## Package 'clusteval'

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Title Evaluation of Clustering Algorithms

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**Description** An R package that provides a suite of tools to evaluate clustering algorithms, clusterings, and individual clusters.

**Depends** R (>= 2.15)

**Imports** parallel, mvtnorm, Rcpp (>= 0.9.13)

LinkingTo Rcpp

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**Collate** 'similarity.r' 'sim\_normal.r' 'sim\_unif.r' 'sim\_student.r' 'clustomit.r' 'sim\_data.r' 'helper-boot.r' 'clusteval.r' 'random\_clustering.r' 'comembership.r'

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boot\_stratified\_omit Creates a list of indices for a stratified nonparametric bootstrap.

#### Description

This function creates a list of indices for a stratified nonparametric bootstrap. Corresponding to our Cluster Omission Stability statistic implemented in clustomit, we omit each group in turn and perform a stratified bootstrap without the group. We denote the number of groups as num\_clusters, which is equal to nlevels(factor(y)). Specifically, suppose that we omit the *k*th group. That is, we ignore all of the observations corresponding to group *k*. Then, we sample with replacement from each of the remaining groups (i.e., every group except for group *k*), yielding a set of bootstrap indices.

## Usage

boot\_stratified\_omit(y, num\_reps = 50)

## Arguments

У	a vector that denotes the grouping of each observation. It must be coercible with as.factor.
num_reps	the number of bootstrap replications to use for each group

## Details

The returned list contains  $K \times num_r eps$  elements.

#### Value

named list containing indices for each bootstrap replication

## Examples

```
set.seed(42)
# We use 4 clusters, each with up to 10 observations. The sample sizes are
# randomly chosen.
num_clusters <- 4
sample_sizes <- sample(10, num_clusters, replace = TRUE)
# Create the cluster labels, y.
y <- unlist(sapply(seq_len(num_clusters), function(k) {
   rep(k, sample_sizes[k])
}))</pre>
```

Index

#### cluster\_similarity

```
# Use 20 reps per group.
boot_stratified_omit(y, num_reps = 20)
# Use the default number of reps per group.
boot_stratified_omit(y)
```

cluster\_similarity Computes the similarity between two clusterings of the same data set.

## Description

For two clusterings of the same data set, this function calculates the similarity statistic specified of the clusterings from the comemberships of the observations. Basically, the comembership is defined as the pairs of observations that are clustered together.

#### Usage

```
cluster_similarity(labels1, labels2,
  similarity = c("jaccard", "rand"),
  method = "independence")
```

#### Arguments

labels1	a vector of n clustering labels
labels2	a vector of n clustering labels
similarity	the similarity statistic to calculate
method	the model under which the statistic was derived

#### Details

To calculate the similarity, we compute the 2x2 contingency table, consisting of the following four cells:

- n\_11 the number of observation pairs where both observations are comembers in both clusterings
- n\_10 the number of observation pairs where the observations are comembers in the first clustering but not the second
- n\_01 the number of observation pairs where the observations are comembers in the second clustering but not the first
- n\_00 the number of observation pairs where neither pair are comembers in either clustering

Currently, we have implemented the following similarity statistics:

- Rand index
- Jaccard coefficient

To compute the contingency table, we use the comembership\_table function.

#### Value

the similarity between the two clusterings

#### Examples

```
# Notice that the number of comemberships is 'n choose 2'.
iris_kmeans <- kmeans(iris[, -5], centers = 3)$cluster
iris_hclust <- cutree(hclust(dist(iris[, -5])), k = 3)
cluster_similarity(iris_kmeans, iris_hclust)
```

clusteval

Evaluation of Clustering Algorithms

## Description

An R package that provides a suite of tools to evaluate clustering algorithms, clusterings, and individual clusters.

clustomit

ClustOmit - Cluster Stability Evaluation via Cluster Omission

#### Description

We provide an implementation of the ClustOmit statistic, which is an approach to evaluating the stability of a clustering determined by a clustering algorithm. As discussed by Hennig (2007), arguably a stable clustering is one in which a perturbation of the original data should yield a similar clustering. However, if a perturbation of the data yields a large change in the clustering, the original clustering is considered unstable. The ClustOmit statistic provides an approach to detecting instability via a stratified, nonparametric resampling scheme. We determine the stability of the clustering via the similarity statistic specified (by default, the Jaccard coefficient).

