# Package 'VGAM' 

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Suggests VGAMdata, MASS, mgcv
Description An implementation of about 6 major classes of statistical regression models. The central algorithm is Fisher scoring and iterative reweighted least squares. At the heart of this package are the vector generalized linear and additive model (VGLM/VGAM) classes. VGLMs can be loosely thought of as multivariate GLMs. VGAMs are data-driven VGLMs that use smoothing. The book "Vector Generalized Linear and Additive Models: With an Implementation in R" (Yee, 2015) [DOI:10.1007/978-1-4939-2818-7](DOI:10.1007/978-1-4939-2818-7) gives details of the statistical framework and the package. Currently only fixed-effects models are implemented. Many (150+) models and distributions are estimated by maximum likelihood estimation (MLE) or penalized MLE. The other classes are RR-VGLMs (reduced-rank VGLMs), quadratic RR-VGLMs, reduced-rank VGAMs, RCIMs (row-column interaction models)---these classes perform constrained and unconstrained quadratic ordination (CQO/UQO) models in ecology, as well as constrained additive ordination (CAO). Note that these functions are subject to change; see the NEWS and ChangeLog files for latest changes.
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VGAM-package

Vector Generalized Linear and Additive Models and Associated Mod
els

## Description

VGAM provides functions for fitting vector generalized linear and additive models (VGLMs and VGAMs), and associated models (Reduced-rank VGLMs, Quadratic RR-VGLMs, Reduced-rank VGAMs). This package fits many models and distributions by maximum likelihood estimation (MLE) or penalized MLE. Also fits constrained ordination models in ecology such as constrained quadratic ordination (CQO).

## Details

This package centers on the iteratively reweighted least squares (IRLS) algorithm. Other key words include Fisher scoring, additive models, reduced-rank regression, penalized likelihood, and constrained ordination. The central modelling functions are vglm, vgam, rrvglm, rcim, cqo, cao. Function vglm operates very similarly to glm but is much more general, and many methods functions such as coef and predict are available. The package uses S4 (see methods-package). A companion package called VGAMdata contains some larger data sets which were shifted from VGAM.

Compared to other similar packages, such as gamlss and mgev, VGAM has more models implemented (150+ of them) and they are not restricted to a location-scale-shape framework or (largely) the 1-parameter exponential family. There is a general statistical framework behind it all, that once grasped, makes regression modelling quite unified. Some features of the package are: (i) most family functions handle multiple responses; (ii) reduced-rank regression is available by operating on latent variables (optimal linear combinations of the explanatory variables); (iii) basic automatic smoothing parameter selection is implemented for VGAMs, although it has to be refined; (iv) smart prediction allows correct prediction of nested terms in the formula provided smart functions are used.
The GLM and GAM classes are special cases of VGLMs and VGAMs. The VGLM/VGAM framework is intended to be very general so that it encompasses as many distributions and models as
possible. VGLMs are limited only by the assumption that the regression coefficients enter through a set of linear predictors. The VGLM class is very large and encompasses a wide range of multivariate response types and models, e.g., it includes univariate and multivariate distributions, categorical data analysis, extreme values, correlated binary data, quantile and expectile regression, time series problems. Potentially, it can handle generalized estimating equations, survival analysis, bioassay data and nonlinear least-squares problems.
Crudely, VGAMs are to VGLMs what GAMs are to GLMs. Two types of VGAMs are implemented: 1st-generation VGAMs with s use vector backfitting, while 2nd-generation VGAMs with sm.os and sm.ps use O -splines and P -splines, do not use the backfitting algorithm, and have automatic smoothing parameter selection. The former is older and is based on Yee and Wild (1996). The latter is more modern (Yee, Somchit and Wild, 2018) but it requires a reasonably large number of observations to work well.
This package is the first to check for the Hauck-Donner effect (HDE) in regression models; see hdeff. This is an aberration of the Wald statistics when the parameter estimates are too close to the boundary of the parameter space. When present the p-value of a regression coefficient is biased upwards so that a highly significant variable might be deemed nonsignificant. Thus the HDE can create havoc for variable selection!
Somewhat related to the previous paragraph, hypothesis testing using the likelihood ratio test, Rao's score test (Lagrange multiplier test) and (modified) Wald's test are all available; see summaryvglm. For all regression coefficients of a model, taken one at a time, all three methods require further IRLS iterations to obtain new values of the other regression coefficients after one of the coefficients has had its value set (usually to 0 ). Hence the computation load is overall significant.
For a complete list of this package, use library (help = "VGAM"). New VGAM family functions are continually being written and added to the package.

## Warning

This package is undergoing continual development and improvement, therefore users should treat everything as subject to change. This includes the family function names, argument names, many of the internals, the use of link functions, and slot names. For example, all link functions may be renamed so that they end in "link", e.g., loglink() instead of loglink(). Some future pain can be avoided by using good programming techniques, e.g., using extractor/accessor functions such as $\operatorname{coef}()$, weights(), vcov(), predict(). Nevertheless, please expect changes in all aspects of the package. See the NEWS file for a list of changes from version to version.

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My website for the VGAM package and book is at https://www.stat.auckland.ac.nz/~yee. There are some resources there, especially as relating to my book and new features added to VGAM.

## See Also

vglm, vgam, rrvglm, rcim, cqo, TypicalVGAMfamilyFunction, CommonVGAMffArguments, Links, hdeff, https://CRAN.R-project.org/package=VGAM.

## Examples

```
# Example 1; proportional odds model
pneumo <- transform(pneumo, let = log(exposure.time))
(fit1 <- vglm(cbind(normal, mild, severe) ~ let, propodds, data = pneumo))
depvar(fit1) # Better than using fit1@y; dependent variable (response)
weights(fit1, type = "prior") # Number of observations
coef(fit1, matrix = TRUE) # p.179, in McCullagh and Nelder (1989)
constraints(fit1) # Constraint matrices
summary(fit1) # HDE could affect these results
summary(fit1, lrt0 = TRUE, score0 = TRUE, wald0 = TRUE) # No HDE
hdeff(fit1) # Check for any Hauck-Donner effect
# Example 2; zero-inflated Poisson model
zdata <- data.frame(x2 = runif(nn <- 2000))
zdata <- transform(zdata, pstr0 = logitlink(-0.5 + 1*x2, inverse = TRUE),
    lambda = loglink( 0.5 + 2*x2, inverse = TRUE))
zdata <- transform(zdata, y = rzipois(nn, lambda, pstr0 = pstr0))
with(zdata, table(y))
fit2 <- vglm(y ~ x2, zipoisson, data = zdata, trace = TRUE)
coef(fit2, matrix = TRUE) # These should agree with the above values
# Example 3; fit a two species GAM simultaneously
fit3 <- vgam(cbind(agaaus, kniexc) ~ s(altitude, df = c(2, 3)),
    binomialff(multiple.responses = TRUE), data = hunua)
```

```
coef(fit3, matrix = TRUE) # Not really interpretable
## Not run: plot(fit3, se = TRUE, overlay = TRUE, lcol = 3:4, scol = 3:4)
000 <- with(hunua, order(altitude))
with(hunua, matplot(altitude[ooo], fitted(fit3)[ooo, ], type = "l",
    lwd = 2, col = 3:4,
    xlab = "Altitude (m)", ylab = "Probability of presence", las = 1,
    main = "Two plant species' response curves", ylim = c(0, 0.8)))
with(hunua, rug(altitude))
## End(Not run)
# Example 4; LMS quantile regression
fit4 <- vgam(BMI ~ s(age, df = c(4, 2)), lms.bcn(zero = 1),
    data = bmi.nz, trace = TRUE)
head(predict(fit4))
head(fitted(fit4))
head(bmi.nz) # Person 1 is near the lower quartile among people his age
head(cdf(fit4))
## Not run: par(mfrow = c(1,1), bty = "l", mar = c(5,4,4,3)+0.1, xpd=TRUE)
qtplot(fit4, percentiles = c(5,50,90,99), main = "Quantiles", las = 1,
        xlim = c(15, 90), ylab = "BMI", lwd=2, lcol=4) # Quantile plot
ygrid <- seq(15, 43, len = 100) # BMI ranges
par(mfrow = c(1, 1), lwd = 2) # Density plot
aa <- deplot(fit4, x0 = 20, y = ygrid, xlab = "BMI", col = "black",
    main = "Density functions at Age=20 (black), 42 (red) and 55 (blue)")
aa
aa <- deplot(fit4, x0 = 42, y = ygrid, add = TRUE, llty = 2, col = "red")
aa <- deplot(fit4, x0 = 55, y = ygrid, add = TRUE, llty = 4, col = "blue",
    Attach = TRUE)
aa@post$deplot # Contains density function values
## End(Not run)
# Example 5; GEV distribution for extremes
(fit5 <- vglm(maxtemp ~ 1, gevff, data = oxtemp, trace = TRUE))
head(fitted(fit5))
coef(fit5, matrix = TRUE)
Coef(fit5)
vcov(fit5)
vcov(fit5, untransform = TRUE)
sqrt(diag(vcov(fit5))) # Approximate standard errors
## Not run: rlplot(fit5)
```


## Description

Estimates the three independent parameters of the the A1A2A3 blood group system.

## Usage

```
A1A2A3(link = "logitlink", inbreeding = FALSE, ip1 = NULL, ip2 = NULL, iF = NULL)
```


## Arguments

link Link function applied to p1, p2 and f. See Links for more choices.
inbreeding Logical. Is there inbreeding?
ip1, ip2, iF Optional initial value for $\mathrm{p} 1, \mathrm{p} 2$ and f .

## Details

The parameters $p 1$ and $p 2$ are probabilities, so that $p 3=1-p 1-p 2$ is the third probability. The parameter f is the third independent parameter if inbreeding $=$ TRUE. If inbreeding $=$ FALSE then $f=0$ and Hardy-Weinberg Equilibrium (HWE) is assumed.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

The input can be a 6 -column matrix of counts, with columns corresponding to A1A1, A1A2, A1A3, A2A2, A2A3, A3A3 (in order). Alternatively, the input can be a 6 -column matrix of proportions (so each row adds to 1 ) and the weights argument is used to specify the total number of counts for each row.

## Author(s)

T. W. Yee

## References

Lange, K. (2002). Mathematical and Statistical Methods for Genetic Analysis, 2nd ed. New York: Springer-Verlag.

## See Also

AA.Aa. aa, AB.Ab.aB.ab, ABO, MNSs.

## Examples

```
ymat <- cbind(108, 196, 429, 143, 513, 559)
fit <- vglm(ymat ~ 1, A1A2A3(link = probitlink), trace = TRUE, crit = "coef")
fit <- vglm(ymat ~ 1, A1A2A3(link = logitlink, ip1 = 0.3, ip2 = 0.3, iF = 0.02),
    trace = TRUE, crit = "coef")
Coef(fit) # Estimated p1 and p2
rbind(ymat, sum(ymat) * fitted(fit))
sqrt(diag(vcov(fit)))
```

AA.Aa. aa The AA-Aa-aa Blood Group System

## Description

Estimates the parameter of the AA-Aa-aa blood group system, with or without Hardy Weinberg equilibrium.

## Usage

AA.Aa.aa(linkp = "logitlink", linkf = "logitlink", inbreeding = FALSE, ipA $=$ NULL, ifp $=$ NULL, zero $=$ NULL)

## Arguments

linkp, linkf Link functions applied to pA and f. See Links for more choices.
$i p A, i f p \quad$ Optional initial values for $p A$ and $f$.
inbreeding Logical. Is there inbreeding?
zero See CommonVGAMffArguments for information.

## Details

This one or two parameter model involves a probability called pA . The probability of getting a count in the first column of the input (an AA) is $\mathrm{pA} * \mathrm{pA}$. When inbreeding = TRUE, an additional parameter f is used. If inbreeding $=$ FALSE then $f=0$ and Hardy-Weinberg Equilibrium (HWE) is assumed. The EIM is used if inbreeding = FALSE.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Warning

Setting inbreeding $=$ FALSE makes estimation difficult with non-intercept-only models. Currently, this code seems to work with intercept-only models.

## Note

The input can be a 3-column matrix of counts, where the columns are $\mathrm{AA}, \mathrm{Ab}$ and aa (in order). Alternatively, the input can be a 3-column matrix of proportions (so each row adds to 1 ) and the weights argument is used to specify the total number of counts for each row.

## Author(s)

T. W. Yee

## References

Weir, B. S. (1996). Genetic Data Analysis II: Methods for Discrete Population Genetic Data, Sunderland, MA: Sinauer Associates, Inc.

## See Also

AB.Ab.aB.ab, ABO, A1A2A3, MNSs.

## Examples

```
y <- cbind(53, 95, 38)
fit1 <- vglm(y ~ 1, AA.Aa.aa, trace = TRUE)
fit2 <- vglm(y ~ 1, AA.Aa.aa(inbreeding = TRUE), trace = TRUE)
rbind(y, sum(y) * fitted(fit1))
Coef(fit1) # Estimated pA
Coef(fit2) # Estimated pA and f
summary(fit1)
```


## Description

Estimates the parameter of the $\mathrm{AB}-\mathrm{Ab}-\mathrm{aB}-\mathrm{ab}$ blood group system.

## Usage

AB.Ab.aB.ab(link = "logitlink", init.p = NULL)

## Arguments

link Link function applied to p . See Links for more choices.
init.p Optional initial value for $p$.

## Details

This one parameter model involves a probability called $p$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

The input can be a 4-column matrix of counts, where the columns are $\mathrm{AB}, \mathrm{Ab}, \mathrm{aB}$ and ab (in order). Alternatively, the input can be a 4-column matrix of proportions (so each row adds to 1 ) and the weights argument is used to specify the total number of counts for each row.

## Author(s)

T. W. Yee

## References

Lange, K. (2002). Mathematical and Statistical Methods for Genetic Analysis, 2nd ed. New York: Springer-Verlag.

## See Also

AA. Aa. aa, ABO, A1A2A3, MNSs.

## Examples

```
ymat <- cbind(AB=1997, Ab=906, aB=904, ab=32) # Data from Fisher (1925)
fit <- vglm(ymat ~ 1, AB.Ab.aB.ab(link = "identitylink"), trace = TRUE)
fit <- vglm(ymat ~ 1, AB.Ab.aB.ab, trace = TRUE)
rbind(ymat, sum(ymat)*fitted(fit))
Coef(fit) # Estimated p
p <- sqrt(4*(fitted(fit)[, 4]))
p*p
summary(fit)
```

ABO The ABO Blood Group System

## Description

Estimates the two independent parameters of the the ABO blood group system.

## Usage

ABO(link. $\mathrm{pA}=$ "logitlink", link. $\mathrm{pB}=$ "logitlink", $\mathrm{ipA}=$ NULL, $\mathrm{ipB}=$ NULL, ip0 = NULL, zero = NULL)

## Arguments

| link. pA, link. pB |  |
| :--- | :--- |
| Link functions applied to pA and pB . See Links for more choices. |  |
| ipA, ipB, ip0 | Optional initial value for pA and pB and pO . A NULL value means values are <br> computed internally. |
| zero | Details at CommonVGAMffArguments. |

## Details

The parameters $p A$ and $p B$ are probabilities, so that $p 0=1-p A-p B$ is the third probability. The probabilities $p A$ and $p B$ correspond to $A$ and $B$ respectively, so that $p O$ is the probability for $O$. It is easier to make use of initial values for $p 0$ than for $p B$. In documentation elsewhere I sometimes use $p A=p$, $p B=q, p 0=r$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

The input can be a 4 -column matrix of counts, where the columns are $\mathrm{A}, \mathrm{B}, \mathrm{AB}, \mathrm{O}$ (in order). Alternatively, the input can be a 4-column matrix of proportions (so each row adds to 1 ) and the weights argument is used to specify the total number of counts for each row.

## Author(s)

T. W. Yee

## References

Lange, K. (2002). Mathematical and Statistical Methods for Genetic Analysis, 2nd ed. New York: Springer-Verlag.

## See Also

AA.Aa. aa, AB.Ab.aB.ab, A1A2A3, MNSs.

## Examples

```
ymat <- cbind(A = 725, B = 258, AB = 72, O = 1073) # Order matters, not the name
fit <- vglm(ymat ~ 1, ABO(link.pA = "identitylink",
                                    link.pB = "identitylink"), trace = TRUE,
    crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit) # Estimated pA and pB
rbind(ymat, sum(ymat) * fitted(fit))
sqrt(diag(vcov(fit)))
```


## Description

Fits an adjacent categories regression model to an ordered (preferably) factor response.

## Usage

acat(link $=$ "loglink", parallel = FALSE, reverse $=$ FALSE, zero $=$ NULL, whitespace $=$ FALSE)

## Arguments

link Link function applied to the ratios of the adjacent categories probabilities. See Links for more choices.
parallel A logical, or formula specifying which terms have equal/unequal coefficients.
reverse Logical. By default, the linear/additive predictors used are $\eta_{j}=\log (P[Y=$ $j+1] / P[Y=j])$ for $j=1, \ldots, M$. If reverse is TRUE then $\eta_{j}=\log (P[Y=$ $j] / P[Y=j+1]$ ) will be used.
zero An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set $\{1,2, \ldots, M\}$.
whitespace See CommonVGAMffArguments for information.

## Details

In this help file the response $Y$ is assumed to be a factor with ordered values $1,2, \ldots, M+1$, so that $M$ is the number of linear/additive predictors $\eta_{j}$.
By default, the log link is used because the ratio of two probabilities is positive.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Warning

No check is made to verify that the response is ordinal if the response is a matrix; see ordered.

## Note

The response should be either a matrix of counts (with row sums that are all positive), or an ordered factor. In both cases, the $y$ slot returned by vglm/vgam/rrvglm is the matrix of counts.
For a nominal (unordered) factor response, the multinomial logit model (multinomial) is more appropriate.

Here is an example of the usage of the parallel argument. If there are covariates $\times 1, x 2$ and $x 3$, then parallel $=$ TRUE $\sim x 1+x 2-1$ and parallel $=$ FALSE $\sim x 3$ are equivalent. This would constrain the regression coefficients for $x 1$ and $\times 2$ to be equal; those of the intercepts and $\times 3$ would be different.

## Author(s)

Thomas W. Yee

## References

Agresti, A. (2013). Categorical Data Analysis, 3rd ed. Hoboken, NJ, USA: Wiley.
Simonoff, J. S. (2003). Analyzing Categorical Data, New York: Springer-Verlag.
Yee, T. W. (2010). The VGAM package for categorical data analysis. Journal of Statistical Software, 32, 1-34. http://www.jstatsoft.org/v32/i10/.

## See Also

cumulative, cratio, sratio, multinomial, margeff, pneumo.

## Examples

```
pneumo <- transform(pneumo, let = log(exposure.time))
(fit <- vglm(cbind(normal, mild, severe) ~ let, acat, data = pneumo))
coef(fit, matrix = TRUE)
constraints(fit)
model.matrix(fit)
```

add1.vglm Add or Drop All Possible Single Terms to/from a Model

## Description

Compute all the single terms in the scope argument that can be added to or dropped from the model, fit those models and compute a table of the changes in fit.

## Usage

```
    ## S3 method for class 'vglm'
```

    add1 (object, scope, test = c("none", "LRT"), k = 2, ...)
    \#\# S3 method for class 'vglm'
    drop1 (object, scope, test \(=c(" n o n e ", " L R T "), k=2, \ldots\) )
    
## Arguments

object a fitted vglm model object.
scope, k See drop1.glm.
test Same as drop1.glm but with fewer choices.
... further arguments passed to or from other methods.

## Details

These functions are a direct adaptation of add1.glm and drop1.glm for vglm-class objects. For drop1 methods, a missing scope is taken to be all terms in the model. The hierarchy is respected when considering terms to be added or dropped: all main effects contained in a second-order interaction must remain, and so on. In a scope formula . means 'what is already there'.
Compared to add1.glm and drop1.glm these functions are simpler, e.g., there is no $C p, \mathrm{~F}$ and Rao (score) tests, $x$ and scale arguments. Most models do not have a deviance, however twice the log-likelihood differences are used to test the significance of terms.
The default output table gives AIC, defined as minus twice $\log$ likelihood plus $2 p$ where $p$ is the rank of the model (the number of effective parameters). This is only defined up to an additive constant (like log-likelihoods).

## Value

An object of class "anova" summarizing the differences in fit between the models.

## Warning

In general, the same warnings in add1.glm and drop1.glm apply here. Furthermore, these functions have not been rigorously tested for all models, so treat the results cautiously and please report any bugs.
Care is needed to check that the constraint matrices of added terms are correct. Also, if object is of the form $\operatorname{vglm}(\ldots$, constraints $=\operatorname{list}(x 1=c m 1, x 2=c m 2))$ then add1.vglm may fail because the constraints argument needs to have the constaint matrices for all terms.

## Note

Most VGAM family functions do not compute a deviance, but instead the likelihood function is evaluated at the MLE. Hence a column name "Deviance" only appears for a few models; and almost always there is a column labelled "logLik".

## See Also

```
step4vglm, vglm, extractAIC.vglm, anova.vglm, backPain2, update.
```


## Examples

```
data("backPain2", package = "VGAM")
summary(backPain2)
fit1 <- vglm(pain ~ x2 + x3 + x4, propodds, data = backPain2)
coef(fit1)
add1(fit1, scope = ~ x2 * x3 * x4, test = "LRT")
drop1(fit1, test = "LRT")
fit2 <- vglm(pain ~ x2 * x3 * x4, propodds, data = backPain2)
drop1(fit2)
```

AICvlm Akaike's Information Criterion

## Description

Calculates the Akaike information criterion for a fitted model object for which a log-likelihood value has been obtained.

## Usage

$$
\begin{aligned}
& \text { AICvlm(object, } \ldots, \text { corrected }=\text { FALSE, } k=2 \text { ) } \\
& \text { AICvgam(object, } \ldots, \mathrm{k}=2) \\
& \text { AICrrvglm(object, } \ldots, \mathrm{k}=2) \\
& \text { AICqrrvglm(object, } \ldots, \mathrm{k}=2) \\
& \text { AICrrvgam(object, } \ldots, \mathrm{k}=2)
\end{aligned}
$$

## Arguments

object Some VGAM object, for example, having class vglm-class.
... Other possible arguments fed into logLik in order to compute the log-likelihood.
corrected Logical, perform the finite sample correction?
$\mathrm{k} \quad$ Numeric, the penalty per parameter to be used; the default is the classical AIC.

## Details

The following formula is used for VGLMs: $-2 \log$-likelihood $+k n_{p a r}$, where $n_{\text {par }}$ represents the number of parameters in the fitted model, and $k=2$ for the usual AIC. One could assign $k=\log (n)$ ( $n$ the number of observations) for the so-called BIC or SBC (Schwarz's Bayesian criterion). This is the function AICvlm().
This code relies on the log-likelihood being defined, and computed, for the object. When comparing fitted objects, the smaller the AIC, the better the fit. The log-likelihood and hence the AIC is only defined up to an additive constant.

Any estimated scale parameter (in GLM parlance) is used as one parameter.
For VGAMs and CAO the nonlinear effective degrees of freedom for each smoothed component is used. This formula is heuristic. These are the functions AICvgam() and AICcao().

The finite sample correction is usually recommended when the sample size is small or when the number of parameters is large. When the sample size is large their difference tends to be negligible. The correction is described in Hurvich and Tsai (1989), and is based on a (univariate) linear model with normally distributed errors.

## Value

Returns a numeric value with the corresponding AIC (or BIC, or .... depending on k ).

## Warning

This code has not been double-checked. The general applicability of AIC for the VGLM/VGAM classes has not been developed fully. In particular, AIC should not be run on some VGAM family functions because of violation of certain regularity conditions, etc.

## Note

AIC has not been defined for QRR-VGLMs, yet.
Using AIC to compare posbinomial models with, e.g., posbernoulli.tb models, requires posbinomial(omit. constant = TRUE). See posbinomial for an example. A warning is given if it suspects a wrong omit. constant value was used.

Where defined, $\operatorname{AICc}(\ldots)$ is the same as $\operatorname{AIC}(\ldots$, corrected $=$ TRUE $)$.

## Author(s)

T. W. Yee.

## References

Hurvich, C. M. and Tsai, C.-L. (1989). Regression and time series model selection in small samples, Biometrika, 76, 297-307.

## See Also

VGLMs are described in vglm-class; VGAMs are described in vgam-class; RR-VGLMs are described in rrvglm-class; AIC, BICvlm, TICvlm, drop1.vglm, extractAIC.vglm.

## Examples

```
pneumo <- transform(pneumo, let = log(exposure.time))
(fit1 <- vglm(cbind(normal, mild, severe) ~ let,
            cumulative(parallel = TRUE, reverse = TRUE), data = pneumo))
coef(fit1, matrix = TRUE)
AIC(fit1)
AICc(fit1) # Quick way
AIC(fit1, corrected = TRUE) # Slow way
(fit2 <- vglm(cbind(normal, mild, severe) ~ let,
    cumulative(parallel = FALSE, reverse = TRUE), data = pneumo))
coef(fit2, matrix = TRUE)
AIC(fit2)
AICc(fit2)
AIC(fit2, corrected = TRUE)
```

```
alaplace Asymmetric Laplace Distribution Family Functions
```


## Description

Maximum likelihood estimation of the 1, 2 and 3-parameter asymmetric Laplace distributions (ALDs). The 2-parameter ALD may, with trepidation and lots of skill, sometimes be used as an approximation of quantile regression.

## Usage

```
alaplace1(tau = NULL, llocation = "identitylink",
    ilocation = NULL, kappa = sqrt(tau/(1 - tau)), Scale.arg = 1,
    ishrinkage = 0.95, parallel.locat = TRUE ~ 0, digt = 4,
    idf.mu = 3, zero = NULL, imethod = 1)
alaplace2(tau = NULL, llocation = "identitylink", lscale = "loglink",
    ilocation = NULL, iscale = NULL, kappa = sqrt(tau/(1 - tau)),
    ishrinkage = 0.95,
    parallel.locat = TRUE ~ 0,
    parallel.scale = FALSE ~ 0,
    digt = 4, idf.mu = 3, imethod = 1, zero = "scale")
alaplace3(llocation = "identitylink", lscale = "loglink",
    lkappa = "loglink", ilocation = NULL, iscale = NULL,
    ikappa = 1, imethod = 1, zero = c("scale", "kappa"))
```


## Arguments

tau, kappa Numeric vectors with $0<\tau<1$ and $\kappa>0$. Most users will only specify tau since the estimated location parameter corresponds to the $\tau$ th regression quantile, which is easier to understand. See below for details.
llocation, lscale, lkappa
Character. Parameter link functions for location parameter $\xi$, scale parameter $\sigma$, asymmetry parameter $\kappa$. See Links for more choices. For example, the argument llocation can help handle count data by restricting the quantiles to be positive (use llocation = "loglink"). However, llocation is best left alone since the theory only works properly with the identity link.
ilocation, iscale, ikappa
Optional initial values. If given, it must be numeric and values are recycled to the appropriate length. The default is to choose the value internally.
parallel.locat, parallel.scale
See the parallel argument of CommonVGAMffArguments. These arguments apply to the location and scale parameters. It generally only makes sense for the
scale parameters to be equal, hence set parallel.scale = TRUE. Note that assigning parallel.locat the value TRUE circumvents the seriously embarrassing quantile crossing problem because all constraint matrices except for the intercept correspond to a parallelism assumption.

| imethod | Initialization method. Either the value 1, 2, 3 or 4. |
| :--- | :--- |
| idf.mu | Degrees of freedom for the cubic smoothing spline fit applied to get an initial <br> estimate of the location parameter. See vsmooth. spline. Used only when <br> imethod $=3$. |
| ishrinkage | How much shrinkage is used when initializing $\xi$. The value must be between 0 <br> and 1 inclusive, and a value of 0 means the individual response values are used, <br> and a value of 1 means the median or mean is used. This argument is used only <br> when imethod = 4. See CommonVGAMffArguments for more information. |
| Scale.arg | The value of the scale parameter $\sigma$. This argument may be used to compute <br> quantiles at different $\tau$ values from an existing fitted alaplace2() model (prac- <br> tical only if it has a single value). If the model has parallel. locat = TRUE then <br> only the intercept need be estimated; use an offset. See below for an example. |
| digt | Passed into Round as the digits argument for the tau values; used cosmetically <br> for labelling. |
| zero | See CommonVGAMffArguments for more information. Where possible, the de- <br> fault is to model all the $\sigma$ and $\kappa$ as an intercept-only term. |

## Details

These VGAM family functions implement one variant of asymmetric Laplace distributions (ALDs) suitable for quantile regression. Kotz et al. (2001) call it the ALD. Its density function is

$$
f(y ; \xi, \sigma, \kappa)=\frac{\sqrt{2}}{\sigma} \frac{\kappa}{1+\kappa^{2}} \exp \left(-\frac{\sqrt{2}}{\sigma \kappa}|y-\xi|\right)
$$

for $y \leq \xi$, and

$$
f(y ; \xi, \sigma, \kappa)=\frac{\sqrt{2}}{\sigma} \frac{\kappa}{1+\kappa^{2}} \exp \left(-\frac{\sqrt{2} \kappa}{\sigma}|y-\xi|\right)
$$

for $y>\xi$. Here, the ranges are for all real $y$ and $\xi$, positive $\sigma$ and positive $\kappa$. The special case $\kappa=1$ corresponds to the (symmetric) Laplace distribution of Kotz et al. (2001). The mean is $\xi+\sigma(1 / \kappa-\kappa) / \sqrt{2}$ and the variance is $\sigma^{2}\left(1+\kappa^{4}\right) /\left(2 \kappa^{2}\right)$. The enumeration of the linear/additive predictors used for alaplace2() is the first location parameter followed by the first scale parameter, then the second location parameter followed by the second scale parameter, etc. For alaplace3(), only a vector response is handled and the last (third) linear/additive predictor is for the asymmetry parameter.
It is known that the maximum likelihood estimate of the location parameter $\xi$ corresponds to the regression quantile estimate of the classical quantile regression approach of Koenker and Bassett (1978). An important property of the ALD is that $P(Y \leq \xi)=\tau$ where $\tau=\kappa^{2} /\left(1+\kappa^{2}\right)$ so that $\kappa=\sqrt{\tau /(1-\tau)}$. Thus alaplace2() might be used as an alternative to rq in the quantreg package, although scoring is really an unsuitable algorithm for estimation here.
Both alaplace1 () and alaplace2() can handle multiple responses, and the number of linear/additive predictors is dictated by the length of tau or kappa. The functions alaplace1() and alaplace2()
can also handle multiple responses (i.e., a matrix response) but only with a single-valued tau or kappa.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

In the extra slot of the fitted object are some list components which are useful, e.g., the sample proportion of values which are less than the fitted quantile curves.

## Warning

These functions are experimental and especially subject to change or withdrawal. The usual MLE regularity conditions do not hold for this distribution so that misleading inferences may result, e.g., in the summary and vcov of the object. The 1-parameter ALD can be approximated by extlogF1 which has continuous derivatives and is recommended over alaplace1.
Care is needed with tau values which are too small, e.g., for count data with llocation = "loglink" and if the sample proportion of zeros is greater than tau.

## Note

These VGAM family functions use Fisher scoring. Convergence may be slow and half-stepping is usual (although one can use trace = TRUE to see which is the best model and then use maxit to choose that model) due to the regularity conditions not holding. Often the iterations slowly crawl towards the solution so monitoring the convergence (set trace $=$ TRUE) is highly recommended. Instead, extlogF1 is recommended.

For large data sets it is a very good idea to keep the length of tau/kappa low to avoid large memory requirements. Then for parallel.locat $=$ FALSE one can repeatedly fit a model with alaplace1 () with one $\tau$ at a time; and for parallel. locat = TRUE one can refit a model with alaplace1() with one $\tau$ at a time but using offsets and an intercept-only model.
A second method for solving the noncrossing quantile problem is illustrated below in Example 3. This is called the accumulative quantile method (AQM) and details are in Yee (2015). It does not make the strong parallelism assumption.

The functions alaplace2() and laplace differ slightly in terms of the parameterizations.

## Author(s)

Thomas W. Yee

## References

Koenker, R. and Bassett, G. (1978). Regression quantiles. Econometrica, 46, 33-50.
Kotz, S., Kozubowski, T. J. and Podgorski, K. (2001). The Laplace distribution and generalizations: a revisit with applications to communications, economics, engineering, and finance, Boston: Birkhauser.

## See Also

ralap, laplace, extlogF1, CommonVGAMffArguments, lms.bcn, amlnormal, sc.studentt2, simulate.vlm.

## Examples

```
## Not run:
# Example 1: quantile regression with smoothing splines
set.seed(123); adata <- data.frame(x2 = sort(runif(n <- 500)))
mymu <- function(x) exp(-2 + 6*sin(2*x-0.2) / (x+0.5)^2)
adata <- transform(adata, y = rpois(n, lambda = mymu(x2)))
mytau <- c(0.25, 0.75); mydof <- 4
fit <- vgam(y ~ s(x2, df = mydof), data=adata, trace=TRUE, maxit = 900,
    alaplace2(tau = mytau, llocat = "loglink",
                    parallel.locat = FALSE))
fitp <- vgam(y ~ s(x2, df = mydof), data = adata, trace=TRUE, maxit=900,
    alaplace2(tau = mytau, llocat = "loglink", parallel.locat = TRUE))
par(las = 1); mylwd <- 1.5
with(adata, plot(x2, jitter(y, factor = 0.5), col = "orange",
                        main = "Example 1; green: parallel.locat = TRUE",
                        ylab = "y", pch = "o", cex = 0.75))
with(adata, matlines(x2, fitted(fit ), col = "blue",
                    lty = "solid", lwd = mylwd))
with(adata, matlines(x2, fitted(fitp), col = "green",
                    lty = "solid", lwd = mylwd))
finexgrid <- seq(0, 1, len = 1001)
for (ii in 1:length(mytau))
    lines(finexgrid, qpois(p = mytau[ii], lambda = mymu(finexgrid)),
            col = "blue", lwd = mylwd)
fit@extra # Contains useful information
# Example 2: regression quantile at a new tau value from an existing fit
# Nb. regression splines are used here since it is easier.
fitp2 <- vglm(y ~ sm.bs(x2, df = mydof), data = adata, trace = TRUE,
            alaplace1(tau = mytau, llocation = "loglink",
                    parallel.locat = TRUE))
newtau <- 0.5 # Want to refit the model with this tau value
fitp3 <- vglm(y ~ 1 + offset(predict(fitp2)[, 1]),
            alaplace1(tau = newtau, llocation = "loglink"), adata)
with(adata, plot(x2, jitter(y, factor = 0.5), col = "orange",
            pch = "o", cex = 0.75, ylab = "y",
            main = "Example 2; parallel.locat = TRUE"))
with(adata, matlines(x2, fitted(fitp2), col = "blue",
            lty = 1, lwd = mylwd))
with(adata, matlines(x2, fitted(fitp3), col = "black",
            lty = 1, lwd = mylwd))
```

\# Example 3: noncrossing regression quantiles using a trick: obtain

```
# successive solutions which are added to previous solutions; use a log
# link to ensure an increasing quantiles at any value of x.
mytau <- seq(0.2, 0.9, by = 0.1)
answer <- matrix(0, nrow(adata), length(mytau)) # Stores the quantiles
adata <- transform(adata, offsety = y*0)
usetau <- mytau
for (ii in 1:length(mytau)) {
# cat("\n\nii = ", ii, "\n")
    adata <- transform(adata, usey = y-offsety)
    iloc <- ifelse(ii == 1, with(adata, median(y)), 1.0) # Well-chosen!
    mydf <- ifelse(ii == 1, 5, 3) # Maybe less smoothing will help
    fit3 <- vglm(usey ~ sm.ns(x2, df = mydf), data = adata, trace = TRUE,
                                    alaplace2(tau = usetau[ii], lloc = "loglink", iloc = iloc))
    answer[, ii] <- (if(ii == 1) 0 else answer[, ii-1]) + fitted(fit3)
    adata <- transform(adata, offsety = answer[, ii])
}
# Plot the results.
with(adata, plot(x2, y, col = "blue",
            main = paste("Noncrossing and nonparallel; tau = ",
                                    paste(mytau, collapse = ", "))))
with(adata, matlines(x2, answer, col = "orange", lty = 1))
# Zoom in near the origin.
with(adata, plot(x2, y, col = "blue", xlim = c(0, 0.2), ylim = 0:1,
    main = paste("Noncrossing and nonparallel; tau = ",
        paste(mytau, collapse = ", "))))
with(adata, matlines(x2, answer, col = "orange", lty = 1))
## End(Not run)
```

alaplaceUC The Laplace Distribution

## Description

Density, distribution function, quantile function and random generation for the 3-parameter asymmetric Laplace distribution with location parameter location, scale parameter scale, and asymmetry parameter kappa.

## Usage

```
dalap(x, location = 0, scale = 1, tau = 0.5, kappa = sqrt(tau/(1-tau)),
    log = FALSE)
palap(q, location = 0, scale = 1, tau = 0.5, kappa = sqrt(tau/(1-tau)),
        lower.tail = TRUE, log.p = FALSE)
qalap(p, location = 0, scale = 1, tau = 0.5, kappa = sqrt(tau/(1-tau)),
        lower.tail = TRUE, log.p = FALSE)
ralap(n, location = 0, scale = 1, tau = 0.5, kappa = sqrt(tau/(1-tau)))
```


## Arguments

| x, q | vector of quantiles. |
| :---: | :---: |
| p | vector of probabilities. |
| n | number of observations. If length $(n)>1$ then the length is taken to be the number required. |
| location | the location parameter $\xi$. |
| scale | the scale parameter $\sigma$. Must consist of positive values. |
| tau | the quantile parameter $\tau$. Must consist of values in $(0,1)$. This argument is used to specify kappa and is ignored if kappa is assigned. |
| kappa | the asymmetry parameter $\kappa$. Must consist of positive values. |
| log | if TRUE, probabilities $p$ are given as $\log (p)$. |
| lower.tail, log.p |  |
|  | Same meaning as in pnorm or qnorm. |

## Details

There are many variants of asymmetric Laplace distributions (ALDs) and this one is known as the ALD by Kotz et al. (2001). See alaplace3, the VGAM family function for estimating the three parameters by maximum likelihood estimation, for formulae and details. The ALD density may be approximated by dextlogF.

## Value

dalap gives the density, palap gives the distribution function, qalap gives the quantile function, and ralap generates random deviates.

## Author(s)

T. W. Yee and Kai Huang

## References

Kotz, S., Kozubowski, T. J. and Podgorski, K. (2001). The Laplace distribution and generalizations: a revisit with applications to communications, economics, engineering, and finance, Boston: Birkhauser.

## See Also

alaplace3, dextlogF, extlogF1.

## Examples

```
x <- seq(-5, 5, by = 0.01)
loc <- 0; sigma <- 1.5; kappa <- 2
## Not run: plot(x, dalap(x, loc, sigma, kappa = kappa), type = "l",
    main = "Blue is density, orange is cumulative distr. function",
    ylim = c(0, 1), sub = "Purple are 5, 10, ..., 95 percentiles",
```

```
    las = 1, ylab = "", cex.main = 0.5, col = "blue")
abline(h = 0, col = "blue", lty = 2)
lines(qalap(seq(0.05, 0.95, by = 0.05), loc, sigma, kappa = kappa),
    dalap(qalap(seq(0.05, 0.95, by = 0.05), loc, sigma, kappa = kappa),
            loc, sigma, kappa = kappa), col="purple", lty=3, type = "h")
lines(x, palap(x, loc, sigma, kappa = kappa), type = "l", col = "orange")
abline(h = 0, lty = 2)
## End(Not run)
pp <- seq(0.05, 0.95, by = 0.05) # Test two functions
max(abs(palap(qalap(pp, loc, sigma, kappa = kappa),
    loc, sigma, kappa = kappa) - pp)) # Should be 0
```


## Description

Return the altered, inflated and truncated values in a GAIT regression from a fitted object

## Usage

```
altered(object, ...)
inflated(object, ...)
truncated(object, ...)
```


## Arguments

object
an object of class "vglm".
. . .
any additional arguments, to future-proof this function.

## Details

Yee and Ma (2020) propose GAIT regression where values from three disjoint sets are referred to as special. These extractor functions return one set each; they are the alter, inflate, truncate (and sometimes max. support) arguments from the family function.

## Value

Returns one type of 'special' sets assoicated with GAIT regression. This is a vector, else a list for truncation. All three sets are returned by specialsvglm.

## References

Yee, T. W. and Ma, C. (2020). Generally-altered, -inflated and -truncated regression, with application to heaped and seeped count data. In preparation.

## See Also

vglm, vglm-class, specialsvglm, gaitpoisson.mix, gaitlog.mix.

## Examples

```
abdata <- data.frame(y = 0:7, w = c(182, 41, 12, 2, 2, 0, 0, 1))
fit1 <- vglm(y ~ 1, gaitpoisson.mix(inflate = 1), abdata, weight = w,
        subset = w > 0)
altered(fit1)
inflated(fit1)
truncated(fit1)
specials(fit1) # All three sets
```

amlbinomial

Binomial Logistic Regression by Asymmetric Maximum Likelihood Estimation

## Description

Binomial quantile regression estimated by maximizing an asymmetric likelihood function.

## Usage

amlbinomial(w.aml = 1, parallel = FALSE, digw = 4, link = "logitlink")

## Arguments

w.aml Numeric, a vector of positive constants controlling the percentiles. The larger the value the larger the fitted percentile value (the proportion of points below the "w-regression plane"). The default value of unity results in the ordinary maximum likelihood (MLE) solution.
parallel If $w$.aml has more than one value then this argument allows the quantile curves to differ by the same amount as a function of the covariates. Setting this to be TRUE should force the quantile curves to not cross (although they may not cross anyway). See CommonVGAMffArguments for more information.
digw Passed into Round as the digits argument for the w.aml values; used cosmetically for labelling.
link See binomialff.

## Details

The general methodology behind this VGAM family function is given in Efron (1992) and full details can be obtained there. This model is essentially a logistic regression model (see binomialff) but the usual deviance is replaced by an asymmetric squared error loss function; it is multiplied by w.aml for positive residuals. The solution is the set of regression coefficients that minimize the sum of these deviance-type values over the data set, weighted by the weights argument (so that it can contain frequencies). Newton-Raphson estimation is used here.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Warning

If w.aml has more than one value then the value returned by deviance is the sum of all the (weighted) deviances taken over all the w. aml values. See Equation (1.6) of Efron (1992).

## Note

On fitting, the extra slot has list components "w.aml" and "percentile". The latter is the percent of observations below the "w-regression plane", which is the fitted values. Also, the individual deviance values corresponding to each element of the argument $\mathrm{w} . \mathrm{aml}$ is stored in the extra slot.
For amlbinomial objects, methods functions for the generic functions qtplot and cdf have not been written yet.
See amlpoisson about comments on the jargon, e.g., expectiles etc.
In this documentation the word quantile can often be interchangeably replaced by expectile (things are informal here).

## Author(s)

Thomas W. Yee

## References

Efron, B. (1992). Poisson overdispersion estimates based on the method of asymmetric maximum likelihood. Journal of the American Statistical Association, 87, 98-107.

## See Also

amlpoisson, amlexponential, amlnormal, extlogF1, alaplace1, denorm.

## Examples

```
# Example: binomial data with lots of trials per observation
set.seed(1234)
sizevec <- rep(100, length = (nn <- 200))
mydat <- data.frame(x = sort(runif(nn)))
mydat <- transform(mydat,
                prob = logitlink(-0 + 2.5*x + x^2, inverse = TRUE))
mydat <- transform(mydat, y = rbinom(nn, size = sizevec, prob = prob))
(fit <- vgam(cbind(y, sizevec - y) ~ s(x, df = 3),
            amlbinomial(w = c(0.01, 0.2, 1, 5, 60)),
            mydat, trace = TRUE))
fit@extra
## Not run:
par(mfrow = c(1,2))
# Quantile plot
```

```
with(mydat, plot(x, jitter(y), col = "blue", las = 1, main =
    paste(paste(round(fit@extra$percentile, digits = 1), collapse = ", "),
        "percentile-expectile curves")))
with(mydat, matlines(x, 100 * fitted(fit), lwd = 2, col = "blue", lty=1))
# Compare the fitted expectiles with the quantiles
with(mydat, plot(x, jitter(y), col = "blue", las = 1, main =
    paste(paste(round(fit@extra$percentile, digits = 1), collapse = ", "),
        "percentile curves are red")))
with(mydat, matlines(x, 100 * fitted(fit), lwd = 2, col = "blue", lty = 1))
for (ii in fit@extra$percentile)
    with(mydat, matlines(x, 100 *
        qbinom(p = ii/100, size = sizevec, prob = prob) / sizevec,
                        col = "red", lwd = 2, lty = 1))
## End(Not run)
```

amlexponential Exponential Regression by Asymmetric Maximum Likelihood Estima-
tion

## Description

Exponential expectile regression estimated by maximizing an asymmetric likelihood function.

## Usage

amlexponential(w.aml = 1, parallel = FALSE, imethod = 1, digw = 4, link = "loglink")

## Arguments

| w.aml | Numeric, a vector of positive constants controlling the expectiles. The larger the <br> value the larger the fitted expectile value (the proportion of points below the "w- <br> regression plane"). The default value of unity results in the ordinary maximum <br> likelihood (MLE) solution. |
| :--- | :--- |
| parallel | If w.aml has more than one value then this argument allows the quantile curves <br> to differ by the same amount as a function of the covariates. Setting this to be <br> TRUE should force the quantile curves to not cross (although they may not cross <br> anyway). See CommonVGAMffArguments for more information. |
| imethod | Integer, either 1 or 2 or 3. Initialization method. Choose another value if con- <br> vergence fails. |
| digw | Passed into Round as the digits argument for the w.aml values; used cosmeti- <br> cally for labelling. |
| link | See exponential and the warning below. |

## Details

The general methodology behind this VGAM family function is given in Efron (1992) and full details can be obtained there.

This model is essentially an exponential regression model (see exponential) but the usual deviance is replaced by an asymmetric squared error loss function; it is multiplied by $w . a m l$ for positive residuals. The solution is the set of regression coefficients that minimize the sum of these deviancetype values over the data set, weighted by the weights argument (so that it can contain frequencies). Newton-Raphson estimation is used here.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Warning

Note that the link argument of exponential and amlexponential are currently different: one is the rate parameter and the other is the mean (expectile) parameter.

If $\mathrm{w} . \mathrm{aml}$ has more than one value then the value returned by deviance is the sum of all the (weighted) deviances taken over all the w. aml values. See Equation (1.6) of Efron (1992).

## Note

On fitting, the extra slot has list components "w.aml" and "percentile". The latter is the percent of observations below the "w-regression plane", which is the fitted values. Also, the individual deviance values corresponding to each element of the argument $\mathrm{w} . \mathrm{aml}$ is stored in the extra slot.
For amlexponential objects, methods functions for the generic functions qtplot and cdf have not been written yet.

See amlpoisson about comments on the jargon, e.g., expectiles etc.
In this documentation the word quantile can often be interchangeably replaced by expectile (things are informal here).

## Author(s)

Thomas W. Yee

## References

Efron, B. (1992). Poisson overdispersion estimates based on the method of asymmetric maximum likelihood. Journal of the American Statistical Association, 87, 98-107.

## See Also

exponential, amlbinomial, amlpoisson, amlnormal, extlogF1, alaplace1, lms.bcg, deexp.

## Examples

```
nn <- 2000
mydat <- data.frame(x = seq(0, 1, length = nn))
mydat <- transform(mydat,
            mu = loglink(-0 + 1.5*x + 0. 2*x^2, inverse = TRUE))
mydat <- transform(mydat, mu = loglink(0 - sin(8*x), inverse = TRUE))
mydat <- transform(mydat, y = rexp(nn, rate = 1/mu))
(fit <- vgam(y ~ s(x, df=5), amlexponential(w=c(0.001, 0.1, 0.5, 5, 60)),
            mydat, trace = TRUE))
fit@extra
## Not run: # These plots are against the sqrt scale (to increase clarity)
par(mfrow = c(1,2))
# Quantile plot
with(mydat, plot(x, sqrt(y), col = "blue", las = 1, main =
    paste(paste(round(fit@extra$percentile, digits = 1), collapse=", "),
            "percentile-expectile curves")))
with(mydat, matlines(x, sqrt(fitted(fit)), lwd = 2, col = "blue", lty=1))
# Compare the fitted expectiles with the quantiles
with(mydat, plot(x, sqrt(y), col = "blue", las = 1, main =
        paste(paste(round(fit@extra$percentile, digits = 1), collapse=", "),
            "percentile curves are orange")))
with(mydat, matlines(x, sqrt(fitted(fit)), lwd = 2, col = "blue", lty=1))
for (ii in fit@extra$percentile)
    with(mydat, matlines(x, sqrt(qexp(p = ii/100, rate = 1/mu)),
    col = "orange"))
## End(Not run)
```

amlnormal Asymmetric Least Squares Quantile Regression

## Description

Asymmetric least squares, a special case of maximizing an asymmetric likelihood function of a normal distribution. This allows for expectile/quantile regression using asymmetric least squares error loss.

## Usage

```
amlnormal(w.aml = 1, parallel = FALSE, lexpectile = "identitylink",
    iexpectile \(=\) NULL, imethod \(=1\), digw = 4)
```


## Arguments

w. aml

Numeric, a vector of positive constants controlling the percentiles. The larger the value the larger the fitted percentile value (the proportion of points below the "w-regression plane"). The default value of unity results in the ordinary least squares (OLS) solution.
parallel If w.aml has more than one value then this argument allows the quantile curves to differ by the same amount as a function of the covariates. Setting this to be TRUE should force the quantile curves to not cross (although they may not cross anyway). See CommonVGAMffArguments for more information.
lexpectile, iexpectile
See CommonVGAMfffarguments for more information.
imethod Integer, either 1 or 2 or 3 . Initialization method. Choose another value if convergence fails.
digw Passed into Round as the digits argument for the w. aml values; used cosmetically for labelling.

## Details

This is an implementation of Efron (1991) and full details can be obtained there. Equation numbers below refer to that article. The model is essentially a linear model (see lm), however, the asymmetric squared error loss function for a residual $r$ is $r^{2}$ if $r \leq 0$ and $w r^{2}$ if $r>0$. The solution is the set of regression coefficients that minimize the sum of these over the data set, weighted by the weights argument (so that it can contain frequencies). Newton-Raphson estimation is used here.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

On fitting, the extra slot has list components "w.aml" and "percentile". The latter is the percent of observations below the "w-regression plane", which is the fitted values.
One difficulty is finding the w.aml value giving a specified percentile. One solution is to fit the model within a root finding function such as uniroot; see the example below.
For amlnormal objects, methods functions for the generic functions qtplot and cdf have not been written yet.
See the note in amlpoisson on the jargon, including expectiles and regression quantiles.
The deviance slot computes the total asymmetric squared error loss (2.5). If w. aml has more than one value then the value returned by the slot is the sum taken over all the w . aml values.
This VGAM family function could well be renamed amlnormal () instead, given the other function names amlpoisson, amlbinomial, etc.
In this documentation the word quantile can often be interchangeably replaced by expectile (things are informal here).

## Author(s)

Thomas W. Yee

## References

Efron, B. (1991). Regression percentiles using asymmetric squared error loss. Statistica Sinica, 1, 93-125.

## See Also

amlpoisson, amlbinomial, amlexponential, bmi.nz, extlogF1, alaplace1, denorm, lms.bcn and similar variants are alternative methods for quantile regression.

## Examples

```
## Not run:
# Example 1
ooo <- with(bmi.nz, order(age))
bmi.nz <- bmi.nz[ooo, ] # Sort by age
(fit <- vglm(BMI ~ sm.bs(age), amlnormal(w.aml = 0.1), data = bmi.nz))
fit@extra # Gives the w value and the percentile
coef(fit, matrix = TRUE)
# Quantile plot
with(bmi.nz, plot(age, BMI, col = "blue", main =
    paste(round(fit@extra$percentile, digits = 1),
            "expectile-percentile curve")))
with(bmi.nz, lines(age, c(fitted(fit)), col = "black"))
# Example 2
# Find the w values that give the 25, 50 and 75 percentiles
find.w <- function(w, percentile = 50) {
    fit2 <- vglm(BMI ~ sm.bs(age), amlnormal(w = w), data = bmi.nz)
    fit2@extra$percentile - percentile
}
# Quantile plot
with(bmi.nz, plot(age, BMI, col = "blue", las = 1, main =
            "25, 50 and 75 expectile-percentile curves"))
for (myp in c(25, 50, 75)) {
# Note: uniroot() can only find one root at a time
    bestw <- uniroot(f = find.w, interval = c(1/10^4, 10^4), percentile = myp)
    fit2 <- vglm(BMI ~ sm.bs(age), amlnormal(w = bestw$root), data = bmi.nz)
    with(bmi.nz, lines(age, c(fitted(fit2)), col = "orange"))
}
# Example 3; this is Example 1 but with smoothing splines and
# a vector w and a parallelism assumption.
ooo <- with(bmi.nz, order(age))
bmi.nz <- bmi.nz[ooo, ] # Sort by age
fit3 <- vgam(BMI ~ s(age, df = 4), data = bmi.nz, trace = TRUE,
                    amlnormal(w = c(0.1, 1, 10), parallel = TRUE))
fit3@extra # The w values, percentiles and weighted deviances
# The linear components of the fit; not for human consumption:
coef(fit3, matrix = TRUE)
# Quantile plot
with(bmi.nz, plot(age, BMI, col="blue", main =
    paste(paste(round(fit3@extra$percentile, digits = 1), collapse = ", "),
            "expectile-percentile curves")))
with(bmi.nz, matlines(age, fitted(fit3), col = 1:fit3@extra$M, lwd = 2))
```

with(bmi.nz, lines(age, c(fitted(fit )), col = "black")) \# For comparison
\#\# End(Not run)
amlpoisson Poisson Regression by Asymmetric Maximum Likelihood Estimation

## Description

Poisson quantile regression estimated by maximizing an asymmetric likelihood function.

## Usage

amlpoisson(w.aml = 1, parallel = FALSE, imethod = 1, digw = 4, link = "loglink")

## Arguments

w.aml Numeric, a vector of positive constants controlling the percentiles. The larger the value the larger the fitted percentile value (the proportion of points below the "w-regression plane"). The default value of unity results in the ordinary maximum likelihood (MLE) solution.
parallel If $w . a m l$ has more than one value then this argument allows the quantile curves to differ by the same amount as a function of the covariates. Setting this to be TRUE should force the quantile curves to not cross (although they may not cross anyway). See CommonVGAMffArguments for more information.
imethod Integer, either 1 or 2 or 3 . Initialization method. Choose another value if convergence fails.
digw Passed into Round as the digits argument for the w. aml values; used cosmetically for labelling.
link See poissonff.

## Details

This method was proposed by Efron (1992) and full details can be obtained there.
The model is essentially a Poisson regression model (see poissonff) but the usual deviance is replaced by an asymmetric squared error loss function; it is multiplied by $w . a m l$ for positive residuals. The solution is the set of regression coefficients that minimize the sum of these deviance-type values over the data set, weighted by the weights argument (so that it can contain frequencies). Newton-Raphson estimation is used here.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Warning

If w.aml has more than one value then the value returned by deviance is the sum of all the (weighted) deviances taken over all the w. aml values. See Equation (1.6) of Efron (1992).

## Note

On fitting, the extra slot has list components "w.aml" and "percentile". The latter is the percent of observations below the "w-regression plane", which is the fitted values. Also, the individual deviance values corresponding to each element of the argument $w$. aml is stored in the extra slot.
For amlpoisson objects, methods functions for the generic functions qtplot and cdf have not been written yet.
About the jargon, Newey and Powell (1987) used the name expectiles for regression surfaces obtained by asymmetric least squares. This was deliberate so as to distinguish them from the original regression quantiles of Koenker and Bassett (1978). Efron (1991) and Efron (1992) use the general name regression percentile to apply to all forms of asymmetric fitting. Although the asymmetric maximum likelihood method very nearly gives regression percentiles in the strictest sense for the normal and Poisson cases, the phrase quantile regression is used loosely in this VGAM documentation.

In this documentation the word quantile can often be interchangeably replaced by expectile (things are informal here).

## Author(s)

Thomas W. Yee

## References

Efron, B. (1991). Regression percentiles using asymmetric squared error loss. Statistica Sinica, 1, 93-125.

Efron, B. (1992). Poisson overdispersion estimates based on the method of asymmetric maximum likelihood. Journal of the American Statistical Association, 87, 98-107.
Koenker, R. and Bassett, G. (1978). Regression quantiles. Econometrica, 46, 33-50.
Newey, W. K. and Powell, J. L. (1987). Asymmetric least squares estimation and testing. Econometrica, 55, 819-847.

## See Also

amlnormal, amlbinomial, extlogF1, alaplace1.

## Examples

```
set.seed(1234)
mydat <- data.frame(x = sort(runif(nn <- 200)))
mydat <- transform(mydat, y = rpois(nn, exp(0 - sin(8*x))))
(fit <- vgam(y ~ s(x), fam = amlpoisson(w.aml = c(0.02, 0.2, 1, 5, 50)),
            mydat, trace = TRUE))
fit@extra
```

```
## Not run:
# Quantile plot
with(mydat, plot(x, jitter(y), col = "blue", las = 1, main =
    paste(paste(round(fit@extra$percentile, digits = 1), collapse = ", "),
        "percentile-expectile curves")))
with(mydat, matlines(x, fitted(fit), lwd = 2))
## End(Not run)
```

anova.vglm

Analysis of Deviance for Vector Generalized Linear Model Fits

## Description

Compute an analysis of deviance table for one or more vector generalized linear model fits.

## Usage

```
## S3 method for class 'vglm'
anova(object, ..., type = c("II", "I", "III", 2, 1, 3),
        test = c("LRT", "none"), trydev = TRUE, silent = TRUE)
```


## Arguments

object, ... objects of class vglm, typically the result of a call to vglm, or a list of objects for the "vglmlist" method. Each model must have an intercept term. If "vglmlist" is used then type $=1$ or type $=" I "$ must be specified.
type character or numeric; any one of the (effectively three) choices given. Note that anova.glm has 1 or "I" as its default; and that Anova.glm has 2 or "II" as its default (and allows for type $=$ "III"), so one can think of this function as a combination of anova.glm and Anova.glm, but with the default of the latter. See Details below for more information.
test a character string, (partially) matching one of "LRT" and "none". In the future it is hoped that "Rao" be also supported, to conduct score tests. The first value is the default.
trydev logical; if TRUE then the deviance is used if possible. Note that only a few VGAM family functions have a deviance that is defined and implemented. Setting it FALSE means the log-likelihood will be used.
silent logical; if TRUE then any warnings will be suppressed. These may arise by IRLS iterations not converging during the fitting of submodels. Setting it FALSE means that any warnings are given.

## Details

anova.vglm is intended to be similar to anova.glm so specifying a single object and type $=1$ gives a sequential analysis of deviance table for that fit. By analysis of deviance, it is meant loosely that if the deviance of the model is not defined or implemented, then twice the difference between the log-likelihoods of two nested models remains asymptotically chi-squared distributed with degrees
of freedom equal to the difference in the number of parameters of the two models. Of course, the usual regularity conditions are assumed to hold. For Type I, the analysis of deviance table has the reductions in the residual deviance as each term of the formula is added in turn are given in as the rows of a table, plus the residual deviances themselves. Type I or sequential tests (as in anova.glm). are computationally the easiest of the three methods. For this, the order of the terms is important, and the each term is added sequentially from first to last.
The Anova() function in car allows for testing Type II and Type III (SAS jargon) hypothesis tests, although the definitions used are not precisely that of SAS. As car notes, Type I rarely test interesting hypotheses in unbalanced designs. Type III enter each term last, keeping all the other terms in the model. Type II tests, according to SAS, add the term after all other terms have been added to the model except terms that contain the effect being tested; an effect is contained in another effect if it can be derived by deleting variables from the latter effect. Type II tests are currently the default.
As in anova.glm, but not as Anova.glm, if more than one object is specified, then the table has a row for the residual degrees of freedom and deviance for each model. For all but the first model, the change in degrees of freedom and deviance is also given. (This only makes statistical sense if the models are nested.) It is conventional to list the models from smallest to largest, but this is up to the user. It is necessary to have type $=1$ with more than one objects are specified.
See anova.glm for more details and warnings. The VGAM package now implements full likelihood models only, therefore no dispersion parameters are estimated.

## Value

An object of class "anova" inheriting from class "data.frame".

## Warning

See anova.glm. Several VGAM family functions implement distributions which do not satisfying the usual regularity conditions needed for the LRT to work. No checking or warning is given for these.
As car says, be careful of Type III tests because they violate marginality. Type II tests (the default) do not have this problem.

## Note

It is possible for this function to stop when type $=2$ or 3, e.g., anova(vglm(cans $\sim$ myfactor, poissonff, data $=$ boxcar)) where myfactor is a factor.
The code was adapted directly from anova.glm and Anova.glm by T. W. Yee. Hence the Type II and Type III tests do not correspond precisely with the SAS definition.

## See Also

anova.glm, stat.anova, stats:::print.anova, Anova.glm if car is installed, vglm, lrtest, add1.vglm, drop1.vglm, lrt.stat.vlm, score.stat.vlm, wald.stat.vlm, backPain2, update.

## Examples

```
# Example 1: a proportional odds model fitted to pneumo.
```

set.seed(1)

```
pneumo <- transform(pneumo, let = log(exposure.time), x3 = runif(8))
fit1 <- vglm(cbind(normal, mild, severe) ~ let , propodds, pneumo)
fit2 <- vglm(cbind(normal, mild, severe) ~ let + x3, propodds, pneumo)
fit3 <- vglm(cbind(normal, mild, severe) ~ let + x3, cumulative, pneumo)
anova(fit1, fit2, fit3, type = 1) # Remember to specify 'type'!!
anova(fit2)
anova(fit2, type = "I")
anova(fit2, type = "III")
# Example 2: a proportional odds model fitted to backPain2.
data("backPain2", package = "VGAM")
summary(backPain2)
fitlogit <- vglm(pain ~ x2 * x3 * x4, propodds, data = backPain2)
coef(fitlogit)
anova(fitlogit)
anova(fitlogit, type = "I")
anova(fitlogit, type = "III")
```


## Description

Maximum likelihood estimation of the three-parameter AR-1 model

## Usage

AR1 (ldrift = "identitylink", lsd = "loglink", lvar = "loglink", lrho = "rhobitlink", idrift $=$ NULL, isd $=$ NULL, ivar $=$ NULL, irho $=$ NULL, imethod $=1$, ishrinkage = 0.95, type.likelihood = c("exact", "conditional"), type.EIM = c("exact", "approximate"), var.arg = FALSE, nodrift = FALSE, print.EIM = FALSE, zero = c(if (var.arg) "var" else "sd", "rho"))

## Arguments

ldrift, lsd, lvar, lrho
Link functions applied to the scaled mean, standard deviation or variance, and correlation parameters. The parameter drift is known as the drift, and it is a scaled mean. See Links for more choices.
idrift, isd, ivar, irho
Optional initial values for the parameters. If failure to converge occurs then try different values and monitor convergence by using trace = TRUE. For a $S$ column response, these arguments can be of length $S$, and they are recycled by the columns first. A value NULL means an initial value for each response is computed internally.
ishrinkage, imethod, zero
See CommonVGAMffArguments for more information. The default for zero assumes there is a drift parameter to be estimated (the default for that argument), so if a drift parameter is suppressed and there are covariates, then zero will need to be assigned the value 1 or 2 or NULL.
var.arg Same meaning as uninormal.
nodrift Logical, for determining whether to estimate the drift parameter. The default is to estimate it. If TRUE, the drift parameter is set to 0 and not estimated.
type.EIM What type of expected information matrix (EIM) is used in Fisher scoring. By default, this family function calls AR1EIM, which recursively computes the exact EIM for the AR process with Gaussian white noise. See Porat and Friedlander (1986) for further details on the exact EIM.

If type.EIM = "approximate" then approximate expression for the EIM of Autoregressive processes is used; this approach holds when the number of observations is large enough. Succinct details about the approximate EIM are delineated at Porat and Friedlander (1987).
print.EIM Logical. If TRUE, then the first few EIMs are printed. Here, the result shown is the sum of each EIM.
type.likelihood
What type of likelihood function is maximized. The first choice (default) is the sum of the marginal likelihood and the conditional likelihood. Choosing the conditional likelihood means that the first observation is effectively ignored (this is handled internally by setting the value of the first prior weight to be some small positive number, e.g., 1.0e-6). See the note below.

## Details

The AR-1 model implemented here has

$$
Y_{1} \sim N\left(\mu, \sigma^{2} /\left(1-\rho^{2}\right)\right)
$$

and

$$
Y_{i}=\mu^{*}+\rho Y_{i-1}+e_{i},
$$

where the $e_{i}$ are i.i.d. $\operatorname{Normal}(0, \mathrm{sd}=\sigma)$ random variates.
Here are a few notes: (1). A test for weak stationarity might be to verify whether $1 / \rho$ lies outside the unit circle. (2). The mean of all the $Y_{i}$ is $\mu^{*} /(1-\rho)$ and these are returned as the fitted values. (3). The correlation of all the $Y_{i}$ with $Y_{i-1}$ is $\rho$. (4). The default link function ensures that $-1<\rho<1$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

Monitoring convergence is urged, i.e., set trace $=$ TRUE.
Moreover, if the exact EIMs are used, set print.EIM = TRUE to compare the computed exact to the approximate EIM.

Under the VGLM/VGAM approach, parameters can be modelled in terms of covariates. Particularly, if the standard deviation of the white noise is modelled in this way, then type.EIM = "exact" may certainly lead to unstable results. The reason is that white noise is a stationary process, and consequently, its variance must remain as a constant. Consequently, the use of variates to model this parameter contradicts the assumption of stationary random components to compute the exact EIMs proposed by Porat and Friedlander (1987).
To prevent convergence issues in such cases, this family function internally verifies whether the variance of the white noise remains as a constant at each Fisher scoring iteration. If this assumption is violated and type.EIM = "exact" is set, then AR1 automatically shifts to type.EIM = "approximate". Also, a warning is accordingly displayed.

## Note

Multiple responses are handled. The mean is returned as the fitted values.

## Author(s)

Victor Miranda (exact method) and Thomas W. Yee (approximate method).

## References

Porat, B. and Friedlander, B. (1987). The Exact Cramer-Rao Bond for Gaussian Autoregressive Processes. IEEE Transactions on Aerospace and Electronic Systems, AES-23(4), 537-542.

## See Also

AR1EIM, vglm. control, dAR1, arima.sim.

## Examples

```
### Example 1: using arima.sim() to generate a 0-mean stationary time series.
nn <- 500
tsdata <- data.frame(x2 = runif(nn))
ar.coef.1 <- rhobitlink(-1.55, inverse = TRUE) # Approx -0.65
ar.coef.2 <- rhobitlink( 1.0, inverse = TRUE) # Approx 0.50
set.seed(1)
tsdata <- transform(tsdata,
    index = 1:nn,
    TS1 = arima.sim(nn, model = list(ar = ar.coef.1),
                        sd = exp(1.5)),
    TS2 = arima.sim(nn, model = list(ar = ar.coef.2),
                sd = exp(1.0 + 1.5 * x2)))
### An autoregressive intercept--only model. ###
### Using the exact EIM, and "nodrift = TRUE" ###
fit1a <- vglm(TS1 ~ 1, data = tsdata, trace = TRUE,
    AR1(var.arg = FALSE, nodrift = TRUE,
            type.EIM = "exact",
            print.EIM = FALSE),
        crit = "coefficients")
Coef(fit1a)
```

```
summary(fit1a)
## Not run:
### Two responses. Here, the white noise standard deviation of TS2 ###
### is modelled in terms of 'x2'. Also, 'type.EIM = exact'. ###
fit1b <- vglm(cbind(TS1, TS2) ~ x2,
    AR1(zero = NULL, nodrift = TRUE,
                var.arg = FALSE,
                type.EIM = "exact"),
            constraints = list("(Intercept)" = diag(4),
                                    "x2" = rbind(0, 0, 1, 0)),
                                    data = tsdata, trace = TRUE, crit = "coefficients")
coef(fit1b, matrix = TRUE)
summary(fit1b)
### Example 2: another stationary time series
nn <- 500
my.rho <- rhobitlink(1.0, inverse = TRUE)
my.mu <- 1.0
my.sd <- exp(1)
tsdata <- data.frame(index = 1:nn, TS3 = runif(nn))
set.seed(2)
for (ii in 2:nn)
    tsdata$TS3[ii] <- my.mu/(1 - my.rho) +
                            my.rho * tsdata$TS3[ii-1] + rnorm(1, sd = my.sd)
tsdata <- tsdata[-(1:ceiling(nn/5)), ] # Remove the burn-in data:
### Fitting an AR(1). The exact EIMs are used.
fit2a <- vglm(TS3 ~ 1, AR1(type.likelihood = "exact", # "conditional",
                type.EIM = "exact"),
            data = tsdata, trace = TRUE, crit = "coefficients")
Coef(fit2a)
summary(fit2a) # SEs are useful to know
Coef(fit2a)["rho"] # Estimate of rho, for intercept-only models
my.rho # The 'truth' (rho)
Coef(fit2a)["drift"] # Estimate of drift, for intercept-only models
my.mu /(1 - my.rho) # The 'truth' (drift)
## End(Not run)
```


## Description

Computation of the exact Expected Information Matrix of the Autoregressive process of order-1 (AR(1)) with Gaussian white noise and stationary random components.

## Usage

```
AR1EIM(x = NULL, var.arg = NULL, p.drift = NULL,
    WNsd \(=\) NULL, ARcoeff1 \(=\) NULL, eps.porat \(=1 \mathrm{e}-2\) )
```


## Arguments

X
A vector of quantiles. The gaussian time series for which the EIMs are computed.
If multiple time series are being analyzed, then $x$ must be a matrix where each column allocates a response. That is, the number of columns (denoted as $N O S$ ) must match the number of responses.
var.arg Logical. Same as with AR1.
p.drift A numeric vector with the scaled mean(s) (commonly referred as drift) of the AR process(es) in turn. Its length matches the number of responses.

WNsd, ARcoeff1 Matrices. The standard deviation of the white noise, and the correlation (coefficient) of the AR(1) model, for each observation.
That is, the dimension for each matrix is $N \times N O S$, where $N$ is the number of observations and $N O S$ is the number of responses. Else, these arguments are recycled.
eps.porat A very small positive number to test whether the standar deviation (WNsd) is close enough to its value estimated in this function.
See below for further details.

## Details

This function implements the algorithm of Porat and Friedlander (1986) to recursively compute the exact expected information matrix (EIM) of Gaussian time series with stationary random components.
By default, when the VGLM/VGAM family function AR1 is used to fit an AR(1) model via vglm, Fisher scoring is executed using the approximate EIM for the AR process. However, this model can also be fitted using the exact EIMs computed by AR1EIM.

Given $N$ consecutive data points, $y_{0}, y_{1}, \ldots, y_{N-1}$ with probability density $f(\boldsymbol{y})$, the Porat and Friedlander algorithm calculates the EIMs $\left[J_{n-1}(\boldsymbol{\theta})\right]$, for all $1 \leq n \leq N$. This is done based on the Levinson-Durbin algorithm for computing the orthogonal polynomials of a Toeplitz matrix. In particular, for the AR(1) model, the vector of parameters to be estimated under the VGAM/VGLM approach is

$$
\boldsymbol{\eta}=\left(\mu^{*}, \log \left(\sigma^{2}\right), \operatorname{rhobit}(\rho)\right),
$$

where $\sigma^{2}$ is the variance of the white noise and $m u^{*}$ is the drift parameter (See AR1 for further details on this).
Consequently, for each observation $n=1, \ldots, N$, the EIM, $J_{n}(\boldsymbol{\theta})$, has dimension $3 \times 3$, where the diagonal elements are:

$$
J_{[n, 1,1]}=E\left[-\partial^{2} \log f(\boldsymbol{y}) / \partial\left(\mu^{*}\right)^{2}\right]
$$

$$
J_{[n, 2,2]}=E\left[-\partial^{2} \log f(\boldsymbol{y}) / \partial\left(\sigma^{2}\right)^{2}\right]
$$

and

$$
J_{[n, 3,3]}=E\left[-\partial^{2} \log f(\boldsymbol{y}) / \partial(\rho)^{2}\right]
$$

As for the off-diagonal elements, one has the usual entries, i.e.,

$$
J_{[n, 1,2]}=J_{[n, 2,1]}=E\left[-\partial^{2} \log f(\boldsymbol{y}) / \partial \sigma^{2} \partial \rho\right]
$$

etc.
If var. arg $=$ FALSE, then $\sigma$ instead of $\sigma^{2}$ is estimated. Therefore, $J_{[n, 2,2]}, J_{[n, 1,2]}$, etc., are correspondingly replaced.
Once these expected values are internally computed, they are returned in an array of dimension $N \times 1 \times 6$, of the form

$$
J[, 1,]=\left[J_{[, 1,1]}, J_{[, 2,2]}, J_{[, 3,3]}, J_{[, 1,2]}, J_{[, 2,3]}, J_{[, 1,3]}\right]
$$

AR1EIM handles multiple time series, say $N O S$. If this happens, then it accordingly returns an array of dimension $N \times N O S \times 6$. Here, $J[, k$,$] , for k=1, \ldots, N O S$, is a matrix of dimension $N \times 6$, which stores the EIMs for the $k^{t h}$ th response, as above, i.e.,

$$
J[, k,]=\left[J_{[, 1,1]}, J_{[, 2,2]}, J_{[, 3,3]}, \ldots\right],
$$

the bandwith form, as per required by AR1.

## Value

An array of dimension $N \times N O S \times 6$, as above.
This array stores the EIMs calculated from the joint density as a function of

$$
\boldsymbol{\theta}=\left(\mu^{*}, \sigma^{2}, \rho\right)
$$

Nevertheless, note that, under the VGAM/VGLM approach, the EIMs must be correspondingly calculated in terms of the linear predictors, $\boldsymbol{\eta}$.

## Asymptotic behaviour of the algorithm

For large enough $n$, the EIMs, $J_{n}(\boldsymbol{\theta})$, become approximately linear in $n$. That is, for some $n_{0}$,

$$
J_{n}(\boldsymbol{\theta}) \equiv J_{n_{0}}(\boldsymbol{\theta})+\left(n-n_{0}\right) \bar{J}(\boldsymbol{\theta}), \quad(* *)
$$

where $\bar{J}(\boldsymbol{\theta})$ is a constant matrix.
This relationsihip is internally considered if a proper value of $n_{0}$ is determined. Different ways can be adopted to find $n_{0}$. In AR1EIM, this is done by checking the difference between the internally
estimated variances and the entered ones at WNsd. If this difference is less than eps.porat at some iteration, say at iteration $n_{0}$, then AR1EIM takes $\bar{J}(\boldsymbol{\theta})$ as the last computed increment of $J_{n}(\boldsymbol{\theta})$, and extraplotates $J_{k}(\boldsymbol{\theta})$, for all $k \geq n_{0}$ using $(*)$. Else, the algorithm will complete the iterations for $1 \leq n \leq N$.
Finally, note that the rate of convergence reasonably decreases if the asymptotic relationship $(*)$ is used to compute $J_{k}(\boldsymbol{\theta}), k \geq n_{0}$. Normally, the number of operations involved on this algorithm is proportional to $N^{2}$.

See Porat and Friedlander (1986) for full details on the asymptotic behaviour of the algorithm.

## Warning

Arguments WNsd, and ARcoeff1 are matrices of dimension $N \times N O S$. Else, these arguments are accordingly recycled.

## Note

For simplicity, one can assume that the time series analyzed has a 0-mean. Consequently, where the family function AR1 calls AR1EIM to compute the EIMs, the argument p.drift is internally set to zero-vector, whereas x is centered by subtracting its mean value.

## Author(s)

V. Miranda and T. W. Yee.

## References

Porat, B. and Friedlander, B. (1986). Computation of the Exact Information Matrix of Gaussian Time Series with Stationary Random Components. IEEE Transactions on Acoustics, Speech, and Signal Processing, 54(1), 118-130.

## See Also

AR1.

## Examples

```
    set.seed(1)
    nn <- 500
    ARcoeff1 <- c(0.3, 0.25) # Will be recycled.
    WNsd <- c(exp(1), exp(1.5)) # Will be recycled.
    p.drift <- c(0, 0) # Zero-mean gaussian time series.
    ### Generate two (zero-mean) AR(1) processes ###
    ts1 <- p.drift[1]/(1 - ARcoeff1[1]) +
        arima.sim(model = list(ar = ARcoeff1[1]), n = nn,
        sd = WNsd[1])
    ts2 <- p.drift[2]/(1 - ARcoeff1[2]) +
        arima.sim(model = list(ar = ARcoeff1[2]), n = nn,
        sd = WNsd[2])
```

```
ARdata <- matrix(cbind(ts1, ts2), ncol = 2)
### Compute the exact EIMs: TWO responses. ###
ExactEIM <- AR1EIM(x = ARdata, var.arg = FALSE, p.drift = p.drift,
                                    WNsd = WNsd, ARcoeff1 = ARcoeff1)
### For response 1:
head(ExactEIM[, 1 ,]) # NOTICE THAT THIS IS A (nn x 6) MATRIX!
### For response 2:
head(ExactEIM[, 2 ,]) # NOTICE THAT THIS IS A (nn x 6) MATRIX!
```

```
auuc Auckland University Undergraduate Counts Data
```


## Description

Undergraduate student enrolments at the University of Auckland in 1990.

## Usage

data(auuc)

## Format

A data frame with 4 observations on the following 5 variables.
Commerce a numeric vector of counts.
Arts a numeric vector of counts.
SciEng a numeric vector of counts.
Law a numeric vector of counts.
Medicine a numeric vector of counts.

## Details

Each student is cross-classified by their colleges (Science and Engineering have been combined) and the socio-economic status (SES) of their fathers ( $1=$ highest, down to $4=$ lowest).

## Source

Dr Tony Morrison.

## References

Wild, C. J. and Seber, G. A. F. (2000). Chance Encounters: A First Course in Data Analysis and Inference, New York: Wiley.

## Examples

```
auuc
## Not run:
round(fitted(grc(auuc)))
round(fitted(grc(auuc, Rank = 2)))
## End(Not run)
```

aux. posbernoulli.t Auxiliary Function for the Positive Bernoulli Family Function with Time Effects

## Description

Returns behavioural effects indicator variables from a capture history matrix.

## Usage

aux. posbernoulli.t(y, check.y = FALSE, rename = TRUE, name = "bei")

## Arguments

y Capture history matrix. Rows are animals, columns are sampling occasions, and values should be 0s and 1s only.
check.y Logical, if TRUE then some basic checking is performed.
rename, name If rename $=$ TRUE then the behavioural effects indicator are named using the value of name as the prefix. If FALSE then use the same column names as $y$.

## Details

This function can help fit certain capture-recapture models (commonly known as $M_{t b}$ or $M_{t b h}$ (no prefix $h$ means it is an intercept-only model) in the literature). See posbernoulli.t for details.

## Value

A list with the following components.
cap.hist1 A matrix the same dimension as y. In any particular row there are 0 s up to the first capture. Then there are 1s thereafter.
cap1 A vector specifying which time occasion the animal was first captured.
$\mathbf{y} \mathbf{0 i}$ Number of noncaptures before the first capture.
$\mathbf{y r 0 i}$ Number of noncaptures after the first capture.
$\mathbf{y r} 1 \mathrm{i}$ Number of recaptures after the first capture.

## See Also

posbernoulli.t, deermice.

## Examples

```
# Fit a M_tbh model to the deermice data:
(pdata <- aux.posbernoulli.t(with(deermice, cbind(y1, y2, y3, y4, y5, y6))))
deermice <- data.frame(deermice,
    bei = 0, # Add this
    pdata$cap.hist1) # Incorporate these
head(deermice) # Augmented with behavioural effect indicator variables
tail(deermice)
```

```
backPain
```

Data on Back Pain Prognosis, from Anderson (1984)

## Description

Data from a study of patients suffering from back pain. Prognostic variables were recorded at presentation and progress was categorised three weeks after treatment.

## Usage

```
    data(backPain)
```


## Format

A data frame with 101 observations on the following 4 variables.
$\mathbf{x} 2$ length of previous attack.
x3 pain change.
x4 lordosis.
pain an ordered factor describing the progress of each patient with levels worse < same < slight.improvement

```
    < moderate.improvement < marked.improvement < complete.relief.
```


## Source

http://ideas.repec.org/c/boc/bocode/s419001.html
The data set and this help file was copied from gnm so that a vignette in VGAM could be run; the analysis is described in Yee (2010).
The data frame backPain2 is a modification of backPain where the variables have been renamed ( x 1 becomes x 2 , x 2 becomes $\times 3, \mathrm{x} 3$ becomes $\times 4$ ) and converted into factors.

## References

Anderson, J. A. (1984). Regression and Ordered Categorical Variables. J. R. Statist. Soc. B, 46(1), 1-30.
Yee, T. W. (2010). The VGAM package for categorical data analysis. Journal of Statistical Software, 32, 1-34. http://www. jstatsoft. org/v32/i10/.

## Examples

```
summary(backPain)
```

summary (backPain2)
beggs Bacon and Eggs Data

## Description

Purchasing of bacon and eggs.

## Usage

```
    data(beggs)
```


## Format

Data frame of a two way table.
$\mathbf{b 0}, \mathbf{b 1}, \mathbf{b 2}, \mathbf{b 3}, \mathbf{b 4}$ The b refers to bacon. The number of times bacon was purchased was $0,1,2,3$, or 4.
$\mathbf{e 0}, \mathbf{e 1}, \mathbf{e} 2, \mathbf{e 3}, \mathbf{e} 4$ The e refers to eggs. The number of times eggs was purchased was $0,1,2,3$, or 4.

## Details

The data is from Information Resources, Inc., a consumer panel based in a large US city [see Bell and Lattin (1998) for further details]. Starting in June 1991, the purchases in the bacon and fresh eggs product categories for a sample of 548 households over four consecutive store trips was tracked. Only those grocery shopping trips with a total basket value of at least five dollars was considered. For each household, the total number of bacon purchases in their four eligible shopping trips and the total number of egg purchases (usually a package of eggs) for the same trips, were counted.

## Source

Bell, D. R. and Lattin, J. M. (1998) Shopping Behavior and Consumer Preference for Store Price Format: Why ‘Large Basket’ Shoppers Prefer EDLP. Marketing Science, 17, 66-88.

## References

Danaher, P. J. and Hardie, B. G. S. (2005). Bacon with Your Eggs? Applications of a New Bivariate Beta-Binomial Distribution. American Statistician, 59(4), 282-286.

## See Also

rrvglm, rcim, grc.

## Examples

beggs
colSums(beggs)
rowSums(beggs)

Bell The Bell Distribution

## Description

Density, and random generation for the Topp-Leone distribution.

## Usage

dbell(x, shape, log = FALSE)
rbell(n, shape)

## Arguments

| $\mathrm{x}, \mathrm{n}$ | Same as Uni form. |
| :--- | :--- |
| shape | the (shape) parameter, which is positive. |
| log | Logical. If log = TRUE then the logarithm of the density is returned. |

## Details

See bell, the VGAM family function for estimating the parameter $s$ by maximum likelihood estimation.

## Value

dbell gives the density, rbell generates random deviates. If shape is large then rbell will become computationally expensive.

## See Also

bell.

## Examples

\#\# Not run:
plot ( $0: 15$, dbell(0:15, shape $=1.5$ ), type $=" \mathrm{h"}$, las = 1 , col = "blue")
\#\# End(Not run)

## Description

Returns the values of the Bell series.

## Usage

bell(n)

## Arguments

$\mathrm{n} \quad$ Vector of non-negative integers. Values greater than 218 return an Inf. Nonintegers or negative values return a NaN .

## Details

The Bell numbers emerge from a series expansion of $\exp \left(e^{x}-1\right)$ for real $x$. The first few values are $B_{0}=1, B_{1}=1, B_{2}=2, B_{3}=5, B_{4}=15$. The series increases quickly so that overflow occurs when its argument is more than 218.

## Value

This function returns $B_{n}$.

## Author(s)

T. W. Yee

## References

Bell, E. T. (1934). Exponential polynomials. Ann. Math., 35, 258-277.
Bell, E. T. (1934). Exponential numbers. Amer. Math. Monthly, 41, 411-419.

## See Also

bellff, rbell.

## Examples

```
    ## Not run:
plot(0:10, bell(0:10), log = "y", type = "h", las = 1, col = "blue")
## End(Not run)
```

bellff Bell Distribution Family Function

## Description

Estimating the shape parameter of the Bell distribution by maximum likelihood estimation.

## Usage

bellff(lshape = "loglink", zero = NULL, gshape = expm1(1.6 * ppoints(7)))

## Arguments

lshape, zero, gshape More information is at CommonVGAMffArguments.

## Details

The Bell distribution has a probability density function that can be written

$$
f(y ; s)=\frac{s^{y} \exp \left(1-e^{s}\right) B_{y}}{y!}
$$

for $y=0(1) \infty$ and shape parameter $0<s$. The mean of $Y$ is $\exp (s) s$ (returned as the fitted values). Fisher-scoring is used. This VGAM family function handles multiple responses.
The function bell returns the first 218 Bell numbers as finite numbers, and returns Inf when its argument has a higher value. Hence this VGAM family function can only handle low-value counts of less than 219.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Author(s)

T. W. Yee

## References

Castellares, F. and Ferrari, S. L. P. and Lemonte, A. J. (2018). On the Bell distribution and its associated regression model for count data. Applied Mathematical Modelling, 56, 172-185.

## See Also

bell, dbell, poissonff.

## Examples

```
bdata <- data.frame(y = rbell(1000, shape = loglink(0.5, inverse = TRUE)))
bfit <- vglm(y ~ 1, bellff, data = bdata, trace = TRUE, crit = "coef")
coef(bfit, matrix = TRUE)
Coef(bfit)
```

Benford Benford's Distribution

## Description

Density, distribution function, quantile function, and random generation for Benford's distribution.

## Usage

```
dbenf(x, ndigits = 1, log = FALSE)
pbenf(q, ndigits = 1, lower.tail = TRUE, log.p = FALSE)
qbenf(p, ndigits = 1, lower.tail = TRUE, log.p = FALSE)
rbenf(n, ndigits = 1)
```


## Arguments

$x, q \quad$ Vector of quantiles. See ndigits.
$p \quad$ vector of probabilities.
$n \quad$ number of observations. A single positive integer. Else if length $(n)>1$ then the length is taken to be the number required.
ndigits Number of leading digits, either 1 or 2. If 1 then the support of the distribution is $\{1, \ldots, 9\}$, else $\{10, \ldots, 99\}$.
$\log , \log . \mathrm{p} \quad$ Logical. If $\log . \mathrm{p}=$ TRUE then all probabilities p are given as $\log (\mathrm{p})$.
lower.tail Same meaning as in pnorm or qnorm.

## Details

Benford's Law (aka the significant-digit law) is the empirical observation that in many naturally occuring tables of numerical data, the leading significant (nonzero) digit is not uniformly distributed in $\{1,2, \ldots, 9\}$. Instead, the leading significant digit ( $=D$, say) obeys the law

$$
P(D=d)=\log _{10}\left(1+\frac{1}{d}\right)
$$

for $d=1, \ldots, 9$. This means the probability the first significant digit is 1 is approximately 0.301 , etc.
Benford's Law was apparently first discovered in 1881 by astronomer/mathematician S. Newcombe. It started by the observation that the pages of a book of logarithms were dirtiest at the beginning and progressively cleaner throughout. In 1938, a General Electric physicist called F. Benford rediscovered the law on this same observation. Over several years he collected data from different
sources as different as atomic weights, baseball statistics, numerical data from Reader's Digest, and drainage areas of rivers.
Applications of Benford's Law has been as diverse as to the area of fraud detection in accounting and the design computers.

## Value

dbenf gives the density, pbenf gives the distribution function, and qbenf gives the quantile function, and rbenf generates random deviates.

## Author(s)

T. W. Yee and Kai Huang

## References

Benford, F. (1938). The Law of Anomalous Numbers. Proceedings of the American Philosophical Society, 78, 551-572.

Newcomb, S. (1881). Note on the Frequency of Use of the Different Digits in Natural Numbers. American Journal of Mathematics, 4, 39-40.

## Examples

```
dbenf(x <- c(0:10, NA, NaN, -Inf, Inf))
pbenf(x)
## Not run:
xx <- 1:9
barplot(dbenf(xx), col = "lightblue", las = 1, xlab = "Leading digit",
            ylab = "Probability", names.arg = as.character(xx),
            main = paste("Benford's distribution", sep = ""))
hist(rbenf(n = 1000), border = "blue", prob = TRUE,
    main = "1000 random variates from Benford's distribution",
    xlab = "Leading digit", sub="Red is the true probability",
    breaks = 0:9 + 0.5, ylim = c(0, 0.35), xlim = c(0, 10.0))
lines(xx, dbenf(xx), col = "red", type = "h")
points(xx, dbenf(xx), col = "red")
## End(Not run)
```


## Benini

The Benini Distribution

## Description

Density, distribution function, quantile function and random generation for the Benini distribution with parameter shape.

## Usage

```
dbenini(x, y0, shape, log = FALSE)
pbenini(q, y0, shape, lower.tail = TRUE, log.p = FALSE)
qbenini ( \(p, y 0\), shape, lower.tail = TRUE, log.p = FALSE)
rbenini(n, y0, shape)
```


## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$n \quad$ number of observations. Same as runif.
y0 the scale parameter $y_{0}$.
shape the positive shape parameter $b$.
$\log \quad$ Logical. If $\log =$ TRUE then the logarithm of the density is returned.
lower.tail, log.p
Same meaning as in pnorm or qnorm.

## Details

See benini1, the VGAM family function for estimating the parameter $s$ by maximum likelihood estimation, for the formula of the probability density function and other details.

## Value

dbenini gives the density, pbenini gives the distribution function, qbenini gives the quantile function, and rbenini generates random deviates.

## Author(s)

T. W. Yee and Kai Huang

## References

Kleiber, C. and Kotz, S. (2003). Statistical Size Distributions in Economics and Actuarial Sciences, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

benini1.

## Examples

```
## Not run:
y0 <- 1; shape <- exp(1)
xx <- seq(0.0, 4, len = 101)
plot(xx, dbenini(xx, y0 = y0, shape = shape), type = "l", col = "blue",
    main = "Blue is density, orange is cumulative distribution function",
    sub = "Purple lines are the 10,20,...,90 percentiles", ylim = 0:1,
```

```
    las = 1, ylab = "", xlab = "x")
    abline(h = 0, col = "blue", lty = 2)
    lines(xx, pbenini(xx, y0 = y0, shape = shape), col = "orange")
    probs <- seq(0.1, 0.9, by = 0.1)
    Q <- qbenini(probs, y0 = y0, shape = shape)
    lines(Q, dbenini(Q, y0 = y0, shape = shape),
        col = "purple", lty = 3, type = "h")
    pbenini(Q, y0 = y0, shape = shape) - probs # Should be all zero
    ## End(Not run)
```

    benini1
    Benini Distribution Family Function

## Description

Estimating the 1-parameter Benini distribution by maximum likelihood estimation.

## Usage

benini1(y0 = stop("argument 'y0' must be specified"), lshape = "loglink", ishape $=$ NULL, imethod $=1$, zero $=$ NULL, parallel $=$ FALSE, type.fitted = c("percentiles", "Qlink"), percentiles = 50)

## Arguments

y0 Positive scale parameter.
lshape Parameter link function and extra argument of the parameter $b$, which is the shape parameter. See Links for more choices. A log link is the default because $b$ is positive.
ishape Optional initial value for the shape parameter. The default is to compute the value internally.
imethod, zero, parallel
Details at CommonVGAMffArguments.
type.fitted, percentiles
See CommonVGAMffArguments for information. Using "Qlink" is for quantilelinks in VGAMextra.

## Details

The Benini distribution has a probability density function that can be written

$$
f(y)=2 s \exp \left(-s\left[\left(\log \left(y / y_{0}\right)\right)^{2}\right]\right) \log \left(y / y_{0}\right) / y
$$

for $0<y_{0}<y$, and shape $s>0$. The cumulative distribution function for $Y$ is

$$
F(y)=1-\exp \left(-s\left[\left(\log \left(y / y_{0}\right)\right)^{2}\right]\right)
$$

Here, Newton-Raphson and Fisher scoring coincide. The median of $Y$ is now returned as the fitted values, by default. This VGAM family function can handle a multiple responses, which is inputted as a matrix.
On fitting, the extra slot has a component called y0 which contains the value of the y0 argument.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

Yet to do: the 2-parameter Benini distribution estimates another shape parameter $a$ too. Hence, the code may change in the future.

## Author(s)

T. W. Yee

## References

Kleiber, C. and Kotz, S. (2003). Statistical Size Distributions in Economics and Actuarial Sciences, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

Benini.

## Examples

```
y0 <- 1; nn <- 3000
bdata <- data.frame(y = rbenini(nn, y0 = y0, shape = exp(2)))
fit <- vglm(y ~ 1, benini1(y0 = y0), data = bdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
fit@extra$y0
c(head(fitted(fit), 1), with(bdata, median(y))) # Should be equal
```


## Description

Density, distribution function, and random generation for the beta-binomial distribution and the inflated beta-binomial distribution.

