

Illustration of the *Wrapped* package

by

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Abstract: The six main functions in *Wrapped* are illustrated through various examples.

1 Introduction

Nadarajah and Zhang (2017) developed the R package *Wrapped*. A description of the program structure of the package is given in Section 2 of Nadarajah and Zhang (2017). There are six main functions: `dwrappedg`, `pwrappedg`, `qwrappedg`, `rwrappedg`, `mwrappedg` and `plotfour`. Various numerical illustrations of these functions are given in Sections 2 to 7.

2 Illustrations of the use of `dwrappedg`

The first illustration plots the PDF of the wrapped beta normal distribution for selected parameter values.

```
x <- -pi+2*pi*seq(0.01,0.99,0.01)
y <- dwrappedg(x, "betanorm", K = 100, shape1 = 0.1, shape2 = 0.9)
plot(x, y, xlab = "x", ylab = "PDF", type = "l")
```

The plot is

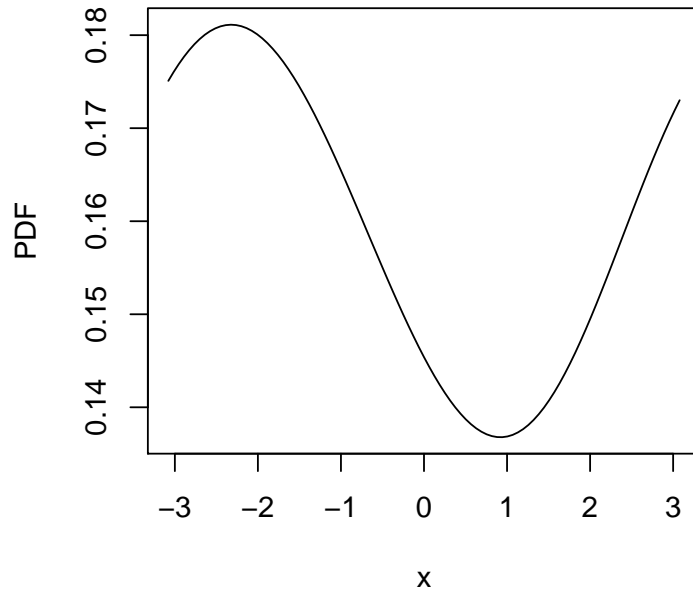


Figure 1: PDF of the wrapped beta normal distribution.

The second illustration plots the PDF of the wrapped skew normal distribution for selected parameter values.

```
x <- -pi+2*pi*seq(0.01,0.99,0.01)
y <- dwrappedg(x, "sn", K = 100, alpha = -2)
plot(x, y, xlab = "x", ylab = "PDF", type = "l")
```

The plot is

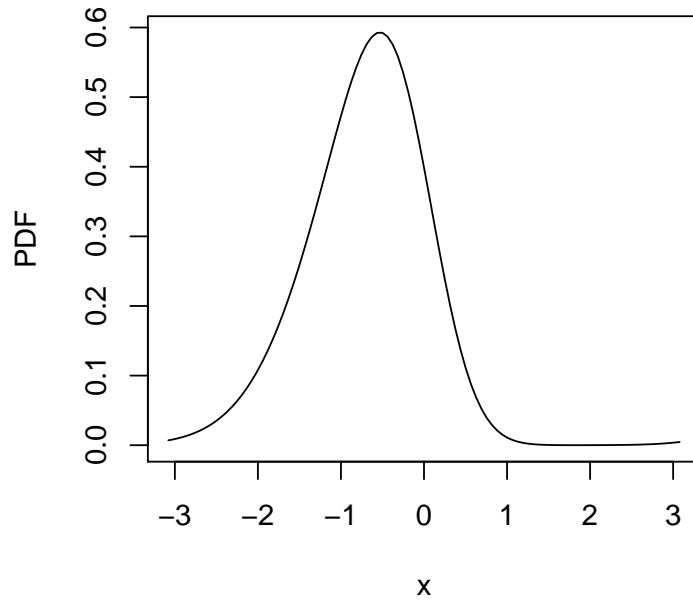


Figure 2: PDF of the wrapped skew normal distribution.

