated Gaussian densities or nonparametrically estimated densities, given samples;

# **Examples**

```
m1 <- c(1,1)
v1 <- matrix(c(4,1,1,9),ncol = 2)
m2 <- c(0,1)
v2 <- matrix(c(1,0,0,1),ncol = 2)
l2dpar(m1,v1,m2,v2)
```

matddchisqsym

Matrix of distances between discrete probability densities given samples

## Description

Computes the matrix of the symmetric Chi-squared distances between several multivariate or univariate discrete probability distributions, estimated from samples.

#### Usage

```
matddchisqsym(x)
```

### **Arguments**

Χ

object of class "folder" containing the data. Its elements are data frames (one data frame per distribution) whose columns are factors.

## Value

Positive symmetric matrix whose order is equal to the number of data frames (or distributions), consisting of the pairwise symmetric chi-squared distances between the distributions.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

### References

Deza, M.M. and Deza E. (2013). Encyclopedia of distances. Springer.

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### See Also

ddchisgsym.

matddchisqsympar for discrete probability densities, given the probabilities on the same support.

### **Examples**

matddchisqsympar

Matrix of distances between discrete probability densities given the probabilities on their common support

### **Description**

Computes the matrix of the symmetric Chi-squared distances between several multivariate or univariate discrete probability distributions on the same support (which can be a Cartesian product of q sets), given the probabilities of the states (which are q-tuples) of the support.

# Usage

```
matddchisqsympar(freq)
```

#### **Arguments**

freq

list of arrays. Their dim attribute is a vector with length q, its elements containing the numbers of levels of the sets. Each array contains the probabilities of the discrete distribution on the same support.

### Value

Positive symmetric matrix whose order is equal to the number of distributions, consisting of the pairwise symmetric chi-squared distances between these distributions.

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### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

#### References

Deza, M.M. and Deza E. (2013). Encyclopedia of distances. Springer.

#### See Also

ddchisqsympar.

matddchisqsym for discrete probability densities which are estimated from the data.

matddhellinger

Matrix of distances between discrete probability densities given samples

# Description

Computes the matrix of the Hellinger (or Matusita) distances between several multivariate or univariate discrete probability distributions, estimated from samples.

### Usage

matddhellinger(x)

#### **Arguments**

Х

object of class "folder" containing the data. Its elements are data frames (one data frame per distribution) whose columns are factors.

#### Value

Positive symmetric matrix whose order is equal to the number of data frames (or distributions), consisting of the pairwise Hellinger distances between the distributions.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

#### References

Deza, M.M. and Deza E. (2013). Encyclopedia of distances. Springer.

#### See Also

ddhellinger.

matddhellingerpar for discrete probability densities, given the probabilities on the same support.

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### **Examples**

matddhellingerpar

Matrix of distances between discrete probability densities given the probabilities on their common support

## **Description**

Computes the matrix of the Hellinger (or Matusita) distances between several multivariate or univariate discrete probability distributions on the same support (which can be a Cartesian product of q sets), given the probabilities of the states (which are q-tuples) of the support.

#### Usage

```
matddhellingerpar(freq)
```

### **Arguments**

freq

list of arrays. Their dim attribute is a vector with length q, its elements containing the numbers of levels of the sets. Each array contains the probabilities of the discrete distribution on the same support.

### Value

Positive symmetric matrix whose order is equal to the number of distributions, consisting of the pairwise Hellinger distances between these distributions.

### Author(s)

matddjeffreys 123

### References

Deza, M.M. and Deza E. (2013). Encyclopedia of distances. Springer.

### See Also

ddhellingerpar.

matddhellinger for discrete probability densities which are estimated from the data.

 ${\sf matddjeffreys}$ 

Matrix of distances between discrete probability densities given samples

# **Description**

Computes the matrix of the Jeffreys divergences between several multivariate or univariate discrete probability distributions, estimated from samples.

## Usage

matddjeffreys(x)

### **Arguments**

Χ

object of class "folder" containing the data. Its elements are data frames (one data frame per distribution) whose columns are factors.

#### Value

Positive symmetric matrix whose order is equal to the number of data frames (or distributions), consisting of the pairwise Jeffreys divergences between the distributions.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

#### References

Deza, M.M. and Dezaz E. (2013). Encyclopedia of distances. Springer.

### See Also

```
ddjeffreys.
```

matddjeffreyspar for discrete probability densities, given the probabilities on the same support.

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### **Examples**

matddjeffreyspar

Matrix of divergences between discrete probability densities given the probabilities on their common support

## **Description**

Computes the matrix of the Jeffreys divergences between several multivariate or univariate discrete probability distributions on the same support (which can be a Cartesian product of q sets), given the probabilities of the states (which are q-tuples) of the support.

#### Usage

```
matddjeffreyspar(freq)
```

### **Arguments**

freq

list of arrays. Their dim attribute is a vector with length q, its elements containing the numbers of levels of the sets. Each array contains the probabilities of the discrete distribution on the same support.

### Value

Positive symmetric matrix whose order is equal to the number of distributions, consisting of the pairwise Jeffreys divergences between these distributions.

### Author(s)

matddjensen 125

### References

Deza, M.M. and Deza E. (2013). Encyclopedia of distances. Springer.

## See Also

ddjeffreyspar.

matddjeffreys for discrete probability densities which are estimated from the data.

matddjensen Matrix of divergences between discrete probability densities given samples

## **Description**

Computes the matrix of the Jensen-Shannon divergences between several multivariate or univariate discrete probability distributions, estimated from samples.

## Usage

matddjensen(x)

### **Arguments**

X

object of class "folder" containing the data. Its elements are data frames (one data frame per distribution) whose columns are factors.

#### Value

Positive symmetric matrix whose order is equal to the number of data frames (or distributions), consisting of the pairwise Jensen-Shannon divergences between the distributions.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

#### References

Deza, M.M. and Deza E. (2013). Encyclopedia of distances. Springer.

### See Also

ddjensen.

matddjensenpar for discrete probability densities, given the probabilities on the same support.

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### **Examples**

matddjensenpar

Matrix of divergences between discrete probability densities given the probabilities on their common support

## **Description**

Computes the matrix of the Jensen-Shannon divergences between several multivariate or univariate discrete probability distributions on the same support (which can be a Cartesian product of q sets), given the probabilities of the states (which are q-tuples) of the support.

#### Usage

```
matddjensenpar(freq)
```

### **Arguments**

freq

list of arrays. Their dim attribute is a vector with length q, its elements containing the numbers of levels of the sets. Each array contains the probabilities of the discrete distribution on the same support.

### Value

Positive symmetric matrix whose order is equal to the number of densities, consisting of the pairwise Jensen-Shannon divergences between the discrete probability densities.

### Author(s)

matddlp 127

### References

Deza, M.M. and Deza E. (2013). Encyclopedia of distances. Springer.

#### See Also

```
ddjensenpar.
```

matddjensen for discrete probability densities which are estimated from the data.

matddlp	Matrix of distances between discrete probability distributions given
	samples

## **Description**

Computes the matrix of the  $L^p$  distances between several multivariate or univariate discrete probability distributions, estimated from samples.

# Usage

```
matddlp(x, p = 1)
```

### **Arguments**

- x object of class "folder" containing the data. Its elements are data frames (one data frame per distribution) whose columns are factors.
- p integer. Parameter of the distance.

## Value

Positive symmetric matrix whose order is equal to the number of data frames (or distributions), consisting of the pairwise  $L^p$  distances between the distributions.

## Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

### References

Deza, M.M. and Deza E. (2013). Encyclopedia of distances. Springer.

#### See Also

ddlp.

matddlppar for discrete probability distributions, given the probabilities on the same support.

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### **Examples**

```
# Example 1
x1 \leftarrow data.frame(x = factor(c("A", "A", "B", "B")))
x2 \leftarrow data.frame(x = factor(c("A", "A", "A", "B", "B")))
x3 <- data.frame(x = factor(c("A", "A", "B", "B", "B", "B")))
xf \leftarrow folder(x1, x2, x3)
matddlp(xf)
matddlp(xf, p = 2)
# Example 2
x1 <- data.frame(x = factor(c("A", "A", "A", "B", "B", "B")),</pre>
                  y = factor(c("a", "a", "a", "b", "b", "b")))
x2 <- data.frame(x = factor(c("A", "A", "A", "B", "B")),</pre>
                  y = factor(c("a", "a", "b", "a", "b")))
x3 <- data.frame(x = factor(c("A", "A", "B", "B", "B", "B")),
                  y = factor(c("a", "b", "a", "b", "a", "b")))
xf \leftarrow folder(x1, x2, x3)
matddlp(xf, p = 1)
```

matddlppar

Matrix of distances between discrete probability densities given the probabilities on their common support

### **Description**

Computes the matrix of the  $L^p$  distances between several multivariate or univariate discrete probability distributions on the same support (which can be a Cartesian product of q sets), given the probabilities of the states (which are q-tuples) of the support.

### Usage

```
matddlppar(freq, p = 1)
```

# **Arguments**

freq list of arrays. Their dim attribute is a vector with length q, its elements containing the numbers of levels of the sets. Each array contains the probabilities of the discrete distribution on the same support.

p integer. Parameter of the distance.

### Value

Positive symmetric matrix whose order is equal to the number of distributions, consisting of the pairwise  $L^p$  distances between these distributions.

## Author(s)

matdistl2d 129

#### References

Deza, M.M. and Deza E. (2013). Encyclopedia of distances. Springer.

#### See Also

ddlppar.

matddlp for discrete probability distributions which are estimated from samples.

matdist12d

Matrix of  $L^2$  distances between probability densities

### **Description**

Computes the matrix of the  $L^2$  distances between several multivariate (p > 1) or univariate (p = 1) probability densities, estimated from samples.

### Usage

```
matdistl2d(x, method = "gaussiand", varwL = NULL)
```

#### **Arguments**

Х

object of class "folder" containing the data. Its elements have only numeric variables (observations of the probability densities). If there are non numeric variables, there is an error.

method

string. It can be:

- "gaussiand" if the densities are considered to be Gaussian.
- "kern" if they are estimated using the Gaussian kernel method.

varwL

list of matrices. The smoothing bandwidths for the estimation of each probability density. If they are omitted, the smoothing bandwidths are computed using the normal reference rule matrix bandwidth (see details of the 12d function).

## Value

Positive symmetric matrix whose order is equal to the number of densities, consisting of the pairwise distances between the probability densities.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

## See Also

distl2d.

matdistl2dpar when the probability densities are Gaussian, given the parameters (means and variances).

matdistl2dnorm

### **Examples**

```
data(roses)
    # Multivariate:
   X <- as.folder(roses[,c("Sha","Den","Sym","rose")], groups = "rose")</pre>
    summary(X)
   mean.X <- mean(X)</pre>
    var.X <- var.folder(X)</pre>
    # Parametrically estimated Gaussian densities:
    matdistl2d(X)
    ## Not run:
   # Estimated densities using the Gaussian kernel method ()normal reference rule bandwidth):
    matdistl2d(X, method = "kern")
    # Estimated densities using the Gaussian kernel method (bandwidth provided):
   matdistl2d(X, method = "kern", varwL = var.X)
## End(Not run)
    # Univariate :
    X1 <- as.folder(roses[,c("Sha","rose")], groups = "rose")</pre>
    summary(X1)
   mean.X1 <- mean(X1)</pre>
    var.X1 <- var.folder(X1)</pre>
    # Parametrically estimated Gaussian densities:
   matdistl2d(X1)
  # Estimated densities using the Gaussian kernel method (normal reference rule bandwidth):
   matdistl2d(X1, method = "kern")
  # Estimated densities using the Gaussian kernel method (normal reference rule bandwidth):
    matdistl2d(X1, method = "kern", varwL = var.X1)
```

matdistl2dnorm

Matrix of  $L^2$  distances between  $L^2$ -normed probability densities

### **Description**

Computes the matrix of the  $L^2$  distances between several multivariate (p > 1) or univariate (p = 1)  $L^2$ -normed probability densities, estimated from samples, where a  $L^2$ -normed probability density is the original probability density function divided by its  $L^2$ -norm.

```
matdistl2dnorm(x, method = "gaussiand", varwL = NULL)
```

matdistl2dnorm 131

#### **Arguments**

Х

object of class "folder" containing the data. Its elements have only numeric variables (observations of the probability densities). If there are non numeric variables, there is an error.

method

string. It can be:

- "gaussiand" if the densities are considered to be Gaussian.
- "kern" if they are estimated using the Gaussian kernel method.

varwL

list of matrices. The smoothing bandwidths for the estimation of each probability density. If they are omitted, the smoothing bandwidths are computed using the normal reference rule matrix bandwidth (see details of the 12d function).

### Value

Positive symmetric matrix whose order is equal to the number of densities, consisting of the pairwise distances between the  $L^2$ -normed probability densities.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### See Also

```
distl2dnorm.
```

matdist12d for the distance matrix between probability densities.

matdistl2dnormpar when the probability densities are Gaussian, given the parameters (means and variances).

