Package 'bpp'

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Type Package

Title Computations Around Bayesian Predictive Power

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Depends mvtnorm, R (>= 2.10)

Suggests knitr, rmarkdown

VignetteBuilder knitr

Description Implements functions to update Bayesian Predictive Power Computations after not stopping a clinical trial at an interim analysis. Such an interim analysis can either be blinded or unblinded. Code is provided for Normally distributed endpoints with known variance, with a prominent example being the hazard ratio.

License GPL (>= 2)

LazyLoad yes

NeedsCompilation no

Repository CRAN

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R topics documented:

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bpp-package *Tools for Computation of Bayesian Predictive Power for a Normally Distributed Endpoint with Known Variance*

Description

Implements functions to update Bayesian Predictive Power Computations after not stopping a clinical trial at an interim analysis, whether blinded or unblinded, for a Normally distributed endpoint with known variance, with a prominent example being the hazard ratio.

Details

Package:	bpp
Type:	Package
Version:	1.0.0
Date:	2016-12-13
License:	GPL (>=2)
LazyLoad:	yes

Author(s)

Kaspar Rufibach (maintainer)
<kaspar.rufibach@roche.com>

References

Rufibach, K., Jordan, P., Abt, M. (2016a). Sequentially Updating the Likelihood of Success of a Phase 3 Pivotal Time-to-Event Trial based on Interim Analyses or External Information. *J. Biopharm. Stat.*, **26**(2), 191–201.

Rufibach, K., Burger, H.U., Abt, M. (2016b). Bayesian Predictive Power: Choice of Prior and some Recommendations for its Use as Probability of Success in Drug Development. *Pharm. Stat.*, **15**, 438–446.

Examples

```
# type ?bpp_1interim for code of all the computations in Rufibach et al (2016a).
```

---

basicPlot *Basic plot functions to illustrate prior and posterior densities when considering a time-to-event endpoint*

Description

Basic plot function, labels are specific to the hazard ratio, i.e. when looking at a time-to-event endpoint.

Usage

```
basicPlot(leg = TRUE, IntEffBoundary = NA, IntFutBoundary = NA, successmean = NA,
       priormean = NA)
```

Arguments

leg	logical, display legend?
IntEffBoundary	Interim efficacy boundary.
IntFutBoundary	Interim futility boundary.
successmean	The mean that defines success at the final analysis. Typically chosen to be the minimal detectable difference, i.e. the critical on the scale of the effect size of interest corresponding to the significance level at the final analysis.
priormean	Mean of the prior.

Value

Empty generic plot.

Author(s)

Kaspar Rufibach (maintainer)
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References

Rufibach, K., Jordan, P., Abt, M. (2016a). Sequentially Updating the Likelihood of Success of a Phase 3 Pivotal Time-to-Event Trial based on Interim Analyses or External Information. *J. Biopharm. Stat.*, **26**(2), 191–201.

Examples

```
# type ?bpp_1interim for code of all the computations in Rufibach et al (2016a).
```

bpp *Bayesian Predictive Power (BPP) for Normally Distributed Endpoint*

Description

Compute BPP for a Normally distributed endpoint, e.g. log(hazard ratio).

Usage

```
bpp(prior = c("normal", "flat"), successmean, finalsigma, priormean, ...)
```

Arguments

`prior`	Prior density on effect sizes.
`successmean`	The mean that defines success at the final analysis. Typically chosen to be the minimal detectable difference, i.e. the critical on the scale of the effect size of interest corresponding to the significance level at the final analysis.
`finalsigma`	(Known) standard deviation at which the final analysis of the study under consideration takes place.
`priormean`	Prior mean.
`...`	Further arguments specific to the chosen prior (see bpp for examples).

Value

A real number, the bpp.

Author(s)

Kaspar Rufibach (maintainer)
<kaspar.rufibach@roche.com>

References

Rufibach, K., Jordan, P., Abt, M. (2016a). Sequentially Updating the Likelihood of Success of a Phase 3 Pivotal Time-to-Event Trial based on Interim Analyses or External Information. *J. Biopharm. Stat.*, **26**(2), 191–201.

Rufibach, K., Burger, H.U., Abt, M. (2016b). Bayesian Predictive Power: Choice of Prior and some Recommendations for its Use as Probability of Success in Drug Development. *Pharm. Stat.*, **15**, 438–446.

Examples

```
# type ?bpp_1interim for code of all the computations in Rufibach et al (2016a).
```

bpp_1interim *Bayesian Predictive Power (BPP) for Normally Distributed Endpoint*

Description

Compute BPP and posterior density for a Normally distributed endpoint, e.g. log(hazard ratio), assuming either an unblinded or blinded interim result.

Usage

```
bpp_1interim(prior = c("normal", "flat"), datasigma, finalsigma, successmean,
                            IntEffBoundary, IntFutBoundary, IntFix, priormean,
                            propA = 0.5, thetas, ...)
```

Arguments

prior	Prior density on effect sizes.
datasigma	(Known) standard error of estimate at interim analysis.
finalsigma	(Known) standard error at which the final analysis of the study under consideration takes place.
successmean	The mean that defines success at the final analysis. Typically chosen to be the minimal detectable difference, i.e. the critical on the scale of the effect size of interest corresponding to the significance level at the final analysis.
IntEffBoundary	Efficacy boundary at the interim analysis.
IntFutBoundary	Futility boundary at the interim analysis.
IntFix	Effect sizes observed at the interim analyis, to compute BPP for an unblinded interim analysis.
priormean	Prior mean.
propA	Proportion of subjects randomized to arm A.
thetas	Grid to compute posterior density on.
...	Further arguments specific to the chosen prior (see bpp_1interim for examples).