The third illustration plots the PDF of the wrapped asymmetric Laplace distribution for selected parameter values.

```
x <- -pi+2*pi*seq(0.01,0.99,0.01)
y <- dwrappedg(x, "asla", K = 100, kappa = 50)
plot(x, y, xlab = "x", ylab = "PDF", type = "l")
```

The plot is

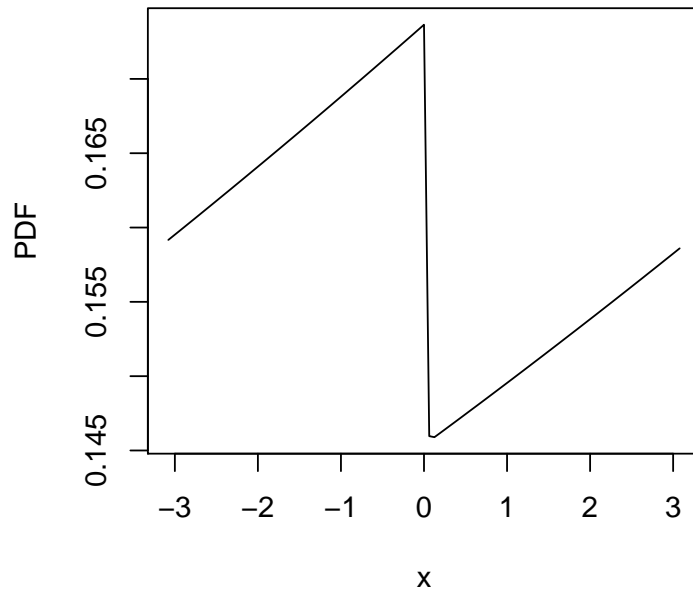


Figure 3: PDF of the wrapped asymmetric Laplace distribution.

The fourth illustration plots the PDF of the wrapped skew t type 3 distribution for selected parameter values.

```
x <- -pi+2*pi*seq(0.01,0.99,0.01)
y <- dwrappedg(x, "ST3", K = 100, nu = 50, tau = 50)
plot(x, y, xlab = "x", ylab = "PDF", type = "l")
```

The plot is

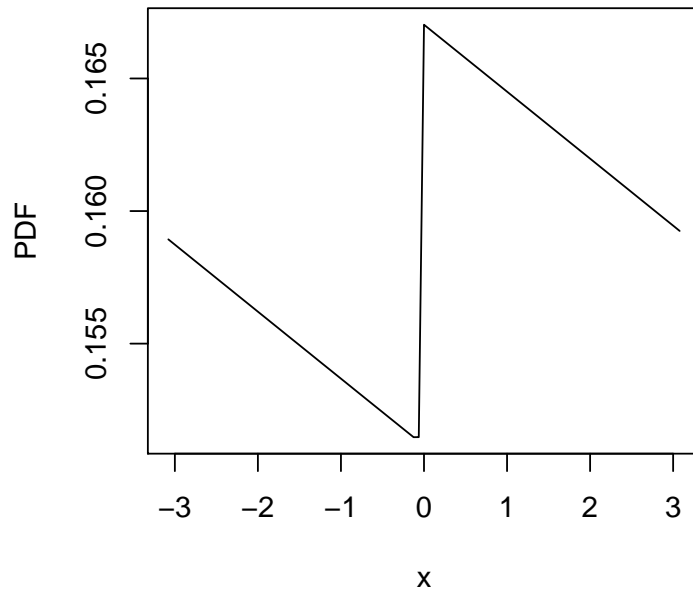


Figure 4: PDF of the wrapped skew t type 3 distribution.

3 Illustrations of the use of `pwrappedg`

The first illustration plots the CDF of the wrapped beta normal distribution for selected parameter values.

```
x <- -pi+2*pi*seq(0.01,0.99,0.01)
y <- pwrappedg(x, "betanorm", K = 100, shape1 = 0.1, shape2 = 0.9)
plot(x, y, xlab = "x", ylab = "CDF", type = "l")
```

The plot is

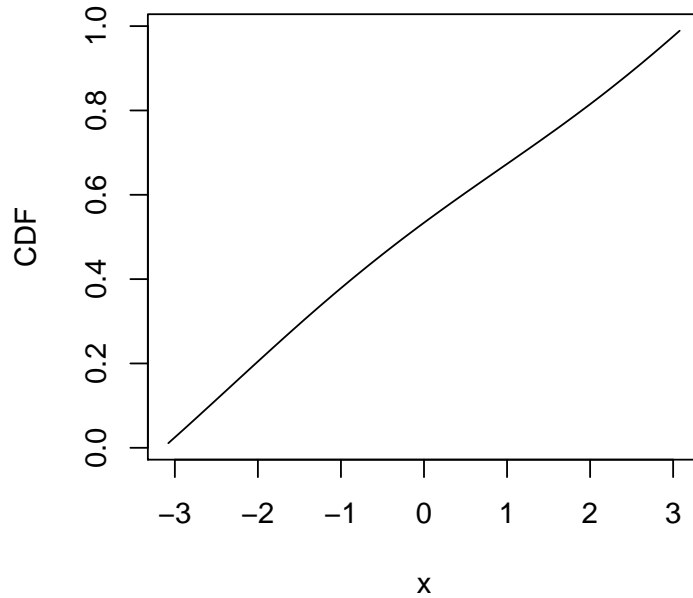


Figure 5: CDF of the wrapped beta normal distribution.