### **Examples**

```
data(roses)

# Multivariate:
X <- as.folder(roses[,c("Sha","Den","Sym","rose")], groups = "rose")
summary(X)
mean.X <- mean(X)
var.X <- var.folder(X)

# Parametrically estimated Gaussian densities:
matdistl2dnorm(X)

## Not run:
# Estimated densities using the Gaussian kernel method ()normal reference rule bandwidth):
matdistl2dnorm(X, method = "kern")

# Estimated densities using the Gaussian kernel method (bandwidth provided):
matdistl2dnorm(X, method = "kern", varwL = var.X)

## End(Not run)</pre>
```

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```
# Univariate :
X1 <- as.folder(roses[,c("Sha","rose")], groups = "rose")
summary(X1)
mean.X1 <- mean(X1)
var.X1 <- var.folder(X1)

# Parametrically estimated Gaussian densities:
matdistl2dnorm(X1)

# Estimated densities using the Gaussian kernel method (normal reference rule bandwidth):
matdistl2dnorm(X1, method = "kern")

# Estimated densities using the Gaussian kernel method (normal reference rule bandwidth):
matdistl2dnorm(X1, method = "kern", varwL = var.X1)</pre>
```

matdistl2dnormpar

Matrix of  $L^2$  distances between  $L^2$ -normed Gaussian densities given their parameters

### **Description**

Computes the matrix of the  $L^2$  distances between several multivariate (p > 1) or univariate (p = 1)  $L^2$ -normed Gaussian densities, given their parameters (mean vectors and covariance matrices if the densities are multivariate, or means and variances if univariate), where a  $L^2$ -normed Gaussian density is the original probability density function divided by its  $L^2$ -norm.

### Usage

```
matdistl2dnormpar(meanL, varL)
```

### **Arguments**

meanL list of the means (p = 1) or vector means (p > 1) of the Gaussian densities.

varL list of the variances (p = 1) or covariance matrices (p > 1) of the Gaussian

densities.

### Value

Positive symmetric matrix whose order is equal to the number of densities, consisting of the pairwise distances between the  $L^2$ -normed probability densities.

#### Author(s)

matdistl2dpar 133

### See Also

```
distl2dnormpar.

matdistl2dpar for the distance matrix between Gaussian densities, given their parameters.

matdistl2dnorm for the distance matrix between normed probability densities which are estimated
```

## **Examples**

from the data.

```
data(roses)
# Multivariate:
X <- roses[,c("Sha","Den","Sym","rose")]
summary(X)
mean.X <- as.list(by(X[, 1:3], X$rose, colMeans))
var.X <- as.list(by(X[, 1:3], X$rose, var))
# Gaussian densities, given parameters
matdistl2dnormpar(mean.X, var.X)
# Univariate :
X1 <- roses[,c("Sha","rose")]
summary(X1)
mean.X1 <- by(X1$Sha, X1$rose, mean)
var.X1 <- by(X1$Sha, X1$rose, var)
# Gaussian densities, given parameters
matdistl2dnormpar(mean.X1, var.X1)</pre>
```

matdistl2dpar

Matrix of  $L^2$  distances between Gaussian densities given their parameters

## **Description**

Computes the matrix of the  $L^2$  distances between several multivariate (p > 1) or univariate (p = 1) Gaussian densities, given their parameters (mean vectors and covariance matrices if the densities are multivariate, or means and variances if univariate).

## Usage

```
matdistl2dpar(meanL, varL)
```

### **Arguments**

meanL list of the means (p=1) or vector means (p>1) of the Gaussian densities. varL list of the variances (p=1) or covariance matrices (p>1) of the Gaussian densities.

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### Value

Positive symmetric matrix whose order is equal to the number of densities, consisting of the pairwise distances between the probability densities.

## Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### See Also

```
distl2dpar.
```

matdist12d for the distance matrix between probability densities which are estimated from the data.

### **Examples**

```
data(roses)

# Multivariate:
X <- roses[,c("Sha","Den","Sym","rose")]
summary(X)
mean.X <- as.list(by(X[, 1:3], X$rose, colMeans))
var.X <- as.list(by(X[, 1:3], X$rose, var))

# Gaussian densities, given parameters
matdistl2dpar(mean.X, var.X)

# Univariate :
X1 <- roses[,c("Sha","rose")]
summary(X1)
mean.X1 <- by(X1$Sha, X1$rose, mean)
var.X1 <- by(X1$Sha, X1$rose, var)

# Gaussian densities, given parameters
matdistl2dpar(mean.X1, var.X1)</pre>
```

mathellinger

Matrix of Hellinger distances between Gaussian densities

# Description

Computes the matrix of the Hellinger distances between several multivariate (p > 1) or univariate (p = 1) Gaussian densities given samples and using hellinger.

```
mathellinger(x)
```

mathellingerpar 135

### **Arguments**

Х

object of class "folder" containing the data. Its elements have only numeric variables (observations of the probability densities). If there are non numeric variables, there is an error.

### Value

Positive symmetric matrix whose order is equal to the number of densities, consisting of the pairwise Hellinger distances between the probability densities.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### See Also

```
hellinger.
```

mathellingerpar when the probability densities are Gaussian, given the parameters (means and variances).

### **Examples**

```
data(roses)
# Multivariate:
X <- as.folder(roses[,c("Sha","Den","Sym","rose")], groups = "rose")
summary(X)
mathellinger(X)
# Univariate :
X1 <- as.folder(roses[,c("Sha","rose")], groups = "rose")
summary(X1)
mathellinger(X1)</pre>
```

mathellingerpar

Matrix of Hellinger distances between Gaussian densities given their parameters

## **Description**

Computes the matrix of the Hellinger distances between several multivariate (p > 1) or univariate (p = 1) Gaussian densities, given their means and variances, using hellingerpar.

```
mathellingerpar(meanL, varL)
```

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### Arguments

meanL list of the means (p = 1) or vector means (p > 1) of the Gaussian densities. varL list of the variances (p = 1) or covariance matrices (p > 1) of the Gaussian densities.

### Value

Positive symmetric matrix whose order is equal to the number of densities, consisting of the pairwise distances between the Gaussian densities.

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### See Also

```
hellingerpar.
```

mathellinger for the distance matrix between probability densities which are estimated from the

# **Examples**

```
data(roses)

# Multivariate:
X <- roses[,c("Sha","Den","Sym","rose")]
summary(X)
mean.X <- as.list(by(X[, 1:3], X$rose, colMeans))
var.X <- as.list(by(X[, 1:3], X$rose, var))
mathellingerpar(mean.X, var.X)

# Univariate :
X1 <- roses[,c("Sha","rose")]
summary(X1)
mean.X1 <- by(X1$Sha, X1$rose, mean)
var.X1 <- by(X1$Sha, X1$rose, var)
mathellingerpar(mean.X1, var.X1)</pre>
```

matipl2d

Matrix of  $L^2$  inner products of probability densities

### Description

Computes the matrix of the  $L^2$  inner products between several multivariate (p > 1) or univariate (p = 1) probability densities, estimated from samples, using 12d.

```
matipl2d(x, method = "gaussiand", varwL = NULL)
```

matipl2d 137

### **Arguments**

Х

object of class "folder" containing the data. Its elements have only numeric variables (observations of the probability densities). If there are non numeric variables, there is an error.

method

string. It can be:

- "gaussiand" if the densities are considered to be Gaussian.
- "kern" if they are estimated using the Gaussian kernel method.

varwL

list of matrices. The smoothing bandwidths for the estimation of each probability density. If they are omitted, the smoothing bandwidths are computed using the normal reference rule matrix bandwidth (see details of the 12d function).

#### Value

Positive symmetric matrix whose order is equal to the number of densities, consisting of the pairwise inner products between the probability densities.

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### See Also

12d.

matipl2dpar when the probability densities are Gaussian, given the parameters (means and variances).

## **Examples**

```
data(roses)
 # Multivariate:
 X <- as.folder(roses[,c("Sha","Den","Sym","rose")], groups = "rose")</pre>
 summary(X)
 mean.X <- mean(X)</pre>
 var.X <- var.folder(X)</pre>
 # Parametrically estimated Gaussian densities:
 matipl2d(X)
# Estimated densities using the Gaussian kernel method (normal reference rule bandwidth):
 matipl2d(X, method = "kern")
 # Estimated densities using the Gaussian kernel method (bandwidth provided):
 matipl2d(X, method = "kern", varwL = var.X)
 # Univariate :
 X1 <- as.folder(roses[,c("Sha","rose")], groups = "rose")</pre>
 summary(X1)
 mean.X1 <- mean(X1)</pre>
```

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```
var.X1 <- var.folder(X1)

# Parametrically estimated Gaussian densities:
matipl2d(X1)

# Estimated densities using the Gaussian kernel method (normal reference rule bandwidth):
matipl2d(X1, method = "kern")

# Estimated densities using the Gaussian kernel method (bandwidth provided):
matipl2d(X1, method = "kern", varwL = var.X1)</pre>
```

matipl2dpar

Matrix of  $L^2$  inner products of Gaussian densities

## **Description**

Computes the matrix of the  $L^2$  inner products between several multivariate (p > 1) or univariate (p = 1) Gaussian densities, given their parameters (mean vectors and covariance matrices if the densities are multivariate, or means and variances if univariate).

### Usage

```
matipl2dpar(meanL, varL)
```

### **Arguments**

meanL list of the means (p = 1) or vector means (p > 1) of the Gaussian densities.

varL list of the variances (p = 1) or covariance matrices (p > 1) of the Gaussian

densities.

### Value

Positive symmetric matrix whose order is equal to the number of densities, consisting of the pairwise inner products between the Gaussian densities.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

## See Also

### 12dpar.

matipl2d for the distance matrix between probability densities which are estimated from the data.

matjeffreys 139

### **Examples**

```
data(roses)
# Multivariate:
X <- roses[,c("Sha","Den","Sym","rose")]
summary(X)
mean.X <- as.list(by(X[, 1:3], X$rose, colMeans))
var.X <- as.list(by(X[, 1:3], X$rose, var))
# Gaussian densities, given parameters
matipl2dpar(mean.X, var.X)
# Univariate :
X1 <- roses[,c("Sha","rose")]
summary(X1)
mean.X1 <- by(X1$Sha, X1$rose, mean)
var.X1 <- by(X1$Sha, X1$rose, var)
# Gaussian densities, given parameters
matipl2dpar(mean.X1, var.X1)</pre>
```

matjeffreys

Matrix of Jeffreys measures (symmetrised Kullback-Leibler divergences) between Gaussian densities

## **Description**

Computes the matrix of the Jeffreys measures between several multivariate (p > 1) or univariate (p = 1) Gaussian densities, given samples.

### Usage

```
matjeffreys(x)
```

### **Arguments**

Х

object of class "folder" containing the data. Its elements have only numeric variables (observations of the probability densities). If there are non numeric variables, there is an error.

## Value

Positive symmetric matrix whose order is equal to the number of densities, consisting of the pairwise Jeffreys measures between the Gaussian densities.

# Author(s)

140 matjeffreyspar

### See Also

matjeffreyspar if the parameters of the Gaussian densities are known.

## **Examples**

```
data(roses)

# Multivariate:
X <- as.folder(roses[,c("Sha","Den","Sym","rose")], groups = "rose")
summary(X)
matjeffreys(X)

# Univariate :
X1 <- as.folder(roses[,c("Sha","rose")], groups = "rose")
summary(X1)
matjeffreys(X1)</pre>
```

matjeffreyspar

Matrix of Jeffreys measures (symmetrised Kullback-Leibler divergences) between Gaussian densities

## **Description**

Computes the matrix of the Jeffreys measures between several multivariate (p > 1) or univariate (p = 1) Gaussian densities, given their parameters (mean vectors and covariance matrices if the densities are multivariate, or means and variances if univariate), using jeffreyspar.

### Usage

```
matjeffreyspar(meanL, varL)
```

# Arguments

meanL list of the means (p = 1) or vector means (p > 1) of the Gaussian densities.

varL list of the variances (p = 1) or covariance matrices (p > 1) of the probability

densities.

### Value

Positive symmetric matrix whose order is equal to the number of densities, consisting of the pairwise Jeffreys measures between the Gaussian densities.

### Author(s)

matwasserstein 141

#### See Also

```
jeffreyspar.
```

matjeffreys for the matrix of Jeffreys divergences between probability densities which are estimated from the data.