## Usage

```
clustomit(x, num_clusters, cluster_method,
  similarity = c("jaccard", "rand"),
  weighted_mean = TRUE, num_reps = 50,
  num_cores = getOption("mc.cores", 2), ...)
```

#### clustomit

#### Arguments

x	data matrix with n observations (rows) and p features (columns)
num_clusters	the number of clusters to find with the clustering algorithm specified in $cluster_method$
cluster_method	a character string or a function specifying the clustering algorithm that will be used. The method specified is matched with the match. fun function. The func- tion given should return only clustering labels for each observation in the matrix x.
similarity	the similarity statistic that is used to compare the original clustering (after a sin- gle cluster and its observations have been omitted) to its resampled counterpart. Currently, we have implemented the Jaccard and Rand similarity statistics and use the Jaccard statistic by default.
weighted_mean	logical value. Should the aggregate similarity score for each bootstrap replica- tion be weighted by the number of observations in each of the observed clusters? By default, yes (i.e., TRUE).
num_reps	the number of bootstrap replicates to draw for each omitted cluster
num_cores	the number of coures to use. If 1 core is specified, then lapply is used without parallelization. See the mc.cores argument in mclapply for more details.
	additional arguments passed to the function specified in cluster_method

## Details

To compute the ClustOmit statistic, we first cluster the data given in x into num\_clusters clusters with the clustering algorithm specified in cluster\_method. We then omit each cluster in turn and all of the observations in that cluster. For the omitted cluster, we resample from the remaining observations and cluster the resampled observations into num\_clusters - 1 clusters again using the clustering algorithm specified in cluster\_method. Next, we compute the similarity between the cluster labels of the original data set and the cluster labels of the bootstrapped sample. We approximate the sampling distribution of the ClustOmit statistic using a stratified, nonparametric bootstrapping scheme and use the apparent variability in the approximated sampling distribution as a diagnostic tool for further evaluation of the ClustOmit statistic to provide a clear interpretation of cluster assessment. The technical details of the ClustOmit statistic can be found in our forthcoming publication entitled "Cluster Stability Evaluation of Gene Expression Data."

The ClustOmit cluster stability statistic is based on the cluster omission admissibility condition from Fisher and Van Ness (1971), who provide decision-theoretic admissibility conditions that a reasonable clustering algorithm should satisfy. The guidelines from Fisher and Van Ness (1971) establish a systematic foundation that is often lacking in the evaluation of clustering algorithms. The ClustOmit statistic is our proposed methodology to evaluate the cluster omission admissibility condition from Fisher and Van Ness (1971).

We require a clustering algorithm function to be specified in the argument cluster\_method. The function given should accept at least two arguments:

**x** matrix of observations to cluster

num\_clusters the number of clusters to find

... additional arguments that can be passed on

Also, the function given should return only clustering labels for each observation in the matrix x. The additional arguments specified in . . . are useful if a wrapper function is used: see the example below for an illustration.

#### Value

object of class clustomit, which contains a named list with elements

**boot\_aggregate:** vector of the aggregated similarity statistics for each bootstrap replicate

**boot\_similarity:** list containing the bootstrapped similarity scores for each cluster omitted

obs\_clusters: the clustering labels determined for the observations in x

num\_clusters: the number of clusters found

**similarity:** the similarity statistic used for comparison between the original clustering and the resampled clusterings

#### References

Fisher, L. and Van Ness, J. (1971), Admissible Clustering Procedures, \_Biometrika\_, 58, 1, 91-104. Hennic, C. (2007), Cluster-wise assessment of cluster stability, \_Computational Statistics and Data Analysis\_, 52, 258-271. http://www.jstor.org/stable/2334320

#### Examples

comembership

Calculates the comemberships of all pairs of a vector of clustering labels.

#### Description

For a set of clustering labels, this function computes the comembership of all pairs of observations. Basically, two observations are said to be comembers if they are clustered together.

#### comembership\_table

## Usage

comembership(labels)

#### Arguments

labels a vector of n clustering labels

## Details

Tibshirani and Walther (2005) use the term 'co-membership', which we shorten to 'comembership'. Some authors instead use the terms 'connectivity' or 'co-occurrence'.

We use the Rcpp package to improve the runtime speed of this function.

#### Value

a vector of choose(n, 2) comembership bits

## References

Tibshirani, R. and Walther, G. (2005), Cluster Validation by Prediction Strength, \_Journal of Computational and Graphical Statistics\_, 14, 3, 511-528. http://amstat.tandfonline.com/doi/ abs/10.1198/106186005X59243.