Value

A list containing the following elements:

initial BPP	BPP based on the prior.
BPP after not stopping at interim interval	BPP after not stopping at a blinded interim.
BPP after not stopping at interim exact	BPP after not stopping at an unblinded interim.
posterior density interval	The posterior density, interval knowledge.

`posterior power interval`
: The posterior power, interval knowledge.

`posterior density exact`
: The posterior density, exact knowledge of interim result.

Author(s)

Kaspar Rufibach (maintainer)
<kaspar.rufibach@roche.com>

References

Rufibach, K., Jordan, P., Abt, M. (2016a). Sequentially Updating the Likelihood of Success of a Phase 3 Pivotal Time-to-Event Trial based on Interim Analyses or External Information. *J. Biopharm. Stat.*, **26**(2), 191–201.

Rufibach, K., Burger, H.U., Abt, M. (2016b). Bayesian Predictive Power: Choice of Prior and some Recommendations for its Use as Probability of Success in Drug Development. *Pharm. Stat.*, **15**, 438–446.

Examples

```
# ------------------------------------------------------------------------------------------
# Reproduce all the computations in Rufibach et al (2016a) for a Normal prior.
# ------------------------------------------------------------------------------------------

# ----------------------------------------
# set all parameters:
# ----------------------------------------
# prior mean / sd
hr0 <- 0.85
sd0 <- 0.11
priormean <- log(hr0)

# specifications for pivotal study
propA <- 0.5   # proportion of patients randomized to arm A
fac <- (propA * (1 - propA)) ^ (-1)
nevents <- c(0.5, 1) * 1600
finalsigma <- sqrt(fac / nevents[2])
alphas <- c(0.001, 0.049)
za <- qnorm(1 - alphas / 2)
hrMDD <- exp(- za * sqrt(fac / nevents))
successmean <- log(hrMDD[2])

# efficacy and futility interim boundary
effi <- log(hrMDD[1])
futi <- log(1.025)

# grid to compute densities on
thetas <- seq(-0.65, 0.3, by = 0.01)
```

```
# ----------------------------------------
# compare Normal and flat prior density
# ----------------------------------------
par(las = 1, mar = c(9, 5, 2, 1), mfrow = c(1, 2))
plot(0, 0, type = "n", xlim = c(-0.6, 0.3), ylim = c(-0.1, 5), xlab = "", ylab = "density",
     main = "")
title(expression("Normal and flat prior density for "*theta), line = 0.7)
basicPlot(leg = FALSE, IntEffBoundary = effi, IntFutBoundary = futi, successmean = successmean,
          priormean = priormean)
lines(thetas, dnorm(thetas, mean = log(hr0), sd = sd0), col = 2, lwd = 2)

# flat prior:
hr0flat <- 0.866
width1 <- 0.21
height1 <- 2.48

lines(thetas, dUniformNormalTails(thetas, mu = log(hr0flat), width = width1, height = height1),
      lwd = 2, col = 3)

# ----------------------------------------
# computations for Normal prior
# ----------------------------------------

# prior probabilities to be below 0.7 or above 1:
lims <- c(0.7, 1)
pnorm1 <- plnorm(lims[1], meanlog = log(hr0), sdlog = sd0, lower.tail = TRUE, log.p = FALSE)
# pnorm(log(lims[1]), mean = log(hr0), sd = sd0)
pnorm2 <- plnorm(lims[2], meanlog = log(hr0), sdlog = sd0, lower.tail = FALSE, log.p = FALSE)
# 1 - pnorm(log(lims[2]), mean = log(hr0), sd = sd0)

# initial bpp
bpp0 <- bpp(prior = "normal", successmean = successmean, finalsigma = finalsigma,
            priormean = log(hr0), priorsigma = sd0)

# update prior with first external study
hr1 <- 0.396
sd1 <- 0.837
up1 <- NormalNormalPosterior(datamean = log(hr1), datasigma = sd1, n = 1,
                             nu = log(hr0), tau = sd0)
bpp1 <- bpp(prior = "normal", successmean = successmean, finalsigma = finalsigma,
            priormean = up1$postmean, priorsigma = up1$postsigma)

# update prior with second external study (result derived from pooled analysis:
# Cox regression on patient level, stratified by study):
hr2 <- 0.287
sd2 <- 0.658
up2 <- NormalNormalPosterior(datamean = log(hr2), datasigma = sd2, n = 1, nu = log(hr0), tau = sd0)
bpp2 <- bpp(prior = "normal", successmean = successmean, finalsigma = finalsigma,
            priormean = up2$postmean, priorsigma = up2$postsigma)

# compute bpp after not stopping at interim:
# assuming both boundaries:
bpp3.tmp <- bpp_1interim(prior = "normal", datasigma = sqrt(fac / nevents[1]),
```

```
                        finalsigma = finalsigma, successmean = successmean,
                        IntEffBoundary = effi, IntFutBoundary = futi, IntFix = 1,
                        priormean = up2$postmean, propA = 0.5, thetas,
                        priorsigma = up2$postsigma)
bpp3 <- bpp3.tmp$"BPP after not stopping at interim interval"
post3 <- bpp3.tmp$"posterior density interval"

# assuming only efficacy boundary:
bpp3_effi_only <- bpp_1interim(prior = "normal", datasigma = sqrt(fac / nevents[1]),
                           finalsigma = finalsigma, successmean = successmean,
                         IntEffBoundary = effi, IntFutBoundary = log(Inf), IntFix = 1,
                           priormean = up2$postmean, propA = 0.5, thetas = thetas,
                           priorsigma =
                          up2$postsigma)$"BPP after not stopping at interim interval"

# assuming only futility boundary:
bpp3_futi_only <- bpp_1interim(prior = "normal", datasigma = sqrt(fac / nevents[1]),
                           finalsigma = finalsigma, successmean = successmean,
                          IntEffBoundary = log(0), IntFutBoundary = futi, IntFix = 1,
                           priormean = up2$postmean, propA = 0.5, thetas = thetas,
                           priorsigma =
                          up2$postsigma)$"BPP after not stopping at interim interval"