## Usage

```
dbetabinom(x, size, prob, rho = 0, log = FALSE)
pbetabinom(q, size, prob, rho = 0, log.p = FALSE)
rbetabinom(n, size, prob, rho = 0)
dbetabinom.ab(x, size, shape1, shape2, log = FALSE, Inf.shape = exp(20),
    limit.prob = 0.5)
pbetabinom.ab(q, size, shape1, shape2, limit.prob \(=0.5\), log.p = FALSE)
rbetabinom.ab(n, size, shape1, shape2, limit. prob \(=0.5\),
    .dontuse.prob \(=\) NULL)
dzoibetabinom(x, size, prob, rho \(=0\), pstr0 \(=0\), pstrsize \(=0, \log =\) FALSE \()\)
pzoibetabinom(q, size, prob, rho, pstr0 \(=0\), pstrsize \(=0\),
    lower.tail = TRUE, log.p = FALSE)
rzoibetabinom(n, size, prob, rho = 0, pstr0 = 0, pstrsize = 0)
dzoibetabinom.ab(x, size, shape1, shape2, pstr0 = 0, pstrsize = 0,
                log = FALSE)
pzoibetabinom.ab(q, size, shape1, shape2, pstr0 = 0, pstrsize = 0,
        lower.tail = TRUE, log.p = FALSE)
rzoibetabinom.ab(n, size, shape1, shape2, pstr0 = 0, pstrsize = 0)
```


## Arguments

$x, q \quad$ vector of quantiles.
size number of trials.
$\mathrm{n} \quad$ number of observations. Same as runif.
prob the probability of success $\mu$. Must be in the unit closed interval $[0,1]$.
rho the correlation parameter $\rho$, which should be in the interval $[0,1)$. The default value of 0 corresponds to the usual binomial distribution with probability prob. Setting rho $=1$ would set both shape parameters equal to 0 , and the ratio $0 / 0$, which is actually NaN , is interpreted by Beta as 0.5 . See the warning below.
shape1, shape2 the two (positive) shape parameters of the standard beta distribution. They are called $a$ and $b$ in beta respectively. Note that shape $1=\operatorname{prob} *(1-r h o) / r h o$ and shape $2=(1-$ prob $) *(1-$ rho $) /$ rho is an important relationship between the parameters, so that the shape parameters are infinite by default because rho $=0$; hence limit. prob = prob is used to obtain the behaviour of the usual binomial distribution.
log, log.p, lower.tail
Same meaning as runif.
Inf.shape Numeric. A large value such that, if shape1 or shape2 exceeds this, then special measures are taken, e.g., calling dbinom. Also, if shape 1 or shape 2 is less than its reciprocal, then special measures are also taken. This feature/approximation is needed to avoid numerical problem with catastrophic cancellation of multiple lbeta calls.
limit.prob Numerical vector; recycled if necessary. If either shape parameters are Inf then the binomial limit is taken, with shape $1 /$ (shape $1+$ shape 2 ) as the probability of success. In the case where both are Inf this probability will be a $\mathrm{NaN}=$ Inf/Inf, however, the value limit. prob is used instead. Hence the default for
dbetabinom. ab() is to assume that both shape parameters are equal as the limit is taken (indeed, Beta uses 0.5). Note that for [dpr]betabinom(), because rho $=0$ by default, then limit. prob $=$ prob so that the beta-binomial distribution behaves like the ordinary binomial distribution with respect to arguments size and prob.
.dontuse.prob An argument that should be ignored and not used.
pstr0 Probability of a structual zero (i.e., ignoring the beta-binomial distribution). The default value of pstr0 corresponds to the response having a beta-binomial distribuion inflated only at size.
pstrsize Probability of a structual maximum value size. The default value of pstrsize corresponds to the response having a beta-binomial distribution inflated only at 0 .

## Details

The beta-binomial distribution is a binomial distribution whose probability of success is not a constant but it is generated from a beta distribution with parameters shape1 and shape2. Note that the mean of this beta distribution is mu = shape $1 /($ shape $1+$ shape 2$)$, which therefore is the mean or the probability of success.
See betabinomial and betabinomialff, the VGAM family functions for estimating the parameters, for the formula of the probability density function and other details.
For the inflated beta-binomial distribution, the probability mass function is

$$
P(Y=y)=(1-p \operatorname{str} 0-p s t r s i z e) \times B B(y)+\operatorname{pstr} 0 \times I[y=0]+\text { pstrsize } \times I[y=\text { size }]
$$

where $B B(y)$ is the probability mass function of the beta-binomial distribution with the same shape parameters (pbetabinom.ab), pstr0 is the inflated probability at 0 and pstrsize is the inflated probability at 1 . The default values of pstr0 and pstrsize mean that these functions behave like the ordinary Betabinom when only the essential arguments are inputted.

## Value

dbetabinom and dbetabinom.ab give the density, pbetabinom and pbetabinom.ab give the distribution function, and rbetabinom and rbetabinom. ab generate random deviates.
dzoibetabinom and dzoibetabinom. ab give the inflated density, pzoibetabinom and pzoibetabinom.ab give the inflated distribution function, and rzoibetabinom and rzoibetabinom. ab generate random inflated deviates.

## Warning

Setting rho $=1$ is not recommended, however the code may be modified in the future to handle this special case.

## Note

pzoibetabinom, pzoibetabinom.ab, pbetabinom and pbetabinom.ab can be particularly slow. The functions here ending in . ab are called from those functions which don't. The simple transformations $\mu=\alpha /(\alpha+\beta)$ and $\rho=1 /(1+\alpha+\beta)$ are used, where $\alpha$ and $\beta$ are the two shape parameters.

## Author(s)

T. W. Yee and Xiangjie Xue

## See Also

betabinomial, betabinomialff, Zoabeta, Beta.

## Examples

```
set.seed(1); rbetabinom(10, 100, prob = 0.5)
set.seed(1); rbinom(10, 100, prob = 0.5) # The same since rho = 0
## Not run: N <- 9; xx <- 0:N; s1 <- 2; s2 <- 3
dy <- dbetabinom.ab(xx, size = N, shape1 = s1, shape2 = s2)
barplot(rbind(dy, dbinom(xx, size = N, prob = s1 / (s1+s2))),
    beside = TRUE, col = c("blue","green"), las = 1,
    main = paste("Beta-binomial (size=",N,", shape1=", s1,
        ", shape2=", s2, ") (blue) vs\n",
    " Binomial(size=", N, ", prob=", s1/(s1+s2), ") (green)",
            sep = ""),
    names.arg = as.character(xx), cex.main = 0.8)
sum(dy * xx) # Check expected values are equal
sum(dbinom(xx, size = N, prob = s1 / (s1+s2)) * xx)
# Should be all 0:
cumsum(dy) - pbetabinom.ab(xx, N, shape1 = s1, shape2 = s2)
y <- rbetabinom.ab(n = 1e4, size = N, shape1 = s1, shape2 = s2)
ty <- table(y)
barplot(rbind(dy, ty / sum(ty)),
    beside = TRUE, col = c("blue", "orange"), las = 1,
    main = paste("Beta-binomial (size=", N, ", shape1=", s1,
        ", shape2=", s2, ") (blue) vs\n",
    " Random generated beta-binomial(size=", N, ", prob=",
    s1/(s1+s2),") (orange)", sep = ""), cex.main = 0.8,
    names.arg = as.character(xx))
N <- 1e5; size <- 20; pstr0 <- 0.2; pstrsize <- 0.2
kk <- rzoibetabinom.ab(N, size, s1, s2, pstr0, pstrsize)
hist(kk, probability = TRUE, border = "blue", ylim = c(0, 0.25),
    main = "Blue/green = inflated; orange = ordinary beta-binomial",
    breaks = -0.5 : (size + 0.5))
sum(kk == 0) / N # Proportion of 0
sum(kk == size) / N # Proportion of size
lines(0 : size,
    dbetabinom.ab(0 : size, size, s1, s2), col = "orange")
lines(0 : size, col = "green", type = "b",
    dzoibetabinom.ab(0 : size, size, s1, s2, pstr0, pstrsize))
## End(Not run)
```

betabinomial Beta-binomial Distribution Family Function

## Description

Fits a beta-binomial distribution by maximum likelihood estimation. The two parameters here are the mean and correlation coefficient.

## Usage

betabinomial(lmu = "logitlink", lrho = "logitlink", irho = NULL, imethod = 1, ishrinkage $=0.95$, nsimEIM $=$ NULL, zero $=$ "rho")

## Arguments

lmu, lrho Link functions applied to the two parameters. See Links for more choices. The defaults ensure the parameters remain in $(0,1)$, however, see the warning below.
irho Optional initial value for the correlation parameter. If given, it must be in $(0,1)$, and is recyled to the necessary length. Assign this argument a value if a convergence failure occurs. Having irho = NULL means an initial value is obtained internally, though this can give unsatisfactory results.
imethod An integer with value 1 or 2 or $\ldots$, which specifies the initialization method for $\mu$. If failure to converge occurs try the another value and/or else specify a value for irho.
zero Specifyies which linear/additive predictor is to be modelled as an intercept only. If assigned, the single value can be either 1 or 2 . The default is to have a single correlation parameter. To model both parameters as functions of the covariates assign zero $=$ NULL. See CommonVGAMffArguments for more information.
ishrinkage, nsimEIM
See CommonVGAMffArguments for more information. The argument ishrinkage is used only if imethod $=2$. Using the argument nsimEIM may offer large advantages for large values of $N$ and/or large data sets.

## Details

There are several parameterizations of the beta-binomial distribution. This family function directly models the mean and correlation parameter, i.e., the probability of success. The model can be written $T \mid P=p \sim \operatorname{Binomial}(N, p)$ where $P$ has a beta distribution with shape parameters $\alpha$ and $\beta$. Here, $N$ is the number of trials (e.g., litter size), $T=N Y$ is the number of successes, and $p$ is the probability of a success (e.g., a malformation). That is, $Y$ is the proportion of successes. Like binomialff, the fitted values are the estimated probability of success (i.e., $E[Y]$ and not $E[T]$ ) and the prior weights $N$ are attached separately on the object in a slot.
The probability function is

$$
P(T=t)=\binom{N}{t} \frac{B e(\alpha+t, \beta+N-t)}{B e(\alpha, \beta)}
$$

where $t=0,1, \ldots, N$, and $B e$ is the beta function with shape parameters $\alpha$ and $\beta$. Recall $Y=$ $T / N$ is the real response being modelled.
The default model is $\eta_{1}=\operatorname{logit}(\mu)$ and $\eta_{2}=\operatorname{logit}(\rho)$ because both parameters lie between 0 and 1 . The mean (of $Y$ ) is $p=\mu=\alpha /(\alpha+\beta)$ and the variance (of $Y$ ) is $\mu(1-\mu)(1+(N-1) \rho) / N$. Here, the correlation $\rho$ is given by $1 /(1+\alpha+\beta)$ and is the correlation between the $N$ individuals within a litter. A litter effect is typically reflected by a positive value of $\rho$. It is known as the over-dispersion parameter.
This family function uses Fisher scoring. Elements of the second-order expected derivatives with respect to $\alpha$ and $\beta$ are computed numerically, which may fail for large $\alpha, \beta, N$ or else take a long time.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm.
Suppose fit is a fitted beta-binomial model. Then fit@y contains the sample proportions $y$, fitted(fit) returns estimates of $E(Y)$, and weights(fit,type="prior") returns the number of trials $N$.

## Warning

If the estimated rho parameter is close to 0 then it pays to try 1 rho $=$ "rhobitlink". One day this may become the default link function.
This family function is prone to numerical difficulties due to the expected information matrices not being positive-definite or ill-conditioned over some regions of the parameter space. If problems occur try setting irho to some numerical value, $\mathrm{nsimEIM}=100$, say, or else use etastart argument of vglm , etc.

## Note

This function processes the input in the same way as binomialff. But it does not handle the case $N=1$ very well because there are two parameters to estimate, not one, for each row of the input. Cases where $N=1$ can be omitted via the subset argument of vglm.
The extended beta-binomial distribution of Prentice (1986) is currently not implemented in the VGAM package as it has range-restrictions for the correlation parameter that are currently too difficult to handle in this package. However, try lrho = "rhobitlink".

## Author(s)

T. W. Yee

## References

Moore, D. F. and Tsiatis, A. (1991). Robust estimation of the variance in moment methods for extra-binomial and extra-Poisson variation. Biometrics, 47, 383-401.

Prentice, R. L. (1986). Binary regression using an extended beta-binomial distribution, with discussion of correlation induced by covariate measurement errors. Journal of the American Statistical Association, 81, 321-327.

## See Also

betabinomialff, Betabinom, binomialff, betaff, dirmultinomial, lirat, simulate.vlm.

## Examples

```
# Example 1
bdata <- data.frame(N = 10, mu = 0.5, rho = 0.8)
bdata <- transform(bdata,
    y = rbetabinom(100, size = N, prob = mu, rho = rho))
fit <- vglm(cbind(y, N-y) ~ 1, betabinomial, data = bdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
head(cbind(depvar(fit), weights(fit, type = "prior")))
# Example 2
fit <- vglm(cbind(R, N-R) ~ 1, betabinomial, lirat,
    trace = TRUE, subset = N > 1)
coef(fit, matrix = TRUE)
Coef(fit)
t(fitted(fit))
t(depvar(fit))
t(weights(fit, type = "prior"))
# Example 3, which is more complicated
lirat <- transform(lirat, fgrp = factor(grp))
summary(lirat) # Only 5 litters in group 3
fit2 <- vglm(cbind(R, N-R) ~ fgrp + hb, betabinomial(zero = 2),
    data = lirat, trace = TRUE, subset = N > 1)
coef(fit2, matrix = TRUE)
## Not run: with(lirat, plot(hb[N > 1], fit2@misc$rho,
                    xlab = "Hemoglobin", ylab = "Estimated rho",
                            pch = as.character(grp[N > 1]), col = grp[N > 1]))
## End(Not run)
## Not run: # cf. Figure 3 of Moore and Tsiatis (1991)
with(lirat, plot(hb, R / N, pch = as.character(grp), col = grp,
                        xlab = "Hemoglobin level", ylab = "Proportion Dead",
                        main = "Fitted values (lines)", las = 1))
smalldf <- with(lirat, lirat[N > 1, ])
for (gp in 1:4) {
    xx <- with(smalldf, hb[grp == gp])
    yy <- with(smalldf, fitted(fit2)[grp == gp])
    000 <- order(xx)
    lines(xx[ooo], yy[ooo], col = gp)
}
## End(Not run)
```


## Description

Fits a beta-binomial distribution by maximum likelihood estimation. The two parameters here are the shape parameters of the underlying beta distribution.

## Usage

betabinomialff(lshape1 = "loglink", lshape2 = "loglink", ishape1 = 1, ishape2 $=$ NULL, imethod $=1$, ishrinkage $=0.95$, nsimEIM = NULL, zero = NULL)

## Arguments

lshape1, lshape2
Link functions for the two (positive) shape parameters of the beta distribution. See Links for more choices.
ishape1, ishape2
Initial value for the shape parameters. The first must be positive, and is recyled to the necessary length. The second is optional. If a failure to converge occurs, try assigning a different value to ishape1 and/or using ishape2.
zero Can be an integer specifying which linear/additive predictor is to be modelled as an intercept only. If assigned, the single value should be either 1 or 2 . The default is to model both shape parameters as functions of the covariates. If a failure to converge occurs, try zero $=2$. See CommonVGAMffArguments for more information.
ishrinkage, nsimEIM, imethod
See CommonVGAMffArguments for more information. The argument ishrinkage is used only if imethod $=2$. Using the argument nsimEIM may offer large advantages for large values of $N$ and/or large data sets.

## Details

There are several parameterizations of the beta-binomial distribution. This family function directly models the two shape parameters of the associated beta distribution rather than the probability of success (however, see Note below). The model can be written $T \mid P=p \sim \operatorname{Binomial}(N, p)$ where $P$ has a beta distribution with shape parameters $\alpha$ and $\beta$. Here, $N$ is the number of trials (e.g., litter size), $T=N Y$ is the number of successes, and $p$ is the probability of a success (e.g., a malformation). That is, $Y$ is the proportion of successes. Like binomialff, the fitted values are the estimated probability of success (i.e., $E[Y]$ and not $E[T]$ ) and the prior weights $N$ are attached separately on the object in a slot.
The probability function is

$$
P(T=t)=\binom{N}{t} \frac{B(\alpha+t, \beta+N-t)}{B(\alpha, \beta)}
$$

where $t=0,1, \ldots, N$, and $B$ is the beta function with shape parameters $\alpha$ and $\beta$. Recall $Y=T / N$ is the real response being modelled.
The default model is $\eta_{1}=\log (\alpha)$ and $\eta_{2}=\log (\beta)$ because both parameters are positive. The mean (of $Y$ ) is $p=\mu=\alpha /(\alpha+\beta$ ) and the variance (of $Y$ ) is $\mu(1-\mu)(1+(N-1) \rho) / N$. Here, the
correlation $\rho$ is given by $1 /(1+\alpha+\beta)$ and is the correlation between the $N$ individuals within a litter. A litter effect is typically reflected by a positive value of $\rho$. It is known as the over-dispersion parameter.
This family function uses Fisher scoring. The two diagonal elements of the second-order expected derivatives with respect to $\alpha$ and $\beta$ are computed numerically, which may fail for large $\alpha, \beta, N$ or else take a long time.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm.

Suppose fit is a fitted beta-binomial model. Then fit@y (better: depvar(fit)) contains the sample proportions $y$, fitted(fit) returns estimates of $E(Y)$, and weights(fit,type $=$ "prior") returns the number of trials $N$.

## Warning

This family function is prone to numerical difficulties due to the expected information matrices not being positive-definite or ill-conditioned over some regions of the parameter space. If problems occur try setting ishape 1 to be some other positive value, using ishape 2 and/or setting zero $=2$.

This family function may be renamed in the future. See the warnings in betabinomial.

## Note

This function processes the input in the same way as binomialff. But it does not handle the case $N=1$ very well because there are two parameters to estimate, not one, for each row of the input. Cases where $N=1$ can be omitted via the subset argument of vglm.
Although the two linear/additive predictors given above are in terms of $\alpha$ and $\beta$, basic algebra shows that the default amounts to fitting a logit link to the probability of success; subtracting the second linear/additive predictor from the first gives that logistic regression linear/additive predictor. That is, $\operatorname{logit}(p)=\eta_{1}-\eta_{2}$. This is illustated in one of the examples below.

The extended beta-binomial distribution of Prentice (1986) is currently not implemented in the VGAM package as it has range-restrictions for the correlation parameter that are currently too difficult to handle in this package.

## Author(s)

T. W. Yee

## References

Moore, D. F. and Tsiatis, A. (1991). Robust estimation of the variance in moment methods for extra-binomial and extra-Poisson variation. Biometrics, 47, 383-401.

Prentice, R. L. (1986). Binary regression using an extended beta-binomial distribution, with discussion of correlation induced by covariate measurement errors. Journal of the American Statistical Association, 81, 321-327.

## See Also

betabinomial, Betabinom, binomialff, betaff, dirmultinomial, lirat, simulate.vlm.

## Examples

```
# Example 1
N <- 10; s1 <- exp(1); s2 <- exp(2)
y <- rbetabinom.ab(n = 100, size = N, shape1 = s1, shape2 = s2)
fit <- vglm(cbind(y,N-y) ~ 1, betabinomialff, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
head(fit@misc$rho) # The correlation parameter
head(cbind(depvar(fit), weights(fit, type = "prior")))
# Example 2
fit <- vglm(cbind(R, N-R) ~ 1, betabinomialff, data = lirat,
    trace = TRUE, subset = N > 1)
coef(fit, matrix = TRUE)
Coef(fit)
fit@misc$rho # The correlation parameter
t(fitted(fit))
t(depvar(fit))
t(weights(fit, type = "prior"))
# A "loglink" link for the 2 shape parameters is a logistic regression:
all.equal(c(fitted(fit)),
as.vector(logitlink(predict(fit)[, 1] -
                            predict(fit)[, 2], inverse = TRUE)))
# Example 3, which is more complicated
lirat <- transform(lirat, fgrp = factor(grp))
summary(lirat) # Only 5 litters in group 3
fit2 <- vglm(cbind(R, N-R) ~ fgrp + hb, betabinomialff(zero = 2),
            data = lirat, trace = TRUE, subset = N > 1)
coef(fit2, matrix = TRUE)
coef(fit2, matrix = TRUE)[, 1] -
coef(fit2, matrix = TRUE)[, 2] # logitlink(p)
## Not run: with(lirat, plot(hb[N > 1], fit2@misc$rho,
                    xlab = "Hemoglobin", ylab = "Estimated rho",
                    pch = as.character(grp[N > 1]), col = grp[N > 1]))
## End(Not run)
## Not run: # cf. Figure 3 of Moore and Tsiatis (1991)
with(lirat, plot(hb, R / N, pch = as.character(grp), col = grp, las = 1,
        xlab = "Hemoglobin level", ylab = "Proportion Dead",
        main = "Fitted values (lines)"))
smalldf <- with(lirat, lirat[N > 1, ])
for (gp in 1:4) {
    xx <- with(smalldf, hb[grp == gp])
    yy <- with(smalldf, fitted(fit2)[grp == gp])
    ooo <- order(xx)
```

```
    lines(xx[ooo], yy[ooo], col = gp)
}
## End(Not run)
```

betaff The Two-parameter Beta Distribution Family Function

## Description

Estimation of the mean and precision parameters of the beta distribution.

## Usage

$\operatorname{betaff}(A=0, B=1,1 m u=" l o g i t l i n k ", ~ l p h i=" l o g l i n k "$, imu = NULL, iphi = NULL, gprobs.y $=$ ppoints(8), gphi $=\exp (-3: 5) / 4$, zero $=$ NULL)

## Arguments

A, B Lower and upper limits of the distribution. The defaults correspond to the standard beta distribution where the response lies between 0 and 1.
lmu, lphi Link function for the mean and precision parameters. The values $A$ and $B$ are extracted from the min and max arguments of extlogitlink. Consequently, only extlogitlink is allowed.
imu, iphi Optional initial value for the mean and precision parameters respectively. A NULL value means a value is obtained in the initialize slot.
gprobs.y, gphi, zero
See CommonVGAMffArguments for more information.

## Details

The two-parameter beta distribution can be written $f(y)=$

$$
(y-A)^{\mu_{1} \phi-1} \times(B-y)^{\left(1-\mu_{1}\right) \phi-1} /\left[\operatorname{beta}\left(\mu_{1} \phi,\left(1-\mu_{1}\right) \phi\right) \times(B-A)^{\phi-1}\right]
$$

for $A<y<B$, and $\operatorname{beta}(.,$.$) is the beta function (see beta). The parameter \mu_{1}$ satisfies $\mu_{1}=$ $(\mu-A) /(B-A)$ where $\mu$ is the mean of $Y$. That is, $\mu_{1}$ is the mean of of a standard beta distribution: $E(Y)=A+(B-A) \times \mu_{1}$, and these are the fitted values of the object. Also, $\phi$ is positive and $A<\mu<B$. Here, the limits $A$ and $B$ are known.
Another parameterization of the beta distribution involving the raw shape parameters is implemented in betaR.
For general $A$ and $B$, the variance of $Y$ is $(B-A)^{2} \times \mu_{1} \times\left(1-\mu_{1}\right) /(1+\phi)$. Then $\phi$ can be interpreted as a precision parameter in the sense that, for fixed $\mu$, the larger the value of $\phi$, the smaller the variance of $Y$. Also, $\mu_{1}=$ shape $1 /($ shape $1+$ shape 2$)$ and $\phi=$ shape $1+$ shape 2 . Fisher scoring is implemented.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

The response must have values in the interval $(A, B)$. The user currently needs to manually choose lmu to match the input of arguments A and B, e.g., with extlogitlink; see the example below.

## Author(s)

Thomas W. Yee

## References

Ferrari, S. L. P. and Francisco C.-N. (2004). Beta regression for modelling rates and proportions. Journal of Applied Statistics, 31, 799-815.

## See Also

betaR,
Beta, dzoabeta, genbetaII, betaII, betabinomialff, betageometric, betaprime, rbetageom, rbetanorm, kumar, extlogitlink, simulate.vlm.

## Examples

```
bdata <- data.frame(y = rbeta(nn <- 1000, shape1 = exp(0), shape2 = exp(1)))
fit1 <- vglm(y ~ 1, betaff, data = bdata, trace = TRUE)
coef(fit1, matrix = TRUE)
Coef(fit1) # Useful for intercept-only models
# General A and B, and with a covariate
bdata <- transform(bdata, x2 = runif(nn))
bdata <- transform(bdata, mu = logitlink(0.5 - x2, inverse = TRUE),
    prec = exp(3.0 + x2)) # prec == phi
bdata <- transform(bdata, shape2 = prec * (1 - mu),
            shape1 = mu * prec)
bdata <- transform(bdata,
    y = rbeta(nn, shape1 = shape1, shape2 = shape2))
bdata <- transform(bdata, Y = 5 + 8 * y) # From 5 to 13, not 0 to 1
fit <- vglm(Y ~ x2, data = bdata, trace = TRUE,
    betaff(A = 5, B = 13, lmu = extlogitlink(min = 5, max = 13)))
coef(fit, matrix = TRUE)
```

Betageom The Beta-Geometric Distribution

## Description

Density, distribution function, and random generation for the beta-geometric distribution.

## Usage

dbetageom( $x$, shape1, shape2, log = FALSE)
pbetageom(q, shape1, shape2, log.p = FALSE)
rbetageom(n, shape1, shape2)

## Arguments

$x, q \quad$ vector of quantiles.
$n \quad$ number of observations. Same as runif.
shape1, shape2 the two (positive) shape parameters of the standard beta distribution. They are called $a$ and $b$ in beta respectively.
$\log , \log . p \quad$ Logical. If TRUE then all probabilities $p$ are given as $\log (p)$.

## Details

The beta-geometric distribution is a geometric distribution whose probability of success is not a constant but it is generated from a beta distribution with parameters shape1 and shape2. Note that the mean of this beta distribution is shape1/(shape1+shape2), which therefore is the mean of the probability of success.

## Value

dbetageom gives the density, pbetageom gives the distribution function, and rbetageom generates random deviates.

## Note

pbetageom can be particularly slow.

## Author(s)

T. W. Yee

## See Also

geometric, betaff, Beta.

## Examples

```
## Not run:
shape1 <- 1; shape2 <- 2; y <- 0:30
proby <- dbetageom(y, shape1, shape2, log = FALSE)
plot(y, proby, type = "h", col = "blue", ylab = "P[Y=y]", main = paste(
    "Y ~ Beta-geometric(shape1=", shape1,", shape2=", shape2, ")", sep = ""))
sum(proby)
## End(Not run)
```

betageometric Beta-geometric Distribution Family Function

## Description

Maximum likelihood estimation for the beta-geometric distribution.

## Usage

betageometric(lprob = "logitlink", lshape = "loglink", iprob $=$ NULL, $\quad$ ishape $=0.1$, moreSummation $=c(2,100)$, tolerance $=1.0 \mathrm{e}-10$, zero $=$ NULL)

## Arguments

lprob, lshape Parameter link functions applied to the parameters $p$ and $\phi$ (called prob and shape below). The former lies in the unit interval and the latter is positive. See Links for more choices.
iprob, ishape Numeric. Initial values for the two parameters. A NULL means a value is computed internally.
moreSummation Integer, of length 2. When computing the expected information matrix a series summation from 0 to moreSummation[1]*max $(y)+$ moreSummation[2] is made, in which the upper limit is an approximation to infinity. Here, $y$ is the response.
tolerance Positive numeric. When all terms are less than this then the series is deemed to have converged.
zero An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. If used, the value must be from the set $\{1,2\}$.

## Details

A random variable $Y$ has a 2-parameter beta-geometric distribution if $P(Y=y)=p(1-p)^{y}$ for $y=0,1,2, \ldots$ where $p$ are generated from a standard beta distribution with shape parameters shape1 and shape2. The parameterization here is to focus on the parameters $p$ and $\phi=$ $1 /($ shape $1+$ shape 2$)$, where $\phi$ is shape. The default link functions for these ensure that the appropriate range of the parameters is maintained. The mean of $Y$ is $E(Y)=\operatorname{shape} 2 /($ shape $1-1)=$ $(1-p) /(p-\phi)$ if shape $1>1$, and if so, then this is returned as the fitted values.

The geometric distribution is a special case of the beta-geometric distribution with $\phi=0$ (see geometric). However, fitting data from a geometric distribution may result in numerical problems because the estimate of $\log (\phi)$ will 'converge' to -Inf.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

The first iteration may be very slow; if practical, it is best for the weights argument of vglm etc. to be used rather than inputting a very long vector as the response, i.e., vglm (y $\sim 1, \ldots$, weights $=w t s)$ is to be preferred over $\operatorname{vglm}(r e p(y, w t s) \sim 1, \ldots)$. If convergence problems occur try inputting some values of argument ishape.

If an intercept-only model is fitted then the misc slot of the fitted object has list components shape 1 and shape2.

## Author(s)

T. W. Yee

## References

Paul, S. R. (2005). Testing goodness of fit of the geometric distribution: an application to human fecundability data. Journal of Modern Applied Statistical Methods, 4, 425-433.

## See Also

geometric, betaff, rbetageom.

## Examples

```
bdata <- data.frame(y = 0:11, wts = c(227,123,72,42,21,31,11,14,6,4,7,28))
fitb <- vglm(y ~ 1, betageometric, data = bdata, weight = wts, trace = TRUE)
fitg <- vglm(y ~ 1, geometric, data = bdata, weight = wts, trace = TRUE)
coef(fitb, matrix = TRUE)
Coef(fitb)
sqrt(diag(vcov(fitb, untransform = TRUE)))
fitb@misc$shape1
fitb@misc$shape2
# Very strong evidence of a beta-geometric:
pchisq(2 * (logLik(fitb) - logLik(fitg)), df = 1, lower.tail = FALSE)
```


## Description

Maximum likelihood estimation of the 3-parameter beta II distribution.

## Usage

betaII(lscale = "loglink", lshape2.p = "loglink", lshape3.q = "loglink", iscale $=$ NULL, ishape2.p $=$ NULL, ishape3.q $=$ NULL, imethod $=1$,
gscale $=\exp (-5: 5)$, gshape2.p $=\exp (-5: 5)$,
gshape3.q $=\operatorname{seq}(0.75,4$, by $=0.25)$,
probs.y $=c(0.25,0.5,0.75)$, zero = "shape")

## Arguments

lscale, lshape2.p, lshape3.q
Parameter link functions applied to the (positive) parameters scale, p and q . See Links for more choices.
iscale, ishape2.p, ishape3.q, imethod, zero
See CommonVGAMffArguments for information.
gscale, gshape2.p, gshape3.q
See CommonVGAMffArguments for information.
probs.y See CommonVGAMffArguments for information.

## Details

The 3-parameter beta II is the 4-parameter generalized beta II distribution with shape parameter $a=1$. It is also known as the Pearson VI distribution. Other distributions which are special cases of the 3-parameter beta II include the Lomax $(p=1)$ and inverse Lomax $(q=1)$. More details can be found in Kleiber and Kotz (2003).
The beta II distribution has density

$$
f(y)=y^{p-1} /\left[b^{p} B(p, q)\{1+y / b\}^{p+q}\right]
$$

for $b>0, p>0, q>0, y \geq 0$. Here, $b$ is the scale parameter scale, and the others are shape parameters. The mean is

$$
E(Y)=b \Gamma(p+1) \Gamma(q-1) /(\Gamma(p) \Gamma(q))
$$

provided $q>1$; these are returned as the fitted values. This family function handles multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

See the notes in genbetaII.

## Author(s)

T. W. Yee

## References

Kleiber, C. and Kotz, S. (2003). Statistical Size Distributions in Economics and Actuarial Sciences, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

betaff, genbetaII, dagum, sinmad, fisk, inv.lomax, lomax, paralogistic, inv. paralogistic.

## Examples

```
bdata <- data.frame(y = rsinmad(2000, shape1.a = 1, shape3.q = exp(2),
    scale = exp(1))) # Not genuine data!
fit <- vglm(y ~ 1, betaII, data = bdata, trace = TRUE)
fit <- vglm(y ~ 1, betaII(ishape2.p = 0.7, ishape3.q = 0.7),
    data = bdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

Betanorm The Beta-Normal Distribution

## Description

Density, distribution function, quantile function and random generation for the univariate betanormal distribution.

## Usage

```
dbetanorm(x, shape1, shape2, mean = 0, sd = 1, log = FALSE)
pbetanorm(q, shape1, shape2, mean = 0, sd = 1,
    lower.tail = TRUE, log.p = FALSE)
qbetanorm(p, shape1, shape2, mean = 0, sd = 1,
    lower.tail = TRUE, log.p = FALSE)
rbetanorm(n, shape1, shape2, mean = 0, sd = 1)
```


## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$n \quad$ number of observations. Same as runif.
shape1, shape2 the two (positive) shape parameters of the standard beta distribution. They are called a and b respectively in beta.
mean, sd the mean and standard deviation of the univariate normal distribution (Normal).
$\log , \log . p \quad$ Logical. If TRUE then all probabilities $p$ are given as $\log (p)$.
lower.tail Logical. If TRUE then the upper tail is returned, i.e., one minus the usual answer.

## Details

The function betauninormal, the VGAM family function for estimating the parameters, has not yet been written.

## Value

dbetanorm gives the density, pbetanorm gives the distribution function, qbetanorm gives the quantile function, and rbetanorm generates random deviates.

## Author(s)

T. W. Yee

## References

Gupta, A. K. and Nadarajah, S. (2004). Handbook of Beta Distribution and Its Applications, pp.146-152. New York: Marcel Dekker.

## Examples

```
## Not run:
shape1 <- 0.1; shape2 <- 4; m <- 1
x <- seq(-10, 2, len = 501)
plot(x, dbetanorm(x, shape1, shape2, m = m), type = "l", ylim = 0:1, las = 1,
    ylab = paste("betanorm(",shape1,", ",shape2,", m=",m, ", sd=1)", sep = ""),
    main = "Blue is density, orange is cumulative distribution function",
    sub = "Gray lines are the 10,20,\ldots.,90 percentiles", col = "blue")
lines(x, pbetanorm(x, shape1, shape2, m = m), col = "orange")
abline(h = 0, col = "black")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qbetanorm(probs, shape1, shape2, m = m)
lines(Q, dbetanorm(Q, shape1, shape2, m = m), col = "gray50", lty = 2, type = "h")
lines(Q, pbetanorm(Q, shape1, shape2, m = m), col = "gray50", lty = 2, type = "h")
abline(h = probs, col = "gray50", lty = 2)
pbetanorm(Q, shape1, shape2, m = m) - probs # Should be all 0
## End(Not run)
```

betaprime The Beta-Prime Distribution

## Description

Estimation of the two shape parameters of the beta-prime distribution by maximum likelihood estimation.

## Usage

betaprime(lshape = "loglink", ishape1 = 2, ishape2 = NULL, zero = NULL)

## Arguments

lshape Parameter link function applied to the two (positive) shape parameters. See Links for more choices.
ishape1, ishape2, zero
See CommonVGAMffArguments.

## Details

The beta-prime distribution is given by

$$
f(y)=y^{\text {shape } 1-1}(1+y)^{- \text {shape } 1-\text { shape } 2} / B(\text { shape } 1, \text { shape } 2)
$$

for $y>0$. The shape parameters are positive, and here, $B$ is the beta function. The mean of $Y$ is shape $1 /($ shape $2-1)$ provided shape $2>1$; these are returned as the fitted values.
If $Y$ has a Beta(shape1, shape2) distribution then $Y /(1-Y)$ and $(1-Y) / Y$ have a Betaprime (shape 1 , shape2) and Betaprime (shape2, shape1) distribution respectively. Also, if $Y_{1}$ has a gamma(shape1) distribution and $Y_{2}$ has a gamma (shape 2$)$ distribution then $Y_{1} / Y_{2}$ has a Betaprime (shape 1 , shape 2 ) distribution.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Note

The response must have positive values only.
The beta-prime distribution is also known as the beta distribution of the second kind or the inverted beta distribution.

## Author(s)

Thomas W. Yee

## References

Johnson, N. L. and Kotz, S. and Balakrishnan, N. (1995). Chapter 25 of: Continuous Univariate Distributions, 2nd edition, Volume 2, New York: Wiley.

## See Also

betaff, Beta.

## Examples

```
nn <- 1000
bdata <- data.frame(shape1 = exp(1), shape2 = exp(3))
bdata <- transform(bdata, yb = rbeta(nn, shape1, shape2))
bdata <- transform(bdata, y1 = (1-yb) / yb,
    y2 = yb / (1-yb),
    y3 = rgamma(nn, exp(3)) / rgamma(nn, exp(2)))
fit1 <- vglm(y1 ~ 1, betaprime, data = bdata, trace = TRUE)
coef(fit1, matrix = TRUE)
fit2 <- vglm(y2 ~ 1, betaprime, data = bdata, trace = TRUE)
coef(fit2, matrix = TRUE)
fit3 <- vglm(y3 ~ 1, betaprime, data = bdata, trace = TRUE)
coef(fit3, matrix = TRUE)
# Compare the fitted values
with(bdata, mean(y3))
head(fitted(fit3))
Coef(fit3) # Useful for intercept-only models
```

betaR The Two-parameter Beta Distribution Family Function

## Description

Estimation of the shape parameters of the two-parameter beta distribution.

## Usage

```
betaR(lshape1 = "loglink", lshape2 = "loglink",
    i1 = NULL, i2 = NULL, trim = 0.05,
    A = 0, B = 1, parallel = FALSE, zero = NULL)
```


## Arguments

lshape1, lshape2, i1, i2
Details at CommonVGAMffArguments. See Links for more choices.
trim An argument which is fed into mean(); it is the fraction (0 to 0.5) of observations to be trimmed from each end of the response $y$ before the mean is computed. This is used when computing initial values, and guards against outliers.
A, B Lower and upper limits of the distribution. The defaults correspond to the standard beta distribution where the response lies between 0 and 1.
parallel, zero See CommonVGAMffArguments for more information.

## Details

The two-parameter beta distribution is given by $f(y)=$

$$
(y-A)^{\text {shape } 1-1} \times(B-y)^{\text {shape } 2-1} /\left[\text { Beta }(\text { shape } 1, \text { shape } 2) \times(B-A)^{\text {shape } 1+\text { shape } 2-1}\right]
$$

for $A<y<B$, and $\operatorname{Beta}(.,$.$) is the beta function (see beta). The shape parameters are pos-$ itive, and here, the limits $A$ and $B$ are known. The mean of $Y$ is $E(Y)=A+(B-A) \times$ shape $1 /($ shape $1+$ shape 2$)$, and these are the fitted values of the object.
For the standard beta distribution the variance of $Y$ is shape $1 \times$ shape $2 /[(1+$ shape $1+$ shape 2$) \times$ (shape $1+$ shape 2$\left.)^{2}\right]$. If $\sigma^{2}=1 /(1+$ shape $1+$ shape 2$)$ then the variance of $Y$ can be written $\sigma^{2} \mu(1-\mu)$ where $\mu=$ shape $1 /($ shape $1+$ shape 2$)$ is the mean of $Y$.
Another parameterization of the beta distribution involving the mean and a precision parameter is implemented in betaff.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Note

The response must have values in the interval $(A, B)$. VGAM 0.7-4 and prior called this function betaff.

## Author(s)

Thomas W. Yee

## References

Johnson, N. L. and Kotz, S. and Balakrishnan, N. (1995). Chapter 25 of: Continuous Univariate Distributions, 2nd edition, Volume 2, New York: Wiley.
Gupta, A. K. and Nadarajah, S. (2004). Handbook of Beta Distribution and Its Applications, New York: Marcel Dekker.

## See Also

```
betaff,
Beta, genbetaII, betaII, betabinomialff, betageometric, betaprime, rbetageom, rbetanorm,
kumar, simulate.vlm.
```


## Examples

```
bdata <- data.frame(y = rbeta(n = 1000, shape1 = exp(0), shape2 = exp(1)))
fit <- vglm(y ~ 1, betaR(lshape1 = "identitylink", lshape2 = "identitylink"),
    data = bdata, trace = TRUE, crit = "coef")
fit <- vglm(y ~ 1, betaR, data = bdata, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit) # Useful for intercept-only models
bdata <- transform(bdata, Y = 5 + 8 * y) # From 5 to 13, not 0 to 1
fit <- vglm(Y ~ 1, betaR(A = 5, B = 13), data = bdata, trace = TRUE)
Coef(fit)
c(meanY = with(bdata, mean(Y)), head(fitted(fit),2))
```

Ali-Mikhail-Haq Bivariate Distribution

## Description

Density, distribution function, and random generation for the (one parameter) bivariate Ali-MikhailHaq distribution.

## Usage

```
dbiamhcop(x1, x2, apar, log = FALSE)
pbiamhcop(q1, q2, apar)
rbiamhcop(n, apar)
```


## Arguments

$x 1, x 2, q 1, q 2$ vector of quantiles.
$\mathrm{n} \quad$ number of observations. Same as runif
apar the association parameter.
log Logical. If TRUE then the logarithm is returned.

## Details

See biamhcop, the VGAM family functions for estimating the parameter by maximum likelihood estimation, for the formula of the cumulative distribution function and other details.

## Value

dbiamhcop gives the density, pbiamhcop gives the distribution function, and rbiamhcop generates random deviates (a two-column matrix).

## Author(s)

T. W. Yee and C. S. Chee

## See Also

biamhcop.

## Examples

```
x <- seq(0, 1, len = (N <- 101)); apar <- 0.7
ox <- expand.grid(x, x)
zedd <- dbiamhcop(ox[, 1], ox[, 2], apar = apar)
## Not run:
contour(x, x, matrix(zedd, N, N), col = "blue")
zedd <- pbiamhcop(ox[, 1], ox[, 2], apar = apar)
contour(x, x, matrix(zedd, N, N), col = "blue")
plot(r <- rbiamhcop(n = 1000, apar = apar), col = "blue")
par(mfrow = c(1, 2))
hist(r[, 1]) # Should be uniform
hist(r[, 2]) # Should be uniform
## End(Not run)
```


## Description

Estimate the association parameter of Ali-Mikhail-Haq's bivariate distribution by maximum likelihood estimation.

## Usage

biamhcop(lapar = "rhobitlink", iapar = NULL, imethod = 1, nsimEIM = 250)

## Arguments

lapar $\quad$ Link function applied to the association parameter $\alpha$, which is real and $-1<$ $\alpha<1$. See Links for more choices.
iapar Numeric. Optional initial value for $\alpha$. By default, an initial value is chosen internally. If a convergence failure occurs try assigning a different value. Assigning a value will override the argument imethod.

| imethod | An integer with value 1 or 2 which specifies the initialization method. If failure <br> to converge occurs try the other value, or else specify a value for iapar. |
| :--- | :--- |
| nsimEIM | See CommonVGAMffArguments for more information. |

## Details

The cumulative distribution function is

$$
P\left(Y_{1} \leq y_{1}, Y_{2} \leq y_{2}\right)=y_{1} y_{2} /\left(1-\alpha\left(1-y_{1}\right)\left(1-y_{2}\right)\right)
$$

for $-1<\alpha<1$. The support of the function is the unit square. The marginal distributions are the standard uniform distributions. When $\alpha=0$ the random variables are independent. This is an Archimedean copula.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

The response must be a two-column matrix. Currently, the fitted value is a matrix with two columns and values equal to 0.5 . This is because each marginal distribution corresponds to a standard uniform distribution.

## Author(s)

T. W. Yee and C. S. Chee

## References

Balakrishnan, N. and Lai, C.-D. (2009). Continuous Bivariate Distributions, 2nd ed. New York: Springer.

## See Also

```
rbiamhcop, bifgmcop, bigumbelIexp, rbilogis, simulate.vlm.
```


## Examples

ymat <- rbiamhcop(1000, apar = rhobitlink(2, inverse = TRUE))
fit <- vglm(ymat ~ 1, biamhcop, trace = TRUE)
coef(fit, matrix $=$ TRUE)
Coef(fit)

## Description

Density and random generation for the (one parameter) bivariate Clayton copula distribution.

## Usage

dbiclaytoncop(x1, x2, apar $=0,1 \mathrm{~g}=$ FALSE)
rbiclaytoncop(n, apar = 0)

## Arguments

$x 1, x 2 \quad$ vector of quantiles. The $x 1$ and $\times 2$ should both be in the interval $(0,1)$.
$n \quad$ number of observations. Same as rnorm.
apar the association parameter. Should be in the interval $[0, \infty)$. The default corresponds to independence.
$\log \quad$ Logical. If TRUE then the logarithm is returned.

## Details

See biclaytoncop, the VGAM family functions for estimating the parameter by maximum likelihood estimation, for the formula of the cumulative distribution function and other details.

## Value

dbiclaytoncop gives the density at point ( $\mathrm{x} 1, \mathrm{x} 2$ ), rbiclaytoncop generates random deviates (a two-column matrix).

## Note

dbiclaytoncop() does not yet handle $\mathrm{x} 1=0$ and/or $\mathrm{x} 2=0$.

## Author(s)

R. Feyter and T. W. Yee

## References

Clayton, D. (1982). A model for association in bivariate survival data. Journal of the Royal Statistical Society, Series B, Methodological, 44, 414-422.

## See Also

biclaytoncop, binormalcop, binormal.

## Examples

```
## Not run: edge <- 0.01 # A small positive value
N <- 101; x <- seq(edge, 1.0 - edge, len = N); Rho <- 0.7
ox <- expand.grid(x, x)
zedd <- dbiclaytoncop(ox[, 1], ox[, 2], apar = Rho, log = TRUE)
par(mfrow = c(1, 2))
contour(x, x, matrix(zedd, N, N), col = "blue", labcex = 1.5, las = 1)
plot(rbiclaytoncop(1000, 2), col = "blue", las = 1)
## End(Not run)
```

biclaytoncop Clayton Copula (Bivariate) Family Function

## Description

Estimate the correlation parameter of the (bivariate) Clayton copula distribution by maximum likelihood estimation.

## Usage

biclaytoncop(lapar = "loglink", iapar = NULL, imethod = 1, parallel = FALSE, zero = NULL)

## Arguments

lapar, iapar, imethod Details at CommonVGAMffArguments. See Links for more link function choices.
parallel, zero Details at CommonVGAMffArguments. If parallel $=$ TRUE then the constraint is also applied to the intercept.

## Details

The cumulative distribution function is

$$
P\left(u_{1}, u_{2} ; \alpha\right)=\left(u_{1}^{-\alpha}+u_{2}^{-\alpha}-1\right)^{-1 / \alpha}
$$

for $0 \leq \alpha$. Here, $\alpha$ is the association parameter. The support of the function is the interior of the unit square; however, values of 0 and/or 1 are not allowed (currently). The marginal distributions are the standard uniform distributions. When $\alpha=0$ the random variables are independent.
This VGAM family function can handle multiple responses, for example, a six-column matrix where the first 2 columns is the first out of three responses, the next 2 columns being the next response, etc.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

The response matrix must have a multiple of two-columns. Currently, the fitted value is a matrix with the same number of columns and values equal to 0.5 . This is because each marginal distribution corresponds to a standard uniform distribution.
This VGAM family function is fragile; each response must be in the interior of the unit square.

## Author(s)

R. Feyter and T. W. Yee

## References

Clayton, D. (1982). A model for association in bivariate survival data. Journal of the Royal Statistical Society, Series B, Methodological, 44, 414-422.
Stober, J. and Schepsmeier, U. (2013). Derivatives and Fisher information of bivariate copulas. Statistical Papers.

## See Also

rbiclaytoncop, dbiclaytoncop, kendall.tau.

## Examples

```
ymat <- rbiclaytoncop(n = (nn <- 1000), apar = exp(2))
bdata <- data.frame(y1 = ymat[, 1],
    y2 = ymat[, 2],
    y3 = ymat[, 1],
    y4 = ymat[, 2],
    x2 = runif(nn))
summary(bdata)
## Not run: plot(ymat, col = "blue")
fit1 <- vglm(cbind(y1, y2, y3, y4) ~ 1, # 2 responses, e.g., (y1,y2) is the first
            biclaytoncop, data = bdata,
            trace = TRUE, crit = "coef") # Sometimes a good idea
coef(fit1, matrix = TRUE)
Coef(fit1)
head(fitted(fit1))
summary(fit1)
# Another example; apar is a function of x2
bdata <- transform(bdata, apar = exp(-0.5 + x2))
ymat <- rbiclaytoncop(n = nn, apar = with(bdata, apar))
bdata <- transform(bdata, y5 = ymat[, 1],
    y6 = ymat[, 2])
fit2 <- vgam(cbind(y5, y6) ~ s(x2), data = bdata,
    biclaytoncop(lapar = "loglink"), trace = TRUE)
## Not run: plot(fit2, lcol = "blue", scol = "orange", se = TRUE, las = 1)
```


## Description

Calculates the Bayesian information criterion (BIC) for a fitted model object for which a loglikelihood value has been obtained.

## Usage

BICvlm(object, ..., k = log(nobs(object)))

## Arguments

object, ... Same as AICvlm.
k
Numeric, the penalty per parameter to be used; the default is $\log (n)$ where $n$ is the number of observations).

## Details

The so-called BIC or SBC (Schwarz's Bayesian criterion) can be computed by calling AICvlm with a different $k$ argument. See AICvlm for information and caveats.

## Value

Returns a numeric value with the corresponding BIC, or ..., depending on $k$.

## Warning

Like AICvlm, this code has not been double-checked. The general applicability of BIC for the VGLM/VGAM classes has not been developed fully. In particular, BIC should not be run on some VGAM family functions because of violation of certain regularity conditions, etc.

Many VGAM family functions such as cumulative can have the number of observations absorbed into the prior weights argument (e.g., weights in vglm), either before or after fitting. Almost all VGAM family functions can have the number of observations defined by the weights argument, e.g., as an observed frequency. BIC simply uses the number of rows of the model matrix, say, as defining $n$, hence the user must be very careful of this possible error. Use at your own risk!!

## Note

BIC, AIC and other ICs can have have many additive constants added to them. The important thing are the differences since the minimum value corresponds to the best model.
BIC has not been defined for QRR-VGLMs yet.

## Author(s)

T. W. Yee.

## See Also

AICvlm, VGLMs are described in vglm-class; VGAMs are described in vgam-class; RR-VGLMs are described in rrvglm-class; BIC, AIC.

## Examples

```
pneumo <- transform(pneumo, let = log(exposure.time))
(fit1 <- vglm(cbind(normal, mild, severe) ~ let,
                cumulative(parallel = TRUE, reverse = TRUE), data = pneumo))
coef(fit1, matrix = TRUE)
BIC(fit1)
(fit2 <- vglm(cbind(normal, mild, severe) ~ let,
    cumulative(parallel = FALSE, reverse = TRUE), data = pneumo))
coef(fit2, matrix = TRUE)
BIC(fit2)
```

Bifgmcop Farlie-Gumbel-Morgenstern's Bivariate Distribution

## Description

Density, distribution function, and random generation for the (one parameter) bivariate Farlie-Gumbel-Morgenstern's distribution.

## Usage

dbifgmcop(x1, x2, apar, log = FALSE)
pbifgmcop(q1, q2, apar)
rbifgmcop(n, apar)

## Arguments

$x 1, x 2, q 1, q 2$ vector of quantiles.
$n \quad$ number of observations. Same as in runif.
apar the association parameter.
$\log \quad$ Logical. If TRUE then the logarithm is returned.

## Details

See bifgmcop, the VGAM family functions for estimating the parameter by maximum likelihood estimation, for the formula of the cumulative distribution function and other details.

## Value

dbifgmcop gives the density, pbifgmcop gives the distribution function, and rbifgmcop generates random deviates (a two-column matrix).

## Author(s)

T. W. Yee

## See Also

bifgmcop.

## Examples

```
## Not run: N <- 101; x <- seq(0.0, 1.0, len = N); apar <- 0.7
ox <- expand.grid(x, x)
zedd <- dbifgmcop(ox[, 1], ox[, 2], apar = apar)
contour(x, x, matrix(zedd, N, N), col = "blue")
zedd <- pbifgmcop(ox[, 1], ox[, 2], apar = apar)
contour(x, x, matrix(zedd, N, N), col = "blue")
plot(r <- rbifgmcop(n = 3000, apar = apar), col = "blue")
par(mfrow = c(1, 2))
hist(r[, 1]) # Should be uniform
hist(r[, 2]) # Should be uniform
## End(Not run)
```

```
bifgmcop
```

Farlie-Gumbel-Morgenstern's Bivariate Distribution Family Function

## Description

Estimate the association parameter of Farlie-Gumbel-Morgenstern's bivariate distribution by maximum likelihood estimation.

## Usage

bifgmcop(lapar = "rhobitlink", iapar = NULL, imethod = 1)

## Arguments

lapar, iapar, imethod
Details at CommonVGAMffArguments. See Links for more link function choices.

## Details

The cumulative distribution function is

$$
P\left(Y_{1} \leq y_{1}, Y_{2} \leq y_{2}\right)=y_{1} y_{2}\left(1+\alpha\left(1-y_{1}\right)\left(1-y_{2}\right)\right)
$$

for $-1<\alpha<1$. The support of the function is the unit square. The marginal distributions are the standard uniform distributions. When $\alpha=0$ the random variables are independent.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

The response must be a two-column matrix. Currently, the fitted value is a matrix with two columns and values equal to 0.5 . This is because each marginal distribution corresponds to a standard uniform distribution.

## Author(s)

T. W. Yee

## References

Castillo, E., Hadi, A. S., Balakrishnan, N. Sarabia, J. S. (2005). Extreme Value and Related Models with Applications in Engineering and Science, Hoboken, NJ, USA: Wiley-Interscience.
Smith, M. D. (2007). Invariance theorems for Fisher information. Communications in StatisticsTheory and Methods, 36(12), 2213-2222.

## See Also

rbifgmcop, bifrankcop, bifgmexp, simulate.vlm.

## Examples

```
ymat <- rbifgmcop(n = 1000, apar = rhobitlink(3, inverse = TRUE))
## Not run: plot(ymat, col = "blue")
fit <- vglm(ymat ~ 1, fam = bifgmcop, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
head(fitted(fit))
```

bifgmexp Bivariate Farlie-Gumbel-Morgenstern Exponential Distribution Family Function

## Description

Estimate the association parameter of FGM bivariate exponential distribution by maximum likelihood estimation.

## Usage

bifgmexp(lapar = "rhobitlink", iapar = NULL, tola0 = 0.01, imethod = 1)

## Arguments

lapar Link function for the association parameter $\alpha$, which lies between -1 and 1. See Links for more choices and other information.
iapar $\quad$ Numeric. Optional initial value for $\alpha$. By default, an initial value is chosen internally. If a convergence failure occurs try assigning a different value. Assigning a value will override the argument imethod.
tola0 Positive numeric. If the estimate of $\alpha$ has an absolute value less than this then it is replaced by this value. This is an attempt to fix a numerical problem when the estimate is too close to zero.
imethod An integer with value 1 or 2 which specifies the initialization method. If failure to converge occurs try the other value, or else specify a value for ia.

## Details

The cumulative distribution function is

$$
P\left(Y_{1} \leq y_{1}, Y_{2} \leq y_{2}\right)=e^{-y_{1}-y_{2}}\left(1+\alpha\left[1-e^{-y_{1}}\right]\left[1-e^{-y_{2}}\right]\right)+1-e^{-y_{1}}-e^{-y_{2}}
$$

for $\alpha$ between -1 and 1. The support of the function is for $y_{1}>0$ and $y_{2}>0$. The marginal distributions are an exponential distribution with unit mean. When $\alpha=0$ then the random variables are independent, and this causes some problems in the estimation process since the distribution no longer depends on the parameter.
A variant of Newton-Raphson is used, which only seems to work for an intercept model. It is a very good idea to set trace $=$ TRUE .

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

The response must be a two-column matrix. Currently, the fitted value is a matrix with two columns and values equal to 1 . This is because each marginal distribution corresponds to a exponential distribution with unit mean.
This VGAM family function should be used with caution.

## Author(s)

T. W. Yee

## References

Castillo, E., Hadi, A. S., Balakrishnan, N. Sarabia, J. S. (2005). Extreme Value and Related Models with Applications in Engineering and Science, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

bifgmcop, bigumbelIexp.

## Examples

```
N <- 1000; mdata <- data.frame(y1 = rexp(N), y2 = rexp(N))
## Not run: plot(ymat)
fit <- vglm(cbind(y1, y2) ~ 1, bifgmexp, data = mdata, trace = TRUE)
fit <- vglm(cbind(y1, y2) ~ 1, bifgmexp, data = mdata, # This may fail
    trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit)
head(fitted(fit))
```

```
bifrankcop Frank's Bivariate Distribution Family Function
```


## Description

Estimate the association parameter of Frank's bivariate distribution by maximum likelihood estimation.

## Usage

bifrankcop(lapar = "loglink", iapar = 2, nsimEIM = 250)

## Arguments

lapar Link function applied to the (positive) association parameter $\alpha$. See Links for more choices.
iapar Numeric. Initial value for $\alpha$. If a convergence failure occurs try assigning a different value.
nsimEIM See CommonVGAMffArguments.

## Details

The cumulative distribution function is

$$
P\left(Y_{1} \leq y_{1}, Y_{2} \leq y_{2}\right)=H_{\alpha}\left(y_{1}, y_{2}\right)=\log _{\alpha}\left[1+\left(\alpha^{y_{1}}-1\right)\left(\alpha^{y_{2}}-1\right) /(\alpha-1)\right]
$$

for $\alpha \neq 1$. Note the logarithm here is to base $\alpha$. The support of the function is the unit square.
When $0<\alpha<1$ the probability density function $h_{\alpha}\left(y_{1}, y_{2}\right)$ is symmetric with respect to the lines $y_{2}=y_{1}$ and $y_{2}=1-y_{1}$. When $\alpha>1$ then $h_{\alpha}\left(y_{1}, y_{2}\right)=h_{1 / \alpha}\left(1-y_{1}, y_{2}\right)$.
If $\alpha=1$ then $H\left(y_{1}, y_{2}\right)=y_{1} y_{2}$, i.e., uniform on the unit square. As $\alpha$ approaches 0 then $H\left(y_{1}, y_{2}\right)=\min \left(y_{1}, y_{2}\right)$. As $\alpha$ approaches infinity then $H\left(y_{1}, y_{2}\right)=\max \left(0, y_{1}+y_{2}-1\right)$.
The default is to use Fisher scoring implemented using rbifrankcop. For intercept-only models an alternative is to set nsimEIM=NULL so that a variant of Newton-Raphson is used.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

The response must be a two-column matrix. Currently, the fitted value is a matrix with two columns and values equal to a half. This is because the marginal distributions correspond to a standard uniform distribution.

## Author(s)

T. W. Yee

## References

Genest, C. (1987). Frank's family of bivariate distributions. Biometrika, 74, 549-555.

## See Also

rbifrankcop, bifgmcop, simulate.vlm.

## Examples

```
## Not run:
ymat <- rbifrankcop(n = 2000, apar = exp(4))
plot(ymat, col = "blue")
fit <- vglm(ymat ~ 1, fam = bifrankcop, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
vcov(fit)
head(fitted(fit))
summary(fit)
## End(Not run)
```


## Description

Estimate the three parameters of McKay's bivariate gamma distribution by maximum likelihood estimation.

## Usage

```
bigamma.mckay(lscale = "loglink", lshape1 = "loglink", lshape2 = "loglink",
    iscale = NULL, ishape1 = NULL, ishape2 = NULL,
    imethod = 1, zero = "shape")
```


## Arguments

lscale, lshape1, lshape2
Link functions applied to the (positive) parameters $a, p$ and $q$ respectively. See Links for more choices.
iscale, ishape1, ishape2
Optional initial values for $a, p$ and $q$ respectively. The default is to compute them internally.
imethod, zero See CommonVGAMffArguments.

## Details

One of the earliest forms of the bivariate gamma distribution has a joint probability density function given by

$$
f\left(y_{1}, y_{2} ; a, p, q\right)=(1 / a)^{p+q} y_{1}^{p-1}\left(y_{2}-y_{1}\right)^{q-1} \exp \left(-y_{2} / a\right) /[\Gamma(p) \Gamma(q)]
$$

for $a>0, p>0, q>0$ and $0<y_{1}<y_{2}$ (Mckay, 1934). Here, $\Gamma$ is the gamma function, as in gamma. By default, the linear/additive predictors are $\eta_{1}=\log (a), \eta_{2}=\log (p), \eta_{3}=\log (q)$.
The marginal distributions are gamma, with shape parameters $p$ and $p+q$ respectively, but they have a common scale parameter $a$. Pearson's product-moment correlation coefficient of $y_{1}$ and $y_{2}$ is $\sqrt{p /(p+q)}$. This distribution is also known as the bivariate Pearson type III distribution. Also, $Y_{2}-y_{1}$, conditional on $Y_{1}=y_{1}$, has a gamma distribution with shape parameter $q$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

The response must be a two column matrix where the first column is $y_{1}$ and the second $y_{2}$. It is necessary that each element of the vectors $y_{1}$ and $y_{2}-y_{1}$ be positive. Currently, the fitted value is a matrix with two columns; the first column has values $a p$ for the marginal mean of $y_{1}$, while the second column has values $a(p+q)$ for the marginal mean of $y_{2}$ (all evaluated at the final iteration).

## Author(s)

T. W. Yee

## References

McKay, A. T. (1934). Sampling from batches. Journal of the Royal Statistical Society-Supplement, 1, 207-216.
Kotz, S. and Balakrishnan, N. and Johnson, N. L. (2000). Continuous Multivariate Distributions Volume 1: Models and Applications, 2nd edition, New York: Wiley.
Balakrishnan, N. and Lai, C.-D. (2009). Continuous Bivariate Distributions, 2nd edition. New York: Springer.

## See Also

gamma2.

## Examples

```
shape1 <- exp(1); shape2 <- exp(2); scalepar <- exp(3)
mdata <- data.frame(y1 = rgamma(nn <- 1000, shape = shape1, scale = scalepar))
mdata <- transform(mdata, zedd = rgamma(nn, shape = shape2, scale = scalepar))
mdata <- transform(mdata, y2 = y1 + zedd) # Z is defined as Y2-y1|Y1=y1
fit <- vglm(cbind(y1, y2) ~ 1, bigamma.mckay, data = mdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
vcov(fit)
colMeans(depvar(fit)) # Check moments
head(fitted(fit), 1)
```

bigumbellexp Gumbel's Type I Bivariate Distribution Family Function

## Description

Estimate the association parameter of Gumbel's Type I bivariate distribution by maximum likelihood estimation.

## Usage

bigumbelIexp(lapar = "identitylink", iapar = NULL, imethod = 1)

## Arguments

lapar Link function applied to the association parameter $\alpha$. See Links for more choices.
iapar Numeric. Optional initial value for $\alpha$. By default, an initial value is chosen internally. If a convergence failure occurs try assigning a different value. Assigning a value will override the argument imethod.
imethod An integer with value 1 or 2 which specifies the initialization method. If failure to converge occurs try the other value, or else specify a value for ia.

## Details

The cumulative distribution function is

$$
P\left(Y_{1} \leq y_{1}, Y_{2} \leq y_{2}\right)=e^{-y_{1}-y_{2}+\alpha y_{1} y_{2}}+1-e^{-y_{1}}-e^{-y_{2}}
$$

for real $\alpha$. The support of the function is for $y_{1}>0$ and $y_{2}>0$. The marginal distributions are an exponential distribution with unit mean.
A variant of Newton-Raphson is used, which only seems to work for an intercept model. It is a very good idea to set trace=TRUE.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

The response must be a two-column matrix. Currently, the fitted value is a matrix with two columns and values equal to 1 . This is because each marginal distribution corresponds to a exponential distribution with unit mean.

This VGAM family function should be used with caution.

## Author(s)

T. W. Yee

## References

Gumbel, E. J. (1960). Bivariate Exponential Distributions. Journal of the American Statistical Association, 55, 698-707.

## See Also

bifgmexp.

## Examples

```
nn <- 1000
gdata <- data.frame(y1 = rexp(nn), y2 = rexp(nn))
## Not run: with(gdata, plot(cbind(y1, y2)))
fit <- vglm(cbind(y1, y2) ~ 1, bigumbelIexp, data = gdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
head(fitted(fit))
```

bilogis Bivariate Logistic Distribution

## Description

Density, distribution function, quantile function and random generation for the 4-parameter bivariate logistic distribution.

## Usage

```
dbilogis(x1, x2, loc1 = 0, scale1 = 1, loc2 = 0, scale2 = 1, log = FALSE)
pbilogis(q1, q2, loc1 = 0, scale1 = 1, loc2 = 0, scale2 = 1)
    rbilogis(n, loc1 = 0, scale1 = 1, loc2 = 0, scale2 = 1)
```


## Arguments

$x 1, x 2, q 1, q 2$ vector of quantiles.
n
number of observations. Same as rlogis.
loc1, loc2 the location parameters $l_{1}$ and $l_{2}$.
scale1, scale2 the scale parameters $s_{1}$ and $s_{2}$.
$\log \quad$ Logical. If $\log =$ TRUE then the logarithm of the density is returned.

## Details

See bilogis, the VGAM family function for estimating the four parameters by maximum likelihood estimation, for the formula of the cumulative distribution function and other details.

## Value

dbilogis gives the density, pbilogis gives the distribution function, and rbilogis generates random deviates (a two-column matrix).

## Note

Gumbel (1961) proposed two bivariate logistic distributions with logistic distribution marginals, which he called Type I and Type II. The Type I is this one. The Type II belongs to the Morgenstern type. The biamhcop distribution has, as a special case, this distribution, which is when the random variables are independent.

## Author(s)

T. W. Yee

## References

Gumbel, E. J. (1961). Bivariate logistic distributions. Journal of the American Statistical Association, 56, 335-349.

## See Also

bilogistic, biamhcop.

## Examples

```
## Not run: par(mfrow = c(1, 3))
ymat <- rbilogis(n = 2000, loc1 = 5, loc2 = 7, scale2 = exp(1))
myxlim <- c(-2, 15); myylim <- c(-10, 30)
plot(ymat, xlim = myxlim, ylim = myylim)
N <- 100
x1 <- seq(myxlim[1], myxlim[2], len = N)
x2 <- seq(myylim[1], myylim[2], len = N)
ox <- expand.grid(x1, x2)
z <- dbilogis(ox[,1], ox[,2], loc1 = 5, loc2 = 7, scale2 = exp(1))
```

```
contour(x1, x2, matrix(z, N, N), main = "density")
z <- pbilogis(ox[,1], ox[,2], loc1 = 5, loc2 = 7, scale2 = exp(1))
contour(x1, x2, matrix(z, N, N), main = "cdf")
## End(Not run)
```

bilogistic
Bivariate Logistic Distribution Family Function

## Description

Estimates the four parameters of the bivariate logistic distribution by maximum likelihood estimation.

## Usage

```
bilogistic(llocation = "identitylink", lscale = "loglink",
    iloc1 = NULL, iscale1 = NULL, iloc2 = NULL, iscale2 = NULL,
    imethod = 1, nsimEIM = 250, zero = NULL)
```


## Arguments

llocation Link function applied to both location parameters $l_{1}$ and $l_{2}$. See Links for more choices.
lscale Parameter link function applied to both (positive) scale parameters $s_{1}$ and $s_{2}$. See Links for more choices.
iloc1, iloc2 Initial values for the location parameters. By default, initial values are chosen internally using imethod. Assigning values here will override the argument imethod.
iscale1, iscale2
Initial values for the scale parameters. By default, initial values are chosen internally using imethod. Assigning values here will override the argument imethod.
imethod An integer with value 1 or 2 which specifies the initialization method. If failure to converge occurs try the other value.
nsimEIM, zero See CommonVGAMffArguments for details.

## Details

The four-parameter bivariate logistic distribution has a density that can be written as

$$
f\left(y_{1}, y_{2} ; l_{1}, s_{1}, l_{2}, s_{2}\right)=2 \frac{\exp \left[-\left(y_{1}-l_{1}\right) / s_{1}-\left(y_{2}-l_{2}\right) / s_{2}\right]}{s_{1} s_{2}\left(1+\exp \left[-\left(y_{1}-l_{1}\right) / s_{1}\right]+\exp \left[-\left(y_{2}-l_{2}\right) / s_{2}\right]\right)^{3}}
$$

where $s_{1}>0$ and $s_{2}>0$ are the scale parameters, and $l_{1}$ and $l_{2}$ are the location parameters. Each of the two responses are unbounded, i.e., $-\infty<y_{j}<\infty$. The mean of $Y_{1}$ is $l_{1}$ etc. The fitted values are returned in a 2 -column matrix. The cumulative distribution function is

$$
F\left(y_{1}, y_{2} ; l_{1}, s_{1}, l_{2}, s_{2}\right)=\left(1+\exp \left[-\left(y_{1}-l_{1}\right) / s_{1}\right]+\exp \left[-\left(y_{2}-l_{2}\right) / s_{2}\right]\right)^{-1}
$$

The marginal distribution of $Y_{1}$ is

$$
P\left(Y_{1} \leq y_{1}\right)=F\left(y_{1} ; l_{1}, s_{1}\right)=\left(1+\exp \left[-\left(y_{1}-l_{1}\right) / s_{1}\right]\right)^{-1}
$$

By default, $\eta_{1}=l_{1}, \eta_{2}=\log \left(s_{1}\right), \eta_{3}=l_{2}, \eta_{4}=\log \left(s_{2}\right)$ are the linear/additive predictors.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Author(s)

T. W. Yee

## References

Gumbel, E. J. (1961). Bivariate logistic distributions. Journal of the American Statistical Association, 56, 335-349.
Castillo, E., Hadi, A. S., Balakrishnan, N. Sarabia, J. S. (2005). Extreme Value and Related Models with Applications in Engineering and Science, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

logistic, rbilogis.

## Examples

```
## Not run:
ymat <- rbilogis(n <- 50, loc1 = 5, loc2 = 7, scale2 = exp(1))
plot(ymat)
bfit <- vglm(ymat ~ 1, family = bilogistic, trace = TRUE)
coef(bfit, matrix = TRUE)
Coef(bfit)
head(fitted(bfit))
vcov(bfit)
head(weights(bfit, type = "work"))
summary(bfit)
## End(Not run)
```

Binom2.or Bivariate Binary Regression with an Odds Ratio

## Description

Density and random generation for a bivariate binary regression model using an odds ratio as the measure of dependency.

## Usage

```
rbinom2.or(n, mu1,
            mu2 = if (exchangeable) mu1 else stop("argument 'mu2' not specified"),
                oratio = 1, exchangeable = FALSE, tol = 0.001, twoCols = TRUE,
                colnames = if (twoCols) c("y1","y2") else c("00", "01", "10", "11"),
                ErrorCheck = TRUE)
dbinom2.or(mu1, mu2 = if (exchangeable) mu1 else stop("'mu2' not specified"),
        oratio = 1, exchangeable = FALSE, tol = 0.001,
        colnames = c("00", "01", "10", "11"), ErrorCheck = TRUE)
```


## Arguments

n
number of observations. Same as in runif. The arguments mu1, mu2, oratio are recycled to this value.
mu1, mu2 The marginal probabilities. Only mu1 is needed if exchangeable = TRUE. Values should be between 0 and 1 .
oratio Odds ratio. Must be numeric and positive. The default value of unity means the responses are statistically independent.
exchangeable Logical. If TRUE, the two marginal probabilities are constrained to be equal.
twoCols Logical. If TRUE, then a $n \times 2$ matrix of 1 s and 0 s is returned. If FALSE, then a $n \times 4$ matrix of 1 s and 0 s is returned.
colnames The dimnames argument of matrix is assigned list(NULL, colnames).
tol Tolerance for testing independence. Should be some small positive numerical value.

ErrorCheck Logical. Do some error checking of the input parameters?

## Details

The function rbinom2. or generates data coming from a bivariate binary response model. The data might be fitted with the VGAM family function binom2. or.
The function dbinom2. or does not really compute the density (because that does not make sense here) but rather returns the four joint probabilities.

## Value

The function rbinom2.or returns either a 2 or 4 column matrix of 1 s and 0 s , depending on the argument twoCols.

The function dbinom2.or returns a 4 column matrix of joint probabilities; each row adds up to unity.

## Author(s)

T. W. Yee

## See Also

binom2.or.

## Examples

```
nn <- 1000 # Example 1
ymat <- rbinom2.or(nn, mu1 = logitlink(1, inv = TRUE), oratio = exp(2), exch = TRUE)
(mytab <- table(ymat[, 1], ymat[, 2], dnn = c("Y1", "Y2")))
(myor <- mytab["0","0"] * mytab["1","1"] / (mytab["1","0"] * mytab["0","1"]))
fit <- vglm(ymat ~ 1, binom2.or(exch = TRUE))
coef(fit, matrix = TRUE)
bdata <- data.frame(x2 = sort(runif(nn))) # Example 2
bdata <- transform(bdata, mu1 = logitlink(-2 + 4 * x2, inverse = TRUE),
    mu2 = logitlink(-1 + 3 * x2, inverse = TRUE))
dmat <- with(bdata, dbinom2.or(mu1 = mu1, mu2 = mu2, oratio = exp(2)))
ymat <- with(bdata, rbinom2.or(n = nn, mu1 = mu1, mu2 = mu2, oratio = exp(2)))
fit2 <- vglm(ymat ~ x2, binom2.or, data = bdata)
coef(fit2, matrix = TRUE)
## Not run:
matplot(with(bdata, x2), dmat, lty = 1:4, col = 1:4, type = "l",
    main = "Joint probabilities", ylim = 0:1,
    ylab = "Probabilities", xlab = "x2", las = 1)
legend("top", lty = 1:4, col = 1:4,
    legend = c("1 = (y1=0, y2=0)", "2 = (y1=0, y2=1)",
                                    "3 = (y1=1, y2=0)", "4 = (y1=1, y2=1)"))
## End(Not run)
```

binom2.or
Bivariate Binary Regression with an Odds Ratio (Family Function)

## Description

Fits a Palmgren (bivariate odds-ratio model, or bivariate logistic regression) model to two binary responses. Actually, a bivariate logistic/probit/cloglog/cauchit model can be fitted. The odds ratio is used as a measure of dependency.

## Usage

binom2.or(lmu = "logitlink", lmu1 = lmu, lmu2 = lmu, loratio = "loglink", imu1 = NULL, imu2 = NULL, ioratio = NULL, zero = "oratio", exchangeable $=$ FALSE, tol $=0.001$, more. robust $=$ FALSE)

## Arguments

lmu
Link function applied to the two marginal probabilities. See Links for more choices. See the note below.
lmu1, lmu2 Link function applied to the first and second of the two marginal probabilities.
loratio Link function applied to the odds ratio. See Links for more choices.
imu1, imu2, ioratio
Optional initial values for the marginal probabilities and odds ratio. See CommonVGAMffArguments for more details. In general good initial values are often required so use these arguments if convergence failure occurs.
zero Which linear/additive predictor is modelled as an intercept only? The default is for the odds ratio. A NULL means none. See CommonVGAMfffarguments for more details.
exchangeable Logical. If TRUE, the two marginal probabilities are constrained to be equal.
tol Tolerance for testing independence. Should be some small positive numerical value.
more. robust Logical. If TRUE then some measures are taken to compute the derivatives and working weights more robustly, i.e., in an attempt to avoid numerical problems. Currently this feature is not debugged if set TRUE.

## Details

Also known informally as the Palmgren model, the bivariate logistic model is a full-likelihood based model defined as two logistic regressions plus $\log$ (oratio) $=$ eta3 where eta3 is the third linear/additive predictor relating the odds ratio to explanatory variables. Explicitly, the default model is

$$
\operatorname{logit}\left[P\left(Y_{j}=1\right)\right]=\eta_{j}, \quad j=1,2
$$

for the marginals, and

$$
\log \left[P\left(Y_{00}=1\right) P\left(Y_{11}=1\right) /\left(P\left(Y_{01}=1\right) P\left(Y_{10}=1\right)\right)\right]=\eta_{3}
$$

specifies the dependency between the two responses. Here, the responses equal 1 for a success and a 0 for a failure, and the odds ratio is often written $\psi=p_{00} p_{11} /\left(p_{10} p_{01}\right)$. The model is fitted by maximum likelihood estimation since the full likelihood is specified. The two binary responses are independent if and only if the odds ratio is unity, or equivalently, the $\log$ odds ratio is 0 . Fisher scoring is implemented.
The default models $\eta_{3}$ as a single parameter only, i.e., an intercept-only model, but this can be circumvented by setting zero $=$ NULL in order to model the odds ratio as a function of all the explanatory variables. The function binom2.or() can handle other probability link functions such as probitlink, clogloglink and cauchitlink links as well, so is quite general. In fact, the two marginal probabilities can each have a different link function. A similar model is the bivariate probit model (binom2 . rho), which is based on a standard bivariate normal distribution, but the bivariate probit model is less interpretable and flexible.
The exchangeable argument should be used when the error structure is exchangeable, e.g., with eyes or ears data.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.
When fitted, the fitted. values slot of the object contains the four joint probabilities, labelled as $\left(Y_{1}, Y_{2}\right)=(0,0),(0,1),(1,0),(1,1)$, respectively. These estimated probabilities should be extracted with the fitted generic function.

Note
At present we call binom2. or families a bivariate odds-ratio model. The response should be either a 4-column matrix of counts (whose columns correspond to $\left(Y_{1}, Y_{2}\right)=(0,0),(0,1),(1,0),(1,1)$ respectively), or a two-column matrix where each column has two distinct values, or a factor with four levels. The function rbinom2. or may be used to generate such data. Successful convergence requires at least one case of each of the four possible outcomes.
By default, a constant odds ratio is fitted because zero $=3$. Set zero = NULL if you want the odds ratio to be modelled as a function of the explanatory variables; however, numerical problems are more likely to occur.
The argument lmu, which is actually redundant, is used for convenience and for upward compatibility: specifying lmu only means the link function will be applied to lmu1 and lmu2. Users who want a different link function for each of the two marginal probabilities should use the lmu1 and lmu2 arguments, and the argument lmu is then ignored. It doesn't make sense to specify exchangeable $=$ TRUE and have different link functions for the two marginal probabilities.
Regarding Yee and Dirnbock (2009), the xij (see vglm. control) argument enables environmental variables with different values at the two time points to be entered into an exchangeable binom2.or model. See the author's webpage for sample code.

## Author(s)

Thomas W. Yee

## References

McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, 2nd ed. London: Chapman \& Hall.
le Cessie, S. and van Houwelingen, J. C. (1994). Logistic regression for correlated binary data. Applied Statistics, 43, 95-108.
Palmgren, J. (1989). Regression Models for Bivariate Binary Responses. Technical Report no. 101, Department of Biostatistics, University of Washington, Seattle.
Yee, T. W. and Dirnbock, T. (2009). Models for analysing species' presence/absence data at two time points. Journal of Theoretical Biology, 259(4), 684-694.

## See Also

rbinom2.or, binom2.rho, loglinb2, zipebcom, coalminers, binomialff, logitlink, probitlink, clogloglink, cauchitlink.

## Examples

```
# Fit the model in Table 6.7 in McCullagh and Nelder (1989)
coalminers <- transform(coalminers, Age = (age - 42) / 5)
fit <- vglm(cbind(nBnW, nBW, BnW, BW) ~ Age,
    binom2.or(zero = NULL), data = coalminers)
fitted(fit)
summary(fit)
coef(fit, matrix = TRUE)
c(weights(fit, type = "prior")) * fitted(fit) # Table 6.8
```

```
## Not run: with(coalminers, matplot(Age, fitted(fit), type = "l", las = 1,
    xlab = "(age - 42) / 5", lwd = 2))
with(coalminers, matpoints(Age, depvar(fit), col=1:4))
legend(x = -4, y = 0.5, lty = 1:4, col = 1:4, lwd = 2,
    legend = c("1 = (Breathlessness=0, Wheeze=0)",
        "2 = (Breathlessness=0, Wheeze=1)",
        "3 = (Breathlessness=1, Wheeze=0)",
        "4 = (Breathlessness=1, Wheeze=1)"))
## End(Not run)
# Another model: pet ownership
## Not run: data(xs.nz, package = "VGAMdata")
# More homogeneous:
petdata <- subset(xs.nz, ethnicity == "European" & age < 70 & sex == "M")
petdata <- na.omit(petdata[, c("cat", "dog", "age")])
summary(petdata)
with(petdata, table(cat, dog)) # Can compute the odds ratio
fit <- vgam(cbind((1-cat) * (1-dog), (1-cat) * dog,
                        cat * (1-dog), cat * dog) ~ s(age, df = 5),
    binom2.or(zero = 3), data = petdata, trace = TRUE)
colSums(depvar(fit))
coef(fit, matrix = TRUE)
## End(Not run)
## Not run: # Plot the estimated probabilities
000 <- order(with(petdata, age))
matplot(with(petdata, age)[ooo], fitted(fit)[ooo, ], type = "l",
    xlab = "Age", ylab = "Probability", main = "Pet ownership",
    ylim = c(0, max(fitted(fit))), las = 1, lwd = 1.5)
legend("topleft", col=1:4, lty = 1:4, leg = c("no cat or dog ",
    "dog only", "cat only", "cat and dog"), lwd = 1.5)
## End(Not run)
```

Binom2.rho

## Bivariate Probit Model

## Description

Density and random generation for a bivariate probit model. The correlation parameter rho is the measure of dependency.

## Usage

rbinom2.rho(n, mu1,
mu2 = if (exchangeable) mu1 else stop("argument 'mu2' not specified"),
rho $=0$, exchangeable $=$ FALSE, twoCols $=$ TRUE,

```
    colnames = if (twoCols) c("y1","y2") else c("00", "01", "10", "11"),
    ErrorCheck = TRUE)
dbinom2.rho(mu1,
    mu2 = if (exchangeable) mu1 else stop("'mu2' not specified"),
    rho = 0, exchangeable = FALSE,
    colnames = c("00", "01", "10", "11"), ErrorCheck = TRUE)
```


## Arguments

n
mu1, mu2 The marginal probabilities. Only mu1 is needed if exchangeable = TRUE. Values should be between 0 and 1 .
rho The correlation parameter. Must be numeric and lie between -1 and 1. The default value of zero means the responses are uncorrelated.
exchangeable Logical. If TRUE, the two marginal probabilities are constrained to be equal.
twoCols Logical. If TRUE, then a $n \times 2$ matrix of 1 s and 0 s is returned. If FALSE, then a $n \times 4$ matrix of 1 s and 0 s is returned.
colnames The dimnames argument of matrix is assigned list(NULL, colnames).
ErrorCheck Logical. Do some error checking of the input parameters?

## Details

The function rbinom2. rho generates data coming from a bivariate probit model. The data might be fitted with the VGAM family function binom2. rho.

The function dbinom2. rho does not really compute the density (because that does not make sense here) but rather returns the four joint probabilities.

## Value

The function rbinom2. rho returns either a 2 or 4 column matrix of 1 s and 0 s, depending on the argument twoCols.

The function dbinom2. rho returns a 4 column matrix of joint probabilities; each row adds up to unity.

## Author(s)

T. W. Yee

## See Also

binom2.rho.

## Examples

```
(myrho <- rhobitlink(2, inverse = TRUE)) # Example 1
ymat <- rbinom2.rho(nn <- 2000, mu1 = 0.8, rho = myrho, exch = TRUE)
(mytab <- table(ymat[, 1], ymat[, 2], dnn = c("Y1", "Y2")))
fit <- vglm(ymat ~ 1, binom2.rho(exch = TRUE))
coef(fit, matrix = TRUE)
bdata <- data.frame(x2 = sort(runif(nn))) # Example 2
bdata <- transform(bdata, mu1 = probitlink(-2+4*x2, inverse = TRUE),
                                    mu2 = probitlink( }-1+3*x2, inverse = TRUE))
dmat <- with(bdata, dbinom2.rho(mu1, mu2, myrho))
ymat <- with(bdata, rbinom2.rho(nn, mu1, mu2, myrho))
fit2 <- vglm(ymat ~ x2, binom2.rho, data = bdata)
coef(fit2, matrix = TRUE)
## Not run: matplot(with(bdata, x2), dmat, lty = 1:4, col = 1:4,
            type = "l", main = "Joint probabilities",
    ylim = 0:1, lwd = 2, ylab = "Probability")
legend(x = 0.25, y = 0.9, lty = 1:4, col = 1:4, lwd = 2,
        legend = c("1 = (y1=0, y2=0)", "2 = (y1=0, y2=1)",
                            "3 = (y1=1, y2=0)", "4 = (y1=1, y2=1)"))
## End(Not run)
```

binom2.rho
Bivariate Probit Model (Family Function)

## Description

Fits a bivariate probit model to two binary responses. The correlation parameter rho is the measure of dependency.

## Usage

binom2.rho(lmu = "probitlink", lrho = "rhobitlink", imu1 = NULL, imu2 = NULL, irho $=$ NULL, imethod $=1$, zero = "rho", exchangeable $=$ FALSE, grho $=\operatorname{seq}(-0.95,0.95$, by $=0.05)$, nsimEIM = NULL)
binom2.Rho(rho = 0, imu1 = NULL, imu2 = NULL,
exchangeable $=$ FALSE, nsimEIM $=$ NULL)

## Arguments

lmu Link function applied to the marginal probabilities. Should be left alone.
lrho Link function applied to the $\rho$ association parameter. See Links for more choices.
imu1, imu2 Optional initial values for the two marginal probabilities. May be a vector.
irho Optional initial value for $\rho$. If given, this should lie between -1 and 1 . See below for more comments.

$$
\begin{aligned}
& \text { zero } \begin{array}{l}
\text { Specifies which linear/additive predictors are modelled as intercept-only. A } \\
\text { NULL means none. Numerically, the } \rho \text { parameter is easiest modelled as an in- } \\
\text { tercept only, hence the default. See CommonVGAMffArguments for more infor- } \\
\text { mation. }
\end{array} \\
& \text { exchangeable } \quad \begin{array}{l}
\text { Logical. If TRUE, the two marginal probabilities are constrained to be equal. } \\
\text { imethod, nsimEIM, grho } \\
\text { See CommonVGAMffArguments for more information. A value of at least } 100 \text { for } \\
\text { nsimEIM is recommended; the larger the value the better. }
\end{array} \\
& \text { rho } \quad \begin{array}{l}
\text { Numeric vector. Values are recycled to the needed length, and ought to be in } \\
\text { range, which is }(-1,1) .
\end{array}
\end{aligned}
$$

## Details

The bivariate probit model was one of the earliest regression models to handle two binary responses jointly. It has a probit link for each of the two marginal probabilities, and models the association between the responses by the $\rho$ parameter of a standard bivariate normal distribution (with zero means and unit variances). One can think of the joint probabilities being $\Phi\left(\eta_{1}, \eta_{2} ; \rho\right)$ where $\Phi$ is the cumulative distribution function of a standard bivariate normal distribution.

Explicitly, the default model is

$$
\operatorname{probit}\left[P\left(Y_{j}=1\right)\right]=\eta_{j}, \quad j=1,2
$$

for the marginals, and

$$
\text { rhobit }[r h o]=\eta_{3} .
$$

The joint probability $P\left(Y_{1}=1, Y_{2}=1\right)=\Phi\left(\eta_{1}, \eta_{2} ; \rho\right)$, and from these the other three joint probabilities are easily computed. The model is fitted by maximum likelihood estimation since the full likelihood is specified. Fisher scoring is implemented.

The default models $\eta_{3}$ as a single parameter only, i.e., an intercept-only model for rho, but this can be circumvented by setting zero $=$ NULL in order to model rho as a function of all the explanatory variables.
The bivariate probit model should not be confused with a bivariate logit model with a probit link (see binom2.or). The latter uses the odds ratio to quantify the association. Actually, the bivariate logit model is recommended over the bivariate probit model because the odds ratio is a more natural way of measuring the association between two binary responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

When fitted, the fitted.values slot of the object contains the four joint probabilities, labelled as $\left(Y_{1}, Y_{2}\right)=(0,0),(0,1),(1,0),(1,1)$, respectively.

## Note

See binom2. or about the form of input the response should have.

By default, a constant $\rho$ is fitted because zero $=$ "rho". Set zero $=$ NULL if you want the $\rho$ parameter to be modelled as a function of the explanatory variables. The value $\rho$ lies in the interval $(-1,1)$, therefore a rhobitlink link is default.

Converge problems can occur. If so, assign irho a range of values and monitor convergence (e.g., set trace $=$ TRUE). Else try imethod. Practical experience shows that local solutions can occur, and that irho needs to be quite close to the (global) solution. Also, imu1 and imu2 may be used.

This help file is mainly about binom2.rho(). binom2.Rho() fits a bivariate probit model with known $\rho$. The inputted rho is saved in the misc slot of the fitted object, with rho as the component name.

In some econometrics applications (e.g., Freedman 2010, Freedman and Sekhon 2010) one response is used as an explanatory variable, e.g., a recursive binomial probit model. Such will not work here. Historically, the bivariate probit model was the first VGAM I ever wrote, based on Ashford and Sowden (1970). I don't think they ever thought of it either! Hence the criticisms raised go beyond the use of what was originally intended.

## Author(s)

Thomas W. Yee

## References

Ashford, J. R. and Sowden, R. R. (1970). Multi-variate probit analysis. Biometrics, 26, 535-546.
Freedman, D. A. (2010). Statistical Models and Causal Inference: a Dialogue with the Social Sciences, Cambridge: Cambridge University Press.

Freedman, D. A. and Sekhon, J. S. (2010). Endogeneity in probit response models. Political Analysis, 18, 138-150.

## See Also

```
rbinom2.rho,rhobitlink, pbinorm, binom2.or, loglinb2, coalminers, binomialff, rhobitlink,
fisherzlink.
```


## Examples

```
coalminers <- transform(coalminers, Age = (age - 42) / 5)
fit <- vglm(cbind(nBnW, nBW, BnW, BW) ~ Age,
    binom2.rho, data = coalminers, trace = TRUE)
summary(fit)
coef(fit, matrix = TRUE)
```


## Description

Family function for fitting generalized linear models to binomial responses

## Usage

binomialff(link = "logitlink", multiple.responses = FALSE, parallel $=$ FALSE, zero $=$ NULL, bred $=$ FALSE, earg.link $=$ FALSE)

## Arguments

link Link function; see Links and CommonVGAMffArguments for more information.
multiple.responses
Multivariate response? If TRUE, then the response is interpreted as $M$ independent binary responses, where $M$ is the number of columns of the response matrix. In this case, the response matrix should have $Q$ columns consisting of counts (successes), and the weights argument should have $Q$ columns consisting of the number of trials (successes plus failures).
If FALSE and the response is a (2-column) matrix, then the number of successes is given in the first column, and the second column is the number of failures.
parallel A logical or formula. Used only if multiple. responses is TRUE. This argument allows for the parallelism assumption whereby the regression coefficients for a variable is constrained to be equal over the $M$ linear/additive predictors. If parallel = TRUE then the constraint is not applied to the intercepts.
zero An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set $\{1,2, \ldots, M\}$, where $M$ is the number of columns of the matrix response. See CommonVGAMffArguments for more information.
earg.link Details at CommonVGAMffArguments.
bred Details at CommonVGAMffArguments. Setting bred = TRUE should work for multiple responses (multiple. responses = TRUE) and all VGAM link functions; it has been tested for logitlink only (and it gives similar results to brglm but not identical), and further testing is required. One result from fitting bias reduced binary regression is that finite regression coefficients occur when the data is separable (see example below). Currently hdeff.vglm does not work when bred = TRUE.

## Details

This function is largely to mimic binomial, however there are some differences.
When used with cqo and cao, it may be preferable to use the clogloglink link.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, vgam, rrvglm, cqo, and cao.

## Warning

See the above note regarding bred.
The maximum likelihood estimate will not exist if the data is completely separable or quasicompletely separable. See Chapter 10 of Altman et al. (2004) for more details, and safeBinaryRegression and hdeff.vglm. Yet to do: add a sepcheck = TRUE, say, argument to further detect this problem and give an appropriate warning.

## Note

If multiple.responses is FALSE (default) then the response can be of one of two formats: a factor (first level taken as failure), or a 2-column matrix (first column = successes) of counts. The argument weights in the modelling function can also be specified as any vector of positive values. In general, 1 means success and 0 means failure (to check, see the $y$ slot of the fitted object). Note that a general vector of proportions of success is no longer accepted.

The notation $M$ is used to denote the number of linear/additive predictors.
If multiple. responses is TRUE, then the matrix response can only be of one format: a matrix of 1 's and 0 's ( $1=$ success ).

Fisher scoring is used. This can sometimes fail to converge by oscillating between successive iterations (Ridout, 1990). See the example below.

## Author(s)

Thomas W. Yee

## References

McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, 2nd ed. London: Chapman \& Hall.

Altman, M. and Gill, J. and McDonald, M. P. (2004). Numerical Issues in Statistical Computing for the Social Scientist, Hoboken, NJ, USA: Wiley-Interscience.

Ridout, M. S. (1990). Non-convergence of Fisher's method of scoring-a simple example. GLIM Newsletter, 20(6).

## See Also

hdeff.vglm, Links, rrvglm, cqo, cao, betabinomial, gtbinomial, posbinomial, zibinomial, double.expbinomial, seq2binomial, amlbinomial, simplex, binomial, simulate.vlm, safeBinaryRegression, residualsvglm.

## Examples