## **Examples**

```
data(roses)

# Multivariate:
X <- roses[,c("Sha","Den","Sym","rose")]
summary(X)
mean.X <- as.list(by(X[, 1:3], X$rose, colMeans))
var.X <- as.list(by(X[, 1:3], X$rose, var))
matjeffreyspar(mean.X, var.X)

# Univariate :
X1 <- roses[,c("Sha","rose")]
summary(X1)
mean.X1 <- by(X1$Sha, X1$rose, mean)
var.X1 <- by(X1$Sha, X1$rose, var)
matjeffreyspar(mean.X1, var.X1)</pre>
```

matwasserstein

Matrix of 2-Wassterstein distance between Gaussian densities

## **Description**

Computes the matrix of the 2-Wassterstein distances between several multivariate (p > 1) or univariate (p = 1) Gaussian densities, given samples.

### Usage

```
matwasserstein(x)
```

### **Arguments**

Χ

object of class "folder" containing the data. Its elements have only numeric variables (observations of the probability densities). If there are non numeric variables, there is an error.

#### Value

Positive symmetric matrix whose order is equal to the number of densities, consisting of the pairwise 2-Wassterstein distance between the Gaussian densities.

## Author(s)

142 matwassersteinpar

### See Also

matwassersteinpar if the parameters of the Gaussian densities are known.

# **Examples**

```
data(roses)

# Multivariate:
X <- as.folder(roses[,c("Sha","Den","Sym","rose")], groups = "rose")
summary(X)
matwasserstein(X)

# Univariate :
X1 <- as.folder(roses[,c("Sha","rose")], groups = "rose")
summary(X1)
matwasserstein(X1)</pre>
```

matwassersteinpar

Matrix of 2-Wasserstein distances between Gaussian densities

## **Description**

Computes the matrix of the 2-Wasserstein distances between several multivariate (p > 1) or univariate (p = 1) Gaussian densities, given their parameters (mean vectors and covariance matrices if the densities are multivariate, or means and variances if univariate), using wassersteinpar.

### Usage

```
matwassersteinpar(meanL, varL)
```

## **Arguments**

meanL list of the means (p = 1) or vector means (p > 1) of the Gaussian densities.

varL list of the variances (p = 1) or covariance matrices (p > 1) of the probability

densities.

### Value

Positive symmetric matrix whose order is equal to the number of densities, consisting of the pairwise 2-Wasserstein distances between the Gaussian densities.

#### Author(s)

mdsdd 143

#### See Also

wasserstein.

matwasserstein for the matrix of 2-Wasserstein distances between probability densities which are estimated from the data.

### **Examples**

```
data(roses)
# Multivariate:
X <- roses[,c("Sha","Den","Sym","rose")]
summary(X)
mean.X <- as.list(by(X[, 1:3], X$rose, colMeans))
var.X <- as.list(by(X[, 1:3], X$rose, var))
matwassersteinpar(mean.X, var.X)
# Univariate :
X1 <- roses[,c("Sha","rose")]
summary(X1)
mean.X1 <- by(X1$Sha, X1$rose, mean)
var.X1 <- by(X1$Sha, X1$rose, var)
matwassersteinpar(mean.X1, var.X1)</pre>
```

mdsdd

Multidimensional scaling of discrete probability distributions

### **Description**

Applies the multidimensional scaling (MDS) method to discrete probability distributions in order to describe T groups of individuals on which are observed q categorical variables. It returns an object of class mdsdd. It applies cmdscale to the distance matrix between the T distributions.

# Usage

```
mdsdd(xf, distance = c("11", "12", "chisqsym", "hellinger", "jeffreys", "jensen", "lp"),
    nb.factors = 3, nb.values = 10,
    association = c("cramer", "tschuprow", "pearson", "phi"),
    sub.title = "", plot.eigen = TRUE, plot.score = FALSE,
    nscore = 1:3, filename = NULL, group.name = "group", add = TRUE, p)
```

### **Arguments**

xf

object of class folder, or list of arrays (or tables).

• If it is a folder, its elements are data frames with q columns (considered as factors). The  $t^{th}$  element (t = 1, ..., T) matches with the  $t^{th}$  group.

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• If it is a list of arrays (or tables), the  $t^{th}$  element  $(t=1,\ldots,T)$  is the table of the joint frequency distribution of q variables within the  $t^{th}$  group. The frequency distribution is expressed with relative or absolute frequencies. These arrays have the same shape:

Each array (or table) xf[[i]] has:

- the same dimension(s). If q=1 (univariate),  $\dim(xf[[i]])$  is an integer. If q>1 (multivariate),  $\dim(xf[[i]])$  is an integer vector of length q.
- the same dimension names dimnames(xf[[i]]) (is non NULL). These dimnames are the names of the variables.

The elements of the arrays are non-negative numbers (if they are not, there is an error).

distance

The distance or divergence used to compute the distance matrix between the discrete distributions (see Details). It can be:

- "11" (default) the  $L^p$  distance with p=1
- "12" the  $L^p$  distance with p=2
- "chisqsym" the symmetric Chi-squared distance
- "hellinger" the Hellinger metric (Matusita distance)
- "jeffreys" the Jeffreys distance (symmetrised Kullback-Leibler divergence)
- "jensen" the Jensen-Shannon distance
- "lp" the  $L^p$  distance with p given by the argument p of the function.

nb.factors

numeric. Number of returned principal coordinates (default nb. factors = 3). This number must be less than T-1.

Warning: The plot.mdsdd and interpret.mdsdd functions cannot take into account more than nb.factors principal factors.

nb.values

numeric. Number of returned eigenvalues (default nb. values = 10).

association

The association measure between two discrete distributions to be used (see Details). It can be:

- "cramer" (default) Cramer's V (see cramer.folder).
- "tschuprow" Tschuprow's T (tschuprow.folder).
- "pearson" Pearson's contingency coefficient (pearson. folder).
- "phi" phi (phi.folder).

sub.title

string. Subtitle for the graphs (default NULL).

plot.eigen

logical. If TRUE (default), the barplot of the eigenvalues is plotted.

plot.score

logical. If TRUE, the graphs of new coordinates are plotted. A new graphic device is opened for each pair of coordinates defined by nscore argument.

nscore

numeric vector. If plot.score = TRUE, the numbers of the principal coordinates which are plotted. By default, nscore = 1:3. Its components cannot be greater than nb.factors.

filename

string. Name of the file in which the results are saved. By default (filename = NULL) they are not saved.

group.name

string. Name of the grouping variable. Default: groupname = "group".

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add logical indicating if an additive constant should be computed and added to the non diagonal dissimilarities such that the modified dissimilarities are Euclidean (default TRUE; see add argument of cmdscale).

integer. Optional. When distance = "lp" ( $L^p$  distance with p > 2), p is the parameter of the distance.

#### **Details**

p

If a folder is given as argument, the T discrete probability distributions  $f_t$  corresponding to the T groups of individuals are estimated from observations. Then the distances/dissimilarities between the estimated distributions are computed, using the distance or divergence defined by the distance argument:

If the distance is "11", "12" or "1p", the distances are computed by the function matddlppar. Otherwise, it can be computed by matddchisqsympar ("chisqsym"), matddhellingerpar ("hellinger"), matddjeffreyspar ("jeffreys") or matddjensenpar ("jensen").

The association measures are computed accordingly to the value of the parameter association The computation uses the corresponding function of the package DescTools (see Assocs). Notice that an association measure between a constant variable with and other variable is set to zero. The association measure between each variable with itself is not computed and the diagonal of the returned association matrices is set to NA.

#### Value

Returns an object of class mdsdd, that is a list including:

inertia data frame of the eigenvalues and the percentages of their sum.

scores data frame of the coordinates along the nb. factors first principal coordinates.

jointp list of arrays. The joint probability distribution for each group.

margins list of two data frames giving respectively:

- The probability distribution of each variable for each group. Each column of the data frame corresponds to one level of one categorical variable and contains the probabilities of this level in each group.
- The joint probability distribution of each pair of variables for each group. Each column of the data frame corresponds to one pair of levels of two categorical variables (one level per variable) and contains the probabilities of this pair of levels in each group.

associations list of T matrices. Each matrix corresponds to a group and gives the pairwise association measures between the q categorical variables.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

#### References

Cox, T.F., Cox, M.A.A. (2001). Multidimensional Scaling, second ed. Chapman & Hall/CRC. Saporta, G. (2006). Probabilit\'es, Analyse des donn\'ees et Statistique. Editions Technip, Paris.

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### See Also

print.mdsdd, plot.mdsdd, interpret.mdsdd

### **Examples**

```
# Example 1 with a folder (10 groups) of 3 factors
# obtained by converting numeric variables
data(roses)
xr = roses[,c("Sha", "Den", "Sym", "rose")]
xf = as.folder(xr, groups = "rose")
xf = cut(xf, breaks = list(c(0, 5, 7, 10), c(0, 4, 6, 10), c(0, 6, 8, 10)), cutcol = 1:3)
a = mdsdd(xf)
print(a)
print(a$jointp)
print(a$margins[[1]]) # equivalent to print(a$margins$margin1)
print(a$margins[[2]])
print(a$associations)
# Example 2 with a list of 7 arrays
data(dspg)
x1 = dspg
mdsdd(x1)
```

mean.folder

Means of a folder of data sets

### Description

Computes the means by column of the elements of an object of class folder.

### Usage

```
## S3 method for class 'folder'
mean(x, ..., na.rm = FALSE)
```

# Arguments

an object of class folder that is a list of data frames with the same column names.
 further arguments passed to or from other methods.
 logical. Should missing values (including NaN) be omitted from the calculations? (see mean or colMeans)

### **Details**

It uses colMeans to compute the mean by numeric column of each element of the folder. If some columns of the data frames are not numeric, there is a warning, and the means are computed on the numeric columns only.

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# Value

A list whose elements are the mean by column of the elements of the folder.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### See Also

folder to create an object of class folder. var. folder, cor. folder, skewness. folder, kurtosis. folder for other statistics for folder objects.

# Examples

```
# First example: iris (Fisher)
data(iris)
iris.fold <- as.folder(iris, "Species")
iris.means <- mean(iris.fold)
print(iris.means)

# Second example: roses
data(roses)
roses.fold <- as.folder(roses, "rose")
roses.means <- mean(roses.fold)
print(roses.means)</pre>
```

mtgcomponents

Components of upper scale of a vertex

# **Description**

For a vertex in an object of class foldermtg, computes its decomposition into vertices of an upper scale.

# Usage

```
mtgcomponents(x, vertex, scale)
```

# Arguments

x an object of class foldermtg.

vertex character. The identifier of a vertex. These identifiers are the rownames of the

data frame x\$topology.

scale integer. The scale of the components of vertex which will be returned.

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#### **Details**

If vertex is a vertex of scale i, then scale (the scale of the returned components of vertex) must be higher than i. For example, if vertex is a vertex of scale 2, then scale > 2, for instance scale = 3. The returned components are then vertices of scale 3 which have a decomposition relationship with vertex.

#### Value

A character vector, containing the idendifiers of the components of vertex.

If there is no component, then the returned vector is empty.

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

```
Cokelaer, T. and Pradal, C. (2010). MTG user guide (Functions for moving in MTGs)
```

#### See Also

```
read.mtg: reads a MTG file and builds an object of class foldermtg. mtgorder, mtgrank.
```

### **Examples**

```
mtgfile <- system.file("extdata/plant1.mtg", package = "dad")
xmtg <- read.mtg(mtgfile)

# Vertex of class "P" (plant, of scale 1), components of class 2 (axes: "A")
mtgcomponents(xmtg, vertex = "v01", scale = 2)

# Vertex of class "P" (plant, of scale 1), components of class 3 ("0", "M" and "I")
mtgcomponents(xmtg, vertex = "v01", scale = 3)

# Vertex of class "A" (stem, of scale 2), components of class 3 ("0", "M" and "I")
mtgcomponents(xmtg, vertex = "v12", scale = 3)</pre>
```

mtgorder

Branching order of vertices

# **Description**

Computes the branching order of vertices contained in an object of class foldermtg. The order of a vertex is the number of the column of topology, which contains this vertex.

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### Usage

```
mtgorder(x, classes = "all", display = FALSE)
```

# **Arguments**

X	an object of class foldermtg.
classes	character vector. The classes of entities for which the branching order is computed. If omitted, the branching orders are computed for all entities.
display	logical. If TRUE, the data frames of x corresponding to classes are displayed. Default: FALSE.

# **Details**

Returns x after appending the branching orders of the vertices of the classes given in the argument classes. The branching orders are appended to the data frames containing the vertices (one data frame per class) and the values of their corresponding features.

### Value

Returns an object of class foldermtg, that is a list of data frames.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

# References

```
Cokelaer, T. and Pradal, C. (2010). MTG user guide
```

### See Also

```
read.mtg: reads a MTG file and builds an object of class foldermtg. mtgorder.
```

```
mtgfile <- system.file("extdata/plant1.mtg", package = "dad")
xmtg <- read.mtg(mtgfile)

# The branching orders
ymtg <- mtgorder(xmtg)
print(ymtg)

# Add the branching orders to the 'foldermtg'
zmtg <- mtgorder(xmtg, display = TRUE)
print(zmtg)</pre>
```

mtgplant1

mtgplant1

Class foldermtg

### **Description**

These data produced by the SAGAH team (Sciences Agronomiques Appliquées à l'Horticulture, now Research Institute on Horticulture and Seeds), provide the topological structure of a rosebush.