# assuming interim efficacy boundary:
bpp4.tmp <- bpp_1interim(prior = "normal", datasigma = sqrt(fac / nevents[1]),
                    finalsigma = finalsigma, successmean = successmean, IntEffBoundary = effi,
                    IntFutBoundary = Inf, IntFix = c(effi, futi), priormean = up2$postmean,
                         propA = 0.5, thetas, priorsigma = up2$postsigma)
bpp4 <- bpp4.tmp$"BPP after not stopping at interim exact"[2, 1]
post4 <- bpp4.tmp$"posterior density exact"[, 1]

# assuming interim futility boundary:
bpp5.tmp <- bpp_1interim(prior = "normal", datasigma = sqrt(fac / nevents[1]),
                    finalsigma = finalsigma, successmean = successmean, IntEffBoundary = effi,
                         IntFutBoundary = Inf, IntFix = futi, priormean = up2$postmean,
                         propA = 0.5, thetas, priorsigma = up2$postsigma)
bpp5 <- bpp5.tmp$"BPP after not stopping at interim exact"[2, 1]
post5 <- bpp5.tmp$"posterior density exact"     # same as post4[, 2]

# ----------------------------------------
# reproduce plots in paper
# ----------------------------------------

# first two updates
par(las = 1, mar = c(9, 5, 2, 1), mfrow = c(1, 2))
plot(0, 0, type = "n", xlim = c(-0.6, 0.3), ylim = c(-0.1, 5), xlab = "", ylab = "density",
     main = "")
title(expression("Normal prior density and corresponding posteriors for "*theta), line = 0.7)
basicPlot(leg = FALSE, IntEffBoundary = effi, IntFutBoundary = futi, successmean = successmean,
          priormean = priormean)
lines(thetas, dnorm(thetas, mean = log(hr0), sd = sd0), col = 2, lwd = 2)
lines(thetas, dnorm(thetas, mean = up1$postmean, sd = up1$postsigma), col = 3, lwd = 2)
lines(thetas, dnorm(thetas, mean = up2$postmean, sd = up2$postsigma), col = 4, lwd = 2)
```

```
lines(thetas, post3, col = 1, lwd = 2)
legend(-0.64, 5.2, c("prior", "posterior after Sub1", "posterior after Sub1 & Sub2",
                     "posterior after Sub1 & Sub2 and not stopping at interim"),
       lty = 1, col = c(2:4, 1), bty = "n", lwd = 2)

# posterior densities for interval knowledge and thetahat equal to boundaries:
plot(0, 0, type = "n", xlim = c(-0.6, 0.3), ylim = c(-0.1, 8), xlab = "", ylab = "density",
     main = "")
title(expression("Posteriors for "*theta*" after not stopping at interim, for Normal prior"),
      line = 0.7)
basicPlot(leg = FALSE, IntEffBoundary = effi, IntFutBoundary = futi, successmean = successmean,
          priormean = priormean)
lines(thetas, post3, col = 1, lwd = 2)
lines(thetas, post4, col = 2, lwd = 2)
lines(thetas, post5, col = 3, lwd = 2)

leg2 <- c("interval knowledge",
          expression(hat(theta)*" = efficacy boundary"),
          expression(hat(theta)*" = futility boundary")
)

legend(-0.62, 8.2, leg2, lty = 1, col = 1:3, lwd = 2, bty = "n",
       title = "posterior after not stopping at interim,")

# ------------------------------------------------------------------------------------------
# Reproduce all the computations in Rufibach et al (2016a) for flat prior.
# ------------------------------------------------------------------------------------------

# ----------------------------------------
# set all parameters first:
# ----------------------------------------

# parameters of flat prior:
priormean <- log(hr0flat)

# ----------------------------------------
# computations for flat prior
# ----------------------------------------

# prior probabilities to be below 0.7 or above 1:
lims <- c(0.7, 1)
flat1 <- pUniformNormalTails(x = log(lims[1]), mu = priormean, width = width1, height = height1)
flat2 <- 1 - pUniformNormalTails(x = log(lims[2]), mu = priormean,
                                 width = width1, height = height1)

# prior
bpp0_1 <- bpp(prior = "flat", successmean = successmean, finalsigma = finalsigma,
              priormean = priormean, width = width1, height = height1)

# update with first external study
hr1 <- 0.396
sd1 <- 0.837
bpp1_1 <- integrate(FlatNormalPosterior, lower = -Inf, upper = Inf, successmean = successmean,
```

```
                    finalsigma = finalsigma, datamean = log(hr1), datasigma = sd1,
                    priormean = priormean, width = width1, height = height1)$value

# update prior (result derived from pooled analysis: Cox regression on patient level,
# stratified by study)
hr2 <- 0.287
sd2 <- 0.658
bpp2_1 <- integrate(FlatNormalPosterior, -Inf, Inf, successmean = successmean,
                    finalsigma = finalsigma, datamean = log(hr2),
                    datasigma = sd2, priormean = priormean,
                    width = width1, height = height1)$value

# update after not stopping at interim
# first compute synthesized prior:
hr0 <- 0.85
sd0 <- 0.11
up2 <- NormalNormalPosterior(datamean = log(hr2), datasigma = sd2, n = 1, nu = log(hr0), tau = sd0)

# assuming both boundaries:
bpp3.tmp_1 <- bpp_1interim(prior = "flat", datasigma = sqrt(fac / nevents[1]),
                        finalsigma = finalsigma, successmean = successmean,
                        IntEffBoundary = effi, IntFutBoundary = futi, IntFix = 1,
                        priormean = up2$postmean, propA = 0.5, thetas,
                        width = width1, height = height1)
bpp3_1 <- bpp3.tmp_1$"BPP after not stopping at interim interval"
post3_1 <- bpp3.tmp_1$"posterior density interval"

# assuming only efficacy boundary:
bpp3_1_effi_only <- bpp_1interim(prior = "flat", datasigma = sqrt(fac / nevents[1]),
                            finalsigma = finalsigma, successmean = successmean,
                          IntEffBoundary = effi, IntFutBoundary = log(Inf), IntFix = 1,
                            priormean = up2$postmean, propA = 0.5, thetas = thetas,
                            width = width1,