The second illustration plots the CDF of the wrapped skew normal distribution for selected parameter values.

```
x <- -pi+2*pi*seq(0.01,0.99,0.01)
y <- pwrappedg(x, "sn", K = 100, alpha = -2)
plot(x, y, xlab = "x", ylab = "CDF", type = "l")
```

The plot is

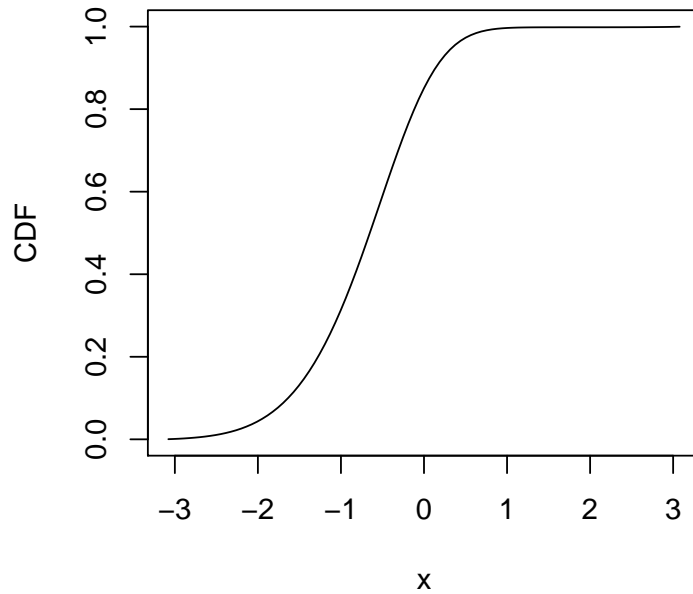


Figure 6: CDF of the wrapped skew normal distribution.

The third illustration plots the CDF of the wrapped asymmetric Laplace distribution for selected parameter values.

```
x <- -pi+2*pi*seq(0.01,0.99,0.01)
y <- pwrappedg(x, "asla", K = 100, kappa = 50)
plot(x, y, xlab = "x", ylab = "CDF", type = "l")
```

The plot is

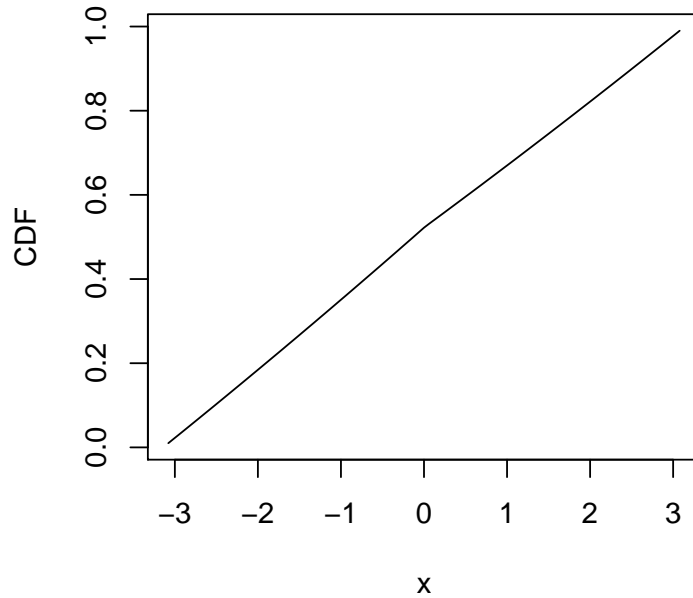


Figure 7: CDF of the wrapped asymmetric Laplace distribution.

The fourth illustration plots the CDF of the wrapped skew t type 3 distribution for selected parameter values.

```
x <- -pi+2*pi*seq(0.01,0.99,0.01)
y <- pwrappedg(x, "ST3", K = 100, nu = 50, tau = 50)
plot(x, y, xlab = "x", ylab = "CDF", type = "l")
```

The plot is

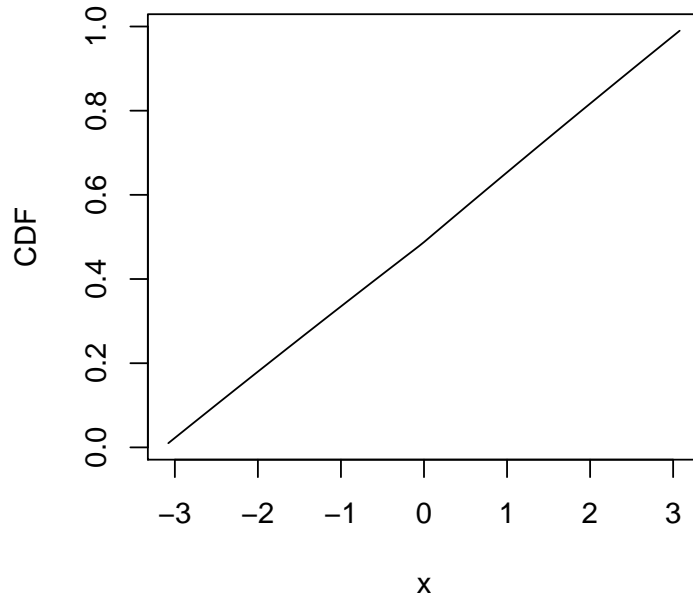


Figure 8: CDF of the wrapped skew t type 3 distribution.