#### Usage

```
data("mtgplant1")
```

#### **Format**

This object of class foldermtg is a list of 10 data frames:

• mtgplant1\$classes: data frame with 6 rows and 5 columns named SYMBOL (factor: the classes of the vertices), SCALE (integer: the scale at which they appear), DECOMPOSITION (factor), INDEXATION (factor) and DEFINITION (factor).

The vertex classes are:

- P: the whole plant (scale 1)
- A: the axes (scale 2)
- 0, M, I: the ..., metamers (phytomers) and inflorescences (scale 3)
- mtgplant1\$description: data frame with 8 rows and 4 columns (factors) named LEFT, RIGHT, RELTYPE and MAX.
- mtgplant1\$features: data frame with 13 rows and 2 columns (factors) named NAME and TYPE.
- mtgplant1\$topology: data frame with 88 rows and 4 columns:
  - order1, order2 and order3 (factors): the codes of the vertices, as they are found in the MTG table of the MTG file. The column on which a code appears gives the branching order of the corresponding vertex.
  - vertex (character): the same codes of vertices, on a single column.
- mtgplant1\$coordinates: data frame with 86 rows and 6 columns (numeric) named XX, YY and 22: cartesian coordinates of the vertices, and AA, BB and CC: an other coordinates system.
- mtgplant1\$P, mtgplant1\$A, mtgplant1\$M, mtgplant1\$I: data frames of the features on the vertices (all numeric).

### **Details**

This object of class foldermtg can be built by reading the data in a MTG file (see examples).

#### References

```
Cokelaer, T. and Pradal, C. (2010). MTG user guide
```

mtgplant2

### See Also

```
read.mtg: to read an MTG file and build an object of class MTG. mtgplant2: an other example of such data.
```

### **Examples**

```
data(mtgplant1)
print(mtgplant1)

# To read these data from a MTG file:
mtgfile1 <- system.file("extdata/plant1.mtg", package = "dad")
mtgplant1 <- read.mtg(mtgfile1)
print(mtgplant1)</pre>
```

mtgplant2

Class foldermtg

### **Description**

These data provides the topology of a bushy plant.

# Usage

```
data("mtgplant2")
```

#### **Format**

This object of class foldermtg is a list of 9 data frames:

• mtgplant2\$classes: data frame with 6 rows and 5 columns named SYMBOL (factor: the classes of the vertices), SCALE (integer: the scale at which they appear), DECOMPOSITION (factor), INDEXATION (factor) and DEFINITION (factor).

The vertex classes are:

- P: the whole plant (scale 1)
- A: the axes (scale 2)
- F, I: the flower and internodes (scale 3)
- mtgplant2\$description: data frame with 4 rows and 4 columns (factors) named LEFT, RIGHT, RELTYPE and MAX.
- mtgplant2\$features: data frame with 9 rows and 2 columns (factors) named NAME and TYPE.
- mtgplant2\$topology: data frame with 14 rows and 3 columns:
  - order1 and order2 (factors): the codes of the vertices, as they are found in the MTG table of the MTG file. The column on which a code appears gives the branching order of the corresponding vertex.
  - vertex (character): the same codes of vertices, on a single column.
- mtgplant2\$coordinates: data frame with 0 rows and 0 columns (there are no spatial coordinates in these MTG data).
- mtgplant2\$P, mtgplant2\$A, mtgplant2\$F and mtgplant2\$I: data frames of the features on the vertices (all numeric).

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#### **Details**

This object of class foldermtg can be built by reading the data in a MTG file (see examples).

#### References

```
Cokelaer, T. and Pradal, C. (2010). MTG user guide
```

### See Also

```
read.mtg: to read an MTG file and build an object of class MTG. mtgplant1: an other example of such data.
```

# **Examples**

```
data(mtgplant2)
print(mtgplant2)

# To read these data from a MTG file:
mtgfile2 <- system.file("extdata/plant2.mtg", package = "dad")
mtgplant2 <- read.mtg(mtgfile2)
print(mtgplant2)</pre>
```

mtgrank

Ranks of vertices in a decomposition

# **Description**

Computes the rank of the vertices contained in an object of class foldermtg. The vertex sequences resulting from a decomposition of other vertices, the rank of the vertices making up the sequences are computed from the beginning of the sequence or from its end. These ranks can be absolute or relative.

For example: ranks of the phytomeres and inflorescences in each stem.

### Usage

```
mtgrank(x, classe, parent.class = NULL, sibling.classes = NULL,
  relative = FALSE, from = c("origin", "end"), rank.name = "Rank",
  display = FALSE)
```

# Arguments

x an object of class foldermtg.

classe character. The class of the vertices for which the ranks are computed.

parent.class character. The class of the parent entities of those for which the ranks are computed. If omitted, the entities of scale maxscal -1, where maxscal is the highest scale in x data.

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sibling.classes

character vector. The classes of vertices appearing at the same scale as classe,

which are used in the computing of the ranks.

If omitted, only the vertices of class classe are used to compute the ranks.

relative logical. If TRUE, the relative ranks are computed, i.e. ranks from 0 to 1. Default:

FALSE.

from character. It can be "origin" (default) or "end".

If from = "origin", the ranks are computed from the origin to the end, i.e. from 1 to its maximum (from 0 to 1 if relative = TRUE). If from = "end", they are computed from the end to the origin, i.e. from the maximum to 1 (from 1 to 0 if

relative = TRUE).

rank.name character. Name of the rank column that is appended to x[[classe]]. The

default is "Rank".

display logical. If TRUE, the data frames of x corresponding to classes are displayed.

Default: FALSE.

#### **Details**

If the branching orders of the entities given by classe, parent.class and, if relevant, sibling.classes are not contained in x, mtgrank() uses mtgorder to compute them. The ranks are appended to the data frames containing the vertices (one data frame per class) and the values of their corresponding features.

### Value

Returns an object of class foldermtg, that is a list of data frames.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

```
Cokelaer, T. and Pradal, C. (2010). MTG user guide
```

#### See Also

```
read.mtg: reads a MTG file and builds an object of class foldermtg. mtgorder.
```

```
mtgfile <- system.file("extdata/plant1.mtg", package = "dad")
xmtg <- read.mtg(mtgfile)

ymtg <- mtgrank(xmtg, "M")
print(ymtg)

mtgrank(xmtg, "M", display = TRUE)</pre>
```

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```
mtgrank(xmtg, "M", parent.class = "A", display = TRUE)
mtgrank(xmtg, "M", parent.class = "A", sibling.classes = c("0", "I"), display = TRUE)
mtgrank(xmtg, "M", relative = TRUE, display = TRUE)
mtgrank(xmtg, "M", from = "origin", display = TRUE)
mtgrank(xmtg, "M", from = "end", display = TRUE)
```

plot.dstatis

Plotting scores of STATIS method (interstructure) analysis

### **Description**

Applies to an object of class dstatis (see details of the dstatis.inter function). Plots the scores.

# Usage

```
## S3 method for class 'dstatis'
plot(x, nscore = 1:3, sub.title = NULL, color = NULL, fontsize.points = 1.5, ...)
```

### Arguments

x object of class dstatis (returned by dstatis.inter).

nscore numeric vector. The numbers of the score vectors to be plotted.

Warning: Its components cannot be greater than the nb.factors argument in

the call of the dstatis.inter function.

sub.title string. Subtitle to be added to each graph.

color When provided, the colour of the symbols of each group. Can be a vector with

length equal to the number of groups.

fontsize.points

Numeric. Expansion of the characters (or symbols) of the groups on the graph.

This works as a multiple of par("cex") (see points).

... optional arguments to plot methods.

#### **Details**

Plots the principal scores returned by the dstatis.inter function. A new graphics window is opened for each pair of principal axes defined by the nscore argument.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### References

Lavit, C., Escoufier, Y., Sabatier, R., Traissac, P. (1994). The ACT (STATIS method). Computational Statistics & Data Analysis, 18 (1994), 97-119.

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### See Also

dstatis.inter; print.dstatis; interpret.dstatis.

### **Examples**

```
data(roses)
rosesf <- as.folder(roses[,c("Sha","Den","Sym","rose")])
# Dual STATIS on the covariance matrices
result <- dstatis.inter(rosesf, data.scaled = FALSE, group.name = "rose")
plot(result)</pre>
```

plot.fhclustd

Plotting a hierarchical clustering

### Description

Applies to an object of class fhelustd (see details of the fhelustd function). Plots the dendogram.

### Usage

# **Arguments**

```
    x object of class fhclustd (returned by fhclustd).
    labels, hang, check, axes, frame.plot, ann, main, sub, xlab, ylab
        Arguments concerning the graphical representation of the dendogram. See plot.hclust.

    Further graphical arguments.
```

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### See Also

fhclustd; print.fhclustd.

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### **Examples**

```
data(castles.dated)
xf <- as.folder(castles.dated$stones)
## Not run:
result <- fhclustd(xf)
plot(result)
plot(result, hang = -1)
## End(Not run)</pre>
```

plot.fmdsd

Plotting scores of multidimensional scaling of density functions

### **Description**

Applies to an object of class fmdsd (see the details section of the fmdsd function). Plots the scores.

### Usage

```
## S3 method for class 'fmdsd'
plot(x, nscore = 1:3, main="MDS of probability density functions",
    sub.title = NULL, color = NULL, fontsize.points = 1.5, ...)
```

# **Arguments**

x object of class fmdsd.

nscore numeric vector. The numbers of the score vectors to be plotted.

Warning: Its components cannot be greater than the nb.factors argument in

the call of the fmdsd function.

main this argument to title has an useful default here.

sub.title string. Subtitle to be added to each graph.

color When provided, the colour of the symbols of each group. Can be a vector with

length equal to the number of groups.

fontsize.points

Numeric. Expansion of the characters (or symbols) of the groups on the graph.

This works as a multiple of par("cex") (see points).

... optional arguments to plot methods.

#### **Details**

Plots the principal scores returned by the function fmdsd. A new graphics window is opened for each pair of principal score vectors defined by the nscore argument.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

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### References

Boumaza, R., Yousfi, S., Demotes-Mainard, S. (2015). Interpreting the principal component analysis of multivariate density functions. Communications in Statistics - Theory and Methods, 44 (16), 3321-3339.

### See Also

fmdsd; print.fmdsd; interpret.fmdsd.

# **Examples**

```
data(roses)
x <- roses[,c("Sha","Den","Sym","rose")]
rosesfold <- as.folder(x)
result <- fmdsd(rosesfold)
plot(result)</pre>
```

plot.foldert

Plotting data of a foldert

# **Description**

Applies to an object of class foldert (called foldert below) that is a list. Plots the longitudinal evolution of a numeric variable for every individuals.

### Usage

# Arguments

x	object of class foldert that is a list of data frames with the same column names, each of them corresponding to a time of observation.
which	character. Name of a column of the data frames of x. It gives the name of the variable to be plotted.
	For each element $x[[k]]$ of $x$ , $x[[k]]$ must be numeric. Otherwise, there is an error
na.inter	logical. If TRUE (default), for each individual, the missing values are deleted before plotting its evolution. If FALSE, the line corresponding to each individual is interrupted if there is a missing value, as for matplot.
type	character string (length 1 vector) or vector of 1-character strings (default "1") indicating the type of plot for each of the individuals followed among time, that is for each line of the data frames in the foldert. For further information about this argument, see matplot.

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ylim	ranges of y axis. xlim is as in matplot. See details.
ylab	a label for the y axis. Default: the name of the plotted variable (which argument). $ \\$
main	an overall title for the plot: see title.
	optional arguments to plot methods.

### **Details**

Internally, plot. foldert builds a matrix mdata containing the data of the variable given by which argument. The element mdata[ind,t] of this matrix is the value of the variable which for the individual ind: x[[t]][ind, which].

If the ylim argument is omitted, the range of y axis is given by range (mdata, na.rm = TRUE)  $\times$  c(0, 1.2).

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

Boumaza, R., Yousfi, S., Demotes-Mainard, S. (2015). Interpreting the principal component analysis of multivariate density functions. Communications in Statistics - Theory and Methods, 44 (16), 3321-3339.

### See Also

foldert: object of class foldert. as.foldert.data.frame: build an object of class foldert from a data frame. as.foldert.array: build an object of class foldert from a 3d-array.

#### **Examples**

plot.fpcad

Plotting scores of principal component analysis of density functions

### **Description**

Applies to an object of class fpcad (see details of the fpcad function). Plots the scores.

#### **Usage**

```
## S3 method for class 'fpcad'
plot(x, nscore = 1:3, main = "PCA of probability density functions",
    sub.title = NULL, color = NULL, fontsize.points = 1.5, ...)
```

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# **Arguments**

x object of class fpcad (returned by fpcad).

nscore numeric vector. The numbers of the score vectors to be plotted.

Warning: Its components cannot be greater than the nb. factors argument in

the call of the fpcad function.

main this argument to title has an useful default here.

sub.title string. Subtitle to be added to each graph.

color When provided, the colour of the symbols of each group. Can be a vector with

length equal to the number of groups.

fontsize.points

Numeric. Expansion of the characters (or symbols) of the groups on the graph.

This works as a multiple of par("cex") (see points).

... optional arguments to plot methods.

### **Details**

Plots the principal scores returned by the fpcad function. A new graphics window is opened for each pair of principal axes defined by the nscore argument.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

Boumaza, R., Yousfi, S., Demotes-Mainard, S. (2015). Interpreting the principal component analysis of multivariate density functions. Communications in Statistics - Theory and Methods, 44 (16), 3321-3339.

### See Also

fpcad; print.fpcad; interpret.fpcad.