```
shunua <- hunua[sort.list(with(hunua, altitude)), ] # Sort by altitude
fit <- vglm(agaaus ~ poly(altitude, 2), binomialff(link = clogloglink),
    data = shunua)
## Not run:
plot(agaaus ~ jitter(altitude), shunua, ylab = "Pr(Agaaus = 1)",
    main = "Presence/absence of Agathis australis", col = 4, las = 1)
with(shunua, lines(altitude, fitted(fit), col = "orange", lwd = 2))
## End(Not run)
# Fit two species simultaneously
fit2 <- vgam(cbind(agaaus, kniexc) ~ s(altitude),
        binomialff(multiple.responses = TRUE), data = shunua)
## Not run:
with(shunua, matplot(altitude, fitted(fit2), type = "l",
    main = "Two species response curves", las = 1))
## End(Not run)
# Shows that Fisher scoring can sometime fail. See Ridout (1990).
ridout <- data.frame(v = c(1000, 100, 10), r = c(4, 3, 3), n = rep(5, 3))
(ridout <- transform(ridout, logv = log(v)))
# The iterations oscillates between two local solutions:
glm.fail <- glm(r / n ~ offset(logv) + 1, weight = n,
    binomial(link = 'cloglog'), ridout, trace = TRUE)
coef(glm.fail)
# vglm()'s half-stepping ensures the MLE of -5.4007 is obtained:
vglm.ok <- vglm(cbind(r, n-r) ~ offset(logv) + 1,
    binomialff(link = clogloglink), ridout, trace = TRUE)
coef(vglm.ok)
# Separable data
set.seed(123)
threshold <- 0
bdata <- data.frame(x2 = sort(rnorm(nn <- 100)))
bdata <- transform(bdata, y1 = ifelse(x2 < threshold, 0, 1))
fit <- vglm(y1 ~ x2, binomialff(bred = TRUE),
    data = bdata, criter = "coef", trace = TRUE)
coef(fit, matrix = TRUE) # Finite!!
summary(fit)
## Not run: plot(depvar(fit) ~ x2, data = bdata, col = "blue", las = 1)
lines(fitted(fit) ~ x2, data = bdata, col = "orange")
abline(v = threshold, col = "gray", lty = "dashed")
## End(Not run)
```

Binorm Bivariate Normal Distribution Cumulative Distribution Function

## Description

Density, cumulative distribution function and random generation for the bivariate normal distribution distribution.

## Usage

```
dbinorm(x1, x2, mean1 = 0, mean2 = 0, var1 = \(1, \operatorname{var} 2=1, \operatorname{cov} 12=0, \log =\) FALSE \()\)
pbinorm(q1, q2, mean1 \(=0\), mean2 \(=0, \operatorname{var} 1=1, \operatorname{var} 2=1, \operatorname{cov} 12=0\) )
\(\operatorname{rbinorm}(n, \quad \operatorname{mean} 1=0, \operatorname{mean} 2=0, \operatorname{var} 1=1, \operatorname{var} 2=1, \operatorname{cov} 12=0)\)
pnorm2 (x1, x2, mean1 = 0, mean2 = 0, var1 = 1, var2 = 1, \(\operatorname{cov} 12=0)\)
```


## Arguments

$x 1, x 2, q 1, q 2$ vector of quantiles.
mean1, mean2, var1, var2, cov12
vector of means, variances and the covariance.
n number of observations. Same as rnorm.
log Logical. If $\log =$ TRUE then the logarithm of the density is returned.

## Details

The default arguments correspond to the standard bivariate normal distribution with correlation parameter $\rho=0$. That is, two independent standard normal distributions. Let sd1 (say) be sqrt (var1) and written $\sigma_{1}$, etc. Then the general formula for the correlation coefficient is $\rho=$ $\operatorname{cov} /\left(\sigma_{1} \sigma_{2}\right)$ where $\operatorname{cov}$ is argument cov12. Thus if arguments var1 and var2 are left alone then cov12 can be inputted with $\rho$.
One can think of this function as an extension of pnorm to two dimensions, however note that the argument names have been changed for VGAM 0.9-1 onwards.

## Value

dbinorm gives the density, pbinorm gives the cumulative distribution function, rbinorm generates random deviates ( $n$ by 2 matrix).

## Warning

Being based on an approximation, the results of pbinorm() may be negative! Also, pnorm2() should be withdrawn soon; use pbinorm() instead because it is identical.

## Note

For rbinorm(), if the $i$ th variance-covariance matrix is not positive-definite then the $i$ th row is all NAs.

## References

pbinorm() is based on Donnelly (1973), the code was translated from FORTRAN to ratfor using struct, and then from ratfor to C manually. The function was originally called bivnor, and TWY only wrote a wrapper function.

Donnelly, T. G. (1973). Algorithm 462: Bivariate Normal Distribution. Communications of the ACM, 16, 638.

## See Also

pnorm, binormal, uninormal.

## Examples

```
yvec <- c(-5, -1.96, 0, 1.96, 5)
ymat <- expand.grid(yvec, yvec)
cbind(ymat, pbinorm(ymat[, 1], ymat[, 2]))
## Not run: rhovec <- seq(-0.95, 0.95, by = 0.01)
plot(rhovec, pbinorm(0, 0, cov12 = rhovec), type = "l", col = "blue", las = 1)
abline(v = 0, h = 0.25, col = "gray", lty = "dashed")
## End(Not run)
```

binormal
Bivariate Normal Distribution Family Function

## Description

Maximum likelihood estimation of the five parameters of a bivariate normal distribution.

## Usage

```
binormal(lmean1 = "identitylink", lmean2 = "identitylink",
    lsd1 = "loglink", lsd2 = "loglink",
    lrho = "rhobitlink",
        imean1 = NULL, imean2 = NULL,
        isd1 = NULL, isd2 = NULL,
        irho = NULL, imethod = 1,
        eq.mean = FALSE, eq.sd = FALSE,
        zero = c("sd", "rho"))
```


## Arguments

lmean1, lmean2, lsd1, lsd2, lrho
Link functions applied to the means, standard deviations and rho parameters. See Links for more choices. Being positive quantities, a log link is the default for the standard deviations.
imean1, imean2, isd1, isd2, irho, imethod, zero
See CommonVGAMfffarguments for more information.
eq.mean, eq.sd Logical or formula. Constrains the means or the standard deviations to be equal.

## Details

For the bivariate normal distribution, this fits a linear model (LM) to the means, and by default, the other parameters are intercept-only. The response should be a two-column matrix. The correlation parameter is rho, which lies between -1 and 1 (thus the rhobitlink link is a reasonable choice). The fitted means are returned as the fitted values, which is in the form of a two-column matrix. Fisher scoring is implemented.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

This function may be renamed to normal2() or something like that at a later date.

## Note

If both equal means and equal standard deviations are desired then use something like constraints $=\operatorname{list}("($ Intercept $) "=\operatorname{matrix}(c(1,1,0,0,0,0,0,1,1,0,0,0,0,0,1), 5,3))$ and maybe zero = NULL etc.

## Author(s)

## T. W. Yee

## See Also

uninormal, trinormal, pbinorm, bistudentt.

## Examples

```
set.seed(123); nn <- 1000
bdata <- data.frame(x2 = runif(nn), x3 = runif(nn))
bdata <- transform(bdata, y1 = rnorm(nn, 1 + 2 * x2),
    y2 = rnorm(nn, 3 + 4 * x2))
fit1 <- vglm(cbind(y1, y2) ~ x2,
    binormal(eq.sd = TRUE), data = bdata, trace = TRUE)
coef(fit1, matrix = TRUE)
constraints(fit1)
summary(fit1)
# Estimated P(Y1 <= y1, Y2 <= y2) under the fitted model
var1 <- loglink(2 * predict(fit1)[, "loglink(sd1)"], inverse = TRUE)
var2 <- loglink(2 * predict(fit1)[, "loglink(sd2)"], inverse = TRUE)
cov12 <- rhobitlink(predict(fit1)[, "rhobitlink(rho)"], inverse = TRUE)
head(with(bdata, pbinorm(y1, y2,
    mean1 = predict(fit1)[, "mean1"],
    mean2 = predict(fit1)[, "mean2"],
    var1 = var1, var2 = var2, cov12 = cov12)))
```


## Description

Estimate the correlation parameter of the (bivariate) Gaussian copula distribution by maximum likelihood estimation.

## Usage

binormalcop(lrho = "rhobitlink", irho = NULL, imethod = 1, parallel = FALSE, zero = NULL)

## Arguments

lrho, irho, imethod
Details at CommonVGAMffArguments. See Links for more link function choices.
parallel, zero Details at CommonVGAMffArguments. If parallel = TRUE then the constraint is applied to the intercept too.

## Details

The cumulative distribution function is

$$
P\left(Y_{1} \leq y_{1}, Y_{2} \leq y_{2}\right)=\Phi_{2}\left(\Phi^{-1}\left(y_{1}\right), \Phi^{-1}\left(y_{2}\right) ; \rho\right)
$$

for $-1<\rho<1, \Phi_{2}$ is the cumulative distribution function of a standard bivariate normal (see pbinorm), and $\Phi$ is the cumulative distribution function of a standard univariate normal (see pnorm).
The support of the function is the interior of the unit square; however, values of 0 and/or 1 are not allowed. The marginal distributions are the standard uniform distributions. When $\rho=0$ the random variables are independent.
This VGAM family function can handle multiple responses, for example, a six-column matrix where the first 2 columns is the first out of three responses, the next 2 columns being the next response, etc.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

The response matrix must have a multiple of two-columns. Currently, the fitted value is a matrix with the same number of columns and values equal to 0.5 . This is because each marginal distribution corresponds to a standard uniform distribution.
This VGAM family function is fragile; each response must be in the interior of the unit square. Setting crit = "coef" is sometimes a good idea because inaccuracies in pbinorm might mean unnecessary half-stepping will occur near the solution.

## Author(s)

T. W. Yee

## References

Schepsmeier, U. and Stober, J. (2013). Derivatives and Fisher information of bivariate copulas. Statistical Papers.

## See Also

rbinormcop, pnorm, kendall.tau.

## Examples

```
    nn <- 1000
    ymat <- rbinormcop(n = nn, rho = rhobitlink(-0.9, inverse = TRUE))
    bdata <- data.frame(y1 = ymat[, 1],
        y2 = ymat[, 2],
        y3 = ymat[, 1],
        y4 = ymat[, 2],
        x2 = runif(nn))
    summary (bdata)
    ## Not run: plot(ymat, col = "blue")
    fit1 <- vglm(cbind(y1, y2, y3, y4) ~ 1, # 2 responses, e.g., (y1,y2) is the first
        fam = binormalcop,
        crit = "coef", # Sometimes a good idea
        data = bdata, trace = TRUE)
    coef(fit1, matrix = TRUE)
    Coef(fit1)
    head(fitted(fit1))
    summary(fit1)
    # Another example; rho is a linear function of x2
    bdata <- transform(bdata, rho = -0.5 + x2)
    ymat <- rbinormcop(n = nn, rho = with(bdata, rho))
    bdata <- transform(bdata, y5 = ymat[, 1],
        y6 = ymat[, 2])
    fit2 <- vgam(cbind(y5, y6) ~ s(x2), data = bdata,
        binormalcop(lrho = "identitylink"), trace = TRUE)
    ## Not run: plot(fit2, lcol = "blue", scol = "orange", se = TRUE, las = 1)
```

    Binormcop Gaussian Copula (Bivariate) Distribution
    
## Description

Density, distribution function, and random generation for the (one parameter) bivariate Gaussian copula distribution.

## Usage

```
dbinormcop(x1, x2, rho = 0, log = FALSE)
pbinormcop(q1, q2, rho = 0)
rbinormcop(n, rho = 0)
```


## Arguments

$x 1, x 2, q 1, q 2 \quad$ vector of quantiles. The $x 1$ and $x 2$ should be in the interval $(0,1)$. Ditto for $q 1$ and q2.
n number of observations. Same as rnorm.
rho the correlation parameter. Should be in the interval $(-1,1)$.
log Logical. If TRUE then the logarithm is returned.

## Details

See binormalcop, the VGAM family functions for estimating the parameter by maximum likelihood estimation, for the formula of the cumulative distribution function and other details.

## Value

dbinormcop gives the density, pbinormcop gives the distribution function, and rbinormcop generates random deviates (a two-column matrix).

## Note

Yettodo: allow $\times 1$ and/or $\times 2$ to have values 1 , and to allow any values for $\times 1$ and/or $\times 2$ to be outside the unit square.

## Author(s)

T. W. Yee

## See Also

binormalcop, binormal.

## Examples

```
## Not run: edge <- 0.01 # A small positive value
N <- 101; x <- seq(edge, 1.0 - edge, len = N); Rho <- 0.7
ox <- expand.grid(x, x)
zedd <- dbinormcop(ox[, 1], ox[, 2], rho = Rho, log = TRUE)
contour(x, x, matrix(zedd, N, N), col = "blue", labcex = 1.5)
zedd <- pbinormcop(ox[, 1], ox[, 2], rho = Rho)
contour(x, x, matrix(zedd, N, N), col = "blue", labcex = 1.5)
## End(Not run)
```

Biplackett Plackett's Bivariate Copula

## Description

Density, distribution function, and random generation for the (one parameter) bivariate Plackett copula.

## Usage

dbiplackcop(x1, x2, oratio, log = FALSE)
pbiplackcop(q1, q2, oratio)
rbiplackcop(n, oratio)

## Arguments

$x 1, x 2, q 1, q 2 \quad$ vector of quantiles.
$n \quad$ number of observations. Same as in runif.
oratio the positive odds ratio $\psi$.
$\log \quad$ Logical. If TRUE then the logarithm is returned.

## Details

See biplackettcop, the VGAM family functions for estimating the parameter by maximum likelihood estimation, for the formula of the cumulative distribution function and other details.

## Value

dbiplackcop gives the density, pbiplackcop gives the distribution function, and rbiplackcop generates random deviates (a two-column matrix).

## Author(s)

T. W. Yee

## References

Mardia, K. V. (1967). Some contributions to contingency-type distributions. Biometrika, 54, 235249.

## See Also

biplackettcop, bifrankcop.

## Examples

```
## Not run: N <- 101; oratio <- exp(1)
x <- seq(0.0, 1.0, len = N)
ox <- expand.grid(x, x)
zedd <- dbiplackcop(ox[, 1], ox[, 2], oratio = oratio)
contour(x, x, matrix(zedd, N, N), col = "blue")
zedd <- pbiplackcop(ox[, 1], ox[, 2], oratio = oratio)
contour(x, x, matrix(zedd, N, N), col = "blue")
plot(rr <- rbiplackcop(n = 3000, oratio = oratio))
par(mfrow = c(1, 2))
hist(rr[, 1]) # Should be uniform
hist(rr[, 2]) # Should be uniform
## End(Not run)
```

biplackettcop Plackett's Bivariate Copula Family Function

## Description

Estimate the association parameter of Plackett's bivariate distribution (copula) by maximum likelihood estimation.

## Usage

biplackettcop(link = "loglink", ioratio = NULL, imethod = 1, nsimEIM = 200)

## Arguments

link Link function applied to the (positive) odds ratio $\psi$. See Links for more choices and information.
ioratio Numeric. Optional initial value for $\psi$. If a convergence failure occurs try assigning a value or a different value.
imethod, nsimEIM
See CommonVGAMffArguments.

## Details

The defining equation is

$$
\psi=H \times\left(1-y_{1}-y_{2}+H\right) /\left(\left(y_{1}-H\right) \times\left(y_{2}-H\right)\right)
$$

where $P\left(Y_{1} \leq y_{1}, Y_{2} \leq y_{2}\right)=H_{\psi}\left(y_{1}, y_{2}\right)$ is the cumulative distribution function. The density function is $h_{\psi}\left(y_{1}, y_{2}\right)=$

$$
\psi\left[1+(\psi-1)\left(y_{1}+y_{2}-2 y_{1} y_{2}\right)\right] /\left(\left[1+(\psi-1)\left(y_{1}+y_{2}\right)\right]^{2}-4 \psi(\psi-1) y_{1} y_{2}\right)^{3 / 2}
$$

for $\psi>0$. Some writers call $\psi$ the cross product ratio but it is called the odds ratio here. The support of the function is the unit square. The marginal distributions here are the standard uniform although it is commonly generalized to other distributions.

If $\psi=1$ then $h_{\psi}\left(y_{1}, y_{2}\right)=y_{1} y_{2}$, i.e., independence. As the odds ratio tends to infinity one has $y_{1}=y_{2}$. As the odds ratio tends to 0 one has $y_{2}=1-y_{1}$.

Fisher scoring is implemented using rbiplackcop. Convergence is often quite slow.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

The response must be a two-column matrix. Currently, the fitted value is a 2-column matrix with 0.5 values because the marginal distributions correspond to a standard uniform distribution.

## Author(s)

T. W. Yee

## References

Plackett, R. L. (1965). A class of bivariate distributions. Journal of the American Statistical Association, 60, 516-522.

## See Also

```
rbiplackcop, bifrankcop.
```


## Examples

```
## Not run:
ymat <- rbiplackcop(n = 2000, oratio = exp(2))
plot(ymat, col = "blue")
fit <- vglm(ymat ~ 1, fam = biplackettcop, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
vcov(fit)
head(fitted(fit))
summary(fit)
## End(Not run)
```

biplot-methods Biplot of Constrained Regression Models

## Description

biplot is a generic function applied to RR-VGLMs and QRR-VGLMs etc. These apply to rank1 and rank-2 models of these only. For RR-VGLMs these plot the second latent variable scores against the first latent variable scores.

## Methods

$\mathbf{x}$ The object from which the latent variables are extracted and/or plotted.

## Note

See lvplot which is very much related to biplots.
Bisa The Birnbaum-Saunders Distribution

## Description

Density, distribution function, and random generation for the Birnbaum-Saunders distribution.

## Usage

```
dbisa(x, scale = 1, shape, log = FALSE)
pbisa(q, scale = 1, shape, lower.tail = TRUE, log.p = FALSE)
qbisa(p, scale = 1, shape, lower.tail = TRUE, log.p = FALSE)
rbisa(n, scale = 1, shape)
```


## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$\mathrm{n} \quad$ Same as in runif.
scale, shape the (positive) scale and shape parameters.
log Logical. If TRUE then the logarithm of the density is returned.
lower.tail, log.p
Same meaning as in pnorm or qnorm.

## Details

The Birnbaum-Saunders distribution is a distribution which is used in survival analysis. See bisa, the VGAM family function for estimating the parameters, for more details.

## Value

dbisa gives the density, pbisa gives the distribution function, and qbisa gives the quantile function, and rbisa generates random deviates.

## Author(s)

T. W. Yee and Kai Huang

## See Also

bisa.

## Examples

```
## Not run:
x <- seq(0, 6, len = 400)
plot(x, dbisa(x, shape = 1), type = "l", col = "blue",
    ylab = "Density", lwd = 2, ylim = c(0,1.3), lty = 3,
    main = "X ~ Birnbaum-Saunders(shape, scale = 1)")
lines(x, dbisa(x, shape = 2), col = "orange", lty = 2, lwd = 2)
lines(x, dbisa(x, shape = 0.5), col = "green", lty = 1, lwd = 2)
legend(x = 3, y = 0.9, legend = paste("shape = ",c(0.5, 1,2)),
        col = c("green","blue","orange"), lty = 1:3, lwd = 2)
shape <- 1; x <- seq(0.0, 4, len = 401)
plot(x, dbisa(x, shape = shape), type = "l", col = "blue", las = 1, ylab = "",
    main = "Blue is density, orange is cumulative distribution function",
    sub = "Purple lines are the 10,20,...,90 percentiles", ylim = 0:1)
abline(h = 0, col = "blue", lty = 2)
lines(x, pbisa(x, shape = shape), col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qbisa(probs, shape = shape)
lines(Q, dbisa(Q, shape = shape), col = "purple", lty = 3, type = "h")
pbisa(Q, shape = shape) - probs # Should be all zero
abline(h = probs, col = "purple", lty = 3)
lines(Q, pbisa(Q, shape = shape), col = "purple", lty = 3, type = "h")
## End(Not run)
```

bisa

Birnbaum-Saunders Distribution Family Function

## Description

Estimates the shape and scale parameters of the Birnbaum-Saunders distribution by maximum likelihood estimation.

## Usage

bisa(lscale = "loglink", lshape = "loglink", iscale = 1, ishape = NULL, imethod = 1, zero = "shape", nowarning = FALSE)

## Arguments

| nowarning | Logical. Suppress a warning? Ignored for VGAM 0.9-7 and higher. |
| :--- | :--- |
| lscale, lshape | Parameter link functions applied to the shape and scale parameters ( $a$ and $b$ <br> below). See Links for more choices. A log link is the default for both because <br> they are positive. |
| iscale, ishape | Initial values for $a$ and $b$. A NULL means an initial value is chosen internally <br> using imethod. |
| imethod | An integer with value 1 or 2 or 3 which specifies the initialization method. If <br> failure to converge occurs try the other value, or else specify a value for ishape <br> and/or iscale. |
| zero | Specifies which linear/additive predictor is modelled as intercept-only. If used, <br> choose one value from the set $\{1,2\} . ~ S e e ~ C o m m o n V G A M f f A r g u m e n t s ~ f o r ~ m o r e ~$ <br> details. |

## Details

The (two-parameter) Birnbaum-Saunders distribution has a cumulative distribution function that can be written as

$$
F(y ; a, b)=\Phi[\xi(y / b) / a]
$$

where $\Phi(\cdot)$ is the cumulative distribution function of a standard normal (see pnorm), $\xi(t)=\sqrt{t}-$ $1 / \sqrt{t}, y>0, a>0$ is the shape parameter, $b>0$ is the scale parameter. The mean of $Y$ (which is the fitted value) is $b\left(1+a^{2} / 2\right)$. and the variance is $a^{2} b^{2}\left(1+\frac{5}{4} a^{2}\right)$. By default, $\eta_{1}=\log (a)$ and $\eta_{2}=\log (b)$ for this family function.
Note that $a$ and $b$ are orthogonal, i.e., the Fisher information matrix is diagonal. This family function implements Fisher scoring, and it is unnecessary to compute any integrals numerically.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Author(s)

T. W. Yee

## References

Lemonte, A. J. and Cribari-Neto, F. and Vasconcellos, K. L. P. (2007). Improved statistical inference for the two-parameter Birnbaum-Saunders distribution. Computational Statistics $\backslash \&$ Data Analysis, 51, 4656-4681.
Birnbaum, Z. W. and Saunders, S. C. (1969). A new family of life distributions. Journal of Applied Probability, 6, 319-327.

Birnbaum, Z. W. and Saunders, S. C. (1969). Estimation for a family of life distributions with applications to fatigue. Journal of Applied Probability, 6, 328-347.
Engelhardt, M. and Bain, L. J. and Wright, F. T. (1981). Inferences on the parameters of the Birnbaum-Saunders fatigue life distribution based on maximum likelihood estimation. Technometrics, 23, 251-256.
Johnson, N. L. and Kotz, S. and Balakrishnan, N. (1995). Continuous Univariate Distributions, 2nd edition, Volume 2, New York: Wiley.

## See Also

pbisa, inv.gaussianff, CommonVGAMffArguments.

## Examples

```
bdata1 <- data.frame(x2 = runif(nn <- 1000))
bdata1 <- transform(bdata1, shape = exp(-0.5 + x2), scale = exp(1.5))
bdata1 <- transform(bdata1, y = rbisa(nn, scale, shape))
fit1 <- vglm(y ~ x2, bisa(zero = 1), data = bdata1, trace = TRUE)
coef(fit1, matrix = TRUE)
## Not run:
bdata2 <- data.frame(shape = exp(-0.5), scale = exp(0.5))
bdata2 <- transform(bdata2, y = rbisa(nn, scale, shape))
fit <- vglm(y ~ 1, bisa, data = bdata2, trace = TRUE)
with(bdata2, hist(y, prob = TRUE, ylim = c(0, 0.5), col = "lightblue"))
coef(fit, matrix = TRUE)
with(bdata2, mean(y))
head(fitted(fit))
x <- with(bdata2, seq(0, max(y), len = 200))
lines(dbisa(x, Coef(fit)[1], Coef(fit)[2]) ~ x, data = bdata2,
    col = "orange", lwd = 2)
## End(Not run)
```

Bistudentt Bivariate Student-t Distribution Cumulative Distribution Function

## Description

Density for the bivariate Student-t distribution distribution.

## Usage

dbistudentt(x1, x2, df, rho = 0, log = FALSE)

## Arguments

$x 1, x 2 \quad$ vector of quantiles.
df, rho vector of degrees of freedom and correlation parameter. For df, a value Inf is currently not working.
log
Logical. If log = TRUE then the logarithm of the density is returned.

## Details

One can think of this function as an extension of $d t$ to two dimensions. See bistudentt for more information.

## Value

dbistudentt gives the density.

## References

Schepsmeier, U. and Stober, J. (2013). Derivatives and Fisher information of bivariate copulas. Statistical Papers.

## See Also

bistudentt, dt.

## Examples

```
## Not run: N <- 101; x <- seq(-4, 4, len = N); Rho <- 0.7; mydf <- 10
ox <- expand.grid(x, x)
zedd <- dbistudentt(ox[, 1], ox[, 2], df = mydf, rho = Rho, log = TRUE)
contour(x, x, matrix(zedd, N, N), col = "blue", labcex = 1.5)
## End(Not run)
```

bistudentt Bivariate Student-t Family Function

## Description

Estimate the degrees of freedom and correlation parameters of the (bivariate) Student-t distribution by maximum likelihood estimation.

## Usage

bistudentt(ldf = "logloglink", lrho = "rhobitlink",
idf $=$ NULL, irho $=$ NULL, imethod $=1$,
parallel = FALSE, zero = "rho")

## Arguments

ldf, lrho, idf, irho, imethod
Details at CommonVGAMffArguments. See Links for more link function choices.
parallel, zero Details at CommonVGAMffArguments.

## Details

The density function is

$$
f\left(y_{1}, y_{2} ; \nu, \rho\right)=\frac{1}{2 \pi \sqrt{1-\rho^{2}}}\left(1+y_{1}^{2}+y_{2}^{2}-2 \rho y_{1} y_{2}\right) /\left(\nu\left(1-\rho^{2}\right)\right)^{(\nu+2) / 2}
$$

for $-1<\rho<1$, and real $y_{1}$ and $y_{2}$.
This VGAM family function can handle multiple responses, for example, a six-column matrix where the first 2 columns is the first out of three responses, the next 2 columns being the next response, etc.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Warning

The working weight matrices have not been fully checked.

## Note

The response matrix must have a multiple of two-columns. Currently, the fitted value is a matrix with the same number of columns and values equal to 0.0 .

## Author(s)

T. W. Yee, with help from Thibault Vatter.

## References

Schepsmeier, U. and Stober, J. (2013). Derivatives and Fisher information of bivariate copulas. Statistical Papers.

## See Also

dbistudentt, binormal, pt.

## Examples

```
nn <- 1000
mydof <- logloglink(1, inverse = TRUE)
ymat <- cbind(rt(nn, df = mydof), rt(nn, df = mydof))
bdata <- data.frame(y1 = ymat[, 1], y2 = ymat[, 2],
                y3 = ymat[, 1], y4 = ymat[, 2], x2 = runif(nn))
summary(bdata)
## Not run: plot(ymat, col = "blue")
fit1 <- vglm(cbind(y1, y2, y3, y4) ~ 1, # 2 responses, e.g., (y1,y2) is the 1st
    fam = bistudentt, # crit = "coef", # Sometimes a good idea
    data = bdata, trace = TRUE)
```

```
coef(fit1, matrix = TRUE)
Coef(fit1)
head(fitted(fit1))
summary(fit1)
```

bmi.nz
Body Mass Index of New Zealand Adults Data

## Description

The body mass indexes and ages from an approximate random sample of 700 New Zealand adults.

## Usage

data(bmi.nz)

## Format

A data frame with 700 observations on the following 2 variables.
age a numeric vector; their age (years).
BMI a numeric vector; their body mass indexes, which is their weight divided by the square of their height $\left(\mathrm{kg} / \mathrm{m}^{2}\right)$.

## Details

They are a random sample from the Fletcher Challenge/Auckland Heart and Health survey conducted in the early 1990s.

There are some outliers in the data set.
A variable gender would be useful, and may be added later.

## Source

Clinical Trials Research Unit, University of Auckland, New Zealand, http://www.ctru. auckland.ac.nz.

## References

MacMahon, S., Norton, R., Jackson, R., Mackie, M. J., Cheng, A., Vander Hoorn, S., Milne, A., McCulloch, A. (1995) Fletcher Challenge-University of Auckland Heart \& Health Study: design and baseline findings. New Zealand Medical Journal, 108, 499-502.

## Examples

```
## Not run: with(bmi.nz, plot(age, BMI, col = "blue"))
fit <- vgam(BMI ~ s(age, df = c(2, 4, 2)), lms.yjn, data = bmi.nz, trace = TRUE)
qtplot(fit, pcol = "blue", tcol = "brown", lcol = "brown")
## End(Not run)
```


## Description

Estimates the parameter of a Borel-Tanner distribution by maximum likelihood estimation.

## Usage

borel.tanner(Qsize = 1, link = "logitlink", imethod = 1)

## Arguments

Qsize A positive integer. It is called $Q$ below and is the initial queue size. The default value $Q=1$ corresponds to the Borel distribution.
link Link function for the parameter; see Links for more choices and for general information.
imethod See CommonVGAMffArguments. Valid values are 1, 2, 3 or 4.

## Details

The Borel-Tanner distribution (Tanner, 1953) describes the distribution of the total number of customers served before a queue vanishes given a single queue with random arrival times of customers (at a constant rate $r$ per unit time, and each customer taking a constant time $b$ to be served). Initially the queue has $Q$ people and the first one starts to be served. The two parameters appear in the density only in the form of the product $r b$, therefore we use $a=r b$, say, to denote the single parameter to be estimated. The density function is

$$
f(y ; a)=\frac{Q}{(y-Q)!} y^{y-Q-1} a^{y-Q} \exp (-a y)
$$

where $y=Q, Q+1, Q+2, \ldots$ The case $Q=1$ corresponds to the Borel distribution (Borel, 1942). For the $Q=1$ case it is necessary for $0<a<1$ for the distribution to be proper. The Borel distribution is a basic Lagrangian distribution of the first kind. The Borel-Tanner distribution is an $Q$-fold convolution of the Borel distribution.
The mean is $Q /(1-a)$ (returned as the fitted values) and the variance is $Q a /(1-a)^{3}$. The distribution has a very long tail unless $a$ is small. Fisher scoring is implemented.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Author(s)

T. W. Yee

## References

Tanner, J. C. (1953). A problem of interference between two queues. Biometrika, 40, 58-69.
Borel, E. (1942). Sur l'emploi du theoreme de Bernoulli pour faciliter le calcul d'une infinite de coefficients. Application au probleme de l'attente a un guichet. Comptes Rendus, Academie des Sciences, Paris, Series A, 214, 452-456.
Johnson N. L., Kemp, A. W. and Kotz S. (2005). Univariate Discrete Distributions, 3rd edition, p.328. Hoboken, New Jersey: Wiley.

Consul, P. C. and Famoye, F. (2006). Lagrangian Probability Distributions, Boston, MA, USA: Birkhauser.

## See Also

rbort, poissonff, felix.

## Examples

```
bdata <- data.frame(y = rbort(n <- 200))
fit <- vglm(y ~ 1, borel.tanner, data = bdata, trace = TRUE, crit = "c")
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

Bort

The Borel-Tanner Distribution

## Description

Density and random generation for the Borel-Tanner distribution.

## Usage

dbort(x, Qsize = 1, a = 0.5, log = FALSE)
$\operatorname{rbort}(\mathrm{n}, \mathrm{Qsize}=1, \mathrm{a}=0.5)$

## Arguments

$x \quad$ vector of quantiles.
$\mathrm{n} \quad$ number of observations. Must be a positive integer of length 1 .
Qsize, a See borel.tanner.
log Logical. If log = TRUE then the logarithm of the density is returned.

## Details

See borel. tanner, the VGAM family function for estimating the parameter, for the formula of the probability density function and other details.

## Value

dbort gives the density, rbort generates random deviates.

## Warning

Looping is used for rbort, therefore values of a close to 1 will result in long (or infinite!) computational times. The default value of $a$ is subjective.

## Author(s)

T. W. Yee

## See Also

borel.tanner.

## Examples

```
## Not run: qsize <- 1; a <- 0.5; x <- qsize:(qsize+10)
plot(x, dbort(x, qsize, a), type = "h", las = 1, col = "blue",
    ylab = paste("fbort(qsize=", qsize, ", a=", a, ")"), log = "y",
    main = "Borel-Tanner density function")
## End(Not run)
```

Brat
Inputting Data to fit a Bradley Terry Model

## Description

Takes in a square matrix of counts and outputs them in a form that is accessible to the brat and bratt family functions.

## Usage

Brat(mat, ties $=0$ * mat, string = c(">", "=="), whitespace = FALSE)

## Arguments

mat Matrix of counts, which is considered $M$ by $M$ in dimension when there are ties, and $M+1$ by $M+1$ when there are no ties. The rows are winners and the columns are losers, e.g., the 2-1 element is now many times Competitor 2 has beaten Competitor 1. The matrices are best labelled with the competitors' names.
ties Matrix of counts. This should be the same dimension as mat. By default, there are no ties. The matrix must be symmetric, and the diagonal should contain NAs.
string Character. The matrices are labelled with the first value of the descriptor, e.g., "NZ > Oz" 'means' NZ beats Australia in rugby. Suggested alternatives include " beats " or " wins against ". The second value is used to handle ties.
whitespace Logical. If TRUE then a white space is added before and after string; it generally enhances readability. See CommonVGAMffArguments for some similar-type information.

## Details

In the VGAM package it is necessary for each matrix to be represented as a single row of data by brat and bratt. Hence the non-diagonal elements of the $M+1$ by $M+1$ matrix are concatenated into $M(M+1)$ values (no ties), while if there are ties, the non-diagonal elements of the $M$ by $M$ matrix are concatenated into $M(M-1)$ values.

## Value

A matrix with 1 row and either $M(M+1)$ or $M(M-1)$ columns.

## Note

This is a data preprocessing function for brat and bratt.
Yet to do: merge InverseBrat into brat.

## Author(s)

T. W. Yee

## References

Agresti, A. (2013). Categorical Data Analysis, 3rd ed. Hoboken, NJ, USA: Wiley.

## See Also <br> brat, bratt, InverseBrat.

## Examples

```
journal <- c("Biometrika", "Comm Statist", "JASA", "JRSS-B")
mat <- matrix(c( NA, 33, 320, 284, 730, NA, 813, 276,
    498, 68, NA, 325, 221, 17, 142, NA), 4, 4)
dimnames(mat) <- list(winner = journal, loser = journal)
Brat(mat) # Less readable
Brat(mat, whitespace = TRUE) # More readable
vglm(Brat(mat, whitespace = TRUE) ~ 1, brat, trace = TRUE)
```

```
brat Bradley Terry Model
```


## Description

Fits a Bradley Terry model (intercept-only model) by maximum likelihood estimation.

## Usage

brat(refgp = "last", refvalue = 1, ialpha = 1)

## Arguments

refgp Integer whose value must be from the set $\{1, \ldots, M+1\}$, where there are $M+1$ competitors. The default value indicates the last competitor is used-but don't input a character string, in general.
refvalue $\quad$ Numeric. A positive value for the reference group.
ialpha Initial values for the $\alpha$ s. These are recycled to the appropriate length.

## Details

The Bradley Terry model involves $M+1$ competitors who either win or lose against each other (no draws/ties allowed in this implementation-see bratt if there are ties). The probability that Competitor $i$ beats Competitor $j$ is $\alpha_{i} /\left(\alpha_{i}+\alpha_{j}\right)$, where all the $\alpha$ s are positive. Loosely, the $\alpha$ s can be thought of as the competitors' 'abilities'. For identifiability, one of the $\alpha_{i}$ is set to a known value refvalue, e.g., 1. By default, this function chooses the last competitor to have this reference value. The data can be represented in the form of a $M+1$ by $M+1$ matrix of counts, where winners are the rows and losers are the columns. However, this is not the way the data should be inputted (see below).

Excluding the reference value/group, this function chooses $\log \left(\alpha_{j}\right)$ as the $M$ linear predictors. The $\log$ link ensures that the $\alpha$ s are positive.
The Bradley Terry model can be fitted by logistic regression, but this approach is not taken here. The Bradley Terry model can be fitted with covariates, e.g., a home advantage variable, but unfortunately, this lies outside the VGLM theoretical framework and therefore cannot be handled with this code.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm.

## Warning

Presently, the residuals are wrong, and the prior weights are not handled correctly. Ideally, the total number of counts should be the prior weights, after the response has been converted to proportions. This would make it similar to family functions such as multinomial and binomialff.

## Note

The function Brat is useful for coercing a $M+1$ by $M+1$ matrix of counts into a one-row matrix suitable for brat. Diagonal elements are skipped, and the usual S order of c(a.matrix) of elements is used. There should be no missing values apart from the diagonal elements of the square matrix. The matrix should have winners as the rows, and losers as the columns. In general, the response should be a 1-row matrix with $M(M+1)$ columns.

Only an intercept model is recommended with brat. It doesn't make sense really to include covariates because of the limited VGLM framework.

Notationally, note that the VGAM family function brat has $M+1$ contestants, while bratt has $M$ contestants.

## Author(s)

T. W. Yee

## References

Agresti, A. (2013). Categorical Data Analysis, 3rd ed. Hoboken, NJ, USA: Wiley.
Stigler, S. (1994). Citation patterns in the journals of statistics and probability. Statistical Science, 9, 94-108.
The BradleyTerry2 package has more comprehensive capabilities than this function.

## See Also

bratt, Brat, multinomial, binomialff.

## Examples

```
# Citation statistics: being cited is a 'win'; citing is a 'loss'
journal <- c("Biometrika", "Comm.Statist", "JASA", "JRSS-B")
mat <- matrix(c( NA, 33, 320, 284,
    730, NA, 813, 276,
    498, 68, NA, 325,
    221, 17, 142, NA), 4, 4)
dimnames(mat) <- list(winner = journal, loser = journal)
fit <- vglm(Brat(mat) ~ 1, brat(refgp = 1), trace = TRUE)
fit <- vglm(Brat(mat) ~ 1, brat(refgp = 1), trace = TRUE, crit = "coef")
summary(fit)
c(0, coef(fit)) # Log-abilities (in order of "journal")
c(1, Coef(fit)) # Abilities (in order of "journal")
fitted(fit) # Probabilities of winning in awkward form
(check <- InverseBrat(fitted(fit))) # Probabilities of winning
check + t(check) # Should be 1's in the off-diagonals
```

bratt Bradley Terry Model With Ties

## Description

Fits a Bradley Terry model with ties (intercept-only model) by maximum likelihood estimation.

## Usage

bratt(refgp = "last", refvalue = 1, ialpha = 1, i0 = 0.01)

## Arguments

refgp Integer whose value must be from the set $\{1, \ldots, M\}$, where there are $M$ competitors. The default value indicates the last competitor is used-but don't input a character string, in general.
refvalue $\quad$ Numeric. A positive value for the reference group.
ialpha Initial values for the $\alpha \mathrm{s}$. These are recycled to the appropriate length.
i0 Initial value for $\alpha_{0}$. If convergence fails, try another positive value.

## Details

There are several models that extend the ordinary Bradley Terry model to handle ties. This family function implements one of these models. It involves $M$ competitors who either win or lose or tie against each other. (If there are no draws/ties then use brat). The probability that Competitor $i$ beats Competitor $j$ is $\alpha_{i} /\left(\alpha_{i}+\alpha_{j}+\alpha_{0}\right)$, where all the $\alpha$ s are positive. The probability that Competitor $i$ ties with Competitor $j$ is $\alpha_{0} /\left(\alpha_{i}+\alpha_{j}+\alpha_{0}\right)$. Loosely, the $\alpha$ s can be thought of as the competitors' 'abilities', and $\alpha_{0}$ is an added parameter to model ties. For identifiability, one of the $\alpha_{i}$ is set to a known value refvalue, e.g., 1 . By default, this function chooses the last competitor to have this reference value. The data can be represented in the form of a $M$ by $M$ matrix of counts, where winners are the rows and losers are the columns. However, this is not the way the data should be inputted (see below).
Excluding the reference value/group, this function chooses $\log \left(\alpha_{j}\right)$ as the first $M-1$ linear predictors. The log link ensures that the $\alpha \mathrm{s}$ are positive. The last linear predictor is $\log \left(\alpha_{0}\right)$.
The Bradley Terry model can be fitted with covariates, e.g., a home advantage variable, but unfortunately, this lies outside the VGLM theoretical framework and therefore cannot be handled with this code.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm.

## Note

The function Brat is useful for coercing a $M$ by $M$ matrix of counts into a one-row matrix suitable for bratt. Diagonal elements are skipped, and the usual $S$ order of $c$ (a.matrix) of elements is used. There should be no missing values apart from the diagonal elements of the square matrix. The matrix should have winners as the rows, and losers as the columns. In general, the response should be a matrix with $M(M-1)$ columns.
Also, a symmetric matrix of ties should be passed into Brat. The diagonal of this matrix should be all NAs.
Only an intercept model is recommended with bratt. It doesn't make sense really to include covariates because of the limited VGLM framework.
Notationally, note that the VGAM family function brat has $M+1$ contestants, while bratt has $M$ contestants.

## Author(s)

T. W. Yee

## References

Torsney, B. (2004). Fitting Bradley Terry models using a multiplicative algorithm. In: Antoch, J. (ed.) Proceedings in Computational Statistics COMPSTAT 2004, Physica-Verlag: Heidelberg. Pages 513-526.

## See Also

brat, Brat, binomialff.

## Examples

```
# citation statistics: being cited is a 'win'; citing is a 'loss'
journal <- c("Biometrika", "Comm.Statist", "JASA", "JRSS-B")
mat <- matrix(c( NA, 33, 320, 284,
            730, NA, 813, 276,
            498, 68, NA, 325,
            221, 17, 142, NA), 4, 4)
dimnames(mat) <- list(winner = journal, loser = journal)
# Add some ties. This is fictitional data.
ties <- 5 + 0 * mat
ties[2, 1] <- ties[1,2] <- 9
# Now fit the model
fit <- vglm(Brat(mat, ties) ~ 1, bratt(refgp = 1), trace = TRUE)
fit <- vglm(Brat(mat, ties) ~ 1, bratt(refgp = 1), trace = TRUE, crit = "coef")
summary(fit)
c(0, coef(fit)) # Log-abilities (in order of "journal"); last is log(alpha0)
c(1, Coef(fit)) # Abilities (in order of "journal"); last is alpha0
fit@misc$alpha # alpha_1,...,alpha_M
```

```
fit@misc$alpha0 # alpha_0
fitted(fit) # Probabilities of winning and tying, in awkward form
predict(fit)
(check <- InverseBrat(fitted(fit))) # Probabilities of winning
qprob <- attr(fitted(fit), "probtie") # Probabilities of a tie
qprobmat <- InverseBrat(c(qprob), NCo = nrow(ties)) # Probabilities of a tie
check + t(check) + qprobmat # Should be 1s in the off-diagonals
```

```
calibrate Model Calibrations
```


## Description

calibrate is a generic function used to produce calibrations from various model fitting functions. The function invokes particular 'methods' which depend on the 'class' of the first argument.

## Usage

calibrate(object, ...)

## Arguments

object An object for which a calibration is desired.
... Additional arguments affecting the calibration produced. Usually the most important argument in . . . is newdata which, for calibrate, contains new response data, $\mathbf{Y}$, say.

## Details

Given a regression model with explanatory variables $\mathbf{X}$ and response $\mathbf{Y}$, calibration involves estimating $\mathbf{X}$ from $\mathbf{Y}$ using the regression model. It can be loosely thought of as the opposite of predict (which takes an $\mathbf{X}$ and returns a $\mathbf{Y}$ of some sort.) In general, the central algorithm is maximum likelihood calibration.

## Value

In general, given a new response $\mathbf{Y}$, some function of the explanatory variables $\mathbf{X}$ are returned. For example, for constrained ordination models such as CQO and CAO models, it is usually not possible to return $\mathbf{X}$, so the latent variables are returned instead (they are linear combinations of the $\mathbf{X})$. See the specific calibrate methods functions to see what they return.

## Note

This function was not called predictx because of the inability of constrained ordination models to return $\mathbf{X}$; they can only return the latent variable values (also known as site scores) instead.

## Author(s)

T. W. Yee

## References

ter Braak, C. J. F. and van Dam, H. (1989). Inferring pH from diatoms: a comparison of old and new calibration methods. Hydrobiologia, 178, 209-223.

## See Also

```
predict, calibrate.rrvglm, calibrate.qrrvglm.
```


## Examples

```
## Not run:
hspider[, 1:6] <- scale(hspider[, 1:6]) # Stdzed environmental vars
set.seed(123)
pcao1 <- cao(cbind(Pardlugu, Pardmont, Pardnigr, Pardpull, Zoraspin) ~
        WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
        family = poissonff, data = hspider, Rank = 1, Bestof = 3,
        df1.nl = c(Zoraspin = 2, 1.9), Crow1positive = TRUE)
siteNos <- 1:2 # Calibrate these sites
cpcao1 <- calibrate(pcao1, trace = TRUE,
        newdata = data.frame(depvar(pcao1)[siteNos, ],
                        model.matrix(pcao1)[siteNos, ]))
# Graphically compare the actual site scores with their calibrated values
persp(pcao1, main = "Site scores: solid=actual, dashed=calibrated",
    label = TRUE, col = "blue", las = 1)
abline(v = latvar(pcao1)[siteNos], col = seq(siteNos)) # Actual scores
abline(v = cpcao1, lty = 2, col = seq(siteNos)) # Calibrated values
## End(Not run)
```

```
calibrate-methods Calibration for Constrained Regression Models
```


## Description

calibrate is a generic function applied to RR-VGLMs, QRR-VGLMs and RR-VGAMs, etc.

## Methods

object The object from which the calibration is performed.

## Description

Performs maximum likelihood calibration for constrained quadratic and additive ordination models (CQO and CAO models are better known as QRR-VGLMs and RR-VGAMs respectively).

## Usage

```
calibrate.qrrvglm(object, newdata = NULL,
    type = c("latvar", "predictors", "response", "vcov", "everything"),
    lr.confint = FALSE, cf.confint = FALSE,
    level = 0.95, initial.vals = NULL, ...)
```


## Arguments

object The fitted CQO/CAO model.
newdata A data frame with new response data, such as new species data. The default is to use the original data used to fit the model; however, the calibration may take a long time to compute because the computations are expensive.
Note that the creation of the model frame associated with newdata is fragile. Factors may not be created properly. If a variable is binary then its best for it to be straightforward and have only 0 and 1 as values.
type What type of result to be returned. The first are the calibrated latent variables or site scores. This is always computed. The "predictors" are the linear/quadratic or additive predictors evaluated at the calibrated latent variables or site scores. The "response" are the fitted values (usually means) evaluated at the calibrated latent variables or site scores. The "vcov" are the Wald-type estimated variance-covariance matrices of the calibrated latent variables or site scores. The "everything" is for all of them, i.e., all types. Note that for CAO models, the "vcov" type is unavailable.
lr.confint, level
Compute approximate likelihood ratio based confidence intervals? If TRUE then level is the confidence level required and one should have type = "latvar" or type = "everything"; and currently only rank-1 models are supported. This option works for CLO and CQO models and not for CAO models. The function uniroot is called to solve for the root of a nonlinear equation to obtain each confidence limit, and this is not entirely reliable. It is assumed that the likelihood function is unimodal about its MLE because only one root is returned if there is more than one. One root is found on each side of the MLE. Technically, the default is to find the value of the latent variable whose difference in deviance (or twice the difference in log-likelihoods) from the optimal model is equal to qchisq(level, $d f=1$ ). The intervals are not true profile likelihood intervals because it is not possible to estimate the regression coefficients of the QRR-VGLM/RR-VGLM based on one response vector. See confint to get the flavour of these two arguments in general.

$$
\begin{array}{ll}
\text { cf.confint } & \begin{array}{l}
\text { Compute approximate characteristic function based confidence intervals? If } \\
\text { TRUE then level is the confidence level required and one should have type = } \\
\\
\text { "latvar" or type = "everything"; and currently only rank-1 models are sup- } \\
\text { ported. This option works for binomialff and poissonff CLO and CQO mod- } \\
\text { els and not for CAO models. The function uniroot is called to solve for the root } \\
\text { of a nonlinear equation to obtain each confidence limit, and this is not entirely } \\
\text { reliable. It is assumed that the likelihood function is unimodal because only } \\
\text { one root is returned if there is more than one. Technically, the CDF of a nor- } \\
\text { malized score statistic is obtained by Gauss-Hermite numerical integration of a } \\
\text { complex-valued integrand, and this is based on the inversion formula described } \\
\text { in Abate and Witt (1992). }
\end{array} \\
\text { initial.vals } \quad \begin{array}{l}
\text { Initial values for the search. For rank-1 models, this should be a vector hav- } \\
\text { ing length equal to nrow(newdata), and for rank-2 models this should be a } \\
\text { two-column matrix with the number of rows equalling the number of rows in }
\end{array} \\
& \begin{array}{l}
\text { newdata. The default is a grid defined by arguments in calibrate. qrrvglm.control. } \\
\text { Arguments that are fed into calibrate.qrrvglm. control. }
\end{array}
\end{array}
$$

## Details

Given a fitted regression CQO/CAO model, maximum likelihood calibration is theoretically easy and elegant. However, the method assumes that all the responses are independent, which is often not true in practice. More details and references are given in Yee (2018) and ch. 6 of Yee (2015).
The function optim is used to search for the maximum likelihood solution. Good initial values are needed, and arguments in calibrate. qrrvglm. control allows the user some control over the choice of these.

## Value

Several methods are implemented to obtain confidence intervals/regions for the calibration estimates. One method is when lr.confint = TRUE, then a 4-column matrix is returned with the confidence limits being the final 2 columns (if type = "everything" then the matrix is returned in the lr.confint list component). Another similar method is when cf.confint = TRUE. There may be some redundancy in whatever is returned. Other methods are returned by using type and they are described as follows.

The argument type determines what is returned. If type = "everything" then all the type values are returned in a list, with the following components. Each component has length nrow(newdata).
latvar Calibrated latent variables or site scores (the default). This may have the attribute "objectiveFunction" which is usually the log-likelihood or the deviance.
predictors linear/quadratic or additive predictors. For example, for Poisson families, this will be on a $\log$ scale, and for binomial families, this will be on a logit scale.
response Fitted values of the response, evaluated at the calibrated latent variables.
vcov
Wald-type estimated variance-covariance matrices of the calibrated latent variables or site scores. Actually, these are stored in a 3-D array whose dimension is c(Rank (object), Rank (object), nrow(newdata)). This type has only been implemented for binomialff and poissonff models with canonical links and noRRR $=\sim 1$ and, for CQOs, I. tolerances $=$ TRUE or eq. tolerances $=$ TRUE.

## Warning

This function is computationally expensive. Setting trace $=$ TRUE to get a running log can be a good idea. This function has been tested but not extensively.

## Note

Despite the name of this function, CAO models are handled as well to a certain extent. Some combinations of parameters are not handled, e.g., lr.confint = TRUE only works for rank-1, type $=$ "vcov" only works for binomialff and poissonff models with canonical links and noRRR $=\sim 1$, and higher-order rank models need eq. tolerances $=$ TRUE or I.tolerances = TRUE as well. For rank-1 objects, lr.confint = TRUE is recommended above type = "vcov" in terms of accuracy and overall generality. For class "qrrvglm" objects it is necessary that all response' tolerance matrices are positive-definite which correspond to bell-shaped response curves/surfaces.
For binomialff and poissonff models the deviance slot is used for the optimization rather than the loglikelihood slot, therefore one can calibrate using real-valued responses. (If the loglikelihood slot were used then functions such as dpois would be used with log = TRUE and then would be restricted to feed in integer-valued response values.)
Maximum likelihood calibration for Gaussian logit regression models may be performed by rioja but this applies to a single environmental variable such as pH in data("SWAP", package $=$ "rioja"). In VGAM calibrate() estimates values of the latent variable rather than individual explanatory variables, hence the setting is more on ordination.

## Author(s)

T. W. Yee. Recent work on the standard errors by David Zucker and Sam Oman at HUJI is gratefully acknowledged-these are returned in the vcov component and provided inspiration for lr.confint and cf.confint. A joint publication is being prepared on this subject.

## References

Abate, J. and Whitt, W. (1992). The Fourier-series method for inverting transforms of probability distributions. Queueing Systems, 10, 5-88.
ter Braak, C. J. F. (1995). Calibration. In: Data Analysis in Community and Landscape Ecology by Jongman, R. H. G., ter Braak, C. J. F. and van Tongeren, O. F. R. (Eds.) Cambridge University Press, Cambridge.

## See Also

calibrate. qrrvglm. control, calibrate.rrvglm, calibrate, cqo, cao, optim, uniroot.

## Examples

```
## Not run:
hspider[, 1:6] <- scale(hspider[, 1:6]) # Stdze environmental variables
set.seed(123)
siteNos <- c(1, 5) # Calibrate these sites
pet1 <- cqo(cbind(Pardlugu, Pardmont, Pardnigr, Pardpull, Zoraspin) ~
            WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
            trace = FALSE,
```

```
    data = hspider[-siteNos, ], # Sites not in fitted model
    family = poissonff, I.toler = TRUE, Crow1positive = TRUE)
y0 <- hspider[siteNos, colnames(depvar(pet1))] # Species counts
(cpet1 <- calibrate(pet1, trace = TRUE, newdata = data.frame(y0)))
(clrpet1 <- calibrate(pet1, lr.confint = TRUE, newdata = data.frame(y0)))
(ccfpet1 <- calibrate(pet1, cf.confint = TRUE, newdata = data.frame(y0)))
(cp1wald <- calibrate(pet1, newdata = y0, type = "everything"))
## End(Not run)
## Not run:
# Graphically compare the actual site scores with their calibrated
# values. 95 percent likelihood-based confidence intervals in green.
persp(pet1, main = "Site scores: solid=actual, dashed=calibrated",
    label = TRUE, col = "gray50", las = 1)
# Actual site scores:
xvars <- rownames(concoef(pet1)) # Variables comprising the latvar
est.latvar <- as.matrix(hspider[siteNos, xvars]) %*% concoef(pet1)
abline(v = est.latvar, col = seq(siteNos))
abline(v = cpet1, lty = 2, col = seq(siteNos)) # Calibrated values
arrows(clrpet1[, 3], c(60, 60), clrpet1[, 4], c(60, 60), # Add CIs
    length = 0.08, col = "orange", angle = 90, code = 3, lwd = 2)
arrows(ccfpet1[, 3], c(70, 70), ccfpet1[, 4], c(70, 70), # Add CIs
    length = 0.08, col = "limegreen", angle = 90, code = 3, lwd = 2)
arrows(cp1wald$latvar - 1.96 * sqrt(cp1wald$vcov), c(65, 65),
    cp1wald$latvar + 1.96 * sqrt(cp1wald$vcov), c(65, 65), # Wald CIs
    length = 0.08, col = "blue", angle = 90, code = 3, lwd = 2)
legend("topright", lwd = 2,
    leg = c("CF interval", "Wald interval", "LR interval"),
    col = c("limegreen", "blue", "orange"), lty = 1)
## End(Not run)
```

calibrate. qrrvglm. control
Control Function for CQO/CAO Calibration

## Description

Algorithmic constants and parameters for running calibrate. qrrvglm are set using this function.

## Usage

calibrate. qrrvglm. control(object, trace = FALSE, method.optim = "BFGS", gridSize = ifelse(Rank == 1, 21, 9), varI.latvar = FALSE, ...)

## Arguments

object The fitted CQO/CAO model. The user should ignore this argument.
trace Logical indicating if output should be produced for each iteration. It is a good idea to set this argument to be TRUE since the computations are expensive.
method.optim Character. Fed into the method argument of optim.
gridSize Numeric, recycled to length Rank. Controls the resolution of the grid used for initial values. For each latent variable, an equally spaced grid of length gridSize is cast from the smallest site score to the largest site score. Then the likelihood function is evaluated on the grid, and the best fit is chosen as the initial value. Thus increasing the value of gridSize increases the chance of obtaining the global solution, however, the computing time increases proportionately.
varI.latvar Logical. For CQO objects only, this argument is fed into Coef. qrrvglm.
... Avoids an error message for extraneous arguments.

## Details

Most CQO/CAO users will only need to make use of trace and gridSize. These arguments should be used inside their call to calibrate. qrrvglm, not this function directly.

## Value

A list which with the following components.

```
trace Numeric (even though the input can be logical).
gridSize Positive integer.
varI.latvar Logical.
```


## Note

Despite the name of this function, CAO models are handled as well.

## References

Yee, T. W. (2020). On constrained and unconstrained quadratic ordination. Manuscript in preparation.

## See Also

calibrate.qrrvglm, Coef.qrrvglm.

## Examples

```
## Not run: hspider[, 1:6] <- scale(hspider[, 1:6]) # Needed for I.tol=TRUE
set.seed(123)
p1 <- cqo(cbind(Alopacce, Alopcune, Pardlugu, Pardnigr,
            Pardpull, Trocterr, Zoraspin) ~
        WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
        family = poissonff, data = hspider, I.tol = TRUE)
```

```
sort(deviance(p1, history = TRUE)) # A history of all the iterations
siteNos <- 3:4 # Calibrate these sites
cp1 <- calibrate(p1, trace = TRUE,
        new = data.frame(depvar(p1)[siteNos, ]))
## End(Not run)
## Not run:
# Graphically compare the actual site scores with their calibrated values
persp(p1, main = "Site scores: solid=actual, dashed=calibrated",
    label = TRUE, col = "blue", las = 1)
abline(v = latvar(p1)[siteNos], col = seq(siteNos)) # Actual site scores
abline(v = cp1, lty = 2, col = seq(siteNos)) # Calibrated values
## End(Not run)
```

```
calibrate.rrvglm
```

Calibration for CLO models ( $R R$-VGLMs)

## Description

Performs maximum likelihood calibration for constrained linear ordination models (CLO models are better known as RR-VGLMs).

## Usage

calibrate.rrvglm(object, newdata = NULL,
type = c("latvar", "predictors", "response", "vcov", "everything"),
lr.confint $=$ FALSE, cf.confint $=$ FALSE,
level = 0.95, initial.vals = NULL, ...)

## Arguments

object The fitted rrvglm model. Note that object should be fitted with corner constraints.
newdata See calibrate.qrrvglm.
type $\quad$ See calibrate. qrrvglm. If type $=$ "vcov" then object should have been fitted using binomialff or poissonff with canonical links, and have noRRR $=\sim$ 1.
lr.confint, cf.confint, level
Same as calibrate. qrrvglm.
initial.vals Same as calibrate.qrrvglm. The default is a grid defined by arguments in calibrate.rrvglm.control.
$\ldots \quad$ Arguments that are fed into calibrate.rrvglm.control.

## Details

Given a fitted regression CLO model, maximum likelihood calibration is theoretically easy and elegant. However, the method assumes that all responses are independent. More details and references are given in Yee (2015).
Calibration requires grouped or non-sparse data as the response. For example, if the family function is multinomial then one cannot usually calibrate $y 0$ if it is a vector of 0 s except for one 1 . Instead, the response vector should be from grouped data so that there are few 0s. Indeed, it is found empirically that the stereotype model (also known as a reduced-rank multinomial logit model) calibrates well only with grouped data, and if the response vector is all 0 s except for one 1 then the MLE will probably be at -Inf or +Inf. As another example, if the family function is poissonff then $y 0$ must not be a vector of all 0 s ; instead, the response vector should have few 0 s ideally. In general, you can use simulation to see what type of data calibrates acceptably.
Internally, this function is a simplification of calibrate. qrrvglm and users should look at that function for details. Good initial values are needed, and a grid is constructed to obtain these. The function calibrate.rrvglm. control allows the user some control over the choice of these.

## Value

See calibrate.qrrvglm. Of course, the quadratic term in the latent variables vanishes for RRVGLMs, so the model is simpler.

## Warning

See calibrate.qrrvglm.

## Note

See calibrate.qrrvglm about, e.g., calibration using real-valued responses.

## Author(s)

T. W. Yee

## See Also

calibrate. qrrvglm, calibrate, rrvglm, weightsvglm, optim, uniroot.

## Examples

```
## Not run: # Example 1
nona.xs.nz <- na.omit(xs.nz) # Overkill!! (Data in VGAMdata package)
nona.xs.nz$dmd <- with(nona.xs.nz, round(drinkmaxday))
nona.xs.nz$feethr <- with(nona.xs.nz, round(feethour))
nona.xs.nz$sleephr <- with(nona.xs.nz, round(sleep))
nona.xs.nz$beats <- with(nona.xs.nz, round(pulse))
p2 <- rrvglm(cbind(dmd, feethr, sleephr, beats) ~ age + smokenow +
    depressed + embarrassed + fedup + hurt + miserable + # 11 psychological
    nofriend + moody + nervous + tense + worry + worrier, # variables
```

```
    noRRR = ~ age + smokenow, trace = FALSE, poissonff, data = nona.xs.nz,
    Rank = 2)
cp2 <- calibrate(p2, newdata = head(nona.xs.nz, 9), trace = TRUE)
cp2
two.cases <- nona.xs.nz[1:2, ] # Another calibration example
two.cases$dmd <- c(4, 10)
two.cases$feethr <- c(4, 7)
two.cases$sleephr <- c(7, 8)
two.cases$beats <- c(62, 71)
(cp2b <- calibrate(p2, newdata = two.cases))
# Example 2
p1 <- rrvglm(cbind(dmd, feethr, sleephr, beats) ~ age + smokenow +
    depressed + embarrassed + fedup + hurt + miserable + # 11 psychological
    nofriend + moody + nervous + tense + worry + worrier, # variables
    noRRR = ~ age + smokenow, trace = FALSE, poissonff, data = nona.xs.nz,
    Rank = 1)
(cp1c <- calibrate(p1, newdata = two.cases, lr.confint = TRUE))
## End(Not run)
```

calibrate.rrvglm. control

Control Function for CLO (RR-VGLM) Calibration

## Description

Algorithmic constants and parameters for running calibrate. rrvglm are set using this function.

## Usage

calibrate.rrvglm.control(object, trace = FALSE, method.optim = "BFGS", gridSize = ifelse(Rank == 1, 17, 9), ...)

## Arguments

object The fitted rrvglm model. The user should ignore this argument.
trace, method.optim
Same as calibrate. qrrvglm. control.
gridSize Same as calibrate.qrrvglm.control.
... Avoids an error message for extraneous arguments.

## Details

Most CLO users will only need to make use of trace and gridSize. These arguments should be used inside their call to calibrate. rrvglm, not this function directly.

## Value

Similar to calibrate. qrrvglm. control.

## See Also

calibrate.rrvglm, Coef.rrvglm.

```
cao Fitting Constrained Additive Ordination (CAO)
```


## Description

A constrained additive ordination (CAO) model is fitted using the reduced-rank vector generalized additive model (RR-VGAM) framework.

## Usage

```
cao(formula, family = stop("argument 'family' needs to be assigned"),
    data = list(),
    weights = NULL, subset = NULL, na.action = na.fail,
    etastart = NULL, mustart = NULL, coefstart = NULL,
    control = cao.control(...), offset = NULL,
    method = "cao.fit", model = FALSE, x.arg = TRUE, y.arg = TRUE,
    contrasts = NULL, constraints = NULL,
    extra = NULL, qr.arg = FALSE, smart = TRUE, ...)
```


## Arguments

formula a symbolic description of the model to be fit. The RHS of the formula is used to construct the latent variables, upon which the smooths are applied. All the variables in the formula are used for the construction of latent variables except for those specified by the argument noRRR, which is itself a formula. The LHS of the formula contains the response variables, which should be a matrix with each column being a response (species).
family a function of class "vglmff" (see vglmff-class) describing what statistical model is to be fitted. This is called a "VGAM family function". See CommonVGAMffArguments for general information about many types of arguments found in this type of function. See cqo for a list of those presently implemented.
data an optional data frame containing the variables in the model. By default the variables are taken from environment (formula), typically the environment from which cao is called.
weights an optional vector or matrix of (prior) weights to be used in the fitting process. For cao, this argument currently should not be used.
subset an optional logical vector specifying a subset of observations to be used in the fitting process.

| na.action | a function which indicates what should happen when the data contain NAs. The <br> default is set by the na. action setting of options, and is na.fail if that is <br> unset. The "factory-fresh" default is na. omit. <br> starting values for the linear predictors. It is a $M$-column matrix. If $M=1$ then <br> it may be a vector. For cao, this argument currently should not be used. <br> etastart <br> starting values for the fitted values. It can be a vector or a matrix. Some family <br> functions do not make use of this argument. For cao, this argument currently <br> should not be used. |
| :--- | :--- |
| mustart | starting values for the coefficient vector. For cao, this argument currently should <br> not be used. |
| coefstart | a list of parameters for controlling the fitting process. See cao. control for <br> details. |
| control | a vector or $M$-column matrix of offset values. These are a priori known and are <br> added to the linear predictors during fitting. For cao, this argument currently <br> should not be used. |
| method | the method to be used in fitting the model. The default (and presently only) <br> method cao.fit uses iteratively reweighted least squares (IRLS) within FOR- <br> TRAN code called from optim. |
| model | a logical value indicating whether the model frame should be assigned in the <br> model slot. |
| x.arg, y.arg logical values indicating whether the model matrix and response vector/matrix |  |

## Details

The arguments of cao are a mixture of those from vgam and cqo, but with some extras in cao. control. Currently, not all of the arguments work properly.

CAO can be loosely be thought of as the result of fitting generalized additive models (GAMs) to several responses (e.g., species) against a very small number of latent variables. Each latent variable is a linear combination of the explanatory variables; the coefficients $\mathbf{C}$ (called $C$ below) are called
constrained coefficients or canonical coefficients, and are interpreted as weights or loadings. The $\mathbf{C}$ are estimated by maximum likelihood estimation. It is often a good idea to apply scale to each explanatory variable first.

For each response (e.g., species), each latent variable is smoothed by a cubic smoothing spline, thus CAO is data-driven. If each smooth were a quadratic then CAO would simplify to constrained quadratic ordination (CQO; formerly called canonical Gaussian ordination or CGO). If each smooth were linear then CAO would simplify to constrained linear ordination (CLO). CLO can theoretically be fitted with cao by specifying $\mathrm{df} 1 . \mathrm{nl}=0$, however it is more efficient to use rrvglm.

Currently, only Rank=1 is implemented, and only noRRR $=\sim 1$ models are handled.
With binomial data, the default formula is

$$
\operatorname{logit}\left(P\left[Y_{s}=1\right]\right)=\eta_{s}=f_{s}(\nu), \quad s=1,2, \ldots, S
$$

where $x_{2}$ is a vector of environmental variables, and $\nu=C^{T} x_{2}$ is a $R$-vector of latent variables. The $\eta_{s}$ is an additive predictor for species $s$, and it models the probabilities of presence as an additive model on the logit scale. The matrix $C$ is estimated from the data, as well as the smooth functions $f_{s}$. The argument noRRR $=\sim 1$ specifies that the vector $x_{1}$, defined for RR-VGLMs and QRR-VGLMs, is simply a 1 for an intercept. Here, the intercept in the model is absorbed into the functions. A clogloglink link may be preferable over a logitlink link.

With Poisson count data, the formula is

$$
\log \left(E\left[Y_{s}\right]\right)=\eta_{s}=f_{s}(\nu)
$$

which models the mean response as an additive models on the log scale.
The fitted latent variables (site scores) are scaled to have unit variance. The concept of a tolerance is undefined for CAO models, but the optimums and maximums are defined. The generic functions Max and Opt should work for CAO objects, but note that if the maximum occurs at the boundary then Max will return a NA. Inference for CAO models is currently undeveloped.

## Value

An object of class "cao" (this may change to "rrvgam" in the future). Several generic functions can be applied to the object, e.g., Coef, concoef, lvplot, summary.

## Warning

CAO is very costly to compute. With version $0.7-8$ it took 28 minutes on a fast machine. I hope to look at ways of speeding things up in the future.

Use set. seed just prior to calling cao() to make your results reproducible. The reason for this is finding the optimal CAO model presents a difficult optimization problem, partly because the log-likelihood function contains many local solutions. To obtain the (global) solution the user is advised to try many initial values. This can be done by setting Bestof some appropriate value (see cao. control). Trying many initial values becomes progressively more important as the nonlinear degrees of freedom of the smooths increase.

Note
CAO models are computationally expensive, therefore setting trace $=$ TRUE is a good idea, as well as running it on a simple random sample of the data set instead.
Sometimes the IRLS algorithm does not converge within the FORTRAN code. This results in warnings being issued. In particular, if an error code of 3 is issued, then this indicates the IRLS algorithm has not converged. One possible remedy is to increase or decrease the nonlinear degrees of freedom so that the curves become more or less flexible, respectively.

## Author(s)

T. W. Yee

## References

Yee, T. W. (2006). Constrained additive ordination. Ecology, 87, 203-213.

## See Also

cao. control, Coef.cao, cqo, latvar, Opt, Max, calibrate. qrrvglm, persp.cao, poissonff, binomialff, negbinomial, gamma2, set.seed, gam, trap0.

## Examples

```
## Not run:
hspider[, 1:6] <- scale(hspider[, 1:6]) # Standardized environmental vars
set.seed(149) # For reproducible results
ap1 <- cao(cbind(Pardlugu, Pardmont, Pardnigr, Pardpull) ~
    WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
    family = poissonff, data = hspider, Rank = 1,
    df1.nl = c(Pardpull= 2.7, 2.5),
    Bestof = 7, Crow1positive = FALSE)
sort(deviance(ap1, history = TRUE)) # A history of all the iterations
Coef(ap1)
concoef(ap1)
par(mfrow = c(2, 2))
plot(ap1) # All the curves are unimodal; some quite symmetric
par(mfrow = c(1, 1), las = 1)
index <- 1:ncol(depvar(ap1))
lvplot(ap1, lcol = index, pcol = index, y = TRUE)
trplot(ap1, label = TRUE, col = index)
abline(a = 0, b = 1, lty = 2)
trplot(ap1, label = TRUE, col = "blue", log = "xy", which.sp = c(1, 3))
abline(a = 0, b = 1, lty = 2)
persp(ap1, col = index, lwd = 2, label = TRUE)
abline(v = Opt(ap1), lty = 2, col = index)
```

```
abline(h = Max(ap1), lty = 2, col = index)
```

\#\# End(Not run)

## Description

Algorithmic constants and parameters for a constrained additive ordination (CAO), by fitting a reduced-rank vector generalized additive model (RR-VGAM), are set using this function. This is the control function for cao.

## Usage

```
cao.control(Rank = 1, all.knots = FALSE, criterion = "deviance", Cinit = NULL,
    Crow1positive = TRUE, epsilon = 1.0e-05, Etamat.colmax = 10,
    GradientFunction = FALSE, iKvector = 0.1, iShape = 0.1,
    noRRR = ~ 1, Norrr = NA,
    SmallNo = 5.0e-13, Use.Init.Poisson.QO = TRUE,
    Bestof = if (length(Cinit)) 1 else 10, maxitl = 10,
    imethod = 1, bf.epsilon = 1.0e-7, bf.maxit = 10,
    Maxit.optim = 250, optim.maxit = 20, sd.sitescores = 1.0,
    sd.Cinit = 0.02, suppress.warnings = TRUE,
    trace = TRUE, df1.nl = 2.5, df2.nl = 2.5,
    spar1 = 0, spar2 = 0, ...)
```


## Arguments

Rank The numerical rank $R$ of the model, i.e., the number of latent variables. Currently only Rank $=1$ is implemented.
all.knots Logical indicating if all distinct points of the smoothing variables are to be used as knots. Assigning the value FALSE means fewer knots are chosen when the number of distinct points is large, meaning less computational expense. See vgam. control for details.
criterion Convergence criterion. Currently, only one is supported: the deviance is minimized.

Cinit Optional initial $\mathbf{C}$ matrix which may speed up convergence.
Crow1positive Logical vector of length Rank (recycled if necessary): are the elements of the first row of $\mathbf{C}$ positive? For example, if Rank is 4, then specifying Crow1positive $=c($ FALSE , TRUE) will force $\mathbf{C}[\mathbf{1 , 1}]$ and $\mathbf{C}[1,3]$ to be negative, and $\mathbf{C}[1,2]$ and $\mathbf{C}[1,4]$ to be positive.
epsilon Positive numeric. Used to test for convergence for GLMs fitted in FORTRAN. Larger values mean a loosening of the convergence criterion.

Etamat.colmax $\quad$| Positive integer, no smaller than Rank. Controls the amount of memory used by |
| :--- |
| .Init.Poisson.Q0(). It is the maximum number of columns allowed for the |
| pseudo-response and its weights. In general, the larger the value, the better the |
| initial value. Used only if Use. Init. Poisson. QO = TRUE. |

GradientFunction
Logical. Whether optim's argument gr is used or not, i.e., to compute gradient
values. Used only if FastAlgorithm is TRUE. Currently, this argument must be
set to FALSE.


#### Abstract

$\mathrm{df} 1 . \mathrm{nl}$, $\mathrm{df} 2 . \mathrm{nl}$ Numeric and non-negative, recycled to length $S$. Nonlinear degrees of freedom for smooths of the first and second latent variables. A value of 0 means the smooth is linear. Roughly, a value between 1.0 and 2.0 often has the approximate flexibility of a quadratic. The user should not assign too large a value to this argument, e.g., the value 4.0 is probably too high. The argument df1.nl is ignored if spar1 is assigned a positive value or values. Ditto for $\mathrm{df} 2 . \mathrm{nl}$. spar1, spar2 Numeric and non-negative, recycled to length $S$. Smoothing parameters of the smooths of the first and second latent variables. The larger the value, the more smooth (less wiggly) the fitted curves. These arguments are an alternative to specifying df1.nl and df2.nl. A value 0 (the default) for spar1 means that $\mathrm{df} 1 . \mathrm{nl}$ is used. Ditto for spar2. The values are on a scaled version of the latent variables. See Green and Silverman (1994) for more information. . Ignored at present.


## Details

Many of these arguments are identical to qrrvglm. control. Here, $R$ is the Rank, $M$ is the number of additive predictors, and $S$ is the number of responses (species). Thus $M=S$ for binomial and Poisson responses, and $M=2 S$ for the negative binomial and 2-parameter gamma distributions.

Allowing the smooths too much flexibility means the CAO optimization problem becomes more difficult to solve. This is because the number of local solutions increases as the nonlinearity of the smooths increases. In situations of high nonlinearity, many initial values should be used, so that Bestof should be assigned a larger value. In general, there should be a reasonable value of $\mathrm{df} 1 . \mathrm{nl}$ somewhere between 0 and about 3 for most data sets.

## Value

A list with the components corresponding to its arguments, after some basic error checking.

## Note

The argument $\mathrm{df} 1 . \mathrm{nl}$ can be inputted in the format $\mathrm{c}(\mathrm{spp} 1=2, \mathrm{spp} 2=3,2.5)$, say, meaning the default value is 2.5 , but two species have alternative values.
If spar1 $=0$ and $\mathrm{df} 1 . \mathrm{nl}=0$ then this represents fitting linear functions (CLO). Currently, this is handled in the awkward manner of setting $\mathrm{df} 1 . \mathrm{nl}$ to be a small positive value, so that the smooth is almost linear but not quite. A proper fix to this special case should done in the short future.

## Author(s)

T. W. Yee

## References

Yee, T. W. (2006). Constrained additive ordination. Ecology, 87, 203-213.
Green, P. J. and Silverman, B. W. (1994). Nonparametric Regression and Generalized Linear Models: A Roughness Penalty Approach, London: Chapman \& Hall.

## See Also

cao.

## Examples

```
    ## Not run:
    hspider[,1:6] <- scale(hspider[,1:6]) # Standardized environmental vars
    set.seed(123)
    ap1 <- cao(cbind(Pardlugu, Pardmont, Pardnigr, Pardpull, Zoraspin) ~
    WaterCon + BareSand + FallTwig +
    CoveMoss + CoveHerb + ReflLux,
    family = poissonff, data = hspider,
    df1.nl = c(Zoraspin = 2.3, 2.1),
    Bestof = 10, Crow1positive = FALSE)
    sort(deviance(ap1, history = TRUE)) # A history of all the iterations
    Coef(ap1)
    par(mfrow = c(2, 3)) # All or most of the curves are unimodal; some are
    plot(ap1, lcol = "blue") # quite symmetric. Hence a CQO model should be ok
    par(mfrow = c(1, 1), las = 1)
    index <- 1:ncol(depvar(ap1)) # lvplot is jagged because only 28 sites
    lvplot(ap1, lcol = index, pcol = index, y = TRUE)
    trplot(ap1, label = TRUE, col = index)
    abline(a = 0, b = 1, lty = 2)
    persp(ap1, label = TRUE, col = 1:4)
    ## End(Not run)
```

    Card
        Cardioid Distribution
    
## Description

Density, distribution function, quantile function and random generation for the cardioid distribution.

## Usage

```
dcard(x, mu, rho, log = FALSE)
pcard(q, mu, rho, lower.tail = TRUE, log.p = FALSE)
qcard(p, mu, rho, tolerance = 1e-07, maxits = 500,
        lower.tail = TRUE, log.p = FALSE)
    rcard(n, mu, rho, ...)
```


## Arguments

| $x, q$ | vector of quantiles. |
| :--- | :--- |
| $p$ | vector of probabilities. |
| $n$ | number of observations. Same as in runif. |
| $m u$, rho | See cardioid for more information. |
| tolerance, maxits, ... |  |

The first two are control parameters for the algorithm used to solve for the roots of a nonlinear system of equations; tolerance controls for the accuracy and maxits is the maximum number of iterations. rcard calls qcard so the . . . can be used to vary the two arguments.
log Logical. If log = TRUE then the logarithm of the density is returned.
lower.tail, log.p
Same meaning as in pnorm or qnorm.

## Details

See cardioid, the VGAM family function for estimating the two parameters by maximum likelihood estimation, for the formula of the probability density function and other details.

## Value

dcard gives the density, pcard gives the distribution function, qcard gives the quantile function, and rcard generates random deviates.

## Note

Convergence problems might occur with rcard.

## Author(s)

Thomas W. Yee and Kai Huang

## See Also

cardioid.

## Examples

```
## Not run:
mu <- 4; rho <- 0.4; x <- seq(0, 2*pi, len = 501)
plot(x, dcard(x, mu, rho), type = "l", las = 1, ylim = c(0, 1), col = "blue",
    ylab = paste("[dp]card(mu=", mu, ", rho=", rho, ")"),
    main = "Blue is density, orange is cumulative distribution function",
    sub = "Purple lines are the 10,20,...,90 percentiles")
lines(x, pcard(x, mu, rho), col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qcard(probs, mu, rho)
lines(Q, dcard(Q, mu, rho), col = "purple", lty = 3, type = "h")
```

```
lines(Q, pcard(Q, mu, rho), col = "purple", lty = 3, type = "h")
abline(h = c(0,probs, 1), v = c(0, 2*pi), col = "purple", lty = 3)
max(abs(pcard(Q, mu, rho) - probs)) # Should be 0
## End(Not run)
```


## cardioid

Cardioid Distribution Family Function

## Description

Estimates the two parameters of the cardioid distribution by maximum likelihood estimation.

## Usage

```
cardioid(lmu = extlogitlink(min = 0, max = 2*pi),
    lrho = extlogitlink(min = -0.5, max = 0.5),
    imu = NULL, irho = 0.3, nsimEIM = 100, zero = NULL)
```


## Arguments

lmu, lrho Parameter link functions applied to the $\mu$ and $\rho$ parameters, respectively. See Links for more choices.
imu, irho Initial values. A NULL means an initial value is chosen internally. See CommonVGAMffArguments for more information.
nsimEIM, zero See CommonVGAMffArguments for more information.

## Details

The two-parameter cardioid distribution has a density that can be written as

$$
f(y ; \mu, \rho)=\frac{1}{2 \pi}(1+2 \rho \cos (y-\mu))
$$

where $0<y<2 \pi, 0<\mu<2 \pi$, and $-0.5<\rho<0.5$ is the concentration parameter. The default link functions enforce the range constraints of the parameters.
For positive $\rho$ the distribution is unimodal and symmetric about $\mu$. The mean of $Y$ (which make up the fitted values) is $\pi+(\rho / \pi)((2 \pi-\mu) \sin (2 \pi-\mu)+\cos (2 \pi-\mu)-\mu \sin (\mu)-\cos (\mu))$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Warning

Numerically, this distribution can be difficult to fit because of a log-likelihood having multiple maximums. The user is therefore encouraged to try different starting values, i.e., make use of imu and irho.

## Note

Fisher scoring using simulation is used.

## Author(s)

T. W. Yee

## References

Jammalamadaka, S. R. and SenGupta, A. (2001). Topics in Circular Statistics, Singapore: World Scientific.

## See Also

rcard, extlogitlink, vonmises.
CircStats and circular currently have a lot more R functions for circular data than the VGAM package.

## Examples

```
## Not run:
cdata <- data.frame(y = rcard(n = 1000, mu = 4, rho = 0.45))
fit <- vglm(y ~ 1, cardioid, data = cdata, trace = TRUE)
coef(fit, matrix=TRUE)
Coef(fit)
c(with(cdata, mean(y)), head(fitted(fit), 1))
summary(fit)
## End(Not run)
```

cauchitlink Cauchit Link Function

## Description

Computes the cauchit (tangent) link transformation, including its inverse and the first two derivatives.

## Usage

```
cauchitlink(theta, bvalue = .Machine$double.eps,
    inverse = FALSE, deriv = 0, short = TRUE, tag = FALSE)
```


## Arguments

| theta | Numeric or character. See below for further details. |
| :--- | :--- |
| bvalue <br> inverse, deriv, | See Links. |
|  | Details at Links. |

## Details

This link function is an alternative link function for parameters that lie in the unit interval. This type of link bears the same relation to the Cauchy distribution as the probit link bears to the Gaussian. One characteristic of this link function is that the tail is heavier relative to the other links (see examples below).
Numerical values of theta close to 0 or 1 or out of range result in Inf, -Inf, NA or NaN.

## Value

For deriv $=0$, the tangent of theta, i.e., $\tan (\mathrm{pi} *($ theta-0.5)) when inverse $=$ FALSE, and if inverse $=$ TRUE then $0.5+\operatorname{atan}($ theta $) / \mathrm{pi}$.
For deriv $=1$, then the function returns $d$ eta $/ d$ theta as a function of theta if inverse $=$ FALSE, else if inverse $=$ TRUE then it returns the reciprocal.

## Note

Numerical instability may occur when theta is close to 1 or 0 . One way of overcoming this is to use bvalue.
As mentioned above, in terms of the threshold approach with cumulative probabilities for an ordinal response this link function corresponds to the Cauchy distribution (see cauchy1).

## Author(s)

Thomas W. Yee

## References

McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, 2nd ed. London: Chapman \& Hall.

## See Also

logitlink, probitlink, clogloglink, loglink, cauchy, cauchy1, Cauchy.

## Examples

```
p <- seq(0.01, 0.99, by = 0.01)
cauchitlink(p)
max(abs(cauchitlink(cauchitlink(p), inverse = TRUE) - p)) # Should be 0
p <- c(seq(-0.02, 0.02, by=0.01), seq(0.97, 1.02, by = 0.01))
cauchitlink(p) # Has no NAs
## Not run:
par(mfrow = c(2, 2), lwd = (mylwd <- 2))
y <- seq(-4, 4, length = 100)
p <- seq(0.01, 0.99, by = 0.01)
for (d in 0:1) {
```

```
    matplot(p, cbind(logitlink(p, deriv = d), probitlink(p, deriv = d)),
                type = "n", col = "purple", ylab = "transformation",
                    las = 1, main = if (d == 0) "Some probability link functions"
            else "First derivative")
    lines(p, logitlink(p, deriv = d), col = "limegreen")
    lines(p, probitlink(p, deriv = d), col = "purple")
    lines(p, clogloglink(p, deriv = d), col = "chocolate")
    lines(p, cauchitlink(p, deriv = d), col = "tan")
    if (d == 0) {
        abline(v = 0.5, h = 0, lty = "dashed")
        legend(0, 4.5, c("logitlink", "probitlink", "clogloglink",
                "cauchitlink"), lwd = mylwd,
                col = c("limegreen", "purple", "chocolate", "tan"))
    } else
        abline(v = 0.5, lty = "dashed")
}
for (d in 0) {
    matplot(y, cbind( logitlink(y, deriv = d, inverse = TRUE),
                    probitlink(y, deriv = d, inverse = TRUE)),
                    type = "n", col = "purple", xlab = "transformation", ylab = "p",
                main = if (d == 0) "Some inverse probability link functions"
                    else "First derivative", las=1)
    lines(y, logitlink(y, deriv = d, inverse = TRUE), col = "limegreen")
    lines(y, probitlink(y, deriv = d, inverse = TRUE), col = "purple")
    lines(y, clogloglink(y, deriv = d, inverse = TRUE), col = "chocolate")
    lines(y, cauchitlink(y, deriv = d, inverse = TRUE), col = "tan")
    if (d == 0) {
        abline(h = 0.5, v = 0, lty = "dashed")
        legend(-4, 1, c("logitlink", "probitlink", "clogloglink",
            "cauchitlink"), lwd = mylwd,
                    col = c("limegreen", "purple", "chocolate", "tan"))
    }
}
par(lwd = 1)
## End(Not run)
```

cauchy
Cauchy Distribution Family Function

## Description

Estimates either the location parameter or both the location and scale parameters of the Cauchy distribution by maximum likelihood estimation.

## Usage

cauchy(llocation = "identitylink", lscale = "loglink", imethod = 1, ilocation = NULL, iscale = NULL,

```
    gprobs.y = ppoints(19), gscale.mux = exp(-3:3), zero = "scale")
cauchy1(scale.arg = 1, llocation = "identitylink", ilocation = NULL,
    imethod = 1, gprobs.y = ppoints(19), zero = NULL)
```


## Arguments

```
llocation, lscale
```

Parameter link functions for the location parameter $a$ and the scale parameter $b$. See Links for more choices.
ilocation, iscale
Optional initial value for $a$ and $b$. By default, an initial value is chosen internally for each.
imethod Integer, either 1 or 2 or 3. Initial method, three algorithms are implemented. The user should try all possible values to help avoid converging to a local solution. Also, choose the another value if convergence fails, or use ilocation and/or iscale.
gprobs.y, gscale.mux, zero
See CommonVGAMffArguments for information.
scale.arg Known (positive) scale parameter, called $b$ below.

## Details

The Cauchy distribution has density function

$$
f(y ; a, b)=\left\{\pi b\left[1+((y-a) / b)^{2}\right]\right\}^{-1}
$$

where $y$ and $a$ are real and finite, and $b>0$. The distribution is symmetric about $a$ and has a heavy tail. Its median and mode are $a$, but the mean does not exist. The fitted values are the estimates of $a$. Fisher scoring is used.
If the scale parameter is known (cauchy1) then there may be multiple local maximum likelihood solutions for the location parameter. However, if both location and scale parameters are to be estimated (cauchy) then there is a unique maximum likelihood solution provided $n>2$ and less than half the data are located at any one point.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

It is well-known that the Cauchy distribution may have local maximums in its likelihood function; make full use of imethod, ilocation, iscale etc.

## Note

Good initial values are needed. By default cauchy searches for a starting value for $a$ and $b$ on a 2-D grid. Likewise, by default, cauchy1 searches for a starting value for $a$ on a 1-D grid. If convergence to the global maximum is not acheieved then it also pays to select a wide range of initial values via the ilocation and/or iscale and/or imethod arguments.

## Author(s)

T. W. Yee

## References

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011). Statistical Distributions, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

Barnett, V. D. (1966). Evaluation of the maximum-likehood estimator where the likelihood equation has multiple roots. Biometrika, 53, 151-165.
Copas, J. B. (1975). On the unimodality of the likelihood for the Cauchy distribution. Biometrika, 62, 701-704.
Efron, B. and Hinkley, D. V. (1978). Assessing the accuracy of the maximum likelihood estimator: Observed versus expected Fisher information. Biometrika, 65, 457-481.

## See Also

Cauchy, cauchit, studentt, simulate.vlm.

## Examples

```
# Both location and scale parameters unknown
set.seed(123)
cdata <- data.frame(x2 = runif(nn <- 1000))
cdata <- transform(cdata, loc = exp(1 + 0.5 * x2), scale = exp(1))
cdata <- transform(cdata, y2 = rcauchy(nn, loc, scale))
fit2 <- vglm(y2 ~ x2, cauchy(lloc = "loglink"), data = cdata, trace = TRUE)
coef(fit2, matrix = TRUE)
head(fitted(fit2)) # Location estimates
summary(fit2)
# Location parameter unknown
cdata <- transform(cdata, scale1 = 0.4)
cdata <- transform(cdata, y1 = rcauchy(nn, loc, scale1))
fit1 <- vglm(y1 ~ x2, cauchy1(scale = 0.4), data = cdata, trace = TRUE)
coef(fit1, matrix = TRUE)
```

cdf. 1 mscreg

Cumulative Distribution Function for LMS Quantile Regression

## Description

Computes the cumulative distribution function (CDF) for observations, based on a LMS quantile regression.

## Usage

cdf.lmscreg(object, newdata $=$ NULL, ...)

## Arguments

object A VGAM quantile regression model, i.e., an object produced by modelling functions such as vglm and vgam with a family function beginning with "lms.".
newdata Data frame where the predictions are to be made. If missing, the original data is used.
... Parameters which are passed into functions such as cdf.lms.yjn.

## Details

The CDFs returned here are values lying in $[0,1]$ giving the relative probabilities associated with the quantiles newdata. For example, a value near 0.75 means it is close to the upper quartile of the distribution.

## Value

A vector of CDF values lying in $[0,1]$.

## Note

The data are treated like quantiles, and the percentiles are returned. The opposite is performed by qtplot.lmscreg.
The CDF values of the model have been placed in @post\$cdf when the model was fitted.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2004). Quantile regression via vector generalized additive models. Statistics in Medicine, 23, 2295-2315.

## See Also

deplot.lmscreg, qtplot. $1 \mathrm{mscreg}, 1 m s . b c n, 1 m s . b c g, 1 m s . y j n$.

## Examples

```
fit <- vgam(BMI ~ s(age, df=c(4, 2)), lms.bcn(zero = 1), data = bmi.nz)
head(fit@post$cdf)
head(cdf(fit)) # Same
head(depvar(fit))
head(fitted(fit))
cdf(fit, data.frame(age = c(31.5, 39), BMI = c(28.4, 24)))
```


## Description

Maximum likelihood estimation of the 2-parameter Gumbel distribution when there are censored observations. A matrix response is not allowed.

## Usage

```
cens.gumbel(llocation = "identitylink", lscale = "loglink", iscale = NULL,
```

                    mean \(=\) TRUE, percentiles \(=\) NULL, zero = "scale")
    
## Arguments

llocation, lscale
Character. Parameter link functions for the location and (positive) scale parameters. See Links for more choices.
iscale Numeric and positive. Initial value for scale. Recycled to the appropriate length. In general, a larger value is better than a smaller value. The default is to choose the value internally.
mean Logical. Return the mean? If TRUE then the mean is returned, otherwise percentiles given by the percentiles argument.
percentiles Numeric with values between 0 and 100. If mean=FALSE then the fitted values are percentiles which must be specified by this argument.
zero An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The value (possibly values) must be from the set $\{1,2\}$ corresponding respectively to location and scale. If zero=NULL then all linear/additive predictors are modelled as a linear combination of the explanatory variables. The default is to fit the shape parameter as an intercept only.

## Details

This VGAM family function is like gumbel but handles observations that are left-censored (so that the true value would be less than the observed value) else right-censored (so that the true value would be greater than the observed value). To indicate which type of censoring, input extra $=$ list (leftcensored = vec1, rightcensored = vec2) where vec1 and vec2 are logical vectors the same length as the response. If the two components of this list are missing then the logical values are taken to be FALSE. The fitted object has these two components stored in the extra slot.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Warning

Numerical problems may occur if the amount of censoring is excessive.

## Note

See gumbel for details about the Gumbel distribution. The initial values are based on assuming all uncensored observations, therefore could be improved upon.

## Author(s)

T. W. Yee

## References

Coles, S. (2001). An Introduction to Statistical Modeling of Extreme Values. London: SpringerVerlag.

## See Also

gumbel, gumbelff, rgumbel, guplot, gev, venice.

## Examples

```
# Example 1
ystar <- venice[["r1"]] # Use the first order statistic as the response
nn <- length(ystar)
L <- runif(nn, 100, 104) # Lower censoring points
U <- runif(nn, 130, 135) # Upper censoring points
y <- pmax(L, ystar) # Left censored
y <- pmin(U, y) # Right censored
extra <- list(leftcensored = ystar < L, rightcensored = ystar > U)
fit <- vglm(y ~ scale(year), data = venice, trace = TRUE, extra = extra,
    fam = cens.gumbel(mean = FALSE, perc = c(5, 25, 50, 75, 95)))
coef(fit, matrix = TRUE)
head(fitted(fit))
fit@extra
# Example 2: simulated data
nn <- 1000
ystar <- rgumbel(nn, loc = 1, scale = exp(0.5)) # The uncensored data
L <- runif(nn, -1, 1) # Lower censoring points
U <- runif(nn, 2, 5) # Upper censoring points
y <- pmax(L, ystar) # Left censored
y <- pmin(U, y) # Right censored
## Not run: par(mfrow = c(1, 2)); hist(ystar); hist(y);
extra <- list(leftcensored = ystar < L, rightcensored = ystar > U)
fit <- vglm(y ~ 1, trace = TRUE, extra = extra, fam = cens.gumbel)
coef(fit, matrix = TRUE)
```


## Description

Maximum likelihood estimation for the normal distribution with left and right censoring.

## Usage

cens.normal(lmu = "identitylink", lsd = "loglink", imethod = 1, zero = "sd")

## Arguments

lmu, lsd Parameter link functions applied to the mean and standard deviation parameters. See Links for more choices. The standard deviation is a positive quantity, therefore a log link is the default.
imethod Initialization method. Either 1 or 2, this specifies two methods for obtaining initial values for the parameters.
zero A vector, e.g., containing the value 1 or 2; if so, the mean or standard deviation respectively are modelled as an intercept only. Setting zero = NULL means both linear/additive predictors are modelled as functions of the explanatory variables. See CommonVGAMffArguments for more information.