#### Examples

```
# We generate K = 3 labels for each of n = 10 observations and compute the
# comembership for all 'n choose 2' pairs.
set.seed(42)
K <- 3
n <- 10
labels <- sample.int(K, n, replace = TRUE)
comembership_out <- comembership(labels)
comembership_out
# Notice that the number of comemberships is 'n choose 2'.
length(comembership_out) == choose(n, 2)
```

comembership_table	Calculates the 2x2 contingency table of agreements and disagreements
	of comemberships from two vectors of clustering labels.

## Description

For two clusterings of the same data set, this function calculates the  $2x^2$  contingency table of agreements and disagreements of the corresponding two vectors of comemberships. Basically, the comembership is defined as the pairs of observations that are clustered together.

#### Usage

comembership\_table(labels1, labels2)

#### Arguments

labels1	a vector of n clustering labels
labels2	a vector of n clustering labels

#### Details

The contingency table calculated is typically utilized in the calculation of a similarity statistic (e.g., Rand index, Jaccard index) between the two clusterings. The 2x2 contingency table consists of the following four cells:

- $n_11$  the number of observation pairs where both observations are comembers in both clusterings
- n\_10 the number of observation pairs where the observations are comembers in the first clustering but not the second
- n\_01 the number of observation pairs where the observations are comembers in the second clustering but not the first
- n\_00 the number of observation pairs where neither pair are comembers in either clustering

Tibshirani and Walther (2005) use the term 'co-membership', which we shorten to 'comembership'. Some authors instead use the terms 'connectivity' or 'co-occurrence'.

We use the Rcpp package to improve the runtime speed of this function.

#### Value

named list containing the calculated contingency table:

- n\_11
- n\_10
- n\_01
- n\_00

#### References

Tibshirani, R. and Walther, G. (2005). Cluster Validation by Prediction Strength. Journal of Computational and Graphical Statistics, 14, 3, 511-528. http://amstat.tandfonline.com/doi/abs/ 10.1198/106186005X59243.

#### Examples

```
# We generate K = 3 labels for each of n = 10 observations and compute the
# comembership for all 'n choose 2' pairs.
set.seed(42)
K <- 3
n <- 10
labels1 <- sample.int(K, n, replace = TRUE)</pre>
```

#### intraclass\_cov

```
labels2 <- sample.int(K, n, replace = TRUE)
comembership_table(labels1, labels2)
# Here, we cluster the \code{\link{iris}} data set with the K-means and
# hierarchical algorithms using the true number of clusters, K = 3.
# Then, we compute the 2x2 contingency table agreements and disagreements of
#' the comemberships.
iris_kmeans <- kmeans(iris[, -5], centers = 3)$cluster
iris_hclust <- cutree(hclust(dist(iris[, -5])), k = 3)
comembership_table(iris_kmeans, iris_hclust)</pre>
```

intraclass\_cov Construct an intraclass covariance matrix.

## Description

We define a  $p \times p$  intraclass covariance (correlation) matrix to be  $\Sigma_m = \sigma^2 (1-\rho) J_p + \rho I_p$ , where  $-(p-1)^{-1} < \rho < 1$ ,  $I_p$  is the  $p \times p$  identity matrix, and  $J_p$  denotes the  $p \times p$  matrix of ones.

## Usage

```
intraclass_cov(p, rho, sigma2 = 1)
```

#### Arguments

р	the dimension of the matrix
rho	the intraclass covariance (correlation) constant
sigma2	the coefficient of the intraclass covariance matrix

## Value

an intraclass covariance matrix matrix of size p

jaccard_indep	Computes the Jaccard similarity coefficient of two clusterings of the
	same data set under the assumption that the two clusterings are inde-
	pendent.

## Description

For two clusterings of the same data set, this function calculates the Jaccard similarity coefficient of the clusterings from the comemberships of the observations. Basically, the comembership is defined as the pairs of observations that are clustered together.