```
data(roses)
rosefold <- as.folder(roses[,c("Sha","Den","Sym","rose")])
result <- fpcad(rosefold)
plot(result)</pre>
```

plot.fpcat

plot.fpcat Plotting scores of principal component analysis of density fun among time	ctions
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# **Description**

Applies to an object of class fpcat (see details of the fpcat function). Plots the scores.

#### **Usage**

```
## S3 method for class 'fpcat'
plot(x, nscore = 1:3, main = "PCA of probability density functions",
    sub.title = NULL, ...)
```

# **Arguments**

x object of class fpcat (returned by fpcat).

nscore numeric vector. The numbers of the score vectors to be plotted.

Warning: Its components cannot be greater than the nb. factors argument in the call of the fpcat function.

main this argument to title has an useful default here.

sub.title string. Subtitle to be added to each graph.

optional arguments to plot methods.

#### **Details**

#### Plots:

- the principal scores returned by the fpcat function with arrows from the point corresponding to each time to the next one; a new graphics window is opened for each pair of principal axes defined by the nscore argument.
- the principal scores among time with arrows from each time to the next one; a new graphics window is opened for each principal axis defined by the nscore argument.

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### References

Boumaza, R., Yousfi, S., Demotes-Mainard, S. (2015). Interpreting the principal component analysis of multivariate density functions. Communications in Statistics - Theory and Methods, 44 (16), 3321-3339.

### See Also

```
fpcat; print.fpcat
```

plot.hclustdd 161

### **Examples**

```
times <- as.Date(c("2017-03-01", "2017-04-01", "2017-05-01", "2017-06-01"))
x1 <- data.frame(z1=rnorm(6,1,5), z2=rnorm(6,3,3))
x2 <- data.frame(z1=rnorm(6,4,6), z2=rnorm(6,5,2))
x3 <- data.frame(z1=rnorm(6,7,2), z2=rnorm(6,8,4))
x4 <- data.frame(z1=rnorm(6,9,3), z2=rnorm(6,10,2))
ft <- foldert(x1, x2, x3, x4, times = times, rows.select="intersect")
print(ft)
result <- fpcat(ft)
plot(result)</pre>
```

plot.hclustdd

Plotting a hierarchical clustering of discrete distributions

### **Description**

Applies to an object of class helustdd (see details of the helustdd function). Plots the dendogram.

#### Usage

### **Arguments**

```
x object of class hclustdd (returned by hclustdd).
labels, hang, check, axes, frame.plot, ann, main, sub, xlab, ylab
Arguments concerning the graphical representation of the dendogram. See plot.hclust.
... Further graphical arguments.
```

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### See Also

hclustdd; print.hclustdd.

```
data(dspg)
xl = dspg
result <- hclustdd(xl)
plot(result)
plot(result, hang = -1)</pre>
```

162 plot.mdsdd

plot.mdsdd	Plotting scores of multidimensional scaling analysis of discrete distributions

# **Description**

Applies to an object of class "mdsdd" (see the details section of the mdsdd function). Plots the scores.

### Usage

```
## S3 method for class 'mdsdd'
plot(x, nscore = 1:2, main="MDS of probability density functions",
    sub.title = NULL, color = NULL, fontsize.points = 1.5, ...)
```

### **Arguments**

x object of class mdsdd.

nscore numeric vector. The numbers of the score vectors to be plotted.

Warning: Its components cannot be greater than the nb.factors argument in

the call of the fmdsd function.

main this argument to title has an useful default here.

sub.title string. Subtitle to be added to each graph.

color When provided, the colour of the symbols of each group. Can be a vector with

length equal to the number of groups.

fontsize.points

Numeric. Expansion of the characters (or symbols) of the groups on the graph.

This works as a multiple of par("cex") (see points).

... optional arguments to plot methods.

### Details

Plots the principal scores returned by the function mdsdd. A new graphics window is opened for each pair of principal score vectors defined by the nscore argument.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

#### See Also

mdsdd; print.mdsdd; interpret.mdsdd.

plotframes 163

### **Examples**

```
# INSEE (France): Diploma x Socio professional group, seven years.
data(dspg)
xlista = dspg
a <- mdsdd(xlista)
plot(a)</pre>
```

plotframes

Plotting of two sets of variables

# **Description**

Plots a set of numeric variables vs. another set and prints the pairwise correlations. It uses the function xyplot of lattice package.

### Usage

```
plotframes(x, y, xlab = NULL, ylab = NULL, font.size = 12, layout = NULL)
```

# Arguments

x	data frame (can also be a tibble). Variables on x coordinates.
У	data frame (or tibble). Variables on y coordinates.
xlab	a label for the x axis, by default the column names of y.
ylab	a label for the y axis (by default there is no label).
font.size	integer. Size of the characters in the strips.
layout	numeric vector of length 2 or 3 giving the number of columns, rows, and optionally pages of the lattice. If omitted, the graphs will be displayed on 3 lines and 3 columns, with a number of pages set to the required number.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

```
require(MASS)
mx <- c(0,0)
vx <- matrix(c(1,0,0,1),ncol = 2)
my <- c(0,1)
vy <- matrix(c(4,1,1,9),ncol = 2)
x <- as.data.frame(mvrnorm(n = 10, mu = mx, Sigma = vx))
y <- as.data.frame(mvrnorm(n = 10, mu = my, Sigma = vy))
colnames(x) <- c("x1", "x2")
colnames(y) <- c("y1", "y2")
plotframes(x, y)</pre>
```

164 print.discdd.misclass

 $\begin{array}{c} \textit{print.discdd.misclass} & \textit{Printing results of discriminant analysis of discrete probability distributions} \\ & \textit{butions} \end{array}$ 

### **Description**

Applies to an object of class "discdd.misclass". Prints the numerical results of discdd.misclass.

# Usage

```
## S3 method for class 'discdd.misclass'
print(x, dist.print=FALSE, prox.print=FALSE, digits=2, ...)
```

#### **Arguments**

X	object of class "discdd.misclass", returned by discdd.misclass.
dist.print	logical. Its default value is FALSE. If TRUE, prints the matrix of distances between, on one side, the groups (densities) and, on the other side, the classes (of groups or densities).
prox.print	logical. Its default value is FALSE. If TRUE, prints the matrix of proximity indices (in percent) )between, on one side, the groups (densities) and, on the other side, the classes (of groups or densities).
digits	numeric. Number of significant digits for the display of numerical results.
	optional arguments to print methods.

### **Details**

By default, are printed the whole misallocation ratio, the confusion matrix (allocations versus origins) with the misallocation ratios per class, and the data frame whose rows are the groups, and whose columns are the origin classes and allocation classes, and a logical variable indicating misclassification.

If dist.print = TRUE or prox.print = TRUE, the distances or proximity indices (in percent) between groups and classes, are displayed.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### References

Rudrauf, J.M., Boumaza, R. (2001). Contribution ? l'étude de l'architecture médiévale: les caractéristiques des pierres à bossage des châteaux forts alsaciens, Centre de Recherches Archéologiques médiévales de Saverne, 5, 5-38.

### See Also

discdd.misclass; print.

print.discdd.predict 165

### **Examples**

```
data("castles.dated")
stones <- castles.dated$stones
periods <- castles.dated$periods
stones$height <- cut(stones$height, breaks = c(19, 27, 40, 71), include.lowest = TRUE)
stones$width <- cut(stones$width, breaks = c(24, 45, 62, 144), include.lowest = TRUE)
stones$edging <- cut(stones$edging, breaks = c(0, 3, 4, 8), include.lowest = TRUE)
stones$boss <- cut(stones$boss, breaks = c(0, 6, 9, 20), include.lowest = TRUE )

castlefh <- folderh(periods, "castle", stones)

res <- discdd.misclass(castlefh, "period")
print(res)</pre>
```

print.discdd.predict Printing results of discriminant analysis of discrete probability distributions

### **Description**

print function, applied to an object of class "discdd.predict", prints numerical results of discdd.predict .

### Usage

```
## S3 method for class 'discdd.predict'
print(x, dist.print=TRUE, prox.print=FALSE, digits=2, ...)
```

# Arguments

X	object of class "discdd.predict", returned by discdd.predict.
dist.print	logical. If TRUE (the default), prints the matrix of distances between, on one side, the groups (densities) and, on the other side, the classes (of groups or densities).
prox.print	logical. Its default value is FALSE. If TRUE, prints the matrix of proximity indices between, on one side, the groups (densities) and, on the other side, the classes (of groups or densities).
digits	numerical. Number of significant digits for the display of numerical results.
	optional arguments to print methods.

# **Details**

By default, are printed:

• if available (if misclass.ratio argument of discdd.predict was TRUE), the whole misallocation ratio, the confusion matrix (allocations versus origins) and the misallocation ratio per class are printed.

print.dstatis

• the data frame the rows of which are the groups, and the columns of which are of the origin (NA if not available) and allocation classes.

If dist.print = TRUE or prox.print = TRUE, the distances or proximity indices between groups and classes, are displayed.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

Rudrauf J.M., Boumaza R. (2001). Contribution ? l'?tude de l'architecture m?di?vale: les caract?ristiques des pierres ? bossage des ch?teaux forts alsaciens. Centre de Recherches Arch?ologiques M?di?vales de Saverne, 5, 5-38.

### See Also

discdd.predict; print.

### **Examples**

```
data(castles.dated)
data(castles.nondated)
stones <- rbind(castles.dated$stones, castles.nondated$stones)
periods <- rbind(castles.dated$periods, castles.nondated$periods)
stones$height <- cut(stones$height, breaks = c(19, 27, 40, 71), include.lowest = TRUE)
stones$width <- cut(stones$width, breaks = c(24, 45, 62, 144), include.lowest = TRUE)
stones$edging <- cut(stones$edging, breaks = c(0, 3, 4, 8), include.lowest = TRUE)
stones$boss <- cut(stones$boss, breaks = c(0, 6, 9, 20), include.lowest = TRUE)

castlesfh <- folderh(periods, "castle", stones)

result <- discdd.predict(castlesfh, "period")
print(result)
print(result, prox.print=TRUE)</pre>
```

print.dstatis

Printing results of STATIS method (interstructure) analysis

# **Description**

Applies to an object of class "dstatis". Prints the numeric results returned by the dstatis.inter function.

### Usage

```
## S3 method for class 'dstatis'
print(x, mean.print = FALSE, var.print = FALSE,
    cor.print = FALSE, skewness.print = FALSE, kurtosis.print = FALSE,
    digits = 2, ...)
```

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# **Arguments**

x	object of class "dstatis", returned by the dstatis.inter function.
mean.print	logical. If TRUE, prints for each group the means and standard deviations of the variables and the norm of the density.
var.print	logical. If TRUE, prints for each group the variances and covariances of the variables.
cor.print	logical. If TRUE, prints for each group the correlations between the variables.
skewness.print	logical. If TRUE, prints for each group the skewness coefficients of the variables.
kurtosis.print	logical. If TRUE, prints for each group the kurtosis coefficients of the variables.
digits	numeric. Number of significant digits for the display of numeric results.
	optional arguments to print methods.

# **Details**

By default, are printed the inertia explained by the nb.values (see dstatis.inter) first principal components, the contributions, the qualities of representation of the densities along the nb.factors (see dstatis.inter) first principal components, and the principal scores.

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

Lavit, C., Escoufier, Y., Sabatier, R., Traissac, P. (1994). The ACT (STATIS method). Computational Statistics & Data Analysis, 18 (1994), 97-119.

### See Also

dstatis.inter; plot.dstatis; interpret.dstatis; print.dstatis.