## Details

This function is like uninormal but handles observations that are left-censored (so that the true value would be less than the observed value) else right-censored (so that the true value would be greater than the observed value). To indicate which type of censoring, input extra = list (leftcensored $=v e c 1$, rightcensored $=v e c 2$ ) where vec1 and vec2 are logical vectors the same length as the response. If the two components of this list are missing then the logical values are taken to be FALSE. The fitted object has these two components stored in the extra slot.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

This function, which is an alternative to tobit, cannot handle a matrix response and uses different working weights. If there are no censored observations then uninormal is recommended instead.

## Author(s)

T. W. Yee

## See Also

tobit, uninormal, double.cens.normal.

## Examples

```
## Not run:
cdata <- data.frame(x2 = runif(nn <- 1000)) # ystar are true values
cdata <- transform(cdata, ystar = rnorm(nn, m = 100 + 15 * x2, sd = exp(3)))
with(cdata, hist(ystar))
cdata <- transform(cdata, L = runif(nn, 80, 90), # Lower censoring points
                            U = runif(nn, 130, 140)) # Upper censoring points
cdata <- transform(cdata, y = pmax(L, ystar)) # Left censored
cdata <- transform(cdata, y = pmin(U, y)) # Right censored
with(cdata, hist(y))
Extra <- list(leftcensored = with(cdata, ystar < L),
rightcensored = with(cdata, ystar > U))
fit1 <- vglm(y ~ x2, cens.normal, data = cdata, crit = "c", extra = Extra)
fit2 <- vglm(y ~ x2, tobit(Lower = with(cdata, L), Upper = with(cdata, U)),
    data = cdata, crit = "c", trace = TRUE)
coef(fit1, matrix = TRUE)
max(abs(coef(fit1, matrix = TRUE) -
            coef(fit2, matrix = TRUE))) # Should be 0
names(fit1@extra)
## End(Not run)
```

```
cens.poisson
```

Censored Poisson Family Function

## Description

Family function for a censored Poisson response.

## Usage

cens.poisson(link = "loglink", imu = NULL, biglambda $=10$, smallno $=1 \mathrm{e}-10$ )

## Arguments

link Link function applied to the mean; see Links for more choices.
imu Optional initial value; see CommonVGAMffArguments for more information.
biglambda, smallno
Used to help robustify the code when lambda is very large and the ppois value is so close to 0 that the first derivative is computed to be a NA or NaN. When this occurs mills.ratio is called.

## Details

Often a table of Poisson counts has an entry $J+$ meaning $\geq J$. This family function is similar to poissonff but handles such censored data. The input requires SurvS4. Only a univariate response is allowed. The Newton-Raphson algorithm is used.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Warning

As the response is discrete, care is required with Surv, especially with "interval" censored data because of the (start, end] format. See the examples below. The examples have $y<L$ as left censored and $y>=U$ (formatted as $U+$ ) as right censored observations, therefore $L<=y<U$ is for uncensored and/or interval censored observations. Consequently the input must be tweaked to conform to the (start, end] format.
A bit of attention has been directed to try robustify the code when lambda is very large, however this currently works for left and right censored data only, not interval censored data. Sometime the fix involves an approximation, hence it is a good idea to set trace $=$ TRUE.

## Note

The function poissonff should be used when there are no censored observations. Also, NAs are not permitted with SurvS4, nor is type = "counting".

## Author(s)

Thomas W. Yee

## References

See survival for background.

## See Also

SurvS4, poissonff, Links, mills.ratio.

## Examples

```
# Example 1: right censored data
set.seed(123); U <- 20
cdata <- data.frame(y = rpois(N <- 100, exp(3)))
cdata <- transform(cdata, cy = pmin(U, y),
                                    rcensored = (y >= U))
cdata <- transform(cdata, status = ifelse(rcensored, 0, 1))
with(cdata, table(cy))
with(cdata, table(rcensored))
with(cdata, table(print(SurvS4(cy, status)))) # Check; U+ means >= U
fit <- vglm(SurvS4(cy, status) ~ 1, cens.poisson, data = cdata,
```

```
    trace = TRUE)
coef(fit, matrix = TRUE)
table(print(depvar(fit))) # Another check; U+ means >= U
# Example 2: left censored data
L <- 15
cdata <- transform(cdata,
                cY = pmax (L, y),
                lcensored = y < L) # Note y < L, not cY == L or y <= L
cdata <- transform(cdata, status = ifelse(lcensored, 0, 1))
with(cdata, table(cY))
with(cdata, table(lcensored))
with(cdata, table(print(SurvS4(cY, status, type = "left")))) # Check
fit <- vglm(SurvS4(cY, status, type = "left") ~ 1, cens.poisson,
    data = cdata, trace = TRUE)
coef(fit, matrix = TRUE)
# Example 3: interval censored data
cdata <- transform(cdata, Lvec = rep(L, len = N),
    Uvec = rep(U, len = N))
cdata <-
    transform(cdata,
                icensored = Lvec <= y & y < Uvec) # Not lcensored or rcensored
with(cdata, table(icensored))
cdata <- transform(cdata, status = rep(3, N)) # 3 == interval censored
cdata <- transform(cdata,
    status = ifelse(rcensored, 0, status)) # 0 means right censored
cdata <- transform(cdata,
            status = ifelse(lcensored, 2, status)) # 2 means left censored
# Have to adjust Lvec and Uvec because of the (start, end] format:
cdata$Lvec[with(cdata,icensored)] <- cdata$Lvec[with(cdata,icensored)]-1
cdata$Uvec[with(cdata,icensored)] <- cdata$Uvec[with(cdata,icensored)]-1
# Unchanged:
cdata$Lvec[with(cdata, lcensored)] <- cdata$Lvec[with(cdata, lcensored)]
cdata$Lvec[with(cdata, rcensored)] <- cdata$Uvec[with(cdata, rcensored)]
with(cdata, # Check
    table(ii <- print(SurvS4(Lvec, Uvec, status, type = "interval"))))
fit <- vglm(SurvS4(Lvec, Uvec, status, type = "interval") ~ 1,
                    cens.poisson, data = cdata, trace = TRUE)
coef(fit, matrix = TRUE)
table(print(depvar(fit))) # Another check
# Example 4: Add in some uncensored observations
index <- (1:N)[with(cdata, icensored)]
index <- head(index, 4)
cdata$status[index] <- 1 # actual or uncensored value
cdata$Lvec[index] <- cdata$y[index]
with(cdata, table(ii <- print(SurvS4(Lvec, Uvec, status,
                                    type = "interval")))) # Check
fit <- vglm(SurvS4(Lvec, Uvec, status, type = "interval") ~ 1,
            cens.poisson, data = cdata, trace = TRUE, crit = "c")
coef(fit, matrix = TRUE)
table(print(depvar(fit))) # Another check
```

cfibrosis Cystic Fibrosis Data

## Description

This data frame concerns families data and cystic fibrosis.

## Usage <br> data(cfibrosis)

## Format

A data frame with 24 rows on the following 4 variables.
siblings, affected, ascertained, families Over ascertained families, the $k$ th ascertained family has $s_{k}$ siblings of whom $r_{k}$ are affected and $a_{k}$ are ascertained.

## Details

The data set allows a classical segregation analysis to be peformed. In particular, to test Mendelian segregation ratios in nuclear family data. The likelihood has similarities with seq2binomial.

## Source

The data is originally from Crow (1965) and appears as Table 2.3 of Lange (2002).
Crow, J. F. (1965) Problems of ascertainment in the analysis of family data. Epidemiology and Genetics of Chronic Disease. Public Health Service Publication 1163, Neel J. V., Shaw M. W., Schull W. J., editors, Department of Health, Education, and Welfare, Washington, DC, USA.

Lange, K. (2002) Mathematical and Statistical Methods for Genetic Analysis. Second Edition. Springer-Verlag: New York, USA.

## Examples

```
cfibrosis
summary(cfibrosis)
```


## Description

Redirects the user to the function cqo.

## Usage

cgo(...)

## Arguments

$$
\ldots \quad \text { Ignored. }
$$

## Details

The former function cgo has been renamed cqo because CGO (for canonical Gaussian ordination) is a confusing and inaccurate name. CQO (for constrained quadratic ordination) is better. This new nomenclature described in Yee (2006).

## Value

Nothing is returned; an error message is issued.

## Warning

The code, therefore, in Yee (2004) will not run without changing the " g " to a " q ".

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2004). A new technique for maximum-likelihood canonical Gaussian ordination. Ecological Monographs, 74, 685-701.

Yee, T. W. (2006). Constrained additive ordination. Ecology, 87, 203-213.

## See Also

cqo.

## Examples

```
## Not run:
cgo()
## End(Not run)
```

chest.nz Chest Pain in NZ Adults Data

## Description

Presence/absence of chest pain in 10186 New Zealand adults.

## Usage

data(chest.nz)

## Format

A data frame with 73 rows and the following 5 variables.
age a numeric vector; age (years).
nolnor a numeric vector of counts; no pain on LHS or RHS.
nolr a numeric vector of counts; no pain on LHS but pain on RHS.
Inor a numeric vector of counts; no pain on RHS but pain on LHS.
Ir a numeric vector of counts; pain on LHS and RHS of chest.

## Details

Each adult was asked their age and whether they experienced any pain or discomfort in their chest over the last six months. If yes, they indicated whether it was on their LHS and/or RHS of their chest.

## Source

MacMahon, S., Norton, R., Jackson, R., Mackie, M. J., Cheng, A., Vander Hoorn, S., Milne, A., McCulloch, A. (1995) Fletcher Challenge-University of Auckland Heart \& Health Study: design and baseline findings. New Zealand Medical Journal, 108, 499-502.

## Examples

```
## Not run:
fit <- vgam(cbind(nolnor, nolr, lnor, lr) ~ s(age, c(4, 3)),
    binom2.or(exchan = TRUE, zero = NULL), data = chest.nz)
coef(fit, matrix = TRUE)
## End(Not run)
## Not run: plot(fit, which.cf = 2, se = TRUE)
```

```
chinese.nz
Chinese Population in New Zealand 1867-2001 Data
```


## Description

The Chinese population in New Zealand from 1867 to 2001, along with the whole of the New Zealand population.

## Usage

data(chinese.nz)

## Format

A data frame with 27 observations on the following 4 variables.
year Year.
male Number of Chinese males.
female Number of Chinese females.
$n z$ Total number in the New Zealand population.

## Details

Historically, there was a large exodus of Chinese from the Guangdong region starting in the mid1800s to the gold fields of South Island of New Zealand, California, and southern Australia, etc. Discrimination then meant that only men were allowed entry, to hinder permanent settlement. In the case of New Zealand, the government relaxed its immigration laws after WWII to allow wives of Chinese already in NZ to join them because China had been among the Allied powers. Gradual relaxation in the immigration and an influx during the 1980s meant the Chinese population became increasingly demographically normal over time.
The NZ total for the years 1867 and 1871 exclude the Maori population. Three modifications have been made to the female column to make the data internally consistent with the original table.

## References

Page 6 of Aliens At My Table: Asians as New Zealanders See Them by M. Ip and N. Murphy, (2005). Penguin Books. Auckland, New Zealand.

## Examples

```
\#\# Not run: par(mfrow \(=c(1,2))\)
plot(female / (male + female) ~ year, chinese.nz, type = "b",
    ylab = "Proportion", col = "blue", las = 1,
    cex \(=0.015 * \operatorname{sqrt}(m a l e+f e m a l e)\),
\# cex \(=0.10 * \operatorname{sqrt}((m a l e ~+~ f e m a l e) \wedge 1.5 ~ / ~ s q r t(f e m a l e) ~ / ~ s q r t(m a l e)), ~\)
    main = "Proportion of NZ Chinese that are female")
abline(h = 0.5, lty = "dashed", col = "gray")
fit1.cnz <- vglm(cbind(female, male) ~ year, binomialff,
    data = chinese.nz)
fit2.cnz <- vglm(cbind(female, male) ~ sm.poly(year, 2), binomialff,
    data \(=\) chinese.nz)
fit4.cnz <- vglm(cbind(female, male) ~ sm.bs(year, 5), binomialff,
    data \(=\) chinese.nz)
lines(fitted(fit1.cnz) ~ year, chinese.nz, col = "purple", lty = 1)
lines(fitted(fit2.cnz) ~ year, chinese.nz, col = "green", lty = 2)
lines(fitted(fit4.cnz) ~ year, chinese.nz, col = "orange", lwd = 2, lty = 1)
legend("bottomright", col = c("purple", "green", "orange"),
        lty \(=c(1,2,1)\), leg \(=c(" l i n e a r ", ~ " q u a d r a t i c ", ~ " B-s p l i n e "))\)
plot(100*(male+female)/nz ~ year, chinese.nz, type = "b", ylab = "Percent",
    ylim \(=c(0, \max (100 *(m a l e+f e m a l e) / n z)), ~ c o l=" b l u e ", ~ l a s=1\),
    main \(=\) "Percent of NZers that are Chinese")
abline(h = 0, lty = "dashed", col = "gray")
\#\# End(Not run)
```

chisq Chi-squared Distribution

## Description

Maximum likelihood estimation of the degrees of freedom for a chi-squared distribution.

## Usage

chisq(link = "loglink", zero = NULL)

## Arguments

link, zero See CommonVGAMffArguments for information.

## Details

The degrees of freedom is treated as a parameter to be estimated, and as real (not integer). Being positive, a log link is used by default. Fisher scoring is used.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

Multiple responses are permitted. There may be convergence problems if the degrees of freedom is very large or close to zero.

## Author(s)

T. W. Yee

## References

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011). Statistical Distributions, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

## See Also

Chisquare. uninormal.

## Examples

```
cdata <- data.frame(x2 = runif(nn <- 1000))
cdata <- transform(cdata, y1 = rchisq(nn, df = exp(1 - 1 * x2)),
    y2 = rchisq(nn, df = exp(2 - 2 * x2)))
fit <- vglm(cbind(y1, y2) ~ x2, chisq, data = cdata, trace = TRUE)
coef(fit, matrix = TRUE)
```

clo Redirects the User to rrvglm()

## Description

Redirects the user to the function rrvglm.

## Usage

clo(...)

## Arguments

. . .

Ignored.

## Details

CLO stands for constrained linear ordination, and is fitted with a statistical class of models called reduced-rank vector generalized linear models (RR-VGLMs). It allows for generalized reducedrank regression in that response types such as Poisson counts and presence/absence data can be handled.
Currently in the VGAM package, rrvglm is used to fit RR-VGLMs. However, the Author's opinion is that linear responses to a latent variable (composite environmental gradient) is not as common as unimodal responses, therefore cqo is often more appropriate.
The new CLO/CQO/CAO nomenclature described in Yee (2006).

## Value

Nothing is returned; an error message is issued.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2006). Constrained additive ordination. Ecology, 87, 203-213.
Yee, T. W. and Hastie, T. J. (2003). Reduced-rank vector generalized linear models. Statistical Modelling, 3, 15-41.

## See Also

> rrvglm, cqo.

## Examples

```
## Not run:
```

clo()
\#\# End(Not run)

## Description

Computes the complementary log-log transformation, including its inverse and the first two derivatives.

## Usage

```
clogloglink(theta, bvalue = NULL, inverse = FALSE, deriv = 0,
    short \(=\) TRUE, tag \(=\) FALSE)
```


## Arguments

$$
\begin{aligned}
& \text { theta } \\
& \text { bvalue } \\
& \text { inverse, deriv, short, tag or character. See below for further details. } \\
& \\
& \\
& \text { Details at Links. }
\end{aligned}
$$

## Details

The complementary log-log link function is commonly used for parameters that lie in the unit interval. But unlike logitlink, probitlink and cauchitlink, this link is not symmetric. It is the inverse CDF of the extreme value (or Gumbel or log-Weibull) distribution. Numerical values of theta close to 0 or 1 or out of range result in Inf, - Inf, NA or NaN .

## Value

For deriv $=0$, the complimentary $\log -\log$ of theta, i.e., $\log (-\log (1-$ theta $))$ when inverse $=$ FALSE, and if inverse $=$ TRUE then $1-\exp (-\exp ($ theta $))$.

For deriv $=1$, then the function returns $d$ eta $/ d$ theta as a function of theta if inverse $=$ FALSE, else if inverse = TRUE then it returns the reciprocal.
Here, all logarithms are natural logarithms, i.e., to base $e$.

## Note

Numerical instability may occur when theta is close to 1 or 0 . One way of overcoming this is to use bvalue.

Changing 1 s to 0 s and 0 s to 1 s in the response means that effectively a $\log \log$ link is fitted. That is, tranform $y$ by $1-y$. That's why only one of clogloglink and logloglink is written.
With constrained ordination (e.g., cqo and cao) used with binomialff, a complementary log-log link function is preferred over the default logitlink, for a good reason. See the example below.
In terms of the threshold approach with cumulative probabilities for an ordinal response this link function corresponds to the extreme value distribution.

## Author(s)

Thomas W. Yee

## References

McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, 2nd ed. London: Chapman \& Hall.
coalminers

## See Also

Links, logitoffsetlink, logitlink, probitlink, cauchitlink, pgumbel.

## Examples

```
p <- seq(0.01, 0.99, by = 0.01)
clogloglink(p)
max(abs(clogloglink(clogloglink(p), inverse = TRUE) - p)) # Should be 0
p <- c(seq(-0.02, 0.02, by = 0.01), seq(0.97, 1.02, by = 0.01))
clogloglink(p) # Has NAs
clogloglink(p, bvalue = .Machine$double.eps) # Has no NAs
## Not run:
p <- seq(0.01, 0.99, by = 0.01)
plot(p, logitlink(p), type = "l", col = "limegreen", lwd = 2, las = 1,
    main = "Some probability link functions", ylab = "transformation")
lines(p, probitlink(p), col = "purple", lwd = 2)
lines(p, clogloglink(p), col = "chocolate", lwd = 2)
lines(p, cauchitlink(p), col = "tan", lwd = 2)
abline(v = 0.5, h = 0, lty = "dashed")
legend(0.1, 4, c("logitlink", "probitlink", "clogloglink", "cauchitlink"),
        col = c("limegreen", "purple", "chocolate", "tan"), lwd = 2)
## End(Not run)
## Not run:
# This example shows that clogloglink is preferred over logitlink
n <- 500; p <- 5; S <- 3; Rank <- 1 # Species packing model:
mydata <- rcqo(n, p, S, eq.tol = TRUE, es.opt = TRUE, eq.max = TRUE,
                    family = "binomial", hi.abundance = 5, seed = 123,
                    Rank = Rank)
fitc <- cqo(attr(mydata, "formula"), I.tol = TRUE, data = mydata,
    fam = binomialff(multiple.responses = TRUE, link = "cloglog"),
    Rank = Rank)
fitl <- cqo(attr(mydata, "formula"), I.tol = TRUE, data = mydata,
    fam = binomialff(multiple.responses = TRUE, link = "logitlink"),
    Rank = Rank)
# Compare the fitted models (cols 1 and 3) with the truth (col 2)
cbind(concoef(fitc), attr(mydata, "concoefficients"), concoef(fitl))
## End(Not run)
```

coalminers Breathlessness and Wheeze Amongst Coalminers Data

## Description

Coalminers who are smokers without radiological pneumoconiosis, classified by age, breathlessness and wheeze.

## Usage

data(coalminers)

## Format

A data frame with 9 age groups with the following 5 columns.
BW Counts with breathlessness and wheeze.
BnW Counts with breathlessness but no wheeze.
nBW Counts with no breathlessness but wheeze.
nBnW Counts with neither breathlessness or wheeze.
age Age of the coal miners (actually, the midpoints of the 5-year category ranges).

## Details

The data were published in Ashford and Sowden (1970). A more recent analysis is McCullagh and Nelder (1989, Section 6.6).

## Source

Ashford, J. R. and Sowden, R. R. (1970) Multi-variate probit analysis. Biometrics, 26, 535-546.

## References

McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models. 2nd ed. London: Chapman \& Hall.

## Examples

str (coalminers)

Coef Computes Model Coefficients and Quantities

## Description

Coef is a generic function which computes model coefficients from objects returned by modelling functions. It is an auxiliary function to coef that enables extra capabilities for some specific models.

## Usage

Coef(object, ...)

## Arguments

object An object for which the computation of other types of model coefficients or quantities is meaningful.
... Other arguments fed into the specific methods function of the model.

## Details

This function can often be useful for vglm objects with just an intercept term in the RHS of the formula, e.g., $\mathrm{y} \sim 1$. Then often this function will apply the inverse link functions to the parameters. See the example below.
For reduced-rank VGLMs, this function can return the $\mathbf{A}, \mathbf{C}$ matrices, etc.
For quadratic and additive ordination models, this function can return ecological meaningful quantities such as tolerances, optimums, maximums.

## Value

The value returned depends specifically on the methods function invoked.

## Warning

This function may not work for all VGAM family functions. You should check your results on some artificial data before applying it to models fitted to real data.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Hastie, T. J. (2003). Reduced-rank vector generalized linear models. Statistical Modelling, 3, 15-41.

## See Also

coef, Coef.vlm, Coef.rrvglm, Coef. qrrvglm, depvar.

## Examples

```
nn <- 1000
bdata <- data.frame(y = rbeta(nn, shape1 = 1, shape2 = 3)) # Original scale
fit <- vglm(y ~ 1, betaR, data = bdata, trace = TRUE) # Intercept-only model
coef(fit, matrix = TRUE) # Both on a log scale
Coef(fit) # On the original scale
```


## Description

This methods function returns important matrices etc. of a QO object.

## Usage

Coef.qrrvglm(object, varI.latvar = FALSE, refResponse = NULL, ...)

## Arguments

object A CQO object. The former has class "qrrvglm".
varI.latvar Logical indicating whether to scale the site scores (latent variables) to have variance-covariance matrix equal to the rank- $R$ identity matrix. All models have uncorrelated site scores (latent variables), and this option stretches or shrinks the ordination axes if TRUE. See below for further details.
refResponse Integer or character. Specifies the reference response or reference species. By default, the reference species is found by searching sequentially starting from the first species until a positive-definite tolerance matrix is found. Then this tolerance matrix is transformed to the identity matrix. Then the sites scores (latent variables) are made uncorrelated. See below for further details.
... Currently unused.

## Details

If I.tolerances=TRUE or eq. tolerances=TRUE (and its estimated tolerance matrix is positivedefinite) then all species' tolerances are unity by transformation or by definition, and the spread of the site scores can be compared to them. Vice versa, if one wishes to compare the tolerances with the sites score variability then setting varI. latvar=TRUE is more appropriate.

For rank-2 QRR-VGLMs, one of the species can be chosen so that the angle of its major axis and minor axis is zero, i.e., parallel to the ordination axes. This means the effect on the latent vars is independent on that species, and that its tolerance matrix is diagonal. The argument refResponse allows one to choose which is the reference species, which must have a positive-definite tolerance matrix, i.e., is bell-shaped. If refResponse is not specified, then the code will try to choose some reference species starting from the first species. Although the refResponse argument could possibly be offered as an option when fitting the model, it is currently available after fitting the model, e.g., in the functions Coef. qrrvglm and lvplot.qrrvglm.

## Value

The A, B1, C, T, D matrices/arrays are returned, along with other slots. The returned object has class "Coef. qrrvglm" (see Coef.qrrvglm-class).

## Note

Consider an equal-tolerances Poisson/binomial CQO model with noRRR $=\sim 1$. For $R=1$ it has about $2 S+p_{2}$ parameters. For $R=2$ it has about $3 S+2 p_{2}$ parameters. Here, $S$ is the number of species, and $p_{2}=p-1$ is the number of environmental variables making up the latent variable. For an unequal-tolerances Poisson/binomial CQO model with noRRR $=\sim 1$, it has about $3 S-1+p_{2}$ parameters for $R=1$, and about $6 S-3+2 p_{2}$ parameters for $R=2$. Since the total number of data points is $n S$, where $n$ is the number of sites, it pays to divide the number of data points by the number of parameters to get some idea about how much information the parameters contain.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2004). A new technique for maximum-likelihood canonical Gaussian ordination. Ecological Monographs, 74, 685-701.

Yee, T. W. (2006). Constrained additive ordination. Ecology, 87, 203-213.

## See Also

cqo, Coef.qrrvglm-class, print.Coef.qrrvglm, lvplot.qrrvglm.

## Examples

```
set.seed(123)
x2 <- rnorm(n <- 100)
x3 <- rnorm(n)
x4 <- rnorm(n)
latvar1 <- 0 + x3 - 2*x4
lambda1 <- exp(3 - 0.5 * ( latvar1-0)^2)
lambda2 <- exp(2 - 0.5 * ( latvar1-1)^2)
lambda3 <- exp(2 - 0.5 * ((latvar1+4)/2)^2) # Unequal tolerances
y1 <- rpois(n, lambda1)
y2 <- rpois(n, lambda2)
y3 <- rpois(n, lambda3)
set.seed(111)
# vvv p1 <- cqo(cbind(y1, y2, y3) ~ x2 + x3 + x4, poissonff, trace = FALSE)
## Not run: lvplot(p1, y = TRUE, lcol = 1:3, pch = 1:3, pcol = 1:3)
# vvv Coef(p1)
# vvv print(Coef(p1), digits=3)
```

```
Coef.qrrvglm-class Class"Coef.qrrvglm"
```


## Description

The most pertinent matrices and other quantities pertaining to a QRR-VGLM (CQO model).

## Objects from the Class

Objects can be created by calls of the form Coef (object, . . .) where object is an object of class "qrrvglm" (created by cqo).

In this document, $R$ is the rank, $M$ is the number of linear predictors and $n$ is the number of observations.

## Slots

A: Of class "matrix", A, which are the linear 'coefficients' of the matrix of latent variables. It is $M$ by $R$.

B1: Of class "matrix", B1. These correspond to terms of the argument noRRR.
C: Of class "matrix", C, the canonical coefficients. It has $R$ columns.
Constrained: Logical. Whether the model is a constrained ordination model.
$D:$ Of class "array", $D[,, j]$ is an order-Rank matrix, for $j=1, \ldots, M$. Ideally, these are negativedefinite in order to make the response curves/surfaces bell-shaped.

Rank: The rank (dimension, number of latent variables) of the RR-VGLM. Called $R$.
latvar: $n$ by $R$ matrix of latent variable values.
latvar.order: Of class "matrix", the permutation returned when the function order is applied to each column of latvar. This enables each column of latvar to be easily sorted.
Maximum: Of class "numeric", the $M$ maximum fitted values. That is, the fitted values at the optimums for noRRR $=\sim 1$ models. If noRRR is not $\sim 1$ then these will be NAs.
NOS: Number of species.
Optimum: Of class "matrix", the values of the latent variables where the optimums are. If the curves are not bell-shaped, then the value will be NA or NaN .
Optimum. order: Of class "matrix", the permutation returned when the function order is applied to each column of Optimum. This enables each row of Optimum to be easily sorted.
bellshaped: Vector of logicals: is each response curve/surface bell-shaped?
dispersion: Dispersion parameter(s).
Dzero: Vector of logicals, is each of the response curves linear in the latent variable(s)? It will be if and only if $D[, j]$ equals $\mathbf{O}$, for $j=1, \ldots, M$.

Tolerance: Object of class "array", Tolerance[, , $j$ ] is an order-Rank matrix, for $j=1, \ldots, M$, being the matrix of tolerances (squared if on the diagonal). These are denoted by $\mathbf{T}$ in Yee (2004). Ideally, these are positive-definite in order to make the response curves/surfaces bellshaped. The tolerance matrices satisfy $T_{s}=-\frac{1}{2} D_{s}^{-1}$.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2004). A new technique for maximum-likelihood canonical Gaussian ordination. Ecological Monographs, 74, 685-701.

## See Also

Coef.qrrvglm, cqo, print. Coef. qrrvglm.

## Examples

```
x2 <- rnorm(n <- 100)
x3 <- rnorm(n)
x4 <- rnorm(n)
latvar1 <- 0 + x3 - 2*x4
lambda1 <- exp(3 - 0.5 * ( latvar1-0)^2)
lambda2 <- exp(2 - 0.5 * ( latvar1-1)^2)
lambda3 <- exp(2 - 0.5 * ((latvar1+4)/2)^2)
y1 <- rpois(n, lambda1)
y2 <- rpois(n, lambda2)
y3 <- rpois(n, lambda3)
yy <- cbind(y1, y2, y3)
# vvv p1 <- cqo(yy ~ x2 + x3 + x4, fam = poissonff, trace = FALSE)
## Not run:
lvplot(p1, y = TRUE, lcol = 1:3, pch = 1:3, pcol = 1:3)
## End(Not run)
# vvv print(Coef(p1), digits = 3)
```

    Coef.rrvglm Returns Important Matrices etc. of a RR-VGLM Object
    
## Description

This methods function returns important matrices etc. of a RR-VGLM object.

## Usage

Coef.rrvglm(object, ...)

## Arguments

object An object of class "rrvglm".
... Currently unused.

## Details

The A, B1, C matrices are returned, along with other slots. See rrvglm for details about RRVGLMs.

## Value

An object of class "Coef.rrvglm" (see Coef.rrvglm-class).

## Note

This function is an alternative to coef.rrvglm.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Hastie, T. J. (2003). Reduced-rank vector generalized linear models. Statistical Modelling, 3, 15-41.

## See Also

```
Coef.rrvglm-class, print.Coef.rrvglm, rrvglm.
```


## Examples

```
# Rank-1 stereotype model of Anderson (1984)
pneumo <- transform(pneumo, let = log(exposure.time), x3 = runif(nrow(pneumo)))
fit <- rrvglm(cbind(normal, mild, severe) ~ let + x3, multinomial, data = pneumo)
coef(fit, matrix = TRUE)
Coef(fit)
```

```
Coef.rrvglm-class Class"Coef.rrvglm"
```


## Description

The most pertinent matrices and other quantities pertaining to a RR-VGLM.

## Objects from the Class

Objects can be created by calls of the form Coef (object, ...) where object is an object of class rrvglm (see rrvglm-class).
In this document, $M$ is the number of linear predictors and $n$ is the number of observations.

## Slots

A: Of class "matrix", A.
B1: Of class "matrix", B1.
C: Of class "matrix", C.
Rank: The rank of the RR-VGLM.
colx1.index: Index of the columns of the "vlm"-type model matrix corresponding to the variables in $\mathbf{x 1}$. These correspond to $\mathbf{B 1}$.
colx2. index: Index of the columns of the "vlm"-type model matrix corresponding to the variables in $\mathbf{x 2}$. These correspond to the reduced-rank regression.
Atilde: Object of class "matrix", the A matrix with the corner rows removed. Thus each of the elements have been estimated. This matrix is returned only if corner constraints were used.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Hastie, T. J. (2003). Reduced-rank vector generalized linear models. Statistical Modelling, 3, 15-41.

## See Also

Coef.rrvglm, rrvglm, rrvglm-class, print. Coef.rrvglm.

## Examples

```
# Rank-1 stereotype model of Anderson (1984)
pneumo <- transform(pneumo, let = log(exposure.time), x3 = runif(nrow(pneumo)))
fit <- rrvglm(cbind(normal, mild, severe) ~ let + x3, multinomial, data = pneumo)
coef(fit, matrix = TRUE)
Coef(fit)
# print(Coef(fit), digits = 3)
```

Coef.vlm Extract Model Coefficients for VLM Objects

## Description

Amongst other things, this function applies inverse link functions to the parameters of intercept-only VGLMs.

## Usage

Coef.vlm(object, ...)

## Arguments

object A fitted model.
... Arguments which may be passed into coef.

## Details

Most VGAM family functions apply a link function to the parameters, e.g., positive parameter are often have a log link, parameters between 0 and 1 have a logit link. This function can back-transform the parameter estimate to the original scale.

## Value

For intercept-only models (e.g., formula is $y \sim 1$ ) the back-transformed parameter estimates can be returned.

## Warning

This function may not work for all VGAM family functions. You should check your results on some artificial data before applying it to models fitted to real data.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Hastie, T. J. (2003). Reduced-rank vector generalized linear models. Statistical Modelling, 3, 15-41.

## See Also

Coef, coef.

## Examples

```
set.seed(123); nn <- 1000
bdata <- data.frame(y = rbeta(nn, shape1 = 1, shape2 = 3))
fit <- vglm(y ~ 1, betaff, data = bdata, trace = TRUE) # intercept-only model
coef(fit, matrix = TRUE) # log scale
Coef(fit) # On the original scale
```

coefvgam Extract Model Coefficients of a vgam() Object

## Description

Extracts the estimated coefficients from vgam() objects.

## Usage

coefvgam(object, type $=c(" l i n e a r ", " n o n l i n e a r "), \ldots)$

## Arguments

object A vgam object.
type $\quad$ Character. The default is the first choice.
... Optional arguments fed into coefvlm.

## Details

For VGAMs, because modified backfitting is performed, each fitted function is decomposed into a linear and nonlinear (smooth) part. The argument type is used to return which one is wanted.

## Value

A vector if type = "linear". A list if type = "nonlinear", and each component of this list corresponds to an sterm; the component contains an S4 object with slot names such as "Bcoefficients", "knots", "xmin", "xmax".

## Author(s)

Thomas W. Yee

## See Also

```
vgam, coefvlm, coef.
```


## Examples

```
fit <- vgam(agaaus ~ s(altitude, df = 2), binomialff, data = hunua)
coef(fit) # Same as coef(fit, type = "linear")
(ii <- coef(fit, type = "nonlinear"))
is.list(ii)
names(ii)
slotNames(ii[[1]])
```

coefvlm Extract Model Coefficients

## Description

Extracts the estimated coefficients from VLM objects such as VGLMs.

## Usage

coefvlm(object, matrix.out $=$ FALSE, label $=$ TRUE, colon $=$ FALSE)

## Arguments

object An object for which the extraction of coefficients is meaningful. This will usually be a vglm object.
matrix.out Logical. If TRUE then a matrix is returned. The explanatory variables are the rows. The linear/additive predictors are the columns. The constraint matrices are used to compute this matrix.
label Logical. If FALSE then the names of the vector of coefficients are set to NULL.
colon Logical. Explanatory variables which appear in more than one linear/additive predictor are labelled with a colon, e.g., age:1, age:2. However, if it only appears in one linear/additive predictor then the $: 1$ is omitted by default. Then setting colon $=$ TRUE will add the $: 1$.

## Details

This function works in a similar way to applying coef() to a lm or glm object. However, for VGLMs, there are more options available.

## Value

A vector usually. A matrix if matrix. out = TRUE.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Hastie, T. J. (2003). Reduced-rank vector generalized linear models. Statistical Modelling, 3, 15-41.

## See Also

vglm, coefvgam, coef.

## Examples

```
zdata <- data.frame(x2 = runif(nn <- 200))
zdata <- transform(zdata, pstr0 = logitlink(-0.5 + 1*x2, inverse = TRUE),
    lambda = loglink( 0.5 + 2*x2, inverse = TRUE))
zdata <- transform(zdata, y2 = rzipois(nn, lambda, pstr0 = pstr0))
fit2 <- vglm(y2 ~ x2, zipoisson(zero = 1), data = zdata, trace = TRUE)
coef(fit2, matrix = TRUE) # Always a good idea
coef(fit2)
coef(fit2, colon = TRUE)
```


## CommonVGAMffArguments Common VGAM Family Function Arguments

## Description

Here is a description of some common and typical arguments found in many VGAM family functions, e.g., lsigma, isigma, gsigma, nsimEI, parallel and zero.

## Usage

TypicalVGAMfamilyFunction(lsigma = "loglink", isigma = NULL, link.list = list("(Default)" = "identitylink",
x2 = "loglink",
x3 = "logofflink",

```
    x4 = "multilogitlink",
    x5 = "multilogitlink"),
earg.list = list("(Default)" = list(),
    x2 = list(),
    x3 = list(offset = -1),
    x4 = list(),
    x5 = list()),
gsigma = exp(-5:5),
parallel = TRUE,
ishrinkage = 0.95,
nointercept = NULL, imethod = 1,
type.fitted = c("mean", "quantiles", "Qlink",
                            "pobs0", "pstr0", "onempstr0"),
percentiles = c(25, 50, 75),
probs.x = c(0.15, 0.85),
probs.y = c(0.25, 0.50, 0.75),
multiple.responses = FALSE, earg.link = FALSE,
whitespace = FALSE, bred = FALSE, lss = TRUE,
oim = FALSE, nsimEIM = 100, byrow.arg = FALSE,
zero = NULL)
```


## Arguments

lsigma Character. Link function applied to a parameter and not necessarily a mean. See Links for a selection of choices. If there is only one parameter then this argument is often called link.
link.list, earg.list
Some VGAM family functions (such as normal.vcm) implement models with potentially lots of parameter link functions. These two arguments allow many such links and extra arguments to be inputted more easily. One has something like link. list = list(" (Default)" = "identitylink", x2 = "loglink", x3 $=$ "logofflink") and earg. list = list("(Default)" = list(), x2 = list(), x3 $=$ "list (offset $=-1$ )"). Then any unnamed terms will have the default link with its corresponding extra argument. Note: the multilogitlink link is also possible, and if so, at least two instances of it are necessary. Then the last term is the baseline/reference group.
isigma Optional initial values can often be inputted using an argument beginning with "i". For example, "isigma" and "ilocation", or just "init" if there is one parameter. A value of NULL means a value is computed internally, i.e., a selfstarting VGAM family function. If a failure to converge occurs make use of these types of arguments.
gsigma Grid-search initial values can be inputted using an argument beginning with "g", e.g., "gsigma", "gshape" and "gscale". If argument isigma is inputted then that has precedence over gsigma, etc.
If the grid search is 2-dimensional then it is advisable not to make the vectors too long as a nested for loop may be used. Ditto for 3-dimensions etc. Sometimes a ". mux" is added as a suffix, e.g., gshape.mux; this means that the grid is created
relatively and not absolutely, e.g., its values are multipled by some single initial estimate of the parameter in order to create the grid on an absolute scale.
Some family functions have an argument called gprobs.y. This is fed into the probs argument of quantile in order to obtain some values of central tendency of the response, i.e., some spread of values in the middle. when imethod $=1$ to obtain an initial value for the mean Some family functions have an argument called iprobs.y, and if so, then these values can overwrite gprobs.y.
parallel A logical, or a simple formula specifying which terms have equal/unequal coefficients. The formula must be simple, i.e., additive with simple main effects terms. Interactions and nesting etc. are not handled. To handle complex formulas use the constraints argument (of vglm etc.); however, there is a lot more setting up involved and things will not be as convenient.
Here are some examples. 1. parallel $=$ TRUE $\sim x 2+x 5$ means the parallelism assumption is only applied to $X_{2}, X_{5}$ and the intercept. 2. parallel $=$ TRUE $\sim-1$ and parallel $=$ TRUE $\sim 0$ mean the parallelism assumption is applied to no variables at all. Similarly, parallel = FALSE $\sim-1$ and parallel $=$ FALSE $\sim$ 0 mean the parallelism assumption is applied to all the variables including the intercept. 3. parallel $=$ FALSE $\sim x 2-1$ and parallel $=$ FALSE $\sim x 2+0$ applies the parallelism constraint to all terms (including the intercept) except for $X_{2}$.
This argument is common in VGAM family functions for categorical responses, e.g., cumulative, acat, cratio, sratio. For the proportional odds model (cumulative) having parallel constraints applied to each explanatory variable (except for the intercepts) means the fitted probabilities do not become negative or greater than 1. However this parallelism or proportional-odds assumption ought to be checked.
nsimEIM Some VGAM family functions use simulation to obtain an approximate expected information matrix (EIM). For those that do, the nsimEIM argument specifies the number of random variates used per observation; the mean of nsimEIM random variates is taken. Thus nsimEIM controls the accuracy and a larger value may be necessary if the EIMs are not positive-definite. For intercept-only models $(y \sim 1)$ the value of nsimEIM can be smaller (since the common value used is also then taken as the mean over the observations), especially if the number of observations is large.
Some VGAM family functions provide two algorithms for estimating the EIM. If applicable, set nsimEIM = NULL to choose the other algorithm.
imethod An integer with value 1 or 2 or 3 or ... which specifies the initialization method for some parameters or a specific parameter. If failure to converge occurs try the next higher value, and continue until success. For example, imethod = 1 might be the method of moments, and imethod $=2$ might be another method. If no value of imethod works then it will be necessary to use arguments such as isigma. For many VGAM family functions it is advisable to try this argument with all possible values to safeguard against problems such as converging to a local solution. VGAM family functions with this argument usually correspond to a model or distribution that is relatively hard to fit successfully, therefore care is needed to ensure the global solution is obtained. So using all possible values that this argument supplies is a good idea.
type.fitted
first choice is always the default. The available choices depends on what kind of family function it is. Using the first few letters of the chosen choice is okay. See fittedvlm for more details.
The choice "Qlink" refers to quantile-links, which was introduced in December 2018 in VGAMextra 0.0-2 for several 1-parameter distributions. Here, either the loglink or logitlink or identitylink of the quantile is the link function (and the choice is dependent on the support of the distribution), and link functions end in "Qlink". A limited amount of support is provided for such links, e.g., fitted(fit) are the fitted quantiles, which is the same as predict (fit, type = "response"). However, fitted(fit, percentiles = 77) will not work.
percentiles Numeric vector, with values between 0 and 100 (although it is not recommended that exactly 0 or 100 be inputted). Used only if type.fitted = "quantiles" or type.fitted = "percentiles", then this argument specifies the values of these quantiles. The argument name tries to reinforce that the values lie between 0 and 100. See fittedvlm for more details.
probs.x, probs.y
Numeric, with values in $(0,1)$. The probabilites that define quantiles with respect to some vector, usually an x or y of some sort. This is used to create two subsets of data corresponding to 'low' and 'high' values of $x$ or $y$. Each value is separately fed into the probs argument of quantile. If the data set size is small then it may be necessary to increase/decrease slightly the first/second values respectively.
lss Logical. This stands for the ordering: location, scale and shape. Should the ordering of the parameters be in this order? Almost all VGAM family functions have this order by default, but in order to match the arguments of existing R functions, one might need to set lss = FALSE. For example, the arguments of weibullR are scale and shape, whereas rweibull are shape and scale. As a temporary measure (from VGAM 0.9-7 onwards but prior to version 1.0-0), some family functions such as sinmad have an lss argument without a default. For these, setting lss = FALSE will work. Later, lss = TRUE will be the default. Be careful for the dpqr-type functions, e.g., rsinmad.
whitespace Logical. Should white spaces (" ") be used in the labelling of the linear/additive predictors? Setting TRUE usually results in more readability but it occupies more columns of the output.
oim Logical. Should the observed information matrices (OIMs) be used for the working weights? In general, setting oim = TRUE means the Newton-Raphson algorithm, and oim = FALSE means Fisher-scoring. The latter uses the EIM, and is usually recommended. If oim $=$ TRUE then nsimEIM is ignored.
zero
Either an integer vector, or a vector of character strings.
If an integer, then it specifies which linear/additive predictor is modelled as intercept-only. That is, the regression coefficients are set to zero for all covariates except for the intercept. If zero is specified then it may be a vector with values from the set $\{1,2, \ldots, M\}$. The value zero $=$ NULL means model all linear/additive predictors as functions of the explanatory variables. Here, $M$ is the number of linear/additive predictors. Technically, if zero contains the value $j$
then the $j$ th row of every constraint matrix (except for the intercept) consists of all 0 values.
Some VGAM family functions allow the zero argument to accept negative values; if so then its absolute value is recycled over each (usual) response. For example, zero $=-2$ for the two-parameter negative binomial distribution would mean, for each response, the second linear/additive predictor is modelled as intercepts-only. That is, for all the $k$ parameters in negbinomial (this VGAM family function can handle a matrix of responses).
Suppose zero = zerovec where zerovec is a vector of negative values. If $G$ is the usual $M$ value for a univariate response then the actual values for argument zero are all values in c (abs (zerovec) , $G+$ abs (zerovec) , $2 * G+a b s$ (zerovec) , $\ldots$ ) lying in the integer range 1 to $M$. For example, setting zero $=-c(2,3)$ for a matrix response of 4 columns with zinegbinomial (which usually has $G=$ $M=3$ for a univariate response) would be equivalent to zero $=c(2,3,5,6,8,9,11,12)$. This example has $M=12$. Note that if zerovec contains negative values then their absolute values should be elements from the set 1:G.
Note: zero may have positive and negative values, for example, setting zero = $c(-2,3)$ in the above example would be equivalent to zero $=c(2,3,5,8,11)$. The argument zero also accepts a character vector (for VGAM 1.0-1 onwards). Each value is fed into grep with fixed = TRUE, meaning that wildcards " $*$ " are not useful. See the example below-all the variants work; those with LOCAT issue a warning that that value is unmatched. Importantly, the parameter names are c("location1","scale1","location2","scale2") because there are 2 responses. Yee (2015) described zero for only numerical input. Allowing character input is particularly important when the number of parameters cannot be determined without having the actual data first. For example, with time series data, an $\operatorname{ARMA}(p, q)$ process might have parameters $\theta_{1}, \ldots, \theta_{p}$ which should be intercept-only by default. Then specifying a numerical default value for zero would be too difficult (there are the drift and scale parameters too). However, it is possible with the character representation: zero = "theta" would achieve this. In the future, most VGAM family functions might be converted to the character representation-the advantage being that it is more readable. When programming a VGAM family function that allows character input, the variable predictors. names must be assigned correctly.
ishrinkage Shrinkage factor $s$ used for obtaining initial values. Numeric, between 0 and 1. In general, the formula used is something like $s \mu+(1-s) y$ where $\mu$ is a measure of central tendency such as a weighted mean or median, and $y$ is the response vector. For example, the initial values are slight perturbations of the mean towards the actual data. For many types of models this method seems to work well and is often reasonably robust to outliers in the response. Often this argument is only used if the argument imethod is assigned a certain value.
nointercept An integer-valued vector specifying which linear/additive predictors have no intercepts. Any values must be from the set $\{1,2, \ldots, M\}$. A value of NULL means no such constraints.
multiple.responses
Logical. Some VGAM family functions allow a multivariate or vector response. If so, then usually the response is a matrix with columns corresponding to the individual response variables. They are all fitted simultaneously. Arguments such
as parallel may then be useful to allow for relationships between the regressions of each response variable. If multiple.responses = TRUE then sometimes the response is interpreted differently, e.g., posbinomial chooses the first column of a matrix response as success and combines the other columns as failure, but when multiple.responses = TRUE then each column of the response matrix is the number of successes and the weights argument is of the same dimension as the response and contains the number of trials.
earg.link This argument should be generally ignored.
byrow.arg Logical. Some VGAM family functions that handle multiple responses have arguments that allow input to be fed in which affect all the responses, e.g., imu for initalizing a mu parameter. In such cases it is sometime more convenient to input one value per response by setting byrow.arg = TRUE; then values are recycled in order to form a matrix of the appropriate dimension. This argument matches byrow in matrix; in fact it is fed into such using matrix (.. , byrow $=$ byrow. arg). This argument has no effect when there is one response.
bred Logical. Some VGAM family functions will allow bias-reduction based on the work by Kosmidis and Firth. Sometimes half-stepping is a good idea; set stepsize $=0.5$ and monitor convergence by setting trace $=$ TRUE.

## Details

Full details will be given in documentation yet to be written, at a later date!

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Warning

The zero argument is supplied for convenience but conflicts can arise with other arguments, e.g., the constraints argument of vglm and vgam. See Example 5 below for an example. If not sure, use, e.g., constraints(fit) and coef(fit, matrix = TRUE) to check the result of a fit fit.
The arguments zero and nointercept can be inputted with values that fail. For example, multinomial (zero $=2$, nointercept $=1: 3$ ) means the second linear/additive predictor is identically zero, which will cause a failure.
Be careful about the use of other potentially contradictory constraints, e.g., multinomial(zero = 2, parallel = TRUE $\sim$ x3). If in doubt, apply constraints() to the fitted object to check.
VGAM family functions with the nsimEIM may have inaccurate working weight matrices. If so, then the standard errors of the regression coefficients may be inaccurate. Thus output from summary (fit), vcov(fit), etc. may be misleading.
Changes relating to the codelss argument have very important consequences and users must beware. Good programming style is to rely on the argument names and not on the order.

## Note

See Links regarding a major change in link functions, for version 0.9-0 and higher (released during the 2 nd half of 2012).

## Author(s)

T. W. Yee

## References

Yee, T. W. (2015). Vector Generalized Linear and Additive Models: With an Implementation in R. New York, USA: Springer.
Kosmidis, I. and Firth, D. (2009). Bias reduction in exponential family nonlinear models. Biometrika, 96(4), 793-804.
Miranda-Soberanis, V. F. and Yee, T. W. (2018). New link functions for distribution-specific quantile regression based on vector generalized linear and additive models. Manuscript in preparation.

## See Also

Links, vglmff-class, UtilitiesVGAM, normal.vcm, multilogitlink, VGAMextra.

## Examples

```
# Example 1
cumulative()
cumulative(link = "probitlink", reverse = TRUE, parallel = TRUE)
# Example 2
wdata <- data.frame(x2 = runif(nn <- 1000))
wdata <- transform(wdata,
    y = rweibull(nn, shape = 2 + exp(1 + x2), scale = exp(-0.5)))
fit <- vglm(y ~ x2, weibullR(lshape = logofflink(offset = -2), zero = 2),
            data = wdata)
coef(fit, mat = TRUE)
# Example 3; multivariate (multiple) response
## Not run:
ndata <- data.frame(x = runif(nn <- 500))
ndata <- transform(ndata,
        y1 = rnbinom(nn, mu = exp(3+x), size = exp(1)), # k is size
    y2 = rnbinom(nn, mu = exp(2-x), size = exp(0)))
fit <- vglm(cbind(y1, y2) ~ x, negbinomial(zero = -2), data = ndata)
coef(fit, matrix = TRUE)
## End(Not run)
# Example 4
## Not run:
# fit1 and fit2 are equivalent
fit1 <- vglm(ymatrix ~ x2 + x3 + x4 + x5,
    cumulative(parallel = FALSE ~ 1 + x3 + x5), data = cdata)
fit2 <- vglm(ymatrix ~ x2 + x3 + x4 + x5,
            cumulative(parallel = TRUE ~ x2 + x4), data = cdata)
## End(Not run)
# Example 5
```

```
udata <- data.frame(x2 = rnorm(nn <- 200))
udata <- transform(udata,
    y1 = rnorm(nn, mean = 1 - 3*x2, sd = exp(1 + 0.2*x2)),
    y2 = rnorm(nn, mean = 1 - 3*x2, sd = exp(1)))
args(uninormal)
fit1 <- vglm(y1 ~ x2, uninormal, data = udata) # This is okay
fit2 <- vglm(y2 ~ x2, uninormal(zero = 2), data = udata) # This is okay
# This creates potential conflict
clist <- list("(Intercept)" = diag(2), "x2" = diag(2))
fit3 <- vglm(y2 ~ x2, uninormal(zero = 2), data = udata,
    constraints = clist) # Conflict!
coef(fit3, matrix = TRUE) # Shows that clist[["x2"]] was overwritten,
constraints(fit3) # i.e., 'zero' seems to override the 'constraints' arg
# Example 6 ('whitespace' argument)
pneumo <- transform(pneumo, let = log(exposure.time))
fit1 <- vglm(cbind(normal, mild, severe) ~ let,
        sratio(whitespace = FALSE, parallel = TRUE), data = pneumo)
fit2 <- vglm(cbind(normal, mild, severe) ~ let,
    sratio(whitespace = TRUE, parallel = TRUE), data = pneumo)
head(predict(fit1), 2) # No white spaces
head(predict(fit2), 2) # Uses white spaces
# Example 7 ('zero' argument with character input)
set.seed(123); n <- 1000
ldata <- data.frame(x2 = runif(n))
ldata <- transform(ldata, y1 = rlogis(n, loc = 5*x2, scale = exp(2)))
ldata <- transform(ldata, y2 = rlogis(n, loc = 5*x2, scale = exp(1*x2)))
ldata <- transform(ldata, w1 = runif(n))
ldata <- transform(ldata, w2 = runif(n))
fit7 <- vglm(cbind(y1, y2) ~ x2,
# logistic(zero = "location1"), # location1 is intercept-only
# logistic(zero = "location2"),
# logistic(zero = "location*"), # Not okay... all is unmatched
# logistic(zero = "scale1"),
# logistic(zero = "scale2"),
# logistic(zero = "scale"), # Both scale parameters are matched
    logistic(zero = c("location", "scale2")), # All but scale1
# logistic(zero = c("LOCAT", "scale2")), # Only scale2 is matched
# logistic(zero = c("LOCAT")), # Nothing is matched
# trace = TRUE,
# weights = cbind(w1, w2),
    weights = w1,
    data = ldata)
coef(fit7, matrix = TRUE)
```


## Description

concoef is a generic function which extracts the constrained (canonical) coefficients from objects returned by certain modelling functions.

## Usage

concoef(object, ...)

## Arguments

object An object for which the extraction of canonical coefficients is meaningful.
... Other arguments fed into the specific methods function of the model.

## Details

For constrained quadratic and ordination models, canonical coefficients are the elements of the $\mathbf{C}$ matrix used to form the latent variables. They are highly interpretable in ecology, and are looked at as weights or loadings.
They are also applicable for reduced-rank VGLMs.

Value
The value returned depends specifically on the methods function invoked.

## Warning

concoef replaces ccoef; the latter is deprecated.
For QO models, there is a direct inverse relationship between the scaling of the latent variables (site scores) and the tolerances. One normalization is for the latent variables to have unit variance. Another normalization is for all the species' tolerances to be unit (provided eq. tolerances is TRUE). These two normalizations cannot simultaneously hold in general. For rank $R$ models with $R>1$ it becomes more complicated because the latent variables are also uncorrelated. An important argument when fitting quadratic ordination models is whether eq. tolerances is TRUE or FALSE. See Yee (2004) for details.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Hastie, T. J. (2003). Reduced-rank vector generalized linear models. Statistical Modelling, 3, 15-41.

Yee, T. W. (2004). A new technique for maximum-likelihood canonical Gaussian ordination. Ecological Monographs, 74, 685-701.
Yee, T. W. (2006). Constrained additive ordination. Ecology, 87, 203-213.

## See Also

concoef-method, concoef.qrrvglm, concoef.cao, coef.

## Examples

```
## Not run: set.seed(111) # This leads to the global solution
hspider[,1:6] <- scale(hspider[,1:6]) # Standardized environmental vars
p1 <- cqo(cbind(Alopacce, Alopcune, Alopfabr, Arctlute, Arctperi,
    Auloalbi, Pardlugu, Pardmont, Pardnigr, Pardpull,
    Trocterr, Zoraspin) ~
        WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
        family = poissonff, data = hspider, Crow1positive = FALSE)
concoef(p1)
## End(Not run)
```

```
concoef-methods Constrained (Canonical) Coefficients
```


## Description

concoef is a generic function used to return the constrained (canonical) coefficients of a constrained ordination model. The function invokes particular methods which depend on the class of the first argument.

## Methods

object The object from which the constrained coefficients are extracted.

```
confintvglm
Confidence Intervals for Parameters of VGLMs
```


## Description

Computes confidence intervals (CIs) for one or more parameters in a fitted model. Currently the object must be a "vglm" object.

## Usage

```
    confintvglm(object, parm, level = 0.95, method = c("wald", "profile"),
    trace = NULL, ...)
```


## Arguments

object A fitted model object.
parm, level, ...
Same as confint.
method Character. The default is the first method. Abbreviations are allowed. Currently "profile" is basically working; and it is likely to be more accurate especially for small samples, as it is based on a profile log likelihood, however it is computationally intensive.
trace Logical. If TRUE then one can monitor the computation as it progresses (because it is expensive). The default is the orginal model's trace value (see vglm. control). Setting FALSE suppresses all intermediate output.

## Details

The default for this methods function is based on confint. default and assumes asymptotic normality. In particular, the coef and vcov methods functions are used for vglm-class objects.

When method = "profile" the function profilevglm is called to do the profiling. The code is very heavily based on profile.glm which was originally written by D. M. Bates and W. N. Venables (For S in 1996) and subsequently corrected by B. D. Ripley. Sometimes the profiling method can give problems, for example, cumulative requires the $M$ linear predictors not to intersect in the data cloud. Such numerical problems are less common when method = "wald", however, it is wellknown that inference based on profile likelihoods is generally more accurate than Wald, especially when the sample size is small. The deviance (deviance (object)) is used if possible, else the difference 2 * (logLik (object) -ell) is computed, where ell are the values of the loglikelihood on a grid.
For Wald CIs and rrvglm-class objects, currently an error message is produced because I haven't gotten around to write the methods function; it's not too hard, but am too busy! An interim measure is to coerce the object into a "vglm" object, but then the confidence intervals will tend to be too narrow because the estimated constraint matrices are treated as known.

For Wald CIs and vgam-class objects, currently an error message is produced because the theory is undeveloped.

## Value

Same as confint.

## Note

The order of the values of argument method may change in the future without notice. The functions plot.profile.glm and pairs.profile.glm from MASS appear to work with output from this function.

## Author(s)

Thomas Yee adapted confint. 1 m to handle "vglm" objects, for Wald-type confidence intervals. Also, profile.glm was originally written by D. M. Bates and W. N. Venables (For S in 1996) and subsequently corrected by B. D. Ripley. This function effectively calls confint.profile.glm() in MASS.

## See Also

vcovvlm, summaryvglm, confint, profile.glm, lrt.stat.vlm, wald.stat, plot.profile.glm, pairs.profile.glm.

## Examples

```
# Example 1: this is based on a glm example
counts <- c(18,17,15,20,10,20,25,13,12)
outcome <- gl(3, 1, 9); treatment <- gl(3, 3)
    glm.D93 <- glm(counts ~ outcome + treatment, family = poisson())
vglm.D93 <- vglm(counts ~ outcome + treatment, family = poissonff)
confint(glm.D93) # needs MASS to be present on the system
confint.default(glm.D93) # based on asymptotic normality
confint(vglm.D93)
confint(vglm.D93) - confint(glm.D93) # Should be all 0s
confint(vglm.D93) - confint.default(glm.D93) # based on asympt. normality
# Example 2: simulated negative binomial data with multiple responses
ndata <- data.frame(x2 = runif(nn <- 100))
ndata <- transform(ndata, y1 = rnbinom(nn, mu = exp(3+x2), size = exp(1)),
    y2 = rnbinom(nn, mu = exp(2-x2), size = exp(0)))
fit1 <- vglm(cbind(y1, y2) ~ x2, negbinomial, data = ndata, trace = TRUE)
coef(fit1)
coef(fit1, matrix = TRUE)
confint(fit1)
confint(fit1, "x2:1") # This might be improved to "x2" some day...
## Not run:
confint(fit1, method = "profile") # Computationally expensive
confint(fit1, "x2:1", method = "profile", trace = FALSE)
## End(Not run)
```

fit2 <- rrvglm(y1 ~ x2, negbinomial(zero = NULL), data = ndata)
confint(as(fit2, "vglm")) \# Too narrow (SEs are biased downwards)
constraints

## Description

Extractor function for the constraint matrices of objects in the VGAM package.

## Usage

```
constraints(object, ...)
constraints.vlm(object, type = c("lm", "term"), all = TRUE, which,
    matrix.out = FALSE, colnames.arg = TRUE,
    rownames.arg = TRUE, ...)
```


## Arguments

$$
\left.\left.\begin{array}{ll}
\text { object } & \begin{array}{l}
\text { Some VGAM object, for example, having class vglmff-class. } \\
\text { type }
\end{array} \\
\text { Character. Whether LM- or term-type constraints are to be returned. The num- } \\
\text { ber of such matrices returned is equal to nvar (object, type ="lm") and the } \\
\text { number of terms, respectively. }
\end{array}\right] \begin{array}{l}
\text { If all = FALSE then which gives the integer index or a vector of logicals speci- } \\
\text { fying the selection. }
\end{array}\right\} \begin{aligned}
& \text { Logical. If TRUE then the constraint matrices are cbind() ed together. The result } \\
& \text { is usually more compact because the default is a list of constraint matrices. }
\end{aligned}
$$

## Details

Constraint matrices describe the relationship of coefficients/component functions of a particular explanatory variable between the linear/additive predictors in VGLM/VGAM models. For example, they may be all different (constraint matrix is the identity matrix) or all the same (constraint matrix has one column and has unit values).

VGLMs and VGAMs have constraint matrices which are known. The class of RR-VGLMs have constraint matrices which are unknown and are to be estimated.

## Value

The extractor function constraints() returns a list comprising of constraint matrices-usually one for each column of the VLM model matrix, and in that order. The list is labelled with the variable names. Each constraint matrix has $M$ rows, where $M$ is the number of linear/additive predictors, and whose rank is equal to the number of columns. A model with no constraints at all has an order $M$ identity matrix as each variable's constraint matrix.
For vglm and vgam objects, feeding in type = "term" constraint matrices back into the same model should work and give an identical model. The default are the "lm"-type constraint matrices; this is a list with one constraint matrix per column of the LM matrix. See the constraints argument of vglm , and the example below.

## Note

In all VGAM family functions zero $=$ NULL means none of the linear/additive predictors are modelled as intercepts-only. Other arguments found in certain VGAM family functions which affect constraint matrices include parallel and exchangeable.
The constraints argument in vglm and vgam allows constraint matrices to be inputted. If so, then constraints(fit, type = "lm") can be fed into the constraints argument of the same object to get the same model.

The xij argument does not affect constraint matrices; rather, it allows each row of the constraint matrix to be multiplied by a specified vector.
corbet

## Author(s)

T. W. Yee

## References

Yee, T. W. and Wild, C. J. (1996). Vector generalized additive models. Journal of the Royal Statistical Society, Series B, Methodological, 58, 481-493.
Yee, T. W. and Hastie, T. J. (2003). Reduced-rank vector generalized linear models. Statistical Modelling, 3, 15-41.

## See Also

is.parallel, is.zero. VGLMs are described in vglm-class; RR-VGLMs are described in rrvglm-class.
Arguments such as zero and parallel found in many VGAM family functions are a way of creating/modifying constraint matrices conveniently, e.g., see zero. See CommonVGAMffArguments for more information.

## Examples

```
# Fit the proportional odds model:
pneumo <- transform(pneumo, let = log(exposure.time))
(fit1 <- vglm(cbind(normal, mild, severe) ~ sm.bs(let, 3),
    cumulative(parallel = TRUE, reverse = TRUE), data = pneumo))
coef(fit1, matrix = TRUE)
constraints(fit1) # Parallel assumption results in this
constraints(fit1, type = "term") # Same as the default ("vlm"-type)
is.parallel(fit1)
# An equivalent model to fit1 (needs the type "term" constraints):
clist.term <- constraints(fit1, type = "term") # "term"-type constraints
# cumulative() has no 'zero' argument to set to NULL (a good idea
# when using the 'constraints' argument):
(fit2 <- vglm(cbind(normal, mild, severe) ~ sm.bs(let, 3), data = pneumo,
                    cumulative(reverse = TRUE), constraints = clist.term))
abs(max(coef(fit1, matrix = TRUE) -
            coef(fit2, matrix = TRUE))) # Should be zero
# Fit a rank-1 stereotype (RR-multinomial logit) model:
fit <- rrvglm(Country ~ Width + Height + HP, multinomial, data = car.all)
constraints(fit) # All except the first are the estimated A matrix
```

```
corbet Corbet's Butterfly Data
```


## Description

About 3300 individual butterflies were caught in Malaya by naturalist Corbet trapping butterflies. They were classified to about 500 species.

## Usage

data(corbet)

## Format

A data frame with 24 observations on the following 2 variables.
species Number of species.
ofreq Observed frequency of individual butterflies of that species.

## Details

In the early 1940s Corbet spent two years trapping butterflies in Malaya. Of interest was the total number of species. Some species were so rare (e.g., 118 species had only one specimen) that it was thought likely that there were many unknown species.

## References

Fisher, R. A., Corbet, A. S. and Williams, C. B. (1943). The Relation Between the Number of Species and the Number of Individuals in a Random Sample of an Animal Population. Journal of Animal Ecology, 12, 42-58.

## Examples

```
summary(corbet)
```


## Description

A constrained quadratic ordination (CQO; formerly called canonical Gaussian ordination or CGO) model is fitted using the quadratic reduced-rank vector generalized linear model (QRR-VGLM) framework.

## Usage

```
    cqo(formula, family = stop("argument 'family' needs to be assigned"),
        data \(=\) list(), weights \(=\) NULL, subset \(=\) NULL,
        na.action \(=\) na.fail, etastart \(=\) NULL, mustart \(=\) NULL,
        coefstart \(=\) NULL, control \(=\) qrrvglm.control(...), offset \(=\) NULL,
        method = "cqo.fit", model = FALSE, x.arg = TRUE, y.arg = TRUE,
        contrasts \(=\) NULL, constraints \(=\) NULL, extra \(=\) NULL,
        smart = TRUE, ...)
```


## Arguments

| formula | a symbolic description of the model to be fit. The RHS of the formula is applied to each linear predictor. Different variables in each linear predictor can be chosen by specifying constraint matrices. |
| :---: | :---: |
| family | a function of class "vglmff" (see vglmff-class) describing what statistical model is to be fitted. This is called a "VGAM family function". See CommonVGAMffArguments for general information about many types of arguments found in this type of function. Currently the following families are supported: poissonff, binomialff (logitlink and clogloglink links available), negbinomial, gamma2. Sometimes special arguments are required for cqo(), e.g., binomialff(multiple. responses = TRUE). |
| data | an optional data frame containing the variables in the model. By default the variables are taken from environment(formula), typically the environment from which cqo is called. |
| weights | an optional vector or matrix of (prior) weights to be used in the fitting process. Currently, this argument should not be used. |
| subset | an optional logical vector specifying a subset of observations to be used in the fitting process. |
| na.action | a function which indicates what should happen when the data contain NAs. The default is set by the na.action setting of options, and is na.fail if that is unset. The "factory-fresh" default is na.omit. |
| etastart | starting values for the linear predictors. It is a $M$-column matrix. If $M=1$ then it may be a vector. Currently, this argument probably should not be used. |
| mustart | starting values for the fitted values. It can be a vector or a matrix. Some family functions do not make use of this argument. Currently, this argument probably should not be used. |
| coefstart | starting values for the coefficient vector. Currently, this argument probably should not be used. |
| control | a list of parameters for controlling the fitting process. See qrrvglm. control for details. |
| offset | This argument must not be used. |
| method | the method to be used in fitting the model. The default (and presently only) method cqo. fit uses iteratively reweighted least squares (IRLS). |
| model | a logical value indicating whether the model frame should be assigned in the model slot. |
| x.arg, y.arg | logical values indicating whether the model matrix and response matrix used in the fitting process should be assigned in the x and y slots. Note the model matrix is the LM model matrix. |
| contrasts | an optional list. See the contrasts.arg of model.matrix. default. |
| constraints | an optional list of constraint matrices. The components of the list must be named with the term it corresponds to (and it must match in character format). Each constraint matrix must have $M$ rows, and be of full-column rank. By default, constraint matrices are the $M$ by $M$ identity matrix unless arguments in the |

family function itself override these values. If constraints is used it must contain all the terms; an incomplete list is not accepted. Constraint matrices for $x_{2}$ variables are taken as the identity matrix.
extra an optional list with any extra information that might be needed by the family function.
smart logical value indicating whether smart prediction (smartpred) will be used. further arguments passed into qrrvglm. control.

## Details

QRR-VGLMs or constrained quadratic ordination (CQO) models are estimated here by maximum likelihood estimation. Optimal linear combinations of the environmental variables are computed, called latent variables (these appear as latvar for $R=1$ else latvar1, latvar2, etc. in the output). Here, $R$ is the rank or the number of ordination axes. Each species' response is then a regression of these latent variables using quadratic polynomials on a transformed scale (e.g., log for Poisson counts, logit for presence/absence responses). The solution is obtained iteratively in order to maximize the log-likelihood function, or equivalently, minimize the deviance.
The central formula (for Poisson and binomial species data) is given by

$$
\eta=B_{1}^{T} x_{1}+A \nu+\sum_{m=1}^{M}\left(\nu^{T} D_{m} \nu\right) e_{m}
$$

where $x_{1}$ is a vector (usually just a 1 for an intercept), $x_{2}$ is a vector of environmental variables, $\nu=C^{T} x_{2}$ is a $R$-vector of latent variables, $e_{m}$ is a vector of 0 s but with a 1 in the $m$ th position. The $\eta$ are a vector of linear/additive predictors, e.g., the $m$ th element is $\eta_{m}=\log \left(E\left[Y_{m}\right]\right)$ for the $m$ th species. The matrices $B_{1}, A, C$ and $D_{m}$ are estimated from the data, i.e., contain the regression coefficients. The tolerance matrices satisfy $T_{s}=-\frac{1}{2} D_{s}^{-1}$. Many important CQO details are directly related to arguments in qrrvglm. control, e.g., the argument noRRR specifies which variables comprise $x_{1}$.
Theoretically, the four most popular VGAM family functions to be used with cqo correspond to the Poisson, binomial, normal, and negative binomial distributions. The latter is a 2-parameter model. All of these are implemented, as well as the 2-parameter gamma.

For initial values, the function. Init.Poisson. QO should work reasonably well if the data is Poisson with species having equal tolerances. It can be quite good on binary data too. Otherwise the Cinit argument in qrrvglm. control can be used.
It is possible to relax the quadratic form to an additive model. The result is a data-driven approach rather than a model-driven approach, so that CQO is extended to constrained additive ordination (CAO) when $R=1$. See cao for more details.
In this documentation, $M$ is the number of linear predictors, $S$ is the number of responses (species). Then $M=S$ for Poisson and binomial species data, and $M=2 S$ for negative binomial and gamma distributed species data.
Incidentally, Unconstrained quadratic ordination (UQO) may be performed by, e.g., fitting a Goodman's RC association model; see uqo and the Yee and Hadi (2014) referenced there. For UQO, the response is the usual site-by-species matrix and there are no environmental variables; the site scores are free parameters. UQO can be performed under the assumption that all species have the same tolerance matrices.

Value
An object of class "qrrvglm".

## Warning

Local solutions are not uncommon when fitting CQO models. To increase the chances of obtaining the global solution, increase the value of the argument Bestof in qrrvglm. control. For reproducibility of the results, it pays to set a different random number seed before calling cqo (the function set. seed does this). The function cqo chooses initial values for $\mathbf{C}$ using. Init. Poisson. Q0() if Use. Init. Poisson. QO = TRUE, else random numbers.

Unless I. tolerances = TRUE or eq. tolerances = FALSE, CQO is computationally expensive with memory and time. It pays to keep the rank down to 1 or 2. If eq. tolerances = TRUE and I. tolerances = FALSE then the cost grows quickly with the number of species and sites (in terms of memory requirements and time). The data needs to conform quite closely to the statistical model, and the environmental range of the data should be wide in order for the quadratics to fit the data well (bellshaped response surfaces). If not, RR-VGLMs will be more appropriate because the response is linear on the transformed scale (e.g., log or logit) and the ordination is called constrained linear ordination or CLO.

Like many regression models, CQO is sensitive to outliers (in the environmental and species data), sparse data, high leverage points, multicollinearity etc. For these reasons, it is necessary to examine the data carefully for these features and take corrective action (e.g., omitting certain species, sites, environmental variables from the analysis, transforming certain environmental variables, etc.). Any optimum lying outside the convex hull of the site scores should not be trusted. Fitting a CAO is recommended first, then upon transformations etc., possibly a CQO can be fitted.
For binary data, it is necessary to have 'enough' data. In general, the number of sites $n$ ought to be much larger than the number of species $S$, e.g., at least 100 sites for two species. Compared to count (Poisson) data, numerical problems occur more frequently with presence/absence (binary) data. For example, if Rank = 1 and if the response data for each species is a string of all absences, then all presences, then all absences (when enumerated along the latent variable) then infinite parameter estimates will occur. In general, setting I. tolerances = TRUE may help.

This function was formerly called cgo. It has been renamed to reinforce a new nomenclature described in Yee (2006).

## Note

The input requires care, preparation and thought-a lot more than other ordination methods. Here is a partial checklist.
(1) The number of species should be kept reasonably low, e.g., 12 max. Feeding in 100+ species wholesale is a recipe for failure. Choose a few species carefully. Using 10 well-chosen species is better than 100+ species thrown in willy-nilly.
(2) Each species should be screened individually first, e.g., for presence/absence is the species totally absent or totally present at all sites? For presence/absence data sort (colMeans(data)) can help avoid such species.
(3) The number of explanatory variables should be kept low, e.g., 7 max.
(4) Each explanatory variable should be screened individually first, e.g., is it heavily skewed or are there outliers? They should be plotted and then transformed where needed. They should not be too highly correlated with each other.
(5) Each explanatory variable should be scaled, e.g., to mean 0 and unit variance. This is especially needed for I. tolerance $=$ TRUE.
(6) Keep the rank low. Only if the data is very good should a rank-2 model be attempted. Usually a rank- 1 model is all that is practically possible even after a lot of work. The rank- 1 model should always be attempted first. Then might be clever and try use this for initial values for a rank-2 model.
(7) If the number of sites is large then choose a random sample of them. For example, choose a maximum of 500 sites. This will reduce the memory and time expense of the computations.
(8) Try I.tolerance = TRUE or eq. tolerance = FALSE if the inputted data set is large, so as to reduce the computational expense. That's because the default, I.tolerance = FALSE and eq. tolerance $=$ TRUE, is very memory hungry.

By default, a rank-1 equal-tolerances QRR-VGLM model is fitted (see qrrvglm. control for the default control parameters). If Rank > 1 then the latent variables are always transformed so that they are uncorrelated. By default, the argument trace is TRUE meaning a running $\log$ is printed out while the computations are taking place. This is because the algorithm is computationally expensive, therefore users might think that their computers have frozen if trace $=$ FALSE
The argument Bestof in qrrvglm. control controls the number of models fitted (each uses different starting values) to the data. This argument is important because convergence may be to a local solution rather than the global solution. Using more starting values increases the chances of finding the global solution. Always plot an ordination diagram (use the generic function lvplot) and see if it looks sensible. Local solutions arise because the optimization problem is highly nonlinear, and this is particularly true for CAO.
Many of the arguments applicable to cqo are common to vglm and rrvglm. control. The most important arguments are Rank, noRRR, Bestof, I.tolerances, eq.tolerances, isd.latvar, and MUXfactor.

When fitting a 2-parameter model such as the negative binomial or gamma, it pays to have eq. tolerances $=$ TRUE and I. tolerances $=$ FALSE. This is because numerical problems can occur when fitting the model far away from the global solution when I. tolerances = TRUE. Setting the two arguments as described will slow down the computation considerably, however it is numerically more stable.
In Example 1 below, an unequal-tolerances rank-1 QRR-VGLM is fitted to the hunting spiders dataset, and Example 2 is the equal-tolerances version. The latter is less likely to have convergence problems compared to the unequal-tolerances model. In Example 3 below, an equal-tolerances rank-2 QRR-VGLM is fitted to the hunting spiders dataset. The numerical difficulties encountered in fitting the rank-2 model suggests a rank-1 model is probably preferable. In Example 4 below, constrained binary quadratic ordination (in old nomenclature, constrained Gaussian logit ordination) is fitted to some simulated data coming from a species packing model. With multivariate binary responses, one must use multiple. responses $=$ TRUE to indicate that the response (matrix) is multivariate. Otherwise, it is interpreted as a single binary response variable. In Example 5 below, the deviance residuals are plotted for each species. This is useful as a diagnostic plot. This is done by (re)regressing each species separately against the latent variable.
Sometime in the future, this function might handle input of the form $c q o(x, y)$, where $x$ and $y$ are matrices containing the environmental and species data respectively.

## Author(s)

Thomas W. Yee. Thanks to Alvin Sou for converting a lot of the original FORTRAN code into C.

## References

Yee, T. W. (2004). A new technique for maximum-likelihood canonical Gaussian ordination. Ecological Monographs, 74, 685-701.
ter Braak, C. J. F. and Prentice, I. C. (1988). A theory of gradient analysis. Advances in Ecological Research, 18, 271-317.

Yee, T. W. (2006). Constrained additive ordination. Ecology, 87, 203-213.

## See Also

qrrvglm. control, Coef.qrrvglm, predictqrrvglm, calibrate.qrrvglm, model.matrixqrrvglm, vcovqrrvglm, rcqo, cao, uqo, rrvglm, poissonff, binomialff, negbinomial, gamma2, lvplot. qrrvglm, perspqrrvglm, trplot.qrrvglm, vglm, set.seed, hspider, trap0.

## Examples

```
## Not run:
# Example 1; Fit an unequal tolerances model to the hunting spiders data
hspider[,1:6] <- scale(hspider[,1:6]) # Standardized environmental variables
set.seed(1234) # For reproducibility of the results
p1ut <- cqo(cbind(Alopacce, Alopcune, Alopfabr, Arctlute, Arctperi,
                    Auloalbi, Pardlugu, Pardmont, Pardnigr, Pardpull,
            Trocterr, Zoraspin) ~
        WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
        fam = poissonff, data = hspider, Crow1positive = FALSE,
        eq.tol = FALSE)
sort(deviance(p1ut, history = TRUE)) # A history of all the iterations
if (deviance(p1ut) > 1177) warning("suboptimal fit obtained")
S <- ncol(depvar(p1ut)) # Number of species
clr <- (1:(S+1))[-7] # Omits yellow
lvplot(p1ut, y = TRUE, lcol = clr, pch = 1:S, pcol = clr,
        las = 1) # Ordination diagram
legend("topright", leg = colnames(depvar(p1ut)), col = clr,
        pch = 1:S, merge = TRUE, bty = "n", lty = 1:S, lwd = 2)
(cp <- Coef(p1ut))
(a <- latvar(cp)[cp@latvar.order]) # Ordered site scores along the gradient
# Names of the ordered sites along the gradient:
rownames(latvar(cp))[cp@latvar.order]
(aa <- Opt(cp)[, cp@Optimum.order]) # Ordered optimums along the gradient
aa <- aa[!is.na(aa)] # Delete the species that is not unimodal
names(aa) # Names of the ordered optimums along the gradient
trplot(p1ut, which.species = 1:3, log = "xy", type = "b", lty = 1, lwd = 2,
        col = c("blue","red","green"), label = TRUE) -> ii # Trajectory plot
legend(0.00005, 0.3, paste(ii$species[, 1], ii$species[, 2], sep = " and "),
        lwd = 2, lty = 1, col = c("blue", "red", "green"))
```

```
abline(a = 0, b = 1, lty = "dashed")
S <- ncol(depvar(p1ut)) # Number of species
clr <- (1:(S+1))[-7] # Omits yellow
persp(p1ut, col = clr, label = TRUE, las = 1) # Perspective plot
# Example 2; Fit an equal tolerances model. Less numerically fraught.
set.seed(1234)
p1et <- cqo(cbind(Alopacce, Alopcune, Alopfabr, Arctlute, Arctperi,
                Auloalbi, Pardlugu, Pardmont, Pardnigr, Pardpull,
                Trocterr, Zoraspin) ~
    WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
    poissonff, data = hspider, Crow1positive = FALSE)
sort(deviance(p1et, history = TRUE)) # A history of all the iterations
if (deviance(p1et) > 1586) warning("suboptimal fit obtained")
S <- ncol(depvar(p1et)) # Number of species
clr <- (1:(S+1))[-7] # Omits yellow
persp(p1et, col = clr, label = TRUE, las = 1)
# Example 3: A rank-2 equal tolerances CQO model with Poisson data
# This example is numerically fraught... need I.toler = TRUE too.
set.seed(555)
p2 <- cqo(cbind(Alopacce, Alopcune, Alopfabr, Arctlute, Arctperi,
                        Auloalbi, Pardlugu, Pardmont, Pardnigr, Pardpull,
            Trocterr, Zoraspin) ~
        WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
        poissonff, data = hspider, Crow1positive = FALSE,
        I.toler = TRUE, Rank = 2, Bestof = 3, isd.latvar = c(2.1, 0.9))
sort(deviance(p2, history = TRUE)) # A history of all the iterations
if (deviance(p2) > 1127) warning("suboptimal fit obtained")
lvplot(p2, ellips = FALSE, label = TRUE, xlim = c(-3,4),
    C = TRUE, Ccol = "brown", sites = TRUE, scol = "grey",
    pcol = "blue", pch = "+", chull = TRUE, ccol = "grey")
# Example 4: species packing model with presence/absence data
set.seed(2345)
n <- 200; p <- 5; S <- 5
mydata <- rcqo(n, p, S, fam = "binomial", hi.abundance = 4,
    eq.tol = TRUE, es.opt = TRUE, eq.max = TRUE)
myform <- attr(mydata, "formula")
set.seed(1234)
b1et <- cqo(myform, binomialff(multiple.responses = TRUE, link = "clogloglink"),
        data = mydata)
sort(deviance(b1et, history = TRUE)) # A history of all the iterations
lvplot(b1et, y = TRUE, lcol = 1:S, pch = 1:S, pcol = 1:S, las = 1)
Coef(b1et)
# Compare the fitted model with the 'truth'
cbind(truth = attr(mydata, "concoefficients"), fitted = concoef(b1et))
```

```
# Example 5: Plot the deviance residuals for diagnostic purposes
set.seed(1234)
p1et <- cqo(cbind(Alopacce, Alopcune, Alopfabr, Arctlute, Arctperi,
                Auloalbi, Pardlugu, Pardmont, Pardnigr, Pardpull,
                Trocterr, Zoraspin) ~
    WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
    poissonff, data = hspider, eq.tol = TRUE, trace = FALSE)
sort(deviance(p1et, history = TRUE)) # A history of all the iterations
if (deviance(p1et) > 1586) warning("suboptimal fit obtained")
S <- ncol(depvar(p1et))
par(mfrow = c(3, 4))
for (ii in 1:S) {
    tempdata <- data.frame(latvar1 = c(latvar(p1et)),
                sppCounts = depvar(p1et)[, ii])
    tempdata <- transform(tempdata, myOffset = -0.5 * latvar1^2)
# For species ii, refit the model to get the deviance residuals
    fit1 <- vglm(sppCounts ~ offset(myOffset) + latvar1, poissonff,
                        data = tempdata, trace = FALSE)
# For checking: this should be 0
# print("max(abs(c(Coef(p1et)@B1[1,ii],Coef(p1et)@A[ii,1])-coef(fit1)))")
# print( max(abs(c(Coef(p1et)@B1[1,ii],Coef(p1et)@A[ii,1])-coef(fit1))) )
# Plot the deviance residuals
    devresid <- resid(fit1, type = "deviance")
    predvalues <- predict(fit1) + fit1@offset
    ooo <- with(tempdata, order(latvar1))
    plot(predvalues + devresid ~ latvar1, data = tempdata, col = "red",
            xlab = "latvar1", ylab = "", main = colnames(depvar(p1et))[ii])
    with(tempdata, lines(latvar1[ooo], predvalues[ooo], col = "blue"))
}
## End(Not run)
```

crashes Crashes on New Zealand Roads in 2009

## Description

A variety of reported crash data cross-classified by time (hour of the day) and day of the week, accumulated over 2009. These include fatalities and injuries (by car), trucks, motor cycles, bicycles and pedestrians. There are some alcohol-related data too.

## Usage

data(crashi)
data(crashf)
data(crashtr)

```
data(crashmc)
data(crashbc)
data(crashp)
data(alcoff)
data(alclevels)
```


## Format

Data frames with hourly times as rows and days of the week as columns. The alclevels dataset has hourly times and alcohol levels.

Mon, Tue, Wed, Thu, Fri, Sat, Sun Day of the week.
$\mathbf{0 - 3 0}, \mathbf{3 1 - 5 0}, 51-80,81-100,101-120,121-150,151-200,201-250,251-300,301-350,350+$ Blood alcohol level (milligrams alcohol per 100 millilitres of blood).

## Details

Each cell is the aggregate number of crashes reported at each hour-day combination, over the 2009 calendar year. The rownames of each data frame is the start time (hourly from midnight onwards) on a 24 hour clock, e.g., 21 means 9.00 pm to 9.59 pm .

For crashes, chrashi are the number of injuries by car, crashf are the number of fatalities by car (not included in chrashi), crashtr are the number of crashes involving trucks, crashmc are the number of crashes involving motorcyclists, crashbc are the number of crashes involving bicycles, and crashp are the number of crashes involving pedestrians. For alcohol-related offences, alcoff are the number of alcohol offenders from breath screening drivers, and alclevels are the blood alcohol levels of fatally injured drivers.

## Source

http://www.transport.govt.nz/research/Pages/Motor-Vehicle-Crashes-in-New-Zealand-2009.aspx. Thanks to Warwick Goold and Alfian F. Hadi for assistance.

## References

Motor Vehicles Crashes in New Zealand 2009; Statistical Statement Calendar Year 2009. Ministry of Transport, NZ Government; Yearly Report 2010. ISSN: 1176-3949

See Also
rrvglm, rcim, grc.

## Examples

```
## Not run: plot(unlist(alcoff), type = "l", frame.plot = TRUE,
    axes = FALSE, col = "blue", bty = "o",
    main = "Alcoholic offenders on NZ roads, aggregated over 2009",
    sub = "Vertical lines at midnight (purple) and noon (orange)",
    xlab = "Day/hour", ylab = "Number of offenders")
axis(1, at = 1 + (0:6) * 24 + 12, labels = colnames(alcoff))
axis(2, las = 1)
```

cratio

```
axis(3:4, labels = FALSE, tick = FALSE)
abline(v = sort(1 + c((0:7) * 24, (0:6) * 24 + 12)), lty = "dashed",
    col = c("purple", "orange"))
## End(Not run)
# Goodmans RC models
## Not run:
fitgrc1 <- grc(alcoff) # Rank-1 model
fitgrc2 <- grc(alcoff, Rank = 2, Corner = FALSE, Uncor = TRUE)
Coef(fitgrc2)
## End(Not run)
## Not run: biplot(fitgrc2, scaleA = 2.3, Ccol = "blue", Acol = "orange",
    Clabels = as.character(1:23), xlim = c(-1.3, 2.3),
        ylim = c(-1.2, 1))
## End(Not run)
```

cratio Ordinal Regression with Continuation Ratios

## Description

Fits a continuation ratio logit/probit/cloglog/cauchit/... regression model to an ordered (preferably) factor response.

## Usage

```
    cratio(link = "logitlink", parallel = FALSE, reverse = FALSE, zero = NULL,
```

        whitespace = FALSE)
    
## Arguments

link Link function applied to the $M$ continuation ratio probabilities. See Links for more choices.
parallel A logical, or formula specifying which terms have equal/unequal coefficients.
reverse Logical. By default, the continuation ratios used are $\eta_{j}=\operatorname{logit}(P[Y>j \mid Y \geq$ $j]$ ) for $j=1, \ldots, M$. If reverse is TRUE, then $\eta_{j}=\operatorname{logit}(P[Y<j+1 \mid Y \leq$ $j+1]$ ) will be used.
zero An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set $\{1,2, \ldots, M\}$. The default value means none are modelled as intercept-only terms.
whitespace See CommonVGAMffArguments for information.

## Details

In this help file the response $Y$ is assumed to be a factor with ordered values $1,2, \ldots, M+1$, so that $M$ is the number of linear/additive predictors $\eta_{j}$.

There are a number of definitions for the continuation ratio in the literature. To make life easier, in the VGAM package, we use continuation ratios and stopping ratios (see sratio). Stopping ratios deal with quantities such as logitlink ( $P[Y=j \mid Y>=j]$ ).

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Warning

No check is made to verify that the response is ordinal if the response is a matrix; see ordered.

## Note

The response should be either a matrix of counts (with row sums that are all positive), or a factor. In both cases, the $y$ slot returned by $\mathrm{vglm} / \mathrm{vgam} / \mathrm{rrvglm}$ is the matrix of counts.

For a nominal (unordered) factor response, the multinomial logit model (multinomial) is more appropriate.

Here is an example of the usage of the parallel argument. If there are covariates $x 1, x 2$ and $x 3$, then parallel $=$ TRUE $\sim x 1+x 2-1$ and parallel $=$ FALSE $\sim x 3$ are equivalent. This would constrain the regression coefficients for $x 1$ and $\times 2$ to be equal; those of the intercepts and $x 3$ would be different.

## Author(s)

Thomas W. Yee

## References

Agresti, A. (2013). Categorical Data Analysis, 3rd ed. Hoboken, NJ, USA: Wiley.
McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, 2nd ed. London: Chapman \& Hall.

Yee, T. W. (2010). The VGAM package for categorical data analysis. Journal of Statistical Software, 32, 1-34. http://www. jstatsoft.org/v32/i10/.

## See Also

sratio, acat, cumulative, multinomial, margeff, pneumo, logitlink, probitlink, clogloglink, cauchitlink.

## Examples

```
pneumo <- transform(pneumo, let = log(exposure.time))
(fit <- vglm(cbind(normal, mild, severe) ~ let,
    cratio(parallel = TRUE), data = pneumo))
coef(fit, matrix = TRUE)
constraints(fit)
predict(fit)
predict(fit, untransform = TRUE)
margeff(fit)
```

```
cumulative Ordinal Regression with Cumulative Probabilities
```


## Description

Fits a cumulative link regression model to a (preferably ordered) factor response.

## Usage

```
    cumulative(link = "logitlink", parallel = FALSE, reverse = FALSE,
                multiple.responses = FALSE, whitespace = FALSE)
```


## Arguments

link Link function applied to the $J$ cumulative probabilities. See Links for more choices, e.g., for the cumulative probitlink/clogloglink/cauchitlink/... models.
parallel A logical or formula specifying which terms have equal/unequal coefficients. See below for more information about the parallelism assumption. The default results in what some people call the generalized ordered logit model to be fitted. If parallel = TRUE then it does not apply to the intercept.
The partial proportional odds model can be fitted by assigning this argument something like parallel $=$ TRUE $\sim-1+x 3+x 5$ so that there is one regression coefficient for $x 3$ and $x 5$. Equivalently, setting parallel $=$ FALSE $\sim 1+x 2+x 4$ means $M$ regression coefficients for the intercept and $\times 2$ and $\times 4$. It is important that the intercept is never parallel.
reverse Logical. By default, the cumulative probabilities used are $P(Y \leq 1), P(Y \leq$ 2), $\ldots, P(Y \leq J)$. If reverse is TRUE then $P(Y \geq 2), P(Y \geq 3), \ldots$, $P(Y \geq J+1)$ are used.
This should be set to TRUE for link= gordlink, pordlink, nbordlink. For these links the cutpoints must be an increasing sequence; if reverse $=$ FALSE for then the cutpoints must be an decreasing sequence.
multiple.responses
Logical. Multiple responses? If TRUE then the input should be a matrix with values $1,2, \ldots, L$, where $L=J+1$ is the number of levels. Each column of the matrix is a response, i.e., multiple responses. A suitable matrix can be obtained from Cut.
whitespace See CommonVGAMffArguments for information.

## Details

In this help file the response $Y$ is assumed to be a factor with ordered values $1,2, \ldots, J+1$. Hence $M$ is the number of linear/additive predictors $\eta_{j}$, for cumulative() one has $M=J$.
This VGAM family function fits the class of cumulative link models to (hopefully) an ordinal response. By default, the non-parallel cumulative logit model is fitted, i.e.,

$$
\eta_{j}=\operatorname{logit}(P[Y \leq j])
$$

where $j=1,2, \ldots, M$ and the $\eta_{j}$ are not constrained to be parallel. This is also known as the non-proportional odds model. If the logit link is replaced by a complementary log-log link (clogloglink) then this is known as the proportional-hazards model.
In almost all the literature, the constraint matrices associated with this family of models are known. For example, setting parallel = TRUE will make all constraint matrices (except for the intercept) equal to a vector of $M 1$ 's. If the constraint matrices are equal, unknown and to be estimated, then this can be achieved by fitting the model as a reduced-rank vector generalized linear model (RRVGLM; see rrvglm). Currently, reduced-rank vector generalized additive models (RR-VGAMs) have not been implemented here.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

No check is made to verify that the response is ordinal if the response is a matrix; see ordered.

## Note

The response should be either a matrix of counts (with row sums that are all positive), or a factor. In both cases, the y slot returned by vglm/vgam/rrvglm is the matrix of counts. The formula must contain an intercept term. Other VGAM family functions for an ordinal response include acat, cratio, sratio. For a nominal (unordered) factor response, the multinomial logit model (multinomial) is more appropriate.
With the logit link, setting parallel = TRUE will fit a proportional odds model. Note that the TRUE here does not apply to the intercept term. In practice, the validity of the proportional odds assumption needs to be checked, e.g., by a likelihood ratio test (LRT). If acceptable on the data, then numerical problems are less likely to occur during the fitting, and there are less parameters. Numerical problems occur when the linear/additive predictors cross, which results in probabilities outside of $(0,1)$; setting parallel = TRUE will help avoid this problem.
Here is an example of the usage of the parallel argument. If there are covariates $\times 2, x 3$ and $x 4$, then parallel $=$ TRUE $\sim x 2+x 3-1$ and parallel $=$ FALSE $\sim x 4$ are equivalent. This would constrain the regression coefficients for $x 2$ and $x 3$ to be equal; those of the intercepts and $x 4$ would be different.

If the data is inputted in long format (not wide format, as in pneumo below) and the self-starting initial values are not good enough then try using mustart, coefstart and/or etatstart. See the example below.

To fit the proportional odds model one can use the VGAM family function propodds. Note that propodds (reverse) is equivalent to cumulative (parallel = TRUE, reverse = reverse) (which is equivalent to cumulative (parallel = TRUE, reverse = reverse, link = "logitlink")). It is for convenience only. A call to cumulative() is preferred since it reminds the user that a parallelism assumption is made, as well as being a lot more flexible.