4 Illustrations of the use of `qwrappedg`

The first illustration plots the quantile function of the wrapped beta normal distribution for selected parameter values.

```
x <- seq(0.01,0.99,0.01)
y <- qwrappedg(x, "betanorm", K = 100, shape1 = 0.1, shape2 = 0.9)
plot(x, y, xlab = "p", ylab = "Quantile function", type = "l")
```

The plot is

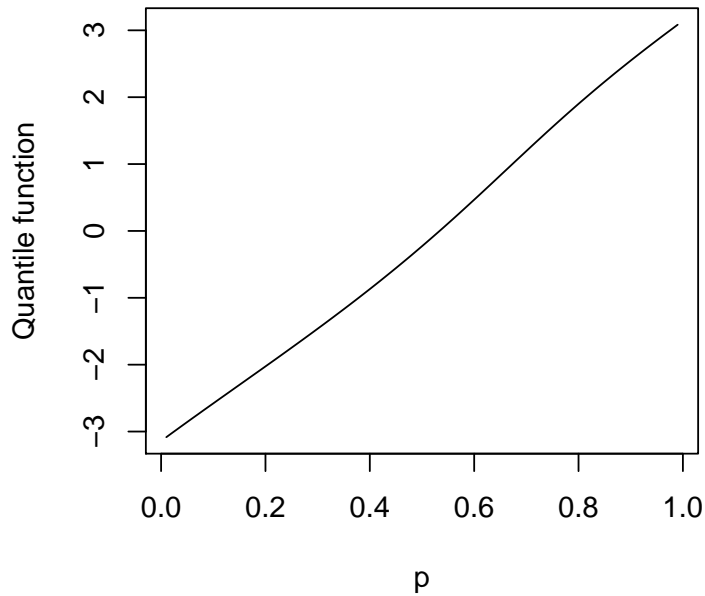


Figure 9: Quantile function of the wrapped beta normal distribution.

The second illustration plots the quantile function of the wrapped skew normal distribution for selected parameter values.

```
x <- seq(0.01,0.99,0.01)
y <- qwrappedg(x, "sn", K = 100, alpha = -2)
plot(x, y, xlab = "p", ylab = "Quantile function", type = "l")
```

The plot is

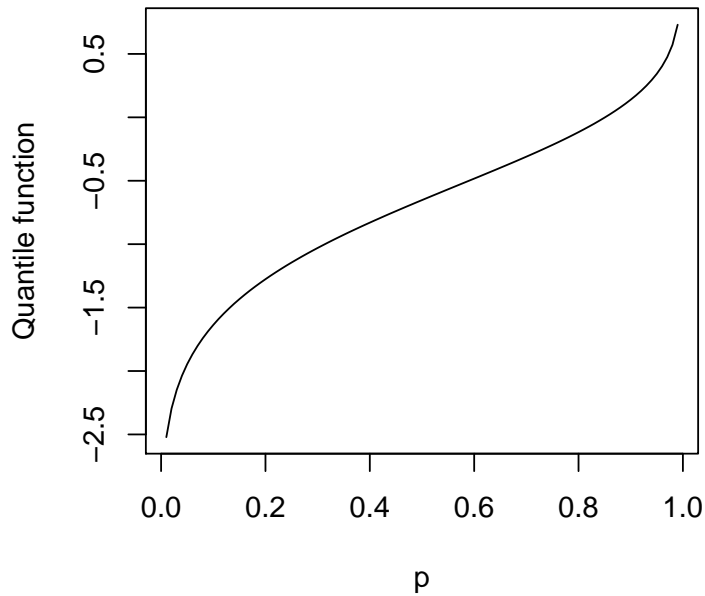


Figure 10: Quantile function of the wrapped skew normal distribution.