```
data(roses)
rosesf <- as.folder(roses[,c("Sha","Den","Sym","rose")])
# Dual STATIS on the covariance matrices
result <- dstatis.inter(rosesf, data.scaled = FALSE, group.name = "rose")
print(result)</pre>
```

168 print.fdiscd.misclass

print.fdiscd.misclass Printing results of discriminant analysis of probability density functions

#### **Description**

Applies to an object of class "fdiscd.misclass". Prints the numerical results of fdiscd.misclass.

# Usage

```
## S3 method for class 'fdiscd.misclass'
print(x, dist.print=FALSE, prox.print=FALSE, digits=2, ...)
```

### **Arguments**

X	object of class "fdiscd.misclass", returned by fdiscd.misclass.
dist.print	logical. Its default value is FALSE. If TRUE, prints the matrix of distances between, on one side, the groups (densities) and, on the other side, the classes (of groups or densities).
prox.print	logical. Its default value is FALSE. If TRUE, prints the matrix of proximity indices (in percent) )between, on one side, the groups (densities) and, on the other side, the classes (of groups or densities).
digits	numeric. Number of significant digits for the display of numerical results.
	optional arguments to print methods.

# Details

By default, are printed the whole misallocation ratio, the confusion matrix (allocations versus origins) with the misallocation ratios per class, and the data frame whose rows are the groups, and whose columns are the origin classes and allocation classes, and a logical variable indicating misclassification.

If dist.print = TRUE or prox.print = TRUE, the distances or proximity indices (in percent) between groups and classes, are displayed.

### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### References

Boumaza, R. (2004). Discriminant analysis with independently repeated multivariate measurements: an  $L^2$  approach. Computational Statistics & Data Analysis, 47, 823-843.

Rudrauf, J.M., Boumaza, R. (2001). Contribution à l'étude de l'architecture médiévale: les caractéristiques des pierres à bossage des châteaux forts alsaciens, Centre de Recherches Archéologiques Médiévales de Saverne, 5, 5-38.

print.fdiscd.predict 169

### See Also

fdiscd.misclass; print.

### **Examples**

```
data(castles.dated)
castlesfh <- folderh(castles.dated$periods, "castle", castles.dated$stones)
result <- fdiscd.misclass(castlesfh, "period")
print(result)
print(result, dist.print=TRUE)
print(result, prox.print=TRUE)</pre>
```

print.fdiscd.predict

Printing results of discriminant analysis of probability density functions

# Description

print function, applied to an object of class "fdiscd.predict", prints numerical results of fdiscd.predict .

#### Usage

```
## S3 method for class 'fdiscd.predict'
print(x, dist.print=TRUE, prox.print=FALSE, digits=2, ...)
```

### **Arguments**

X	object of class "fdiscd.predict", returned by fdiscd.predict.
dist.print	logical. If TRUE (the default), prints the matrix of distances between, on one side, the groups (densities) and, on the other side, the classes (of groups or densities).
prox.print	logical. Its default value is FALSE. If TRUE, prints the matrix of proximity indices between, on one side, the groups (densities) and, on the other side, the classes (of groups or densities).
digits	numerical. Number of significant digits for the display of numerical results.
	optional arguments to print methods.

# **Details**

By default, are printed:

- if available (if misclass.ratio argument of fdiscd.predict was TRUE), the whole misallocation ratio, the confusion matrix (allocations versus origins) and the misallocation ratio per class are printed.
- the data frame the rows of which are the groups, and the columns of which are of the origin (NA if not available) and allocation classes.

If dist.print = TRUE or prox.print = TRUE, the distances or proximity indices between groups and classes, are displayed.

170 print.fhclustd

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

Boumaza, R. (2004). Discriminant analysis with independently repeated multivariate measurements: an  $L^2$  approach. Computational Statistics & Data Analysis, 47, 823-843.

Rudrauf J.M., Boumaza R. (2001). Contribution à l'étude de l'architecture médiévale: les caractéristiques des pierres à bossage des châteaux forts alsaciens. Centre de Recherches Archéologiques Médiévales de Saverne, 5, 5-38.

#### See Also

fdiscd.predict; print.

# **Examples**

```
data(castles.dated)
data(castles.nondated)
castles.stones <- rbind(castles.dated$stones, castles.nondated$stones)
castles.periods <- rbind(castles.dated$periods, castles.nondated$periods)
castlesfh <- folderh(castles.periods, "castle", castles.stones)
result <- fdiscd.predict(castlesfh, "period")
print(result)
print(result, prox.print=TRUE)</pre>
```

print.fhclustd

Printing results of a hierarchical clustering of probability density functions

### Description

print function, applied to an object of class "fhclustd", prints numerical results of fhclustd .

# Usage

```
## S3 method for class 'fhclustd'
print(x, dist.print=FALSE, digits=2, ...)
```

### **Arguments**

```
x object of class "fhclustd", returned by fhclustd.

dist.print logical. If TRUE (default: FALSE), prints the matrix of distances between the groups (densities).

digits numerical. Number of significant digits for the display of numerical results.

optional arguments to print methods.
```

print.fmdsd 171

### **Details**

If dist.print = TRUE, the distances between groups are displayed.

By default, the result of the clustering is printed. The display is the same as that of the print.hclust function.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

# See Also

fhclustd; print.

### **Examples**

```
data(castles.dated)
xf <- as.folder(castles.dated$stones)
## Not run:
result <- fhclustd(xf)
print(result)
print(result, dist.print = TRUE)
## End(Not run)</pre>
```

print.fmdsd

Printing results of a multidimensional scaling analysis of probability densities

# **Description**

Applies to an object of class "fmdsd". Prints the numeric results returned by the fmdsd function.

# Usage

```
## S3 method for class 'fmdsd'
print(x, mean.print = FALSE, var.print = FALSE,
    cor.print = FALSE, skewness.print = FALSE, kurtosis.print = FALSE,
    digits = 2, ...)
```

### **Arguments**

X	object of class "fmdsd", returned by the fmdsd function.
mean.print	logical. If TRUE, prints for each group the means and standard deviations of the variables and the norm of the density.
var.print	logical. If TRUE, prints for each group the variances and covariances of the variables.
cor.print	logical. If TRUE, prints for each group the correlations between the variables.

172 print.foldermtg

```
skewness.print logical. If TRUE, prints for each group the skewness coefficients of the variables. kurtosis.print logical. If TRUE, prints for each group the kurtosis coefficients of the variables. digits numeric. Number of significant digits for the display of numeric results. optional arguments to print methods.
```

#### **Details**

By default, are printed the inertia explained by the nb.values (see fmdsd) first coordinates and the nb.factors (see fmdsd) coordinates of the densities.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

Boumaza, R., Yousfi, S., Demotes-Mainard, S. (2015). Interpreting the principal component analysis of multivariate density functions. Communications in Statistics - Theory and Methods, 44 (16), 3321-3339.

#### See Also

fmdsd; plot.fmdsd; interpret.fmdsd; print.

# **Examples**

```
data(roses)
x <- roses[,c("Sha","Den","Sym","rose")]
rosesfold <- as.folder(x)
result <- fmdsd(rosesfold)
print(result)
print(result, mean.print = TRUE)</pre>
```

print.foldermtg

Printing an object of class foldermtg

### **Description**

print function, applied to an object of class "foldermtg", prints an MTG (Multiscale Tree Graph) folder, as returned by foldermtg function.

### Usage

```
## S3 method for class 'foldermtg'
print(x, classes = TRUE, description = FALSE, features = TRUE,
topology = FALSE, coordinates = FALSE, ...)
```

print.foldermtg 173

# Arguments

X	an object of class foldermtg.
classes	logical. If TRUE (default), prints the data frame describing the classes (CLASSES: table in the MTG file).
description	logical. If TRUE (default: FALSE), prints the description data frame (DESCRIPTION: table in the MTG file).
features	logical. If TRUE (default), prints the data frame of the features and their types (FEATURES: table in the MTG file).
topology	logical. If TRUE (default: FALSE), prints the data frame of the plant topology.
coordinates	logical. If TRUE (default: $FALSE$ ), prints the spatial coordinates of the entities of the plant.
• • •	optional arguments to print methods.

### **Details**

If classes, description or features are TRUE, the corresponding data frames are displayed.

If topology = TRUE, the plant structure is displayed; and if coordinates = TRUE, the spatial coordinates are displayed.

By default, the data frames containing the features on the vertices per class are printed.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

```
Cokelaer, T. and Pradal, C. (2010). MTG user guide
```

### See Also

read.mtg: reads a MTG file and creates an object of class "foldermtg".

```
mtgfile1 <- system.file("extdata/plant1.mtg", package = "dad")
xmtg1 <- read.mtg(mtgfile1)
print(xmtg1)
print(xmtg1, topology = TRUE)
print(xmtg1, coordinates = TRUE)

mtgfile2 <- system.file("extdata/plant2.mtg", package = "dad")
xmtg2 <- read.mtg(mtgfile2)
print(xmtg2)
print(xmtg2, topology = TRUE)
print(xmtg2, coordinates = TRUE)</pre>
```

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print.foldert

Printing an object of class foldert

### **Description**

print function, applied to an object of class "foldert", prints a foldert, as returned by foldert or as. foldert function.

# Usage

```
## S3 method for class 'foldert'
print(x, ...)
```

# **Arguments**

x an object of class foldert.

... optional arguments to print methods.

### **Details**

The foldert is printed. In any data frame x[[t]] of this foldert, if a row is entirely NA (which means that the corresponding individual was not observed at time t), this row are not printed.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### See Also

foldert: object of class foldert. as.foldert.data.frame: build an object of class foldert from a data frame. as.foldert.array: build an object of class foldert from a 3d-array.

```
data(floribundity)
ft <- foldert(floribundity, cols.select = "union", rows.select = "union")
print(ft)</pre>
```

print.fpcad 175

print.fpcad	Printing results of a functional PCA of probability densities

### **Description**

Applies to an object of class "fpcad". Prints the numeric results returned by the fpcad function.

# Usage

```
## S3 method for class 'fpcad'
print(x, mean.print = FALSE, var.print = FALSE,
    cor.print = FALSE, skewness.print = FALSE, kurtosis.print = FALSE,
    digits = 2, ...)
```

# Arguments

X	object of class "fpcad", returned by the fpcad function.
mean.print	logical. If TRUE, prints for each group the means and standard deviations of the variables and the norm of the density.
var.print	logical. If TRUE, prints for each group the variances and covariances of the variables.
cor.print	logical. If TRUE, prints for each group the correlations between the variables.
skewness.print	logical. If TRUE, prints for each group the skewness coefficients of the variables.
kurtosis.print	logical. If TRUE, prints for each group the kurtosis coefficients of the variables.
digits	numeric. Number of significant digits for the display of numeric results.
	optional arguments to print methods.

#### **Details**

By default, are printed the inertia explained by the nb.values (see fpcad) first principal components, the contributions, the qualities of representation of the densities along the nb.factors (see fpcad) first principal components, and the principal scores.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

Boumaza, R., Yousfi, S., Demotes-Mainard, S. (2015). Interpreting the principal component analysis of multivariate density functions. Communications in Statistics - Theory and Methods, 44 (16), 3321-3339.

### See Also

fpcad; plot.fpcad; interpret.fpcad; print.

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### **Examples**

```
data(roses)
rosefold <- as.folder(roses[,c("Sha","Den","Sym","rose")])
result <- fpcad(rosefold)
print(result)
print(result, mean.print = TRUE)</pre>
```

print.fpcat

Printing results of a functional PCA of probability densities among time

#### **Description**

Applies to an object of class "fpcat". Prints the numeric results returned by the fpcat function.

# Usage

```
## S3 method for class 'fpcat'
print(x, mean.print = FALSE, var.print = FALSE,
    cor.print = FALSE, skewness.print = FALSE, kurtosis.print = FALSE,
    digits = 2, ...)
```

# **Arguments**

Χ	object of class "fpcat", returned by the fpcat function.
mean.print	logical. If TRUE, prints for each observation time the means and standard deviations of the variables and the norm of the density.
var.print	logical. If TRUE, prints for each observation time the variances and covariances of the variables.
cor.print	logical. If TRUE, prints for each observation time the correlations between the variables.
skewness.print	logical. If TRUE, prints for each observation time the skewness coefficients of the variables.
kurtosis.print	logical. If TRUE, prints for each observation time the kurtosis coefficients of the variables.
digits	numeric. Number of significant digits for the display of numeric results.
	optional arguments to print methods.

### **Details**

By default, are printed the vector of observation times (numeric, ordered factor or object of class "Date"), the inertia explained by the nb.values (see fpcat) first principal components, the contributions, the qualities of representation of the densities along the nb.factors (see fpcat) first principal components, and the principal scores.

print.hclustdd 177

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

Boumaza, R., Yousfi, S., Demotes-Mainard, S. (2015). Interpreting the principal component analysis of multivariate density functions. Communications in Statistics - Theory and Methods, 44 (16), 3321-3339.

#### See Also

fpcat; plot.fpcat; print.

### **Examples**