#### Usage

```
jaccard_indep(labels1, labels2)
```

#### Arguments

labels1	a vector of n clustering labels
labels2	a vector of n clustering labels

## Details

To calculate the Rand index, we compute the 2x2 contingency table, consisting of the following four cells:

- $n_11$  the number of observation pairs where both observations are comembers in both clusterings
- n\_10 the number of observation pairs where the observations are comembers in the first clustering but not the second
- n\_01 the number of observation pairs where the observations are comembers in the second clustering but not the first
- $n_0$  the number of observation pairs where neither pair are comembers in either clustering

The Jaccard similarity coefficient is defined as:

$$J = \frac{n_{11}}{n_{11} + n_{10} + n_{01}}$$

In the special case that the Jaccard coefficient results in 0/0, we define J = 0. For instance, this case can occur when both clusterings consist of all singleton clusters.

To compute the contingency table, we use the comembership\_table function.

#### Value

the Jaccard coefficient for the two sets of cluster labels (See Details.)

#### Examples

```
## Not run:
# We generate K = 3 labels for each of n = 10 observations and compute the
# Jaccard similarity coefficient between the two clusterings.
set.seed(42)
K <- 3
n <- 10
labels1 <- sample.int(K, n, replace = TRUE)</pre>
labels2 <- sample.int(K, n, replace = TRUE)</pre>
jaccard_indep(labels1, labels2)
# Here, we cluster the \code{\link{iris}} data set with the K-means and
# hierarchical algorithms using the true number of clusters, K = 3.
# Then, we compute the Jaccard similarity coefficient between the two
# clusterings.
iris_kmeans <- kmeans(iris[, -5], centers = 3)$cluster</pre>
iris_hclust <- cutree(hclust(dist(iris[, -5])), k = 3)</pre>
jaccard_indep(iris_kmeans, iris_hclust)
```

## End(Not run)

random\_clustering Randomly cluster a data set into K clusters.

#### Description

For each observation (row) in 'x', one of K labels is randomly generated. By default, the probabilities of selecting each clustering label are equal, but this can be altered by specifying 'prob', a vector of probabilities for each cluster.

#### Usage

random\_clustering(x, K, prob = NULL)

#### Arguments

x	a matrix containing the data to cluster. The rows are the sample observations, and the columns are the features.
К	the number of clusters
prob	a vector of probabilities to generate each cluster label. If NULL, each cluster label has an equal chance of being selected.

## Details

Random clustering is often utilized as a baseline comparison clustering against which other clustering algorithms are employed to identify structure within the data. Typically, comparisons are made in terms of proposed clustering assessment and evaluation methods as well as clustering similarity measures. For the former, a specified clustering evaluation method is computed for the considered clustering algorithms as well as random clustering. If the clusters determined by a considered clustering algorithm do not differ significantly from the random clustering, we might conclude that the found clusters are no better than naively choosing clustering labels for each observation at random. Likewise, a similarity measure can be computed to compare the clusterings are significantly similar, once again, we might conclude the clusters found via the considered clustering algorithm do not differ significantly from those found at random. In either case, the clusters are unlikely to provide meaningful results on which the user can better understand the inherent structure within the data.

#### Value

a vector of clustering labels for each observation in 'x'.

rand\_indep

Computes the Rand similarity index of two clusterings of the same data set under the assumption that the two clusterings are independent.

## Description

For two clusterings of the same data set, this function calculates the Rand similarity coefficient of the clusterings from the comemberships of the observations. Basically, the comembership is defined as the pairs of observations that are clustered together.

## Usage

```
rand_indep(labels1, labels2)
```

## Arguments

labels1	a vector of n clustering labels
labels2	a vector of n clustering labels

#### Details

To calculate the Rand index, we compute the 2x2 contingency table, consisting of the following four cells:

- **n\_11** the number of observation pairs where both observations are comembers in both clusterings
- n\_10 the number of observation pairs where the observations are comembers in the first clustering but not the second
- n\_01 the number of observation pairs where the observations are comembers in the second clustering but not the first
- n\_00 the number of observation pairs where neither pair are comembers in either clustering

The Rand similarity index is defined as:

$$R = \frac{n_{11} + n_{00}}{n_{11} + n_{10} + n_{01} + n_{00}}$$

To compute the contingency table, we use the comembership\_table function.

## Value

the Rand index for the two sets of cluster labels

#### sim\_data

## Examples

```
## Not run:
# We generate K = 3 labels for each of n = 10 observations and compute the
# Rand similarity index between the two clusterings.
set.seed(42)
K <- 3
n <- 10
labels1 <- sample.int(K, n, replace = TRUE)
labels2 <- sample.int(K, n, replace = TRUE)
rand_indep(labels1, labels2)
# Here, we cluster the \code{\link{iris}} data set with the K-means and
# hierarchical algorithms using the true number of clusters, K = 3.
# Then, we compute the Rand similarity index between the two clusterings.
iris_kmeans <- kmeans(iris[, -5], centers = 3)$cluster
iris_hclust <- cutree(hclust(dist(iris[, -5])), k = 3)
rand_indep(iris_kmeans, iris_hclust)
```

## End(Not run)

sim_data	Wrapper function to generate data from a variety of data-generating
	models.