The third illustration plots the quantile function of the wrapped asymmetric Laplace distribution for selected parameter values.

```
x <- seq(0.01,0.99,0.01)
y <- qwrappedg(x, "asla", K = 100, kappa = 50)
plot(x, y, xlab = "p", ylab = "Quantile function", type = "l")
```

The plot is

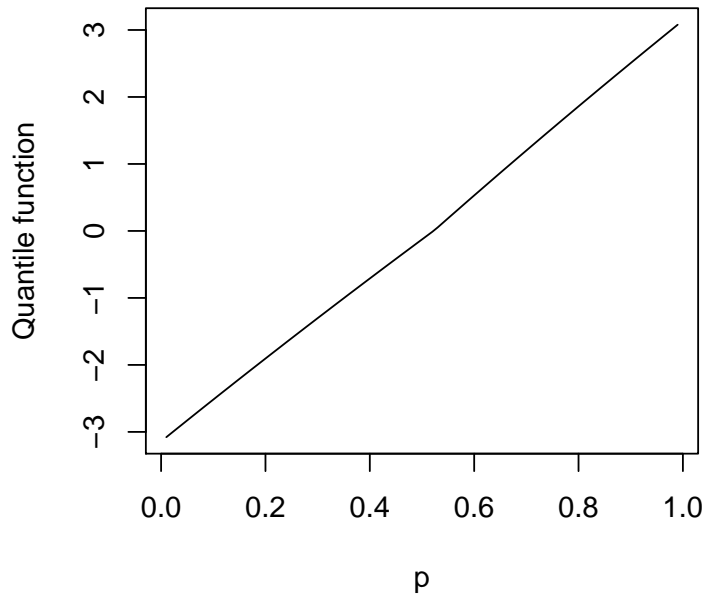


Figure 11: Quantile function of the wrapped asymmetric Laplace distribution.

The fourth illustration plots the quantile function of the wrapped skew t type 3 distribution for selected parameter values.

```
x <- seq(0.01,0.99,0.01)
y <- qwrappedg(x, "ST3", K = 100, nu = 50, tau = 50)
plot(x, y, xlab = "p", ylab = "Quantile function", type = "l")
```

The plot is

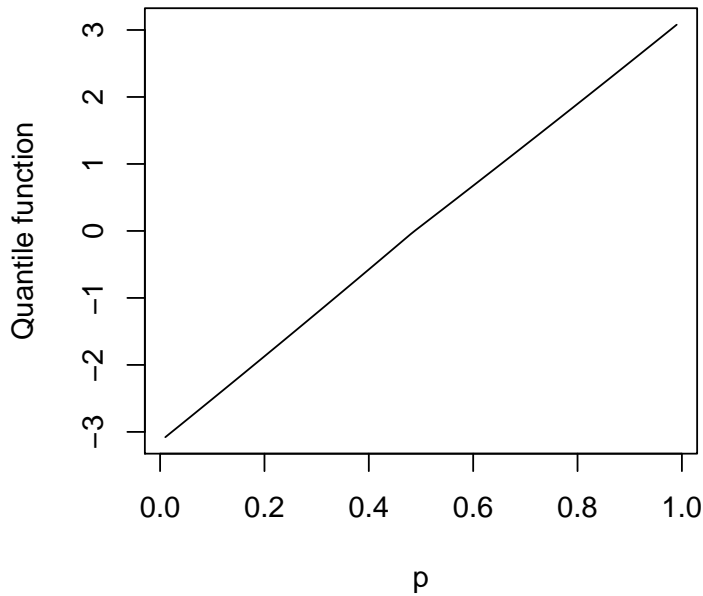


Figure 12: Quantile function of the wrapped skew t type 3 distribution.

5 Illustrations of the use of `rwrappedg`

The first illustration generates random numbers from the wrapped beta normal distribution for selected parameter values.

```
y <- rwrappedg(10, "betanorm", shape1 = 0.1, shape2 = 0.9)
y
```

The output is

```

          cc          ss
[1,] 0.83946691 0.54341081
[2,] 0.39540021 0.91850894
[3,] 0.04224134 -0.99910744
[4,] -0.72772234 -0.68587185
[5,] -0.99644429 0.08425424
[6,] 0.65401088 -0.75648514
[7,] -0.60078553 0.79941025
[8,] -0.48238208 -0.87596092
[9,] -0.99733337 0.07298044
[10,] 0.84894119 -0.52848733
```

The second illustration generates random numbers from the wrapped skew normal distribution for selected parameter values.

```
y <- rwrappedg(10, "sn", alpha = -2)
y
```

The output is

```
          cc          ss
[1,] -0.4294329 -0.9030988
[2,]  0.2692447 -0.9630718
[3,]  0.9914533 -0.1304623
[4,]  0.4694726 -0.8829471
[5,]  0.9759357  0.2180586
[6,]  0.4242805 -0.9055308
[7,]  0.8751958 -0.4837690
[8,] -0.3903690 -0.9206585
[9,]  0.8103032 -0.5860108
[10,] 0.7150983 -0.6990239
```

The third illustration generates random numbers from the wrapped asymmetric Laplace distribution for selected parameter values.

```
y <- rwrappedg(10, "asla", kappa = 50)
y
```