                          height = height1)$"BPP after not stopping at interim interval"

# assuming only futility boundary:
bpp3_1_futi_only <- bpp_1interim(prior = "flat", datasigma = sqrt(fac / nevents[1]),
                            finalsigma = finalsigma, successmean = successmean,
                         IntEffBoundary = log(0), IntFutBoundary = futi, IntFix = 1,
                            priormean = up2$postmean, propA = 0.5, thetas = thetas,
                            width = width1,
                          height = height1)$"BPP after not stopping at interim interval"

# assuming interim efficacy boundary:
bpp4_1.tmp <- bpp_1interim(prior = "flat", datasigma = sqrt(fac / nevents[1]),
                            finalsigma = finalsigma, successmean = successmean,
                          IntEffBoundary = log(0), IntFutBoundary = effi, IntFix = effi,
                            priormean = up2$postmean, propA = 0.5, thetas = thetas,
                            width = width1, height = height1)
bpp4_1 <- bpp4_1.tmp$"BPP after not stopping at interim exact"[2, 1]
post4_1 <- bpp4_1.tmp$"posterior density exact"

# assuming interim futility boundary:
```

```
bpp5_1 <- integrate(Vectorize(estimate_toIntegrate), lower = -Inf, upper = Inf, prior = "flat",
                    successmean = successmean, finalsigma = finalsigma, datamean = futi,
                datasigma = sqrt(fac / nevents[1]), priormean = up2$postmean, width = width1,
                    height = height1)$value


bpp5_1.tmp <- bpp_1interim(prior = "flat", datasigma = sqrt(fac / nevents[1]),
                               finalsigma = finalsigma, successmean = successmean,
                           IntEffBoundary = log(0), IntFutBoundary = effi, IntFix = futi,
                               priormean = up2$postmean, propA = 0.5, thetas = thetas,
                               width = width1, height = height1)
bpp5_1 <- bpp5_1.tmp$"BPP after not stopping at interim exact"[2, 1]
post5_1 <- bpp5_1.tmp$"posterior density exact"


# ----------------------------------------
# plots for flat prior
# ----------------------------------------

# first two updates with external studies
# compute posteriors
flatpost1 <- rep(NA, length(thetas))
flatpost2 <- flatpost1
for (i in 1:length(thetas)){
  flatpost1[i] <- estimate_posterior(x = thetas[i], prior = "flat", datamean = log(hr1),
                                     datasigma = sd1, priormean = priormean,
                                     width = width1, height = height1)
  flatpost2[i] <- estimate_posterior(x = thetas[i], prior = "flat", datamean = log(hr2),
                                     datasigma = sd2, priormean = priormean,
                                     width = width1, height = height1)
}

par(las = 1, mar = c(9, 5, 2, 1), mfrow = c(1, 2))
plot(0, 0, type = "n", xlim = c(-0.6, 0.3), ylim = c(-0.10, 5), xlab = "", ylab = "density",
     main = "")
title(expression("Flat prior density and corresponding posteriors for "*theta), line = 0.7)
basicPlot(leg = FALSE, IntEffBoundary = effi, IntFutBoundary = futi, successmean = successmean,
          priormean = priormean)
lines(thetas, dUniformNormalTails(thetas, mu = priormean, width = width1, height = height1),
      lwd = 2, col = 2)
lines(thetas, flatpost1, col = 3, lwd = 2)
lines(thetas, flatpost2, col = 4, lwd = 2)
lines(thetas, post3_1, col = 1, lwd = 2)

legend(-0.64, 5.2, c("prior", "posterior after Sub1", "posterior after Sub1 & Sub2",
                     "posterior after Sub1 & Sub2 and not stopping at interim"), lty = 1,
                      col = c(2:4, 1), bty = "n", lwd = 2)

# posterior densities for interval knowledge and thetahat equal to boundaries:
plot(0, 0, type = "n", xlim = c(-0.6, 0.3), ylim = c(-0.10, 8), xlab = "", ylab = "density",
     main = "")
title(expression("Posteriors for "*theta*" after not stopping at interim, for Flat prior"),
      line = 0.7)
```

```
basicPlot(leg = FALSE, IntEffBoundary = effi, IntFutBoundary = futi, successmean = successmean,
          priormean = priormean)
lines(thetas, post3_1, col = 1, lwd = 2)
lines(thetas, post4_1, col = 2, lwd = 2)
lines(thetas, post5_1, col = 3, lwd = 2)

leg.flat <- c("interval knowledge",
              expression(hat(theta)*" = efficacy boundary"),
              expression(hat(theta)*" = futility boundary")
)

legend(-0.62, 8.2, leg.flat, lty = 1, col = 1:3, lwd = 2, bty = "n",
       title = "posterior after not stopping at interim,")

# ----------------------------------------
# reproduce Table 1 in Rufibach et al (2016a)
# ----------------------------------------
mat <- matrix(NA, ncol = 2, nrow = 10)
mat[, 1] <- c(pnorm1, pnorm2, bpp0, bpp1, bpp2, bpp3, bpp3_futi_only, bpp3_effi_only,
              bpp4, bpp5)
mat[, 2] <- c(flat1, flat2, bpp0_1, bpp1_1, bpp2_1, bpp3_1, bpp3_1_futi_only,
              bpp3_1_effi_only, bpp4_1, bpp5_1)
colnames(mat) <- c("Normal prior", "Flat prior")
rownames(mat) <- c(paste("Probability for hazard ratio to be $le$ ", lims[1], sep = ""),
paste("Probability for hazard ratio to be $ge$ ", lims[2], sep = ""),
"PoS based on prior distribution", "PoS after Sub1", "PoS after Sub1 and Sub2",
"PoS after not stopping at interim, assuming $inte{hat theta} in [effi{theta}, futi{theta}]$",
"PoS after not stopping at interim, assuming $inte{hat theta} in [-infty, futi{theta}]$",
"PoS after not stopping at interim, assuming $inte{hat theta} in [effi{theta}, infty]$",
"PoS after not stopping at interim, assuming $inte{hat theta} = effi{theta}$",
"PoS after not stopping at interim, assuming $inte{hat theta} = futi{theta}$")
as.data.frame(format(mat, digits = 2))
```