```
times <- as.Date(c("2017-03-01", "2017-04-01", "2017-05-01", "2017-06-01"))
x1 <- data.frame(z1=rnorm(6,1,5), z2=rnorm(6,3,3))
x2 <- data.frame(z1=rnorm(6,4,6), z2=rnorm(6,5,2))
x3 <- data.frame(z1=rnorm(6,7,2), z2=rnorm(6,8,4))
x4 <- data.frame(z1=rnorm(6,9,3), z2=rnorm(6,10,2))
ft <- foldert(x1, x2, x3, x4, times = times, rows.select="intersect")
print(ft)
result <- fpcat(ft)

print(result, mean.print = TRUE, var.print = TRUE)</pre>
```

print.hclustdd

Printing results of a hierarchical clustering of discrete distributions

### **Description**

print function, applied to an object of class "hclustdd", prints numerical results of hclustdd .

### Usage

```
## S3 method for class 'hclustdd'
print(x, dist.print=FALSE, digits=2, ...)
```

### **Arguments**

```
x object of class "hclustdd", returned by hclustdd.

dist.print logical. If TRUE (default: FALSE), prints the matrix of distances between the groups (densities).

digits numerical. Number of significant digits for the display of numerical results.

optional arguments to print methods.
```

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# **Details**

If dist.print = TRUE, the distances between groups are displayed.

By default, the result of the clustering is printed. The display is the same as that of the print.hclust function.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### See Also

hclustdd; plot.hclustdd.

# **Examples**

```
data(dspg)
xl = dspg
result <- hclustdd(xl)
print(result)
print(result, dist.print = TRUE)</pre>
```

print.mdsdd

Printing results of a multidimensional scaling analysis of discrete distributions

# Description

Applies to an object of class "mdsdd". Prints the numeric results returned by the mdsdd function.

### Usage

# Arguments

X	object of class "mdsdd", returned by the mdsdd function.
joint	logical. If TRUE, prints for each group the table of estimated joint distribution.
margin1	logical. If TRUE, prints for each group the data frame of estimated marginal distributions.
margin2	logical. If TRUE, prints for each group the data frame of the estimated marginal distributions per combination of two variables.
association	logical. If TRUE, prints for each group the matrix of the pairwise association measures of the variables.
	optional arguments to print methods.

read.mtg 179

#### **Details**

By default, are printed the inertia explained by the nb.values (see mdsdd) first coordinates and the nb.factors (see mdsdd) coordinates of the densities.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Sabine Demotes-Mainard

#### See Also

```
mdsdd; plot.mdsdd; interpret.mdsdd
```

### **Examples**

```
# INSEE (France): Diploma x Socio professional group, seven years.
data(dspg)
xlista = dspg
a <- mdsdd(xlista)
print(a, joint = TRUE, margin1 = TRUE, margin2 = TRUE)</pre>
```

read.mtg

Read a MTG (Multiscale Tree Graph) file

# Description

Reads an MTG (Multiscale Tree Graph) file and returns an object of class foldermtg, that is a list of data frames (see Details).

### Usage

```
read.mtg(file, ...)
```

# Arguments

```
file character. Path of the MTG file.
... optional arguments to print methods.
```

#### **Details**

Recalling that a MTG file is a text file that can be opened with a spreadsheet (Excel, LibreOffice-Calc...). Its 4 tables are:

- **CLASSES:** In this table the first column, named SYMBOL, contains the symbolic character denoting each botanical entity (or vertex class, plant component...) used in the MTG (for example, P for plant, A for axis...). The second column, named SCALE, represents the scale at which each entity appears in the MTG (for example 1 for P, 2 for axis...).
- **DESCRIPTION:** This table displays the relations between the vertices: + (branching relationship) or < (successor relationship).

180 read.mtg

• **FEATURES:** This table contains the features that can be attached to the vertices and their types: INT (integer), REAL (real numbers), STRING (character)...

• MTG: This table describes the plant topology, that is the vertices (one vertex per row) and their relations, the spatial coordinates of each vertex and the values taken by each vertex on the above listed features.

Each vertex is labelled by its class, designating its botanical entity, and its index, designating its position among its immediate neighbours having the same scale. Each vertex label is preceded by + or <, seen above, or by the symbol / (decomposition relationship) that means that the corresponding vertex is the first vertex of the decomposition of the vertex which precedes /.

Notice that the column number of a vertex matches with its branching order. The vertices of scale k resulting from the decomposition of a vertex of scale k-1, named parent vertex, have the same order as that of the parent vertex.

See the example below.

#### Value

read.mtg returns an object, say x, of class fodermtg, that is a list of at least 6 data frames:

classes the table CLASSES: in the MTG file.

description the table DESCRIPTION: in the MTG file.

features the table FEATURES: in the MTG file.

topology data frame containing the first columns of the "MTG:" table of the MTG file. If

the maximum branching order of the elements of the MTG is p, then x\$topology

has p columns.

If the i-th vertex appears on the j-th column, it means that its branching order

is j, that is it belongs to a vertex of the j-th order.

coordinates data frame of the spatial coordinates of the entities. It has six columns: XX,

YY, ZZ (cartesian coordinates), AA, BB, CC (angle coordinates). If there are no

coordinates in the MTG file, this data frame has 0 row.

The sixth and following elements are nclass data frames, nclass being the number of classes in the MTG file. Each data frame matches with a vertex class, such as "P" (plant), "A" (axes), "M" (metamers or phytomers), and contains the features on the corresponing vertices.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

```
Cokelaer, T. and Pradal, C. (2010). MTG user guide
```

#### See Also

```
print.foldermtg
mtgorder
```

rmcol.folder 181

## **Examples**

```
mtgfile1 <- system.file("extdata/plant1.mtg", package = "dad")
x1 <- read.mtg(mtgfile1)
print(x1)

mtgfile2 <- system.file("extdata/plant2.mtg", package = "dad")
x2 <- read.mtg(mtgfile2)
print(x2)</pre>
```

rmcol.folder

Remove columns in all elements of a folder

## **Description**

Remove some columns in all data frames of a folder.

#### Usage

```
rmcol.folder(object, name)
```

# Arguments

object of class folder that is a list of data frames with the same column names.

name character vector. The names of the columns to be removed in each data frame

of the folder.

## Value

A folder with the same number of elements as object. Its  $k^{th}$  element is a data frame, and its columns are the columns of object[[k]], except those given by name.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### See Also

```
folder: object of class folder.
getcol.folder: select columns in all elements of a folder.
getrow.folder: select rows in all elements of a folder.
rmrow.folder: remove rows in all elements of a folder.
```

```
data(iris)
iris.fold <- as.folder(iris, "Species")
rmcol.folder(iris.fold, c("Petal.Length", "Petal.Width"))</pre>
```

182 rmcol.foldert

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rmcc	١١	† 0	١c	ler	^†

Remove cols in all elements of a foldert

# **Description**

Remove some columns in all data frames of a foldert.

## Usage

```
rmcol.foldert(object, name)
```

## **Arguments**

object object of class foldert that is a list of data frames with the same column names,

each of them corresponding to a time of observation.

name character vector. The names of the columns to be removed in each data frame

of the foldert.

#### Value

A foldert with the same number of elements as object. Its  $k^{th}$  element is a data frame, and its columns are the columns of object[[k]], except those given by name.

## Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### See Also

```
foldert: object of class foldert.
getcol.foldert: select columns in all elements of a foldert.
getrow.foldert: get rows in all elements of a foldert.
rmrow.foldert: remove rows in all elements of a foldert.
```

```
data(floribundity)
ft0 <- foldert(floribundity, cols.select = "union", rows.select = "union")
ft0
rmcol.foldert(ft0, c("area"))</pre>
```

rmrow.folder 183

rmrow	fo1	dor

Remove rows in all elements of a folder

## **Description**

Remove some rows in all data frames of a folder.

# Usage

```
rmrow.folder(object, name)
```

## **Arguments**

object of class folder that is a list of data frames with the same column names.

name character vector. The names of the rows to be removed in each data frame of the

folder.

#### Value

A folder with the same number of elements as object. Its  $k^{th}$  element is a data frame, and its rows are the rows of object[[k]], except those given by name.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

### See Also

```
folder: object of class folder.

getrow. folder: select rows in all elements of a folder.

getcol. folder: select columns in all elements of a folder.

rmcol. folder: remove columns in all elements of a folder.
```

```
data(iris)
iris.fold <- as.folder(iris, "Species")
rmrow.folder(iris.fold, as.character(seq(1, 150, by = 2)))</pre>
```

184 rmrow.foldert

rmrow.foldert

Remove rows in all elements of a foldert

# Description

Remove some rows in all data frames of a foldert.

## Usage

```
rmrow.foldert(object, name)
```

## **Arguments**

object object of class foldert that is a list of data frames with the same column names,

each of them corresponding to a time of observation.

name character vector. The names of the rows to be removed in each data frame of the

foldert.

#### Value

A foldert with the same number of elements as object. Its  $k^{th}$  element is a data frame, and its rows are the rows of object[[k]], except those given by name.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### See Also

```
foldert: object of class foldert.
getrow.foldert: select rows in all elements of a foldert.
getcol.foldert: select columns in all elements of a foldert.
rmcol.foldert: remove columns in all elements of a foldert.
```

```
data(floribundity)
ft0 <- foldert(floribundity, cols.select = "union", rows.select = "union")
ft0
rmrow.foldert(ft0, c("rose", c("16", "51")))</pre>
```

roseflowers 185

roseflowers

Rose flowers

## **Description**

The data are extracted from measures on roses from an agronomic experiment in a greenhouse and outdoors.

# Usage

```
data(roseflowers)
```

#### **Format**

roseflowers is a list of two data frames:

- roseflowers\$variety: this first data frame has 5 rows and 3 columns (factors) named place, rose and variety.
- roseflowers\$flower: this second data frame has 11 cases and 5 columns named numflower (the order number of the flower), rose, diameter and height (the diameter and height of the flower), and nleaves (the number of the leaves of the axis).

## **Examples**

```
data(roseflowers)
summary(roseflowers$variety)
summary(roseflowers$flower)
```

roseleaves

Rose leaves

## **Description**

The data are extracted from measures on roses from an agronomic experiment in a greenhouse and outdoors.

## Usage

```
data("roseleaves")
```

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#### **Format**

roseleaves is a list of four data frames:

• roseflowers\$rose: data frame with 7 rows and 3 columns (factors) named rose, place and variety.

- roseflowers\$stem: data frame with 12 rows and 5 columns named rose, stem, date, order (the ramification order of the stem) and nleaves (the number of leaves of the stem).
- roseflowers\$leaf: data frame with 35 rows and 5 columns named stem, leaf, rank (the rank of the leaf on the stem), nleaflets and lrachis (the number of leaflets of the leaf and the length of its rachis).
- roseflowers\$leaflet: data frame with 221 rows and 4 columns named leaf, leaflet, lleaflet and wleaflet (the length and width of the leaflet).

Each row (rose) in roseleaves\$rose pertains to several rows (stems) in roseleaves\$stem.

Each row (stem) in roseleaves\$rose pertains to several rows (leaves) in roseleaves\$leaf.

Each row (leaf) in roseleaves\$rose pertains to several rows (leaflets) in roseleaves\$leaflet.

#### **Examples**

```
data(roseleaves)
summary(roseleaves$rose)
summary(roseleaves$stem)
summary(roseleaves$leaf)
summary(roseleaves$leaflet)
```

rosephytomer

Rose leaf and internode dynamics

#### **Description**

These data are extracted from measures on rosebushes during a study on leaf and internode expansion dynamics. For four rosebushes, on each metamer, the length of the terminal leaflet and the length of the internode were measured on several days, from the 24 april 2010 to the 19 july 2010.

The metamers which have no leaflets are omitted.

## Usage

```
data("rosephytomer")
```

#### **Format**

A data frame with 643 rows (4 plants, 7, 8 or 9 metamers per plant, 37 days of observation) and 6 columns:

```
date a POSIXct
```

nplant a factor with levels 113 114 118 121. Numbers of the plants.

roses 187

rank numeric. Rank of the metamer on the stem.

lleaflet, linternode numeric. Length of the terminal leaflet, length of the internode. phytomer factor. Identifiers of the metamers.

#### Source

Demotes-Mainard, S., Bertheloot, J., Boumaza, R., Huché-Thélier, L., Guéritaine, G., Guérin, V. and Andrieu, B. (2013). Rose bush leaf and internode expansion dynamics: analysis and development of a model capturing interplant variability. Frontiers in Plant Science 4: 418. Doi: 10.3389/fpls.2013.00418

## **Examples**

```
data(rosephytomer)
as.foldert(rosephytomer, method = 1, ind = "phytomer", timecol = "date", same.rows = TRUE)
```

roses

Roses data

# Description

Sensory data characterising the visual aspect of 10 rosebushes

#### Usage

data(roses)

### Format

roses is a data frame of sensory data with 420 rows (10 products, 14 assessors, 3 sessions) and 17 columns. The first 16 columns are numeric and correspond to 16 visual characteristics of rose-bushes. The last column is a factor giving the name of the corresponding rosebush.

- Sha: top sided shape
- Den: foliage thickness
- Sym: plant symmetry
- Vgr: stem vigour
- Qrm: quantity of stems
- Htr: branching level
- Qf1: quantity of flowers
- Ef1: staggering of flowering
- Mvf1: flower enhancement
- Difl: flower size
- Qfr: quantity of faded flowers/fruits

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```
• Qbt: quantity of floral buds
```

• Defl: density of flower petals

• Vcfl: intensity of flower colour

• Tfe: leaf size

· Vfe: darkness of leaf colour

• rose: factor with 10 levels: A, B, C, D, E, F, G, H, I and J

#### Source

Boumaza, R., Huché-Thélier, L., Demotes-Mainard, S., Le Coz, E., Leduc, N., Pelleschi-Travier, S., Qannari, E.M., Sakr, S., Santagostini, P., Symoneaux, R., Guérin, V. (2010). Sensory profile and preference analysis in ornamental horticulture: The case of rosebush. Food Quality and Preference, 21, 987-997.