The output is

```
          cc          ss
[1,]  0.16742717  0.98588445
[2,]  0.01786582  0.99984039
[3,]  0.61352794 -0.78967301
[4,]  0.87713424 -0.48024528
[5,]  0.88737847 -0.46104169
[6,] -0.99789982  0.06477620
[7,]  0.14408957 -0.98956465
[8,] -0.94946120  0.31388443
[9,] -0.80854181  0.58843873
[10,] 0.99940181 -0.03458343
```

The fourth illustration generates random numbers from the wrapped skew t type 3 distribution for selected parameter values.

```
y <- rwrappedg(10, "ST3", nu = 50, tau = 50)
y
```

The output is

```

          cc          ss
[1,]  0.9291279 -0.36975853
[2,]  0.9140545 -0.40559139
[3,]  0.8973898 -0.44123855
[4,]  0.7227332  0.69112710
[5,] -0.9996830 -0.02517933
[6,] -0.8283459  0.56021697
[7,]  0.3805908 -0.92474357
[8,] -0.8576298  0.51426748
[9,]  0.5786188 -0.81559815
[10,] 0.4398901  0.89805162

```

6 Illustrations of the use of mwrappedg

Consider the following data on 30 cross-beds azimuths of palaeocurrents. The data on degrees is converted to radii.

```

x = c(294, 177, 257, 301, 257, 267, 329, 177, 241, 315,
+ 229, 239, 277, 250, 287, 281, 166, 229, 254, 232,
+ 290, 245, 245, 214, 272, 224, 215, 242, 186, 224)
x = 2 * pi * x / 360 - pi

```

The first illustration is the fit of the wrapped normal distribution to the data.

```
mwrappedg("norm", data = x, starts = c(mean(x), 1), K = 100, method = "BFGS")
```

The output is

```

$Estimates
      MLE Std. Dev. Inf. 95% CI Sup. 95% CI
[1,] 1.1728617 0.12682025  0.9242986  1.421425
[2,] 0.6946231 0.08967455  0.5188642  0.870382

$Measures
      AIC      CAIC      BIC      HQIC      W      A Min(-log(Likelihood))
67.27319 67.71764 70.07559 68.1697 0.02754482 0.2095757 31.6366

$`Kolmogorov-Smirnov Test`
  KS Statistic KS p-value
  0.08226683  0.9872238

$`Convergence Status`

"Algorithm Converged"

```

The second illustration is the fit of the wrapped logistic distribution to the data.

```
mwrappedg("logis", data = x, starts = c(mean(x), 1), K = 100, method = "BFGS")
```

The output is

```
$Estimates
      MLE Std. Dev. Inf. 95% CI Sup. 95% CI
[1,] 1.181547 0.12672650 0.9331680 1.4299267
[2,] 0.398243 0.06068586 0.2793009 0.5171851

$Measures
      AIC      CAIC      BIC      HQIC      W      A Min(-log(Likelihood))
68.10648 68.55092 70.90887 69.00299 0.02303805 0.1835242      32.05324

$'Kolmogorov-Smirnov Test'
  KS Statistic KS p-value
 0.06864052 0.9989203

$'Convergence Status'

"Algorithm Converged"
```

The third illustration is the fit of the wrapped Gumbel distribution to the data.

```
mwrappedg("gumbel", data = x, starts = c(mean(x), 1), K = 100, method = "BFGS")
```

The output is

```
$Estimates
      MLE Std. Dev. Inf. 95% CI Sup. 95% CI
[1,] 0.8245372 0.13432556 0.5612639 1.0878105
[2,] 0.6860317 0.09273898 0.5042667 0.8677968

$Measures
      AIC      CAIC      BIC      HQIC      W      A Min(-log(Likelihood))
71.98751 72.43195 74.7899 72.88402 0.04632486 0.380803      33.99375

$'Kolmogorov-Smirnov Test'
  KS Statistic KS p-value
 0.1710141 0.3441205

$'Convergence Status'

"Algorithm Converged"
```

The fourth illustration is the fit of the wrapped Laplace distribution to the data.

```
mwrappedg("laplace", data = x, starts = c(mean(x), 1), K = 100, method = "BFGS")
```


The output is

```
$Estimates
      MLE Std. Dev. Inf. 95% CI Sup. 95% CI
[1,] 1.1344640 0.01651829 1.1020887 1.1668392
[2,] 0.5456715 0.10041207 0.3488675 0.7424756

$Measures
      AIC CAIC BIC HQIC W A Min(-log(Likelihood))
69.10345 69.5479 71.90585 69.99996 0.02765046 0.2146212 32.55173

$'Kolmogorov-Smirnov Test'
KS Statistic KS p-value
0.1016237 0.9160182

$'Convergence Status'

"Algorithm Converged"
```