`bpp_2interim` *Bayesian Predictive Power (BPP) for Normally Distributed Endpoint*

Description

Compute BPP and posterior density for a Normally distributed endpoint, e.g. log(hazard ratio), assuming the trial did not stop at two blinded interim analyses.

Usage

```
bpp_2interim(prior = "normal", datasigma, finalsigma, successmean, IntEffBoundary,
                    IntFutBoundary, priormean, thetas, ...)
```

Arguments

prior	Prior density on effect sizes. So far, this function only accomodates a Normal prior, as opposed to bpp_1interim where also the pessimistic prior introduced in Rufibach et al (2016a) can be specified.
datasigma	(Known) standard error of estimate at interim analysis.
finalsigma	(Known) standard error at which the final analysis of the study under consideration takes place.
successmean	The mean that defines success at the final analysis. Typically chosen to be the minimal detectable difference, i.e. the critical on the scale of the effect size of interest corresponding to the significance level at the final analysis.
IntEffBoundary	2-d vector of efficacy boundaries at the interim analyses.
IntFutBoundary	2-d vector of futility boundary at the interim analyses.
priormean	Prior mean.
thetas	Grid to compute posterior density on.
...	Further arguments specific to the chosen prior (see bpp_1interim for examples).

Value

A list containing the following elements:

initial BPP	BPP based on the prior.
BPP after not stopping at interim interval	BPP after not stopping at a blinded interim.
posterior density interval	The posterior density, interval knowledge.

Author(s)

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References

Rufibach, K., Jordan, P., Abt, M. (2016a). Sequentially Updating the Likelihood of Success of a Phase 3 Pivotal Time-to-Event Trial based on Interim Analyses or External Information. *J. Biopharm. Stat.*, **26**(2), 191–201.

Rufibach, K., Burger, H.U., Abt, M. (2016b). Bayesian Predictive Power: Choice of Prior and some Recommendations for its Use as Probability of Success in Drug Development. *Pharm. Stat.*, **15**, 438–446.