## Description

We provide a wrapper function to generate from three data-generating models:

sim\_unif Five multivariate uniform distributions

sim\_normal Multivariate normal distributions with intraclass covariance matrices

sim\_student Multivariate Student's t distributions each with a common covariance matrix

#### Usage

```
sim_data(family = c("uniform", "normal", "student"), ...)
```

#### Arguments

family	the family of distributions from which to generate data
	optional arguments that are passed to the data-generating function

## Details

For each data-generating model, we generate  $n_m$  observations (m = 1, ..., M) from each of M multivariate distributions so that the Euclidean distance between each of the population centroids and the origin is equal and scaled by  $\Delta \ge 0$ . For each model, the argument delta controls this separation.

This wrapper function is useful for simulation studies, where the efficacy of supervised and unsupervised learning methods and algorithms are evaluated as a the population separation is increased.

#### Value

named list containing:

- **x:** A matrix whose rows are the observations generated and whose columns are the p features (variables)
- y: A vector denoting the population from which the observation in each row was generated.

## Examples

```
set.seed(42)
uniform_data <- sim_data(family = "uniform")
normal_data <- sim_data(family = "normal", delta = 2)
student_data <- sim_data(family = "student", delta = 1, df = 1:5)</pre>
```

sim_normal	Generates random variates from multivariate normal populations with
	intraclass covariance matrices.

## Description

We generate  $n_m$  observations (m = 1, ..., M) from each of M multivariate normal distributions such that the Euclidean distance between each of the means and the origin is equal and scaled by  $\Delta \ge 0$ .

## Usage

sim\_normal(n = rep(25, 5), p = 50, rho = rep(0.9, 5), delta = 0, sigma2 = 1, seed = NULL)

#### Arguments

n	a vector (of length M) of the sample sizes for each population
р	the dimension of the multivariate normal populations
rho	a vector (of length M) of the intraclass constants for each population
delta	the fixed distance between each population and the origin
sigma2	the coefficient of each intraclass covariance matrix
seed	seed for random number generation (If NULL, does not set seed)

## Details

Let  $\Pi_m$  denote the *m*th population with a *p*-dimensional multivariate normal distribution,  $N_p(\mu_m, \Sigma_m)$  with mean vector  $\mu_m$  and covariance matrix  $\Sigma_m$ . Also, let  $e_m$  be the *m*th standard basis vector (i.e., the *m*th element is 1 and the remaining values are 0). Then, we define

$$\mu_m = \Delta \sum_{j=1}^{p/M} e_{(p/M)(m-1)+j}.$$

Note that p must be divisible by M. By default, the first 10 dimensions of  $\mu_1$  are set to delta with all remaining dimensions set to 0, the second 10 dimensions of  $\mu_2$  are set to delta with all remaining dimensions set to 0, and so on.

Also, we consider intraclass covariance (correlation) matrices such that  $\Sigma_m = \sigma^2(1 - \rho_m)J_p + \rho_m I_p$ , where  $-(p-1)^{-1} < \rho_m < 1$ ,  $I_p$  is the  $p \times p$  identity matrix, and  $J_p$  denotes the  $p \times p$  matrix of ones.

By default, we let M = 5,  $\Delta = 0$ , and  $\sigma^2 = 1$ . Furthermore, we generate 25 observations from each population by default.

For  $\Delta = 0$  and  $\rho_m = \rho$ ,  $m = 1, \dots, M$ , the M populations are equal.

## Value

named list containing:

- **x:** A matrix whose rows are the observations generated and whose columns are the p features (variables)
- y: A vector denoting the population from which the observation in each row was generated.

#### Examples

sim\_student

Generates random variates from multivariate Student's t populations.

## Description

We generate  $n_m$  observations (m = 1, ..., M) from each of M multivariate Student's t distributions such that the Euclidean distance between each of the means and the origin is equal and scaled by  $\Delta \ge 0$ .