The fifth illustration is the fit of the wrapped Student's t distribution to the data.

```
mwrappedg("t.scaled", data = x, starts = c(mean(x), sd(x), 50), K = 100, method = "BFGS")
```

The output is

```
$Estimates
      MLE Std. Dev. Inf. 95% CI Sup. 95% CI
[1,] 1.1740348 0.12720525 0.9247171 1.4233525
[2,] 0.6864103 0.09429592 0.5015937 0.8712269
[3,] 67.5447148 224.76746687 -372.9914252 508.0808548

$Measures
      AIC CAIC BIC HQIC W A Min(-log(Likelihood))
69.36032 70.2834 73.56391 70.70509 0.02669693 0.2049471 31.68016

$'Kolmogorov-Smirnov Test'
KS Statistic KS p-value
0.0808072 0.9895772

$'Convergence Status'

"Algorithm Converged"
```

For each of the five fitted wrapped distributions, the output gives the parameter estimates, standard errors, 95 percent confidence intervals, value of Akaike Information Criterion, value of Consistent Akaike Information Criterion, value of Bayesian Information Criterion, value of Hannan Quinn Information Criterion, Cramer von Misses statistic value, Anderson Darling statistic value,

minimum value of the negative log likelihood, Kolmogorov Smirnov statistic value, its p value and convergence status of the minimization of the negative log likelihood.

The standard errors appear smaller compared to the parameter estimates for each fitted distribution. An exception is the Student's t wrapped distribution. Also the p -value for each fitted distribution appears acceptable at the five percent significance level.

The wrapped normal distribution gives the smallest values for Cramer von Misses statistic, Anderson Darling statistic, Akaike Information Criterion, Consistent Akaike Information Criterion, Bayesian Information Criterion, Hannan Quinn information criterion, minimum of the negative log-likelihood function. But the wrapped logistic distribution gives the smallest Kolmogorov Smirnov test statistic and the largest p -value.

7 Illustrations of the use of plotfour

The first illustration plots the PDFs of the wrapped beta normal distribution for selected parameter values.

```
plotfour("betanorm",K=100,para=list(c(0,1,0.1,9),c(0,1,0.1,20),c(0,1,20,0.1),c(0,1,3,3)),
plotit="pdf")
```

The plots are

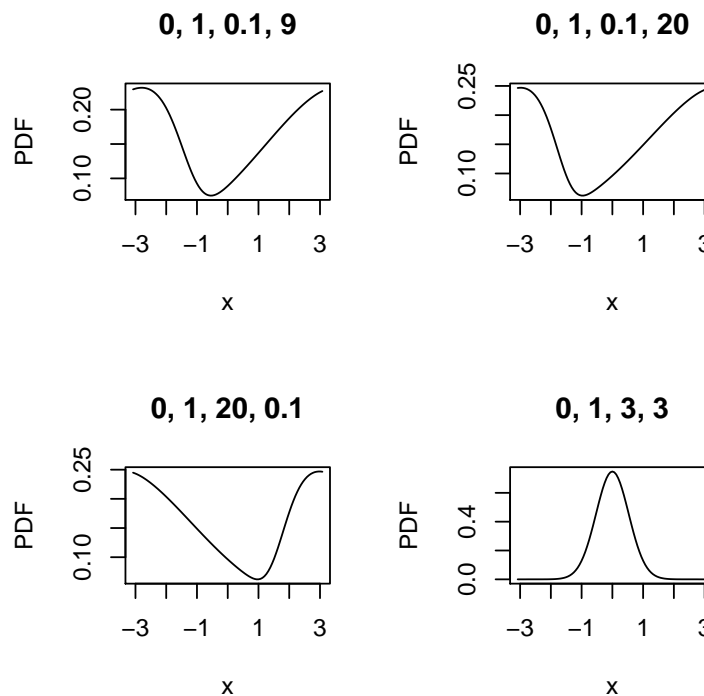


Figure 13: PDFs of the wrapped beta normal distributions for selected parameter values.

The second illustration plots the CDFs of the wrapped skew normal distributions for selected parameter values.

```
plotfour("sn",K=100,para=list(c(0,1,-2),c(0,1,0),c(0,1,2),c(0,1,10)),
plotit="cdf")
```