Examples

```
# ------------------------------------------------------------------------------------------
# Illustrate the update after two passed interims using the Gallium clinical trial
# ------------------------------------------------------------------------------------------

# ----------------------------------------
# set all parameters:
# ----------------------------------------
# prior mean / sd
hr0 <- 0.9288563
priormean <- log(hr0)
priorsigma <- sqrt(4 / 12)

# specifications for pivotal study
propA <- 0.5   # proportion of patients randomized to arm A
fac <- (propA * (1 - propA)) ^ (-1)
nevents <- c(111, 248, 370)
datasigma <- sqrt(fac / nevents[1:2])
finalsigma <- sqrt(fac / nevents[3])
za <- c(3.9285726330559, 2.5028231888636, 1.9936294555664)
alphas <- 2 * (1 - pnorm(za))
hrMDD <- exp(- za * sqrt(fac / nevents))
successmean <- log(hrMDD[3])

# efficacy and futility interim boundary
effi <- log(c(0, hrMDD[2]))
futi <- log(c(1, Inf))

# grid to compute densities on
thetas <- seq(-0.65, 0.3, by = 0.01)

bpp_2interim(prior = "normal", datasigma = datasigma, finalsigma = finalsigma,
             successmean = successmean, IntEffBoundary = effi, IntFutBoundary = futi,
             priormean = priormean, thetas = thetas, priorsigma = priorsigma)
```

`estimate_posterior` *Posterior density conditional on known interim result*

Description

If we update the prior with a known estimate at an interim analysis, we get this density.

Usage

```
estimate_posterior(x, prior = c("normal", "flat"), datamean, datasigma, priormean, ...)
```

Arguments

x	Value at which to evaluate the function.
prior	Prior density on effect sizes.
datamean	Mean of the data.
datasigma	(Known) standard deviation of datamean.
priormean	Prior mean.
...	Further arguments specific to the chosen prior (see bpp for examples).

Value

Value of the function, a real number.

Author(s)

Kaspar Rufibach (maintainer)
<kaspar.rufibach@roche.com>

References

Rufibach, K., Jordan, P., Abt, M. (2016a). Sequentially Updating the Likelihood of Success of a Phase 3 Pivotal Time-to-Event Trial based on Interim Analyses or External Information. *J. Biopharm. Stat.*, **26**(2), 191–201.

Examples

```
# type ?bpp_1interim for code of all the computations in Rufibach et al (2016a).
```

---

estimate_posterior_nominator

Posterior density conditional on interim result is proportional to the value of this function

---

Description

If we update the prior with a known estimate at an interim analysis, we get a density that is proportional to the value of this function.

Usage

```
estimate_posterior_nominator(x, prior = c("normal", "flat"), datamean,
                                    datasigma, priormean, ...)
```

Arguments

x	Value at which to evaluate the function.
prior	Prior density on effect sizes.
datamean	Mean of the data.
datasigma	(Known) standard deviation of datamean.
priormean	Prior mean.
...	Further arguments specific to the chosen prior (see bpp for examples).

Value

Value of the function, a real number.

Author(s)

Kaspar Rufibach (maintainer)
<kaspar.rufibach@roche.com>

References

Rufibach, K., Jordan, P., Abt, M. (2016a). Sequentially Updating the Likelihood of Success of a Phase 3 Pivotal Time-to-Event Trial based on Interim Analyses or External Information. *J. Biopharm. Stat.*, **26**(2), 191–201.

Examples

```
# type ?bpp_1interim for code of all the computations in Rufibach et al (2016a).
```

---

estimate_toIntegrate *Product of posterior density and conditional power for known interim result*

Description

Product of posterior density and conditional power for known interim result, integrate over this function to get BPP.

Usage

```
estimate_toIntegrate(x, prior = c("normal", "flat"), successmean,
            finalsigma, datamean, datasigma, priormean, propA = 0.5, ...)
```

Arguments

x	Value at which to evaluate the function.
prior	Prior density on effect sizes.
successmean	The mean that defines success at the final analysis. Typically chosen to be the minimal detectable difference, i.e. the critical on the scale of the effect size of interest corresponding to the significance level at the final analysis.
finalsigma	(Known) standard deviation at which the final analysis of the study under consideration takes place.
datamean	Mean of the data.
datasigma	(Known) standard deviation of datamean.
priormean	Prior mean.
propA	Proportion of subjects randomized to arm A.
...	Further arguments specific to the chosen prior (see bpp for examples).

Value

Value of the function, a real number.

Author(s)

Kaspar Rufibach (maintainer)
<kaspar.rufibach@roche.com>

References

Rufibach, K., Jordan, P., Abt, M. (2016a). Sequentially Updating the Likelihood of Success of a Phase 3 Pivotal Time-to-Event Trial based on Interim Analyses or External Information. *J. Biopharm. Stat.*, **26**(2), 191–201.

Examples

```
# type ?bpp_1interim for code of all the computations in Rufibach et al (2016a).
```

FlatNormalPosterior *Integrand to compute Bayesian Predictive Power when flat prior has been updated with likelihood*

Description

Assume we have a flat prior on our effect, update it with a Normal likelihood and then want to compute Bayesian Predictive Power. This function provides the integrand for that computation, i.e. the product of the power function and the posterior.

Usage

```
FlatNormalPosterior(x, successmean, finalsigma, datamean, datasigma, priormean,
                          width, height)
```

Arguments

x	Value at which to evaluate the function.
successmean	The mean that defines success at the final analysis. Typically chosen to be the minimal detectable difference, i.e. the critical on the scale of the effect size of interest corresponding to the significance level at the final analysis.
finalsigma	(Known) standard deviation at which the final analysis of the study under consideration takes place.
datamean	Mean of the data.
datasigma	(Known) standard deviation of datamean.
priormean	Prior mean.
width	Width of the flat part of the prior.
height	Height of the flat part of the prior.