## Author(s)

Thomas W. Yee

## References

Agresti, A. (2013). Categorical Data Analysis, 3rd ed. Hoboken, NJ, USA: Wiley.
Agresti, A. (2010). Analysis of Ordinal Categorical Data, 2nd ed. Hoboken, NJ, USA: Wiley.
Dobson, A. J. and Barnett, A. (2008). An Introduction to Generalized Linear Models, 3rd ed. Boca Raton, FL, USA: Chapman \& Hall/CRC Press.

McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, 2nd ed. London: Chapman \& Hall.
Simonoff, J. S. (2003). Analyzing Categorical Data, New York: Springer-Verlag.
Yee, T. W. (2010). The VGAM package for categorical data analysis. Journal of Statistical Software, 32, 1-34. http://www.jstatsoft.org/v32/i10/.
Yee, T. W. and Wild, C. J. (1996). Vector generalized additive models. Journal of the Royal Statistical Society, Series B, Methodological, 58, 481-493.

## See Also

propodds, R2latvar, ordsup, prplot, margeff, acat, cratio, sratio, multinomial, pneumo, Links, hdeff.vglm, logitlink, probitlink, clogloglink, cauchitlink, gordlink, pordlink, nbordlink, logistic1.

## Examples

```
# Fit the proportional odds model, p.179, in McCullagh and Nelder (1989)
pneumo <- transform(pneumo, let = log(exposure.time))
(fit <- vglm(cbind(normal, mild, severe) ~ let,
    cumulative(parallel = TRUE, reverse = TRUE), data = pneumo))
depvar(fit) # Sample proportions (good technique)
fit@y # Sample proportions (bad technique)
weights(fit, type = "prior") # Number of observations
coef(fit, matrix = TRUE)
constraints(fit) # Constraint matrices
apply(fitted(fit), 1, which.max) # Classification
apply(predict(fit, newdata = pneumo, type = "response"),
        1, which.max) # Classification
R2latvar(fit)
# Check that the model is linear in let -----------------------
fit2 <- vgam(cbind(normal, mild, severe) ~ s(let, df = 2),
    cumulative(reverse = TRUE), data = pneumo)
```

```
## Not run: plot(fit2, se = TRUE, overlay = TRUE, lcol = 1:2, scol = 1:2)
# Check the proportional odds assumption with a LRT ----------
(fit3 <- vglm(cbind(normal, mild, severe) ~ let,
            cumulative(parallel = FALSE, reverse = TRUE), data = pneumo))
pchisq(2 * (logLik(fit3) - logLik(fit)),
            df = length(coef(fit3)) - length(coef(fit)), lower.tail = FALSE)
lrtest(fit3, fit) # More elegant
# A factor() version of fit ----------------------------------
# This is in long format (cf. wide format above)
Nobs <- round(depvar(fit) * c(weights(fit, type = "prior")))
sumNobs <- colSums(Nobs) # apply(Nobs, 2, sum)
pneumo.long <-
    data.frame(symptoms = ordered(rep(rep(colnames(Nobs), nrow(Nobs)),
                                    times = c(t(Nobs))),
                                    levels = colnames(Nobs)),
            let = rep(rep(with(pneumo, let), each = ncol(Nobs)),
                    times = c(t(Nobs))))
with(pneumo.long, table(let, symptoms)) # Should be same as pneumo
(fit.long1 <- vglm(symptoms ~ let, data = pneumo.long, trace = TRUE,
    cumulative(parallel = TRUE, reverse = TRUE)))
coef(fit.long1, matrix = TRUE) # Should be as coef(fit, matrix = TRUE)
# Could try using mustart if fit.long1 failed to converge.
mymustart <- matrix(sumNobs / sum(sumNobs),
                            nrow(pneumo.long), ncol(Nobs), byrow = TRUE)
fit.long2 <- vglm(symptoms ~ let, mustart = mymustart,
    cumulative(parallel = TRUE, reverse = TRUE),
    data = pneumo.long, trace = TRUE)
coef(fit.long2, matrix = TRUE) # Should be as coef(fit, matrix = TRUE)
```

Dagum The Dagum Distribution

## Description

Density, distribution function, quantile function and random generation for the Dagum distribution with shape parameters a and $p$, and scale parameter scale.

## Usage

ddagum(x, scale = 1, shape1.a, shape2.p, log = FALSE)
pdagum(q, scale = 1, shape1.a, shape2.p, lower.tail = TRUE, log.p = FALSE)
qdagum(p, scale = 1, shape1.a, shape2.p, lower.tail = TRUE, log.p = FALSE)
rdagum(n, scale $=1$, shape1.a, shape2.p)

## Arguments

```
    x,q vector of quantiles.
    p vector of probabilities.
    n number of observations. If length (n) > 1, the length is taken to be the number
        required.
    shape1.a, shape2.p
                            shape parameters.
    scale scale parameter.
    log Logical. If log = TRUE then the logarithm of the density is returned.
    lower.tail, log.p
        Same meaning as in pnorm or qnorm.
```


## Details

See dagum, which is the VGAM family function for estimating the parameters by maximum likelihood estimation.

## Value

ddagum gives the density, pdagum gives the distribution function, qdagum gives the quantile function, and rdagum generates random deviates.

## Note

The Dagum distribution is a special case of the 4-parameter generalized beta II distribution.

## Author(s)

T. W. Yee and Kai Huang

## References

Kleiber, C. and Kotz, S. (2003). Statistical Size Distributions in Economics and Actuarial Sciences, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

dagum, genbetaII.

## Examples

```
probs <- seq(0.1, 0.9, by = 0.1)
shape1.a <- 1; shape2.p <- 2
# Should be 0:
max(abs(pdagum(qdagum(p = probs, shape1.a = shape1.a, shape2.p = shape2.p),
                                    shape1.a = shape1.a, shape2.p = shape2.p) - probs))
## Not run: par(mfrow = c(1, 2))
```

```
x <- seq(-0.01, 5, len = 401)
plot(x, dexp(x), type = "l", col = "black", ylab = "", las = 1, ylim = c(0, 1),
    main = "Black is standard exponential, others are ddagum(x, ...)")
lines(x, ddagum(x, shape1.a = shape1.a, shape2.p = 1), col = "orange")
lines(x, ddagum(x, shape1.a = shape1.a, shape2.p = 2), col = "blue")
lines(x, ddagum(x, shape1.a = shape1.a, shape2.p = 5), col = "green")
legend("topright", col = c("orange","blue","green"), lty = rep(1, len = 3),
        legend = paste("shape1.a =", shape1.a, ", shape2.p =", c(1, 2, 5)))
plot(x, pexp(x), type = "l", col = "black", ylab = "", las = 1,
    main = "Black is standard exponential, others are pdagum(x, ...)")
lines(x, pdagum(x, shape1.a = shape1.a, shape2.p = 1), col = "orange")
lines(x, pdagum(x, shape1.a = shape1.a, shape2.p = 2), col = "blue")
lines(x, pdagum(x, shape1.a = shape1.a, shape2.p = 5), col = "green")
legend("bottomright", col = c("orange","blue","green"), lty = rep(1, len = 3),
        legend = paste("shape1.a =", shape1.a, ", shape2.p =", c(1, 2, 5)))
## End(Not run)
```

dagum Dagum Distribution Family Function

## Description

Maximum likelihood estimation of the 3-parameter Dagum distribution.

## Usage

```
dagum(lscale = "loglink", lshape1.a = "loglink", lshape2.p = "loglink",
        iscale = NULL, ishape1.a = NULL, ishape2.p = NULL, imethod = 1,
        lss = TRUE, gscale = exp(-5:5), gshape1.a = seq(0.75, 4, by = 0.25),
        gshape2.p = exp(-5:5), probs.y = c(0.25, 0.5, 0.75), zero = "shape")
```


## Arguments

lss See CommonVGAMffArguments for important information.
lshape1.a, lscale, lshape2.p
Parameter link functions applied to the (positive) parameters a, scale, and p. See Links for more choices.
iscale, ishape1.a, ishape2.p, imethod, zero
See CommonVGAMffArguments for information. For imethod $=2$ a good initial value for ishape2. p is needed to obtain a good estimate for the other parameter.
gscale, gshape1.a, gshape2.p
See CommonVGAMffArguments for information.
probs.y See CommonVGAMffArguments for information.

## Details

The 3-parameter Dagum distribution is the 4-parameter generalized beta II distribution with shape parameter $q=1$. It is known under various other names, such as the Burr III, inverse Burr, betaK , and 3-parameter kappa distribution. It can be considered a generalized log-logistic distribution. Some distributions which are special cases of the 3-parameter Dagum are the inverse Lomax ( $a=$ 1), Fisk $(p=1)$, and the inverse paralogistic $(a=p)$. More details can be found in Kleiber and Kotz (2003).
The Dagum distribution has a cumulative distribution function

$$
F(y)=\left[1+(y / b)^{-a}\right]^{-p}
$$

which leads to a probability density function

$$
f(y)=a p y^{a p-1} /\left[b^{a p}\left\{1+(y / b)^{a}\right\}^{p+1}\right]
$$

for $a>0, b>0, p>0, y \geq 0$. Here, $b$ is the scale parameter scale, and the others are shape parameters. The mean is

$$
E(Y)=b \Gamma(p+1 / a) \Gamma(1-1 / a) / \Gamma(p)
$$

provided $-a p<1<a$; these are returned as the fitted values. This family function handles multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

See the notes in genbetaII.
From Kleiber and Kotz (2003), the MLE is rather sensitive to isolated observations located sufficiently far from the majority of the data. Reliable estimation of the scale parameter require $n>7000$, while estimates for $a$ and $p$ can be considered unbiased for $n>2000$ or 3000 .

## Author(s)

T. W. Yee

## References

Kleiber, C. and Kotz, S. (2003). Statistical Size Distributions in Economics and Actuarial Sciences, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

Dagum, genbetaII, betaII, sinmad, fisk, inv.lomax, lomax, paralogistic, inv. paralogistic, simulate.vlm.

## Examples

```
ddata <- data.frame(y = rdagum(n = 3000, scale = exp(2),
    shape1 = exp(1), shape2 = exp(1)))
fit <- vglm(y ~ 1, dagum(lss = FALSE), data = ddata, trace = TRUE)
fit <- vglm(y ~ 1, dagum(lss = FALSE, ishape1.a = exp(1)),
            data = ddata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```


## Description

Density for the AR-1 model.

## Usage

dAR1 (x, drift $=0$, var.error $=1$, ARcoef1 $=0.0$,
type.likelihood = c("exact", "conditional"), log = FALSE)

## Arguments

$x$, vector of quantiles.
drift the scaled mean (also known as the drift parameter), $\mu^{*}$. Note that the mean is $\mu^{*} /(1-\rho)$. The default corresponds to observations that have mean 0 .
log Logical. If TRUE then the logarithm of the density is returned.
type.likelihood, var.error, ARcoef1
See AR1. The argument ARcoef1 is $\rho$. The argument var. error is the variance of the i.i.d. random noise, i.e., $\sigma^{2}$. If type.likelihood = "conditional" then the first element or row of the result is currently assigned NA-this is because the density of the first observation is effectively ignored.

## Details

Most of the background to this function is given in AR1. All the arguments are converted into matrices, and then all their dimensions are obtained. They are then coerced into the same size: the number of rows is the maximum of all the single rows, and ditto for the number of columns.

## Value

dAR1 gives the density.

## Author(s)

T. W. Yee and Victor Miranda

## See Also

AR1.

## Examples

```
nn <- 100; set.seed(1)
tdata <- data.frame(index = 1:nn,
    TS1 = arima.sim(nn, model = list(ar = -0.50),
        sd = exp(1)))
fit1 <- vglm(TS1 ~ 1, AR1, data = tdata, trace = TRUE)
rhobitlink(-0.5)
coef(fit1, matrix = TRUE)
(Cfit1 <- Coef(fit1))
summary(fit1) # SEs are useful to know
logLik(fit1)
sum(dAR1(depvar(fit1), drift = Cfit1[1], var.error = (Cfit1[2])^2,
    ARcoef1 = Cfit1[3], log = TRUE))
fit2 <- vglm(TS1 ~ 1, AR1(type.likelihood = "cond"), data = tdata, trace = TRUE)
(Cfit2 <- Coef(fit2)) # Okay for intercept-only models
logLik(fit2)
head(keep <- dAR1(depvar(fit2), drift = Cfit2[1], var.error = (Cfit2[2])^2,
    ARcoef1 = Cfit2[3], type.likelihood = "cond", log = TRUE))
sum(keep[-1])
```

deermice

Captures of Peromyscus maniculatus (Also Known as Deer Mice).

## Description

## Captures of Peromyscus maniculatus collected at East Stuart Gulch, Colorado, USA.

## Usage

data(deermice)

## Format

The format is a data frame.

## Details

Peromyscus maniculatus is a rodent native to North America. The deer mouse is small in size, only about 8 to 10 cm long, not counting the length of the tail.

Originally, the columns of this data frame represent the sex ( $m$ or $f$ ), the ages ( $y$ : young, sa: semiadult, a: adult), the weights in grams, and the capture histories of 38 individuals over 6 trapping occasions (1: captured, 0 : not captured).

The data set was collected by V. Reid and distributed with the CAPTURE program of Otis et al. (1978).
deermice has 38 deermice whereas Perom had 36 deermice (Perom has been withdrawn.) In deermice the two semi-adults have been classified as adults. The sex variable has 1 for female, and 0 for male.

## References

Huggins, R. M. (1991). Some practical aspects of a conditional likelihood approach to capture experiments. Biometrics, 47, 725-732.
Otis, D. L. et al. (1978). Statistical inference from capture data on closed animal populations, Wildlife Monographs, 62, 3-135.

## See Also

posbernoulli.b, posbernoulli.t, fill1.

## Examples

```
head(deermice)
## Not run:
fit1 <- vglm(cbind(y1, y2, y3, y4, y5, y6) ~ sex + age,
    posbernoulli.t(parallel.t = TRUE), data = deermice, trace = TRUE)
coef(fit1)
coef(fit1, matrix = TRUE)
## End(Not run)
```

```
deplot.lmscreg Density Plot for LMS Quantile Regression
```


## Description

Plots a probability density function associated with a LMS quantile regression.

## Usage

deplot.lmscreg(object, newdata $=$ NULL, $x 0, y . a r g$, show.plot $=$ TRUE, ...)

## Arguments

object A VGAM quantile regression model, i.e., an object produced by modelling functions such as vglm and vgam with a family function beginning with "lms.", e.g., lms.yjn.
newdata Optional data frame containing secondary variables such as sex. It should have a maximum of one row. The default is to use the original data.
x0 Numeric. The value of the primary variable at which to make the 'slice'.

| y.arg | Numerical vector. The values of the response variable at which to evaluate the <br> density. This should be a grid that is fine enough to ensure the plotted curves are <br> smooth. |
| :--- | :--- |
| show.plot | Logical. Plot it? If FALSE no plot will be done. |
| $\ldots$ | Graphical parameter that are passed into plotdeplot. Imscreg. |

## Details

This function calls, e.g., deplot. lms.yjn in order to compute the density function.

## Value

The original object but with a list placed in the slot post, called @post\$deplot. The list has components

| newdata | The argument newdata above, or a one-row data frame constructed out of the $x 0$ argument. |
| :---: | :---: |
| y | The argument y.arg above. |
| density | Vector of the density function values evaluated at y . arg. |

## Note

plotdeplot.lmscreg actually does the plotting.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2004). Quantile regression via vector generalized additive models. Statistics in Medicine, 23, 2295-2315.

## See Also

plotdeplot.lmscreg, qtplot. 1 mscreg , $1 \mathrm{~ms} . \mathrm{bcn}, 1 \mathrm{~ms} . \mathrm{bcg}, 1 \mathrm{~ms} . y j n$.

## Examples

```
## Not run:
fit <- vgam(BMI ~ s(age, df = c(4, 2)), fam = lms.bcn(zero = 1), data = bmi.nz)
ygrid <- seq(15, 43, by = 0.25)
deplot(fit, x0 = 20, y = ygrid, xlab = "BMI", col = "green", llwd = 2,
            main = "BMI distribution at ages 20 (green), 40 (blue), 60 (red)")
deplot(fit, x0 = 40, y = ygrid, add = TRUE, col = "blue", llwd = 2)
deplot(fit, x0 = 60, y = ygrid, add = TRUE, col = "red", llwd = 2) -> a
names(a@post$deplot)
a@post$deplot$newdata
head(a@post$deplot$y)
```

    head(a@post\$deplot\$density)
    \#\# End(Not run)
    depvar Response Variable Extracted
    
## Description

A generic function that extracts the response/dependent variable from objects.

## Usage

depvar(object, ...)

## Arguments

object An object that has some response/dependent variable.
... Other arguments fed into the specific methods function of the model. In particular, sometimes type $=c(" l m ", " l m 2 ")$ is available, in which case the first one is chosen if the user does not input a value. The latter value corresponds to argument form2, and sometimes a response for that is optional.

## Details

By default this function is preferred to calling fit@y, say.

## Value

The response/dependent variable, usually as a matrix or vector.

## Author(s)

Thomas W. Yee

## See Also

model.matrix, vglm.

## Examples

```
pneumo <- transform(pneumo, let = log(exposure.time))
(fit <- vglm(cbind(normal, mild, severe) ~ let, propodds, data = pneumo))
fit@y # Sample proportions (not recommended)
depvar(fit) # Better than using fit@y; dependent variable (response)
weights(fit, type = "prior") # Number of observations
```

```
dextlogF Extended log-F Distribution
```


## Description

Density for the extended log-F distribution.

## Usage

```
dextlogF(x, lambda, tau, location = 0, scale = 1, log = FALSE)
```


## Arguments

x
lambda, tau
location, scale
See extlogF1.
$\log \quad$ If TRUE then the log density is returned, else the density.

## Details

The details are given in extlogF1.

## Value

dextlogF gives the density.

## Author(s)

T. W. Yee

## See Also

extlogF1, dalap.

## Examples

```
## Not run: x <- seq(-2, 8, by = 0.1); mytau <- 0.25; mylambda <- 0.2
plot(x, dextlogF(x, mylambda, tau = mytau), type = "l",
    las = 1, col = "blue", ylab = "PDF (log-scale)", log = "y",
    main = "Extended log-F density function is blue",
    sub = "Asymmetric Laplace is orange dashed")
lines(x, dalap(x, tau = mytau, scale = 3.5), col = "orange", lty = 2)
abline(v = 0, col = "gray", lty = 2)
## End(Not run)
```

```
df.residual Residual Degrees-of-Freedom
```


## Description

Returns the residual degrees-of-freedom extracted from a fitted VGLM object.

## Usage

df.residual_vlm(object, type = c("vlm", "lm"), ...)

## Arguments

object an object for which the degrees-of-freedom are desired, e.g., a vglm object.
type the type of residual degrees-of-freedom wanted. In some applications the 'usual' LM-type value may be more appropriate. The default is the first choice.
... additional optional arguments.

## Details

When a VGLM is fitted, a large (VLM) generalized least squares (GLS) fit is done at each IRLS iteration. To do this, an ordinary least squares (OLS) fit is performed by transforming the GLS using Cholesky factors. The number of rows is $M$ times the 'ordinary' number of rows of the LM-type model: $n M$. Here, $M$ is the number of linear/additive predictors. So the formula for the VLM-type residual degrees-of-freedom is $n M-p^{*}$ where $p^{*}$ is the number of columns of the 'big' VLM matrix. The formula for the LM-type residual degrees-of-freedom is $n-p_{j}$ where $p_{j}$ is the number of columns of the 'ordinary' LM matrix corresponding to the $j$ th linear/additive predictor.

## Value

The value of the residual degrees-of-freedom extracted from the object. When type = "vlm" this is a single integer, and when type $=" l m "$ this is a $M$-vector of integers.

## See Also

vglm, deviance, lm, anova.vglm,

## Examples

```
pneumo <- transform(pneumo, let = log(exposure.time))
(fit <- vglm(cbind(normal, mild, severe) ~ let, propodds, data = pneumo))
head(model.matrix(fit, type = "vlm"))
head(model.matrix(fit, type = "lm"))
df.residual(fit, type = "vlm") # n * M - p_VLM
nobs(fit, type = "vlm") # n * M
nvar(fit, type = "vlm") # p_VLM
```

```
df.residual(fit, type = "lm") # n - p_LM(j); Useful in some situations
nobs(fit, type = "lm") # n
nvar(fit, type = "lm") # p_LM
    nvar_vlm(fit, type = "lm") # p_LM(j) (<= p_LM elementwise)
```

    dgenpois The Generalized Poisson Distribution
    
## Description

Density for the Generalized Poisson Distribution.

## Usage

dgenpois(x, lambda $=0$, theta, log $=$ FALSE)

## Arguments

$x$, vector of quantiles.
lambda, theta See genpoisson. The default value of lambda corresponds to an ordinary Poisson distribution.
$\log \quad$ Logical. If TRUE then the logarithm of the density is returned.

## Details

Most of the background to this function is given in genpoisson. Some warnings relevant to this distribution are given there, especially relating to the complicated range of the parameter lambda about or near -1 .

Note that numerical round off errors etc. can occur; see below for an example.

## Value

dgenpois gives the density. The value NaN is returned for elements not satisfying the parameter restrictions, e.g., if $\lambda>1$.

## Author(s)

T. W. Yee

## See Also

genpoisson, dpois.

## Examples

```
sum(dgenpois(0:1000, lambda = -0.5, theta = 2)) # Not perfect...
## Not run:
lambda <- -0.2; theta <- 2; y <- 0:10
proby <- dgenpois(y, lambda = lambda, theta = theta, log = FALSE)
plot(y, proby, type = "h", col = "blue", lwd = 2, ylab = "P[Y=y]",
    main = paste("Y ~ Generalized Poisson(lambda=", lambda,
            ", theta=", theta, ")", sep = ""), las = 1,
    sub = "Orange is the Poisson probability function")
sum(proby)
lines(y + 0.1, dpois(y, theta), type = "h", lwd = 2, col = "orange")
## End(Not run)
```

dhuber Huber's Least Favourable Distribution

## Description

Density, distribution function, quantile function and random generation for Huber's least favourable distribution, see Huber and Ronchetti (2009).

## Usage

dhuber ( $\mathrm{x}, \mathrm{k}=0.862$, mu $=0$, sigma $=1$, log $=$ FALSE)
edhuber (x, k = 0.862, mu $=0$, sigma $=1$, log $=$ FALSE) $\operatorname{rhuber}(\mathrm{n}, \mathrm{k}=0.862$, mu $=0$, sigma $=1$ ) qhuber $(\mathrm{p}, \mathrm{k}=0.862, \mathrm{mu}=0$, sigma $=1$, lower.tail $=$ TRUE, log. $\mathrm{p}=$ FALSE $)$ $\operatorname{phuber}(\mathrm{q}, \mathrm{k}=0.862, \mathrm{mu}=0$, sigma $=1$, lower.tail $=$ TRUE, log. $\mathrm{p}=$ FALSE $)$

## Arguments

$x, q \quad$ numeric vector, vector of quantiles.
$p \quad$ vector of probabilities.
$\mathrm{n} \quad$ number of random values to be generated. If length $(\mathrm{n})>1$ then the length is taken to be the number required.
$\mathrm{k} \quad$ numeric. Borderline value of central Gaussian part of the distribution. This is known as the tuning constant, and should be positive. For example, $k=0.862$ refers to a $20 \%$ contamination neighborhood of the Gaussian distribution. If $k=$ 1.40 then this is $5 \%$ contamination.
mu numeric. distribution mean.
sigma numeric. Distribution scale (sigma $=1$ defines the distribution in standard form, with standard Gaussian centre).
$\log \quad$ Logical. If log = TRUE then the logarithm of the result is returned.
lower.tail, log.p
Same meaning as in pnorm or qnorm.

## Details

Details are given in huber2, the VGAM family function for estimating the parameters mu and sigma.

## Value

dhuber gives out a vector of density values.
edhuber gives out a list with components val (density values) and eps (contamination proportion).
rhuber gives out a vector of random numbers generated by Huber's least favourable distribution.
phuber gives the distribution function, qhuber gives the quantile function.

## Author(s)

Christian Hennig wrote [d, ed, r]huber() (from smoothmest) and slight modifications were made by T. W. Yee to replace looping by vectorization and addition of the log argument. Arash Ardalan wrote [pq]huber (), and two arguments for these were implemented by Kai Huang. This helpfile was adapted from smoothmest.

## See Also

huber2.

## Examples

```
set.seed(123456)
edhuber(1:5, k = 1.5)
rhuber(5)
## Not run: mu <- 3; xx <- seq(-2, 7, len = 100) # Plot CDF and PDF
plot(xx, dhuber(xx, mu = mu), type = "l", col = "blue", las = 1, ylab = "",
    main = "blue is density, orange is cumulative distribution function",
    sub = "Purple lines are the 10,20,...,90 percentiles",
    ylim = 0:1)
abline(h = 0, col = "blue", lty = 2)
lines(xx, phuber(xx, mu = mu), type = "l", col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qhuber(probs, mu = mu)
lines(Q, dhuber(Q, mu = mu), col = "purple", lty = 3, type = "h")
lines(Q, phuber(Q, mu = mu), col = "purple", lty = 3, type = "h")
abline(h = probs, col = "purple", lty = 3)
phuber(Q, mu = mu) - probs # Should be all 0s
## End(Not run)
```


## Description

Density, distribution function, quantile function, and random generation for the differenced zeta distribution.

## Usage

ddiffzeta(x, shape, start $=1$, log $=$ FALSE)
pdiffzeta(q, shape, start $=1$, lower.tail = TRUE)
qdiffzeta(p, shape, start = 1)
rdiffzeta(n, shape, start = 1)

## Arguments

| $x, q, p, n$ | Same as in runif. |
| :--- | :--- |
| shape, start | Details at diffzeta. |
| log, lower.tail |  |
|  | Same as in runif. |

## Details

This distribution appears to work well on the distribution of English words in such texts. Some more details are given in diffzeta.

## Value

ddiffzeta gives the density, pdiffzeta gives the distribution function, qdiffzeta gives the quantile function, and rdiffzeta generates random deviates.

## Note

Given some response data, the VGAM family function diffzeta estimates the parameter shape.
Function pdiffzeta() suffers from the problems that plog sometimes has, i.e., when $p$ is very close to 1 .

## Author(s)

T. W. Yee

## See Also

```
    diffzeta, zetaff, zipf, Oizeta.
```


## Examples

```
ddiffzeta(1:20, 0.5, start = 2)
rdiffzeta(20, 0.5)
## Not run: shape <- 0.8; x <- 1:10
plot(x, ddiffzeta(x, shape = shape), type = "h", ylim = 0:1,
    sub = "shape=0.8", las = 1, col = "blue", ylab = "Probability",
    main = "Differenced zeta distribution: blue=PMF; orange=CDF")
lines(x + 0.1, pdiffzeta(x, shape = shape), col = "orange", lty = 3, type = "h")
## End(Not run)
```

diffzeta Differenced Zeta Distribution Family Function

## Description

Estimates the parameter of the differenced zeta distribution.

## Usage

diffzeta(start = 1, lshape = "loglink", ishape = NULL)

## Arguments

lshape, ishape Same as zetaff.
start Smallest value of the support of the distribution. Must be a positive integer.

## Details

The PMF is

$$
P(Y=y)=(a / y)^{s}-(a /(1+y))^{s}, s>0, y=a, a+1, \ldots
$$

where $s$ is the positive shape parameter, and $a$ is start. According to Moreno-Sanchez et al. (2016), this model fits quite well to about 40 percent of all the English books in the Project Gutenberg data base (about 30,000 texts). Multiple responses are handled.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Author(s)

T. W. Yee

## References

Moreno-Sanchez, I., Font-Clos, F. and Corral, A. (2016). Large-Scale Analysis of Zipf's Law in English Texts, PLoS ONE, 11(1), 1-19.

## See Also

Diffzeta, zetaff, zeta, zipf, zipf.

## Examples

```
odata <- data.frame(x2 = runif(nn <- 1000)) # Artificial data
odata <- transform(odata, shape = loglink(-0.25 + x2, inverse = TRUE))
odata <- transform(odata, y1 = rdiffzeta(nn, shape))
with(odata, table(y1))
ofit <- vglm(y1 ~ x2, diffzeta, data = odata, trace = TRUE, crit = "coef")
coef(ofit, matrix = TRUE)
```

```
dirichlet Fitting a Dirichlet Distribution
```


## Description

Fits a Dirichlet distribution to a matrix of compositions.

## Usage

dirichlet(link = "loglink", parallel = FALSE, zero = NULL, imethod = 1)

## Arguments

link Link function applied to each of the $M$ (positive) shape parameters $\alpha_{j}$. See Links for more choices. The default gives $\eta_{j}=\log \left(\alpha_{j}\right)$.
parallel, zero, imethod
See CommonVGAMffArguments for more information.

## Details

In this help file the response is assumed to be a $M$-column matrix with positive values and whose rows each sum to unity. Such data can be thought of as compositional data. There are $M$ linear/additive predictors $\eta_{j}$.

The Dirichlet distribution is commonly used to model compositional data, including applications in genetics. Suppose $\left(Y_{1}, \ldots, Y_{M}\right)^{T}$ is the response. Then it has a Dirichlet distribution if $\left(Y_{1}, \ldots, Y_{M-1}\right)^{T}$ has density

$$
\frac{\Gamma\left(\alpha_{+}\right)}{\prod_{j=1}^{M} \Gamma\left(\alpha_{j}\right)} \prod_{j=1}^{M} y_{j}^{\alpha_{j}-1}
$$

where $\alpha_{+}=\alpha_{1}+\cdots+\alpha_{M}, \alpha_{j}>0$, and the density is defined on the unit simplex

$$
\Delta_{M}=\left\{\left(y_{1}, \ldots, y_{M}\right)^{T}: y_{1}>0, \ldots, y_{M}>0, \sum_{j=1}^{M} y_{j}=1\right\}
$$

One has $E\left(Y_{j}\right)=\alpha_{j} / \alpha_{+}$, which are returned as the fitted values. For this distribution Fisher scoring corresponds to Newton-Raphson.
The Dirichlet distribution can be motivated by considering the random variables $\left(G_{1}, \ldots, G_{M}\right)^{T}$ which are each independent and identically distributed as a gamma distribution with density $f\left(g_{j}\right)=$ $g_{j}^{\alpha_{j}-1} e^{-g_{j}} / \Gamma\left(\alpha_{j}\right)$. Then the Dirichlet distribution arises when $Y_{j}=G_{j} /\left(G_{1}+\cdots+G_{M}\right)$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.
When fitted, the fitted.values slot of the object contains the $M$-column matrix of means.

## Note

The response should be a matrix of positive values whose rows each sum to unity. Similar to this is count data, where probably a multinomial logit model (multinomial) may be appropriate. Another similar distribution to the Dirichlet is the Dirichlet-multinomial (see dirmultinomial).

## Author(s)

Thomas W. Yee

## References

Lange, K. (2002). Mathematical and Statistical Methods for Genetic Analysis, 2nd ed. New York: Springer-Verlag.

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011). Statistical Distributions, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

## See Also

rdiric, dirmultinomial, multinomial, simplex.

## Examples

```
ddata <- data.frame(rdiric(n = 1000,
                                    shape = exp(c(y1 = -1, y2 = 1, y3 = 0))))
fit <- vglm(cbind(y1, y2, y3) ~ 1, dirichlet,
    data = ddata, trace = TRUE, crit = "coef")
Coef(fit)
coef(fit, matrix = TRUE)
head(fitted(fit))
```


## Description

Fits a Dirichlet-multinomial distribution to a matrix of non-negative integers.

## Usage

```
dirmul.old(link = "loglink", ialpha = 0.01, parallel = FALSE, zero = NULL)
```


## Arguments

link Link function applied to each of the $M$ (positive) shape parameters $\alpha_{j}$ for $j=$ $1, \ldots, M$. See Links for more choices. Here, $M$ is the number of columns of the response matrix.
ialpha Numeric vector. Initial values for the alpha vector. Must be positive. Recycled to length $M$.
parallel A logical, or formula specifying which terms have equal/unequal coefficients.
zero An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set $\{1,2, \ldots, M\}$.

## Details

The Dirichlet-multinomial distribution, which is somewhat similar to a Dirichlet distribution, has probability function

$$
P\left(Y_{1}=y_{1}, \ldots, Y_{M}=y_{M}\right)=\binom{2 y_{*}}{y_{1}, \ldots, y_{M}} \frac{\Gamma\left(\alpha_{+}\right)}{\Gamma\left(2 y_{*}+\alpha_{+}\right)} \prod_{j=1}^{M} \frac{\Gamma\left(y_{j}+\alpha_{j}\right)}{\Gamma\left(\alpha_{j}\right)}
$$

for $\alpha_{j}>0, \alpha_{+}=\alpha_{1}+\cdots+\alpha_{M}$, and $2 y_{*}=y_{1}+\cdots+y_{M}$. Here, $\binom{a}{b}$ means " $a$ choose $b$ " and refers to combinations (see choose). The (posterior) mean is

$$
E\left(Y_{j}\right)=\left(y_{j}+\alpha_{j}\right) /\left(2 y_{*}+\alpha_{+}\right)
$$

for $j=1, \ldots, M$, and these are returned as the fitted values as a $M$-column matrix.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Note

The response should be a matrix of non-negative values. Convergence seems to slow down if there are zero values. Currently, initial values can be improved upon.
This function is almost defunct and may be withdrawn soon. Use dirmultinomial instead.

## Author(s)

Thomas W. Yee

## References

Lange, K. (2002). Mathematical and Statistical Methods for Genetic Analysis, 2nd ed. New York: Springer-Verlag.
Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011). Statistical Distributions, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.
Paul, S. R., Balasooriya, U. and Banerjee, T. (2005). Fisher information matrix of the Dirichletmultinomial distribution. Biometrical Journal, 47, 230-236.
Tvedebrink, T. (2010). Overdispersion in allelic counts and $\theta$-correction in forensic genetics. Theoretical Population Biology, 78, 200-210.

## See Also

dirmultinomial, dirichlet, betabinomialff, multinomial.

## Examples

```
# Data from p. }50\mathrm{ of Lange (2002)
alleleCounts <- c(2, 84, 59, 41, 53, 131, 2, 0,
            0, 50, 137, 78, 54, 51, 0, 0,
            0, 80, 128, 26, 55, 95, 0, 0,
            0, 16, 40, 8, 68, 14, 7, 1)
dim(alleleCounts) <- c(8, 4)
alleleCounts <- data.frame(t(alleleCounts))
dimnames(alleleCounts) <- list(c("White","Black","Chicano","Asian"),
                            paste("Allele", 5:12, sep = ""))
set.seed(123) # @initialize uses random numbers
fit <- vglm(cbind(Allele5,Allele6,Allele7,Allele8,Allele9,
                    Allele10,Allele11,Allele12) ~ 1, dirmul.old,
        trace = TRUE, crit = "c", data = alleleCounts)
(sfit <- summary(fit))
vcov(sfit)
round(eta2theta(coef(fit), fit@misc$link, fit@misc$earg), digits = 2) # not preferred
round(Coef(fit), digits = 2) # preferred
round(t(fitted(fit)), digits = 4) # 2nd row of Table 3.5 of Lange (2002)
coef(fit, matrix = TRUE)
pfit <- vglm(cbind(Allele5,Allele6,Allele7,Allele8,Allele9,
            Allele10,Allele11,Allele12) ~ 1,
        dirmul.old(parallel = TRUE), trace = TRUE,
        data = alleleCounts)
round(eta2theta(coef(pfit, matrix = TRUE), pfit@misc$link,
            pfit@misc$earg), digits = 2) # 'Right' answer
round(Coef(pfit), digits = 2) # 'Wrong' answer due to parallelism constraint
```


## Description

Fits a Dirichlet-multinomial distribution to a matrix response.

## Usage

dirmultinomial(lphi = "logitlink", iphi = 0.10, parallel = FALSE, zero = "M")

## Arguments

lphi Link function applied to the $\phi$ parameter, which lies in the open unit interval $(0,1)$. See Links for more choices.
iphi $\quad$ Numeric. Initial value for $\phi$. Must be in the open unit interval $(0,1)$. If a failure to converge occurs then try assigning this argument a different value.
parallel A logical (formula not allowed here) indicating whether the probabilities $\pi_{1}, \ldots, \pi_{M-1}$ are to be equal via equal coefficients. Note $\pi_{M}$ will generally be different from the other probabilities. Setting parallel = TRUE will only work if you also set zero $=$ NULL because of interference between these arguments (with respect to the intercept term).
zero An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set $\{1,2, \ldots, M\}$. If the character " M " then this means the numerical value $M$, which corresponds to linear/additive predictor associated with $\phi$. Setting zero $=$ NULL means none of the values from the set $\{1,2, \ldots, M\}$.

## Details

The Dirichlet-multinomial distribution arises from a multinomial distribution where the probability parameters are not constant but are generated from a multivariate distribution called the Dirichlet distribution. The Dirichlet-multinomial distribution has probability function

$$
P\left(Y_{1}=y_{1}, \ldots, Y_{M}=y_{M}\right)=\binom{N_{*}}{y_{1}, \ldots, y_{M}} \frac{\prod_{j=1}^{M} \prod_{r=1}^{y_{j}}\left(\pi_{j}(1-\phi)+(r-1) \phi\right)}{\prod_{r=1}^{N_{*}}(1-\phi+(r-1) \phi)}
$$

where $\phi$ is the over-dispersion parameter and $N_{*}=y_{1}+\cdots+y_{M}$. Here, $\binom{a}{b}$ means " $a$ choose $b "$ and refers to combinations (see choose). The above formula applies to each row of the matrix response. In this VGAM family function the first $M-1$ linear/additive predictors correspond to the first $M-1$ probabilities via

$$
\eta_{j}=\log (P[Y=j] / P[Y=M])=\log \left(\pi_{j} / \pi_{M}\right)
$$

where $\eta_{j}$ is the $j$ th linear/additive predictor ( $\eta_{M}=0$ by definition for $P[Y=M]$ but not for $\phi$ ) and $j=1, \ldots, M-1$. The $M$ th linear/additive predictor corresponds to lphi applied to $\phi$.

Note that $E\left(Y_{j}\right)=N_{*} \pi_{j}$ but the probabilities (returned as the fitted values) $\pi_{j}$ are bundled together as a $M$-column matrix. The quantities $N_{*}$ are returned as the prior weights.
The beta-binomial distribution is a special case of the Dirichlet-multinomial distribution when $M=$ 2; see betabinomial. It is easy to show that the first shape parameter of the beta distribution is shape $1=\pi(1 / \phi-1)$ and the second shape parameter is shape $2=(1-\pi)(1 / \phi-1)$. Also, $\phi=1 /(1+$ shape $1+$ shape 2$)$, which is known as the intra-cluster correlation coefficient.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

If the model is an intercept-only model then @misc (which is a list) has a component called shape which is a vector with the $M$ values $\pi_{j}(1 / \phi-1)$.

## Warning

This VGAM family function is prone to numerical problems, especially when there are covariates.

## Note

The response can be a matrix of non-negative integers, or else a matrix of sample proportions and the total number of counts in each row specified using the weights argument. This dual input option is similar to multinomial.

To fit a 'parallel' model with the $\phi$ parameter being an intercept-only you will need to use the constraints argument.
Currently, Fisher scoring is implemented. To compute the expected information matrix a for loop is used; this may be very slow when the counts are large. Additionally, convergence may be slower than usual due to round-off error when computing the expected information matrices.

## Author(s)

Thomas W. Yee

## References

Paul, S. R., Balasooriya, U. and Banerjee, T. (2005). Fisher information matrix of the Dirichletmultinomial distribution. Biometrical Journal, 47, 230-236.

Tvedebrink, T. (2010). Overdispersion in allelic counts and $\theta$-correction in forensic genetics. Theoretical Population Biology, 78, 200-210.
Yu, P. and Shaw, C. A. (2014). An Efficient Algorithm for Accurate Computation of the DirichletMultinomial Log-Likelihood Function. Bioinformatics, 30, 1547-54.

## See Also

dirmul.old, betabinomial, betabinomialff, dirichlet, multinomial.

## Examples

```
    nn <- 5; M <- 4; set.seed(1)
    ydata <- data.frame(round(matrix(runif(nn * M, max = 100), nn, M))) # Integer counts
    colnames(ydata) <- paste("y", 1:M, sep = "")
    fit <- vglm(cbind(y1, y2, y3, y4) ~ 1, dirmultinomial,
            data = ydata, trace = TRUE)
    head(fitted(fit))
    depvar(fit) # Sample proportions
    weights(fit, type = "prior", matrix = FALSE) # Total counts per row
    ## Not run:
    ydata <- transform(ydata, x2 = runif(nn))
    fit <- vglm(cbind(y1, y2, y3, y4) ~ x2, dirmultinomial,
    data = ydata, trace = TRUE)
    Coef(fit)
    coef(fit, matrix = TRUE)
    (sfit <- summary(fit))
    vcov(sfit)
    ## End(Not run)
```

    dlogF log F Distribution
    
## Description

Density for the $\log \mathrm{F}$ distribution.

## Usage

$d \log F(x$, shape1, shape2, $\log =F A L S E)$

## Arguments

X
Vector of quantiles.
shape1, shape2 Positive shape parameters.
log if TRUE then the log density is returned, else the density.

## Details

The details are given in logF.

## Value

dlogF gives the density.

## Author(s)

T. W. Yee

## See Also

hypersecant, dextlogF.

## Examples

```
## Not run: shape1 <- 1.5; shape2 <- 0.5; x <- seq(-5, 8, length = 1001)
plot(x, dlogF(x, shape1, shape2), type = "l",
    las = 1, col = "blue", ylab = "pdf",
    main = "log F density function")
## End(Not run)
```

double.cens.normal Univariate Normal Distribution with Double Censoring

## Description

Maximum likelihood estimation of the two parameters of a univariate normal distribution when there is double censoring.

## Usage

double.cens.normal(r1 = 0, r2 = 0, lmu = "identitylink", lsd = "loglink", imu = NULL, isd = NULL, zero = "sd")

## Arguments

$r 1, r 2 \quad$ Integers. Number of smallest and largest values censored, respectively.
lmu, lsd Parameter link functions applied to the mean and standard deviation. See Links for more choices.
imu, isd, zero See CommonVGAMfffarguments for more information.

## Details

This family function uses the Fisher information matrix given in Harter and Moore (1966). The matrix is not diagonal if either $r 1$ or $r 2$ are positive.
By default, the mean is the first linear/additive predictor and the log of the standard deviation is the second linear/additive predictor.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

Note
This family function only handles a vector or one-column matrix response. The weights argument, if used, are interpreted as frequencies, therefore it must be a vector with positive integer values.

With no censoring at all (the default), it is better (and equivalent) to use uninormal.

## Author(s)

T. W. Yee

## References

Harter, H. L. and Moore, A. H. (1966). Iterative maximum-likelihood estimation of the parameters of normal populations from singly and doubly censored samples. Biometrika, 53, 205-213.

## See Also

uninormal, cens.normal, tobit.

## Examples

```
## Not run: # Repeat the simulations described in Harter and Moore (1966)
SIMS <- 100 # Number of simulations (change this to 1000)
mu.save <- sd.save <- rep(NA, len = SIMS)
r1 <- 0; r2 <- 4; nn <- 20
for (sim in 1:SIMS) {
    y <- sort(rnorm(nn))
    y <- y[(1+r1):(nn-r2)] # Delete r1 smallest and r2 largest
    fit <- vglm(y ~ 1, double.cens.normal(r1 = r1, r2 = r2))
    mu.save[sim] <- predict(fit)[1, 1]
    sd.save[sim] <- exp(predict(fit)[1, 2]) # Assumes a log link and ~ 1
}
c(mean(mu.save), mean(sd.save)) # Should be c(0,1)
c(sd(mu.save), sd(sd.save))
## End(Not run)
# Data from Sarhan and Greenberg (1962); MLEs are mu = 9.2606, sd = 1.3754
strontium90 <- data.frame(y = c(8.2, 8.4, 9.1, 9.8, 9.9))
fit <- vglm(y ~ 1, double.cens.normal(r1 = 2, r2 = 3, isd = 6),
    data = strontium90, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
```


## Description

Fits a double exponential binomial distribution by maximum likelihood estimation. The two parameters here are the mean and dispersion parameter.

## Usage

double.expbinomial(lmean = "logitlink", ldispersion = "logitlink", idispersion = 0.25, zero = "dispersion")

## Arguments

lmean, ldispersion
Link functions applied to the two parameters, called $\mu$ and $\theta$ respectively below. See Links for more choices. The defaults cause the parameters to be restricted to $(0,1)$.
idispersion Initial value for the dispersion parameter. If given, it must be in range, and is recyled to the necessary length. Use this argument if convergence failure occurs.
zero A vector specifying which linear/additive predictor is to be modelled as interceptonly. If assigned, the single value can be either 1 or 2 . The default is to have a single dispersion parameter value. To model both parameters as functions of the covariates assign zero $=$ NULL. See CommonVGAMffArguments for more details.

## Details

This distribution provides a way for handling overdispersion in a binary response. The double exponential binomial distribution belongs the family of double exponential distributions proposed by Efron (1986). Below, equation numbers refer to that original article. Briefly, the idea is that an ordinary one-parameter exponential family allows the addition of a second parameter $\theta$ which varies the dispersion of the family without changing the mean. The extended family behaves like the original family with sample size changed from $n$ to $n \theta$. The extended family is an exponential family in $\mu$ when $n$ and $\theta$ are fixed, and an exponential family in $\theta$ when $n$ and $\mu$ are fixed. Having $0<\theta<1$ corresponds to overdispersion with respect to the binomial distribution. See Efron (1986) for full details.

This VGAM family function implements an approximation (2.10) to the exact density (2.4). It replaces the normalizing constant by unity since the true value nearly equals 1 . The default model fitted is $\eta_{1}=\operatorname{logit}(\mu)$ and $\eta_{2}=\operatorname{logit}(\theta)$. This restricts both parameters to lie between 0 and 1 , although the dispersion parameter can be modelled over a larger parameter space by assigning the arguments ldispersion and edispersion.

Approximately, the mean (of $Y$ ) is $\mu$. The effective sample size is the dispersion parameter multiplied by the original sample size, i.e., $n \theta$. This family function uses Fisher scoring, and the two estimates are asymptotically independent because the expected information matrix is diagonal.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm.

## Warning

Numerical difficulties can occur; if so, try using idispersion.

## Note

This function processes the input in the same way as binomialff, however multiple responses are not allowed (binomialff(multiple. responses $=$ FALSE) $)$.

## Author(s)

T. W. Yee

## References

Efron, B. (1986). Double exponential families and their use in generalized linear regression. Journal of the American Statistical Association, 81, 709-721.

## See Also

binomialff, toxop, CommonVGAMffArguments.

## Examples

```
# This example mimics the example in Efron (1986).
# The results here differ slightly.
# Scale the variables
toxop <- transform(toxop,
    phat = positive / ssize,
    srainfall = scale(rainfall), # (6.1)
    sN = scale(ssize)) # (6.2)
# A fit similar (should be identical) to Section 6 of Efron (1986).
# But does not use poly(), and M = 1.25 here, as in (5.3)
cmlist <- list("(Intercept)" = diag(2),
            "I(srainfall)" = rbind(1, 0),
            "I(srainfall^2)" = rbind(1, 0),
            "I(srainfall^3)" = rbind(1, 0),
            "I(sN)" = rbind(0, 1),
            "I(sN^2)" = rbind(0, 1))
fit <- vglm(cbind(phat, 1 - phat) * ssize ~
    I(srainfall) + I(srainfall^2) + I(srainfall^3) +
    I(sN) + I(sN^2),
        double.expbinomial(ldisp = extlogitlink(min = 0, max = 1.25),
                    idisp = 0.2, zero = NULL),
        toxop, trace = TRUE, constraints = cmlist)
```

```
# Now look at the results
coef(fit, matrix = TRUE)
head(fitted(fit))
summary(fit)
vcov(fit)
sqrt(diag(vcov(fit))) # Standard errors
# Effective sample size (not quite the last column of Table 1)
head(predict(fit))
Dispersion <- extlogitlink(predict(fit)[,2], min = 0, max = 1.25, inverse = TRUE)
c(round(weights(fit, type = "prior") * Dispersion, digits = 1))
# Ordinary logistic regression (gives same results as (6.5))
ofit <- vglm(cbind(phat, 1 - phat) * ssize ~
    I(srainfall) + I(srainfall^2) + I(srainfall^3),
    binomialff, toxop, trace = TRUE)
# Same as fit but it uses poly(), and can be plotted (cf. Figure 1)
cmlist2 <- list("(Intercept)" = diag(2),
    "poly(srainfall, degree = 3)" = rbind(1, 0),
    "poly(sN, degree = 2)" = rbind(0, 1))
fit2 <- vglm(cbind(phat, 1 - phat) * ssize ~
    poly(srainfall, degree = 3) + poly(sN, degree = 2),
    double.expbinomial(ldisp = extlogitlink(min = 0, max = 1.25),
                        idisp = 0.2, zero = NULL),
    toxop, trace = TRUE, constraints = cmlist2)
## Not run: par(mfrow = c(1, 2))
plot(as(fit2, "vgam"), se = TRUE, lcol = "blue", scol = "orange") # Cf. Figure 1
# Cf. Figure 1(a)
par(mfrow = c(1,2))
ooo <- with(toxop, sort.list(rainfall))
with(toxop, plot(rainfall[ooo], fitted(fit2)[ooo], type = "l",
    col = "blue", las = 1, ylim = c(0.3, 0.65)))
with(toxop, points(rainfall[ooo], fitted(ofit)[ooo], col = "orange",
                    type = "b", pch = 19))
# Cf. Figure 1(b)
000 <- with(toxop, sort.list(ssize))
with(toxop, plot(ssize[ooo], Dispersion[ooo], type = "l", col = "blue",
    las = 1, xlim = c(0, 100)))
## End(Not run)
```

ducklings

## Description

Relative frequencies of serum proteins in white Pekin ducklings as determined by electrophoresis.

## Usage

```
data(ducklings)
```


## Format

The format is: chr "ducklings"

## Details

Columns p1, p2, p3 stand for pre-albumin, albumin, globulins respectively. These were collected from 3-week old white Pekin ducklings. Let $Y_{1}$ be proportional to the total milligrams of prealbumin in the blood serum of a duckling. Similarly, let $Y_{2}$ and $Y_{3}$ be directly proportional to the same factor as $Y_{1}$ to the total milligrams respectively of albumin and globulins in its blood serum. The proportion of pre-albumin is given by $Y_{1} /\left(Y_{1}+Y_{2}+Y_{3}\right)$, and similarly for the others.

## Source

Mosimann, J. E. (1962) On the compound multinomial distribution, the multivariate $\beta$-distribution, and correlations among proportions, Biometrika, 49, 65-82.

## See Also

dirichlet.

## Examples

```
print(ducklings)
```

eCDF Empirical Cumulative Distribution Function

## Description

Returns the desired quantiles of quantile regression object such as an extlogF1() or lms.ben() VGLM object

## Usage

eCDF.vglm(object, all = FALSE, ...)

## Arguments

object an object such as a vglm object with family function extlogF1 or lms.bcn.
all Logical. Return all other information? If true, the empirical CDF is returned.
... additional optional arguments. Currently unused.

## Details

This function was specifically written for a vglm object with family function extlogF1 or lms.ben. It returns the proportion of data lying below each of the fitted quantiles, and optionally the desired quantiles (arguments tau or percentiles / 100 in the family function). The output is coerced to be comparable between family functions by calling the columns by the same names.

## Value

A vector with each value lying in $(0,1)$. If all $=$ TRUE then a 2 -column matrix with the second column being the tau values or equivalent.

## See Also

extlogF1, lms.bcn, vglm.

## Examples

```
fit1 <- vglm(BMI ~ ns(age, 4), extlogF1, data = bmi.nz) # trace = TRUE
eCDF(fit1)
eCDF(fit1, all = TRUE)
```

    enzyme Enzyme Data
    
## Description

Enzyme velocity and substrate concentration.

## Usage

data(enzyme)

## Format

A data frame with 12 observations on the following 2 variables.
conc a numeric explanatory vector; substrate concentration
velocity a numeric response vector; enzyme velocity

## Details

Sorry, more details need to be included later.

## Source

Sorry, more details need to be included later.

## References

Watts, D. G. (1981). An introduction to nonlinear least squares. In: L. Endrenyi (Ed.), Kinetic Data Analysis: Design and Analysis of Enzyme and Pharmacokinetic Experiments, pp.1-24. New York: Plenum Press.

## See Also

micmen.

## Examples

```
## Not run:
fit <- vglm(velocity ~ 1, micmen, data = enzyme, trace = TRUE,
                    form2 = ~ conc - 1, crit = "crit")
    summary(fit)
    ## End(Not run)
```

    erf
    Error Function, and variants
    
## Description

Computes the error function, or its inverse, based on the normal distribution. Also computes the complement of the error function, or its inverse,

## Usage

$\operatorname{erf}(x$, inverse $=$ FALSE $)$
$\operatorname{erfc}(x$, inverse $=$ FALSE $)$

## Arguments

x
Numeric.
inverse
Logical. Of length 1.

## Details

$\operatorname{Erf} f(x)$ is defined as

$$
\operatorname{Erf}(x)=\frac{2}{\sqrt{\pi}} \int_{0}^{x} \exp \left(-t^{2}\right) d t
$$

so that it is closely related to pnorm. The inverse function is defined for $x$ in $(-1,1)$.

## Value

Returns the value of the function evaluated at $x$.

## Note

Some authors omit the term $2 / \sqrt{\pi}$ from the definition of $\operatorname{Erf}(x)$. Although defined for complex arguments, this function only works for real arguments.
The complementary error function $\operatorname{erfc}(x)$ is defined as $1-\operatorname{erf}(x)$, and is implemented by erfc. Its inverse function is defined for $x$ in $(0,2)$.

## Author(s)

T. W. Yee

## References

Abramowitz, M. and Stegun, I. A. (1972). Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, New York: Dover Publications Inc.

## See Also

pnorm.

## Examples

```
    ## Not run:
    curve(erf, -3, 3, col = "orange", ylab = "", las = 1)
    curve(pnorm, -3, 3, add = TRUE, col = "blue", lty = "dotted", lwd = 2)
    abline(v = 0, h = 0, lty = "dashed")
    legend("topleft", c("erf(x)", "pnorm(x)"), col = c("orange", "blue"),
        lty = c("solid", "dotted"), lwd = 1:2)
    ## End(Not run)
```

    erlang Erlang Distribution
    
## Description

Estimates the scale parameter of the Erlang distribution by maximum likelihood estimation.

## Usage

erlang(shape.arg, lscale = "loglink", imethod = 1, zero = NULL)

## Arguments

shape.arg The shape parameters. The user must specify a positive integer, or integers for multiple responses. They are recycled by. row = TRUE according to matrix.
lscale Link function applied to the (positive) scale parameter. See Links for more choices.
imethod, zero See CommonVGAMffArguments for more details.

## Details

The Erlang distribution is a special case of the gamma distribution with shape that is a positive integer. If shape. $\arg =1$ then it simplifies to the exponential distribution. As illustrated in the example below, the Erlang distribution is the distribution of the sum of shape. arg independent and identically distributed exponential random variates.
The probability density function of the Erlang distribution is given by

$$
f(y)=\exp (-y / \text { scale }) y^{\text {shape }-1} \text { scale } e^{- \text {shape }} / \Gamma(\text { shape })
$$

for known positive integer shape, unknown scale $>0$ and $y>0$. Here, $\Gamma$ (shape) is the gamma function, as in gamma. The mean of $Y$ is $\mu=$ shape $\times$ scale and its variance is shape $\times$ scale $^{2}$. The linear/additive predictor, by default, is $\eta=\log ($ scale $)$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

Multiple responses are permitted. The rate parameter found in gammaR is $1 /$ scale here-see also rgamma.

## Author(s)

T. W. Yee

## References

Most standard texts on statistical distributions describe this distribution, e.g.,
Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011). Statistical Distributions, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

## See Also

gammaR, exponential, simulate.vlm.

## Examples

```
rate <- exp(2); myshape <- 3
edata <- data.frame(y = rep(0, nn <- 1000))
for (ii in 1:myshape)
    edata <- transform(edata, y = y + rexp(nn, rate = rate))
fit <- vglm(y ~ 1, erlang(shape = myshape), data = edata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit) # Answer = 1/rate
1/rate
summary(fit)
```


## Description

Density function, distribution function, and expectile function and random generation for the distribution associated with the expectiles of an exponential distribution.

## Usage

deexp (x, rate $=1, \log =$ FALSE $)$
$\operatorname{peexp}(q$, rate $=1$, lower.tail $=$ TRUE, log. $\mathrm{p}=$ FALSE $)$
qeexp (p, rate $=1$, Maxit. $\mathrm{nr}=10$, Tol. $\mathrm{nr}=1.0 \mathrm{e}-6$,
lower.tail $=$ TRUE, log.p $=$ FALSE)
$\operatorname{reexp}(n$, rate $=1)$

## Arguments

$x, p, q \quad$ See deunif.
$n$, rate, log See rexp.
lower.tail, log.p
Same meaning as in pexp or qexp.
Maxit.nr, Tol.nr
See deunif.

## Details

General details are given in deunif including a note regarding the terminology used. Here, exp corresponds to the distribution of interest, $F$, and eexp corresponds to $G$. The addition of "e" is for the 'other' distribution associated with the parent distribution. Thus deexp is for $g$, peexp is for $G$, qeexp is for the inverse of $G$, reexp generates random variates from $g$.
For qeexp the Newton-Raphson algorithm is used to solve for $y$ satisfying $p=G(y)$. Numerical problems may occur when values of $p$ are very close to 0 or 1 .

## Value

deexp ( x ) gives the density function $g(x)$. peexp (q) gives the distribution function $G(q)$. qeexp (p) gives the expectile function: the value $y$ such that $G(y)=p$. reexp(n) gives $n$ random variates from $G$.

## Author(s)

T. W. Yee and Kai Huang

## See Also

> deunif, denorm, dexp.

## Examples

```
my.p <- 0.25; y <- rexp(nn <- 1000)
(myexp <- qeexp(my.p))
sum(myexp - y[y <= myexp]) / sum(abs(myexp - y)) # Should be my.p
## Not run: par(mfrow = c(2,1))
yy <- seq(-0, 4, len = nn)
plot(yy, deexp(yy), col = "blue", ylim = 0:1, xlab = "y", ylab = "g(y)",
    type = "l", main = "g(y) for Exp(1); dotted green is f(y) = dexp(y)")
lines(yy, dexp(yy), col = "darkgreen", lty = "dotted", lwd = 2) # 'original'
plot(yy, peexp(yy), type = "l", col = "blue", ylim = 0:1,
    xlab = "y", ylab = "G(y)", main = "G(y) for Exp(1)")
abline(v = 1, h = 0.5, col = "red", lty = "dashed")
lines(yy, pexp(yy), col = "darkgreen", lty = "dotted", lwd = 2)
## End(Not run)
```

Expectiles-Normal Expectiles of the Normal Distribution

## Description

Density function, distribution function, and expectile function and random generation for the distribution associated with the expectiles of a normal distribution.

## Usage

```
denorm(x, mean \(=0\), sd \(=1\), log \(=\) FALSE)
penorm( \(q\), mean \(=0\), \(s d=1\), lower.tail \(=\) TRUE, log. \(p=\) FALSE)
qenorm( \(p\), mean \(=0\), \(s d=1\), Maxit. \(n r=10\), Tol. \(n r=1.0 \mathrm{e}-6\),
    lower.tail \(=\) TRUE, log. \(\mathrm{p}=\) FALSE)
\(\operatorname{renorm}(n\), mean \(=0, s d=1)\)
```


## Arguments

$x, p, q \quad$ See deunif.
n, mean, sd, log
See rnorm.
lower.tail, log.p
Same meaning as in pnorm or qnorm.
Maxit.nr, Tol.nr
See deunif.

## Details

General details are given in deunif including a note regarding the terminology used. Here, norm corresponds to the distribution of interest, $F$, and enorm corresponds to $G$. The addition of "e" is for the 'other' distribution associated with the parent distribution. Thus denorm is for $g$, penorm is for $G$, qenorm is for the inverse of $G$, renorm generates random variates from $g$.
For qenorm the Newton-Raphson algorithm is used to solve for $y$ satisfying $p=G(y)$. Numerical problems may occur when values of $p$ are very close to 0 or 1 .

## Value

denorm(x) gives the density function $g(x)$. penorm(q) gives the distribution function $G(q)$. qenorm (p) gives the expectile function: the value $y$ such that $G(y)=p$. renorm(n) gives $n$ random variates from $G$.

## Author(s)

T. W. Yee and Kai Huang

## See Also

deunif, deexp, dnorm, amlnormal, lms.bcn.

## Examples

```
my.p <- 0.25; y <- rnorm(nn <- 1000)
(myexp <- qenorm(my.p))
sum(myexp - y[y <= myexp]) / sum(abs(myexp - y)) # Should be my.p
# Non-standard normal
mymean <- 1; mysd <- 2
yy <- rnorm(nn, mymean, mysd)
(myexp <- qenorm(my.p, mymean, mysd))
sum(myexp - yy[yy <= myexp]) / sum(abs(myexp - yy)) # Should be my.p
penorm(-Inf, mymean, mysd) # Should be 0
penorm( Inf, mymean, mysd) # Should be 1
penorm(mean(yy), mymean, mysd) # Should be 0.5
abs(qenorm(0.5, mymean, mysd) - mean(yy)) # Should be 0
abs(penorm(myexp, mymean, mysd) - my.p) # Should be 0
integrate(f = denorm, lower = -Inf, upper = Inf,
            mymean, mysd) # Should be 1
## Not run:
par(mfrow = c(2, 1))
yy <- seq(-3, 3, len = nn)
plot(yy, denorm(yy), type = "l", col="blue", xlab = "y", ylab = "g(y)",
    main = "g(y) for N(0,1); dotted green is f(y) = dnorm(y)")
lines(yy, dnorm(yy), col = "darkgreen", lty = "dotted", lwd = 2) # 'original'
plot(yy, penorm(yy), type = "l", col = "blue", ylim = 0:1,
    xlab = "y", ylab = "G(y)", main = "G(y) for N(0,1)")
abline(v = 0, h = 0.5, col = "red", lty = "dashed")
```

```
lines(yy, pnorm(yy), col = "darkgreen", lty = "dotted", lwd = 2)
## End(Not run)
```


## Description

Density function, distribution function, and quantile/expectile function and random generation for the scaled Student t distribution with 2 degrees of freedom.

## Usage

dsc.t2(x, location $=0$, scale $=1$, log $=$ FALSE)
psc.t2(q, location $=0$, scale $=1$, lower.tail $=$ TRUE, log.p = FALSE)
qsc.t2(p, location $=0$, scale $=1$, lower.tail $=$ TRUE, log. $p=$ FALSE)
rsc.t2(n, location $=0$, scale $=1$ )

## Arguments

$x, q \quad$ Vector of expectiles/quantiles. See the terminology note below.
$\mathrm{p} \quad$ Vector of probabilities. These should lie in $(0,1)$.
$n, \log \quad$ See runif.
location, scale
Location and scale parameters. The latter should have positive values. Values of these vectors are recyled.
lower.tail, log.p
Same meaning as in pt or qt.

## Details

A Student-t distribution with 2 degrees of freedom and a scale parameter of sqrt(2) is equivalent to the standard form of this distribution (called Koenker's distribution below). Further details about this distribution are given in sc.studentt2.

## Value

dsc.t2 ( $x$ ) gives the density function. psc.t2(q) gives the distribution function. qsc.t2(p) gives the expectile and quantile function. $\mathrm{rsc} . \mathrm{t} 2(\mathrm{n})$ gives $n$ random variates.

## Author(s)

T. W. Yee and Kai Huang

See Also
dt, sc.studentt2.

## Examples

```
my.p <- 0.25; y <- rsc.t2(nn <- 5000)
(myexp <- qsc.t2(my.p))
sum(myexp - y[y <= myexp]) / sum(abs(myexp - y)) # Should be my.p
# Equivalently:
I1 <- mean(y <= myexp) * mean( myexp - y[y <= myexp])
I2 <- mean(y > myexp) * mean(-myexp + y[y > myexp])
I1 / (I1 + I2) # Should be my.p
# Or:
I1 <- sum( myexp - y[y <= myexp])
I2 <- sum(-myexp + y[y > myexp])
# Non-standard Koenker distribution
myloc <- 1; myscale <- 2
yy <- rsc.t2(nn, myloc, myscale)
(myexp <- qsc.t2(my.p, myloc, myscale))
sum(myexp - yy[yy <= myexp]) / sum(abs(myexp - yy)) # Should be my.p
psc.t2(mean(yy), myloc, myscale) # Should be 0.5
abs(qsc.t2(0.5, myloc, myscale) - mean(yy)) # Should be 0
abs(psc.t2(myexp, myloc, myscale) - my.p) # Should be 0
integrate(f = dsc.t2, lower = -Inf, upper = Inf,
            locat = myloc, scale = myscale) # Should be 1
y <- seq(-7, 7, len = 201)
max(abs(dsc.t2(y) - dt(y / sqrt(2), df = 2) / sqrt(2))) # Should be 0
## Not run: plot(y, dsc.t2(y), type = "l", col = "blue", las = 1,
    ylim = c(0, 0.4), main = "Blue = Koenker; orange = N(0, 1)")
lines(y, dnorm(y), type = "l", col = "orange")
abline(h = 0, v = 0, lty = 2)
## End(Not run)
```

Expectiles-Uniform Expectiles of the Uniform Distribution

## Description

Density function, distribution function, and expectile function and random generation for the distribution associated with the expectiles of a uniform distribution.

## Usage

deunif(x, min $=0, \max =1, \log =$ FALSE $)$
peunif( $q, \min =0, \max =1$, lower.tail = TRUE, log.p = FALSE)
qeunif(p, min = 0, max = 1, Maxit.nr = 10, Tol.nr = 1.0e-6, lower.tail $=$ TRUE, log. $\mathrm{p}=$ FALSE)
reunif(n, min $=0, \max =1$ )

## Arguments

$$
\begin{array}{ll}
\mathrm{x}, \mathrm{q} & \text { Vector of expectiles. See the terminology note below. } \\
\mathrm{p} \\
\mathrm{n}, \mathrm{~min}, \text { max, log } & \text { Vector of probabilities. These should lie in }(0,1) . \\
\text { lower.tail, log. } \mathrm{p}
\end{array} \begin{aligned}
& \text { See runif. } \\
& \text { Maxit.nr }
\end{aligned} \begin{aligned}
& \text { Same meaning as in punif or qunif. } \\
& \text { Numeric. Maximum number of Newton-Raphson iterations allowed. A warning } \\
& \text { is issued if convergence is not obtained for all p values. }
\end{aligned}
$$

## Details

Jones (1994) elucidated on the property that the expectiles of a random variable $X$ with distribution function $F(x)$ correspond to the quantiles of a distribution $G(x)$ where $G$ is related by an explicit formula to $F$. In particular, let $y$ be the $p$-expectile of $F$. Then $y$ is the $p$-quantile of $G$ where

$$
p=G(y)=(P(y)-y F(y)) /(2[P(y)-y F(y)]+y-\mu),
$$

and $\mu$ is the mean of $X$. The derivative of $G$ is

$$
g(y)=(\mu F(y)-P(y)) /(2[P(y)-y F(y)]+y-\mu)^{2} .
$$

Here, $P(y)$ is the partial moment $\int_{-\infty}^{y} x f(x) d x$ and $0<p<1$. The 0.5 -expectile is the mean $\mu$ and the 0.5 -quantile is the median.
A note about the terminology used here. Recall in the $S$ language there are the dpqr-type functions associated with a distribution, e.g., dunif, punif, qunif, runif, for the uniform distribution. Here, unif corresponds to $F$ and eunif corresponds to $G$. The addition of "e" (for expectile) is for the 'other' distribution associated with the parent distribution. Thus deunif is for $g$, peunif is for $G$, qeunif is for the inverse of $G$, reunif generates random variates from $g$.
For qeunif the Newton-Raphson algorithm is used to solve for $y$ satisfying $p=G(y)$. Numerical problems may occur when values of $p$ are very close to 0 or 1 .

## Value

deunif $(x)$ gives the density function $g(x)$. peunif(q) gives the distribution function $G(q)$. qeunif (p) gives the expectile function: the expectile $y$ such that $G(y)=p$. reunif ( n ) gives $n$ random variates from $G$.

## Author(s)

T. W. Yee and Kai Huang

## References

Jones, M. C. (1994). Expectiles and M-quantiles are quantiles. Statistics and Probability Letters, 20, 149-153.

Yee, T. W. (2012). Vector generalized linear and additive quantile and expectile regression. In preparation.

## See Also

deexp, denorm, dunif, dsc.t2.

## Examples

```
my.p <- 0.25; y <- runif(nn <- 1000)
(myexp <- qeunif(my.p))
sum(myexp - y[y <= myexp]) / sum(abs(myexp - y)) # Should be my.p
# Equivalently:
I1 <- mean(y <= myexp) * mean( myexp - y[y <= myexp])
I2 <- mean(y > myexp) * mean(-myexp + y[y > myexp])
I1 / (I1 + I2) # Should be my.p
# Or:
I1 <- sum( myexp - y[y <= myexp])
I2 <- sum(-myexp + y[y > myexp])
# Non-standard uniform
mymin <- 1; mymax <- 8
yy <- runif(nn, mymin, mymax)
(myexp <- qeunif(my.p, mymin, mymax))
sum(myexp - yy[yy <= myexp]) / sum(abs(myexp - yy)) # Should be my.p
peunif(mymin, mymin, mymax) # Should be 0
peunif(mymax, mymin, mymax) # Should be 1
peunif(mean(yy), mymin, mymax) # Should be 0.5
abs(qeunif(0.5, mymin, mymax) - mean(yy)) # Should be 0
abs(qeunif(0.5, mymin, mymax) - (mymin+mymax)/2) # Should be 0
abs(peunif(myexp, mymin, mymax) - my.p) # Should be 0
integrate(f = deunif, lower = mymin - 3, upper = mymax + 3,
                    min = mymin, max = mymax) # Should be 1
## Not run:
par(mfrow = c(2,1))
yy <- seq(0.0, 1.0, len = nn)
plot(yy, deunif(yy), type = "l", col = "blue", ylim = c(0, 2),
    xlab = "y", ylab = "g(y)", main = "g(y) for Uniform(0,1)")
lines(yy, dunif(yy), col = "darkgreen", lty = "dotted", lwd = 2) # 'original'
plot(yy, peunif(yy), type = "l", col = "blue", ylim = 0:1,
    xlab = "y", ylab = "G(y)", main = "G(y) for Uniform(0,1)")
abline(a = 0.0, b = 1.0, col = "darkgreen", lty = "dotted", lwd = 2)
abline(v = 0.5, h = 0.5, col = "red", lty = "dashed")
## End(Not run)
```

expexpff Exponentiated Exponential Distribution

## Description

Estimates the two parameters of the exponentiated exponential distribution by maximum likelihood estimation.

## Usage

```
expexpff(lrate = "loglink", lshape = "loglink",
    irate = NULL, ishape = 1.1, tolerance = 1.0e-6, zero = NULL)
```


## Arguments

lshape, lrate Parameter link functions for the $\alpha$ and $\lambda$ parameters. See Links for more choices. The defaults ensure both parameters are positive.
ishape Initial value for the $\alpha$ parameter. If convergence fails try setting a different value for this argument.
irate Initial value for the $\lambda$ parameter. By default, an initial value is chosen internally using ishape.
tolerance $\quad$ Numeric. Small positive value for testing whether values are close enough to 1 and 2.
zero An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The default is none of them. If used, choose one value from the set $\{1,2\}$.

## Details

The exponentiated exponential distribution is an alternative to the Weibull and the gamma distributions. The formula for the density is

$$
f(y ; \lambda, \alpha)=\alpha \lambda(1-\exp (-\lambda y))^{\alpha-1} \exp (-\lambda y)
$$

where $y>0, \lambda>0$ and $\alpha>0$. The mean of $Y$ is $(\psi(\alpha+1)-\psi(1)) / \lambda$ (returned as the fitted values) where $\psi$ is the digamma function. The variance of $Y$ is $\left(\psi^{\prime}(1)-\psi^{\prime}(\alpha+1)\right) / \lambda^{2}$ where $\psi^{\prime}$ is the trigamma function.

This distribution has been called the two-parameter generalized exponential distribution by Gupta and Kundu (2006). A special case of the exponentiated exponential distribution: $\alpha=1$ is the exponential distribution.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Warning

Practical experience shows that reasonably good initial values really helps. In particular, try setting different values for the ishape argument if numerical problems are encountered or failure to convergence occurs. Even if convergence occurs try perturbing the initial value to make sure the global solution is obtained and not a local solution. The algorithm may fail if the estimate of the shape parameter is too close to unity.

## Note

Fisher scoring is used, however, convergence is usually very slow. This is a good sign that there is a bug, but I have yet to check that the expected information is correct. Also, I have yet to implement Type-I right censored data using the results of Gupta and Kundu (2006).
Another algorithm for fitting this model is implemented in expexpff1.

## Author(s)

T. W. Yee

## References

Gupta, R. D. and Kundu, D. (2001). Exponentiated exponential family: an alternative to gamma and Weibull distributions, Biometrical Journal, 43, 117-130.
Gupta, R. D. and Kundu, D. (2006). On the comparison of Fisher information of the Weibull and GE distributions, Journal of Statistical Planning and Inference, 136, 3130-3144.

## See Also

expexpff1, gammaR, weibullR, CommonVGAMffArguments.

## Examples

```
# A special case: exponential data
edata <- data.frame(y = rexp(n <- 1000))
fit <- vglm(y ~ 1, fam = expexpff, data = edata, trace = TRUE, maxit = 99)
coef(fit, matrix = TRUE)
Coef(fit)
# Ball bearings data (number of million revolutions before failure)
edata <- data.frame(bbearings = c(17.88, 28.92, 33.00, 41.52, 42.12, 45.60,
48.80, 51.84, 51.96, 54.12, 55.56, 67.80, 68.64, 68.64,
68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92,
128.04, 173.40))
fit <- vglm(bbearings ~ 1, fam = expexpff(irate = 0.05, ish = 5),
    trace = TRUE, maxit = 300, data = edata)
coef(fit, matrix = TRUE)
Coef(fit) # Authors get c(rate=0.0314, shape=5.2589)
logLik(fit) # Authors get -112.9763
# Failure times of the airconditioning system of an airplane
eedata <- data.frame(acplane = c(23, 261, 87, 7, 120, 14, 62, 47,
225, 71, 246, 21, 42, 20, 5, 12, 120, 11, 3, 14,
71, 11, 14, 11, 16, 90, 1, 16, 52, 95))
fit <- vglm(acplane ~ 1, fam = expexpff(ishape = 0.8, irate = 0.15),
    trace = TRUE, maxit = 99, data = eedata)
coef(fit, matrix = TRUE)
Coef(fit) # Authors get c(rate=0.0145, shape=0.8130)
logLik(fit) # Authors get log-lik -152.264
```


## Description

Estimates the two parameters of the exponentiated exponential distribution by maximizing a profile (concentrated) likelihood.

## Usage

expexpff1(lrate = "loglink", irate = NULL, ishape = 1)

## Arguments

lrate Parameter link function for the (positive) $\lambda$ parameter. See Links for more choices.
irate Initial value for the $\lambda$ parameter. By default, an initial value is chosen internally using ishape.
ishape Initial value for the $\alpha$ parameter. If convergence fails try setting a different value for this argument.

## Details

See expexpff for details about the exponentiated exponential distribution. This family function uses a different algorithm for fitting the model. Given $\lambda$, the MLE of $\alpha$ can easily be solved in terms of $\lambda$. This family function maximizes a profile (concentrated) likelihood with respect to $\lambda$. Newton-Raphson is used, which compares with Fisher scoring with expexpff.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Warning

The standard errors produced by a summary of the model may be wrong.

## Note

This family function works only for intercept-only models, i.e., y $\sim 1$ where $y$ is the response.
The estimate of $\alpha$ is attached to the misc slot of the object, which is a list and contains the component shape.

As Newton-Raphson is used, the working weights are sometimes negative, and some adjustment is made to these to make them positive.
Like expexpff, good initial values are needed. Convergence may be slow.

## Author(s)

T. W. Yee

## References

Gupta, R. D. and Kundu, D. (2001). Exponentiated exponential family: an alternative to gamma and Weibull distributions, Biometrical Journal, 43, 117-130.

## See Also

expexpff, CommonVGAMffArguments.

## Examples

```
# Ball bearings data (number of million revolutions before failure)
edata <- data.frame(bbearings = c(17.88, 28.92, 33.00, 41.52, 42.12, 45.60,
48.80, 51.84, 51.96, 54.12, 55.56, 67.80, 68.64, 68.64,
68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92,
128.04, 173.40))
fit <- vglm(bbearings ~ 1, expexpff1(ishape = 4), trace = TRUE,
    maxit = 250, checkwz = FALSE, data = edata)
coef(fit, matrix = TRUE)
Coef(fit) # Authors get c(0.0314, 5.2589) with log-lik -112.9763
logLik(fit)
fit@misc$shape # Estimate of shape
```

\# Failure times of the airconditioning system of an airplane
eedata <- data.frame(acplane $=\mathrm{c}(23,261,87,7,120,14,62,47$,
$225,71,246,21,42,20,5,12,120,11,3,14$,
71, 11, 14, 11, 16, 90, 1, 16, 52, 95))
fit <- vglm(acplane ~ 1, expexpff1(ishape = 0.8), trace = TRUE,
maxit $=50$, checkwz $=$ FALSE, data $=$ eedata )
coef(fit, matrix = TRUE)
Coef(fit) \# Authors get c(0.0145, 0.8130) with log-lik -152.264
$\operatorname{logLik}(f i t)$
fit@misc\$shape \# Estimate of shape

## Description

Density, distribution function, quantile function and random generation for the exponential geometric distribution.

## Usage

dexpgeom(x, scale $=1$, shape, log = FALSE)
pexpgeom (q, scale $=1$, shape)
qexpgeom ( p , scale $=1$, shape)
rexpgeom(n, scale $=1$, shape)

## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$n \quad$ number of observations. If length $(n)>1$ then the length is taken to be the number required.
scale, shape positive scale and shape parameters.
$\log \quad$ Logical. If $\log =$ TRUE then the logarithm of the density is returned.

## Details

See expgeometric, the VGAM family function for estimating the parameters, for the formula of the probability density function and other details.

## Value

dexpgeom gives the density, pexpgeom gives the distribution function, qexpgeom gives the quantile function, and rexpgeom generates random deviates.

## Note

We define scale as the reciprocal of the scale parameter used by Adamidis and Loukas (1998).

## Author(s)

J. G. Lauder and T. W. Yee

## See Also

```
expgeometric, exponential,geometric.
```


## Examples

```
## Not run:
shape <- 0.5; scale <- 1; nn <- 501
x <- seq(-0.10, 3.0, len = nn)
plot(x, dexpgeom(x, scale, shape), type = "l", las = 1, ylim = c(0, 2),
    ylab = paste("[dp]expgeom(shape = ", shape, ", scale = ", scale, ")"),
    col = "blue", cex.main = 0.8,
    main = "Blue is density, red is cumulative distribution function",
    sub = "Purple lines are the 10,20,\ldots.,90 percentiles")
lines(x, pexpgeom(x, scale, shape), col = "red")
probs <- seq(0.1, 0.9, by = 0.1)
```

```
Q <- qexpgeom(probs, scale, shape)
lines(Q, dexpgeom(Q, scale, shape), col = "purple", lty = 3, type = "h")
lines(Q, pexpgeom(Q, scale, shape), col = "purple", lty = 3, type = "h")
abline(h = probs, col = "purple", lty = 3)
max(abs(pexpgeom(Q, scale, shape) - probs)) # Should be 0
## End(Not run)
```

expgeometric Exponential Geometric Distribution Family Function

## Description

Estimates the two parameters of the exponential geometric distribution by maximum likelihood estimation.

## Usage

expgeometric(lscale = "loglink", lshape = "logitlink",
iscale $=$ NULL, $\quad$ ishape $=$ NULL,
tol12 $=1 \mathrm{e}-05$, zero $=1$, nsimEIM $=400$ )

## Arguments

lscale, lshape Link function for the two parameters. See Links for more choices.
iscale, ishape Numeric. Optional initial values for the scale and shape parameters.
tol12 Numeric. Tolerance for testing whether a parameter has value 1 or 2.
zero, nsimEIM See CommonVGAMffArguments.

## Details

The exponential geometric distribution has density function

$$
f(y ; c=\text { scale }, s=\text { shape })=(1 / c)(1-s) e^{-y / c}\left(1-s e^{-y / c}\right)^{-2}
$$

where $y>0, c>0$ and $s \in(0,1)$. The mean, $(c(s-1) / s) \log (1-s)$ is returned as the fitted values. Note the median is $c \log (2-s)$. Simulated Fisher scoring is implemented.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

We define scale as the reciprocal of the scale parameter used by Adamidis and Loukas (1998).

## Author(s)

J. G. Lauder and T. W. Yee

## References

Adamidis, K., Loukas, S. (1998). A lifetime distribution with decreasing failure rate. Statistics and Probability Letters, 39, 35-42.

## See Also

dexpgeom, exponential, geometric.

## Examples

```
## Not run:
Scale <- exp(2); shape = logitlink(-1, inverse = TRUE);
edata <- data.frame(y = rexpgeom(n = 2000, scale = Scale, shape = shape))
fit <- vglm(y ~ 1, expgeometric, edata, trace = TRUE)
c(with(edata, mean(y)), head(fitted(fit), 1))
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
## End(Not run)
```

expint

The Exponential Integral and Variants

## Description

Computes the exponential integral $E i(x)$ for real values, as well as $\exp (-x) \times E i(x)$ and $E_{1}(x)$ and their derivatives (up to the 3rd derivative).

## Usage

expint(x, deriv = 0)
expexpint (x, deriv = 0)
expint.E1 (x, deriv = 0)

## Arguments

| $x$ | Numeric. Ideally a vector of positive reals. |
| :--- | :--- |
| deriv | Integer. Either $0,1,2$ or 3. |

## Details

The exponential integral $E i(x)$ function is the integral of $\exp (t) / t$ from 0 to $x$, for positive real $x$. The function $E_{1}(x)$ is the integral of $\exp (-t) / t$ from $x$ to infinity, for positive real $x$.

## Value

Function expint (x, deriv $=\mathrm{n}$ ) returns the $n$th derivative of $E i(x)$ (up to the 3 rd ), function expexpint (x, deriv $=\mathrm{n}$ ) returns the $n$th derivative of $\exp (-x) \times E i(x)$ (up to the 3rd), function expint. E1 (x, deriv $=\mathrm{n}$ ) returns the $n$th derivative of $E_{1}(x)$ (up to the 3 rd ).

## Warning

These functions have not been tested thoroughly.

## Author(s)

T. W. Yee has simply written a small wrapper function to call the NETLIB FORTRAN code. Xiangjie Xue modified the functions to calculate derivatives. Higher derivatives can actually be calculated—please let me know if you need it.

## References

http://www.netlib.org/specfun/ei.

## See Also

log, exp. There is also a package called expint.

## Examples

```
    ## Not run:
par(mfrow = c(2, 2))
curve(expint, 0.01, 2, xlim = c(0, 2), ylim = c(-3, 5),
    las = 1, col = "orange")
abline(v = (-3):5, h = (-4):5, lwd = 2, lty = "dotted", col = "gray")
abline(h = 0, v = 0, lty = "dashed", col = "blue")
curve(expexpint, 0.01, 2, xlim = c(0, 2), ylim = c(-3, 2),
            las = 1, col = "orange")
abline(v = (-3):2, h = (-4):5, lwd = 2, lty = "dotted", col = "gray")
abline(h = 0, v = 0, lty = "dashed", col = "blue")
curve(expint.E1, 0.01, 2, xlim = c(0, 2), ylim = c(0, 5),
            las = 1, col = "orange")
abline(v = (-3):2, h = (-4):5, lwd = 2, lty = "dotted", col = "gray")
abline(h = 0, v = 0, lty = "dashed", col = "blue")
## End(Not run)
```

explink Exponential Link Function

## Description

Computes the exponential transformation, including its inverse and the first two derivatives.

## Usage

explink(theta, bvalue $=$ NULL, inverse $=$ FALSE, deriv $=0$, short $=$ TRUE, tag $=$ FALSE $)$

## Arguments

theta Numeric or character. See below for further details.
bvalue See clogloglink.
inverse, deriv, short, tag
Details at Links.

## Details

The exponential link function is potentially suitable for parameters that are positive. Numerical values of theta close to negative or positive infinity may result in 0 , Inf, -Inf, NA or NaN.

## Value

For explink with deriv=0, the exponential of theta, i.e., $\exp (t h e t a)$ when inverse $=$ FALSE. And if inverse $=$ TRUE then $\log ($ theta $)$; if theta is not positive then it will return NaN .
For deriv $=1$, then the function returns $d$ eta $/ d$ theta as a function of theta if inverse $=$ FALSE, else if inverse = TRUE then it returns the reciprocal.
Here, all logarithms are natural logarithms, i.e., to base $e$.

## Note

This function has particular use for computing quasi-variances when used with rcim and uninormal.
Numerical instability may occur when theta is close to negative or positive infinity. One way of overcoming this (one day) is to use bvalue.

## Author(s)

Thomas W. Yee

## See Also

Links, loglink, rcim, Qvar, uninormal.

## Examples

```
theta <- rnorm(30)
explink(theta)
max(abs(explink(explink(theta), inverse = TRUE) - theta)) # Should be 0
```

explog The Exponential Logarithmic Distribution

## Description

Density, distribution function, quantile function and random generation for the exponential logarithmic distribution.

## Usage

dexplog(x, scale $=1$, shape, log = FALSE)
$\operatorname{pexplog}(q$, scale $=1$, shape)
qexplog(p, scale $=1$, shape)
$\operatorname{rexplog}(n$, scale $=1$, shape)

## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$n \quad$ number of observations. If length $(n)>1$ then the length is taken to be the number required.
scale, shape positive scale and shape parameters.
log Logical. If log = TRUE then the logarithm of the density is returned.

## Details

See explogff, the VGAM family function for estimating the parameters, for the formula of the probability density function and other details.

## Value

dexplog gives the density, pexplog gives the distribution function, qexplog gives the quantile function, and rexplog generates random deviates.

## Note

We define scale as the reciprocal of the scale parameter used by Tahmasabi and Rezaei (2008).

## Author(s)

J. G. Lauder and T. W. Yee

## See Also

explogff, exponential.

## Examples

```
## Not run:
shape <- 0.5; scale <- 2; nn <- 501
x <- seq(-0.50, 6.0, len = nn)
plot(x, dexplog(x, scale, shape), type = "l", las = 1, ylim = c(0, 1.1),
    ylab = paste("[dp]explog(shape = ", shape, ", scale = ", scale, ")"),
    col = "blue", cex.main = 0.8,
    main = "Blue is density, orange is cumulative distribution function",
    sub = "Purple lines are the 10,20,...,90 percentiles")
lines(x, pexplog(x, scale, shape), col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qexplog(probs, scale, shape = shape)
lines(Q, dexplog(Q, scale, shape = shape), col = "purple", lty = 3, type = "h")
lines(Q, pexplog(Q, scale, shape = shape), col = "purple", lty = 3, type = "h")
abline(h = probs, col = "purple", lty = 3)
max(abs(pexplog(Q, scale, shape = shape) - probs)) # Should be 0
## End(Not run)
```

explogff

Exponential Logarithmic Distribution Family Function

## Description

Estimates the two parameters of the exponential logarithmic distribution by maximum likelihood estimation.

## Usage

```
explogff(lscale = "loglink", lshape = "logitlink",
    iscale = NULL, ishape = NULL,
    tol12 = 1e-05, zero = 1, nsimEIM = 400)
```


## Arguments

lscale, lshape See CommonVGAMffArguments for information.
tol12 Numeric. Tolerance for testing whether a parameter has value 1 or 2.
iscale, ishape, zero, nsimEIM
See CommonVGAMffArguments.

## Details

The exponential logarithmic distribution has density function

$$
f(y ; c, s)=(1 /(-\log p))\left(\left((1 / c)(1-s) e^{-y / c}\right) /\left(1-(1-s) e^{-y / c}\right)\right)
$$

where $y>0$, scale parameter $c>0$, and shape parameter $s \in(0,1)$. The mean, ( $-\operatorname{polylog}(2,1-$ $p) c) / \log (s)$ is not returned as the fitted values. Note the median is $c \log (1+\sqrt{s})$ and it is currently returned as the fitted values. Simulated Fisher scoring is implemented.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

We define scale as the reciprocal of the rate parameter used by Tahmasabi and Sadegh (2008).
Yet to do: find a polylog() function.

## Author(s)

J. G. Lauder and T. W .Yee

## References

Tahmasabi, R., Sadegh, R. (2008). A two-parameter lifetime distribution with decreasing failure rate. Computational Statistics and Data Analysis, 52, 3889-3901.

## See Also

dexplog, exponential,

## Examples

```
## Not run: Scale <- exp(2); shape <- logitlink(-1, inverse = TRUE)
edata <- data.frame(y = rexplog( }\textrm{n}=2000, scale = Scale, shape = shape))
fit <- vglm(y ~ 1, explogff, data = edata, trace = TRUE)
c(with(edata, median(y)), head(fitted(fit), 1))
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
## End(Not run)
```


## Description

Maximum likelihood estimation for the exponential distribution.

## Usage

exponential(link = "loglink", location $=0$, expected $=$ TRUE, type.fitted = c("mean", "percentiles", "Qlink"), percentiles = 50, ishrinkage $=0.95$, parallel $=$ FALSE, zero $=$ NULL)

## Arguments

link Parameter link function applied to the positive parameter rate. See Links for more choices.
location Numeric of length 1, the known location parameter, $A$, say.
expected Logical. If TRUE Fisher scoring is used, otherwise Newton-Raphson. The latter is usually faster.
ishrinkage, parallel, zero
See CommonVGAMffArguments for information.
type.fitted, percentiles
See CommonVGAMffArguments for information.

## Details

The family function assumes the response $Y$ has density

$$
f(y)=\lambda \exp (-\lambda(y-A))
$$

for $y>A$, where $A$ is the known location parameter. By default, $A=0$. Then $E(Y)=A+1 / \lambda$ and $\operatorname{Var}(Y)=1 / \lambda^{2}$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

Suppose $A=0$. For a fixed time interval, the number of events is Poisson with mean $\lambda$ if the time between events has a geometric distribution with mean $\lambda^{-1}$. The argument rate in exponential is the same as rexp etc. The argument lambda in rpois is somewhat the same as rate here.

## Author(s)

T. W. Yee

## References

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011). Statistical Distributions, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

## See Also

amlexponential, gpd, laplace, expgeometric, explogff, poissonff, mix2exp, freund61, simulate.vlm, Exponential.

## Examples

```
edata <- data.frame(x2 = runif(nn <- 100) - 0.5)
edata <- transform(edata, x3 = runif(nn) - 0.5)
edata <- transform(edata, eta = 0.2 - 0.7 * x2 + 1.9 * x3)
edata <- transform(edata, rate = exp(eta))
edata <- transform(edata, y = rexp(nn, rate = rate))
with(edata, stem(y))
fit.slow <- vglm(y ~ x2 + x3, exponential, data = edata, trace = TRUE)
fit.fast <- vglm(y ~ x2 + x3, exponential(exp = FALSE), data = edata,
            trace = TRUE, crit = "coef")
coef(fit.slow, mat = TRUE)
summary(fit.slow)
# Compare results with a GPD. Has a threshold.
threshold <- 0.5
gdata <- data.frame(y1 = threshold + rexp(n = 3000, rate = exp(1.5)))
fit.exp <- vglm(y1 ~ 1, exponential(location = threshold), data = gdata)
coef(fit.exp, matrix = TRUE)
Coef(fit.exp)
logLik(fit.exp)
fit.gpd <- vglm(y1 ~ 1, gpd(threshold = threshold), data = gdata)
coef(fit.gpd, matrix = TRUE)
Coef(fit.gpd)
logLik(fit.gpd)
```

exppois The Exponential Poisson Distribution

## Description

Density, distribution function, quantile function and random generation for the exponential poisson distribution.

## Usage

```
dexppois(x, rate \(=1\), shape, log = FALSE)
pexppois(q, rate \(=1\), shape, lower.tail = TRUE, log.p = FALSE)
qexppois(p, rate \(=1\), shape, lower.tail = TRUE, log.p = FALSE)
rexppois(n, rate \(=1\), shape)
```


## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$n \quad$ number of observations. If length $(n)>1$ then the length is taken to be the number required.
shape, rate positive parameters.
log Logical. If log = TRUE then the logarithm of the density is returned.
lower.tail, log.p
Same meaning as in pnorm or qnorm.

## Details

See exppoisson, the VGAM family function for estimating the parameters, for the formula of the probability density function and other details.

## Value

dexppois gives the density, pexppois gives the distribution function, qexppois gives the quantile function, and rexppois generates random deviates.

## Author(s)

Kai Huang and J. G. Lauder

## See Also

exppoisson.

## Examples

```
## Not run: rate <- 2; shape <- 0.5; nn <- 201
x <- seq(-0.05, 1.05, len = nn)
plot(x, dexppois(x, rate = rate, shape), type = "l", las = 1, ylim = c(0, 3),
    ylab = paste("fexppoisson(rate = ", rate, ", shape = ", shape, ")"),
    col = "blue", cex.main = 0.8,
    main = "Blue is the density, orange the cumulative distribution function",
    sub = "Purple lines are the 10,20,...,90 percentiles")
lines(x, pexppois(x, rate = rate, shape), col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qexppois(probs, rate = rate, shape)
lines(Q, dexppois(Q, rate = rate, shape), col = "purple", lty = 3, type = "h")
lines(Q, pexppois(Q, rate = rate, shape), col = "purple", lty = 3, type = "h")
```

```
abline(h = probs, col = "purple", lty = 3); abline(h = 0, col = "gray50")
max(abs(pexppois(Q, rate = rate, shape) - probs)) # Should be 0
## End(Not run)
```

exppoisson Exponential Poisson Distribution Family Function

## Description

Estimates the two parameters of the exponential Poisson distribution by maximum likelihood estimation.

## Usage

exppoisson(lrate = "loglink", lshape = "loglink", irate $=2$, ishape $=1.1, \quad$ zero $=$ NULL)

## Arguments

lshape, lrate Link function for the two positive parameters. See Links for more choices.
ishape, irate Numeric. Initial values for the shape and rate parameters. Currently this function is not intelligent enough to obtain better initial values.
zero See CommonVGAMffArguments.

## Details

The exponential Poisson distribution has density function

$$
f(y ; \beta=\text { rate }, \lambda=\text { shape })=\frac{\lambda \beta}{1-e^{-\lambda}} e^{-\lambda-\beta y+\lambda \exp (-\beta y)}
$$

where $y>0$, and the parameters shape, $\lambda$, and rate, $\beta$, are positive. The distribution implies a population facing discrete hazard rates which are multiples of a base hazard. This VGAM family function requires the hypergeo package (to use their genhypergeo function). The median is returned as the fitted value.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Warning

This VGAM family function does not work properly!

## Author(s)

J. G. Lauder, jamesglauder@gmail.com

## References

Kus, C., (2007). A new lifetime distribution. Computational Statistics and Data Analysis, 51, 4497-4509.

## See Also

dexppois, exponential, poisson.

## Examples

```
## Not run:
shape <- exp(1); rate <- exp(2)
rdata <- data.frame(y = rexppois(n = 1000, rate = rate, shape = shape))
library("hypergeo") # Required!
fit <- vglm(y ~ 1, exppoisson, data = rdata, trace = FALSE, maxit = 1200)
c(with(rdata, median(y)), head(fitted(fit), 1))
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
## End(Not run)
```

```
extlogF1 Extended log-F Distribution Family Function
```


## Description

Maximum likelihood estimation of the 1-parameter extended log-F distribution.

## Usage

```
extlogF1(tau = c(0.25, 0.5, 0.75), parallel = TRUE ~ 0,
    seppar = 0, tol0 = -0.001,
    llocation = "identitylink", ilocation = NULL,
    lambda.arg = NULL, scale.arg = 1, ishrinkage = 0.95,
    digt = 4, idf.mu = 3, imethod = 1)
```


## Arguments

tau Numeric, the desired quantiles. A strictly increasing sequence, each value must be in $(0,1)$. The default values are the three quartiles, matching lms.ben.
parallel Similar to alaplace1, applying to the location parameters. One can try fix up the quantile-crossing problem after fitting the model by calling fix.crossing. Use is.crossing to see if there is a problem. The default for parallel is totally FALSE, i.e., FALSE for every variable including the intercept. Quantilecrossing can occur when values of tau are too close, given the data. How the quantiles are modelled with respect to the covariates also has a big effect, e.g.,
if they are too flexible or too inflexible then the problem is likely to occur. For example, using bs with $\mathrm{df}=10$ is likely to create problems.
Setting parallel $=$ TRUE results in a totally parallel model; all quantiles are parallel and this assumption can be too strong for some data sets. Instead, fix.crossing only repairs the quantiles that cross. So one must carefully choose values of tau for fitting the original fit.
seppar, tol0 Numeric, both of unit length and nonnegative, the separation and shift parameters. If seppar is positive then any crossing quantile is penalized by the difference cubed multiplied by seppar. The log-likelihood subtracts the penalty. The shift parameter ensures that the result is strictly noncrossing when seppar is large enough; otherwise if tol0 $=0$ and seppar is large then the crossing quantiles remain crossed even though the offending amount becomes small but never exactly 0 . Informally, tol0 pushes the adjustment enough so that is.crossing should return FALSE.
If tol0 is positive then that is the shift in absolute terms. But tol0 may be assigned a negative value, in which case it is interpreted multiplicatively relative to the midspread of the response; tol0 <-abs(tol0) * midspread. Regardless, fit@extra\$tol0 is the amount in absolute terms.
If avoiding the quantile crossing problem is of concern to you, try increasing seppar to decrease the amount of crossing. Probably it is best to choose the smallest value of seppar so that is.crossing returns FALSE. Increasing tol0 relatively or absolutely means the fitted quantiles are allowed to move apart more. However, tau must be considered when choosing tol0.
llocation, ilocation
See Links for more choices and CommonVGAMffArguments for more information. Choosing loglink should usually be good for counts. And choosing logitlink should be a reasonable for proportions. However, avoid choosing tau values close to the boundary, for example, if $p_{0}$ is the proportion of 0 s then choose $p_{0} \ll \tau$. For proportions grouped data is much better than ungrouped data, and the bigger the groups the more the granularity so that the empirical proportion can approximate tau more closely.
lambda.arg Positive tuning parameter which controls the sharpness of the cusp. The limit as it approaches 0 is probably very similar to dalap. The default is to choose the value internally. If scale. arg increases, then probably lambda. arg needs to increase accordingly. If lambda. arg is too large then the empirical quantiles may not be very close to tau. If lambda. arg is too close to 0 then the convergence behaviour will not be good and local solutions found, as well as numerical problems in general. Monitoring convergence is recommended when varying lambda. arg.
scale.arg Positive scale parameter and sometimes called scale. The transformation used is ( $y$-location) / scale. This function should be okay for response variables having a moderate range ( $0-100$, say), but if very different from this then experimenting with this argument will be a good idea.
ishrinkage, idf.mu, digt
Similar to alaplace1.
imethod Initialization method. Either the value 1, 2, or .... See CommonVGAMffArguments for more information.

## Details

This is an experimental family function for quantile regression. Fasiolo et al. (2020) propose an extended log-F distribution (ELF) however this family function only estimates the location parameter. The distribution has a scale parameter which can be inputted (default value is unity). One location parameter is estimated for each tau value and these are the estimated quantiles. For quantile regression it is not necessary to estimate the scale parameter since the log-likelihood function is triangle shaped.
The ELF is used as an approximation of the asymmetric Laplace distribution (ALD). The latter cannot be estimated properly using Fisher scoring/IRLS but the ELF holds promise because it has continuous derivatives and therefore fewer problems with the regularity conditions. Because the ELF is fitted to data to obtain an empirical result the convergence behaviour may not be gentle and smooth. Hence there is a function-specific control function called extlogF1. control which has something like stepsize $=0.5$ and maxits $=100$. It has been found that slowing down the rate of convergence produces greater stability during the estimation process. Regardless, convergence should be monitored carefully always.
This function accepts a vector response but not a matrix response.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

Changes will occur in the future to fine-tune things. In general setting trace $=$ TRUE is strongly encouraged because it is needful to check that convergence occurs properly.
If seppar $>0$ then logLik(fit) will return the penalized log-likelihood.

## Author(s)

Thomas W. Yee

## References

Fasiolo, M., Wood, S. N., Zaffran, M., Nedellec, R. and Goude, Y. (2020). Fast calibrated additive quantile regression. J. Amer. Statist. Assoc., in press.
Yee, T. W. (2020). On quantile regression based on the 1-parameter extended log-F distribution. In preparation.

## See Also

dextlogF, is.crossing, fix.crossing, eCDF, vglm. control, logF, alaplace1, dalap, lms.bcn.

## Examples

```
nn <- 1000; mytau <- c(0.25, 0.75)
edata <- data.frame(x2 = sort(rnorm(nn)))
edata <- transform(edata, y1 = 1 + x2 + rnorm(nn, sd = exp(-1)),
```

```
    y2 = cos(x2) / (1 + abs(x2)) + rnorm(nn, sd = exp(-1)))
fit1 <- vglm(y1 ~ x2, extlogF1(tau = mytau), data = edata) # trace = TRUE
fit2 <- vglm(y2 ~ bs(x2, 6), extlogF1(tau = mytau), data = edata)
coef(fit1, matrix = TRUE)
fit2@extra$percentile # Empirical percentiles here
summary(fit2)
c(is.crossing(fit1), is.crossing(fit2))
head(fitted(fit1))
## Not run: plot(y2 ~ x2, edata, col = "blue")
matlines(with(edata, x2), fitted(fit2), col="orange", lty = 1, lwd = 2)
## End(Not run)
```

familyname Family Function Name

## Description

Extractor function for the name of the family function of an object in the VGAM package.

## Usage

familyname(object, ...)
familyname.vlm(object, all = FALSE, ...)

## Arguments

object Some VGAM object, for example, having class vglmff-class.
all If all = TRUE then all of the vfamily slot is returned; this contains subclasses the object might have. The default is the return the first value only.
... Other possible arguments for the future.

## Details

Currently VGAM implements over 150 family functions. This function returns the name of the function assigned to the family argument, for modelling functions such as vglm and vgam. Sometimes a slightly different answer is returned, e.g., propodds really calls cumulative with some arguments set, hence the output returned by this function is "cumulative" (note that one day this might change, however).

## Value

A character string or vector.

## Note

Arguments used in the invocation are not included. Possibly this is something to be done in the future.

## See Also

vglmff-class, vglm-class.

## Examples

```
    pneumo <- transform(pneumo, let = log(exposure.time))
    fit1 <- vglm(cbind(normal, mild, severe) ~ let,
        cumulative(parallel = TRUE, reverse = TRUE), data = pneumo)
    familyname(fit1)
    familyname(fit1, all = TRUE)
    familyname(propodds()) # "cumulative"
```

    Felix
        The Felix Distribution
    
## Description

Density for the Felix distribution.

## Usage

dfelix $(x$, rate $=0.25, \log =$ FALSE $)$

## Arguments

x
rate See felix.
$\log \quad$ Logical. If $\log =$ TRUE then the logarithm of the density is returned.

## Details

See felix, the VGAM family function for estimating the parameter, for the formula of the probability density function and other details.

## Value

dfelix gives the density.

## Warning

The default value of rate is subjective.

## Author(s)

T. W. Yee

See Also
felix.

## Examples

```
    \#\# Not run:
    rate <- 0.25; x <- 1:15
    plot(x, dfelix(x, rate), type = "h", las = 1, col = "blue",
        ylab = paste("dfelix(rate=", rate, ")"),
        main = "Felix density function")
    \#\# End(Not run)
```

    felix
    Felix Distribution Family Function
    
## Description

Estimates the parameter of a Felix distribution by maximum likelihood estimation.

## Usage

felix(lrate $=$ extlogitlink(min $=0, \max =0.5)$, imethod $=1$ )

## Arguments

lrate Link function for the parameter, called $a$ below; see Links for more choices and for general information.
imethod See CommonVGAMffArguments. Valid values are 1, 2, 3 or 4.

## Details

The Felix distribution is an important basic Lagrangian distribution. The density function is

$$
f(y ; a)=\frac{1}{((y-1) / 2)!} y^{(y-3) / 2} a^{(y-1) / 2} \exp (-a y)
$$

where $y=1,3,5, \ldots$ and $0<a<0.5$. The mean is $1 /(1-2 a)$ (returned as the fitted values). Fisher scoring is implemented.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Author(s)

T. W. Yee

## References

Consul, P. C. and Famoye, F. (2006). Lagrangian Probability Distributions, Boston, USA: Birkhauser.

## See Also

dfelix, borel.tanner.

## Examples

```
fdata <- data.frame(y = 2 * rpois(n = 200, 1) + 1) # Not real data!
fit <- vglm(y ~ 1, felix, data = fdata, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

fff F Distribution Family Function

## Description

Maximum likelihood estimation of the (2-parameter) F distribution.