#### Usage

#### Arguments

n	a vector (of length M) of the sample sizes for each population
р	the dimension of the multivariate Student's t distributions
df	a vector (of length M) of the degrees of freedom for each population
delta	the fixed distance between each population and the origin
Sigma	the common covariance matrix
seed	seed for random number generation (If NULL, does not set seed)

## Details

Let  $\Pi_m$  denote the *m*th population with a *p*-dimensional multivariate Student's t distribution,  $T_p(\mu_m, \Sigma_m, c_m)$ , where  $\mu_m$  is the population location vector,  $\Sigma_m$  is the positive-definite covariance matrix, and  $c_m$  is the degrees of freedom.

Let  $e_m$  be the *m*th standard basis vector (i.e., the *m*th element is 1 and the remaining values are 0). Then, we define

$$\mu_m = \Delta \sum_{j=1}^{p/M} e_{(p/M)(m-1)+j}.$$

Note that p must be divisible by M. By default, the first 10 dimensions of  $\mu_1$  are set to delta with all remaining dimensions set to 0, the second 10 dimensions of  $\mu_2$  are set to delta with all remaining dimensions set to 0, and so on.

We use a common covariance matrix  $\Sigma_m = \Sigma$  for all populations.

For small values of  $c_m$ , the tails are heavier, and, therefore, the average number of outlying observations is increased.

By default, we let M = 5,  $\Delta = 0$ ,  $\Sigma_m = I_p$ , and  $c_m = 6$ , m = 1, ..., M, where  $I_p$  denotes the  $p \times p$  identity matrix. Furthermore, we generate 25 observations from each population by default.

For  $\Delta = 0$  and  $c_m = c, m = 1, \dots, M$ , the M populations are equal.

#### Value

named list containing:

- **x:** A matrix whose rows are the observations generated and whose columns are the p features (variables)
- y: A vector denoting the population from which the observation in each row was generated.

#### Examples

## sim\_unif

colMeans(x[i,])
}))
(sample\_means <- do.call(rbind, sample\_means))</pre>

sim\_unif

Generates random variates from five multivariate uniform populations.

## Description

We generate n observations from each of four trivariate distributions such that the Euclidean distance between each of the populations is a fixed constant, delta > 0.

## Usage

sim\_unif(n = rep(25, 5), delta = 0, seed = NULL)

### Arguments

n	a vector (of length $M = 5$ ) of the sample sizes for each population
delta	the fixed distance between each population and the origin
seed	Seed for random number generation. (If NULL, does not set seed)

#### Details

To define the populations, let  $x = (X_1, \ldots, X_p)'$  be a multivariate uniformly distributed random vector such that  $X_j \sim U(a_j, b_j)$  is an independently distributed uniform random variable with  $a_j < b_j$  for  $j = 1, \ldots, p$ . Let  $Pi_m$  denote the *m*th population  $(m = 1, \ldots, 5)$ . Then, we have the five populations:

$$\begin{split} \Pi_1 &= U(-1/2, 1/2) \times U(\Delta - 1/2, \Delta + 1/2) \times U(-1/2, 1/2) \times U(-1/2, 1/2), \\ \Pi_2 &= U(\Delta - 1/2, \Delta + 1/2) \times U(-1/2, 1/2) \times U(-1/2, 1/2) \times U(-1/2, 1/2), \\ \Pi_3 &= U(-1/2, 1/2) \times U(-\Delta - 1/2, -\Delta + 1/2) \times U(-1/2, 1/2) \times U(-1/2, 1/2), \\ \Pi_4 &= U(-1/2, 1/2) \times U(-1/2, 1/2) \times U(-\Delta - 1/2, -\Delta + 1/2) \times U(-1/2, 1/2), \\ \Pi_5 &= U(-1/2, 1/2) \times U(-1/2, 1/2) \times U(-1/2, 1/2) \times U(\Delta - 1/2, \Delta + 1/2). \end{split}$$

We generate  $n_m$  observations from population  $\Pi_m$ .

For  $\Delta = 0$  and  $\rho_m = \rho$ ,  $m = 1, \dots, M$ , the M populations are equal.

Notice that the support of each population is a unit hypercube with 4 features. Moreover, for  $\Delta \ge 1$ , the populations are mutually exclusive and entirely separated.

## Value

named list containing:

- **x:** A matrix whose rows are the observations generated and whose columns are the p features (variables)
- y: A vector denoting the population from which the observation in each row was generated.

## Examples

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