Value

Value of the function, a real number.

Author(s)

Kaspar Rufibach (maintainer)
<kaspar.rufibach@roche.com>

References

Rufibach, K., Jordan, P., Abt, M. (2016a). Sequentially Updating the Likelihood of Success of a Phase 3 Pivotal Time-to-Event Trial based on Interim Analyses or External Information. *J. Biopharm. Stat.*, **26**(2), 191–201.

Examples

```
# type ?bpp_1interim for code of all the computations in Rufibach et al (2016a).
```

`interval_posterior_nominator` *Posterior density conditional on interim result, only known as interval, is proportional to the value of this function*

Description

If we update the prior with the knowledge that the interim estimate was between a futility and efficacy boundary at an interim analysis, we get a density that is proportional to the value of this function.

Usage

```
interval_posterior_nominator(x, prior = c("normal", "flat"),
          IntEffBoundary, IntFutBoundary, datasigma, priormean, ...)
```

Arguments

`x`	Value at which to evaluate the function.
`prior`	Prior density on effect sizes.
`IntEffBoundary`	Efficacy boundary at the interim analysis.
`IntFutBoundary`	Futility boundary at the interim analysis.
`datasigma`	(Known) standard deviation of `datamean`, i.e. at interim analysis.
`priormean`	Prior mean.
`...`	Further arguments specific to the chosen prior (see `bpp` for examples).

Value

Value of the function, a real number.

Author(s)

Kaspar Rufibach (maintainer)
<kaspar.rufibach@roche.com>

References

Rufibach, K., Jordan, P., Abt, M. (2016a). Sequentially Updating the Likelihood of Success of a Phase 3 Pivotal Time-to-Event Trial based on Interim Analyses or External Information. *J. Biopharm. Stat.*, **26**(2), 191–201.

Examples

```
# type ?bpp_1interim for code of all the computations in Rufibach et al (2016a).
```

`interval_posterior_nominator2` *Posterior density conditional on two interim results, both only known as intervals, is proportional to the value of this function*

Description

If we update the prior with the knowledge that two interim estimates were between a futility and efficacy boundary, we get a density that is proportional to the value of this function.

Usage

```
interval_posterior_nominator2(x, prior = "normal",
          IntEffBoundary, IntFutBoundary, datasigma, priormean, ...)
```

Arguments

`x`	Value at which to evaluate the function.
`prior`	Prior density on effect sizes.
`IntEffBoundary`	Efficacy boundary at the interim analysis.
`IntFutBoundary`	Futility boundary at the interim analysis.
`datasigma`	(Known) standard deviation of `datamean`, i.e. at interim analysis.
`priormean`	Prior mean.
`...`	Further arguments specific to the chosen prior (see `bpp` for examples).

Value

Value of the function, a real number.

Author(s)

Kaspar Rufibach (maintainer)
<kaspar.rufibach@roche.com>

References

Rufibach, K., Jordan, P., Abt, M. (2016a). Sequentially Updating the Likelihood of Success of a Phase 3 Pivotal Time-to-Event Trial based on Interim Analyses or External Information. *J. Biopharm. Stat.*, **26**(2), 191–201.

Examples

```
# type ?bpp_2interim for code of all the computations in Rufibach et al (2016a).
```

`interval_toIntegrate` *Product of posterior density and conditional power for blinded interim result*

Description

Product of posterior density and conditional power for blinded interim result, integrate over this function to get BPP.

Usage

```
interval_toIntegrate(x, prior = c("normal", "flat"), datasigma,
             finalsigma, successmean, IntEffBoundary, IntFutBoundary,
             priormean, ...)
```

Arguments

`x`	Value at which to evaluate the function.
`prior`	Prior density on effect sizes.
`datasigma`	(Known) standard deviation of `datamean`, i.e. at interim analysis.
`finalsigma`	(Known) standard deviation at which the final analysis of the study under consideration takes place.
`successmean`	The mean that defines success at the final analysis. Typically chosen to be the minimal detectable difference, i.e. the critical on the scale of the effect size of interest corresponding to the significance level at the final analysis.
`IntEffBoundary`	Efficacy boundary at the interim analysis.
`IntFutBoundary`	Futility boundary at the interim analysis.
`priormean`	Prior mean.
`...`	Further arguments specific to the chosen prior (see `bpp` for examples).

Value

Value of the function, a real number.

Author(s)

Kaspar Rufibach (maintainer)
<kaspar.rufibach@roche.com>

References

Rufibach, K., Jordan, P., Abt, M. (2016a). Sequentially Updating the Likelihood of Success of a Phase 3 Pivotal Time-to-Event Trial based on Interim Analyses or External Information. *J. Biopharm. Stat.*, **26**(2), 191–201.

Examples

```
# type ?bpp_1interim for code of all the computations in Rufibach et al (2016a).
```

`interval_toIntegrate2` *Product of posterior density and conditional power for blinded interim result*

Description

Product of posterior density and conditional power for two blinded interim results, integrate over this function to get BPP.