## Usage

fff(link = "loglink", idf1 = NULL, idf2 = NULL, nsimEIM = 100, imethod $=1$, zero $=$ NULL)

## Arguments

link Parameter link function for both parameters. See Links for more choices. The default keeps the parameters positive.
idf1, idf2 Numeric and positive. Initial value for the parameters. The default is to choose each value internally.
nsimEIM, zero See CommonVGAMfffarguments for more information.
imethod Initialization method. Either the value 1 or 2. If both fail try setting values for idf1 and idf2.

## Details

The F distribution is named after Fisher and has a density function that has two parameters, called df 1 and df2 here. This function treats these degrees of freedom as positive reals rather than integers. The mean of the distribution is $d f 2 /(d f 2-2)$ provided $d f 2>2$, and its variance is $2 d f 2^{2}(d f 1+$ $d f 2-2) /\left(d f 1(d f 2-2)^{2}(d f 2-4)\right)$ provided $d f 2>4$. The estimated mean is returned as the fitted values. Although the F distribution can be defined to accommodate a non-centrality parameter ncp, it is assumed zero here. Actually it shouldn't be too difficult to handle any known ncp; something to do in the short future.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Warning

Numerical problems will occur when the estimates of the parameters are too low or too high.

## Author(s)

T. W. Yee

## References

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011). Statistical Distributions, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

## See Also

FDist.

## Examples

```
## Not run:
fdata <- data.frame(x2 = runif(nn <- 2000))
fdata <- transform(fdata, df1 = exp(2+0.5*x2),
    df2 = exp(2-0.5*x2))
fdata <- transform(fdata, y = rf(nn, df1, df2))
fit <- vglm(y ~ x2, fff, data = fdata, trace = TRUE)
coef(fit, matrix = TRUE)
## End(Not run)
```

fill

Creates a Matrix of Appropriate Dimension

## Description

A support function for the argument $x i j$, it generates a matrix of an appropriate dimension.

## Usage

fill(x, values $=0, n \operatorname{col} x=n \operatorname{col}(x))$

## Arguments

x
A vector or matrix which is used to determine the dimension of the answer, in particular, the number of rows. After converting $x$ to a matrix if necessary, the answer is a matrix of values values, of dimension nrow( $x$ ) by ncolx.
values Numeric. The answer contains these values, which are recycled columnwise if necessary, i.e., as matrix (values, . . . , byrow=TRUE).
ncolx The number of columns of the returned matrix. The default is the number of columns of x .

## Details

The xij argument for vglm allows the user to input variables specific to each linear/additive predictor. For example, consider the bivariate logit model where the first/second linear/additive predictor is the logistic regression of the first/second binary response respectively. The third linear/additive predictor is $\log (O R)=e t a 3$, where $O R$ is the odds ratio. If one has ocular pressure as a covariate in this model then xij is required to handle the ocular pressure for each eye, since these will be different in general. [This contrasts with a variable such as age, the age of the person, which has a common value for both eyes.] In order to input these data into vglm one often finds that functions fill, fill1, etc. are useful.

All terms in the xij and formula arguments in vglm must appear in the form2 argument too.

## Value

matrix (values, nrow=nrow ( $x$ ) , ncol=ncolx), i.e., a matrix consisting of values values, with the number of rows matching $x$, and the default number of columns is the number of columns of $x$.

## Note

The effect of the xij argument is after other arguments such as exchangeable and zero. Hence xij does not affect constraint matrices.

Additionally, there are currently 3 other identical fill functions, called fill1, fill2 and fill3; if you need more then assign fill4 $=\mathrm{fill5}=\mathrm{fill1}$ etc. The reason for this is that if more than one fill function is needed then they must be unique. For example, if $M=4$ then $\mathrm{xij}=\mathrm{op} \sim$ lop $+r o p+f i l l(m o p)+f i l l(m o p)$ would reduce to $x i j=o p \sim l o p+r o p+f i l l(m o p)$, whereas $x i j$ $=\mathrm{op} \sim$ lop + rop + fill1 (mop) + fill2 (mop) would retain all $M$ terms, which is needed.
In Examples 1 to 3 below, the xij argument illustrates covariates that are specific to a linear predictor. Here, lop/rop are the ocular pressures of the left/right eye in an artificial dataset, and mop is their mean. Variables leye and reye might be the presence/absence of a particular disease on the LHS/RHS eye respectively.
In Example 3, the xij argument illustrates fitting the (exchangeable) model where there is a common smooth function of the ocular pressure. One should use regression splines since $s$ in vgam does not handle the xij argument. However, regression splines such as bs and ns need to have the same basis functions here for both functions, and Example 3 illustrates a trick involving a function BS to obtain this, e.g., same knots. Although regression splines create more than a single column per term in the model matrix, fill(BS(lop, rop)) creates the required (same) number of columns.

## Author(s)

T. W. Yee

## See Also

vglm. control, vglm, multinomial, Select.

## Examples

```
fill(runif(5))
fill(runif(5), ncol = 3)
```

```
fill(runif(5), val = 1, ncol = 3)
# Generate eyes data for the examples below. Eyes are independent (OR=1).
nn <- 1000 # Number of people
eyesdata <- data.frame(lop = round(runif(nn), 2),
            rop = round(runif(nn), 2),
            age = round(rnorm(nn, 40, 10)))
eyesdata <- transform(eyesdata,
    mop = (lop + rop) / 2, # Mean ocular pressure
    op = (lop + rop) / 2, # Value unimportant unless plotting
# op = lop, # Choose this if plotting
    eta1 = 0 - 2*lop + 0.04*age, # Linear predictor for left eye
    eta2 = 0 - 2*rop + 0.04*age) # Linear predictor for right eye
eyesdata <- transform(eyesdata,
    leye = rbinom(nn, size = 1, prob = logitlink(eta1, inverse = TRUE)),
    reye = rbinom(nn, size = 1, prob = logitlink(eta2, inverse = TRUE)))
# Example 1. All effects are linear.
fit1 <- vglm(cbind(leye,reye) ~ op + age,
            family = binom2.or(exchangeable = TRUE, zero = 3),
            data = eyesdata, trace = TRUE,
            xij = list(op ~ lop + rop + fill(lop)),
            form2 = ~ op + lop + rop + fill(lop) + age)
head(model.matrix(fit1, type = "lm")) # LM model matrix
head(model.matrix(fit1, type = "vlm")) # Big VLM model matrix
coef(fit1)
coef(fit1, matrix = TRUE) # Unchanged with 'xij'
constraints(fit1)
max(abs(predict(fit1)-predict(fit1, new = eyesdata))) # Predicts correctly
summary(fit1)
## Not run:
plotvgam(fit1, se = TRUE) # Wrong, e.g., because it plots against op, not lop.
# So set op = lop in the above for a correct plot.
## End(Not run)
# Example 2. This model uses regression splines on ocular pressure.
# It uses a trick to ensure common basis functions.
BS <- function(x, ...)
    sm.bs(c(x,...), df = 3)[1:length(x), , drop = FALSE] # trick
fit2 <- vglm(cbind(leye,reye) ~ BS(lop,rop) + age,
    family = binom2.or(exchangeable = TRUE, zero = 3),
    data = eyesdata, trace = TRUE,
    xij = list(BS(lop,rop) ~ BS(lop,rop) +
                                    BS(rop,lop) +
                                    fill(BS(lop,rop))),
    form2 = ~ BS(lop,rop) + BS(rop,lop) + fill(BS(lop,rop)) +
        lop + rop + age)
head(model.matrix(fit2, type = "lm")) # LM model matrix
head(model.matrix(fit2, type = "vlm")) # Big VLM model matrix
coef(fit2)
coef(fit2, matrix = TRUE)
```

```
summary(fit2)
fit2@smart.prediction
max(abs(predict(fit2) - predict(fit2, new = eyesdata))) # Predicts correctly
predict(fit2, new = head(eyesdata)) # Note the 'scalar' OR, i.e., zero=3
max(abs(head(predict(fit2)) -
                predict(fit2, new = head(eyesdata)))) # Should be 0
## Not run:
plotvgam(fit2, se = TRUE, xlab = "lop") # Correct
## End(Not run)
# Example 3. Capture-recapture model with ephemeral and enduring
# memory effects. Similar to Yang and Chao (2005), Biometrics.
deermice <- transform(deermice, Lag1 = y1)
M.tbh.lag1 <-
    vglm(cbind(y1, y2, y3, y4, y5, y6) ~ sex + weight + Lag1,
        posbernoulli.tb(parallel.t = FALSE ~ 0,
                            parallel.b = FALSE ~ 0,
                            drop.b = FALSE ~ 1),
        xij = list(Lag1 ~ fill(y1) + fill(y2) + fill(y3) + fill(y4) +
                        fill(y5) + fill(y6) +
                            y1 + y2 + y3 + y4 + y5),
        form2 = ~ sex + weight + Lag1 +
            fill(y1) + fill(y2) + fill(y3) + fill(y4) +
            fill(y5) + fill(y6) +
            y1 + y2 + y3 + y4 + y5 + y6,
        data = deermice, trace = TRUE)
coef(M.tbh.lag1)
```

finney44 Toxicity trial for insects

## Description

A data frame of a toxicity trial.

## Usage

data(finney44)

## Format

A data frame with 6 observations on the following 3 variables.
pconc a numeric vector, percent concentration of pyrethrins.
hatched number of eggs that hatched.
unhatched number of eggs that did not hatch.

## Details

Finney (1944) describes a toxicity trial of five different concentrations of pyrethrins (percent) plus a control that were administered to eggs of Ephestia kuhniella. The natural mortality rate is large, and a common adjustment is to use Abbott's formula.

## References

Finney, D. J. (1944). The application of the probit method to toxicity test data adjusted for mortality in the controls. Annals of Applied Biology, 31, 68-74.
Abbott, W. S. (1925). A method of computing the effectiveness of an insecticide. Journal of Economic Entomology, 18, 265-7.

## Examples

data(finney44)
transform(finney44, mortality = unhatched / (hatched + unhatched))

## Description

Computes the Fisher Z transformation, including its inverse and the first two derivatives.

## Usage

fisherzlink(theta, bminvalue = NULL, bmaxvalue = NULL, inverse $=$ FALSE, deriv $=0$, short $=$ TRUE, tag $=$ FALSE)

## Arguments

theta Numeric or character. See below for further details.
bminvalue, bmaxvalue
Optional boundary values. Values of theta which are less than or equal to -1 can be replaced by bminvalue before computing the link function value. Values of theta which are greater than or equal to 1 can be replaced by bmaxvalue before computing the link function value. See Links.
inverse, deriv, short, tag
Details at Links.

## Details

The fisherz link function is commonly used for parameters that lie between -1 and 1 . Numerical values of theta close to -1 or 1 or out of range result in Inf, $-\mathrm{Inf}, \mathrm{NA}$ or NaN .

## Value

For deriv $=0,0.5 * \log ((1+$ theta $) /(1-$ theta $))(\operatorname{same}$ as atanh $($ theta $))$ when inverse $=$ FALSE, and if inverse $=$ TRUE then $(\exp (2 *$ theta $)-1) /(\exp (2 *$ theta $)+1)($ same as tanh (theta $))$.

For deriv $=1$, then the function returns $d$ eta $/ d$ theta as a function of theta if inverse $=$ FALSE, else if inverse $=$ TRUE then it returns the reciprocal.

Here, all logarithms are natural logarithms, i.e., to base $e$.

Note
Numerical instability may occur when theta is close to -1 or 1 . One way of overcoming this is to use, e.g., bminvalue.

The link function rhobitlink is very similar to fisherzlink, e.g., just twice the value of fisherzlink. This link function may be renamed to atanhlink in the near future.

## Author(s)

Thomas W. Yee

## References

McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, 2nd ed. London: Chapman \& Hall.

## See Also

Links, rhobitlink, logitlink.

## Examples

```
theta <- seq(-0.99, 0.99, by = 0.01)
y <- fisherzlink(theta)
## Not run: plot(theta, y, type = "l", las = 1, ylab = "",
    main = "fisherzlink(theta)", col = "blue")
abline(v = (-1):1, h = 0, lty = 2, col = "gray")
## End(Not run)
x <- c(seq(-1.02, -0.98, by = 0.01), seq(0.97, 1.02, by = 0.01))
fisherzlink(x) # Has NAs
fisherzlink(x, bminvalue = -1 + .Machine$double.eps,
    bmaxvalue = 1 - .Machine$double.eps) # Has no NAs
```

Fisk The Fisk Distribution

## Description

Density, distribution function, quantile function and random generation for the Fisk distribution with shape parameter a and scale parameter scale.

## Usage

dfisk(x, scale = 1, shape1.a, log = FALSE)
pfisk(q, scale = 1, shape1.a, lower.tail = TRUE, log.p = FALSE)
qfisk(p, scale = 1, shape1.a, lower.tail = TRUE, log.p = FALSE)
rfisk(n, scale $=1$, shape1.a)

## Arguments

$x, q \quad$ vector of quantiles.
$p$ vector of probabilities.
$n \quad$ number of observations. If length $(n)>1$ then the length is taken to be the number required.
shape1.a shape parameter.
scale scale parameter.
log Logical. If log = TRUE then the logarithm of the density is returned.
lower.tail, log.p
Same meaning as in pnorm or qnorm.

## Details

See fisk, which is the VGAM family function for estimating the parameters by maximum likelihood estimation.

## Value

dfisk gives the density, pfisk gives the distribution function, qfisk gives the quantile function, and rfisk generates random deviates.

## Note

The Fisk distribution is a special case of the 4-parameter generalized beta II distribution.

## Author(s)

T. W. Yee and Kai Huang

## References

Kleiber, C. and Kotz, S. (2003). Statistical Size Distributions in Economics and Actuarial Sciences, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

fisk, genbetaII.

## Examples

```
fdata <- data.frame(y = rfisk(n = 1000, shape = exp(1), scale = exp(2)))
fit <- vglm(y ~ 1, fisk(lss = FALSE), data = fdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
```

    fisk Fisk Distribution family function
    
## Description

Maximum likelihood estimation of the 2-parameter Fisk distribution.

## Usage

fisk(lscale = "loglink", lshape1.a = "loglink", iscale = NULL, ishape1.a $=$ NULL, imethod $=1$, lss $=$ TRUE, $\operatorname{gscale}=\exp (-5: 5)$, gshape1.a $=\operatorname{seq}(0.75,4$, by $=0.25)$, probs.y $=c(0.25,0.5,0.75)$, zero = "shape")

## Arguments

lss
See CommonVGAMffArguments for important information.
lshape1.a, lscale
Parameter link functions applied to the (positive) parameters $a$ and scale. See Links for more choices.
iscale, ishape1.a, imethod, zero
See CommonVGAMffArguments for information. For imethod $=2$ a good initial value for iscale is needed to obtain a good estimate for the other parameter.
gscale, gshape1.a
See CommonVGAMffArguments for information.
probs.y See CommonVGAMffArguments for information.

## Details

The 2-parameter Fisk (aka log-logistic) distribution is the 4-parameter generalized beta II distribution with shape parameter $q=p=1$. It is also the 3-parameter Singh-Maddala distribution with shape parameter $q=1$, as well as the Dagum distribution with $p=1$. More details can be found in Kleiber and Kotz (2003).
The Fisk distribution has density

$$
f(y)=a y^{a-1} /\left[b^{a}\left\{1+(y / b)^{a}\right\}^{2}\right]
$$

for $a>0, b>0, y \geq 0$. Here, $b$ is the scale parameter scale, and $a$ is a shape parameter. The cumulative distribution function is

$$
F(y)=1-\left[1+(y / b)^{a}\right]^{-1}=\left[1+(y / b)^{-a}\right]^{-1}
$$

The mean is

$$
E(Y)=b \Gamma(1+1 / a) \Gamma(1-1 / a)
$$

provided $a>1$; these are returned as the fitted values. This family function handles multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

See the notes in genbetaII.

## Author(s)

T. W. Yee

## References

Kleiber, C. and Kotz, S. (2003). Statistical Size Distributions in Economics and Actuarial Sciences, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

Fisk, genbetaII, betaII, dagum, sinmad, inv.lomax, lomax, paralogistic, inv.paralogistic, simulate.vlm.

## Examples

```
fdata <- data.frame(y = rfisk(n = 200, shape = exp(1), scale = exp(2)))
fit <- vglm(y ~ 1, fisk(lss = FALSE), data = fdata, trace = TRUE)
fit <- vglm(y ~ 1, fisk(ishape1.a = exp(2)), data = fdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

fittedvlm Fitted Values of a VLM object

## Description

Extractor function for the fitted values of a model object that inherits from a vector linear model (VLM), e.g., a model of class "vglm".

## Usage

fittedvlm(object, drop $=$ FALSE, type.fitted $=$ NULL, percentiles $=$ NULL, ...)

## Arguments

object a model object that inherits from a VLM.
drop Logical. If FALSE then the answer is a matrix. If TRUE then the answer is a vector.
type.fitted Character. Some VGAM family functions have a type.fitted argument. If so then a different type of fitted value can be returned. It is recomputed from the model after convergence. Note: this is an experimental feature and not all VGAM family functions have this implemented yet. See CommonVGAMffArguments for more details.
percentiles See CommonVGAMffArguments for details.
... Currently unused.

## Details

The "fitted values" usually corresponds to the mean response, however, because the VGAM package fits so many models, this sometimes refers to quantities such as quantiles. The mean may even not exist, e.g., for a Cauchy distribution.
Note that the fitted value is output from the @linkinv slot of the VGAM family function, where the eta argument is the $n \times M$ matrix of linear predictors.

## Value

The fitted values evaluated at the final IRLS iteration.

## Note

This function is one of several extractor functions for the VGAM package. Others include coef, deviance, weights and constraints etc. This function is equivalent to the methods function for the generic function fitted.values.
If fit is a VLM or VGLM then fitted(fit) and predict(fit, type = "response") should be equivalent (see predictvglm). The latter has the advantage in that it handles a newdata argument so that the fitted values can be computed for a different data set.

## Author(s)

Thomas W. Yee

## References

Chambers, J. M. and T. J. Hastie (eds) (1992). Statistical Models in S. Wadsworth \& Brooks/Cole.

## See Also

fitted, predictvglm, vglmff-class.

## Examples

```
# Categorical regression example 1
pneumo <- transform(pneumo, let = log(exposure.time))
(fit1 <- vglm(cbind(normal, mild, severe) ~ let, propodds, data = pneumo))
fitted(fit1)
# LMS quantile regression example 2
fit2 <- vgam(BMI ~ s(age, df = c(4, 2)),
    lms.bcn(zero = 1), data = bmi.nz, trace = TRUE)
head(predict(fit2, type = "response")) # Equal to the the following two:
head(fitted(fit2))
predict(fit2, type = "response", newdata = head(bmi.nz))
# Zero-inflated example 3
zdata <- data.frame(x2 = runif(nn <- 1000))
zdata <- transform(zdata, pstr0.3 = logitlink(-0.5 , inverse = TRUE),
    lambda.3 = loglink(-0.5 + 2*x2, inverse = TRUE))
zdata <- transform(zdata, y1 = rzipois(nn, lambda = lambda.3, pstr0 = pstr0.3))
fit3 <- vglm(y1 ~ x2, zipoisson(zero = NULL), data = zdata, trace = TRUE)
head(fitted(fit3, type.fitted = "mean" )) # E(Y), which is the default
head(fitted(fit3, type.fitted = "pobs0")) # P(Y = 0)
head(fitted(fit3, type.fitted = "pstr0")) # Prob of a structural 0
head(fitted(fit3, type.fitted = "onempstr0")) # 1 - prob of a structural 0
```

fix.crossing Fixing a Quantile Regression having Crossing

## Description

Returns a similar object fitted with columns of the constraint matrices amalgamated so it is a partially parallel VGLM object. The columns combined correspond to certain crossing quantiles. This applies especially to an extlogF1() VGLM object.

## Usage

fix.crossing.vglm(object, maxit $=100$, trace $=$ FALSE, $\ldots$ )

## Arguments

object an object such as a vglm object with family function extlogF1.
maxit, trace values for overwriting components in vglm. control. Setting these to NULL will mean the values in vglm. control on object will be retained.
$\ldots \quad$ additional optional arguments. Currently unused.

## Details

The quantile crossing problem has been described as disturbing and embarrassing. This function was specifically written for a vglm with family function extlogF1. It examines the fitted quantiles of object to see if any cross. If so, then a pair of columns is combined to make those two quantiles parallel. After fitting the submodel it then repeats testing for crossing quantiles and repairing them, until there is no more quantile crossing detected. Note that it is possible that the quantiles cross in some subset of the covariate space not covered by the data-see is.crossing.

This function is fragile and likely to change in the future. For extlogF1 models, it is assumed that argument data has been assigned a data frame, and that the default values of the argument parallel has been used; this means that the second constraint matrix is diag(M). The constraint matrix of the intercept term remains unchanged as diag(M).

## Value

An object very similar to the original object, but with possibly different constraint matrices (partially parallel) so as to remove any quantile crossing.

## See Also

extlogF1, is.crossing, lms.ben. vglm.

## Examples

```
## Not run: ooo <- with(bmi.nz, order(age))
bmi.nz <- bmi.nz[ooo, ] # Sort by age
with(bmi.nz, plot(age, BMI, col = "blue"))
mytau <- c(50, 93, 95, 97) / 100 # Some quantiles are quite close
fit1 <- vglm(BMI ~ ns(age, 7), extlogF1(mytau), bmi.nz, trace = TRUE)
plot(BMI ~ age, bmi.nz, col = "blue", las = 1,
    main = "Partially parallel (darkgreen) & nonparallel quantiles",
    sub = "Crossing quantiles are orange")
fix.crossing(fit1)
matlines(with(bmi.nz, age), fitted(fit1), lty = 1, col = "orange")
fit2 <- fix.crossing(fit1) # Some quantiles have been fixed
constraints(fit2)
matlines(with(bmi.nz, age), fitted(fit2), lty = "dashed",
            col = "darkgreen", lwd = 2)
## End(Not run)
```

flourbeetle Mortality of Flour Beetles from Carbon Disulphide

## Description

The flourbeetle data frame has 8 rows and 4 columns. Two columns are explanatory, the other two are responses.

## Usage

data(flourbeetle)

## Format

This data frame contains the following columns:
logdose $\log 10$ applied to CS2mgL.
CS2mgL a numeric vector, the concentration of gaseous carbon disulphide in mg per litre.
exposed a numeric vector, counts; the number of beetles exposed to the poison.
killed a numeric vector, counts; the numbers killed.

## Details

These data were originally given in Table IV of Bliss (1935) and are the combination of two series of toxicological experiments involving Tribolium confusum, also known as the flour beetle. Groups of such adult beetles were exposed for 5 hours of gaseous carbon disulphide at different concentrations, and their mortality measured.

## Source

Bliss, C.I., 1935. The calculation of the dosage-mortality curve. Annals of Applied Biology, 22, 134-167.

## See Also

binomialff, probitlink.

## Examples

```
fit1 <- vglm(cbind(killed, exposed - killed) ~ logdose,
            binomialff(link = probitlink), flourbeetle, trace = TRUE)
summary(fit1)
```

Foldnorm The Folded-Normal Distribution

## Description

Density, distribution function, quantile function and random generation for the (generalized) foldednormal distribution.

## Usage

dfoldnorm(x, mean $=0, \mathrm{sd}=1$, $\mathrm{a} 1=1, \mathrm{a} 2=1$, log $=$ FALSE)
pfoldnorm(q, mean $=0, s d=1, a 1=1$, $a 2=1$,
lower.tail = TRUE, log.p = FALSE)
qfoldnorm(p, mean $=0, s d=1, a 1=1, a 2=1$,
lower.tail $=$ TRUE, log.p $=$ FALSE,... )
rfoldnorm(n, mean $=0, s d=1, a 1=1, a 2=1$ )

## Arguments

| $\mathrm{x}, \mathrm{q}$ | vector of quantiles. |
| :--- | :--- |
| p | vector of probabilities. |
| n | number of observations. Same as rnorm. |
| mean, sd | see rnorm. |
| a 1, a2 | see foldnormal. |
| log | Logical. If TRUE then the log density is returned. |
| lower.tail, log.p | Same meaning as in pnorm or qnorm. |
|  | Arguments that can be passed into uniroot. |

## Details

See foldnormal, the VGAM family function for estimating the parameters, for the formula of the probability density function and other details.

## Value

dfoldnorm gives the density, pfoldnorm gives the distribution function, qfoldnorm gives the quantile function, and rfoldnorm generates random deviates.

## Note

qfoldnorm() runs very slowly because it calls uniroot for each value of the argument $p$. The solution is consequently not exact; the . . . can be used to obtain a more accurate solution if necessary.

## Author(s)

T. W. Yee and Kai Huang

## See Also

foldnormal, uniroot.

## Examples

```
## Not run:
m <- 1.5; SD <- exp(0)
x<- seq(-1, 4, len = 501)
plot(x, dfoldnorm(x, m = m, sd = SD), type = "l", ylim = 0:1, las = 1,
    ylab = paste("foldnorm(m = ", m, ", sd = ", round(SD, digits = 3), ")"),
    main = "Blue is density, orange is cumulative distribution function",
    sub = "Purple lines are the 10,20,\ldots.,90 percentiles", col = "blue")
abline(h = 0, col = "gray50")
lines(x, pfoldnorm(x, m = m, sd = SD), col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qfoldnorm(probs, m = m, sd = SD)
lines(Q, dfoldnorm(Q, m = m, sd = SD), col = "purple", lty = 3, type = "h")
lines(Q, pfoldnorm(Q, m = m, sd = SD), col = "purple", lty = 3, type = "h")
abline(h = probs, col = "purple", lty = 3)
max(abs(pfoldnorm(Q, m = m, sd = SD) - probs)) # Should be 0
## End(Not run)
```


## Description

Fits a (generalized) folded (univariate) normal distribution.

## Usage

```
foldnormal(lmean = "identitylink", lsd = "loglink", imean = NULL, isd = NULL,
    a1 = 1, a2 = 1, nsimEIM = 500, imethod = 1, zero = NULL)
```


## Arguments

lmean, lsd Link functions for the mean and standard deviation parameters of the usual univariate normal distribution. They are $\mu$ and $\sigma$ respectively. See Links for more choices.
imean, isd Optional initial values for $\mu$ and $\sigma$. A NULL means a value is computed internally. See CommonVGAMffArguments.
a1, a2 Positive weights, called $a_{1}$ and $a_{2}$ below. Each must be of length 1.
nsimEIM, imethod, zero
See CommonVgAMffArguments.

## Details

If a random variable has an ordinary univariate normal distribution then the absolute value of that random variable has an ordinary folded normal distribution. That is, the sign has not been recorded; only the magnitude has been measured.
More generally, suppose $X$ is normal with mean mean and standard deviation sd. Let $Y=$ $\max \left(a_{1} X,-a_{2} X\right)$ where $a_{1}$ and $a_{2}$ are positive weights. This means that $Y=a_{1} X$ for $X>0$, and $Y=a_{2} X$ for $X<0$. Then $Y$ is said to have a generalized folded normal distribution. The ordinary folded normal distribution corresponds to the special case $a_{1}=a_{2}=1$.
The probability density function of the ordinary folded normal distribution can be written dnorm ( y , mean, sd ) $+\operatorname{dnorm}(y,-m e a n, s d)$ for $y \geq 0$. By default, mean and $\log (s d)$ are the linear/additive predictors. Having mean $=0$ and $s d=1$ results in the half-normal distribution. The mean of an ordinary folded normal distribution is

$$
E(Y)=\sigma \sqrt{2 / \pi} \exp \left(-\mu^{2} /\left(2 \sigma^{2}\right)\right)+\mu[1-2 \Phi(-\mu / \sigma)]
$$

and these are returned as the fitted values. Here, $\Phi()$ is the cumulative distribution function of a standard normal (pnorm).

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Warning

Under- or over-flow may occur if the data is ill-conditioned. It is recommended that several different initial values be used to help avoid local solutions.

## Note

The response variable for this family function is the same as uninormal except positive values are required. Reasonably good initial values are needed. Fisher scoring using simulation is implemented.
See CommonVGAMffArguments for general information about many of these arguments.
Yet to do: implement the results of Johnson (1962) which gives expressions for the EIM, albeit, under a different parameterization. Also, one element of the EIM appears to require numerical integration.

## Author(s)

Thomas W. Yee

## References

Lin, P. C. (2005). Application of the generalized folded-normal distribution to the process capability measures. International Journal of Advanced Manufacturing Technology, 26, 825-830.
Johnson, N. L. (1962). The folded normal distribution: accuracy of estimation by maximum likelihood. Technometrics, 4, 249-256.

## See Also

rfoldnorm, uninormal, dnorm, skewnormal.

## Examples

```
## Not run: m <- 2; SD <- exp(1)
fdata <- data.frame(y = rfoldnorm(n <- 1000, m = m, sd = SD))
hist(with(fdata, y), prob = TRUE, main = paste("foldnormal(m = ", m,
    ", sd = ", round(SD, 2), ")"))
fit <- vglm(y ~ 1, foldnormal, data = fdata, trace = TRUE)
coef(fit, matrix = TRUE)
(Cfit <- Coef(fit))
# Add the fit to the histogram:
mygrid <- with(fdata, seq(min(y), max(y), len = 200))
lines(mygrid, dfoldnorm(mygrid, Cfit[1], Cfit[2]), col = "orange")
## End(Not run)
```

foldsqrtlink Folded Square Root Link Function

## Description

Computes the folded square root transformation, including its inverse and the first two derivatives.

## Usage

foldsqrtlink(theta, min = 0, max = 1, mux = sqrt(2),
inverse $=$ FALSE, deriv $=0$, short $=$ TRUE, tag $=$ FALSE)

## Arguments

theta Numeric or character. See below for further details.
min, max, mux These are called $L, U$ and $K$ below.
inverse, deriv, short, tag
Details at Links.

## Details

The folded square root link function can be applied to parameters that lie between $L$ and $U$ inclusive. Numerical values of theta out of range result in NA or NaN .

## Value

For foldsqrtlink with deriv = 0: $K(\sqrt{\theta-L}-\sqrt{U-\theta})$ or mux * (sqrt (theta-min) -sqrt (max-theta) ) when inverse $=$ FALSE, and if inverse $=$ TRUE then some more complicated function that returns a NA unless theta is between-mux*sqrt(max-min) and mux*sqrt(max-min).
For deriv $=1$, then the function returns $d$ eta $/ d$ theta as a function of theta if inverse $=$ FALSE, else if inverse $=$ TRUE then it returns the reciprocal.

## Note

The default has, if theta is 0 or 1 , the link function value is $-\operatorname{sqrt}(2)$ and $+\operatorname{sqrt}(2)$ respectively. These are finite values, therefore one cannot use this link function for general modelling of probabilities because of numerical problem, e.g., with binomialff, cumulative. See the example below.

## Author(s)

Thomas W. Yee

## See Also

Links.

## Examples

```
p <- seq(0.01, 0.99, by = 0.01)
foldsqrtlink(p)
max(abs(foldsqrtlink(foldsqrtlink(p), inverse = TRUE) - p)) # Should be 0
p <- c(seq(-0.02, 0.02, by = 0.01), seq(0.97, 1.02, by = 0.01))
foldsqrtlink(p) # Has NAs
## Not run:
p <- seq(0.01, 0.99, by = 0.01)
par(mfrow = c(2, 2), lwd = (mylwd <- 2))
y <- seq(-4, 4, length = 100)
for (d in 0:1) {
    matplot(p, cbind(logitlink(p, deriv = d), foldsqrtlink(p, deriv = d)),
                type = "n", col = "purple", ylab = "transformation", las = 1,
                main = if (d == 0) "Some probability link functions"
                else "First derivative")
    lines(p, logitlink(p, deriv = d), col = "limegreen")
    lines(p, probitlink(p, deriv = d), col = "purple")
    lines(p, clogloglink(p, deriv = d), col = "chocolate")
    lines(p, foldsqrtlink(p, deriv = d), col = "tan")
    if (d == 0) {
        abline(v = 0.5, h = 0, lty = "dashed")
        legend(0, 4.5, c("logitlink", "probitlink", "clogloglink", "foldsqrtlink"),
            lwd = 2, col = c("limegreen", "purple", "chocolate", "tan"))
    } else
        abline(v = 0.5, lty = "dashed")
}
for (d in 0) {
    matplot(y, cbind(logitlink(y, deriv = d, inverse = TRUE),
                                    foldsqrtlink(y, deriv = d, inverse = TRUE)),
            type = "n", col = "purple", xlab = "transformation", ylab = "p",
            lwd = 2, las = 1,
            main = if (d == 0) "Some inverse probability link functions"
            else "First derivative")
    lines(y, logitlink(y, deriv = d, inverse = TRUE), col = "limegreen")
```

```
    lines(y, probitlink(y, deriv = d, inverse = TRUE), col = "purple")
    lines(y, clogloglink(y, deriv = d, inverse = TRUE), col = "chocolate")
    lines(y, foldsqrtlink(y, deriv = d, inverse = TRUE), col = "tan")
    if (d == 0) {
        abline(h = 0.5, v = 0, lty = "dashed")
        legend(-4, 1, c("logitlink", "probitlink", "clogloglink", "foldsqrtlink"),
            lwd = 2, col = c("limegreen", "purple", "chocolate", "tan"))
    }
}
par(lwd = 1)
## End(Not run)
# This is lucky to converge
fit.h <- vglm(agaaus ~ sm.bs(altitude), binomialff(link = foldsqrtlink(mux = 5)),
            data = hunua, trace = TRUE)
## Not run:
plotvgam(fit.h, se = TRUE, lcol = "orange", scol = "orange",
        main = "Orange is Hunua, Blue is Waitakere")
## End(Not run)
head(predict(fit.h, hunua, type = "response"))
## Not run:
# The following fails.
pneumo <- transform(pneumo, let = log(exposure.time))
fit <- vglm(cbind(normal, mild, severe) ~ let,
    cumulative(link = foldsqrtlink(mux = 10), par = TRUE, rev = TRUE),
    data = pneumo, trace = TRUE, maxit = 200)
## End(Not run)
```

formulavlm Model Formulae and Term Names for VGLMs

## Description

The methods function for formula to extract the formula from a fitted object, as well as a methods function to return the names of the terms in the formula.

## Usage

```
## S3 method for class 'vlm'
formula(x, ...)
formulavlm(x, form.number = 1, ...)
term.names(model, ...)
term.namesvlm(model, form.number = 1, ...)
```


## Arguments

| x, model | A fitted model object. |
| :--- | :--- |
| form. number | Formula number, is 1 or 2. which correspond to the arguments formula and <br> form2 respectively. |
| $\ldots$ | Same as formula. |

## Details

The formula methods function is based on formula.

## Value

The formula methods function should return something similar to formula. The term.names methods function should return a character string with the terms in the formula; this includes any intercept (which is denoted by "(Intercept)" as the first element.)

## Author(s)

Thomas W. Yee

## See Also

has.interceptvlm.

## Examples

```
# Example: this is based on a glm example
counts <- c(18,17,15,20,10,20,25,13,12)
outcome <- gl(3, 1, 9); treatment <- gl(3, 3)
vglm.D93 <- vglm(counts ~ outcome + treatment, family = poissonff)
formula(vglm.D93)
pdata <- data.frame(counts, outcome, treatment) # Better style
vglm.D93 <- vglm(counts ~ outcome + treatment, poissonff, data = pdata)
formula(vglm.D93)
term.names(vglm.D93)
responseName(vglm.D93)
has.intercept(vglm.D93)
```

Frank
Frank's Bivariate Distribution

## Description

Density, distribution function, and random generation for the (one parameter) bivariate Frank distribution.

## Usage

dbifrankcop(x1, x2, apar, log = FALSE)
pbifrankcop(q1, q2, apar)
rbifrankcop(n, apar)

## Arguments

$x 1, x 2, q 1, q 2$ vector of quantiles.
$n \quad$ number of observations. Same as in runif.
apar the positive association parameter.
log Logical. If log = TRUE then the logarithm of the density is returned.

## Details

See bifrankcop, the VGAM family functions for estimating the association parameter by maximum likelihood estimation, for the formula of the cumulative distribution function and other details.

## Value

dbifrankcop gives the density, pbifrankcop gives the distribution function, and rbifrankcop generates random deviates (a two-column matrix).

## Author(s)

T. W. Yee

## References

Genest, C. (1987). Frank's family of bivariate distributions. Biometrika, 74, 549-555.

## See Also

bifrankcop.

## Examples

```
## Not run: N <- 100; apar <- exp(2)
xx <- seq(-0.30, 1.30, len = N)
ox <- expand.grid(xx, xx)
zedd <- dbifrankcop(ox[, 1], ox[, 2], apar = apar)
contour(xx, xx, matrix(zedd, N, N))
zedd <- pbifrankcop(ox[, 1], ox[, 2], apar = apar)
contour(xx, xx, matrix(zedd, N, N))
plot(rr <- rbifrankcop(n = 3000, apar = exp(4)))
par(mfrow = c(1, 2))
hist(rr[, 1]); hist(rr[, 2]) # Should be uniform
## End(Not run)
```


## Frechet The Frechet Distribution

## Description

Density, distribution function, quantile function and random generation for the three parameter Frechet distribution.

## Usage

```
dfrechet(x, location = 0, scale = 1, shape, log = FALSE)
pfrechet(q, location = 0, scale = 1, shape,
            lower.tail = TRUE, log.p = FALSE)
qfrechet(p, location = 0, scale = 1, shape,
            lower.tail = TRUE, log.p = FALSE)
rfrechet(n, location = 0, scale = 1, shape)
```


## Arguments

| $\mathrm{x}, \mathrm{q}$ | vector of quantiles. |
| :--- | :--- |
| p | vector of probabilities. |
| n | number of observations. Passed into runif. <br> location, scale, shape <br> the location parameter $a$, scale parameter $b$, and shape parameter $s$. |
| log Logical. If log = TRUE then the logarithm of the density is returned. <br> lower.tail, log.  <br>  Same meaning as in punif or qunif. |  |

## Details

See frechet, the VGAM family function for estimating the 2 parameters (without location parameter) by maximum likelihood estimation, for the formula of the probability density function and range restrictions on the parameters.

## Value

dfrechet gives the density, pfrechet gives the distribution function, qfrechet gives the quantile function, and rfrechet generates random deviates.

## Author(s)

T. W. Yee and Kai Huang

## References

Castillo, E., Hadi, A. S., Balakrishnan, N. Sarabia, J. S. (2005). Extreme Value and Related Models with Applications in Engineering and Science, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

frechet.

## Examples

```
## Not run: shape <- 5
x <- seq(-0.1, 3.5, length = 401)
plot(x, dfrechet(x, shape = shape), type = "l", ylab = "", las = 1,
    main = "Frechet density divided into 10 equal areas; orange = cdf")
abline(h = 0, col = "blue", lty = 2)
qq <- qfrechet(seq(0.1, 0.9, by = 0.1), shape = shape)
lines(qq, dfrechet(qq, shape = shape), col = "purple", lty = 3, type = "h")
lines(x, pfrechet(q = x, shape = shape), col = "orange")
## End(Not run)
```

frechet Frechet Distribution Family Function

## Description

Maximum likelihood estimation of the 2-parameter Frechet distribution.

## Usage

frechet(location = 0, lscale = "loglink", lshape = logofflink(offset = -2), iscale $=$ NULL, ishape $=$ NULL, nsimEIM $=250$, zero $=$ NULL)

## Arguments

location Numeric. Location parameter. It is called $a$ below.
lscale, lshape Link functions for the parameters; see Links for more choices.
iscale, ishape, zero, nsimEIM
See CommonVGAMffArguments for information.

## Details

The (3-parameter) Frechet distribution has a density function that can be written

$$
f(y)=\frac{s b}{(y-a)^{2}}[b /(y-a)]^{s-1} \exp \left[-(b /(y-a))^{s}\right]
$$

for $y>a$ and scale parameter $b>0$. The positive shape parameter is $s$. The cumulative distribution function is

$$
F(y)=\exp \left[-(b /(y-a))^{s}\right] .
$$

The mean of $Y$ is $a+b \Gamma(1-1 / s)$ for $s>1$ (these are returned as the fitted values). The variance of $Y$ is $b^{2}\left[\Gamma(1-2 / s)-\Gamma^{2}(1-1 / s)\right]$ for $s>2$.
Family frechet has $a$ known, and $\log (b)$ and $\log (s-2)$ are the default linear/additive predictors. The working weights are estimated by simulated Fisher scoring.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Warning

Family function frechet may fail for low values of the shape parameter, e.g., near 2 or lower.

## Author(s)

T. W. Yee

## References

Castillo, E., Hadi, A. S., Balakrishnan, N. Sarabia, J. S. (2005). Extreme Value and Related Models with Applications in Engineering and Science, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

rfrechet, gev.

## Examples

```
## Not run:
set.seed(123)
fdata <- data.frame(y1 = rfrechet(nn <- 1000, shape = 2 + exp(1)))
with(fdata, hist(y1))
fit2 <- vglm(y1 ~ 1, frechet, data = fdata, trace = TRUE)
coef(fit2, matrix = TRUE)
Coef(fit2)
head(fitted(fit2))
with(fdata, mean(y1))
head(weights(fit2, type = "working"))
vcov(fit2)
## End(Not run)
```

freund61
Freund's (1961) Bivariate Extension of the Exponential Distribution

## Description

Estimate the four parameters of the Freund (1961) bivariate extension of the exponential distribution by maximum likelihood estimation.

## Usage

```
freund61(la = "loglink", lap = "loglink", lb = "loglink", lbp = "loglink",
    ia = NULL, iap = NULL, ib = NULL, ibp = NULL,
    independent = FALSE, zero = NULL)
```


## Arguments

la, lap, lb, lbp
Link functions applied to the (positive) parameters $\alpha, \alpha^{\prime}, \beta$ and $\beta^{\prime}$, respectively (the "p" stands for "prime"). See Links for more choices.
ia, iap, ib, ibp
Initial value for the four parameters respectively. The default is to estimate them all internally.
independent Logical. If TRUE then the parameters are constrained to satisfy $\alpha=\alpha^{\prime}$ and $\beta=\beta^{\prime}$, which implies that $y_{1}$ and $y_{2}$ are independent and each have an ordinary exponential distribution.
zero A vector specifying which linear/additive predictors are modelled as intercepts only. The values can be from the set $\{1,2,3,4\}$. The default is none of them. See CommonVGAMffarguments for more information.

## Details

This model represents one type of bivariate extension of the exponential distribution that is applicable to certain problems, in particular, to two-component systems which can function if one of the components has failed. For example, engine failures in two-engine planes, paired organs such as peoples' eyes, ears and kidneys. Suppose $y_{1}$ and $y_{2}$ are random variables representing the lifetimes of two components $A$ and $B$ in a two component system. The dependence between $y_{1}$ and $y_{2}$ is essentially such that the failure of the $B$ component changes the parameter of the exponential life distribution of the $A$ component from $\alpha$ to $\alpha^{\prime}$, while the failure of the $A$ component changes the parameter of the exponential life distribution of the $B$ component from $\beta$ to $\beta^{\prime}$.
The joint probability density function is given by

$$
f\left(y_{1}, y_{2}\right)=\alpha \beta^{\prime} \exp \left(-\beta^{\prime} y_{2}-\left(\alpha+\beta-\beta^{\prime}\right) y_{1}\right)
$$

for $0<y_{1}<y_{2}$, and

$$
f\left(y_{1}, y_{2}\right)=\beta \alpha^{\prime} \exp \left(-\alpha^{\prime} y_{1}-\left(\alpha+\beta-\alpha^{\prime}\right) y_{2}\right)
$$

for $0<y_{2}<y_{1}$. Here, all four parameters are positive, as well as the responses $y_{1}$ and $y_{2}$. Under this model, the probability that component $A$ is the first to fail is $\alpha /(\alpha+\beta)$. The time to the first failure is distributed as an exponential distribution with rate $\alpha+\beta$. Furthermore, the distribution of the time from first failure to failure of the other component is a mixture of Exponential $\left(\alpha^{\prime}\right)$ and Exponential $\left(\beta^{\prime}\right)$ with proportions $\beta /(\alpha+\beta)$ and $\alpha /(\alpha+\beta)$ respectively.
The marginal distributions are, in general, not exponential. By default, the linear/additive predictors are $\eta_{1}=\log (\alpha), \eta_{2}=\log \left(\alpha^{\prime}\right), \eta_{3}=\log (\beta), \eta_{4}=\log \left(\beta^{\prime}\right)$.
A special case is when $\alpha=\alpha^{\prime}$ and $\beta=\beta^{\prime}$, which means that $y_{1}$ and $y_{2}$ are independent, and both have an ordinary exponential distribution with means $1 / \alpha$ and $1 / \beta$ respectively.
Fisher scoring is used, and the initial values correspond to the MLEs of an intercept model. Consequently, convergence may take only one iteration.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

To estimate all four parameters, it is necessary to have some data where $y_{1}<y_{2}$ and $y_{2}<y_{1}$.
The response must be a two-column matrix, with columns $y_{1}$ and $y_{2}$. Currently, the fitted value is a matrix with two columns; the first column has values $\left(\alpha^{\prime}+\beta\right) /\left(\alpha^{\prime}(\alpha+\beta)\right)$ for the mean of $y_{1}$, while the second column has values $\left(\beta^{\prime}+\alpha\right) /\left(\beta^{\prime}(\alpha+\beta)\right)$ for the mean of $y_{2}$. The variance of $y_{1}$ is

$$
\frac{\left(\alpha^{\prime}\right)^{2}+2 \alpha \beta+\beta^{2}}{\left(\alpha^{\prime}\right)^{2}(\alpha+\beta)^{2}}
$$

the variance of $y_{2}$ is

$$
\frac{\left(\beta^{\prime}\right)^{2}+2 \alpha \beta+\alpha^{2}}{\left(\beta^{\prime}\right)^{2}(\alpha+\beta)^{2}}
$$

the covariance of $y_{1}$ and $y_{2}$ is

$$
\frac{\alpha^{\prime} \beta^{\prime}-\alpha \beta}{\alpha^{\prime} \beta^{\prime}(\alpha+\beta)^{2}}
$$

## Author(s)

T. W. Yee

## References

Freund, J. E. (1961). A bivariate extension of the exponential distribution. Journal of the American Statistical Association, 56, 971-977.

## See Also

exponential.

## Examples

```
fdata <- data.frame(y1 = rexp(nn <- 1000, rate = exp(1)))
fdata <- transform(fdata, y2 = rexp(nn, rate = exp(2)))
fit1 <- vglm(cbind(y1, y2) ~ 1, fam = freund61, data = fdata, trace = TRUE)
coef(fit1, matrix = TRUE)
Coef(fit1)
vcov(fit1)
head(fitted(fit1))
summary(fit1)
# y1 and y2 are independent, so fit an independence model
fit2 <- vglm(cbind(y1, y2) ~ 1, freund61(indep = TRUE),
    data = fdata, trace = TRUE)
coef(fit2, matrix = TRUE)
constraints(fit2)
pchisq(2 * (logLik(fit1) - logLik(fit2)), # p-value
    df = df.residual(fit2) - df.residual(fit1), lower.tail = FALSE)
lrtest(fit1, fit2) # Better alternative
```

```
gabinomial.mlm
```

Generally-Altered Binomial Regression Family Function (multinomial logit model based; GA-binom-MLM)

## Description

Fits a generally-altered binomial regression (using a multinomial logit model for the altered values).

## Usage

$$
\begin{aligned}
& \text { gabinomial.mlm(alter }=0 \text {, zero }=N U L L, ~ l p r o b=" l o g i t l i n k ", ~ \\
& \text { type.fitted }=c(" m e a n ", " p r o b ", ~ " p o b s . a ", ~ " P o b s . a "), ~ \\
& \text { imethod }=1 \text {, iprob }=N U L L)
\end{aligned}
$$

## Arguments

alter Vector of altered values, i.e., nonnegative integers. Must have unique values only. May be a NULL, which stands for empty set (same as binomialff). The default is the 0 -altered binomial
lprob See Links for more choices and information.
type.fitted See CommonVGAMffArguments for information. The choice "pobs.a" is the probability of an altered value. See gaitpoisson.mlm for more details.
imethod, iprob See CommonVGAMffArguments for information.
zero See CommonVGAMffArguments for information. Setting zero = "pobs" will model the multinomial probabilities as simple as possible (intercept-only), hence should be more numerically stable than the default, and this is recommended for many analyses especially when there are many explanatory variables..

## Details

The generally-altered binomial distribution is the ordinary binomial distribution with the probability of certain values (alter argument) being modelled using a multinomial logit model. The other values are modelled using a generally-truncated binomial distribution. The 0 -altered binomial distribution is a special case and is the default (it is called a hurdle model by some people).
This function implements Fisher scoring and currently does not handle multiple responses. Compared to gibinomial (not complete yet) this family function handles deflation and inflation, therefore handles a wider range of data.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.
The fitted.values slot of the fitted object, which should be extracted by the generic function fitted, returns the mean $\mu$ (default). The choice type.fitted = "pobs.a" returns the sum of all the altered probabilities. See gaitpoisson.mlm for more details.

## Note

This family function does not yet have the robustness of multinomial when computing the working weight matrices. Several improvements are needed, e.g., better labelling and initial values and handling multiple responses.
The default value of zero is not the best for most data sets; so assign it a value (see above).
This family function effectively combines gtbinomial and multinomial into one family function.

## Author(s)

## T. W. Yee and Chenchen Ma

## See Also

Gaitbinom.mlm, gtbinomial, zabinomial, multinomial, rposbinom, CommonVGAMffArguments, simulate.vlm.

## Examples

```
avec <- c(0, 5) # Alter these values
Size <- 10
gdata <- data.frame(x2 = runif(nn <- 1000))
gdata <- transform(gdata, x3 = runif(nn))
gdata <- transform(gdata,
                prob1 = logitlink(1 + 2 * x2 + .5 * x3, inverse = TRUE),
                prob2 = logitlink(1 - 1 * x2 + .5 * x3, inverse = TRUE),
                prob3 = logitlink(1, inverse = TRUE))
gdata <- transform(gdata,
    y1 = rgaitbinom.mlm(nn, prob = prob1, Size, pobs.a = c(0.2, 0.3),
                                    byrow = TRUE, alter = avec),
        y2 = rgaitbinom.mlm(nn, prob = prob2, Size, pobs.a = c(0.2, 0.3),
                                    byrow = TRUE, alter = avec),
        y3 = rgaitbinom.mlm(nn, prob = prob3, Size, pobs.a = c(0.2, 0.3),
                                    byrow = TRUE, alter = avec))
gabinomial.mlm(alter = avec)
with(gdata, table(y1))
with(gdata, table(y2))
with(gdata, table(y3))
fit1 <- vglm(cbind(y1, Size-y1) ~ x2 + x3, gabinomial.mlm(alter = avec),
            crit = "coef", trace = TRUE, data = gdata)
head(fitted(fit1))
head(predict(fit1))
coef(fit1, matrix = TRUE)
summary(fit1)
# Another example -----------------------------
fit3 <- vglm(cbind(y3, Size-y3) ~ 1, gabinomial.mlm(alter = avec),
            gdata, trace = TRUE)
coef(fit3, matrix = TRUE)
```


## Description

Density, distribution function, quantile function and random generation for the generally-altered, -inflated and -truncated binomial distribution based on the multinomial logit model (MLM). This distribution is sometimes abbreviated as GAIT-Binom-MLM-MLM.

## Usage

dgaitbinom.mlm(x, size, prob, alter = NULL, inflate = NULL, truncate $=$ NULL, pobs.a $=0$, pstr. $\mathrm{i}=0$, byrow.arg = FALSE, log.arg = FALSE, .errorcheck = TRUE)
pgaitbinom.mlm(q, size, prob, alter = NULL, inflate = NULL, truncate $=$ NULL, pobs.a $=0$, pstr.i $=0$, byrow.arg = FALSE, .errorcheck = TRUE)
qgaitbinom.mlm(p, size, prob, alter = NULL, inflate = NULL, truncate $=$ NULL, pobs.a $=0$, pstr.i $=0$, byrow.arg $=$ FALSE)
rgaitbinom.mlm(n, size, prob, alter = NULL, inflate = NULL, truncate $=$ NULL, pobs.a $=0$, pstr.i $=0$, byrow.arg $=$ FALSE)

## Arguments

$\mathrm{x}, \mathrm{q}, \mathrm{p}, \mathrm{n}, \mathrm{log} . \arg$
Same meaning as in rbinom.
size, prob Same meaning as in rbinom, i.e., for an ordinary binomial distribution. Short vectors are recycled.
alter, inflate, truncate
Vectors of nonnegative integers; the altered, inflated and truncated values. Each argument must have unique values only. The default value of NULL means an empty set.
pstr.i, byrow.arg
The first argument can be a length(inflate)-vector of probabilities; else a matrix of dimension $n \times$ length (inflate) of probabilities. If a vector then this matrix is constructed from the vector using byrow.arg to determine the enumeration of the elements (similar to matrix). These arguments are not used unless inflate is assigned.
pobs.a This argument is similar to pstr.i but is used when alter is assigned a vector. The argument byrow.arg is used similarly to construct a matrix of dimension $\mathrm{n} \times$ length(alter) of probabilities. This argument is not used unless alter is assigned.
.errorcheck Logical. Should be ignored by the user.

## Details

See Gaitpois for generic details applying to both distributions.
These functions do what dgabinom, dgibinom, dgtbinom, pgabinom, pgibinom, pgtbinom, qgabinom, qgibinom, qgtbinom, rgabinom, rgibinom, rgtbinom collectively did because the arguments alter, inflate and truncate have been combined.

## Value

dgaitbinom.mlm gives the density, pgaitbinom.mlm gives the distribution function, qgaitbinom.mlm gives the quantile function, and rgaitbinom.mlm generates random deviates. The default values of the arguments correspond to ordinary dbinom, pbinom, qbinom, rbinom respectively.

## Author(s)

T. W. Yee.

## See Also

Gaitnbinom.mlm, Posbinom, Binomial.

## Examples

```
avec <- 5:6 # Alter these (special) values
ivec <- 9:10 # Inflate these (special) values
tvec <- 2:4 # Truncate these (special) values
pobs.a <- c(0.1, 0.2)
pstr.i <- (1:2)/10
size <- 10; prob <- 0.5; xx <- 0:11
y <- rgaitbinom.mlm(1000, size, prob, alter = avec, inflate = ivec,
    truncate = tvec, pstr.i = pstr.i, pobs.a = pobs.a, byrow = TRUE)
table(y)
(ii <- dgaitbinom.mlm(xx, size, prob, alter = avec, inflate = ivec,
        truncate = tvec, pstr.i = pstr.i, pobs.a = pobs.a, byrow = TRUE))
```

Gaitlog Generally-Altered, -Inflated and -Truncated Logarithmic Distribu-
tion

## Description

Density, distribution function, quantile function and random generation for the generally-altered, -inflated and -truncated logarithmic distribution. Both parametric and nonparametric variants are supported; these are based on finite mixtures of the parent with itself and the multinomial logit model (MLM) respectively. Altogether it can be abbreviated as GAAIIT-Log(shape.p)-Log(shape.a)-MLM-Log(shape.i)-MLM, and it is also known as the GAIT-Log PNP combo.

## Usage

```
dgaitlog(x, shape.p, alter.mix = NULL, alter.mlm = NULL,
            inflate.mix = NULL, inflate.mlm = NULL, truncate = NULL,
            max.support = Inf, pobs.mix.a = 0, pobs.mlm.a = 0,
            pstr.mix.i = 0, pstr.mlm.i = 0, shape.a = shape.p,
            shape.i = shape.p, byrow.arg = FALSE, deflation = FALSE,
            log.arg = FALSE)
pgaitlog(q, shape.p, alter.mix = NULL, alter.mlm = NULL,
                inflate.mix = NULL, inflate.mlm = NULL, truncate = NULL,
            max.support = Inf, pobs.mix.a = 0, pobs.mlm.a = 0,
            pstr.mix.i = 0, pstr.mlm.i = 0, shape.a = shape.p,
            shape.i = shape.p, byrow.arg = FALSE)
qgaitlog(p, shape.p, alter.mix = NULL, alter.mlm = NULL,
            inflate.mix = NULL, inflate.mlm = NULL, truncate = NULL,
            max.support = Inf, pobs.mix.a = 0, pobs.mlm.a = 0,
            pstr.mix.i = 0, pstr.mlm.i = 0, shape.a = shape.p,
            shape.i = shape.p, byrow.arg = FALSE)
rgaitlog(n, shape.p, alter.mix = NULL, alter.mlm = NULL,
        inflate.mix = NULL, inflate.mlm = NULL, truncate = NULL,
        max.support = Inf, pobs.mix.a = 0, pobs.mlm.a = 0,
        pstr.mix.i = 0, pstr.mlm.i = 0, shape.a = shape.p,
        shape.i = shape.p, byrow.arg = FALSE)
```


## Arguments

$\mathrm{x}, \mathrm{q}, \mathrm{p}, \mathrm{n}, \mathrm{log} . \arg$
Same meaning as in dlog.
shape.p, shape.a, shape.i
Same meaning as shape for dlog, i.e., for an ordinary logarithmic distribution. See Gaitpois for generic information.
truncate, max. support
See Gaitpois for generic information.
alter.mix, inflate.mix
See Gaitpois for generic information.
alter.mlm, inflate.mlm
See Gaitpois for generic information.
pobs.mlm.a, pstr.mlm.i, byrow.arg
See Gaitpois for generic information.
pobs.mix.a, pstr.mix.i
See Gaitpois for generic information.
deflation See Gaitpois for generic information.

## Details

These functions allow any combination of 3 operator types: truncation, alteration and inflation. See Gaitpois for generic information. These functions do what Oalog, Oilog, Otlog collectively did plus much more.

In the notation of Yee and Ma (2020) these functions allow for the special cases: (i) GAIT$\log$ (shape.p)-Log(shape.a, alter.mix, pobs.mix.a)-Log(shape.i, inflate.mix, pstr.mix.i); (ii) GAIT-Log(shape.p)-MLM(alter.mlm, pobs.mlm.a)-MLM(inflate.mlm, pstr.mlm.i). Model (i) is totally parametric while model (ii) is the most nonparametric possible.

## Value

dgaitlog gives the density, pgaitlog gives the distribution function, qgaitlog gives the quantile function, and rgaitlog generates random deviates. The default values of the arguments correspond to ordinary dlog, plog, qlog, rlog respectively.

## Note

See Gaitpois for general information also relevant to this parent distribution.

## Author(s)

T. W. Yee.

## See Also

Gaitpois, multinomial, gaitlog.mix, Oalog, Oilog, Otlog.

## Examples

```
ivec <- c(2, 10); avec <- ivec + 4; shape <- 0.95; xgrid <- 0:29
tvec <- 15; max.support <- 25; pobs.a <- 0.10; pstr.i <- 0.15
(ddd <- dgaitlog(xgrid, shape, truncate = tvec,
    max.support = max.support, pobs.mix.a = pobs.a,
    alter.mix = avec, pstr.mix.i = pstr.i, inflate.mix = ivec))
## Not run: plot(xgrid, ddd, type = "n", ylab = "Probability", xlab = "x",
            main = "GAIT PMF---Logarithmic Parent")
mylwd <- 0.5
abline(v = avec, col = 'green', lwd = mylwd)
abline(v = ivec, col = 'red', lwd = mylwd)
abline(v = tvec, col = 'tan', lwd = mylwd)
abline(v = max.support, col = 'magenta', lwd = mylwd)
abline(h = c(pobs.a, pstr.i, 0:1), col = 'gray', lty = "dashed")
lines(xgrid, dlog(xgrid, shape), col = 'gray', lty = "dashed") # f_{\pi}
lines(xgrid, ddd, type = "h", col = "blue", lwd = 3) # GAIT PMF
points(xgrid[ddd == 0], ddd[ddd == 0], pch = 16)
## End(Not run)
```

```
gaitlog.mix
```

Generally-Altered, -Inflated and -Truncated Logarithmic Regression Family Function (GAIT-Log-Log-Log Mixture Variant)

## Description

Fits a generally-altered, -inflated and -truncated logarithmic regression (mixtures of logarithmic distributions on nested and/or partitioned supports).

## Usage

gaitlog.mix(alter = NULL, inflate = NULL, truncate $=$ NULL, max. support = Inf, zero = c("pobs.a", "pstr.i"), eq.ap = FALSE, eq.ip = FALSE, lshape.p = "logitlink", lpobs.a = "logitlink", lshape.a = "logitlink", lpstr.i = "logitlink", lshape.i = "logitlink", type.fitted = c("mean", "pobs.a", "pstr.i", "Pobs.a", "Pstr.i", "prob.a", "prob.i", "prob.t", "lhs.prob"), imethod $=1$, gshape.p = -expm1 (-7 * ppoints(12)), ishape.p = NULL, ishape.a = NULL, ishape.i = NULL, ipobs.a $=$ NULL, ipstr.i $=$ NULL, ishrinkage $=0.95$, probs.y $=0.35$ )

## Arguments

alter, inflate, truncate
See gaitpoisson.mix.
lshape.p, lshape.a, lshape.i
Link functions. See gaitpoisson.mix and Links for more choices and information.
lpobs.a, lpstr.i
Link functions. See gaitpoisson.mix and Links for more choices and information.
eq.ap, eq.ip Single logical each. See gaitpoisson.mix
type.fitted, max.support
See gaitpoisson.mix.
imethod, ipobs.a, ipstr.i
See CommonVGAMffArguments for information.
gshape.p, ishape.p
See CommonVGAMffArguments for information. The former is used only if the latter is not given. Practical experience has shown that good initial values are needed, so if convergence is not obtained then try a finer grid.
ishape.a, ishape.i
See CommonVGAMffArguments for information.
probs.y, ishrinkage
See CommonVGAMffArguments for information.
zero See gaitpoisson.mix and CommonVGAMffArguments for information.

## Details

Although the full GAIT-Log-Log-Log model may be fitted, the two submodels that may be fitted can be abbreviated GAT-Log-Log or GIT-Log-Log, which is where the inner distribution for ordinary values is the logarithmic distribution, and the outer distribution for the altered or inflated values is another logarithmic distribution with a different shape parameter by default. Thus for the GAT model the distribution being fitted is a (spliced) mixture of two logarithmic with differing (partitioned) support. Likewise, for the GIT model the distribution being fitted is a mixture of two logarithmic with nested support. Two shape parameters at a time may be constrained to be equal using eq.ap or eq.ip.
For the GIT model, by default, a logistic regression models the (structural) probability pstr.i that the response is inflated.
This function currently does not handle multiple responses. Further details are at Gaitlog.
For the GIT model, the ordering of the linear/additive predictors corresponds to length(inflate) equalling 0,1 , and more than 1 ; the dimension grows accordingly. The same idea holds for the GAT model.

Apart from the order of the linear/additive predictors, the following are (or should be) equivalent: gaitlog.mix() and logff(), gaitlog.mix(alter = 1) and oalog(zero = "pobs1"), gaitlog.mix(inflate $=1)$ and oilog(zero = "pstr1"), gaitlog.mix(truncate $=1)$ and otlog().

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

See gaitpoisson.mix

## Note

See gaitpoisson.mix

## Author(s)

T. W. Yee

## See Also

Gaitlog, gaitpoisson.mix, gaitpoisson.mlm, oalog, oilog, otlog, CommonVGAMffArguments, rootogram4, simulate.vlm.

## Examples

```
avec <- c(5, 10) # Alter these values
ivec <- c(3, 15) # Inflate these values
tvec <- c(6, 7) # Truncate these values
max.support <- 20; set.seed(1)
pobs.a <- logitlink(-1, inverse = TRUE) # About 0.27
```

```
pstr.i <- logitlink(-1, inverse = TRUE) # About 0.27
gdata <- data.frame(x2 = runif(nn <- 1000))
gdata <- transform(gdata, shape.p = logitlink(2+0.5*x2, inverse = TRUE))
gdata <- transform(gdata,
    y1 = rgaitlog(nn, shape.p, alter.mix = avec, pobs.mix.a = pobs.a,
            inflate.mix = ivec, pstr.mix.i = pstr.i, truncate = tvec,
            max.support = max.support))
gaitlog.mix(alter = avec, inflate = ivec, max.support = max.support)
with(gdata, table(y1))
gaitlxfit <- vglm(y1 ~ x2, crit = "coef", trace = TRUE, data = gdata,
    gaitlog.mix(inflate = ivec, truncate = tvec,
    max.support = max.support,
    alter = avec, eq.ap = TRUE, eq.ip = TRUE))
head(fitted(gaitlxfit, type.fitted = "Pstr.i"))
head(predict(gaitlxfit))
coef(gaitlxfit, matrix = TRUE)
summary(gaitlxfit)
```

Gaitnbinom.mix

Generally-Altered, -Inflated and -Truncated Negative Binomial Distribution (GAIT-NB-NB-NB mixture)

## Description

Density, distribution function, quantile function and random generation for the generally-altered, -inflated and -truncated negative binomial (NB) distribution, based on mixtures of NB distributions having different and/or nested support. This distribution is sometimes abbreviated as GAIT-NB-NB-NB.

## Usage

dgaitnbinom.mix(x, size.p, prob.p = NULL, munb.p = NULL, alter $=$ NULL, inflate $=$ NULL, truncate $=$ NULL, max.support $=$ Inf, pobs.a $=0$, pstr.i = 0, size.a = size.p, size.i = size.p, prob.a $=$ prob.p, prob.i = prob.p, munb.a $=$ munb.p, munb. $i=$ munb.p, log.arg $=$ FALSE)
pgaitnbinom.mix(q, size.p, prob.p = NULL, munb.p = NULL, alter $=$ NULL, inflate $=$ NULL, truncate $=$ NULL, max.support $=$ Inf, pobs.a $=0$, pstr.i $=0$, size.a $=$ size.p, size.i = size.p, prob.a $=$ prob.p, prob.i = prob.p,
munb. $a=$ munb. $p$, munb. $i=$ munb. $p$ )
qgaitnbinom.mix(p, size.p, prob.p = NULL, munb.p = NULL, alter $=$ NULL, inflate $=$ NULL, truncate $=$ NULL, max.support $=$ Inf, pobs.a = 0, pstr.i = 0, size.a = size.p, size.i = size.p, prob.a = prob.p, prob.i = prob.p, munb.a $=$ munb. $p$, munb. $i=$ munb. $p$ )
rgaitnbinom.mix(n, size.p, prob.p = NULL, munb.p $=$ NULL, alter $=$ NULL, inflate $=$ NULL, truncate $=$ NULL, max.support $=\operatorname{Inf}$,

```
pobs.a = 0, pstr.i = 0, size.a = size.p, size.i = size.p,
prob.a = prob.p, prob.i = prob.p,
munb.a = munb.p, munb.i = munb.p)
```


## Arguments

$\mathrm{x}, \mathrm{p}, \mathrm{q}, \mathrm{n}, \mathrm{log} . \arg$
Same meaning as in NegBinomial.
size.p, size.a, size.i
Same meaning as size in NegBinomial, i.e., for an ordinary NB distribution. The first is for the main parent (inner) distribution, and the outer distribution(s) (usually spikes) concern the altered and/or inflated values. Short vectors are recycled.
prob.p, prob.a, prob.i
Same meaning as prob in NegBinomial.
munb.p, munb. a, munb.i
Same meaning as mu in NegBinomial. Only one of the prob-type and munb-type arguments should be specified. Short vectors are recycled.
alter, inflate, truncate
See Gaitnbinom.mlm. The order of precedence is the same, viz. truncation first, then altering, and finally inflation. If alter and pobs. a are both of unit length then the default probability mass function (PMF) evaluated at alter is pobs.a.
pobs.a, pstr.i See Gaitpois.
max.support Same as Gaitnbinom.mlm.

## Details

These functions will be combined soon. See Gaitpois for parallel information. This distribution may be suitable for heaped data whose parent distribution is overdispersed with respect to the Poisson.
In the notation of Yee and Ma (2020) these functions concern the GAIT-NB-NB-NB distribution. For information on the GAIT-NB-MLM-MLM distribution see Gaitnbinom.mlm.

## Value

dgaitnbinom.mix gives the density (PMF), pgaitnbinom. mix gives the distribution function, qgaitnbinom.mix gives the quantile function, and rgaitnbinom.mix generates random deviates.

## Note

It is intended that the minimal defaults of these functions are equivalent to NegBinomial, e.g., dgaitnbinom.mix (x, size, munb. $p=$ munb ) and dnbinom ( $x$, size, mu = munb) should be identical , and ditto for the other 3 functions.

## Author(s)

T. W. Yee.

## See Also

Gaitnbinom.mlm for the GAIT-NB-MLM-MLM distribution, NegBinomial.

## Examples

```
ivec <- c(5, 15, 10); avec <- ivec; size = 10; munb <- 10
max.support <- 20; pobs.a <- 0.35; xvec <- 0:max.support
# GAT-NB-NB mixture
(pmf.a <- dgaitnbinom.mix(xvec, size.p = size, munb.p = munb,
    max.support = max.support, pobs.a = pobs.a, alter = avec))
sum(pmf.a) # Should be 1
## Not run:
ind4 <- match(xvec, avec, nomatch = 0) > 0 # xvec %in% avec
plot(xvec[ ind4], pmf.a[ ind4], type = "h", col = "orange", lwd = 1.1,
    las = 1, xlim = range(xvec), main = "GAT-NB-NB",
    ylim = c(0, max(pmf.a)), xlab = "y", ylab = "Probability") # Spikes
lines(xvec[!ind4], pmf.a[!ind4], type = "h", col = "blue")
## End(Not run)
# GIT-NB-NB mixture
pstr.i <- 0.15
(pmf.i <- dgaitnbinom.mix(xvec, size.p = size, munb.p = munb,
    max.support = max.support, pstr.i = pstr.i, inflate = ivec))
sum(pmf.i) # Should be 1
## Not run: # Plot the components of pmf.i
plot(xvec, (1 - pstr.i) * dnbinom(xvec, size, mu = munb), type = "h",
    col = "blue", las = 1, xlim = range(xvec),
    main = "GIT-NB-NB", # The inner distribution
    ylim = c(0, max(pmf.i)), xlab = "y", ylab = "Probability")
spikes <- dnbinom(ivec, size, mu = munb) * pstr.i / sum(
            dnbinom(ivec, size, mu = munb))
start.pt <- dnbinom(ivec, size, mu = munb) *
            (1 - pstr.i) / pnbinom(max.support, size, mu = munb)
segments(ivec, start.pt, # The outer distribution
            ivec, start.pt + spikes, col = "orange", lwd = 1.1)
## End(Not run)
```

Gaitnbinom.mlm
Generally-Altered, -Inflated and -Truncated Negative Binomial Dis-
tribution (GAIT-NB-MLM-MLM)

## Description

Density, distribution function, quantile function and random generation for the generally-altered, inflated and -truncated negative binomial distribution, based on the multinomial logit model (MLM). This distribution is sometimes abbreviated as GAIT-NB-MLM-MLM.

## Usage

```
dgaitnbinom.mlm(x, size, prob = NULL, munb = NULL, alter = NULL,
                        inflate \(=\) NULL, truncate \(=\) NULL, max.support \(=\) Inf,
    pobs.a \(=0\), pstr.i \(=0\), byrow.arg \(=\) FALSE,
    log.arg = FALSE)
pgaitnbinom.mlm(q, size, prob \(=\) NULL, munb \(=\) NULL, alter \(=\) NULL,
        inflate \(=\) NULL, truncate \(=\) NULL, max.support \(=\) Inf,
        pobs.a \(=0\), pstr.i \(=0\), byrow.arg \(=\) FALSE)
qgaitnbinom.mlm(p, size, prob \(=\) NULL, munb \(=\) NULL, alter \(=\) NULL,
        inflate \(=\) NULL, truncate \(=\) NULL, max.support \(=\operatorname{Inf}\),
        pobs.a \(=0\), pstr.i \(=0\), byrow.arg \(=\) FALSE)
rgaitnbinom.mlm(n, size, prob = NULL, munb = NULL, alter = NULL,
        inflate \(=\) NULL, truncate \(=\) NULL, max.support \(=\) Inf,
        pobs.a \(=0\), pstr.i \(=0\), byrow.arg \(=\) FALSE)
```


## Arguments

$\mathrm{x}, \mathrm{q}, \mathrm{p}, \mathrm{n}, \mathrm{log} . \arg$
Same meaning as in NegBinomial.
size, prob, munb
Same meaning as in NegBinomial, i.e., for an ordinary negative binomial distribution. Argument mu has been renamed to munb to emphasize that it refers to the negative binomial mean. Only one of prob and munb should be specified. Short vectors are recycled.
alter, inflate, truncate
Se Gaitpois.
pstr.i, byrow.arg
See Gaitpois.
pobs.a, max.support
See Gaitpois.

## Details

Largely similar to Gaitpois except a negative binomial distribution replaces the Poisson distribution. Special cases of these functions include dposnegbin, dzanegbin, dzinegbin, pposnegbin, pzanegbin, pzinegbin, qposnegbin, qzanegbin, qzinegbin, rposnegbin, rzanegbin, rzinegbin corresponding to the special value 0 . However, now arguments alter, inflate and truncate have been supplied in one function.

## Value

dgaitnbinom.mlm gives the density (PMF), pgaitnbinom.mlm gives the distribution function, qgaitnbinom.mlm gives the quantile function, and rgaitnbinom.mlm generates random deviates. The default values of the arguments correspond to ordinary dnegbin, pnegbin, qnegbin, rnegbin respectively.

## Author(s)

T. W. Yee.

## See Also

gatnbinomial.mlm, Gaitpois, NegBinomial.

## Examples

```
avec <- c(5, 10) # Alter these (special) values
ivec <- c(15, 20) # Inflate these (special) values
tvec <- 0 # Truncate this (special) value
pobs.a <- c(0.1, 0.2)
pstr.i <- c(0.1, 0.1); size <- 3; munb <- 15; x <- 0:22
y <- rgaitnbinom.mlm(1000, size, munb = munb, alter = avec,
                                    inflate = ivec, truncate = tvec, max.support = 20,
                                    pstr.i = pstr.i, pobs.a = pobs.a, byrow = TRUE)
table(y)
(pmf <- dgaitnbinom.mlm(x, size, munb = munb, alter = avec,
                    inflate = ivec, truncate = tvec, max.support = 20,
                    pstr.i = pstr.i, pobs.a = pobs.a, byrow = TRUE))
## Not run: plot(x, pmf, type = "h", col = "blue", las = 1,
                        main = "Heaped data that is also upper-truncated")
## End(Not run)
```


## Description

Density, distribution function, quantile function and random generation for the generally-altered, -inflated and -truncated Poisson distribution. Both parametric and nonparametric variants are supported; these are based on finite mixtures of the parent with itself and the multinomial logit model (MLM) respectively. Altogether it can be abbreviated as GAAIIT-Pois(lambda.p)-Pois(lambda.a)-MLM-Pois(lambda.i)-MLM, and it is also known as the GAIT-Pois PNP combo.

## Usage

dgaitpois(x, lambda.p, alter.mix = NULL, alter.mlm = NULL,
inflate.mix $=$ NULL, inflate.mlm $=$ NULL, truncate $=$ NULL, max.support $=$ Inf, pobs.mix.a $=0$, pobs.mlm.a $=0$, pstr.mix.i $=0$, pstr.mlm.i $=0$, lambda.a $=$ lambda.p, lambda.i = lambda.p, byrow.arg = FALSE, deflation = FALSE, log.arg = FALSE)
pgaitpois(q, lambda.p, alter.mix $=$ NULL, alter.mlm $=$ NULL,
inflate.mix $=$ NULL, inflate.mlm $=$ NULL, truncate $=$ NULL, max.support $=$ Inf, pobs.mix.a $=0$, pobs.mlm.a $=0$, pstr.mix.i = 0, pstr.mlm.i = 0, lambda.a = lambda.p, lambda.i = lambda.p, byrow.arg = FALSE)
qgaitpois(p, lambda.p, alter.mix = NULL, alter.mlm = NULL, inflate.mix = NULL, inflate.mlm = NULL, truncate = NULL, max. support $=$ Inf, pobs.mix.a $=0$, pobs.mlm.a $=0$,

```
    pstr.mix.i = 0, pstr.mlm.i = 0, lambda.a = lambda.p,
    lambda.i = lambda.p, byrow.arg = FALSE)
rgaitpois(n, lambda.p, alter.mix = NULL, alter.mlm = NULL,
    inflate.mix = NULL, inflate.mlm = NULL, truncate = NULL,
    max.support = Inf, pobs.mix.a = 0, pobs.mlm.a = 0,
    pstr.mix.i = 0, pstr.mlm.i = 0, lambda.a = lambda.p,
    lambda.i = lambda.p, byrow.arg = FALSE)
```


## Arguments

$\mathrm{x}, \mathrm{q}, \mathrm{p}, \mathrm{n}, \mathrm{log} . \arg$
Same meaning as in rpois.
lambda.p, lambda.a, lambda.i
Same meaning as in rpois, i.e., for an ordinary Poisson distribution. The first is for the main parent (inner) distribution. The other two concern the parametric variant and these outer distributions (usually spikes) may be altered and/or inflated. Short vectors are recycled.
truncate, max.support
numeric; these specify the set of truncated values. The default value of NULL means an empty set for the former. The latter is the maximum support value so that any value larger has been truncated (necessary because truncate = (max. support
$+1)$ : Inf is not allowed), hence is needed for truncating the upper tail of the distribution. Note that max (truncate) < max. support must be satisfied otherwise an error message will be issued.
alter.mix, inflate.mix
Vectors of nonnegative integers; the altered, inflated and truncated values for the parametric variant. Each argument must have unique values only. Assigning argument alter.mix means that pobs.mix. a will be used. Assigning argument inflate.mix means that pstr.mix.i will be used. If alter.mix is of unit length then the default probability mass function (PMF) evaluated at alter.mix will be pobs.mix.a. So having alter.mix $=0$ corresponds to the zero-inflated Poisson distribution (see Zipois).
alter.mlm, inflate.mlm
Similar to the above, but for the nonparametric (MLM) variant. Assigning argument alter.mlm means that pobs.mlm.a will be used. Assigning argument inflate.mlm means that pstr.mlm.i will be used. Collectively, the above 6 arguments represent 5 disjoint sets of special values and they are a proper subset of the support of the distribution.
pobs.mlm.a, pstr.mlm.i, byrow.arg
These arguments are coerced into a matrix of probabilities using byrow. arg to determine the order of the elements (similar to matrix). The first argument is recycled if necessary to become $\mathrm{n} \times$ length(alter.mlm). The second argument becomes $n \times$ length (inflate.mlm). Thus these arguments are not used unless alter.mlm and inflate.mlm are assigned. If deflation = TRUE then pstr.mlm.i may be negative.
pobs.mix.a, pstr.mix.i
Vectors of probabilities that are recycled if necessary to length $n$. The first argument is used when alter.mix is not NULL. The second argument is used
when inflate.mix is not NULL.
deflation Logical. If TRUE then pstr.mlm. i is allowed to have negative values, however, not too negative so that the final PMF becomes negative. Of course, if the values are negative then they cannot be interpreted as probabilities. In theory, one could also allow pstr.mix.i to be negative, but currently this is disallowed.

## Details

These functions allow any combination of 3 operator types: truncation, alteration and inflation. The precedence is truncation, then alteration and lastly inflation. This order minimizes the potential interference among the operators. Loosely, a set of probabilities is set to 0 by truncation and the remaining probabilities are scaled up. Then a different set of probabilities are set to some values pobs.mix. a and/or pobs.mlm. a and the remaining probabilities are rescaled up. Then another different set of probabilities is inflated by an amount pstr.mlm.i and/or proportional to pstr.mix.i so that individual elements in this set have two sources. Then all the probabilities are rescaled so that they sum to unity.

Both parametric and nonparametric variants are implemented. They usually have arguments with suffix .mix and .mlm respectively. The MLM is a loose coupling that effectively separates the parent distribution from the altered values. Values inflated nonparametrically effectively have their spikes shaved off. The .mix variant has associated with it lambda. a and lambda.i because it is mixture of 3 Poisson distributions with partitioned or nested support.
Any value of the support of the distribution that is altered, inflated or truncated is called a special value. A special value that is altered may mean that its probability increases or decreases relative to the parent distribution. An inflated special value means that its probability has increased, provided alteration elsewhere has not made it decrease in the first case.

Jargonwise, the outer distributions concern those special values which are altered or inflated, and the inner distribution concerns the remaining support points that correspond directly to the parent distribution. These functions do what Zapois, Zipois, Pospois collectively did plus much more.
In the notation of Yee and Ma (2020) these functions allow for the special cases: (i) GAIT-Pois(lambda.p)-Pois(lambda.a, alter.mix, pobs.mix.a)-Pois(lambda.i, inflate.mix, pstr.mix.i); (ii) GAIT-Pois(lambda.p)-MLM(alter.mlm, pobs.mlm.a)-MLM(inflate.mlm, pstr.mlm.i). Model (i) is totally parametric while model (ii) is the most nonparametric possible.

## Value

dgaitpois gives the density, pgaitpois gives the distribution function, qgaitpois gives the quantile function, and rgaitpois generates random deviates. The default values of the arguments correspond to ordinary dpois, ppois, qpois, rpois respectively.

## Author(s)

T. W. Yee.

## References

Yee, T. W. and Ma, C. (2020). Generally-altered, -inflated and -truncated regression, with application to heaped and seeped count data. In preparation.

## See Also

multinomial, gaitpoisson.mlm, gaitpoisson.mix, Zapois, Zipois, Pospois Poisson; Gaitlog, Gaitzeta.

## Examples