The plots are

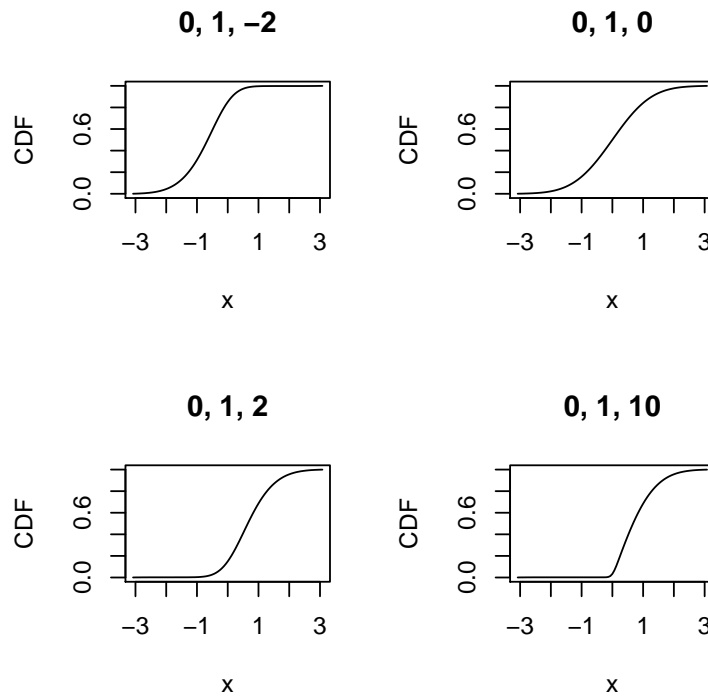


Figure 14: CDFs of the wrapped skew normal distributions for selected parameter values.

The third illustration plots the quantile functions of the wrapped asymmetric Laplace distributions for selected parameter values.

```
plotfour("asla",K=100,para=list(c(0,1,1),c(0,2,1),c(0,5,1),c(0,50,1)),
plotit="quantile")
```

The plots are

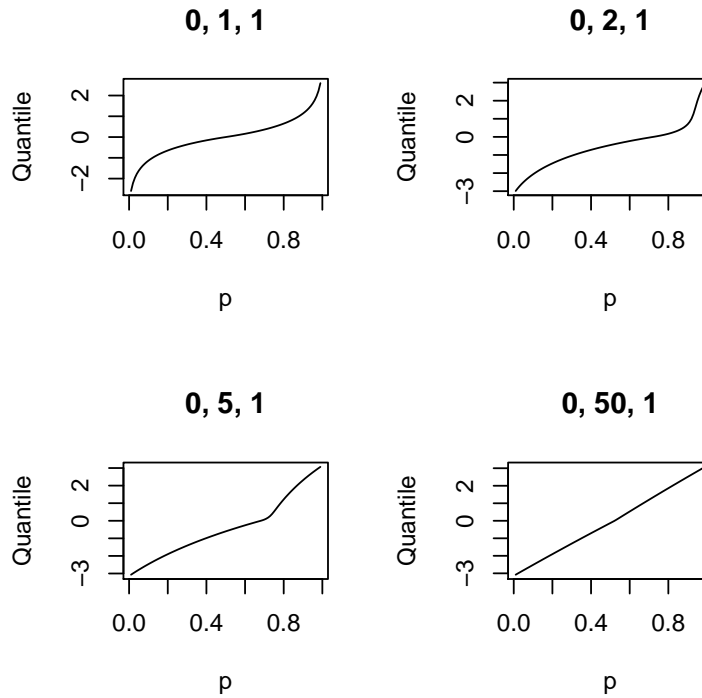


Figure 15: Qunatile functions of the wrapped asymmetric Laplace distributions for selected parameter values.

The fourth illustration plots the histograms of the radii of the 100 random numbers generated from the wrapped skew t type 3 distributions for selected parameter values.

```
plotfour("ST3",K=100,para=list(c(0,1,1,1),c(0,1,2,5),c(0,1,0.1,5),c(0,1,50,50)),
plotit="random")
```

The plots are

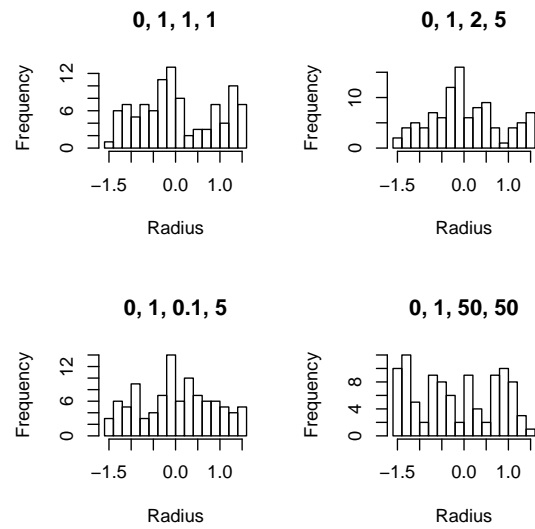


Figure 16: Histograms of 100 random numbers generated from the wrapped skew t type 3 distributions for selected parameter values.

References

- [1] Nadarajah, S. and Zhang, S. (2017). Wrapped: An R Package for Circular Data. Under review for PLOS One.