Usage

```
interval_toIntegrate2(x, prior = "normal", datasigma, finalsigma, successmean,
                              IntEffBoundary, IntFutBoundary, priormean, ...)
```

Arguments

`x`	Value at which to evaluate the function.
`prior`	Prior density on effect sizes.
`datasigma`	(Known) standard deviation of `datamean`, i.e. at interim analysis.
`finalsigma`	(Known) standard deviation at which the final analysis of the study under consideration takes place.
`successmean`	The mean that defines success at the final analysis. Typically chosen to be the minimal detectable difference, i.e. the critical on the scale of the effect size of interest corresponding to the significance level at the final analysis.
`IntEffBoundary`	Efficacy boundary at the interim analysis.
`IntFutBoundary`	Futility boundary at the interim analysis.
`priormean`	Prior mean.
`...`	Further arguments specific to the chosen prior (see `bpp` for examples).

Value

Value of the function, a real number.

Author(s)

Kaspar Rufibach (maintainer)
<kaspar.rufibach@roche.com>

References

Rufibach, K., Jordan, P., Abt, M. (2016a). Sequentially Updating the Likelihood of Success of a Phase 3 Pivotal Time-to-Event Trial based on Interim Analyses or External Information. *J. Biopharm. Stat.*, **26**(2), 191–201.

Examples

```
# type ?bpp_2interim for code of all the computations in Rufibach et al (2016a).
```

`NormalNormalPosterior` *Normal-Normal Posterior in conjugate normal model, for known sigma*

Description

Compute the posterior distribution in a conjugate normal model for known variance: Let $X_1, \ldots, X_n$ be a sample from a $N(\mu, \sigma^2)$ distribution, with σ assumed known. We assume a prior distribution on μ, namely $N(\nu, \tau^2)$. The posterior distribution is then $\mu|x \sim N(\mu_p, \sigma_p^2)$ with

$$\mu_p = (1/(\sigma^2/n) + \tau^{-2})^{-1}(\bar{x}/(\sigma^2/n) + \nu/\tau^2)$$

and

$$\sigma_p = (1/(\sigma^2/n) + \tau^{-2})^{-1}.$$

These formulas are available e.g. in Held (2008, p. 147ff).

Usage

```
NormalNormalPosterior(datamean, datasigma, n, nu, tau)
```

Arguments

`datamean`	Mean of the data.
`datasigma`	(Known) standard deviation of `datamean`.
`n`	Number of observations.
`nu`	Prior mean.
`tau`	Prior standard deviation.

Value

A list with the entries:

`postmean`	Posterior mean.
`postsigma`	Posterior standard deviation.

Author(s)

Kaspar Rufibach (maintainer)
<kaspar.rufibach@roche.com>

References

Held, L., Sabanes-Bove, D. (2014). *Applied Statistical Inference*. Springer.

Examples

```
## data:
n <- 25
sd0 <- 3
x <- rnorm(n, mean = 2, sd = sd0)

## prior:
nu <- 0
tau <- 2

## posterior:
NormalNormalPosterior(datamean = mean(x), datasigma = sd0, n = 77, nu = nu, tau = tau)
```

---

post_power *Conditional power conditioning on a blinded interim*

Description

Conditional power conditioning on a blinded interim, i.e. the estimate after the interim is only known to lie in an interval.

Usage

```
post_power(x, datasigma, finalsigma, successmean, IntEffBoundary, IntFutBoundary)
```

Arguments

x	Value at which to evaluate the function.
datasigma	(Known) standard deviation of datamean.
finalsigma	(Known) standard deviation at which the final analysis of the study under consideration takes place.
successmean	The mean that defines success at the final analysis. Typically chosen to be the minimal detectable difference, i.e. the critical on the scale of the effect size of interest corresponding to the significance level at the final analysis.
IntEffBoundary	Efficacy boundary at the interim analysis.
IntFutBoundary	Futility boundary at the interim analysis.

Value

Value of the function, a real number.

Author(s)

Kaspar Rufibach (maintainer)
<kaspar.rufibach@roche.com>

References

Rufibach, K., Jordan, P., Abt, M. (2016a). Sequentially Updating the Likelihood of Success of a Phase 3 Pivotal Time-to-Event Trial based on Interim Analyses or External Information. *J. Biopharm. Stat.*, **26**(2), 191–201.

Examples

```
# type ?bpp_1interim for code of all the computations in Rufibach et al (2016a).
```

UniformNormalTails *Density and CDF for Uniform Distribution with Normal tails*

Description

Density function and cumulative distribution function for a Uniform density with Normal tails. Introduced in Rufibach et al (2016a) as pessimistic distribution to compute Bayesian Predictive Power.

Usage

```
dUniformNormalTails(x, mu, width, height)
pUniformNormalTails(x, mu, width, height)
```

Arguments

x	Vector of quantiles.
mu	Mean of the pessimistic prior.
width	Width of the flat part of the prior.
height	Height of the flat part of the prior.

Value

Density at x.

Author(s)

Kaspar Rufibach (maintainer)
<kaspar.rufibach@roche.com>

References

Rufibach, K., Jordan, P., Abt, M. (2016a). Sequentially Updating the Likelihood of Success of a Phase 3 Pivotal Time-to-Event Trial based on Interim Analyses or External Information. *J. Biopharm. Stat.*, **26**(2), 191–201.

Examples

```
# type ?bpp_1interim for code of all the computations in Rufibach et al (2016a).
```

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