```
ivec <- c(6, 14); avec <- c(8, 11); lambda <- 10; xgrid <- 0:25
tvec <- 15; max.support <- 20; pobs.a <- 0.05; pstr.i <- 0.25
(ddd <- dgaitpois(xgrid, lambda, lambda.a = lambda + 5,
    truncate = tvec, max.support = max.support, pobs.mix.a = pobs.a,
    pobs.mlm.a = pobs.a, alter.mlm = avec,
    pstr.mix.i = pstr.i, inflate.mix = ivec))
## Not run: plot(xgrid, ddd, type = "n", ylab = "Probability", xlab = "x",
                main = "GAIT PNP Combo PMF---Poisson Parent")
mylwd <- 0.5
abline(v = avec, col = 'green', lwd = mylwd)
abline(v = ivec, col = 'red', lwd = mylwd)
abline(v = tvec, col = 'tan', lwd = mylwd)
abline(v = max.support, col = 'magenta', lwd = mylwd)
abline(h = c(pobs.a, pstr.i, 0:1), col = 'gray', lty = "dashed")
lines(xgrid, dpois(xgrid, lambda), col = 'gray', lty = "dashed") # f_{\pi}
lines(xgrid, ddd, type = "h", col = "blue", lwd = 3) # GAIT PNP combo PMF
points(xgrid[ddd == 0], ddd[ddd == 0], pch = 16)
## End(Not run)
```

gaitpoisson.mix Generally-Altered, -Inflated and -Truncated Poisson Regression Family Function (GAIT-Pois-Pois-Pois Mixture Variant)

## Description

Fits a generally-altered, -inflated and -truncated Poisson regression (mixtures of Poissons on nested and/or partitioned supports). The truncation may include values in the upper tail.

## Usage

gaitpoisson.mix(alter = NULL, inflate $=$ NULL, truncate $=$ NULL, max.support = Inf, zero = c("pobs.a", "pstr.i"),
eq.ap = FALSE, eq.ip = FALSE, llambda.p = "loglink",
lpobs.a = "logitlink", llambda.a = "loglink",
lpstr.i = "logitlink", llambda.i = "loglink",
type.fitted = c("mean", "pobs.a", "pstr.i", "Pobs.a", "Pstr.i",
"prob.a", "prob.i", "prob.t", "lhs.prob"),
imethod $=1$, ilambda.p = NULL, ilambda.a $=$ NULL, ilambda.i = NULL,
ipobs.a $=$ NULL, ipstr.i $=$ NULL, ishrinkage $=0.95$, probs.y $=0.35$ )

## Arguments

```
    alter, inflate, truncate
```

Vector of altered, inflated and truncated values, i.e., nonnegative integers. A NULL stands for an empty set so the default is effectively equivalent to poissonff. The parameter lambda. $p$ is always estimated. If length(alter) is 1 then the parameter pobs.a is estimated too. If length(inflate) is 1 then the parameter pstr. $i$ is estimated too. If length(alter) is 2 or more then the parameter lambda.a is estimated too, corresponding to an outer distribution. If length(inflate) is 2 or more then the parameter lambda. $i$ is estimated too, corresponding to an outer distribution.
Due to its flexibility, it is easy to misuse this function and ideally the values of these arguments should be well justified by the application on hand. Adding unnecessary values to these arguments willy-nilly is a recipe for disaster, especially for inflate. Using alter effectively removes a subset of the data from the main analysis, therefore may result in a substantial loss of efficiency. For seeped values, alter should be used rather than inflate. Heaped values can be handled by alter and inflate.
llambda.p, llambda.a, llambda.i
Link functions; the suffixes .p, a and .i refer to the parent, altered and inflated distributions respectively. See Links for more choices and information.
lpobs.a, lpstr.i
Link functions; See Links for more choices and information.
eq.ap, eq.ip Single logical each. Constrain the rate parameters to be equal? See CommonVGAMffArguments for information. For the GIT-Pois-Pois, after plotting the responses, if the distribution of the spikes above the nominal probabilities has roughly the same shape as the ordinary values then setting eq.ip = TRUE would be a good idea (so that lambda. $i==$ lambda.p). And if inflate is of length 2 or thereabouts, then TRUE should definitely be entertained. Likewise, for heaped or seeped data, setting eq. $\mathrm{ap}=$ TRUE (so that lambda. $\mathrm{p}==$ lambda. p ) would be a good idea for the GAT-Pois-Pois if the shape of the altered probabilities is roughly the same as the (inner) parent distribution.
type.fitted, max. support
See CommonVGAMffArguments and gaitpoisson.mlm for information.
The choice "lhs.prob" is the 1 minus the probability of value greater than "max. support", using the parent distribution.
imethod, ipobs.a, ipstr.i
See CommonVGAMffArguments for information.
ilambda.p, ilambda.a, ilambda.i
See CommonVGAMffArguments for information.
probs.y, ishrinkage
See CommonVGAMffArguments for information.
zero See CommonVGAMffArguments for information. For the GIT-Pois-Pois, having zero = "pstr.i" will model the mixing probability as simple as possible (intercept-only), hence should be more numerically stable than NULL; and zero = "pstr. i " is recommended for many analyses especially when there are many explanatory variables. Likewise, for the GAT-Pois-Pois, having zero =
"pobs.a" will model that probability as simple as possible. The default vector is pruned of any irrelevant values.

## Details

Although the full GAIT-Pois-Pois-Pois model may be fitted, the two submodels that may be fitted can be abbreviated GAT-Pois-Pois or GIT-Pois-Pois, which is where the inner distribution for ordinary values is the Poisson distribution, and the outer distribution for the altered or inflated values is another Poisson distribution with a different rate parameter by default. Thus for the GAT model the distribution being fitted is a (spliced) mixture of two Poissons with differing (partitioned) support. Likewise, for the GIT model the distribution being fitted is a mixture of two Poissons with nested support. The two rate parameters may be constrained to be equal using eq.ap or eq.ip.
For the GIT model, by default, a logistic regression models the (structural) probability pstr.i that the response is inflated.
This function currently does not handle multiple responses. Further details are at Gaitpois. An alternative variant of this distribution, more unstructured in nature, is based on the multinomial logit model—see gaitpoisson.mlm.

For the GIT model, the ordering of the linear/additive predictors corresponds to length(inflate) equalling 0,1 , and more than 1 ; the dimension grows accordingly. The same idea holds for the GAT model.
Apart from the order of the linear/additive predictors, the following are (or should be) equivalent: gaitpoisson.mix() and poissonff(), gaitpoisson.mix(alter =0) and zapoisson(zero = "pobs0"), gaitpoisson.mix(inflate = 0) and zipoisson(zero = "pstr0"), gaitpoisson.mix(truncate $=0)$ and pospoisson().

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.
The fitted.values slot of the fitted object, which should be extracted by the generic function fitted, are similar to gaitpoisson.mlm.

## Warning

Amateurs have the tendency to be overzealous fitting zero-inflated models when the fitted mean is low-the warning of ziP should be heeded and it applies here to all inflated values.

Fitting a GIT model requires more caution than for the GAT hurdle model because ideally gross inflation is needed in the data for it to work properly. Deflation or no inflation will produce numerical problems such as extreme coefficient values, hence set trace $=$ TRUE to monitor convergence. It is often a good idea to set eq.ip = TRUE, especially when length(inflate) is low or the values of inflate are not spread over the range of the response. That is, if the inflate values form a single small cluster then this can easily create estimation difficulties-the idea is somewhat similar to multicollinearity.

## Note

The defaults for this family function may change in the future as more experience is obtained using it. If length(inflate) is very low then it is probably a good idea to set eq.ip = TRUE so that the
estimation can borrow strength from both the inflated and non-inflated values. Numerical problems can easily arise because of the flexibility of this distribution and/or the lack of sizeable inflation; it is a good idea to gain experience with simulated data first before applying it to real data.

See gaitpoisson.mlm for other general details.

## Author(s)

T. W. Yee

## References

Yee, T. W. and Ma, C. (2020). Generally-altered, -inflated and -truncated regression, with application to heaped and seeped count data. In preparation.

## See Also

Gaitpois, gaitpoisson.mlm, gatnbinomial.mix, zipoisson, pospoisson, gaitlog.mix, CommonVGAMffArguments, rootogram4, simulate.vlm.

## Examples

```
avec <- c(5, 10) # Alter these values
ivec <- c(3, 15) # Inflate these values
tvec <- c(6, 7) # Truncate these values
pobs.a <- logitlink(-1, inverse = TRUE) # About 0.27
pstr.i <- logitlink(-1, inverse = TRUE) # About 0.27
max.support <- 20; set.seed(1)
gdata <- data.frame(x2 = runif(nn <- 1000))
gdata <- transform(gdata, lambda.p = exp(2 + 0.5 * x2))
gdata <- transform(gdata,
    y1 = rgaitpois(nn, lambda.p, alter.mix = avec, pobs.mix.a = pobs.a,
                        inflate.mix = ivec, pstr.mix.i = pstr.i,
                            truncate = tvec, max.support = max.support))
gaitpoisson.mix(alter = avec, inflate = ivec)
with(gdata, table(y1))
gaitpxfit <- vglm(y1 ~ x2, crit = "coef", trace = TRUE, data = gdata,
    gaitpoisson.mix(alter = avec, inflate = ivec,
                                    truncate = tvec, eq.ap = TRUE,
                                    eq.ip = TRUE, max.support = max.support))
head(fitted(gaitpxfit, type.fitted = "Pstr.i"))
head(predict(gaitpxfit))
coef(gaitpxfit, matrix = TRUE)
summary(gaitpxfit)
```


## Description

Fits a generally-altered, -inflated and -truncated Poisson regression (using a multinomial logit model for the altered or inflated values). The truncation may include values in the upper tail. Currently only one of altered or inflated are allowed in a single model.

## Usage

gaitpoisson.mlm(alter = NULL, inflate = NULL, truncate $=$ NULL, max.support $=\operatorname{Inf}$,
zero = c("pobs", "pstr"), parallel.ap = FALSE, parallel.ip = FALSE,
llambda = "loglink", type.fitted = c("mean", "pobs.a", "pstr.i",
"Pobs.a", "Pstr.i", "prob.a", "prob.i", "prob.t", "lhs.prob"),
imethod $=1$, mux.inflate $=0.5$, ilambda $=$ NULL,
ishrinkage $=0.95$, probs.y $=0.35$ )

## Arguments

alter, inflate, truncate
Vector of altered, inflated and truncated values, i.e., nonnegative integers. A NULL stands for an empty set so the default is effectively equivalent to poissonff. Currently al ter and inflate cannot both be used at the same time. If length(alter) is 1 or more then the multinomial logit model (MLM) probabilities of observing those values are modelled-see multinomial. If length(alter) is 1 then effectively a logistic regression is estimated as a special case. Likewise, if length (inflate) is 1 or more then the MLM structural probabilities are modelledsee multinomial. And if length(inflate) is 1 then effectively a logistic regression is estimated as a special case.
See gaitpoisson.mix for more information about these arguments.
llambda Link function for the parent distribution. See Links for more choices and information.
parallel.ap, parallel.ip
Single logical each. Constrain the MLM probabilities to be equal? If so then this applies to all length(alter) pobs probabilities or all length(inflate) pstr probabilities. See CommonVGAMfffarguments for information. The default means that the probabilities are generally unconstrained and unstructured.
type.fitted See CommonVGAMffArguments and below for information. Choosing an irrelevant value may result in an obscure error message, e.g., "Pstr.i" for a GAT model.
The choice "lhs.prob" is the 1 minus the probability of value greater than "max. support", using the parent distribution.

> max. support $\begin{aligned} & \text { See Gaitpois for information. This enables RHS-truncation, i.e., something } \\ & \text { equivalent to truncate }=(U+1): \text { Inf for some upper support point U. }\end{aligned}$ imethod, ilambda, mux.inflate See CommonVGAMffArguments for information. probs.y, ishrinkage See CommonVGAMffArguments for information. $\quad \begin{aligned} & \text { See CommonVGAMffArguments for information. For the GIT-Pois-MLM sub- } \\ & \text { model, having zero = "pstr" will model the mixing probabilities as simple as } \\ & \text { possible (intercept-only), hence should be more numerically stable than NULL; } \\ & \text { and zero = "pstr" is recommended for many analyses especially when there } \\ & \text { are many explanatory variables. Likewise, for the GAT-Pois-MLM submodel, } \\ & \\ & \text { setting zero = "pobs" will model the multinomial probabilities as simple as pos- } \\ & \text { sible (intercept-only), hence should be more numerically stable than the default, } \\ & \text { and this is recommended for many analyses especially when there are many } \\ & \text { explanatory variables. The default vector is pruned of irrelevant values. }\end{aligned}$

## Details

The full distribution being considered can be abbreviated GAIT-Pois-MLM-MLM, which is where the inner distribution for ordinary values is the Poisson distribution, and the outer distributions for the altered and inflated values are unstructured probabilities. Thus the distribution being fitted is a mixture of a Poisson and two MLMs with the support of one of the MLMs being equal to the set of altered values and the other for the inflated values. Hence the probability for each inflated value comes from two sources: the parent distribution and a MLM.
The two submodels that may be fitted can be abbreviated GAT-Pois-MLM or GIT-Pois-MLM. An alternative variant of this distribution, more structured in nature, replaces the MLMs by Poisson distributions with potentially different means-see gaitpoisson.mix. This family function may be described as nonparametric while gaitpoisson.mix is parametric.

For the GIT model, the ordering of the linear/additive predictors corresponds to length(inflate) equalling 0 and more than 0 ; the dimension grows accordingly. The same idea holds for the GAT model.
This function currently does not handle multiple responses. Further details are at Gaitpois.
Apart from the order of the linear/additive predictors, the following are (or should be) equivalent: gaitpoisson.mlm() and poissonff(), gaitpoisson.mlm(alter =0) and zapoisson(zero = "pobs0"), gaitpoisson.mlm(inflate = 0) and zipoisson(zero = "pstr0"), gaitpoisson.mlm(truncate $=0)$ and pospoisson().

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

The fitted.values slot of the fitted object, which should be extracted by the generic function fitted, is similar to gaitpoisson.mix. The default is to return the mean $\mu$. The choice type. fitted $=$ "pobs.a" returns a matrix of all the altered probabilities (Greek symbol omega); each column is an omega, and the last column is labelled "(Others)". The choice type.fitted = "Pobs.a" returns the same.

The choice "pstr.i" returns a matrix of all the inflated probabilities (Greek symbol phi); each column is a phi, and the last column is labelled "(Others)". The choice "Pstr.i" returns a length(inflate)-column matrix with probabilities coming from the two sources.
The choice "prob.a" returns the sum of the probability of having an altered value, given the estimate of lambda from the parent distribution; i.e., it is summed over alter and evaluated at the parent PMF. The choice "prob.i" is similar to "prob.a" but summed over inflate. Likewise, the choice "prob.t" summed over truncate.

## Warning

Using inflate requires more caution than alter because gross inflation is ideally needed for it to work safely. Deflation or no inflation will produce numerical problems, hence set trace $=$ TRUE to monitor convergence.
See gaitpoisson.mix for other general details and warnings.

## Author(s)

T. W. Yee

## References

Yee, T. W. and Ma, C. (2020). Generally-altered, -inflated and -truncated regression, with application to heaped and seeped count data. In preparation.

## See Also

Gaitpois, gaitpoisson.mix, gatnbinomial.mlm, zipoisson, pospoisson, CommonVGAMffArguments, rootogram4, simulate.vlm.

## Examples

```
ivec <- c(3, 4) # Inflate these values
tvec <- c(5, 7) # Truncate these values
max.support <- 20 # Values beyond here are truncated
pstr.i <- 0.1 # Common value for all values of ivec
gdata <- data.frame(x2 = runif(nn <- 1000))
gdata <- transform(gdata, lambda = exp(1.5 + 0.5 * x2))
gdata <- transform(gdata,
    y1 = rgaitpois(nn, lambda, pstr.mlm.i = pstr.i, inflate.mlm = ivec,
                    truncate = tvec, max.support = max.support))
gaitpoisson.mlm(inflate = ivec)
with(gdata, table(y1))
fit1 <- vglm(y1 ~ x2, crit = "coef", trace = TRUE, data = gdata,
                        gaitpoisson.mlm(inflate = ivec, truncate = tvec,
                    parallel.ip = TRUE, # Good idea here
                    max.support = max.support))
head(fitted(fit1, type.fitted = "Pstr.i"))
head(predict(fit1))
log(pstr.i / (1 - length(ivec) * pstr.i)) # Value of some of the etas
coef(fit1, matrix = TRUE)
summary(fit1)
```


## Description

Density, distribution function, quantile function and random generation for the generally-altered, -inflated and -truncated zeta distribution. Both parametric and nonparametric variants are supported; these are based on finite mixtures of the parent with itself and the multinomial logit model (MLM) respectively. Altogether it can be abbreviated as GAAIIT-Zeta(shape.p)-Zeta(shape.a)-MLM-Zeta(shape.i)-MLM, and it is also known as the GAIT-Zeta PNP combo.

## Usage

dgaitzeta(x, shape.p, alter.mix = NULL, alter.mlm = NULL,
inflate.mix = NULL, inflate.mlm = NULL, truncate $=$ NULL,
max.support $=$ Inf, pobs.mix.a $=0$, pobs.mlm.a $=0$,
pstr.mix.i $=0$, pstr.mlm.i $=0$, shape.a $=$ shape.p,
shape.i = shape.p, byrow.arg = FALSE, deflation = FALSE,
log.arg = FALSE)
pgaitzeta(q, shape.p, alter.mix = NULL, alter.mlm = NULL,
inflate.mix $=$ NULL, inflate.mlm $=$ NULL, truncate $=$ NULL,
max.support $=I n f$, pobs.mix.a $=0$, pobs.mlm.a $=0$,
pstr.mix.i $=0$, pstr.mlm.i $=0$, shape.a $=$ shape.p, shape.i = shape.p, byrow.arg = FALSE)
qgaitzeta(p, shape.p, alter.mix = NULL, alter.mlm = NULL,
inflate.mix $=$ NULL, inflate.mlm $=$ NULL, truncate $=$ NULL,
max.support $=$ Inf, pobs.mix.a $=0$, pobs.mlm.a $=0$,
pstr.mix.i $=0$, pstr.mlm.i $=0$, shape.a $=$ shape.p,
shape.i = shape.p, byrow.arg = FALSE)
rgaitzeta(n, shape.p, alter.mix = NULL, alter.mlm = NULL,
inflate.mix $=$ NULL, inflate.mlm $=$ NULL, truncate $=$ NULL,
max.support $=$ Inf, pobs.mix.a $=0$, pobs.mlm.a $=0$,
pstr.mix.i $=0$, pstr.mlm.i $=0$, shape.a $=$ shape.p,
shape.i = shape.p, byrow.arg = FALSE)

## Arguments

$\mathrm{x}, \mathrm{q}, \mathrm{p}, \mathrm{n}, \mathrm{log} . \arg$
Same meaning as in dzeta.
shape.p, shape.a, shape.i
Same meaning as shape for dzeta, i.e., for an ordinary zeta distribution. See Gaitpois for generic information.
truncate, max. support
See Gaitpois for generic information.
alter.mix, inflate.mix
See Gaitpois for generic information.

```
alter.mlm, inflate.mlm
                            See Gaitpois for generic information.
pobs.mlm.a, pstr.mlm.i, byrow.arg
    See Gaitpois for generic information.
pobs.mix.a, pstr.mix.i
    See Gaitpois for generic information.
deflation See Gaitpois for generic information.
```


## Details

These functions allow any combination of 3 operator types: truncation, alteration and inflation. See Gaitpois for generic information. These functions do what Oazeta, Oizeta, Otzeta collectively did plus much more.
In the notation of Yee and Ma (2020) these functions allow for the special cases: (i) GAIT-Zeta(shape.p)-Zeta(shape.a, alter.mix, pobs.mix.a)-Zeta(shape.i, inflate.mix, pstr.mix.i);
(ii) GAIT-Zeta(shape.p)-MLM(alter.mlm, pobs.mlm.a)-MLM(inflate.mlm, pstr.mlm.i). Model (i) is totally parametric while model (ii) is the most nonparametric possible.

## Value

dgaitzeta gives the density, pgaitzeta gives the distribution function, qgaitzeta gives the quantile function, and rgaitzeta generates random deviates. The default values of the arguments correspond to ordinary dzeta, pzeta, qzeta, rzeta respectively.

## Note

See Gaitpois for general information also relevant to this parent distribution.

## Author(s)

T. W. Yee.

## See Also

Gaitpois, multinomial, gaitzeta.mix, Oazeta, Oizeta, Otzeta.

## Examples

```
ivec <- c(2, 10); avec <- ivec + 4; shape <- 0.95; xgrid <- 0:29
tvec <- 15; max.support <- 25; pobs.a <- 0.10; pstr.i <- 0.15
(ddd <- dgaitzeta(xgrid, shape, truncate = tvec,
    max.support = max.support, pobs.mix.a = pobs.a,
    alter.mix = avec, pstr.mix.i = pstr.i, inflate.mix = ivec))
## Not run: plot(xgrid, ddd, type = "n", ylab = "Probability", xlab = "x",
            main = "GAIT PMF---Zeta Parent")
mylwd <- 0.5
abline(v = avec, col = 'green', lwd = mylwd)
abline(v = ivec, col = 'red', lwd = mylwd)
abline(v = tvec, col = 'tan', lwd = mylwd)
abline(v = max.support, col = 'magenta', lwd = mylwd)
```

```
    abline(h = c(pobs.a, pstr.i, 0:1), col = 'gray', lty = "dashed")
    lines(xgrid, dzeta(xgrid, shape), col = 'gray', lty = "dashed") # f_{\pi}
    lines(xgrid, ddd, type = "h", col = "blue", lwd = 3) # GAIT PMF
    points(xgrid[ddd == 0], ddd[ddd == 0], pch = 16)
    ## End(Not run)
```

    gamma1 1-parameter Gamma Regression Family Function
    
## Description

Estimates the 1-parameter gamma distribution by maximum likelihood estimation.

## Usage

gamma1(link = "loglink", zero = NULL, parallel = FALSE, type.fitted = c("mean", "percentiles", "Qlink"), percentiles = 50)

## Arguments

link Link function applied to the (positive) shape parameter. See Links for more choices and general information.
zero, parallel Details at CommonVGAMffArguments.
type.fitted, percentiles
See CommonVGAMffArguments for information. Using "Qlink" is for quantilelinks in VGAMextra.

## Details

The density function is given by

$$
f(y)=\exp (-y) \times y^{\text {shape }-1} / \Gamma(\text { shape })
$$

for shape $>0$ and $y>0$. Here, $\Gamma$ (shape $)$ is the gamma function, as in gamma. The mean of $Y$ (returned as the default fitted values) is $\mu=$ shape, and the variance is $\sigma^{2}=$ shape.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

This VGAM family function can handle a multiple responses, which is inputted as a matrix.
The parameter shape matches with shape in rgamma. The argument rate in rgamma is assumed 1 for this family function, so that scale $=1$ is used for calls to dgamma, qgamma, etc.
If rate is unknown use the family function gammaR to estimate it too.

## Author(s)

T. W. Yee

## References

Most standard texts on statistical distributions describe the 1-parameter gamma distribution, e.g.,
Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011). Statistical Distributions, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

## See Also

gammaR for the 2-parameter gamma distribution, lgamma1, lindley, simulate.vlm.

## Examples

```
gdata <- data.frame(y = rgamma(n = 100, shape = exp(3)))
fit <- vglm(y ~ 1, gamma1, data = gdata, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

gamma2 2-parameter Gamma Regression Family Function

## Description

Estimates the 2-parameter gamma distribution by maximum likelihood estimation.

## Usage

```
gamma2(lmu = "loglink", lshape = "loglink",
    imethod = 1, ishape = NULL,
    parallel = FALSE, deviance.arg = FALSE, zero = "shape")
```


## Arguments

lmu, lshape Link functions applied to the (positive) $m u$ and shape parameters (called $\mu$ and $a$ respectively). See Links for more choices.
ishape Optional initial value for shape. A NULL means a value is computed internally. If a failure to converge occurs, try using this argument. This argument is ignored if used within cqo; see the iShape argument of qrrvglm. control instead.
imethod An integer with value 1 or 2 which specifies the initialization method for the $\mu$ parameter. If failure to converge occurs try another value (and/or specify a value for ishape).
deviance.arg Logical. If TRUE, the deviance function is attached to the object. Under ordinary circumstances, it should be left alone because it really assumes the shape parameter is at the maximum likelihood estimate. Consequently, one cannot use that criterion to minimize within the IRLS algorithm. It should be set TRUE only when used with cqo under the fast algorithm.
zero See CommonVGAMffArguments for information.
parallel Details at CommonVGAMffArguments. If parallel = TRUE then the constraint is not applied to the intercept.

## Details

This distribution can model continuous skewed responses. The density function is given by

$$
f(y ; \mu, a)=\frac{\exp (-a y / \mu) \times(a y / \mu)^{a-1} \times a}{\mu \times \Gamma(a)}
$$

for $\mu>0, a>0$ and $y>0$. Here, $\Gamma(\cdot)$ is the gamma function, as in gamma. The mean of $Y$ is $\mu=\mu$ (returned as the fitted values) with variance $\sigma^{2}=\mu^{2} / a$. If $0<a<1$ then the density has a pole at the origin and decreases monotonically as $y$ increases. If $a=1$ then this corresponds to the exponential distribution. If $a>1$ then the density is zero at the origin and is unimodal with mode at $y=\mu-\mu / a$; this can be achieved with lshape="logloglink".
By default, the two linear/additive predictors are $\eta_{1}=\log (\mu)$ and $\eta_{2}=\log (a)$. This family function implements Fisher scoring and the working weight matrices are diagonal.
This VGAM family function handles multivariate responses, so that a matrix can be used as the response. The number of columns is the number of species, say, and zero=-2 means that all species have a shape parameter equalling a (different) intercept only.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

The response must be strictly positive. A moment estimator for the shape parameter may be implemented in the future.
If mu and shape are vectors, then rgamma $(n=n$, shape $=$ shape, $s c a l e=m u /$ shape $)$ will generate random gamma variates of this parameterization, etc.; see GammaDist.

## Author(s)

T. W. Yee

## References

The parameterization of this VGAM family function is the 2-parameter gamma distribution described in the monograph
McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, 2nd ed. London: Chapman \& Hall.

## See Also

gamma1 for the 1-parameter gamma distribution, gammaR for another parameterization of the 2parameter gamma distribution that is directly matched with rgamma, bigamma. mckay for $a$ bivariate gamma distribution, expexpff, GammaDist, gordlink, CommonVGAMffArguments, simulate.vlm, negloglink.

## Examples

```
# Essentially a 1-parameter gamma
gdata <- data.frame(y = rgamma(n = 100, shape = exp(1)))
fit1 <- vglm(y ~ 1, gamma1, data = gdata)
fit2 <- vglm(y ~ 1, gamma2, data = gdata, trace = TRUE, crit = "coef")
coef(fit2, matrix = TRUE)
c(Coef(fit2), colMeans(gdata))
# Essentially a 2-parameter gamma
gdata <- data.frame(y = rgamma(n = 500, rate = exp(-1), shape = exp(2)))
fit2 <- vglm(y ~ 1, gamma2, data = gdata, trace = TRUE, crit = "coef")
coef(fit2, matrix = TRUE)
c(Coef(fit2), colMeans(gdata))
summary(fit2)
```


## Description

Estimate the parameter of a gamma hyperbola bivariate distribution by maximum likelihood estimation.

## Usage

gammahyperbola(ltheta = "loglink", itheta = NULL, expected = FALSE)

## Arguments

ltheta Link function applied to the (positive) parameter $\theta$. See Links for more choices.
itheta Initial value for the parameter. The default is to estimate it internally.
expected Logical. FALSE means the Newton-Raphson (using the observed information matrix) algorithm, otherwise the expected information matrix is used (Fisher scoring algorithm).

## Details

The joint probability density function is given by

$$
f\left(y_{1}, y_{2}\right)=\exp \left(-e^{-\theta} y_{1} / \theta-\theta y_{2}\right)
$$

for $\theta>0, y_{1}>0, y_{2}>1$. The random variables $Y_{1}$ and $Y_{2}$ are independent. The marginal distribution of $Y_{1}$ is an exponential distribution with rate parameter $\exp (-\theta) / \theta$. The marginal distribution of $Y_{2}$ is an exponential distribution that has been shifted to the right by 1 and with rate parameter $\theta$. The fitted values are stored in a two-column matrix with the marginal means, which are $\theta \exp (\theta)$ and $1+1 / \theta$.
The default algorithm is Newton-Raphson because Fisher scoring tends to be much slower for this distribution.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

The response must be a two-column matrix.

## Author(s)

T. W. Yee

## References

Reid, N. (2003). Asymptotics and the theory of inference. Annals of Statistics, 31, 1695-1731.

## See Also

exponential.

## Examples

```
gdata <- data.frame(x2 = runif(nn <- 1000))
gdata <- transform(gdata, theta = exp(-2 + x2))
gdata <- transform(gdata, y1 = rexp(nn, rate = exp(-theta)/theta),
    y2 = rexp(nn, rate = theta) + 1)
fit <- vglm(cbind(y1, y2) ~ x2, gammahyperbola(expected = TRUE), data = gdata)
coef(fit, matrix = TRUE)
Coef(fit)
head(fitted(fit))
summary(fit)
```


## Description

Estimates the 2-parameter gamma distribution by maximum likelihood estimation.

## Usage

gammaR(lrate = "loglink", lshape = "loglink", irate = NULL, ishape = NULL, lss = TRUE, zero = "shape")

## Arguments

lrate, lshape Link functions applied to the (positive) rate and shape parameters. See Links for more choices.
irate, ishape Optional initial values for rate and shape. A NULL means a value is computed internally. If a failure to converge occurs, try using these arguments.
zero, lss Details at CommonVGAMffArguments.

## Details

The density function is given by

$$
f(y ; \text { rate }, \text { shape })=\exp (- \text { rate } \times y) \times y^{\text {shape }-1} \times \text { rate }{ }^{\text {shape }} / \Gamma(\text { shape })
$$

for shape $>0$, rate $>0$ and $y>0$. Here, $\Gamma$ (shape) is the gamma function, as in gamma. The mean of $Y$ is $\mu=$ shape/rate (returned as the fitted values) with variance $\sigma^{2}=\mu^{2} /$ shape $=$ shape/rate ${ }^{2}$. By default, the two linear/additive predictors are $\eta_{1}=\log ($ rate $)$ and $\eta_{2}=\log$ (shape).

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

The parameters rate and shape match with the arguments rate and shape of rgamma. The order of the arguments agree too. Here, scale $=1 /$ rate is used, so one can use negloglink. Multiple responses are handled.
If rate $=1$ use the family function gamma1 to estimate shape.
The reciprocal of a 2-parameter gamma random variate has an inverse gamma distribution. One might write a VGAM family function called invgammaR() to estimate this, but for now, just feed in the reciprocal of the response.

## Author(s)

T. W. Yee

## References

Most standard texts on statistical distributions describe the 2-parameter gamma distribution, e.g.,
Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011). Statistical Distributions, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

## See Also

gamma1 for the 1-parameter gamma distribution, gamma2 for another parameterization of the 2parameter gamma distribution, bigamma.mckay for $a$ bivariate gamma distribution, expexpff, simulate.vlm, rgamma, negloglink.

## Examples

```
# Essentially a 1-parameter gamma
gdata <- data.frame(y1 = rgamma(n <- 100, shape = exp(1)))
fit1 <- vglm(y1 ~ 1, gamma1, data = gdata, trace = TRUE)
fit2 <- vglm(y1 ~ 1, gammaR, data = gdata, trace = TRUE, crit = "coef")
coef(fit2, matrix = TRUE)
Coef(fit2)
# Essentially a 2-parameter gamma
gdata <- data.frame(y2 = rgamma(n = 500, rate = exp(1), shape = exp(2)))
fit2 <- vglm(y2 ~ 1, gammaR, data = gdata, trace = TRUE, crit = "coef")
coef(fit2, matrix = TRUE)
Coef(fit2)
summary(fit2)
```

garma GARMA (Generalized Autoregressive Moving-Average) Models

## Description

Fits GARMA models to time series data.

## Usage

garma(link = "identitylink", p.ar.lag = 1, q.ma.lag = 0, coefstart = NULL, step = 1)

## Arguments

link Link function applied to the mean response. The default is suitable for continuous responses. The link loglink should be chosen if the data are counts. The link reciprocal can be chosen if the data are counts and the variance assumed for this is $\mu^{2}$. The links logitlink, probitlink, clogloglink, and cauchitlink are supported and suitable for binary responses.
Note that when the log or logit link is chosen: for log and logit, zero values can be replaced by bvalue. See loglink and logitlink etc. for specific information about each link function.

| p.ar.lag | A positive integer, the lag for the autoregressive component. Called $p$ below. |
| :--- | :--- |
| q.ma.lag | A non-negative integer, the lag for the moving-average component. Called $q$ <br> below. |
| coefstart | Starting values for the coefficients. Assigning this argument is highly recom- <br> mended. For technical reasons, the argument coefstart in vglm cannot be <br> used. |
| step | Numeric. Step length, e.g., 0.5 means half-stepsizing. |

## Details

This function draws heavily on Benjamin et al. (1998). See also Benjamin et al. (2003). GARMA models extend the ARMA time series model to generalized responses in the exponential family, e.g., Poisson counts, binary responses. Currently, this function is rudimentary and can handle only certain continuous, count and binary responses only. The user must choose an appropriate link for the link argument.
The $\operatorname{GARMA}(p, q)$ model is defined by firstly having a response belonging to the exponential family

$$
f\left(y_{t} \mid D_{t}\right)=\exp \left\{\frac{y_{t} \theta_{t}-b\left(\theta_{t}\right)}{\phi / A_{t}}+c\left(y_{t}, \phi / A_{t}\right)\right\}
$$

where $\theta_{t}$ and $\phi$ are the canonical and scale parameters respectively, and $A_{t}$ are known prior weights. The mean $\mu_{t}=E\left(Y_{t} \mid D_{t}\right)=b^{\prime}\left(\theta_{t}\right)$ is related to the linear predictor $\eta_{t}$ by the link function $g$. Here, $D_{t}=\left\{x_{t}, \ldots, x_{1}, y_{t-1}, \ldots, y_{1}, \mu_{t-1}, \ldots, \mu_{1}\right\}$ is the previous information set. Secondly, the $\operatorname{GARMA}(p, q)$ model is defined by

$$
g\left(\mu_{t}\right)=\eta_{t}=x_{t}^{T} \beta+\sum_{k=1}^{p} \phi_{k}\left(g\left(y_{t-k}\right)-x_{t-k}^{T} \beta\right)+\sum_{k=1}^{q} \theta_{k}\left(g\left(y_{t-k}\right)-\eta_{t-k}\right) .
$$

Parameter vectors $\beta, \phi$ and $\theta$ are estimated by maximum likelihood.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm.

## Warning

This VGAM family function is 'non-standard' in that the model does need some coercing to get it into the VGLM framework. Special code is required to get it running. A consequence is that some methods functions may give wrong results when applied to the fitted object.

## Note

This function is unpolished and is requires lots of improvements. In particular, initialization is very poor. Results appear very sensitive to quality of initial values. A limited amount of experience has shown that half-stepsizing is often needed for convergence, therefore choosing crit = "coef" is not recommended.
Overdispersion is not handled. For binomial responses it is currently best to input a vector of 1 s and 0 s rather than the cbind(successes, failures) because the initialize slot is rudimentary.

## Author(s)

T. W. Yee

## References

Benjamin, M. A., Rigby, R. A. and Stasinopoulos, M. D. (1998). Fitting Non-Gaussian Time Series Models. Pages 191-196 in: Proceedings in Computational Statistics COMPSTAT 1998 by Payne, R. and P. J. Green. Physica-Verlag.

Benjamin, M. A., Rigby, R. A. and Stasinopoulos, M. D. (2003). Generalized Autoregressive Moving Average Models. Journal of the American Statistical Association, 98: 214-223.
Zeger, S. L. and Qaqish, B. (1988). Markov regression models for time series: a quasi-likelihood approach. Biometrics, 44: 1019-1031.

## Examples

```
gdata <- data.frame(interspike = c(68, 41, 82, 66, 101, 66, 57, 41, 27, 78,
59, 73, 6, 44, 72, 66, 59, 60, 39, 52,
50, 29, 30, 56, 76, 55, 73, 104, 104, 52,
25, 33, 20, 60, 47, 6, 47, 22, 35, 30,
29, 58, 24, 34, 36, 34, 6, 19, 28, 16,
36, 33, 12, 26, 36, 39, 24, 14, 28, 13,
    2, 30, 18, 17, 28, 9, 28, 20, 17, 12,
    19, 18, 14, 23, 18, 22, 18, 19, 26, 27,
    23, 24, 35, 22, 29, 28, 17, 30, 34, 17,
    20, 49, 29, 35, 49, 25, 55, 42, 29, 16)) # See Zeger and Qaqish (1988)
    gdata <- transform(gdata, spikenum = seq(interspike))
    bvalue <- 0.1 # .Machine$double.xmin # Boundary value
    fit <- vglm(interspike ~ 1, trace = TRUE, data = gdata,
        garma(loglink(bvalue = bvalue),
        p = 2, coefstart = c(4, 0.3, 0.4)))
    summary(fit)
    coef(fit, matrix = TRUE)
    Coef(fit) # A bug here
    ## Not run: with(gdata, plot(interspike, ylim = c(0, 120), las = 1,
        xlab = "Spike Number", ylab = "Inter-Spike Time (ms)", col = "blue"))
    with(gdata, lines(spikenum[-(1:fit@misc$plag)], fitted(fit), col = "orange"))
    abline(h = mean(with(gdata, interspike)), lty = "dashed", col = "gray")
    ## End(Not run)
```

gatnbinomial.mix

Generally-Altered and -Truncated Negative Binomial Regression Family Function (GAT-NB-NB Mixture Variant)

## Description

Fits a generally-altered and -truncated negative binomial regression (mixture of 2 NBs on differing support). The truncation may include values in the upper tail.

## Usage

```
gatnbinomial.mix(alter = NULL, truncate = NULL,
    zero = c("pobs.a", "size"), parallel = FALSE,
    lmunb.p = "loglink", lsize.p = "loglink", lpobs.a = "logitlink",
    lmunb.a = "loglink", lsize.a = "loglink",
    type.fitted = c("mean", "pobs.a", "Pobs.a",
            "prob.a", "prob.i", "prob.t", "lhs.prob"),
    imethod = 1, imunb.p = NULL, isize.p = NULL,
    ipobs.a = NULL, imunb.a = imunb.p, isize.a = isize.p,
    ishrinkage \(=0.95\), probs.y \(=0.35\), cutoff. prob \(=0.999\),
    eps.trig \(=1 \mathrm{e}-7\), max.chunk. \(\mathrm{MB}=30\) )
```


## Arguments

alter, truncate
Very similar to gaitpoisson.mix. Note: alter must be assigned a vector of length 3 or more for this function to work properly. The default settings are effectively equivalent to negbinomial. If length(alter) $==2$ then this function will not work due to identifiability problems. In general, the regression tends to be more stable when length(alter) is not too small and the values of alter are spread out over the entire support.
lpobs.a, lmunb.p, lmunb.a
Link functions; the .p and . a refer to the parent and altered distributions respectively. See Links for more choices and information.
lsize.p, lsize.a
Same as above.
parallel, type.fitted
the latter is Very similar to gaitpoisson.mix; the former concerns: constrain the mean and size parameters to be equal? Setting parallel = TRUE is probably a good idea for many data sets, especially when length(alter) is low and the values of alter are not spread out.
imethod, imunb.p, isize.p
See CommonVGAMffArguments for information.
imunb.a, isize.a, ipobs.a
See CommonVGAMffArguments for information.
probs.y, ishrinkage
See CommonVGAMffArguments for information.
zero See CommonVGAMffArguments and gaitpoisson.mix for information.
cutoff.prob, eps.trig, max.chunk.MB
See negbinomial for information.

## Details

This distribution, also known as the GAT-NB-NB mixture, is an alternative to gaitpoisson.mix and allows for overdispersion relative to the Poisson distribution. Because the size parameters are estimated this family function is more difficult to fit and is numerically fraught, in comparison.

The distribution is also a more structured model compared to gatnbinomial.mlm because the (outer) distribution of the altered values is also an NB too.
Apart from the ordering of the linear/additive predictors, here are some special cases: gatnbinomial.mix() is the same as negbinomial, gatnbinomial. $\mathrm{mix}(a l t e r=0)$ is the same as zanegbinomial, gatnbinomial.mix (truncate $=0$ ) is the same as posnegbinomial. See gatnbinomial.mlm for similar information.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.
The fitted.values slot of the fitted object, which should be extracted by the generic function fitted, are similar to gatnbinomial.mlm.

## Warning

The same caution needed for gatnbinomial.mlm and gaitpoisson.mix applies here, however this function is a bit more parametric (structured) in comparison to . mlm functions, especially when parallel = TRUE.

## Note

Convergence is rather slow because of an infinite series that is approximated with finite terms. This family function is quite expensive. See also gaitpoisson.mix. This family function offers potential for regressing heaped count data.

## Author(s)

T. W. Yee

## References

Yee, T. W. and Ma, C. (2020). Generally-altered, -inflated and -truncated regression, with application to heaped and seeped count data. In preparation.

## See Also

Gaitnbinom.mix, gatnbinomial.mlm, gaitpoisson.mix, negbinomial, posnegbinomial, zanegbinomial, CommonVGAMffArguments, rootogram4, simulate.vlm.

## Examples

```
avec <- seq(5, 25, by = 5) # Alter these values
tvec <- c(4, 7, 22) # Truncate these values
pobs.a <- logitlink(-0.5, inverse = TRUE) # About 0.5
gdata <- data.frame(x2 = runif(nn <- 1000))
gdata <- transform(gdata, munb.p = exp(2 + 0 * x2), size.p = exp(1))
gdata <- transform(gdata,
    y1 = rgaitnbinom.mix(nn, size.p = size.p, munb.p = munb.p,
                pobs.a = pobs.a, truncate = tvec, alter = avec))
gatnbinomial.mix(alter = avec)
```

```
(ty1 <- with(gdata, table(y1)))
## Not run:
plot(as.numeric(names(ty1)), c(ty1) / sum(ty1), xlab = "y",
    ylab = "Proportion", las = 1, type = "h", col = "blue")
fit1 <- vglm(y1 ~ 1, trace = TRUE, data = gdata,
        gatnbinomial.mix(alter = avec, truncate = tvec,
        parallel = TRUE))
head(fitted(fit1, type.fitted = "Pobs.a"))
head(predict(fit1))
coef(fit1, matrix = TRUE)
summary(fit1)
## End(Not run)
```

gatnbinomial.mlm Generally-Altered and -Truncated Negative Binomial Regression
Family Function (GAT-NB-MLM Variant)

## Description

Fits a generally-altered and -truncated negative binomial regression (using a multinomial logit model for the altered values).

## Usage

```
gatnbinomial.mlm(alter = NULL, truncate = NULL, zero = "size",
    lmunb = "loglink", lsize = "loglink", type.fitted = c("mean",
    "pobs.a", "Pobs.a", "prob.a", "prob.i", "prob.t", "lhs.prob"),
    imethod = 1, imunb = NULL, isize = exp(1), ishrinkage = 0.95,
    probs.y = 0.35, cutoff.prob = 0.999, eps.trig = 1e-7,
    max.chunk.MB = 30)
```


## Arguments

alter, truncate
Same as gaitpoisson.mlm.
lmunb, lsize See Links for more choices and information. Similar to negbinomial.
type.fitted, zero
See CommonVGAMffArguments and gaitpoisson.mlm for information.
imethod, imunb, isize
See CommonVGAMffArguments for information.
probs.y, ishrinkage
See CommonVGAMffArguments for information.
cutoff.prob, eps.trig
See negbinomial for information.
max.chunk.MB See negbinomial for information.

## Details

The generally-truncated (GT) negative binomial distribution is an ordinary negative binomial distribution with the probability of certain values (given by the truncate argument) being zero. Thus the other probabilities are scaled up. The (0-truncated) positive-negative binomial distribution is a special case (posnegbinomial).
The generally-altered (GA) negative binomial distribution is an ordinary negative binomial distribution with the probability of certain values (given by the alter argument) being modelled using a multinomial logit model (see multinomial). The 0 -altered negative binomial distribution is a special case (zanegbinomial) and it is called a hurdle model by some people.
This function can fit both the GA and GT models simultaneously, called the GAT-NB-MLM. It might be applied to heaped data. That is, each special value can be altered or truncated but not both. The default settings make this family function equivalent to negbinomial.
This function implements Fisher scoring and currently does not handle multiple responses for GT or the GA model. Compared to what could be ginbinomial.mlm this family function handles deflation and inflation, therefore handles a wider range of data. For further details please see Gaitnbinom.mlm.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam. See gaitpoisson.mlm for more information.

## Warning

Due to its flexibility, it is easy to misuse this function; the truncate vector should ideally be not very long and have values that can be justified by the application on hand. Likewise, the alter vector should be short too, and each value should have good justification for being included. Adding unnecessary values to these two arguments willy-nilly is a recipe for disaster.

Regarding truncation, under- or over-flow may occur if the data is ill-conditioned. The response is checked to see that no values equal any values of the truncate vector.
Compared to gaitpoisson.mlm this family function is even more difficult to fit because it is more flexible and involves approximating an infinite series when computing the expected information matrix.

## Note

See gaitpoisson.mlm for more information about robustness.
This family function effectively combines what could have been ganbinomial.mlm() and gtnbinomial.mlm() together. The former would have effectively included multinomial inside it.
It is possible in the near future that zanegbinomial, zanegbinomialff, Zanegbin, posnegbinomial, Posnegbin be moved into VGAMdata. Likewise for zapoisson, zapoissonff, Zapois.

An argument max. support is not implemented because the upper RHS tail of the NB distribution does not admit practical formulas that can be computed. However, instead of using something like max. support = A one could try something like truncate $=(A+1):(A+l a r g e)$ for some positive integer large that is large enough, but within reason. For example, instead of max. support $=20$ one could try truncate $=21: 30$ or truncate $=21: 40$.

## Author(s)

T. W. Yee

## See Also

Gaitnbinom.mlm, gaitpoisson.mlm, gatnbinomial.mix, multinomial, negbinomial, posnegbinomial, zanegbinomial, CommonVGAMffArguments, rootogram4, simulate.vlm.

## Examples

```
## Not run:
avec <- c(10, 20, 30) # Alter these values
tvec <- 0 # Truncate this value
pobs.a <- logitlink(-(2:4), inverse = TRUE) # Between 0.02 and 0.12
size1 <- exp(1)
set.seed(1)
gdata <- data.frame(x2 = runif(nn <- 1000))
gdata <- transform(gdata, lambda1 = exp(2))
gdata <- transform(gdata,
        y1 = rgaitnbinom.mlm(nn, size1, mu = lambda1, pobs.a = pobs.a,
                            truncate = tvec, byrow = TRUE, alter = avec))
gatnbinomial.mlm(alter = avec)
(ty1 <- with(gdata, table(y1)))
propn1 <- c(ty1) / sum(ty1)
plot(as.numeric(names(ty1)), propn1, las = 1, xlab = "y",
            yaxs = "i", ylim = c(0, max(propn1) * 1.1), main = "Heaped data",
            ylab = "Proportion", lwd = 3, type = "h", col = "blue")
fit1 <- vglm(y1 ~ 1, trace = TRUE, data = gdata, crit = "coef",
                    gatnbinomial.mlm(alter = avec, truncate = tvec))
head(fitted(fit1))
head(predict(fit1))
coef(fit1, matrix = TRUE)
summary(fit1)
## End(Not run)
```

GenbetaII

The Generalized Beta II Distribution

## Description

Density for the generalized beta II distribution with shape parameters a and pand q, and scale parameter scale.

## Usage

dgenbetaII (x, scale $=1$, shape1.a, shape2.p, shape3.q, log = FALSE)

## Arguments

$x \quad$ vector of quantiles.
shape1.a, shape2.p, shape3.q positive shape parameters.
scale positive scale parameter.
$\log \quad$ Logical. If $\log =$ TRUE then the logarithm of the density is returned.

## Details

See genbetaII, which is the VGAM family function for estimating the parameters by maximum likelihood estimation. Several distributions, such as the Singh-Maddala, are special case of this flexible 4-parameter distribution. The product of shape1. a and shape2.p determines the behaviour of the density at the origin.

## Value

dgenbetaII gives the density.

## Author(s)

T. W. Yee

## References

Kleiber, C. and Kotz, S. (2003). Statistical Size Distributions in Economics and Actuarial Sciences, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

genbetaII.

## Examples

```
dgenbetaII(0, shape1.a = 1/4, shape2.p = 4, shape3.q = 3)
dgenbetaII(0, shape1.a = 1/4, shape2.p = 2, shape3.q = 3)
dgenbetaII(0, shape1.a = 1/4, shape2.p = 8, shape3.q = 3)
```


## Description

Maximum likelihood estimation of the 4-parameter generalized beta II distribution.

## Usage

```
genbetaII(lscale = "loglink", lshape1.a = "loglink", lshape2.p = "loglink",
            lshape3.q = "loglink", iscale = NULL, ishape1.a = NULL,
            ishape2.p = NULL, ishape3.q = NULL, lss = TRUE,
            gscale = exp(-5:5), gshape1.a = exp(-5:5),
            gshape2.p = exp(-5:5), gshape3.q = exp(-5:5),
            zero = "shape")
```


## Arguments

lss
See CommonVGAMffArguments for important information.
lshape1.a, lscale, lshape2.p, lshape3.q
Parameter link functions applied to the shape parameter a, scale parameter scale, shape parameter $p$, and shape parameter $q$. All four parameters are positive. See Links for more choices.
iscale, ishape1.a, ishape2.p, ishape3.q
Optional initial values for the parameters. A NULL means a value is computed internally using the arguments gscale, gshape1.a, etc.
gscale, gshape1.a, gshape2.p, gshape3.q
See CommonVGAMffArguments for information. Replaced by iscale, ishape1.a etc. if given.
zero The default is to set all the shape parameters to be intercept-only. See CommonVGAMffArguments for information.

## Details

This distribution is most useful for unifying a substantial number of size distributions. For example, the Singh-Maddala, Dagum, Fisk (log-logistic), Lomax (Pareto type II), inverse Lomax, beta distribution of the second kind distributions are all special cases. Full details can be found in Kleiber and Kotz (2003), and Brazauskas (2002). The argument names given here are used by other families that are special cases of this family. Fisher scoring is used here and for the special cases too.
The 4-parameter generalized beta II distribution has density

$$
f(y)=a y^{a p-1} /\left[b^{a p} B(p, q)\left\{1+(y / b)^{a}\right\}^{p+q}\right]
$$

for $a>0, b>0, p>0, q>0, y \geq 0$. Here $B$ is the beta function, and $b$ is the scale parameter scale, while the others are shape parameters. The mean is

$$
E(Y)=b \Gamma(p+1 / a) \Gamma(q-1 / a) /(\Gamma(p) \Gamma(q))
$$

provided $-a p<1<a q$; these are returned as the fitted values.
This family function handles multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

This distribution is very flexible and it is not generally recommended to use this family function when the sample size is small-numerical problems easily occur with small samples. Probably several hundred observations at least are needed in order to estimate the parameters with any level of confidence. Neither is the inclusion of covariates recommended at all-not unless there are several thousand observations. The mean is finite only when $-a p<1<a q$, and this can be easily violated by the parameter estimates for small sample sizes. Try fitting some of the special cases of this distribution (e.g., sinmad, fisk, etc.) first, and then possibly use those models for initial values for this distribution.

## Note

The default is to use a grid search with respect to all four parameters; this is quite costly and is time consuming. If the self-starting initial values fail, try experimenting with the initial value arguments. Also, the constraint $-a p<1<a q$ may be violated as the iterations progress so it pays to monitor convergence, e.g., set trace = TRUE. Successful convergence depends on having very good initial values. This is rather difficult for this distribution so that a grid search is conducted by default. One suggestion for increasing the estimation reliability is to set stepsize $=0.5$ and maxit $=100$; see vglm. control.

## Author(s)

T. W. Yee, with help from Victor Miranda.

## References

Kleiber, C. and Kotz, S. (2003). Statistical Size Distributions in Economics and Actuarial Sciences, Hoboken, NJ, USA: Wiley-Interscience.
Brazauskas, V. (2002). Fisher information matrix for the Feller-Pareto distribution. Statistics \& Probability Letters, 59, 159-167.

## See Also

dgenbetaII, betaff, betaII, dagum, sinmad, fisk, lomax, inv.lomax, paralogistic, inv. paralogistic, lino, CommonVGAMffArguments, vglm. control.

## Examples

```
## Not run:
gdata <- data.frame(y = rsinmad(3000, shape1 = exp(1), scale = exp(2),
            shape3 = exp(1))) # A special case!
fit <- vglm(y ~ 1, genbetaII(lss = FALSE), data = gdata, trace = TRUE)
fit <- vglm(y ~ 1, data = gdata, trace = TRUE,
    genbetaII(ishape1.a = 3, iscale = 7, ishape3.q = 2.3))
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
## End(Not run)
```


## Description

Estimation of the 3-parameter generalized gamma distribution proposed by Stacy (1962).

## Usage

gengamma.stacy(lscale = "loglink", ld = "loglink", lk = "loglink", iscale $=$ NULL, id $=$ NULL, $i k=$ NULL, imethod $=1$, gscale.mux $=\exp ((-4: 4) / 2)$, gshape1.d $=\exp ((-5: 5) / 2)$, gshape2.k $=\exp ((-5: 5) / 2), \operatorname{probs.y}=0.3$, zero $=c(" d ", " k "))$

## Arguments

lscale, ld, lk Parameter link function applied to each of the positive parameters $b, d$ and $k$, respectively. See Links for more choices.
iscale, id, ik Initial value for $b, d$ and $k$, respectively. The defaults mean an initial value is determined internally for each.
gscale.mux, gshape1.d, gshape2.k
See CommonVGAMffArguments for information. Replaced by iscale, id etc. if given.
imethod, probs.y, zero
See CommonVGAMffArguments for information.

## Details

The probability density function can be written

$$
f(y ; b, d, k)=d b^{-d k} y^{d k-1} \exp \left[-(y / b)^{d}\right] / \Gamma(k)
$$

for scale parameter $b>0$, and Weibull-type shape parameter $d>0$, gamma-type shape parameter $k>0$, and $y>0$. The mean of $Y$ is $b \times \Gamma(k+1 / d) / \Gamma(k)$ (returned as the fitted values), which equals $b k$ if $d=1$.
There are many special cases, as given in Table 1 of Stacey and Mihram (1965). In the following, the parameters are in the order $b, d, k$. The special cases are: Exponential $f(y ; b, 1,1)$, Gamma $f(y ; b, 1, k)$, Weibull $f(y ; b, d, 1)$, Chi Squared $f(y ; 2,1, a / 2)$ with $a$ degrees of freedom, Chi $f(y ; \sqrt{2}, 2, a / 2)$ with $a$ degrees of freedom, Half-normal $f(y ; \sqrt{2}, 2,1 / 2)$, Circular normal $f(y ; \sqrt{2}, 2,1)$, Spherical normal $f(y ; \sqrt{2}, 2,3 / 2)$, Rayleigh $f(y ; c \sqrt{2}, 2,1)$ where $c>0$. Also the log-normal distribution corresponds to when $\mathrm{k}=\mathrm{Inf}$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

Several authors have considered maximum likelihood estimation for the generalized gamma distribution and have found that the Newton-Raphson algorithm does not work very well and that the existence of solutions to the log-likelihood equations is sometimes in doubt. Although Fisher scoring is used here, it is likely that the same problems will be encountered. It appears that large samples are required, for example, the estimator of $k$ became asymptotically normal only with 400 or more observations. It is not uncommon for maximum likelihood estimates to fail to converge even with two or three hundred observations. With covariates, even more observations are needed to increase the chances of convergence. Using covariates is not advised unless the sample size is at least a few thousand, and even if so, modelling 1 or 2 parameters as intercept-only is a very good idea (e.g., zero $=2: 3$ ). Monitoring convergence is also a very good idea (e.g., set trace $=$ TRUE). Half-stepping is not uncommon, and if this occurs, then the results should be viewed with more suspicion.

## Note

The notation used here differs from Stacy (1962) and Prentice (1974). Poor initial values may result in failure to converge so if there are covariates and there are convergence problems, try using or checking the zero argument (e.g., zero $=2: 3$ ) or the ik argument or the imethod argument, etc.

## Author(s)

T. W. Yee

## References

Stacy, E. W. (1962). A generalization of the gamma distribution. Annals of Mathematical Statistics, 33(3), 1187-1192.

Stacy, E. W. and Mihram, G. A. (1965). Parameter estimation for a generalized gamma distribution. Technometrics, 7, 349-358.

Prentice, R. L. (1974). A log gamma model and its maximum likelihood estimation. Biometrika, 61, 539-544.

## See Also

rgengamma.stacy, gamma1, gamma2, prentice74, simulate.vlm, chisq, lognormal, rayleigh, weibullR.

## Examples

```
k <- exp(-1); Scale <- exp(1); dd <- exp(0.5); set.seed(1)
gdata <- data.frame(y = rgamma(2000, shape = k, scale = Scale))
gfit <- vglm(y ~ 1, gengamma.stacy, data = gdata, trace = TRUE)
coef(gfit, matrix = TRUE)
```


## Description

Density, distribution function, quantile function and random generation for the generalized gamma distribution with scale parameter scale, and parameters $d$ and $k$.

## Usage

dgengamma.stacy (x, scale $=1, \mathrm{~d}, \mathrm{k}, \log =$ FALSE)
pgengamma.stacy(q, scale $=1$, d, k,
lower.tail $=$ TRUE, log. $\mathrm{p}=$ FALSE)
qgengamma.stacy ( p, scale $=1, \mathrm{~d}, \mathrm{k}$,
lower.tail $=$ TRUE, log. $p=$ FALSE)
rgengamma.stacy ( n , scale $=1, \mathrm{~d}, \mathrm{k}$ )

## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$n \quad$ number of observations. Same as in runif.
scale the (positive) scale parameter $b$.
$\mathrm{d}, \mathrm{k} \quad$ the (positive) parameters $d$ and $k$. Both can be thought of as shape parameters, where $d$ is of the Weibull-type and $k$ is of the gamma-type.
log Logical. If log = TRUE then the logarithm of the density is returned.
lower.tail, log.p
Same meaning as in pnorm or qnorm.

## Details

See gengamma. stacy, the VGAM family function for estimating the generalized gamma distribution by maximum likelihood estimation, for formulae and other details. Apart from n, all the above arguments may be vectors and are recyled to the appropriate length if necessary.

## Value

dgengamma. stacy gives the density, pgengamma. stacy gives the distribution function, qgengamma. stacy gives the quantile function, and rgengamma. stacy generates random deviates.

## Author(s)

T. W. Yee and Kai Huang

## References

Stacy, E. W. and Mihram, G. A. (1965). Parameter estimation for a generalized gamma distribution. Technometrics, 7, 349-358.

## See Also

gengamma. stacy.

## Examples

```
## Not run: x <- seq(0, 14, by = 0.01); d <- 1.5; Scale <- 2; k <- 6
plot(x, dgengamma.stacy(x, Scale, d = d, k = k), type = "l",
    col = "blue", ylim = 0:1,
    main = "Blue is density, orange is cumulative distribution function",
    sub = "Purple are 5,10,...,95 percentiles", las = 1, ylab = "")
abline(h = 0, col = "blue", lty = 2)
lines(qgengamma.stacy(seq(0.05, 0.95, by = 0.05), Scale, d = d, k = k),
    dgengamma.stacy(qgengamma.stacy(seq(0.05, 0.95, by = 0.05),
                                    Scale, d = d, k = k),
    Scale, d = d, k = k), col = "purple", lty = 3, type = "h")
lines(x, pgengamma.stacy(x, Scale, d = d, k = k), col = "orange")
abline(h = 0, lty = 2)
## End(Not run)
```

genpoisson

Generalized Poisson distribution

## Description

Estimation of the two-parameter generalized Poisson distribution.

## Usage

genpoisson(llambda = "rhobitlink", ltheta = "loglink",
ilambda $=$ NULL, itheta $=$ NULL,
use. approx $=$ TRUE, imethod $=1$, ishrinkage $=0.95$, zero = "lambda")

## Arguments

llambda, ltheta
Parameter link functions for $\lambda$ and $\theta$. See Links for more choices. The $\lambda$ parameter lies at least within the interval $[-1,1]$; see below for more details, and an alternative link is rhobitlink. The $\theta$ parameter is positive, therefore the default is the log link.
ilambda, itheta
Optional initial values for $\lambda$ and $\theta$. The default is to choose values internally.

```
use.approx
Logical. If TRUE then an approximation to the expected information matrix is used, otherwise Newton-Raphson is used.
imethod An integer with value 1 or 2 or 3 which specifies the initialization method for the parameters. If failure to converge occurs try another value and/or else specify a value for ilambda and/or itheta.
ishrinkage, zero
See CommonVGAMffArguments for information.
```


## Details

The generalized Poisson distribution has density

$$
f(y)=\theta(\theta+\lambda y)^{y-1} \exp (-\theta-\lambda y) / y!
$$

for $\theta>0$ and $y=0,1,2, \ldots$. Now $\max (-1,-\theta / m) \leq \lambda \leq 1$ where $m(\geq 4)$ is the greatest positive integer satisfying $\theta+m \lambda>0$ when $\lambda<0$ [and then $P(Y=y)=0$ for $y>m$ ]. Note the complicated support for this distribution means, for some data sets, the default link for llambda will not always work, and some tinkering may be required to get it running.

As Consul and Famoye (2006) state on p.165, the lower limits on $\lambda$ and $m \geq 4$ are imposed to ensure that there are at least 5 classes with nonzero probability when $\lambda$ is negative.
An ordinary Poisson distribution corresponds to $\lambda=0$. The mean (returned as the fitted values) is $E(Y)=\theta /(1-\lambda)$ and the variance is $\theta /(1-\lambda)^{3}$.

For more information see Consul and Famoye (2006) for a summary and Consul (1989) for full details.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

Monitor convergence! This family function is fragile. Don't get confused because theta (and not lambda) here really matches more closely with lambda of dpois.

## Note

This family function handles multiple responses. This distribution is potentially useful for dispersion modelling. Convergence problems may occur when lambda is very close to 0 or 1 . If a failure occurs then you might want to try something like llambda $=$ extlogitlink (min $=-0.9, \max =1$ ) to handle the LHS complicated constraint, and if that doesn't work, try llambda = extlogitlink(min $=-0.8, \max =1$ ), etc.

## Author(s)

T. W. Yee

## References

Consul, P. C. and Famoye, F. (2006). Lagrangian Probability Distributions, Boston, USA: Birkhauser.
Jorgensen, B. (1997). The Theory of Dispersion Models. London: Chapman \& Hall
Consul, P. C. (1989). Generalized Poisson Distributions: Properties and Applications. New York, USA: Marcel Dekker.

## See Also

poissonff, dpois. dgenpois, rhobitlink, extlogitlink.

## Examples

```
gdata <- data.frame(x2 = runif(nn <- 200))
gdata <- transform(gdata, y1 = rpois(nn, exp(2 - x2))) # Poisson data
fit <- vglm(y1 ~ x2, genpoisson, data = gdata, trace = TRUE)
coef(fit, matrix = TRUE)
summary(fit)
```

genray The Generalized Rayleigh Distribution

## Description

Density, distribution function, quantile function and random generation for the generalized Rayleigh distribution.

## Usage

dgenray (x, scale $=1$, shape, log = FALSE)
pgenray (q, scale = 1, shape, lower.tail = TRUE, log.p = FALSE)
qgenray ( p , scale $=1$, shape, lower.tail = TRUE, log.p = FALSE)
rgenray(n, scale = 1, shape)

## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$n \quad$ number of observations. If length $(n)>1$ then the length is taken to be the number required.
scale, shape positive scale and shape parameters.
log Logical. If log = TRUE then the logarithm of the density is returned.
lower.tail, log.p
Same meaning as in pnorm or qnorm.

## Details

See genrayleigh, the VGAM family function for estimating the parameters, for the formula of the probability density function and other details.

## Value

dgenray gives the density, pgenray gives the distribution function, qgenray gives the quantile function, and rgenray generates random deviates.

## Note

We define scale as the reciprocal of the scale parameter used by Kundu and Raqab (2005).

## Author(s)

Kai Huang and J. G. Lauder and T. W. Yee

## See Also

genrayleigh, rayleigh.

## Examples

```
## Not run:
shape <- 0.5; Scale <- 1; nn <- 501
x <- seq(-0.10, 3.0, len = nn)
plot(x, dgenray(x, shape, scale = Scale), type = "l", las = 1, ylim = c(0, 1.2),
        ylab = paste("[dp]genray(shape = ", shape, ", scale = ", Scale, ")"),
    col = "blue", cex.main = 0.8,
    main = "Blue is density, orange is cumulative distribution function",
    sub = "Purple lines are the 10,20,...,90 percentiles")
lines(x, pgenray(x, shape, scale = Scale), col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qgenray(probs, shape, scale = Scale)
lines(Q, dgenray(Q, shape, scale = Scale), col = "purple", lty = 3, type = "h")
lines(Q, pgenray(Q, shape, scale = Scale), col = "purple", lty = 3, type = "h")
abline(h = probs, col = "purple", lty = 3)
max(abs(pgenray(Q, shape, scale = Scale) - probs)) # Should be 0
## End(Not run)
```

```
genrayleigh
```


## Description

Estimates the two parameters of the generalized Rayleigh distribution by maximum likelihood estimation.

## Usage

$$
\begin{aligned}
& \text { genrayleigh(lscale }=\text { "loglink", lshape }=" l o g l i n k ", ~ \\
& \text { iscale }=\text { NULL, ishape }=\text { NULL, } \\
& \text { tol12 }=1 \mathrm{e}-05, \text { nsimEIM }=300, \text { zero }=2)
\end{aligned}
$$

## Arguments

lscale, lshape Link function for the two positive parameters, scale and shape. See Links for more choices.
iscale, ishape Numeric. Optional initial values for the scale and shape parameters.
nsimEIM, zero See CommonVGAMffArguments.
tol12 Numeric and positive. Tolerance for testing whether the second shape parameter is either 1 or 2 . If so then the working weights need to handle these singularities.

## Details

The generalized Rayleigh distribution has density function

$$
f(y ; b=\text { scale }, s=\text { shape })=\left(2 s y / b^{2}\right) e^{-(y / b)^{2}}\left(1-e^{-(y / b)^{2}}\right)^{s-1}
$$

where $y>0$ and the two parameters, $b$ and $s$, are positive. The mean cannot be expressed nicely so the median is returned as the fitted values. Applications of the generalized Rayleigh distribution include modeling strength data and general lifetime data. Simulated Fisher scoring is implemented.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

We define scale as the reciprocal of the scale parameter used by Kundu and Raqab (2005).

## Author(s)

J. G. Lauder and T. W. Yee

## References

Kundu, D., Raqab, M. C. (2005). Generalized Rayleigh distribution: different methods of estimations. Computational Statistics and Data Analysis, 49, 187-200.

## See Also

dgenray, rayleigh.

## Examples

```
Scale <- exp(1); shape <- exp(1)
rdata <- data.frame(y = rgenray(n = 1000, scale = Scale, shape = shape))
fit <- vglm(y ~ 1, genrayleigh, data = rdata, trace = TRUE)
c(with(rdata, mean(y)), head(fitted(fit), 1))
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

geometric

Geometric (Truncated and Untruncated) Distributions

## Description

Maximum likelihood estimation for the geometric and truncated geometric distributions.

## Usage

geometric(link = "logitlink", expected = TRUE, imethod = 1, iprob = NULL, zero = NULL)
truncgeometric(upper.limit = Inf, link = "logitlink", expected = TRUE, imethod = 1, iprob = NULL, zero = NULL)

## Arguments

link Parameter link function applied to the probability parameter $p$, which lies in the unit interval. See Links for more choices.
expected Logical. Fisher scoring is used if expected = TRUE, else Newton-Raphson.
iprob, imethod, zero
See CommonVGAMffArguments for details.
upper.limit Numeric. Upper values. As a vector, it is recycled across responses first. The default value means both family functions should give the same result.

## Details

A random variable $Y$ has a 1-parameter geometric distribution if $P(Y=y)=p(1-p)^{y}$ for $y=0,1,2, \ldots$. Here, $p$ is the probability of success, and $Y$ is the number of (independent) trials that are fails until a success occurs. Thus the response $Y$ should be a non-negative integer. The mean of $Y$ is $E(Y)=(1-p) / p$ and its variance is $\operatorname{Var}(Y)=(1-p) / p^{2}$. The geometric distribution is a special case of the negative binomial distribution (see negbinomial). The geometric distribution is also a special case of the Borel distribution, which is a Lagrangian distribution. If $Y$ has a geometric distribution with parameter $p$ then $Y+1$ has a positive-geometric distribution with the same parameter. Multiple responses are permitted.
For truncgeometric(), the (upper) truncated geometric distribution can have response integer values from 0 to upper. limit. It has density prob * (1-prob)^y / [1-(1-prob)^(1+upper.limit)].
For a generalized truncated geometric distribution with integer values $L$ to $U$, say, subtract $L$ from the response and feed in $U-L$ as the upper limit.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Author(s)

T. W. Yee. Help from Viet Hoang Quoc is gratefully acknowledged.

## References

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011). Statistical Distributions, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

## See Also

negbinomial, Geometric, betageometric, expgeometric, zageometric, zigeometric, rbetageom, simulate.vlm.

## Examples

```
gdata <- data.frame(x2 = runif(nn <- 1000) - 0.5)
gdata <- transform(gdata, x3 = runif(nn) - 0.5,
    x4 = runif(nn) - 0.5)
gdata <- transform(gdata, eta = -1.0 - 1.0 * x2 + 2.0 * x3)
gdata <- transform(gdata, prob = logitlink(eta, inverse = TRUE))
gdata <- transform(gdata, y1 = rgeom(nn, prob))
with(gdata, table(y1))
fit1 <- vglm(y1 ~ x2 + x3 + x4, geometric, data = gdata, trace = TRUE)
coef(fit1, matrix = TRUE)
summary(fit1)
# Truncated geometric (between 0 and upper.limit)
upper.limit <- 5
tdata <- subset(gdata, y1 <= upper.limit)
nrow(tdata) # Less than nn
fit2 <- vglm(y1 ~ x2 + x3 + x4, truncgeometric(upper.limit),
    data = tdata, trace = TRUE)
coef(fit2, matrix = TRUE)
# Generalized truncated geometric (between lower.limit and upper.limit)
lower.limit <- 1
upper.limit <- 8
gtdata <- subset(gdata, lower.limit <= y1 & y1 <= upper.limit)
with(gtdata, table(y1))
nrow(gtdata) # Less than nn
fit3 <- vglm(y1 - lower.limit ~ x2 + x3 + x4,
    truncgeometric(upper.limit - lower.limit),
    data = gtdata, trace = TRUE)
coef(fit3, matrix = TRUE)
```

```
    get.smart Retrieve One Component of ".smart.prediction"
```


## Description

Retrieve one component of the list .smart.prediction from smartpredenv.

## Usage

get.smart()

## Details

get.smart is used in "read" mode within a smart function: it retrieves parameters saved at the time of fitting, and is used for prediction. get.smart is only used in smart functions such as sm. poly; get.smart.prediction is only used in modelling functions such as 1 m and glm. The function get.smart gets only a part of .smart. prediction whereas get. smart.prediction gets the entire .smart.prediction.

## Value

Returns with one list component of . smart. prediction from smartpredenv, in fact, . smart. prediction[[. smart.predi The whole procedure mimics a first-in first-out stack (better known as a queue).

## Side Effects

The variable .smart.prediction. counter in smartpredenv is incremented beforehand, and then written back to smartpredenv.

## See Also

```
get.smart.prediction.
```


## Examples

```
print(sm.min1)
```

```
get.smart.prediction Retrieves ".smart.prediction"
```


## Description

Retrieves.smart.prediction from smartpredenv.

## Usage

get.smart.prediction()

## Details

A smart modelling function such as $1 m$ allows smart functions such as sm. bs to write to a data structure called .smart.prediction in smartpredenv. At the end of fitting, get.smart.prediction retrieves this data structure. It is then attached to the object, and used for prediction later.

## Value

Returns with the list . smart. prediction from smartpredenv.

## See Also

get.smart, lm.

## Examples

```
## Not run:
fit$smart <- get.smart.prediction() # Put at the end of lm()
## End(Not run)
```

gev

Generalized Extreme Value Regression Family Function

## Description

Maximum likelihood estimation of the 3-parameter generalized extreme value (GEV) distribution.

## Usage

```
gev(llocation = "identitylink", lscale = "loglink",
    lshape = logofflink(offset = 0.5), percentiles = c(95, 99),
    ilocation = NULL, iscale = NULL, ishape = NULL, imethod = 1,
    gprobs.y = (1:9)/10, gscale.mux = exp((-5:5)/6),
    gshape = (-5:5) / 11 + 0.01,
    iprobs.y = NULL, tolshape0 = 0.001,
    type.fitted = c("percentiles", "mean"),
    zero = c("scale", "shape"))
gevff(llocation = "identitylink", lscale = "loglink",
    lshape = logofflink(offset = 0.5), percentiles = c(95, 99),
    ilocation = NULL, iscale = NULL, ishape = NULL, imethod = 1,
    gprobs.y = (1:9)/10, gscale.mux = exp((-5:5)/6),
    gshape = (-5:5) / 11 + 0.01,
    iprobs.y = NULL, tolshape0 = 0.001,
    type.fitted = c("percentiles", "mean"), zero = c("scale", "shape"))
```


## Arguments

llocation, lscale, lshape
Parameter link functions for $\mu, \sigma$ and $\xi$ respectively. See Links for more choices. For the shape parameter, the default logofflink link has an offset called $A$ below; and then the linear/additive predictor is $\log (\xi+A)$ which means that $\xi>-A$. For technical reasons (see Details) it is a good idea for $A=0.5$.
percentiles Numeric vector of percentiles used for the fitted values. Values should be between 0 and 100. This argument is ignored if type.fitted = "mean".
type.fitted See CommonVGAMffArguments for information. The default is to use the percentiles argument. If "mean" is chosen, then the mean $\mu+\sigma(\Gamma(1-\xi)-1) / \xi$ is returned as the fitted values, and these are only defined for $\xi<1$.
ilocation, iscale, ishape
Numeric. Initial value for the location parameter, $\sigma$ and $\xi$. A NULL means a value is computed internally. The argument ishape is more important than the other two. If a failure to converge occurs, or even to obtain initial values occurs, try assigning ishape some value (positive or negative; the sign can be very important). Also, in general, a larger value of iscale tends to be better than a smaller value.
imethod Initialization method. Either the value 1 or 2 . If both methods fail then try using ishape. See CommonVGAMfffarguments for information.
gshape $\quad$ Numeric vector. The values are used for a grid search for an initial value for $\xi$. See CommonVGAMffArguments for information.
gprobs.y, gscale.mux, iprobs.y
Numeric vectors, used for the initial values. See CommonVGAMffArguments for information.
tolshape0 Passed into dgev when computing the log-likelihood.
zero A specifying which linear/additive predictors are modelled as intercepts only. The values can be from the set $\{1,2,3\}$ corresponding respectively to $\mu, \sigma, \xi$.

If zero $=$ NULL then all linear/additive predictors are modelled as a linear combination of the explanatory variables. For many data sets having zero $=3$ is a good idea. See CommonVGAMffArguments for information.

## Details

The GEV distribution function can be written

$$
G(y)=\exp \left(-[(y-\mu) / \sigma]_{+}^{-1 / \xi}\right)
$$

where $\sigma>0,-\infty<\mu<\infty$, and $1+\xi(y-\mu) / \sigma>0$. Here, $x_{+}=\max (x, 0)$. The $\mu, \sigma, \xi$ are known as the location, scale and shape parameters respectively. The cases $\xi>0, \xi<0, \xi=0$ correspond to the Frechet, reverse Weibull, and Gumbel types respectively. It can be noted that the Gumbel (or Type I) distribution accommodates many commonly-used distributions such as the normal, lognormal, logistic, gamma, exponential and Weibull.

For the GEV distribution, the $k$ th moment about the mean exists if $\xi<1 / k$. Provided they exist, the mean and variance are given by $\mu+\sigma\{\Gamma(1-\xi)-1\} / \xi$ and $\sigma^{2}\left\{\Gamma(1-2 \xi)-\Gamma^{2}(1-\xi)\right\} / \xi^{2}$ respectively, where $\Gamma$ is the gamma function.
Smith (1985) established that when $\xi>-0.5$, the maximum likelihood estimators are completely regular. To have some control over the estimated $\xi$ try using lshape $=$ logofflink (offset $=0.5$ ), say, or lshape $=$ extlogitlink (min $=-0.5, \max =0.5)$, say.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

Currently, if an estimate of $\xi$ is too close to 0 then an error may occur for gev() with multivariate responses. In general, gevff() is more reliable than gev().
Fitting the GEV by maximum likelihood estimation can be numerically fraught. If $1+\xi(y-\mu) / \sigma \leq$ 0 then some crude evasive action is taken but the estimation process can still fail. This is particularly the case if vgam with s is used; then smoothing is best done with vglm with regression splines (bs or ns) because vglm implements half-stepsizing whereas vgam doesn't (half-stepsizing helps handle the problem of straying outside the parameter space.)

## Note

The VGAM family function gev can handle a multivariate (matrix) response, cf. multiple responses. If so, each row of the matrix is sorted into descending order and NAs are put last. With a vector or one-column matrix response using gevff will give the same result but be faster and it handles the $\xi=0$ case. The function gev implements Tawn (1988) while gevff implements Prescott and Walden (1980).

Function egev() has been replaced by the new family function gevff(). It now conforms to the usual VGAM philosophy of having M1 linear predictors per (independent) response. This is the usual way multiple responses are handled. Hence $\operatorname{vglm}(\operatorname{cbind}(\mathrm{y} 1, \mathrm{y} 2) \ldots$, gevff, ...) will have 6 linear predictors and it is possible to constrain the linear predictors so that the answer is similar
to $\operatorname{gev}()$. Missing values in the response of $\operatorname{gevff}()$ will be deleted; this behaviour is the same as with almost every other VGAM family function.
The shape parameter $\xi$ is difficult to estimate accurately unless there is a lot of data. Convergence is slow when $\xi$ is near -0.5 . Given many explanatory variables, it is often a good idea to make sure zero $=3$. The range restrictions of the parameter $\xi$ are not enforced; thus it is possible for a violation to occur.

Successful convergence often depends on having a reasonably good initial value for $\xi$. If failure occurs try various values for the argument ishape, and if there are covariates, having zero $=3$ is advised.

## Author(s)

T. W. Yee

## References

Yee, T. W. and Stephenson, A. G. (2007). Vector generalized linear and additive extreme value models. Extremes, 10, 1-19.
Tawn, J. A. (1988). An extreme-value theory model for dependent observations. Journal of Hydrology, 101, 227-250.
Prescott, P. and Walden, A. T. (1980). Maximum likelihood estimation of the parameters of the generalized extreme-value distribution. Biometrika, 67, 723-724.
Smith, R. L. (1985). Maximum likelihood estimation in a class of nonregular cases. Biometrika, 72, 67-90.

## See Also

rgev, gumbel, gumbelff, guplot, rlplot.gevff, gpd, weibullR, frechet, extlogitlink, oxtemp, venice, CommonVGAMffArguments.

## Examples

```
## Not run:
# Multivariate example
fit1 <- vgam(cbind(r1, r2) ~ s(year, df = 3), gev(zero = 2:3),
            data = venice, trace = TRUE)
coef(fit1, matrix = TRUE)
head(fitted(fit1))
par(mfrow = c(1, 2), las = 1)
plot(fit1, se = TRUE, lcol = "blue", scol = "forestgreen",
    main = "Fitted mu(year) function (centered)", cex.main = 0.8)
with(venice, matplot(year, depvar(fit1)[, 1:2], ylab = "Sea level (cm)",
    col = 1:2, main = "Highest 2 annual sea levels", cex.main = 0.8))
with(venice, lines(year, fitted(fit1)[,1], lty = "dashed", col = "blue"))
legend("topleft", lty = "dashed", col = "blue", "Fitted 95 percentile")
# Univariate example
(fit <- vglm(maxtemp ~ 1, gevff, data = oxtemp, trace = TRUE))
head(fitted(fit))
```

```
coef(fit, matrix = TRUE)
Coef(fit)
vcov(fit)
vcov(fit, untransform = TRUE)
sqrt(diag(vcov(fit))) # Approximate standard errors
rlplot(fit)
## End(Not run)
```

gevUC The Generalized Extreme Value Distribution

## Description

Density, distribution function, quantile function and random generation for the generalized extreme value distribution (GEV) with location parameter location, scale parameter scale and shape parameter shape.

## Usage

```
dgev(x, location = 0, scale = 1, shape = 0, log = FALSE,
        tolshape0 = sqrt(.Machine$double.eps))
    pgev(q, location = 0, scale = 1, shape = 0, lower.tail = TRUE, log.p = FALSE)
    qgev(p, location = 0, scale = 1, shape = 0, lower.tail = TRUE, log.p = FALSE)
    rgev(n, location = 0, scale = 1, shape = 0)
```


## Arguments

| $\mathrm{x}, \mathrm{q}$ | vector of quantiles. |
| :---: | :---: |
| p | vector of probabilities. |
| n | number of observations. If length $(n)>1$ then the length is taken to be the number required. |
| location | the location parameter $\mu$. |
| scale | the (positive) scale parameter $\sigma$. Must consist of positive values. |
| shape | the shape parameter $\xi$. |
| log | Logical. If log = TRUE then the logarithm of the density is returned. |
| lower.tail, log.p |  |
|  | Same meaning as in punif or qunif. |
| tolshape0 | Positive numeric. Threshold/tolerance value for resting whether $\xi$ is zero. If the absolute value of the estimate of $\xi$ is less than this value then it will be assumed zero and a Gumbel distribution will be used. |

## Details

See gev, the VGAM family function for estimating the 3 parameters by maximum likelihood estimation, for formulae and other details. Apart from n, all the above arguments may be vectors and are recyled to the appropriate length if necessary.

## Value

dgev gives the density, pgev gives the distribution function, qgev gives the quantile function, and rgev generates random deviates.

Note
The default value of $\xi=0$ means the default distribution is the Gumbel.
Currently, these functions have different argument names compared with those in the evd package.

## Author(s)

T. W. Yee

## References

Coles, S. (2001). An Introduction to Statistical Modeling of Extreme Values. London: SpringerVerlag.

## See Also

gev, gevff, vglm. control.

## Examples

```
    loc <- 2; sigma <- 1; xi <- -0.4
    pgev(qgev(seq(0.05, 0.95, by = 0.05), loc, sigma, xi), loc, sigma, xi)
    ## Not run: x <- seq(loc - 3, loc + 3, by = 0.01)
    plot(x, dgev(x, loc, sigma, xi), type = "l", col = "blue", ylim = c(0, 1),
        main = "Blue is density, orange is cumulative distribution function",
        sub = "Purple are 10,...,90 percentiles", ylab = "", las = 1)
    abline(h = 0, col = "blue", lty = 2)
    lines(qgev(seq(0.1, 0.9, by = 0.1), loc, sigma, xi),
        dgev(qgev(seq(0.1, 0.9, by = 0.1), loc, sigma, xi), loc, sigma, xi),
        col = "purple", lty = 3, type = "h")
    lines(x, pgev(x, loc, sigma, xi), type = "l", col = "orange")
    abline(h = (0:10)/10, lty = 2, col = "gray50")
    ## End(Not run)
```

gew
General Electric and Westinghouse Data

## Description

General Electric and Westinghouse capital data.

## Usage

data(gew)

## Format

A data frame with 20 observations on the following 7 variables. All variables are numeric vectors. Variables ending in .g correspond to General Electric and those ending in .w are Westinghouse.
year The observations are the years from 1934 to 1953
invest.g, invest.w investment figures. These are $I=$ Gross investment $=$ additions to plant and equipment plus maintenance and repairs in millions of dollars deflated by $P_{1}$.
capital.g, capital.w capital stocks. These are $C=$ The stock of plant and equipment $=$ accumulated sum of net additions to plant and equipment deflated by $P_{1}$ minus depreciation allowance deflated by $P_{3}$.
value.g, value.w market values. These are $F=$ Value of the firm $=$ price of common and preferred shares at December 31 (or average price of December 31 and January 31 of the following year) times number of common and preferred shares outstanding plus total book value of debt at December 31 in millions of dollars deflated by $P_{2}$.

## Details

These data are a subset of a table in Boot and de Wit (1960), also known as the Grunfeld data. It is used a lot in econometrics, e.g., for seemingly unrelated regressions (see SURff).
Here, $P_{1}=$ Implicit price deflator of producers durable equipment (base 1947), $P_{2}=$ Implicit price deflator of G.N.P. (base 1947), $P_{3}=$ Depreciation expense deflator $=$ ten years moving average of wholesale price index of metals and metal products (base 1947).

## Source

Table 10 of: Boot, J. C. G. and de Wit, G. M. (1960) Investment Demand: An Empirical Contribution to the Aggregation Problem. International Economic Review, 1, 3-30.
Grunfeld, Y. (1958) The Determinants of Corporate Investment. Unpublished PhD Thesis (Chicago).

## References

Zellner, A. (1962). An efficient method of estimating seemingly unrelated regressions and tests for aggregation bias. Journal of the American Statistical Association, 57, 348-368.

## See Also

SURff, http://statmath.wu.ac.at/~zeileis/grunfeld (the link might now be stale).

## Examples

```
str(gew)
```


## Description

Density, cumulative distribution function, quantile function and random generation for the Gompertz distribution.

## Usage

dgompertz(x, scale $=1$, shape, log = FALSE)
pgompertz(q, scale = 1, shape, lower.tail = TRUE, log.p = FALSE)
qgompertz( p , scale = 1, shape, lower.tail = TRUE, log.p = FALSE)
rgompertz(n, scale = 1, shape)

## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$n \quad$ number of observations. Same as in runif.
log Logical. If log = TRUE then the logarithm of the density is returned.
lower.tail, log.p
Same meaning as in pnorm or qnorm.
scale, shape positive scale and shape parameters.

## Details

See gompertz for details.

## Value

dgompertz gives the density, pgompertz gives the cumulative distribution function, qgompertz gives the quantile function, and rgompertz generates random deviates.

## Author(s)

T. W. Yee and Kai Huang

## See Also

gompertz, dgumbel, dmakeham.

## Examples

```
    probs <- seq(0.01, 0.99, by = 0.01)
    Shape <- exp(1); Scale <- exp(1)
    max(abs(pgompertz(qgompertz(p = probs, Scale, shape = Shape),
                Scale, shape = Shape) - probs)) # Should be 0
    ## Not run: x <- seq(-0.1, 1.0, by = 0.001)
    plot(x, dgompertz(x, Scale,shape = Shape), type = "l", col = "blue", las = 1,
    main = "Blue is density, orange is cumulative distribution function",
    sub = "Purple lines are the 10,20,\ldots.,90 percentiles",
    ylab = "")
abline(h = 0, col = "blue", lty = 2)
lines(x, pgompertz(x, Scale, shape = Shape), col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qgompertz(probs, Scale, shape = Shape)
lines(Q, dgompertz(Q, Scale, shape = Shape), col = "purple",
    lty = 3, type = "h")
pgompertz(Q, Scale, shape = Shape) - probs # Should be all zero
abline(h = probs, col = "purple", lty = 3)
## End(Not run)
```

```
gompertz Gompertz Regression Family Function
```


## Description

Maximum likelihood estimation of the 2-parameter Gompertz distribution.

## Usage

```
gompertz(lscale = "loglink", lshape = "loglink",
                iscale = NULL, ishape = NULL,
                nsimEIM = 500, zero = NULL, nowarning = FALSE)
```


## Arguments

nowarning Logical. Suppress a warning? Ignored for VGAM 0.9-7 and higher.
lshape, lscale Parameter link functions applied to the shape parameter a, scale parameter scale. All parameters are positive. See Links for more choices.
ishape, iscale Optional initial values. A NULL means a value is computed internally.
nsimEIM, zero See CommonVGAMffArguments.

## Details

The Gompertz distribution has a cumulative distribution function

$$
F(x ; \alpha, \beta)=1-\exp [-(\alpha / \beta) \times(\exp (\beta x)-1)]
$$

which leads to a probability density function

$$
f(x ; \alpha, \beta)=\alpha \exp (\beta x) \exp [-(\alpha / \beta) \times(\exp (\beta x)-1)]
$$

for $\alpha>0, \beta>0, x>0$. Here, $\beta$ is called the scale parameter scale, and $\alpha$ is called the shape parameter (one could refer to $\alpha$ as a location parameter and $\beta$ as a shape parameter-see Lenart (2012)). The mean is involves an exponential integral function. Simulated Fisher scoring is used and multiple responses are handled.
The Makeham distibution has an additional parameter compared to the Gompertz distribution. If $X$ is defined to be the result of sampling from a Gumbel distribution until a negative value $Z$ is produced, then $X=-Z$ has a Gompertz distribution.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

The same warnings in makeham apply here too.

## Author(s)

T. W. Yee

## References

Lenart, A. (2012). The moments of the Gompertz distribution and maximum likelihood estimation of its parameters. Scandinavian Actuarial Journal, in press.

## See Also

dgompertz, makeham, simulate.vlm.

## Examples

```
## Not run:
gdata <- data.frame(x2 = runif(nn <- 1000))
gdata <- transform(gdata, eta1 = -1,
    eta2 = -1 + 0.2* x2,
    ceta1 = 1,
    ceta2 = -1 + 0.2 * x2)
gdata <- transform(gdata, shape1 = exp(eta1),
    shape2 = exp(eta2),
    scale1 = exp(ceta1),
    scale2 = exp(ceta2))
gdata <- transform(gdata, y1 = rgompertz(nn, scale = scale1, shape = shape1),
    y2 = rgompertz(nn, scale = scale2, shape = shape2))
fit1 <- vglm(y1 ~ 1, gompertz, data = gdata, trace = TRUE)
fit2 <- vglm(y2 ~ x2, gompertz, data = gdata, trace = TRUE)
```

```
    coef(fit1, matrix = TRUE)
    Coef(fit1)
    summary(fit1)
    coef(fit2, matrix = TRUE)
    summary(fit2)
    ## End(Not run)
```

    gordlink
    
## Description

Computes the gamma-ordinal transformation, including its inverse and the first two derivatives.

## Usage

gordlink(theta, lambda = 1, cutpoint = NULL, inverse = FALSE, deriv $=0$, short $=$ TRUE, tag $=$ FALSE)

## Arguments

theta Numeric or character. See below for further details.
lambda, cutpoint
The former is the shape parameter in gamma2. cutpoint is optional; if NULL then cutpoint is ignored from the GOLF definition. If given, the cutpoints should be non-negative integers. If gordlink() is used as the link function in cumulative then, if the cutpoints are known, then one should choose reverse $=$ TRUE, parallel = FALSE ~ -1 . If the cutpoints are unknown, then choose reverse = TRUE, parallel = TRUE.
inverse, deriv, short, tag
Details at Links.

## Details

The gamma-ordinal link function (GOLF) can be applied to a parameter lying in the unit interval. Its purpose is to link cumulative probabilities associated with an ordinal response coming from an underlying 2-parameter gamma distribution.
See Links for general information about VGAM link functions.

## Value

See Yee (2019) for details.

## Warning

Prediction may not work on vglm or vgam etc. objects if this link function is used.

## Note

Numerical values of theta too close to 0 or 1 or out of range result in large positive or negative values, or maybe 0 depending on the arguments. Although measures have been taken to handle cases where theta is too close to 1 or 0 , numerical instabilities may still arise.
In terms of the threshold approach with cumulative probabilities for an ordinal response this link function corresponds to the gamma distribution (see gamma2) that has been recorded as an ordinal response using known cutpoints.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2020). Ordinal ordination with normalizing link functions for count data, (in preparation).

## See Also

Links, gamma2, pordlink, nbordlink, cumulative.

## Examples

```
## Not run:
gordlink("p", lambda = 1, short = FALSE)
gordlink("p", lambda = 1, tag = TRUE)
p <- seq(0.02, 0.98, len = 201)
y <- gordlink(p, lambda = 1)
y. <- gordlink(p, lambda = 1, deriv = 1, inverse = TRUE)
max(abs(gordlink(y, lambda = 1, inverse = TRUE) - p)) # Should be 0
#\ dontrun{par(mfrow = c(2, 1), las = 1)
#plot(p, y, type = "l", col = "blue", main = "gordlink()")
#abline(h = 0, v = 0.5, col = "orange", lty = "dashed")
#plot(p, y., type = "l", col = "blue",
# main = "(Reciprocal of) first GOLF derivative")
#}
# Another example
gdata <- data.frame(x2 = sort(runif(nn <- 1000)))
gdata <- transform(gdata, x3 = runif(nn))
gdata <- transform(gdata, mymu = exp( 3 + 1 * x2 - 2 * x3))
lambda <- 4
gdata <- transform(gdata,
                            y1 = rgamma(nn, shape = lambda, scale = mymu / lambda))
cutpoints <- c(-Inf, 10, 20, Inf)
gdata <- transform(gdata, cuty = Cut(y1, breaks = cutpoints))
#\ dontrun{ par(mfrow = c(1, 1), las = 1)
#with(gdata, plot(x2, x3, col = cuty, pch = as.character(cuty))) }
```