## **Examples**

```
data(roses)
summary(roses)
```

skewness.folder

Skewness coefficients of a folder of data sets

## **Description**

Computes the skewness coefficient by column of the elements of an object of class folder.

#### Usage

```
skewness.folder(x, na.rm = FALSE, type = 3)
```

## Arguments

X	an object of class folder that is a list of data frames with the same column
	names.
na.rm	logical. Should missing values be omitted from the calculations? (see skewness)
type	an integer between 1 and 3 (see skewness).

#### **Details**

It uses skewness to compute the mean by numeric column of each element of the folder. If some columns of the data frames are not numeric, there is a warning, and the means are computed on the numeric columns only.

#### Value

A list whose elements are the skewness coefficients by column of the elements of the folder.

sqrtmatrix 189

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### See Also

folder to create an object is of class folder. mean. folder, var. folder, cor. folder, kurtosis. folder for other statistics for folder objects.

# **Examples**

```
# First example: iris (Fisher)
data(iris)
iris.fold <- as.folder(iris, "Species")
iris.skewness <- skewness.folder(iris.fold)
print(iris.skewness)

# Second example: roses
data(roses)
roses.fold <- as.folder(roses, "rose")
roses.skewness <- skewness.folder(roses.fold)
print(roses.skewness)</pre>
```

sqrtmatrix

Square root of a symmetric, positive semi-definite matrix

#### **Description**

Calculation of the square root of a positive semi-definite matrix (see Details for the definition of such a matrix).

### Usage

```
sqrtmatrix(mat)
```

#### **Arguments**

mat

numeric matrix.

# Details

The matrix mat must be symmetric and positive semi-definite. Otherwise, there is an error.

The square root of the matrix mat is the positive semi-definite matrix M such as t(M) % % M = mat. Do not confuse with sqrt(mat), which returns the square root of the elements of mat.

The computation is based on the diagonalisation of mat. The eigenvalues smaller than 10^-16 are identified as null values.

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## Value

Matrix: the square root of the matrix mat.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

## **Examples**

```
M2 <- matrix(c(5, 4, 4, 5), nrow = 2)
M <- sqrtmatrix(M2)
M</pre>
```

summary.folder

Summarize a folder

# **Description**

Summarize an object of class folder.

#### Usage

```
## S3 method for class 'folder'
summary(object, ...)
```

#### **Arguments**

object of class folder that is a list of data frames with the same column names.
... further arguments passed to or from other methods.

#### Value

A list, each element of it contains the summary of the corresponding element of object. This list has an attribute attr(,"same.rows").

## Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### See Also

folder: object of class folder. as.folder.data.frame: build an object of class folder from a data frame.

```
data(iris)
iris.fold <- as.folder(iris, "Species")
summary(iris.fold)</pre>
```

summary.folderh 191

summary.folderh

Summarize a folderh

#### **Description**

Summarize an object of class folderh.

# Usage

```
## S3 method for class 'folderh'
summary(object, ...)
```

## **Arguments**

object of class folderh that is a list of data frames.
... further arguments passed to or from other methods.

#### Value

A list, each element of it containing the summary of the corresponding element of object. This list has an attribute attr(,"keys") (see folderh).

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

# See Also

folderh: object of class folderh.

```
# First example
mtgfile <- system.file("extdata/plant1.mtg", package = "dad")
x <- read.mtg(mtgfile)
fh1 <- as.folderh(x, classes = c("P", "A", "M"))
summary(fh1)

# Second example
data(roseleaves)
roses <- roseleaves$rose
stems <- roseleaves$stem
leaves <- roseleaves$leaf
leaflets <- roseleaves$leaf
leaflets <- roseleaves$leaflet
fh2 <- folderh(roses, "rose", stems, "stem", leaves, "leaf", leaflets)
summary(fh2)</pre>
```

192 summary.foldermtg

summary.foldermtg

Summary of an object of class foldermtg

## **Description**

Summary method for S3 class foldermtg.

## Usage

```
## S3 method for class 'foldermtg'
summary(object, ...)
```

# Arguments

```
object an object of class foldermtg.... optional arguments to summary methods.
```

## Value

The summary of the data frames containing the vertices of each class and the values of the features on these vertices.

# Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

```
Cokelaer, T. and Pradal, C. (2010). MTG user guide
```

## See Also

read.mtg: reads a MTG file and creates an object of class "foldermtg".

```
mtgfile1 <- system.file("extdata/plant1.mtg", package = "dad")
x1 <- read.mtg(mtgfile1)
summary(x1)

mtgfile2 <- system.file("extdata/plant2.mtg", package = "dad")
x2 <- read.mtg(mtgfile2)
summary(x2)</pre>
```

summary.foldert 193

summary.foldert

Summarize a foldert

## **Description**

Summarize an object of class foldert.

# Usage

```
## S3 method for class 'foldert'
summary(object, ...)
```

## Arguments

object of class foldert that is a list of data frames organised according to time.

... further arguments passed to or from other methods.

#### Value

A list, each element of it contains the summary of the corresponding element of object. This list has two attributes attr(,"times") and attr(,"same.rows").

## Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### See Also

foldert: object of class foldert. as.foldert.data.frame: build an object of class foldert from a data frame. as.foldert.array: build an object of class foldert from a 3d-array.

```
# 1st example
data(floribundity)
ftflor <- foldert(floribundity, cols.select = "union", rows.select = "union")
summary(ftflor)</pre>
```

194 var.folder

var.folder	Variance matrices of a folder of data sets	
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## **Description**

Computes the variance matrices of the elements of an object of class folder.

## Usage

```
var.folder(x, na.rm = FALSE, use = "everything")
```

# Arguments

x	an object of class folder that is a list of data frames with the same column names.
na.rm	logical. Should missing values be removed? (see var)
use	an optional character string giving a method for computing covariances in the presence of missing values. This must be (an abbreviation of) one of the strings "everything", "all.obs", "complete.obs", "na.or.complete", or "pairwise.complete.obs" (see var).

#### **Details**

It uses var to compute the variance matrix of the numeric columns of each element of the folder. If some columns of the data frames are not numeric, there is a warning, and the variances are computed on the numeric columns only.

#### Value

A list whose elements are the variance matrices of the elements of the folder.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

## See Also

folder to create an object is of class folder. mean.folder, cor.folder, skewness.folder, kurtosis.folder for other statistics for folder objects.

```
# First example: iris (Fisher)
data(iris)
iris.fold <- as.folder(iris, "Species")
iris.vars <- var.folder(iris.fold)
print(iris.vars)</pre>
```

varietyleaves 195

```
# Second example: roses
data(roses)
roses.fold <- as.folder(roses, "rose")
roses.vars <- var.folder(roses.fold)
print(roses.vars)</pre>
```

varietyleaves

Rose variety leaves

#### **Description**

The data are extracted from measures on roses from an agronomic experiment in a greenhouse and outdoors.

#### Usage

```
data("varietyleaves")
```

#### **Format**

varietyleaves is an object of class "folderh", that is a list of two data frames:

- varietyleaves\$variety: data frame with 31 rows and 2 columns (factors) named rose and variety.
- varietyleaves\$leaves: data frame with 581 rows and 5 columns named rose, nleaflet (number of leaflets), lrachis (length of the rachis), lleaflet (length of the principal leaflet) and wleaflet (width of the principal leaflet).

## **Examples**

```
data(varietyleaves)
summary(varietyleaves)
```

wasserstein

2-Wasserstein distance between Gaussian densities

## **Description**

The 2-Wasserstein distance between two multivariate (p > 1) or univariate (p = 1) Gaussian densities (see Details).

# Usage

```
wasserstein(x1, x2, check = FALSE)
```

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## **Arguments**

x1	a matrix or data frame of $n_1$ rows (observations) and $p$ columns (variables) (can also be a tibble) or a vector of length $n_1$ .
x2	matrix or data frame (or tibble) of $n_2$ rows and $p$ columns or vector of length $n_2$ .
check	logical. When TRUE (the default is FALSE) the function checks if the covariance matrices are not degenerate (multivariate case) or if the variances are not zero (univariate case).

#### **Details**

The Wasserstein distance between the two Gaussian densities is computed by using the wassersteinpar function and the density parameters estimated from samples.

#### Value

Returns the 2-Wasserstein distance between the two probability densities.

Be careful! If check = FALSE and one smoothing bandwidth matrix is degenerate, the result returned can not be considered.

#### Author(s)

Rachid Boumaza, Pierre Santagostini, Smail Yousfi, Gilles Hunault, Sabine Demotes-Mainard

#### References

Peterson, A., Mueller, H.G. (2016). Functional Data Analysis for Density Functions by Transformation to a Hilbert Space. The annals of Statistics, 44 (1), 183-218. DOI: 10.1214/15-AOS1363

Dowson, D.C., Ladau, B.V. (1982). The Fréchet Distance between Multivariate Normal Distributions. Journal of Multivariate Analysis, 12, 450-455.

#### See Also

wassersteinpar: 2-Wasserstein distance between Gaussian densities, given their parameters.

```
require(MASS)
m1 <- c(0,0)
v1 <- matrix(c(1,0,0,1),ncol = 2)
m2 <- c(0,1)
v2 <- matrix(c(4,1,1,9),ncol = 2)
x1 <- mvrnorm(n = 3,mu = m1,Sigma = v1)
x2 <- mvrnorm(n = 5, mu = m2, Sigma = v2)
wasserstein(x1, x2)</pre>
```

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wassersteinpar	2-Wasserstein distance between Gaussian densities given their param-
	eters

## **Description**

The 2-Wasserstein distance between two multivariate (p > 1) or univariate (p = 1) Gaussian densities given their parameters (mean vectors and covariance matrices if the densities are multivariate, or means and variances if univariate) (see Details).

## Usage

```
wassersteinpar(mean1, var1, mean2, var2, check = FALSE)
```

## **Arguments**

mean1	p-length numeric vector: the mean of the first Gaussian density.
var1	$p \times p$ symmetric numeric matrix $(p > 1)$ or numeric $(p = 1)$ : the covariance matrix $(p > 1)$ or the variance $(p = 1)$ of the first Gaussian density.
mean2	p-length numeric vector: the mean of the second Gaussian density.
var2	$p \times p$ symmetric numeric matrix $(p > 1)$ or numeric $(p = 1)$ : the covariance matrix $(p > 1)$ or the variance $(p = 1)$ of the second Gaussian density.
check	logical. When TRUE (the default is FALSE) the function checks if the covariance matrices are not degenerate (multivariate case) or if the variances are not zero (univariate case).

#### **Details**

The mean vectors (m1 and m2) and variance matrices (v1 and v2) given as arguments (mean1, mean2, var1 and var2) are used to compute the 2-Wasserstein distance between the two Gaussian densities, equal to:

$$(||m1 - m2||_2^2 + trace((v1 + v2) - 2*(v2^{1/2}v1v2^{1/2})^{1/2}))^{1/2}$$

$$((m1-m2)^2 + v1 + v2 - 2 * (v1 * v2)^{1/2})^{1/2}$$

#### Value

If p = 1:.

The 2-Wasserstein distance between two Gaussian densities.

Be careful! If check = FALSE and one covariance matrix is degenerated (multivariate case) or one variance is zero (univariate case), the result returned must not be considered.

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#### References

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Dowson, D.C., Ladau, B.V. (1982). The Fréchet Distance between Multivariate Normal Distributions. Journal of Multivariate Analysis, 12, 450-455.

#### See Also

wasserstein: 2-Wasserstein distance between Gaussian densities estimated from samples.

```
m1 <- c(1,1)
v1 <- matrix(c(4,1,1,9),ncol = 2)
m2 <- c(0,1)
v2 <- matrix(c(1,0,0,1),ncol = 2)
wassersteinpar(m1,v1,m2,v2)
```

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