```
with(gdata, table(cuty) / sum(table(cuty)))
fit <- vglm(cuty ~ x2 + x3, cumulative(multiple.responses = TRUE,
    reverse = TRUE, parallel = FALSE ~ -1,
    link = gordlink(cutpoint = cutpoints[2:3], lambda = lambda)),
    data = gdata, trace = TRUE)
    head(depvar(fit))
    head(fitted(fit))
    head(predict(fit))
    coef(fit)
    coef(fit, matrix = TRUE)
    constraints(fit)
    fit@misc
    ## End(Not run)
```

    gpd Generalized Pareto Distribution Regression Family Function
    
## Description

Maximum likelihood estimation of the 2-parameter generalized Pareto distribution (GPD).

## Usage

gpd(threshold = 0, lscale = "loglink", lshape = logofflink(offset = 0.5), percentiles $=c(90,95)$, iscale $=$ NULL, ishape $=$ NULL, tolshape0 = 0.001, type.fitted = c("percentiles", "mean"), imethod = 1, zero = "shape")

## Arguments

threshold Numeric, values are recycled if necessary. The threshold value(s), called $\mu$ below.
lscale Parameter link function for the scale parameter $\sigma$. See Links for more choices.
lshape Parameter link function for the shape parameter $\xi$. See Links for more choices. The default constrains the parameter to be greater than -0.5 because if $\xi \leq$ -0.5 then Fisher scoring does not work. See the Details section below for more information.
For the shape parameter, the default logofflink link has an offset called $A$ below; and then the second linear/additive predictor is $\log (\xi+A)$ which means that $\xi>-A$. The working weight matrices are positive definite if $A=0.5$.
percentiles Numeric vector of percentiles used for the fitted values. Values should be between 0 and 100. See the example below for illustration. This argument is ignored if type.fitted = "mean".
type.fitted See CommonVGAMffArguments for information. The default is to use the percentiles argument. If "mean" is chosen, then the mean $\mu+\sigma /(1-\xi)$ is returned as the fitted values, and these are only defined for $\xi<1$.
iscale, ishape Numeric. Optional initial values for $\sigma$ and $\xi$. The default is to use imethod and compute a value internally for each parameter. Values of ishape should be between -0.5 and 1. Values of iscale should be positive.
tolshape0 Passed into dgpd when computing the log-likelihood.
imethod Method of initialization, either 1 or 2. The first is the method of moments, and the second is a variant of this. If neither work, try assigning values to arguments ishape and/or iscale.
zero Can be an integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. For one response, the value should be from the set $\{1,2\}$ corresponding respectively to $\sigma$ and $\xi$. It is often a good idea for the $\sigma$ parameter only to be modelled through a linear combination of the explanatory variables because the shape parameter is probably best left as an intercept only: zero $=2$. Setting zero $=$ NULL means both parameters are modelled with explanatory variables. See CommonVGAMffArguments for more details.

## Details

The distribution function of the GPD can be written

$$
G(y)=1-[1+\xi(y-\mu) / \sigma]_{+}^{-1 / \xi}
$$

where $\mu$ is the location parameter (known, with value threshold), $\sigma>0$ is the scale parameter, $\xi$ is the shape parameter, and $h_{+}=\max (h, 0)$. The function $1-G$ is known as the survivor function. The limit $\xi \rightarrow 0$ gives the shifted exponential as a special case:

$$
G(y)=1-\exp [-(y-\mu) / \sigma]
$$

The support is $y>\mu$ for $\xi>0$, and $\mu<y<\mu-\sigma / \xi$ for $\xi<0$.
Smith (1985) showed that if $\xi<=-0.5$ then this is known as the nonregular case and problems/difficulties can arise both theoretically and numerically. For the (regular) case $\xi>-0.5$ the classical asymptotic theory of maximum likelihood estimators is applicable; this is the default.

Although for $\xi<-0.5$ the usual asymptotic properties do not apply, the maximum likelihood estimator generally exists and is superefficient for $-1<\xi<-0.5$, so it is "better" than normal. When $\xi<-1$ the maximum likelihood estimator generally does not exist as it effectively becomes a two parameter problem.
The mean of $Y$ does not exist unless $\xi<1$, and the variance does not exist unless $\xi<0.5$. So if you want to fit a model with finite variance use lshape = "extlogitlink".

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam. However, for this VGAM family function, vglm is probably preferred over vgam when there is smoothing.

## Warning

Fitting the GPD by maximum likelihood estimation can be numerically fraught. If $1+\xi(y-\mu) / \sigma \leq$ 0 then some crude evasive action is taken but the estimation process can still fail. This is particularly
the case if vgam with $s$ is used. Then smoothing is best done with vglm with regression splines (bs or $n s$ ) because vglm implements half-stepsizing whereas vgam doesn't. Half-stepsizing helps handle the problem of straying outside the parameter space.

## Note

The response in the formula of vglm and vgam is $y$. Internally, $y-\mu$ is computed. This VGAM family function can handle a multiple responses, which is inputted as a matrix. The response stored on the object is the original uncentred data.
With functions rgpd, dgpd, etc., the argument location matches with the argument threshold here.

## Author(s)

T. W. Yee

## References

Yee, T. W. and Stephenson, A. G. (2007). Vector generalized linear and additive extreme value models. Extremes, 10, 1-19.

Coles, S. (2001). An Introduction to Statistical Modeling of Extreme Values. London: SpringerVerlag.
Smith, R. L. (1985). Maximum likelihood estimation in a class of nonregular cases. Biometrika, 72, 67-90.

## See Also

rgpd, meplot, gev, paretoff, vglm, vgam, s.

## Examples

```
# Simulated data from an exponential distribution (xi = 0)
Threshold <- 0.5
gdata <- data.frame(y1 = Threshold + rexp(n = 3000, rate = 2))
fit <- vglm(y1 ~ 1, gpd(threshold = Threshold), data = gdata, trace = TRUE)
head(fitted(fit))
summary(depvar(fit)) # The original uncentred data
coef(fit, matrix = TRUE) # xi should be close to 0
Coef(fit)
summary(fit)
head(fit@extra$threshold) # Note the threshold is stored here
# Check the 90 percentile
ii <- depvar(fit) < fitted(fit)[1, "90%"]
100 * table(ii) / sum(table(ii)) # Should be 90%
# Check the 95 percentile
ii <- depvar(fit) < fitted(fit)[1, "95%"]
100 * table(ii) / sum(table(ii)) # Should be 95%
```

```
    ## Not run: plot(depvar(fit), col = "blue", las = 1,
    main = "Fitted 90% and 95% quantiles")
    matlines(1:length(depvar(fit)), fitted(fit), lty = 2:3, lwd = 2)
    ## End(Not run)
    # Another example
    gdata <- data.frame(x2 = runif(nn <- 2000))
    Threshold <- 0; xi <- exp(-0.8) - 0.5
    gdata <- transform(gdata, y2 = rgpd(nn, scale = exp(1 + 0.1*x2), shape = xi))
    fit <- vglm(y2 ~ x2, gpd(Threshold), data = gdata, trace = TRUE)
    coef(fit, matrix = TRUE)
    ## Not run: # Nonparametric fits
    # Not so recommended:
    fit1 <- vgam(y2 ~ s(x2), gpd(Threshold), data = gdata, trace = TRUE)
    par(mfrow = c(2, 1))
    plot(fit1, se = TRUE, scol = "blue")
    # More recommended:
    fit2 <- vglm(y2 ~ sm.bs(x2), gpd(Threshold), data = gdata, trace = TRUE)
    plot(as(fit2, "vgam"), se = TRUE, scol = "blue")
## End(Not run)
```

gpdUC The Generalized Pareto Distribution

## Description

Density, distribution function, quantile function and random generation for the generalized Pareto distribution (GPD) with location parameter location, scale parameter scale and shape parameter shape.

## Usage

dgpd(x, location $=0$, scale $=1$, shape $=0$, log $=$ FALSE, tolshape0 $=$ sqrt(.Machine\$double.eps))
$\operatorname{pgpd}(q$, location $=0$, scale $=1$, shape $=0$, lower.tail = TRUE, log.p = FALSE)
$\operatorname{qgpd}(\mathrm{p}$, location $=0$, scale $=1$, shape $=0$, lower.tail = TRUE, log.p = FALSE)
$\operatorname{rgpd}(\mathrm{n}$, location $=0$, scale $=1$, shape $=0)$

## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$n \quad$ number of observations. If length $(n)>1$ then the length is taken to be the number required.
location the location parameter $\mu$.
scale the (positive) scale parameter $\sigma$.
shape the shape parameter $\xi$.
$\log \quad$ Logical. If $\log =$ TRUE then the logarithm of the density is returned.
lower.tail, log.p
Same meaning as in punif or qunif.
tolshape0 Positive numeric. Threshold/tolerance value for resting whether $\xi$ is zero. If the absolute value of the estimate of $\xi$ is less than this value then it will be assumed zero and an exponential distribution will be used.

## Details

See gpd, the VGAM family function for estimating the two parameters by maximum likelihood estimation, for formulae and other details. Apart from $n$, all the above arguments may be vectors and are recyled to the appropriate length if necessary.

## Value

dgpd gives the density, pgpd gives the distribution function, qgpd gives the quantile function, and rgpd generates random deviates.

## Note

The default values of all three parameters, especially $\xi=0$, means the default distribution is the exponential.

Currently, these functions have different argument names compared with those in the evd package.

## Author(s)

T. W. Yee and Kai Huang

## References

Coles, S. (2001). An Introduction to Statistical Modeling of Extreme Values. London: SpringerVerlag.

## See Also

gpd, Exponential.

## Examples

```
## Not run: loc <- 2; sigma <- 1; xi <- -0.4
x <- seq(loc - 0.2, loc + 3, by = 0.01)
plot(x, dgpd(x, loc, sigma, xi), type = "l", col = "blue", ylim = c(0, 1),
    main = "Blue is density, red is cumulative distribution function",
    sub = "Purple are 5,10,...,95 percentiles", ylab = "", las = 1)
abline(h = 0, col = "blue", lty = 2)
lines(qgpd(seq(0.05, 0.95, by = 0.05), loc, sigma, xi),
    dgpd(qgpd(seq(0.05, 0.95, by = 0.05), loc, sigma, xi), loc, sigma, xi),
    col = "purple", lty = 3, type = "h")
lines(x, pgpd(x, loc, sigma, xi), type = "l", col = "red")
abline(h = 0, lty = 2)
pgpd(qgpd(seq(0.05, 0.95, by = 0.05), loc, sigma, xi), loc, sigma, xi)
## End(Not run)
```

grain.us Grain Prices Data in USA

## Description

A 4-column matrix.

## Usage

data(grain.us)

## Format

The columns are:
wheat.flour numeric
corn numeric
wheat numeric
rye numeric

## Details

Monthly averages of grain prices in the United States for wheat flour, corn, wheat, and rye for the period January 1961 through October 1972. The units are US dollars per 100 pound sack for wheat flour, and per bushel for corn, wheat and rye.

## Source

Ahn and Reinsel (1988).

## References

Ahn, S. K and Reinsel, G. C. (1988). Nested reduced-rank autoregressive models for multiple time series. Journal of the American Statistical Association, 83, 849-856.

## Examples

```
## Not run:
cgrain <- scale(grain.us, scale = FALSE) # Center the time series only
fit <- vglm(cgrain ~ 1, rrar(Rank = c(4, 1)),
    epsilon = 1e-3, stepsize = 0.5, trace = TRUE, maxit = 50)
summary(fit)
## End(Not run)
```

grc Row-Column Interaction Models including Goodman's RC Associa- tion Model and Unconstrained Quadratic Ordination

## Description

Fits a Goodman's RC association model (GRC) to a matrix of counts, and more generally, rowcolumn interaction models (RCIMs). RCIMs allow for unconstrained quadratic ordination (UQO).

## Usage

```
grc(y, Rank = 1, Index.corner = 2:(1 + Rank),
    str0 = 1, summary.arg = FALSE, h.step = 1e-04, ...)
rcim(y, family = poissonff, Rank = 0, M1 = NULL,
    weights = NULL, which.linpred = 1,
    Index.corner = ifelse(is.null(str0), 0, max(str0)) + 1:Rank,
    rprefix = "Row.", cprefix = "Col.", iprefix = "X2.",
    offset = 0, str0 = if (Rank) 1 else NULL,
    summary.arg = FALSE, h.step = 0.0001,
    rbaseline = 1, cbaseline = 1,
    has.intercept = TRUE, M = NULL,
    rindex = 2:nrow(y), cindex = 2:ncol(y), iindex = 2:nrow(y), ...)
```


## Arguments

$y \quad$ For $\operatorname{grc}()$ : a matrix of counts. For $\operatorname{rcim}()$ : a general matrix response depending on family. Output from table() is acceptable; it is converted into a matrix. Note that $y$ should be at least 3 by 3 in dimension.
family A VGAM family function. By default, the first linear/additive predictor is fitted using main effects plus an optional rank-Rank interaction term. Not all family functions are suitable or make sense. All other linear/additive predictors are fitted using an intercept-only, so it has a common value over all rows and columns.

For example, zipoissonff may be suitable for counts but not zipoisson because of the ordering of the linear/additive predictors. If the VGAM family function does not have an infos slot then M1 needs to be inputted (the number of linear predictors for an ordinary (usually univariate) response, aka $M$ ). The VGAM family function also needs to be able to handle multiple responses (currently not all of them can do this).
$\begin{array}{ll}\text { Rank } & \begin{array}{l}\text { An integer from the set }\{0, \ldots, \min (\operatorname{nrow}(\mathrm{y}), \mathrm{ncol}(\mathrm{y}))\} \text {. This is the dimension } \\ \text { of the fit in terms of the interaction. For } \operatorname{grc}() \text { this argument must be positive. A } \\ \text { value of } 0 \text { means no interactions (i.e., main effects only); each row and column } \\ \text { is represented by an indicator variable. }\end{array} \\ \text { weights } & \begin{array}{l}\text { Prior weights. Fed into rrvglm or vglm. }\end{array} \\ \text { which.linpred } & \begin{array}{l}\text { Single integer. Specifies which linear predictor is modelled as the sum of an } \\ \text { intercept, row effect, column effect plus an optional interaction term. It should } \\ \text { be one value from the set 1:M1. }\end{array} \\ \text { Index.corner } & \begin{array}{l}\text { A vector of Rank integers. These are used to store the Rank by Rank identity } \\ \text { matrix in the A matrix; corner constraints are used }\end{array}\end{array}$ rprefix, cprefix, iprefix

Character, for rows and columns and interactions respectively. For labelling the indicator variables.
offset Numeric. Either a matrix of the right dimension, else a single numeric expanded into such a matrix.
str0 Ignored if Rank $=0$, else an integer from the set $\{1, \ldots, \min (\operatorname{nrow}(y), \operatorname{ncol}(y))\}$, specifying the row that is used as the structural zero. Passed into rrvglm. control if Rank >0. Set str0 $=$ NULL for none .
summary.arg Logical. If TRUE then a summary is returned. If TRUE then y may be the output (fitted object) of grc().
h.step A small positive value that is passed into summary.rrvglm(). Only used when summary. arg = TRUE.
$\ldots \quad$ Arguments that are passed into rrvglm.control().
M1 The number of linear predictors of the VGAM family function for an ordinary (univariate) response. Then the number of linear predictors of the rcim() fit is usually the number of columns of y multiplied by M1. The default is to evaluate the infos slot of the VGAM family function to try to evaluate it; see $\mathrm{vglmff}-\mathrm{class}$. If this information is not yet supplied by the family function then the value needs to be inputted manually using this argument.
rbaseline, cbaseline
Baseline reference levels for the rows and columns. Currently stored on the object but not used.
has.intercept Logical. Include an intercept?
$M$, cindex $\quad M$ is the usual VGAM $M$, viz. the number of linear/additive predictors in total. Also, cindex means column index, and these point to the columns of $y$ which are part of the vector of linear/additive predictor main effects.
For family = multinomial it is necessary to input these arguments as $M=\operatorname{ncol}(y)-1$ and cindex $=2:(n \operatorname{col}(y)-1)$.
rindex, iindex rindex means row index, and these are similar to cindex. iindex means interaction index, and these are similar to cindex.

## Details

Goodman's RC association model fits a reduced-rank approximation to a table of counts. A Poisson model is assumed. The log of each cell mean is decomposed as an intercept plus a row effect plus a column effect plus a reduced-rank component. The latter can be collectively written $A \% * \% t(C)$, the product of two 'thin' matrices. Indeed, A and C have Rank columns. By default, the first column and row of the interaction matrix $\mathrm{A} \% * \% \mathrm{t}(\mathrm{C})$ is chosen to be structural zeros, because str0 $=1$. This means the first row of $A$ are all zeros.
This function uses options()\$contrasts to set up the row and column indicator variables. In particular, Equation (4.5) of Yee and Hastie (2003) is used. These are called Row. and Col. (by default) followed by the row or column number.

The function $\operatorname{rcim}()$ is more general than $\operatorname{grc}()$. Its default is a no-interaction model of $\operatorname{grc}()$, i.e., rank-0 and a Poisson distribution. This means that each row and column has a dummy variable associated with it. The first row and first column are baseline. The power of rcim() is that many VGAM family functions can be assigned to its family argument. For example, uninormal fits something in between a 2-way ANOVA with and without interactions, alaplace2 with Rank $=0$ is something like medpolish. Others include zipoissonff and negbinomial. Hopefully one day all VGAM family functions will work when assigned to the family argument, although the result may not have meaning.

Unconstrained quadratic ordination (UQO) can be performed using rcim() and grc(). This has been called unconstrained Gaussian ordination in the literature, however the word Gaussian has two meanings which is confusing; it is better to use quadratic because the bell-shape response surface is meant. UQO is similar to CQO (cqo) except there are no environmental/explanatory variables. Here, a GLM is fitted to each column (species) that is a quadratic function of hypothetical latent variables or gradients. Thus each row of the response has an associated site score, and each column of the response has an associated optimum and tolerance matrix. UQO can be performed here under the assumption that all species have the same tolerance matrices. See Yee and Hadi (2014) for details. It is not recommended that presence/absence data be inputted because the information content is so low for each site-species cell. The example below uses Poisson counts.

## Value

An object of class "grc", which currently is the same as an "rrvglm" object. Currently, a rank-0 rcim() object is of class rcim0-class, else of class "rcim" (this may change in the future).

## Warning

The function $\operatorname{rcim}()$ is experimental at this stage and may have bugs. Quite a lot of expertise is needed when fitting and in its interpretion thereof. For example, the constraint matrices applies the reduced-rank regression to the first (see which.linpred) linear predictor and the other linear predictors are intercept-only and have a common value throughout the entire data set. This means that, by default, family = zipoissonff is appropriate but not family = zipoisson. Else set family $=$ zipoisson and which.linpred $=2$. To understand what is going on, do examine the constraint matrices of the fitted object, and reconcile this with Equations (4.3) to (4.5) of Yee and Hastie (2003).

The functions temporarily create a permanent data frame called .grc.df or .rcim.df, which used to be needed by summary. $r \mathrm{rvglm}()$. Then these data frames are deleted before exiting the function. If an error occurs then the data frames may be present in the workspace.

## Note

These functions set up the indicator variables etc. before calling rrvglm or vglm. The . . . is passed into rrvglm. control or vglm. control, This means, e.g., Rank $=1$ is default for $\operatorname{grc}()$.
The data should be labelled with rownames and colnames. Setting trace = TRUE is recommended to monitor convergence. Using criterion = "coefficients" can result in slow convergence.
If summary = TRUE then y can be a "grc" object, in which case a summary can be returned. That is, $\operatorname{grc}(y$, summary $=$ TRUE $)$ is equivalent to summary $(\operatorname{grc}(y))$. It is not possible to plot a grc ( $y$, summary $=$ TRUE $)$ or rcim( $y$, summary $=$ TRUE) object.

## Author(s)

Thomas W. Yee, with assistance from Alfian F. Hadi.

## References

Yee, T. W. and Hastie, T. J. (2003). Reduced-rank vector generalized linear models. Statistical Modelling, 3, 15-41.
Yee, T. W. and Hadi, A. F. (2014). Row-column interaction models, with an R implementation. Computational Statistics, 29, 1427-1445.
Goodman, L. A. (1981). Association models and canonical correlation in the analysis of crossclassifications having ordered categories. Journal of the American Statistical Association, 76, 320334.

## See Also

rrvglm, rrvglm. control, rrvglm-class, summary.grc, moffset, Rcim, Select, Qvar, plotrcim0, cqo, multinomial, alcoff, crashi, auuc, olym08, olym12, poissonff, medpolish.

## Examples

```
# Example 1: Undergraduate enrolments at Auckland University in 1990
fitted(grc1 <- grc(auuc))
summary(grc1)
grc2 <- grc(auuc, Rank = 2, Index.corner = c(2, 5))
fitted(grc2)
summary(grc2)
model3 <- rcim(auuc, Rank = 1, fam = multinomial,
                        M = ncol(auuc)-1, cindex = 2:(ncol(auuc)-1), trace = TRUE)
fitted(model3)
summary(model3)
# Median polish but not 100 percent reliable. Maybe call alaplace2()...
## Not run:
rcim0 <- rcim(auuc, fam = alaplace1(tau = 0.5), trace=FALSE, maxit = 500)
round(fitted(rcim0), digits = 0)
round(100 * (fitted(rcim0) - auuc) / auuc, digits = 0) # Discrepancy
depvar(rcim0)
```

```
round(coef(rcim0, matrix = TRUE), digits = 2)
Coef(rcim0, matrix = TRUE)
# constraints(rcim0)
names(constraints(rcim0))
# Compare with medpolish():
(med.a <- medpolish(auuc))
fv <- med.a$overall + outer(med.a$row, med.a$col, "+")
round(100 * (fitted(rcim0) - fv) / fv) # Hopefully should be all 0s
## End(Not run)
```

\# Example 2: 2012 Summer Olympic Games in London
\#\# Not run: top10 <- head(olym12, 10)
grc1.oly12 <- with(top10, grc(cbind(gold, silver, bronze)))
round(fitted(grc1.oly12))
round(resid(grc1.oly12, type = "response"), digits = 1) \# Resp. resids
summary (grc1.oly12)
Coef(grc1.oly12)
\#\# End(Not run)
\# Example 3: UQO; see Yee and Hadi (2014)
\#\# Not run:
n <- 100; p <- 5; S <- 10
pdata <- rcqo(n, p, S, es.opt = FALSE, eq.max = FALSE,
eq.tol $=$ TRUE, sd.latvar $=0.75$ ) \# Poisson counts
true.nu <- attr(pdata, "latvar") \# The 'truth'; site scores
attr(pdata, "tolerances") \# The 'truth'; tolerances
Y <- Select(pdata, "y", sort = FALSE) \# Y matrix ( $n \times S$ ); the "y" vars
uqo.rcim1 <- rcim(Y, Rank = 1,
str0 = NULL, \# Delta covers entire $n \times \mathrm{M}$ matrix
iindex = 1: $\operatorname{nrow}(\mathrm{Y})$, \# RRR covers the entire $Y$
has.intercept $=$ FALSE) \# Suppress the intercept
\# Plot 1
$\operatorname{par}(m f r o w=c(2,2))$
plot(attr(pdata, "optimums"), Coef(uqo.rcim1)@A,
col = "blue", type = "p", main = "(a) UQO optimums",
xlab = "True optimums", ylab = "Estimated (UQO) optimums")
mylm <- $\operatorname{lm}($ Coef(uqo.rcim1)@A ~ attr(pdata, "optimums"))

\# Plot 2
fill.val <- NULL \# Choose this for the new parameterization
plot(attr(pdata, "latvar"), c(fill.val, concoef(uqo.rcim1)),
las = 1, col = "blue", type = "p", main = "(b) UQO site scores",
xlab = "True site scores", ylab = "Estimated (UQO) site scores" )
mylm <- lm(c(fill.val, concoef(uqo.rcim1)) ~ attr(pdata, "latvar"))
abline (coef = coef(mylm), col = "orange", lty = "dashed")

```
# Plots 3 and 4
myform <- attr(pdata, "formula")
p1ut <- cqo(myform, family = poissonff,
            eq.tol = FALSE, trace = FALSE, data = pdata)
c1ut <- cqo(Select(pdata, "y", sort = FALSE) ~ scale(latvar(uqo.rcim1)),
            family = poissonff, eq.tol = FALSE, trace = FALSE, data = pdata)
lvplot(p1ut, lcol = 1:S, y = TRUE, pcol = 1:S, pch = 1:S, pcex = 0.5,
        main = "(c) CQO fitted to the original data",
        xlab = "Estimated (CQO) site scores")
lvplot(c1ut, lcol = 1:S, y = TRUE, pcol = 1:S, pch = 1:S, pcex = 0.5,
        main = "(d) CQO fitted to the scaled UQO site scores",
        xlab = "Estimated (UQO) site scores")
## End(Not run)
```

gtbinomial

## Description

Fits a generally-truncated binomial distribution.

## Usage

gtbinomial(truncate = 0, zero = NULL, link = "logitlink", type.fitted = c("mean", "prob", "prob.t"), multiple.responses = FALSE, parallel = FALSE)

## Arguments

truncate Vector of truncated values, i.e., integers between 0 and size inclusive. Must have unique values only. May be a NULL, which stands for empty set (same as binomialff). The default is the 0 -truncated or positive-binomial.
link, zero See CommonVGAMffArguments for information.
multiple.responses, parallel
See CommonVGAMffArguments for information.
type.fitted See CommonVGAMffArguments for information. The choice "prob.t" means the probability of having a truncated value, given the estimate of prob from an ordinary binomial distribution. The mean can be be obtained by the extractor function fitted applied to the object (the default).

## Details

The generally-truncated binomial distribution is the ordinary binomial distribution with the probability of certain values (truncate argument) being zero. Thus the other probabilities are scaled up. The positive-binomial ( 0 -truncated) distribution is a special case and is the default.
Note that the argument truncate is integer-valued even though the response is converted into a sample proportion.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Warning

Under- or over-flow may occur if the data is ill-conditioned. The response is checked to see that no values equal any values of the truncate vector. It is easy to misuse this function; the truncate vector should ideally not be very long and have values that can be justified by the application on hand.

## Note

This family function can handle multiple responses.

## Author(s)

Thomas W. Yee and Chenchen Ma

## See Also

Gaitbinom.mlm, rposbinom, posbinomial, binomialff, simulate.vlm.

## Examples

```
nnn <- 1000; tvec <- c(2, 3, 6); size <- 10
pdata <- data.frame(x2 = runif(nnn), x3 = runif(nnn))
pdata <- transform(pdata,
    y1 = rgaitbinom.mlm(nnn, prob = logitlink(-1, inverse = TRUE),
                truncate = tvec, size = size),
    y2 = rgaitbinom.mlm(nnn, prob = logitlink(-1 + x2 + x3, inverse = TRUE),
                        truncate = tvec, size = size))
with(pdata, table(y1))
with(pdata, table(y2))
head(pdata)
fit1 <- vglm(cbind(y1, size - y1) ~ 1,
                            gtbinomial(truncate = tvec), trace = TRUE, data = pdata)
coef(fit1, matrix = TRUE)
summary(fit1)
fit2 <- vglm(cbind(y2, size - y2) ~ x2 + x3, crit = "coef",
    gtbinomial(truncate = tvec), trace = TRUE, data = pdata)
coef(fit2, matrix = TRUE)
summary(fit2)
```


## Description

Maximum likelihood estimation of the 2-parameter Gumbel distribution.

## Usage

```
    gumbel(llocation = "identitylink", lscale = "loglink",
            iscale = NULL, R = NA, percentiles = c(95, 99),
            mpv = FALSE, zero = NULL)
    gumbelff(llocation = "identitylink", lscale = "loglink",
                iscale = NULL, R = NA, percentiles = c(95, 99),
        zero = "scale", mpv = FALSE)
```


## Arguments

llocation, lscale
Parameter link functions for $\mu$ and $\sigma$. See Links for more choices.
iscale Numeric and positive. Optional initial value for $\sigma$. Recycled to the appropriate length. In general, a larger value is better than a smaller value. A NULL means an initial value is computed internally.
R Numeric. Maximum number of values possible. See Details for more details.
percentiles Numeric vector of percentiles used for the fitted values. Values should be between 0 and 100 . This argument uses the argument $R$ if assigned. If percentiles $=$ NULL then the mean will be returned as the fitted values.
mpv Logical. If $\mathrm{mpv}=$ TRUE then the median predicted value (MPV) is computed and returned as the (last) column of the fitted values. This argument is ignored if percentiles $=$ NULL. See Details for more details.
zero A vector specifying which linear/additive predictors are modelled as intercepts only. The value (possibly values) can be from the set $\{1,2\}$ corresponding respectively to $\mu$ and $\sigma$. By default all linear/additive predictors are modelled as a linear combination of the explanatory variables. See CommonVGAMffArguments for more details.

## Details

The Gumbel distribution is a generalized extreme value (GEV) distribution with shape parameter $\xi=0$. Consequently it is more easily estimated than the GEV. See gev for more details.

The quantity $R$ is the maximum number of observations possible, for example, in the Venice data below, the top 10 daily values are recorded for each year, therefore $R=365$ because there are about 365 days per year. The MPV is the value of the response such that the probability of obtaining a value greater than the MPV is 0.5 out of $R$ observations. For the Venice data, the MPV is the sea level such that there is an even chance that the highest level for a particular year exceeds the MPV.

If mpv = TRUE then the column labelled "MPV" contains the MPVs when fitted() is applied to the fitted object.
The formula for the mean of a response $Y$ is $\mu+\sigma \times$ Euler where Euler is a constant that has value approximately equal to 0.5772 . The formula for the percentiles are (if R is not given) $\mu-\sigma \times \log [-\log (P / 100)]$ where $P$ is the percentile argument value(s). If R is given then the percentiles are $\mu-\sigma \times \log [R(1-P / 100)]$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

When $R$ is not given (the default) the fitted percentiles are that of the data, and not of the overall population. For example, in the example below, the 50 percentile is approximately the running median through the data, however, the data are the highest sea level measurements recorded each year (it therefore equates to the median predicted value or MPV).

## Note

Like many other usual VGAM family functions, gumbelff() handles (independent) multiple responses.
gumbel () can handle more of a multivariate response, i.e., a matrix with more than one column. Each row of the matrix is sorted into descending order. Missing values in the response are allowed but require na. action = na.pass. The response matrix needs to be padded with any missing values. With a multivariate response one has a matrix $y$, say, where $y[, 2]$ contains the second order statistics, etc.

## Author(s)

T. W. Yee

## References

Yee, T. W. and Stephenson, A. G. (2007). Vector generalized linear and additive extreme value models. Extremes, 10, 1-19.
Smith, R. L. (1986). Extreme value theory based on the $r$ largest annual events. Journal of Hydrology, 86, 27-43.
Rosen, O. and Cohen, A. (1996). Extreme percentile regression. In: Haerdle, W. and Schimek, M. G. (eds.), Statistical Theory and Computational Aspects of Smoothing: Proceedings of the COMPSTAT '94 Satellite Meeting held in Semmering, Austria, 27-28 August 1994, pp.200-214, Heidelberg: Physica-Verlag.
Coles, S. (2001). An Introduction to Statistical Modeling of Extreme Values. London: SpringerVerlag.

## See Also

rgumbel, dgumbelII, cens.gumbel, guplot, gev, gevff, venice.

## Examples

```
# Example 1: Simulated data
gdata <- data.frame(y1 = rgumbel(n = 1000, loc = 100, scale = exp(1)))
fit1 <- vglm(y1 ~ 1, gumbelff(perc = NULL), data = gdata, trace = TRUE)
coef(fit1, matrix = TRUE)
Coef(fit1)
head(fitted(fit1))
with(gdata, mean(y1))
# Example 2: Venice data
(fit2 <- vglm(cbind(r1, r2, r3, r4, r5) ~ year, data = venice,
                gumbel(R = 365, mpv = TRUE), trace = TRUE))
head(fitted(fit2))
coef(fit2, matrix = TRUE)
sqrt(diag(vcov(summary(fit2)))) # Standard errors
# Example 3: Try a nonparametric fit -----------------------
# Use the entire data set, including missing values
# Same as as.matrix(venice[, paste0("r", 1:10)]):
Y <- Select(venice, "r", sort = FALSE)
fit3 <- vgam(Y ~ s(year, df = 3), gumbel(R = 365, mpv = TRUE),
    data = venice, trace = TRUE, na.action = na.pass)
depvar(fit3)[4:5, ] # NAs used to pad the matrix
## Not run: # Plot the component functions
par(mfrow = c(2, 3), mar = c(6, 4, 1, 2) + 0.3, xpd = TRUE)
plot(fit3, se = TRUE, lcol = "blue", scol = "limegreen", lty = 1,
    lwd = 2, slwd = 2, slty = "dashed")
# Quantile plot --- plots all the fitted values
qtplot(fit3, mpv = TRUE, lcol = c(1, 2, 5), tcol = c(1, 2, 5), lwd = 2,
        pcol = "blue", tadj = 0.1, ylab = "Sea level (cm)")
# Plot the 99 percentile only
year <- venice[["year"]]
matplot(year, Y, ylab = "Sea level (cm)", type = "n")
matpoints(year, Y, pch = "*", col = "blue")
lines(year, fitted(fit3)[, "99%"], lwd = 2, col = "orange")
# Check the 99 percentiles with a smoothing spline.
# Nb. (1-0.99) * 365 = 3.65 is approx. 4, meaning the 4th order
# statistic is approximately the 99 percentile.
plot(year, Y[, 4], ylab = "Sea level (cm)", type = "n",
    main = "Orange is 99 percentile, Green is a smoothing spline")
points(year, Y[, 4], pch = "4", col = "blue")
lines(year, fitted(fit3)[, "99%"], lty = 1, col = "orange")
lines(smooth.spline(year, Y[, 4], df = 4), col = "limegreen", lty = 2)
## End(Not run)
```

```
Gumbel-II The Gumbel-II Distribution
```


## Description

Density, cumulative distribution function, quantile function and random generation for the GumbelII distribution.

## Usage

dgumbelII(x, scale $=1$, shape, log = FALSE)
pgumbelII(q, scale = 1, shape, lower.tail = TRUE, log.p = FALSE)
qgumbelII( p , scale $=1$, shape, lower.tail = TRUE, log.p = FALSE)
rgumbelII( $n$, scale $=1$, shape)

## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
n number of observations. Same as in runif.
$\log \quad$ Logical. If $\log =$ TRUE then the logarithm of the density is returned.
lower.tail, log.p
Same meaning as in pnorm or qnorm.
shape, scale positive shape and scale parameters.

## Details

See gumbelII for details.

## Value

dgumbelII gives the density, pgumbelII gives the cumulative distribution function, qgumbelII gives the quantile function, and rgumbelII generates random deviates.

## Author(s)

T. W. Yee and Kai Huang

## See Also

gumbelII, dgumbel.

## Examples

```
probs <- seq(0.01, 0.99, by = 0.01)
Scale <- exp(1); Shape <- exp( 0.5);
max(abs(pgumbelII(qgumbelII(p = probs, shape = Shape, Scale),
                            shape = Shape, Scale) - probs)) # Should be 0
## Not run: x <- seq(-0.1, 10, by = 0.01);
plot(x, dgumbelII(x, shape = Shape, Scale), type = "l", col = "blue", las = 1,
    main = "Blue is density, orange is cumulative distribution function",
    sub = "Purple lines are the 10,20,\ldots.,90 percentiles",
    ylab = "", ylim = 0:1)
abline(h = 0, col = "blue", lty = 2)
lines(x, pgumbelII(x, shape = Shape, Scale), col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qgumbelII(probs, shape = Shape, Scale)
lines(Q, dgumbelII(Q, Scale, Shape), col = "purple", lty = 3, type = "h")
pgumbelII(Q, shape = Shape, Scale) - probs # Should be all zero
abline(h = probs, col = "purple", lty = 3)
## End(Not run)
```


## Description

Maximum likelihood estimation of the 2-parameter Gumbel-II distribution.

## Usage

```
gumbelII(lscale = "loglink", lshape = "loglink", iscale = NULL, ishape = NULL,
    probs.y = c(0.2, 0.5, 0.8), perc.out = NULL, imethod = 1,
    zero = "shape", nowarning = FALSE)
```


## Arguments

nowarning Logical. Suppress a warning?
lshape, lscale Parameter link functions applied to the (positive) shape parameter (called $s$ below) and (positive) scale parameter (called $b$ below). See Links for more choices.
Parameter link functions applied to the
ishape, iscale Optional initial values for the shape and scale parameters.
imethod See weibullR.
zero, probs.y Details at CommonVGAMffArguments.
perc.out If the fitted values are to be quantiles then set this argument to be the percentiles of these, e.g., 50 for median.

## Details

The Gumbel-II density for a response $Y$ is

$$
f(y ; b, s)=s y^{s-1} \exp \left[-(y / b)^{s}\right] /\left(b^{s}\right)
$$

for $b>0, s>0, y>0$. The cumulative distribution function is

$$
F(y ; b, s)=\exp \left[-(y / b)^{-s}\right] .
$$

The mean of $Y$ is $b \Gamma(1-1 / s)$ (returned as the fitted values) when $s>1$, and the variance is $b^{2} \Gamma(1-2 / s)$ when $s>2$. This distribution looks similar to weibullR, and is due to Gumbel (1954).

This VGAM family function currently does not handle censored data. Fisher scoring is used to estimate the two parameters. Probably similar regularity conditions hold for this distribution compared to the Weibull distribution.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

See weibullR. This VGAM family function handles multiple responses.

## Author(s)

T. W. Yee

## References

Gumbel, E. J. (1954). Statistical theory of extreme values and some practical applications. Applied Mathematics Series, volume 33, U.S. Department of Commerce, National Bureau of Standards, USA.

## See Also

dgumbelII, gumbel, gev.

## Examples

```
gdata <- data.frame(x2 = runif(nn <- 1000))
gdata <- transform(gdata, heta1 = +1,
    heta2 = -1 + 0.1 * x2,
    ceta1 = 0,
    ceta2 = 1)
gdata <- transform(gdata, shape1 = exp(heta1),
    shape2 = exp(heta2),
    scale1 = exp(ceta1),
    scale2 = exp(ceta2))
gdata <- transform(gdata,
```

y1 = rgumbelII(nn, scale = scale1, shape = shape1),
y2 = rgumbelII (nn, scale = scale2, shape = shape2))

```
fit <- vglm(cbind(y1, y2) ~ x2,
    gumbelII(zero = c(1, 2, 3)), data = gdata, trace = TRUE)
coef(fit, matrix = TRUE)
vcov(fit)
summary(fit)
```

gumbelUC The Gumbel Distribution

## Description

Density, distribution function, quantile function and random generation for the Gumbel distribution with location parameter location and scale parameter scale.

## Usage

dgumbel(x, location $=0$, scale $=1$, log $=$ FALSE $)$
pgumbel(q, location $=0$, scale $=1$, lower.tail $=$ TRUE, log. $p=$ FALSE)
qgumbel ( p , location $=0$, scale $=1$, lower.tail $=$ TRUE, log. $\mathrm{p}=$ FALSE)
rgumbel(n, location $=0$, scale $=1$ )

## Arguments

| $\mathrm{x}, \mathrm{q}$ | vector of quantiles. |
| :--- | :--- |
| p |  |
| n | vector of probabilities. <br> number of observations. If length $(\mathrm{n})>1$ then the length is taken to be the <br> number required. |
| location | the location parameter $\mu$. This is not the mean of the Gumbel distribution (see <br> Details below). |
| scale | the scale parameter $\sigma$. This is not the standard deviation of the Gumbel distri- <br> bution (see Details below). |
| log | Logical. If log $=$ TRUE then the logarithm of the density is returned. <br> lower.tail, log. $p$ <br> Same meaning as in punif or qunif. |

## Details

The Gumbel distribution is a special case of the generalized extreme value (GEV) distribution where the shape parameter $\xi=0$. The latter has 3 parameters, so the Gumbel distribution has two. The Gumbel distribution function is

$$
G(y)=\exp \left(-\exp \left[-\frac{y-\mu}{\sigma}\right]\right)
$$

where $-\infty<y<\infty,-\infty<\mu<\infty$ and $\sigma>0$. Its mean is

$$
\mu-\sigma * \gamma
$$

and its variance is

$$
\sigma^{2} * \pi^{2} / 6
$$

where $\gamma$ is Euler's constant (which can be obtained as -digamma(1)).
See gumbel, the VGAM family function for estimating the two parameters by maximum likelihood estimation, for formulae and other details. Apart from $n$, all the above arguments may be vectors and are recyled to the appropriate length if necessary.

## Value

dgumbel gives the density, pgumbel gives the distribution function, qgumbel gives the quantile function, and rgumbel generates random deviates.

## Note

The VGAM family function gumbel can estimate the parameters of a Gumbel distribution using maximum likelihood estimation.

## Author(s)

T. W. Yee

## References

Coles, S. (2001). An Introduction to Statistical Modeling of Extreme Values. London: SpringerVerlag.

## See Also

gumbel, gumbelff, gev, dgompertz.

## Examples

```
mu <- 1; sigma <- 2;
y <- rgumbel(n = 100, loc = mu, scale = sigma)
c(mean(y), mu - sigma * digamma(1)) # Sample and population means
c(var(y), sigma^2 * pi^2 / 6) # Sample and population variances
## Not run: x <- seq(-2.5, 3.5, by = 0.01)
loc <- 0; sigma <- 1
plot(x, dgumbel(x, loc, sigma), type = "l", col = "blue", ylim = c(0, 1),
    main = "Blue is density, red is cumulative distribution function",
    sub = "Purple are 5,10,\ldots.,95 percentiles", ylab = "", las = 1)
abline(h = 0, col = "blue", lty = 2)
lines(qgumbel(seq(0.05, 0.95, by = 0.05), loc, sigma),
    dgumbel(qgumbel(seq(0.05, 0.95, by = 0.05), loc, sigma), loc, sigma),
    col = "purple", lty = 3, type = "h")
lines(x, pgumbel(x, loc, sigma), type = "l", col = "red")
```

```
abline(h = 0, lty = 2)
```

\#\# End(Not run)
guplot Gumbel Plot

## Description

Produces a Gumbel plot, a diagnostic plot for checking whether the data appears to be from a Gumbel distribution.

## Usage

```
guplot(object, ...)
guplot.default(y, main = "Gumbel Plot",
    xlab = "Reduced data", ylab = "Observed data", type = "p", ...)
    guplot.vlm(object, ...)
```


## Arguments

y A numerical vector. NAs etc. are not allowed.
main Character. Overall title for the plot.
$x l a b \quad$ Character. Title for the x axis.
ylab Character. Title for the y axis.
type Type of plot. The default means points are plotted.
object An object that inherits class "vlm", usually of class vglm-class or vgam-class.
... Graphical argument passed into plot. See par for an exhaustive list. The arguments xlim and ylim are particularly useful.

## Details

If $Y$ has a Gumbel distribution then plotting the sorted values $y_{i}$ versus the reduced values $r_{i}$ should appear linear. The reduced values are given by

$$
r_{i}=-\log \left(-\log \left(p_{i}\right)\right)
$$

where $p_{i}$ is the $i$ th plotting position, taken here to be $(i-0.5) / n$. Here, $n$ is the number of observations. Curvature upwards/downwards may indicate a Frechet/Weibull distribution, respectively. Outliers may also be detected using this plot.
The function guplot is generic, and guplot. default and guplot.vlm are some methods functions for Gumbel plots.

## Value

A list is returned invisibly with the following components.
$x \quad$ The reduced data.
$y \quad$ The sorted y data.

## Note

The Gumbel distribution is a special case of the GEV distribution with shape parameter equal to zero.

## Author(s)

T. W. Yee

## References

Coles, S. (2001). An Introduction to Statistical Modeling of Extreme Values. London: SpringerVerlag.
Gumbel, E. J. (1958). Statistics of Extremes. New York, USA: Columbia University Press.

## See Also

gumbel, gumbelff, gev, venice.

## Examples

```
## Not run: guplot(rnorm(500), las = 1) -> ii
names(ii)
guplot(with(venice, r1), col = "blue") # Venice sea levels data
## End(Not run)
```

has.interceptvlm Has a Fitted VGLM Got an Intercept Term?

## Description

Looks at the formula to see if it has an intercept term.

## Usage

has.intercept(object, ...)
has.interceptvlm(object, form.number = 1, ...)

## Arguments

object A fitted model object.
form. number
Formula number, is 1 or 2 . which correspond to the arguments formula and form2 respectively.
... Arguments that are might be passed from one function to another.

## Details

This methods function is a simple way to determine whether a fitted vglm object etc. has an intercept term or not. It is not entirely foolproof because one might suppress the intercept from the formula and then add in a variable in the formula that has a constant value.

## Value

Returns a single logical.

## Author(s)

Thomas W. Yee

## See Also

formulavlm, termsvlm.

## Examples

```
# Example: this is based on a glm example
counts <- c(18,17,15,20,10,20,25,13,12)
outcome <- gl(3, 1, 9); treatment <- gl(3, 3)
pdata <- data.frame(counts, outcome, treatment) # Better style
vglm.D93 <- vglm(counts ~ outcome + treatment, poissonff, data = pdata)
formula(vglm.D93)
term.names(vglm.D93)
responseName(vglm.D93)
has.intercept(vglm.D93)
```

hatvalues

## Description

When complete, a suite of functions that can be used to compute some of the regression (leave-oneout deletion) diagnostics, for the VGLM class.

## Usage

```
hatvalues(model, ...)
hatvaluesvlm(model, type = c("diagonal", "matrix", "centralBlocks"), ...)
hatplot(model, ...)
hatplot.vlm(model, multiplier = c(2, 3), lty = "dashed",
            xlab = "Observation", ylab = "Hat values", ylim = NULL, ...)
dfbetavlm(model, maxit.new = 1,
            trace. new \(=\) FALSE,
            smallno = 1.0e-8, ...)
```


## Arguments

model
an $R$ object, typically returned by vglm.
type
Character. The default is the first choice, which is a $n M \times n M$ matrix. If type $=$ "matrix" then the entire hat matrix is returned. If type $=$ "centralBlocks" then $n$ central $M \times M$ block matrices, in matrix-band format.
multiplier Numeric, the multiplier. The usual rule-of-thumb is that values greater than two or three times the average leverage (at least for the linear model) should be checked.
lty, xlab, ylab, ylim
Graphical parameters, see par etc. The default of ylim is c(0, max (hatvalues(model))) which means that if the horizontal dashed lines cannot be seen then there are no particularly influential observations.
maxit.new, trace.new, smallno
Having maxit. new $=1$ will give a one IRLS step approximation from the ordinary solution (and no warnings!). Else having maxit. new $=10$, say, should usually mean convergence will occur for all observations when they are removed one-at-a-time. Else having maxit.new = 2, say, should usually mean some lack of convergence will occur when observations are removed one-at-a-time. Setting trace. new = TRUE will produce some running output at each IRLS iteration and for each individual row of the model matrix. The argument smallno multiplies each value of the original prior weight (often unity); setting it identically to zero will result in an error, but setting a very small value effectively removes that observation.
... further arguments, for example, graphical parameters for hatplot.vlm().

## Details

The invocation hatvalues (vglmObject) should return a $n \times M$ matrix of the diagonal elements of the hat (projection) matrix of a vglm object. To do this, the QR decomposition of the object is retrieved or reconstructed, and then straightforward calculations are performed.
The invocation hatplot (vglmObject) should plot the diagonal of the hat matrix for each of the $M$ linear/additive predictors. By default, two horizontal dashed lines are added; hat values higher than these ought to be checked.

## Note

It is hoped, soon, that the full suite of functions described at influence.measures will be written for VGLMs. This will enable general regression deletion diagnostics to be available for the entire VGLM class.

## Author(s)

T. W. Yee.

## See Also

vglm, cumulative, influence.measures.

## Examples

```
# Proportional odds model, p.179, in McCullagh and Nelder (1989)
pneumo <- transform(pneumo, let = log(exposure.time))
fit <- vglm(cbind(normal, mild, severe) ~ let, cumulative, data = pneumo)
hatvalues(fit) # n x M matrix, with positive values
all.equal(sum(hatvalues(fit)), fit@rank) # Should be TRUE
## Not run: par(mfrow = c(1, 2))
hatplot(fit, ylim = c(0, 1), las = 1, col = "blue")
## End(Not run)
```

hdeff Hauck-Donner Effects: A Detection Test for Wald Tests

## Description

A detection test for the Hauck-Donner effect on each regression coefficient of a VGLM regression.

```
Usage
    hdeff(object, ...)
    hdeff.vglm(object, derivative = NULL, se.arg = FALSE,
        subset = NULL, hstep = 0.005, fd.only = FALSE, ...)
```


## Arguments

object A vglm object. Although only a limited number of family functions have an analytical solution to the HDE detection test (binomialff, borel. tanner, cumulative, erlang, felix, lindley, poissonff, topple, uninormal, zipoissonff, and zipoisson; hopefully some more will be implemented in the short future!) the finite-differences (FDs) method can be applied to almost all VGAM family functions to get a numerical solution.
derivative Numeric. Either 1 or 2. Currently only a few models having one linear predictor are handled analytically for derivative $=2$, e.g., binomialff, poissonff. However, the numerical method can return the first two derivatives for almost all models.
se.arg Logical. If TRUE then the derivatives of the standard errors are returned as well, because usually the derivatives of the Wald statistics are of central interest. Requires derivative to be assigned the value 1 or 2 for this argument to operate.
subset Logical or vector of indices, to select the regression coefficients of interest. The default is to select all coefficients. Recycled if necessary if logical. If numeric then they should comprise elements from 1:length (coef(object)). This argument can be useful for computing the derivatives of a Cox regression (coxph) fitted using artificially created Poisson data; then there are many coefficients that are effectively nuisance parameters.

hstep | Positive numeric and recycled to length 2 ; it is the so-called step size when using |
| :--- |
| finite-differences and is often called $h$ in the calculus literature, e.g., $f^{\prime}(x)$ is |
| approximately $(f(x+h)-f(x)) / h$. For the 2 nd-order partial derivatives, there |
| are two step sizes and hence this argument is recycled to length 2 . The default |
| is to have the same values. The 1 st-order derivatives use the first value only. It |
| is recommended that a few values of this argument be tried because values of |
| the first and second derivatives can vary accordingly. If any values are too large |
| then the derivatives may be inaccurate; and if too small then the derivatives may |
| be unstable and subject to too much round-off/cancellation error (in fact it may |
| create an error or a NA). |

fd.only

| Logical; if TRUE then finite-differences are used to estimate the derivatives even |
| :--- |
| if an analytical solution has been coded, By default, finite-differences will be |
| used when an analytical solution has not been implemented. |


$\ldots$ | currently unused but may be used in the future for further arguments passed into |
| :--- |
| the other methods functions. |

## Details

Almost all of statistical inference based on the likelihood assumes that the parameter estimates are located in the interior of the parameter space. The nonregular case of being located on the boundary is not considered very much and leads to very different results from the regular case. Practically, an important question is: how close is close to the boundary? One might answer this as: the parameter estimates are too close to the boundary when the Hauck-Donner effect (HDE) is present, whereby the Wald statistic becomes aberrant.

Hauck and Donner (1977) first observed an aberration of the Wald test statistic not monotonically increasing as a function of increasing distance between the parameter estimate and the null value. This "disturbing" and "undesirable" underappreciated effect has since been observed in other regression models by various authors. This function computes the first, and possibly second, derivative of the Wald statistic for each regression coefficient. A negative value of the first derivative is indicative of the HDE being present. More information can be obtained from hdeffsev regarding HDE severity: there may be none, faint, weak, moderate, strong and extreme amounts of HDE present.

In general, most models have derivatives that are computed numerically using finite-difference approximations. The reason is that it takes a lot of work to program in the analytical solution (this includes a few very common models, such as poissonff and binomialff, where the first two derivatives have been implemented).

## Value

By default this function returns a labelled logical vector; a TRUE means the HDE is affirmative for that coefficient (negative slope). Hence ideally all values are FALSE. Any TRUE values suggests that the MLE is too near the boundary of the parameter space, and that the p -value for that regression coefficient is biased upwards. When present a highly significant variable might be deemed nonsignificant, and thus the HDE can create havoc for variable selection. If the HDE is present then more accurate p-values can generally be obtained by conducting a likelihood ratio test (see lrt.stat.vlm) or Rao's score test (see score.stat.vlm); indeed the default of wald.stat.vlm does not suffer from the HDE.

Setting deriv = 1 returns a numerical vector of first derivatives of the Wald statistics. Setting deriv $=2$ returns a 2 -column matrix of first and second derivatives of the Wald statistics. Then setting se $. \arg =$ TRUE returns an additional 1 or 2 columns.

Some 2nd derivatives are NA if only a partial analytic solution has been programmed in.
For those VGAM family functions whose HDE test has not yet been implemented explicitly (the vast majority of them), finite-difference approximations to the derivatives will be used-see the arguments hstep and fd. only for getting some control on them.

## Note

The function summaryvglm conducts the HDE detection test if possible and prints out a line at the bottom if the HDE is detected for some regression coefficients. By "if possible", only a few family functions are exempt and they have an infos slot with component hadof = FALSE; such as normal.vcm, rec.normal because it uses the BFGS-IRLS method for computing the working weights. For these few a NULL is returned by hdeff.

If the second derivatives are of interest then it is recommended that crit = "c" be added to the fitting so that a slightly more accurate model results (usually one more IRLS iteration). This is because the FD approximation is very sensitive to values of the working weights, so they need to be computed accurately. Occasionally, if the coefficient is close to 0 , then its Wald statistic's second derivative may be unusually large in magnitude (this could be due to something such as roundoff error).

This function is currently under development and may change a little in the short future. For HDE severity measures see hdeffsev.
Yee (2018) gives details about HDE detection for the entire VGLM class, and proves a tipping point theorem with tipping points $1 / 4$ and $3 / 5$. The HDE severity measures allow partitioning of the parameter space into 6 regions from the interior and going outwards towards the boundary edges. It is also shown that with 1-parameter binary regression the HDE cannot occur unless the log odds ratio is at least 2.40 , which corresponds to an odds ratio of 11.0 or more.

## Author(s)

Thomas W. Yee.

## References

Hauck, J. W. W. and A. Donner (1977). Wald's test as applied to hypotheses in logit analysis. Journal of the American Statistical Association, 72, 851-853. Corrigenda: JASA, 75, 482.

Yee, T. W. (2018) On the Hauck-Donner effect in Wald tests: Detection, and parameter space characterization (submitted for publication).

## See Also

## Examples

```
pneumo <- transform(pneumo, let = log(exposure.time))
fit <- vglm(cbind(normal, mild, severe) ~ let, data = pneumo,
                    trace = TRUE, crit = "c", # Get some more accuracy
    cumulative(reverse = TRUE, parallel = TRUE))
cumulative()@infos()$hadof # Analytical solution implemented
hdeff(fit)
hdeff(fit, deriv = 1) # Analytical solution
hdeff(fit, deriv = 2) # It is a partial analytical solution
hdeff(fit, deriv = 2, se.arg = TRUE,
        fd.only = TRUE) # All derivatives solved numerically by FDs
```

hdeffsev Hauck-Donner Effects: Severity Measures

## Description

Computes the severity of the Hauck-Donner effect for each regression coefficient of a VGLM regression.

## Usage

hdeffsev (x, y, dy, ddy, allofit = FALSE, tol0 = 0.1, severity.table = c("None", "Faint", "Weak", "Moderate", "Strong", "Extreme", "Undetermined"))

## Arguments

$\mathrm{x}, \mathrm{y} \quad$ Numeric vectors; x are the estimates, and y are the Wald statistics.
dy, ddy Numeric vectors; the first and second derivatives of the Wald statistics. They can be computed by hdeff.
allofit Logical. If TRUE then other quantities are returned in a list. The default is a vector with elements selected from the argument severity.table.
severity.table Character vector with 7 values. The last value is used for initialization. Usually users should not assign anything to arguments severity. table or tol0.
tol0 Numeric. Any estimate whose absolute value is less than tol0 is assigned the first value of the argument severity.table, i.e., none. This is to handle a singularity at the origin: the estimates might be extremely close to 0 .

## Details

This function is rough-and-ready. It is possible to use the first two derivatives obtained from hdeff to categorize the severity of the the Hauck-Donner effect (HDE). It is effectively assumed that, starting at the origin and going right, the curve is made up of a convex segment followed by a concave segment and then the convex segment. Midway in the concave segment the derivative is 0 , and beyond that the HDE is really manifest because the derivative is negative.

For "none" the estimate lies on the convex part of the curve near the origin, hence there is no HDE at all.
For "faint" and "weak" the estimate lies on the concave part of the curve but the Wald statistic is still increasing as estimate gets away from 0 , hence it is only a mild HDE.
For "moderate", "strong" and "extreme" the Wald statistic is decreasing as the estimate gets away from 0 , hence it really does exhibit the HDE. It is recommended that lrt.stat be used to compute LRT p-values, as they do not suffer from the HDE.

## Value

By default this function returns a labelled vector with elements selected from severity.table. If allofit = TRUE then Yee (2018) gives details about the other list components: a quantity called zeta is the normal line projected onto the x -axis, and its first derivative gives additional information about the position of the estimate along the curve.

## Note

This function is likely to change in the short future because it is experimental and far from complete. Improvements are intended.
See hdeff; Yee (2018) gives details on VGLM HDE detection, severity measures, two tipping points ( $1 / 4$ and $3 / 5$ ), parameter space partitioning into several regions, and a bound for the HDE for 1-parameter binary regression, etc.

## Author(s)

Thomas W. Yee.

## See Also

seglines, hdeff.

## Examples

```
deg <- 4 # myfun is a function that approximates the HDE
myfun <- function(x, deriv = 0) switch(as.character(deriv),
    '0' = x^deg * exp(-x),
    '1' = (deg * x^(deg-1) - x^deg) * exp(-x),
    '2' = (deg*(deg-1)*x^(deg-2) - 2*deg*x^(deg-1) + x^deg)*exp(-x))
xgrid <- seq(0, 10, length = 101)
ansm <- hdeffsev(xgrid, myfun(xgrid), myfun(xgrid, deriv = 1),
            myfun(xgrid, deriv = 2), allofit = TRUE)
digg <- 4
cbind(severity = ansm$sev,
            fun = round(myfun(xgrid), digg),
            deriv1 = round(myfun(xgrid, deriv = 1), digg),
            deriv2 = round(myfun(xgrid, deriv = 2), digg),
            zderiv1 = round(1 + (myfun(xgrid, deriv = 1) )^2 +
                            myfun(xgrid, deriv = 2) * myfun(xgrid), digg))
```

hormone Hormone Assay Data

## Description

A hormone assay data set from Carroll and Ruppert (1988).

## Usage <br> data(hormone)

## Format

A data frame with 85 observations on the following 2 variables.
$X$ a numeric vector, suitable as the x -axis in a scatter plot. The reference method.
$Y$ a numeric vector, suitable as the $y$-axis in a scatter plot. The test method.

## Details

The data is given in Table 2.4 of Carroll and Ruppert (1988), and was downloaded from http://www. stat. tamu. edu/~carr prior to 2019. The book describes the data as follows. The data are the results of two assay methods for hormone data; the scale of the data as presented is not particularly meaningful, and the original source of the data refused permission to divulge further information. As in a similar example of Leurgans (1980), the old or reference method is being used to predict the new or test method. The overall goal is to see whether we can reproduce the test-method measurements with the referencemethod measurements. Thus calibration might be of interest for the data.

## References

Carroll, R. J. and Ruppert, D. (1988). Transformation and Weighting in Regression. New York, USA: Chapman \& Hall.

Leurgans, S. (1980). Evaluating laboratory measurement techniques. Biostatistics Casebook. Eds.: Miller, R. G. Jr., and Efron, B. and Brown, B. W. Jr., and Moses, L. New York, USA: Wiley.

Yee, T. W. (2014). Reduced-rank vector generalized linear models with two linear predictors. Computational Statistics and Data Analysis, 71, 889-902.

## See Also

uninormal, rrvglm.

## Examples

```
## Not run:
data(hormone)
summary(hormone)
modelI <-rrvglm(Y ~ 1 + X, data = hormone, trace = TRUE,
    uninormal(zero = NULL, lsd = "identitylink", imethod = 2))
# Alternative way to fit modelI
modelI.other <- vglm(Y ~ 1 + X, data = hormone, trace = TRUE,
    uninormal(zero = NULL, lsd = "identitylink"))
# Inferior to modelI
modelII <- vglm(Y ~ 1 + X, data = hormone, trace = TRUE,
    family = uninormal(zero = NULL))
logLik(modelI)
logLik(modelII) # Less than logLik(modelI)
# Reproduce the top 3 equations on p. }65\mathrm{ of Carroll and Ruppert (1988).
# They are called Equations (1)--(3) here.
# Equation (1)
hormone <- transform(hormone, rX = 1 / X)
clist <- list("(Intercept)" = diag(2), X = diag(2), rX = rbind(0, 1))
fit1 <- vglm(Y ~ 1 + X + rX, family = uninormal(zero = NULL),
    constraints = clist, data = hormone, trace = TRUE)
coef(fit1, matrix = TRUE)
summary(fit1) # Actually, the intercepts do not seem significant
plot(Y ~ X, hormone, col = "blue")
lines(fitted(fit1) ~ X, hormone, col = "orange")
# Equation (2)
fit2 <- rrvglm(Y ~ 1 + X, uninormal(zero = NULL), hormone, trace = TRUE)
coef(fit2, matrix = TRUE)
plot(Y ~ X, hormone, col = "blue")
lines(fitted(fit2) ~ X, hormone, col = "red")
# Add +- 2 SEs
lines(fitted(fit2) + 2 * exp(predict(fit2)[, "loglink(sd)"]) ~ X,
    hormone, col = "orange")
lines(fitted(fit2) - 2 * exp(predict(fit2)[, "loglink(sd)"]) ~ X,
    hormone, col = "orange")
# Equation (3)
# Does not fit well because the loglink link for the mean is not good.
fit3 <- rrvglm(Y ~ 1 + X, maxit = 300, data = hormone, trace = TRUE,
    uninormal(lmean = "loglink", zero = NULL))
coef(fit3, matrix = TRUE)
plot(Y ~ X, hormone, col = "blue") # Does not look okay.
lines(exp(predict(fit3)[, 1]) ~ X, hormone, col = "red")
# Add +- 2 SEs
```

```
lines(fitted(fit3) + 2 * exp(predict(fit3)[, "loglink(sd)"]) ~ X,
    hormone, col = "orange")
lines(fitted(fit3) - 2 * exp(predict(fit3)[, "loglink(sd)"]) ~ X,
    hormone, col = "orange")
## End(Not run)
```

hspider Hunting Spider Data

## Description

Abundance of hunting spiders in a Dutch dune area.

## Usage

data(hspider)

## Format

A data frame with 28 observations (sites) on the following 18 variables.
WaterCon Log percentage of soil dry mass.
BareSand Log percentage cover of bare sand.
FallTwig Log percentage cover of fallen leaves and twigs.
CoveMoss Log percentage cover of the moss layer.
CoveHerb Log percentage cover of the herb layer.
Refllux Reflection of the soil surface with cloudless sky.
Alopacce Abundance of Alopecosa accentuata.
Alopcune Abundance of Alopecosa cuneata.
Alopfabr Abundance of Alopecosa fabrilis.
Arctlute Abundance of Arctosa lutetiana.
Arctperi Abundance of Arctosa perita.
Auloalbi Abundance of Aulonia albimana.
Pardlugu Abundance of Pardosa lugubris.
Pardmont Abundance of Pardosa monticola.
Pardnigr Abundance of Pardosa nigriceps.
Pardpull Abundance of Pardosa pullata.
Trocterr Abundance of Trochosa terricola.
Zoraspin Abundance of Zora spinimana.

## Details

The data, which originally came from Van der Aart and Smeek-Enserink (1975) consists of abundances (numbers trapped over a 60 week period) and 6 environmental variables. There were 28 sites.

This data set has been often used to illustrate ordination, e.g., using canonical correspondence analysis (CCA). In the example below, the data is used for constrained quadratic ordination (CQO; formerly called canonical Gaussian ordination or CGO), a numerically intensive method that has many superior qualities. See cqo for details.

## References

Van der Aart, P. J. M. and Smeek-Enserink, N. (1975). Correlations between distributions of hunting spiders (Lycosidae, Ctenidae) and environmental characteristics in a dune area. Netherlands Journal of Zoology, 25, 1-45.

## Examples

```
summary(hspider)
## Not run:
# Standardize the environmental variables:
hspider[, 1:6] <- scale(subset(hspider, select = WaterCon:ReflLux))
# Fit a rank-1 binomial CAO
hsbin <- hspider # Binary species data
hsbin[, -(1:6)] <- as.numeric(hsbin[, -(1:6)] > 0)
set.seed(123)
ahsb1 <- cao(cbind(Alopcune, Arctlute, Auloalbi, Zoraspin) ~
            WaterCon + ReflLux,
            family = binomialff(multiple.responses = TRUE),
            df1.nl = 2.2, Bestof = 3, data = hsbin)
par(mfrow = 2:1, las = 1)
lvplot(ahsb1, type = "predictors", llwd=2, ylab="logitlink(p)", lcol=1:9)
persp(ahsb1, rug = TRUE, col = 1:10, lwd = 2)
coef(ahsb1)
## End(Not run)
```


## Description

M-estimation of the two parameters of Huber's least favourable distribution. The one parameter case is also implemented.

## Usage

huber1(llocation = "identitylink", k = 0.862, imethod = 1)
huber2(llocation = "identitylink", lscale = "loglink",
$k=0.862$, imethod $=1$, zero $="$ scale")

## Arguments

llocation, lscale

| Link functions applied to the location and scale parameters. See Links for more |
| :--- |
| choices. |

$k$

imethod, zero $\quad$| Tuning constant. See rhuber for more information. |
| :--- |

| See CommonVGAMffArguments for information. The default value of zero means |
| :--- |
| the scale parameter is modelled as intercept-only. |

## Details

Huber's least favourable distribution family function is popular for resistant/robust regression. The center of the distribution is normal and its tails are double exponential.

By default, the mean is the first linear/additive predictor (returned as the fitted values; this is the location parameter), and the log of the scale parameter is the second linear/additive predictor. The Fisher information matrix is diagonal; Fisher scoring is implemented.
The VGAM family function huber 1 () estimates only the location parameter. It assumes a scale parameter of unit value.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

Warning: actually, huber2() may be erroneous since the first derivative is not continuous when there are two parameters to estimate. huber1() is fine in this respect.
The response should be univariate.

## Author(s)

T. W. Yee. Help was given by Arash Ardalan.

## References

Huber, P. J. and Ronchetti, E. (2009). Robust Statistics, 2nd ed. New York: Wiley.

## See Also

rhuber, uninormal, laplace, CommonVGAMffArguments.

## Examples

```
set.seed(1231); NN <- 30; coef1 <- 1; coef2 <- 10
hdata <- data.frame(x2 = sort(runif(NN)))
hdata <- transform(hdata, y = rhuber(NN, mu = coef1 + coef2 * x2))
hdata$x2[1] <- 0.0 # Add an outlier
hdata$y[1] <- 10
fit.huber2 <- vglm(y ~ x2, huber2(imethod = 3), data = hdata, trace = TRUE)
fit.huber1 <- vglm(y ~ x2, huber1(imethod = 3), data = hdata, trace = TRUE)
coef(fit.huber2, matrix = TRUE)
summary(fit.huber2)
## Not run: # Plot the results
plot(y ~ x2, data = hdata, col = "blue", las = 1)
lines(fitted(fit.huber2) ~ x2, data = hdata, col = "darkgreen", lwd = 2)
fit.lm <- lm(y ~ x2, hdata) # Compare to a LM:
lines(fitted(fit.lm) ~ x2, data = hdata, col = "lavender", lwd = 3)
# Compare to truth:
lines(coef1 + coef2 * x2 ~ x2, data = hdata, col = "orange", lwd = 2, lty = "dashed")
legend("bottomright", legend = c("truth", "huber", "lm"),
        col = c("orange", "darkgreen", "lavender"),
        lty = c("dashed", "solid", "solid"), lwd = c(2, 2, 3))
## End(Not run)
```


## Description

Simulated capture data set for the linear logistic model depending on an occasion covariate and an individual covariate for 10 trapping occasions and 20 individuals.

## Usage

data(Huggins89table1)
data(Huggins89.t1)

## Format

The format is a data frame.

## Details

Table 1 of Huggins (1989) gives this toy data set. Note that variables t1,..., t10 are occasionspecific variables. They correspond to the response variables $\mathrm{y} 1, \ldots, \mathrm{y} 10$ which have values 1 for capture and 0 for not captured.
Both Huggins89table1 and Huggins89.t1 are identical. The latter used variables beginning with $z$, not $t$, and may be withdrawn very soon.

## References

Huggins, R. M. (1989). On the statistical analysis of capture experiments. Biometrika, 76, 133-140.

## Examples

```
Huggins89table1 <- transform(Huggins89table1, x3.tij = t01,
    T02 = t02, T03 = t03, T04 = t04, T05 = t05, T06 = t06,
    T07 = t07, T08 = t08, T09 = t09, T10 = t10)
small.table1 <- subset(Huggins89table1,
    y01 + y02 + y03 + y04 + y05 + y06 + y07 + y08 + y00 + y10 > 0)
# fit.tbh is the bottom equation on p.133.
# It is a M_tbh model.
fit.tbh <-
    vglm(cbind(y01, y02, y03, y04, y05, y06, y07, y08, y09, y10) ~ x2 + x3.tij,
        xij = list(x3.tij ~ t01 + t02 + t03 + t04 + t05 + t06 + t07 + t08 + t09 + t10 +
                        T02 + T03 + T04 + T05 + T06 + T07 + T08 + T09 + T10 - 1),
        posbernoulli.tb(parallel.t = TRUE ~ x2 + x3.tij),
        data = small.table1, trace = TRUE,
        form2 = ~ x2 + x3.tij +
            t01 + t02 + t03 + t04 + t05 + t06 + t07 + t08 + t09 + t10 +
            T02 + T03 + T04 + T05 + T06 + T07 + T08 + T09 + T10)
# These results differ a bit from Huggins (1989), probably because
# two animals had to be removed here (they were never caught):
coef(fit.tbh) # First element is the behavioural effect
sqrt(diag(vcov(fit.tbh))) # SEs
constraints(fit.tbh, matrix = TRUE)
summary(fit.tbh, presid = FALSE)
fit.tbh@extra$N.hat # Estimate of the population site N; cf. 20.86
fit.tbh@extra$SE.N.hat # Its standard error; cf. 1.87 or 4.51
fit.th <- vglm(cbind(y01, y02, y03, y04, y05, y06, y07, y08, y09, y10) ~ x2,
    posbernoulli.t, data = small.table1, trace = TRUE)
coef(fit.th)
constraints(fit.th)
coef(fit.th, matrix = TRUE) # M_th model
summary(fit.th, presid = FALSE)
fit.th@extra$N.hat # Estimate of the population size N
fit.th@extra$SE.N.hat # Its standard error
fit.bh <- vglm(cbind(y01, y02, y03, y04, y05, y06, y07, y08, y09, y10) ~ x2,
    posbernoulli.b(I2 = FALSE), data = small.table1, trace = TRUE)
coef(fit.bh)
```

```
constraints(fit.bh)
coef(fit.bh, matrix = TRUE) # M_bh model
summary(fit.bh, presid = FALSE)
fit.bh@extra$N.hat
fit.bh@extra$SE.N.hat
fit.h <- vglm(cbind(y01, y02, y03, y04, y05, y06, y07, y08, y09, y10) ~ x2,
posbernoulli.b, data = small.table1, trace = TRUE)
coef(fit.h, matrix = TRUE) # M_h model (version 1)
coef(fit.h)
summary(fit.h, presid = FALSE)
fit.h@extra$N.hat
fit.h@extra$SE.N.hat
Fit.h <- vglm(cbind(y01, y02, y03, y04, y05, y06, y07, y08, y09, y10) ~ x2,
    posbernoulli.t(parallel.t = TRUE ~ x2),
    data = small.table1, trace = TRUE)
coef(Fit.h)
coef(Fit.h, matrix = TRUE) # M_h model (version 2)
summary(Fit.h, presid = FALSE)
Fit.h@extra$N.hat
Fit.h@extra$SE.N.hat
```

hunua Hunua Ranges Data

## Description

The hunua data frame has 392 rows and 18 columns. Altitude is explanatory, and there are binary responses (presence/absence $=1 / 0$ respectively) for 17 plant species.

## Usage

data(hunua)

## Format

This data frame contains the following columns:
agaaus Agathis australis, or Kauri
beitaw Beilschmiedia tawa, or Tawa
corlae Corynocarpus laevigatus
cyadea Cyathea dealbata
cyamed Cyathea medullaris
daccup Dacrydium cupressinum
dacdac Dacrycarpus dacrydioides
eladen Elaecarpus dentatus
hedarb Hedycarya arborea
hohpop Species name unknown
kniexc Knightia excelsa, or Rewarewa
kuneri Kunzea ericoides
lepsco Leptospermum scoparium
metrob Metrosideros robusta
neslan Nestegis lanceolata
rhosap Rhopalostylis sapida
vitluc Vitex lucens, or Puriri
altitude meters above sea level

## Details

These were collected from the Hunua Ranges, a small forest in southern Auckland, New Zealand. At 392 sites in the forest, the presence/absence of 17 plant species was recorded, as well as the altitude. Each site was of area size $200 \mathrm{~m}^{2}$.

## Source

Dr Neil Mitchell, University of Auckland.

## See Also

waitakere.

## Examples

```
# Fit a GAM using vgam() and compare it with the Waitakere Ranges one
fit.h <- vgam(agaaus ~ s(altitude, df = 2), binomialff, data = hunua)
## Not run:
plot(fit.h, se = TRUE, lcol = "orange", scol = "orange",
    llwd = 2, slwd = 2, main = "Orange is Hunua, Blue is Waitakere")
## End(Not run)
head(predict(fit.h, hunua, type = "response"))
fit.w <- vgam(agaaus ~ s(altitude, df = 2), binomialff, data = waitakere)
## Not run:
plot(fit.w, se = TRUE, lcol = "blue", scol = "blue", add = TRUE)
## End(Not run)
head(predict(fit.w, hunua, type = "response")) # Same as above?
```

hyperg Hypergeometric Family Function

## Description

Family function for a hypergeometric distribution where either the number of white balls or the total number of white and black balls are unknown.

## Usage

$\operatorname{hyperg}(\mathrm{N}=\mathrm{NULL}, \mathrm{D}=\mathrm{NULL}, \operatorname{lprob}=$ "logitlink", iprob = NULL)

## Arguments

N
Total number of white and black balls in the urn. Must be a vector with positive values, and is recycled, if necessary, to the same length as the response. One of $N$ and D must be specified.

D Number of white balls in the urn. Must be a vector with positive values, and is recycled, if necessary, to the same length as the response. One of $N$ and $D$ must be specified.
lprob Link function for the probabilities. See Links for more choices.
iprob Optional initial value for the probabilities. The default is to choose initial values internally.

## Details

Consider the scenario from dhyper where there are $N=m+n$ balls in an urn, where $m$ are white and $n$ are black. A simple random sample (i.e., without replacement) of $k$ balls is taken. The response here is the sample proportion of white balls. In this document, N is $N=m+n, \mathrm{D}$ is $m$ (for the number of "defectives", in quality control terminology, or equivalently, the number of marked individuals). The parameter to be estimated is the population proportion of white balls, viz. prob $=m /(m+n)$.
Depending on which one of $N$ and $D$ is inputted, the estimate of the other parameter can be obtained from the equation $p r o b=m /(m+n)$, or equivalently, $\mathrm{prob}=\mathrm{D} / \mathrm{N}$. However, the log-factorials are computed using lgamma and both $m$ and $n$ are not restricted to being integer. Thus if an integer $N$ is to be estimated, it will be necessary to evaluate the likelihood function at integer values about the estimate, i.e., at trunc(Nhat) and ceiling(Nhat) where Nhat is the (real) estimate of $N$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, vgam, rrvglm, cqo, and cao.

## Warning

No checking is done to ensure that certain values are within range, e.g., $k \leq N$.

## Note

The response can be of one of three formats: a factor (first level taken as success), a vector of proportions of success, or a 2 -column matrix (first column $=$ successes) of counts. The argument weights in the modelling function can also be specified. In particular, for a general vector of proportions, you will need to specify weights because the number of trials is needed.

## Author(s)

Thomas W. Yee

## References

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011). Statistical Distributions, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

## See Also

dhyper, binomialff.

## Examples

```
nn <- 100
m <- 5 # Number of white balls in the population
k <- rep(4, len = nn) # Sample sizes
n <- 4 # Number of black balls in the population
y <- rhyper(nn = nn, m = m, n = n, k = k)
yprop <- y / k # Sample proportions
# N is unknown, D is known. Both models are equivalent:
fit <- vglm(cbind(y,k-y) ~ 1, hyperg(D = m), trace = TRUE, crit = "c")
fit <- vglm(yprop ~ 1, hyperg(D = m), weight = k, trace = TRUE, crit = "c")
# N is known, D is unknown. Both models are equivalent:
fit <- vglm(cbind(y, k-y) ~ 1, hyperg(N = m+n), trace = TRUE, crit = "l")
fit <- vglm(yprop ~ 1, hyperg(N = m+n), weight = k, trace = TRUE, crit = "l")
coef(fit, matrix = TRUE)
Coef(fit) # Should be equal to the true population proportion
unique(m / (m+n)) # The true population proportion
fit@extra
head(fitted(fit))
summary(fit)
```

hypersecant

Hyperbolic Secant Regression Family Function

## Description

Estimation of the parameter of the hyperbolic secant distribution.

## Usage

hypersecant(link.theta $=$ extlogitlink(min $=-\mathrm{pi} / 2, \max =\mathrm{pi} / 2$ ), init.theta $=$ NULL)
hypersecant01(link.theta $=$ extlogitlink( $\min =-\mathrm{pi} / 2, \max =\mathrm{pi} / 2$ ), init.theta $=$ NULL)

## Arguments

link. theta Parameter link function applied to the parameter $\theta$. See Links for more choices.
init.theta Optional initial value for $\theta$. If failure to converge occurs, try some other value. The default means an initial value is determined internally.

## Details

The probability density function of the hyperbolic secant distribution is given by

$$
f(y ; \theta)=\exp (\theta y+\log (\cos (\theta))) /(2 \cosh (\pi y / 2))
$$

for parameter $-\pi / 2<\theta<\pi / 2$ and all real $y$. The mean of $Y$ is $\tan (\theta)$ (returned as the fitted values). Morris (1982) calls this model NEF-HS (Natural Exponential Family-Hyperbolic Secant). It is used to generate NEFs, giving rise to the class of NEF-GHS (G for Generalized).
Another parameterization is used for hypersecant01(): let $Y=(\operatorname{logit} U) / \pi$. Then this uses

$$
f(u ; \theta)=(\cos (\theta) / \pi) \times u^{-0.5+\theta / \pi} \times(1-u)^{-0.5-\theta / \pi}
$$

for parameter $-\pi / 2<\theta<\pi / 2$ and $0<u<1$. Then the mean of $U$ is $0.5+\theta / \pi$ (returned as the fitted values) and the variance is $\left(\pi^{2}-4 \theta^{2}\right) /\left(8 \pi^{2}\right)$.
For both parameterizations Newton-Raphson is same as Fisher scoring.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Author(s)

T. W. Yee

## References

Jorgensen, B. (1997). The Theory of Dispersion Models. London: Chapman \& Hall.
Morris, C. N. (1982). Natural exponential families with quadratic variance functions. The Annals of Statistics, 10(1), 65-80.

## See Also

extlogitlink.

## Examples

```
hdata <- data.frame(x2 = rnorm(nn <- 200))
hdata <- transform(hdata, y = rnorm(nn)) # Not very good data!
fit1 <- vglm(y ~ x2, hypersecant, data = hdata, trace = TRUE, crit = "coef")
coef(fit1, matrix = TRUE)
fit1@misc$earg
# Not recommended:
fit2 <- vglm(y ~ x2, hypersecant(link = "identitylink"), data = hdata, trace = TRUE)
coef(fit2, matrix = TRUE)
fit2@misc$earg
```

Hzeta Haight's Zeta Distribution

## Description

Density, distribution function, quantile function and random generation for Haight's zeta distribution with parameter shape.

## Usage

dhzeta(x, shape, log = FALSE)
phzeta(q, shape, log.p = FALSE)
qhzeta( $p$, shape)
rhzeta( $n$, shape)

## Arguments

| $\mathrm{x}, \mathrm{q}, \mathrm{p}, \mathrm{n}$ | Same meaning as runif. |
| :--- | :--- |
| shape | The positive shape parameter. Called $\alpha$ below. |
| log, log.p | Same meaning as in pnorm or qnorm. |

## Details

The probability function is

$$
f(x)=(2 x-1)^{(-\alpha)}-(2 x+1)^{(-\alpha)},
$$

where $\alpha>0$ and $x=1,2, \ldots$.

## Value

dhzeta gives the density, phzeta gives the distribution function, qhzeta gives the quantile function, and rhzeta generates random deviates.

Note
Given some response data, the VGAM family function hzeta estimates the parameter shape.

## Author(s)

T. W. Yee and Kai Huang

## See Also

hzeta, zeta, zetaff, simulate.vlm.

## Examples

```
dhzeta(1:20, 2.1)
rhzeta(20, 2.1)
round(1000 * dhzeta(1:8, 2))
table(rhzeta(1000, 2))
## Not run: shape <- 1.1; x <- 1:10
plot(x, dhzeta(x, shape = shape), type = "h", ylim = 0:1, lwd = 2,
    sub = paste("shape =", shape), las = 1, col = "blue", ylab = "Probability",
    main = "Haight's zeta: blue = density; orange = distribution function")
lines(x+0.1, phzeta(x, shape = shape), col = "orange", lty = 3, lwd = 2,
    type = "h")
## End(Not run)
```

hzeta Haight's Zeta Family Function

## Description

Estimating the parameter of Haight's zeta distribution

## Usage

hzeta(lshape = "logloglink", ishape = NULL, nsimEIM = 100)

## Arguments

lshape Parameter link function for the parameter, called $\alpha$ below. See Links for more choices. Here, a log-log link keeps the parameter greater than one, meaning the mean is finite.
ishape, nsimEIM See CommonVGAMffArguments for more information.

## Details

The probability function is

$$
f(y)=(2 y-1)^{(-\alpha)}-(2 y+1)^{(-\alpha)},
$$

where the parameter $\alpha>0$ and $y=1,2, \ldots$. The function dhzeta computes this probability function. The mean of $Y$, which is returned as fitted values, is $\left(1-2^{-\alpha}\right) \zeta(\alpha)$ provided $\alpha>1$, where $\zeta$ is Riemann's zeta function. The mean is a decreasing function of $\alpha$. The mean is infinite if $\alpha \leq 1$, and the variance is infinite if $\alpha \leq 2$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Author(s)

T. W. Yee

## References

Johnson N. L., Kemp, A. W. and Kotz S. (2005). Univariate Discrete Distributions, 3rd edition, pp.533-4. Hoboken, New Jersey: Wiley.

## See Also

Hzeta, zeta, zetaff, loglog, simulate.vlm.

## Examples

```
shape <- exp(exp(-0.1)) # The parameter
hdata <- data.frame(y = rhzeta(n = 1000, shape))
fit <- vglm(y ~ 1, hzeta, data = hdata, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit) # Useful for intercept-only models; should be same as shape
c(with(hdata, mean(y)), head(fitted(fit), 1))
summary(fit)
```

iam

Index from Array to Matrix

## Description

Maps the elements of an array containing symmetric positive-definite matrices to a matrix with sufficient columns to hold them (called matrix-band format.)

## Usage

iam(j, k, M, both = FALSE, diag = TRUE)

## Arguments

$j \quad$ An integer from the set $\{1: M\}$ giving the row number of an element.
$k \quad$ An integer from the set $\{1: M\}$ giving the column number of an element.
M The number of linear/additive predictors. This is the dimension of each positivedefinite symmetric matrix.
both Logical. Return both the row and column indices? See below for more details.
diag Logical. Return the indices for the diagonal elements? If FALSE then only the strictly upper triangular part of the matrix elements are used.

## Details

Suppose we have $n$ symmetric positive-definite square matrices, each $M$ by $M$, and these are stored in an array of dimension $c(n, M, M)$. Then these can be more compactly represented by a matrix of dimension $c(n, K)$ where $K$ is an integer between $M$ and $M *(M+1) / 2$ inclusive. The mapping between these two representations is given by this function. It firstly enumerates by the diagonal elements, followed by the band immediately above the diagonal, then the band above that one, etc. The last element is ( $1, M$ ). This function performs the mapping from elements ( $j, k$ ) of symmetric positive-definite square matrices to the columns of another matrix representing such. This is called the matrix-band format and is used by the VGAM package.

## Value

This function has a dual purpose depending on the value of both. If both=FALSE then the column number corresponding to the $j-k$ element of the matrix is returned. If both $=$ TRUE then $j$ and $k$ are ignored and a list with the following components are returned.
row. index The row indices of the upper triangular part of the matrix (This may or may not include the diagonal elements, depending on the argument diagonal).
col.index The column indices of the upper triangular part of the matrix (This may or may not include the diagonal elements, depending on the argument diagonal).

## Note

This function is used in the weight slot of many VGAM family functions (see vglmff-class), especially those whose $M$ is determined by the data, e.g., dirichlet, multinomial.

## Author(s)

T. W. Yee

## See Also

```
vglmff-class.
```


## Examples

```
iam(1, 2, M = 3) # The 4th column represents element (1,2) of a 3x3 matrix
iam(NULL, NULL, M = 3, both = TRUE) # Return the row and column indices
dirichlet()@weight
M <- 4
temp1 <- iam(NA, NA, M = M, both = TRUE)
mat1 <- matrix(NA, M, M)
mat1[cbind(temp1$row, temp1$col)] = 1:length(temp1$row)
mat1 # More commonly used
temp2 <- iam(NA, NA, M = M, both = TRUE, diag = FALSE)
mat2 <- matrix(NA, M, M)
mat2[cbind(temp2$row, temp2$col)] = 1:length(temp2$row)
mat2 # Rarely used
```

```
identitylink Identity Link Function
```


## Description

Computes the identity transformation, including its inverse and the first two derivatives.

## Usage

```
    identitylink(theta, inverse = FALSE, deriv = 0, short = TRUE, tag = FALSE)
```

negidentitylink(theta, inverse = FALSE, deriv = 0, short = TRUE, tag = FALSE)

## Arguments

theta Numeric or character. See below for further details.
inverse, deriv, short, tag
Details at Links.

## Details

The identity link function $g(\theta)=\theta$ should be available to every parameter estimated by the VGAM library. However, it usually results in numerical problems because the estimates lie outside the permitted range. Consequently, the result may contain Inf, -Inf, NA or NaN.

The function negidentitylink is the negative-identity link function and corresponds to $g(\theta)=$ $-\theta$. This is useful for some models, e.g., in the literature supporting the gevff function it seems that half of the authors use $\xi=-k$ for the shape parameter and the other half use $k$ instead of $\xi$.

## Value

For identitylink(): for deriv=0, the identity of theta, i.e., theta when inverse = FALSE, and if inverse $=$ TRUE then theta. For deriv $=1$, then the function returns $d$ eta / $d$ theta as a function of theta if inverse $=$ FALSE, else if inverse $=$ TRUE then it returns the reciprocal.
For negidentitylink(): the results are similar to identitylink() except for a sign change in most cases.

## Author(s)

Thomas W. Yee

## References

McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, 2nd ed. London: Chapman \& Hall.

## See Also

Links, loglink, logitlink, probitlink, powerlink.

## Examples

```
identitylink((-5):5)
identitylink((-5):5, deriv = 1)
identitylink((-5):5, deriv = 2)
negidentitylink((-5):5)
negidentitylink((-5):5, deriv = 1)
negidentitylink((-5):5, deriv = 2)
```

```
inv.binomial Inverse Binomial Distribution Family Function
```


## Description

Estimates the two parameters of an inverse binomial distribution by maximum likelihood estimation.

## Usage

inv.binomial(lrho = extlogitlink(min = 0.5, max = 1), llambda $=$ "loglink", irho = NULL, ilambda $=$ NULL, zero $=$ NULL)

## Arguments

lrho, llambda Link function for the $\rho$ and $\lambda$ parameters. See Links for more choices.
irho, ilambda Numeric. Optional initial values for $\rho$ and $\lambda$.
zero See CommonVGAMffArguments.

## Details

The inverse binomial distribution of Yanagimoto (1989) has density function

$$
f(y ; \rho, \lambda)=\frac{\lambda \Gamma(2 y+\lambda)}{\Gamma(y+1) \Gamma(y+\lambda+1)}\{\rho(1-\rho)\}^{y} \rho^{\lambda}
$$

where $y=0,1,2, \ldots$ and $\frac{1}{2}<\rho<1$, and $\lambda>0$. The first two moments exist for $\rho>\frac{1}{2}$; then the mean is $\lambda(1-\rho) /(2 \rho-1)$ (returned as the fitted values) and the variance is $\lambda \rho(1-\rho) /(2 \rho-1)^{3}$. The inverse binomial distribution is a special case of the generalized negative binomial distribution of Jain and Consul (1971). It holds that $\operatorname{Var}(Y)>E(Y)$ so that the inverse binomial distribution is overdispersed compared with the Poisson distribution.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

This VGAM family function only works reasonably well with intercept-only models. Good initial values are needed; if convergence failure occurs use irho and/or ilambda.
Some elements of the working weight matrices use the expected information matrix while other elements use the observed information matrix. Yet to do: using the mean and the reciprocal of $\lambda$ results in an EIM that is diagonal.

## Author(s)

T. W. Yee

## References

Yanagimoto, T. (1989). The inverse binomial distribution as a statistical model. Communications in Statistics: Theory and Methods, 18, 3625-3633.

Jain, G. C. and Consul, P. C. (1971). A generalized negative binomial distribution. SIAM Journal on Applied Mathematics, 21, 501-513.
Jorgensen, B. (1997). The Theory of Dispersion Models. London: Chapman \& Hall

## See Also

negbinomial, poissonff.

## Examples

```
idata <- data.frame(y = rnbinom(n <- 1000, mu = exp(3), size = exp(1)))
fit <- vglm(y ~ 1, inv.binomial, data = idata, trace = TRUE)
with(idata, c(mean(y), head(fitted(fit), 1)))
summary(fit)
coef(fit, matrix = TRUE)
Coef(fit)
```

sum(weights(fit)) \# Sum of the prior weights
sum(weights(fit, type = "work")) \# Sum of the working weights

Inv.gaussian The Inverse Gaussian Distribution

## Description

Density, distribution function and random generation for the inverse Gaussian distribution.

## Usage

dinv.gaussian(x, mu, lambda, log = FALSE)
pinv.gaussian(q, mu, lambda)
rinv.gaussian(n, mu, lambda)

## Arguments

$x, q \quad$ vector of quantiles.
$n \quad$ number of observations. If length $(n)>1$ then the length is taken to be the number required.
mu the mean parameter.
lambda the $\lambda$ parameter.
$\log \quad$ Logical. If $\log =$ TRUE then the logarithm of the density is returned.

## Details

See inv.gaussianff, the VGAM family function for estimating both parameters by maximum likelihood estimation, for the formula of the probability density function.

## Value

dinv.gaussian gives the density, pinv.gaussian gives the distribution function, and rinv.gaussian generates random deviates.

## Note

Currently qinv.gaussian is unavailable.

## Author(s)

T. W. Yee

## References

Johnson, N. L. and Kotz, S. and Balakrishnan, N. (1994). Continuous Univariate Distributions, 2nd edition, Volume 1, New York: Wiley.

Taraldsen, G. and Lindqvist, B. H. (2005). The multiple roots simulation algorithm, the inverse Gaussian distribution, and the sufficient conditional Monte Carlo method. Preprint Statistics No. 4/2005, Norwegian University of Science and Technology, Trondheim, Norway.

## See Also

```
inv.gaussianff, waldff.
```


## Examples

```
## Not run: x <- seq(-0.05, 4, len = 300)
plot(x, dinv.gaussian(x, mu = 1, lambda = 1), type = "l",
    col = "blue",las = 1, main =
    "blue is density, orange is cumulative distribution function")
abline(h = 0, col = "gray", lty = 2)
lines(x, pinv.gaussian(x, mu = 1, lambda = 1), type = "l", col = "orange")
## End(Not run)
```

inv.gaussianff Inverse Gaussian Distribution Family Function

## Description

Estimates the two parameters of the inverse Gaussian distribution by maximum likelihood estimation.

## Usage

inv.gaussianff(lmu = "loglink", llambda = "loglink", imethod = 1, ilambda = NULL, parallel $=$ FALSE, ishrinkage $=0.99$, zero $=$ NULL)

## Arguments

lmu, llambda Parameter link functions for the $\mu$ and $\lambda$ parameters. See Links for more choices.
ilambda, parallel
See CommonVGAMffArguments for more information. If parallel = TRUE then the constraint is not applied to the intercept.
imethod, ishrinkage, zero See CommonVGAMffArguments for information.

## Details

The standard ("canonical") form of the inverse Gaussian distribution has a density that can be written as

$$
f(y ; \mu, \lambda)=\sqrt{\lambda /\left(2 \pi y^{3}\right)} \exp \left(-\lambda(y-\mu)^{2} /\left(2 \mu^{2} y\right)\right)
$$

where $y>0, \mu>0$, and $\lambda>0$. The mean of $Y$ is $\mu$ and its variance is $\mu^{3} / \lambda$. By default, $\eta_{1}=\log (\mu)$ and $\eta_{2}=\log (\lambda)$. The mean is returned as the fitted values. This VGAM family function can handle multiple responses (inputted as a matrix).

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Note

The inverse Gaussian distribution can be fitted (to a certain extent) using the usual GLM framework involving a scale parameter. This family function is different from that approach in that it estimates both parameters by full maximum likelihood estimation.

## Author(s)

T. W. Yee

## References

Johnson, N. L. and Kotz, S. and Balakrishnan, N. (1994). Continuous Univariate Distributions, 2nd edition, Volume 1, New York: Wiley.

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011). Statistical Distributions, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

## See Also

Inv.gaussian, waldff, bisa.
The R package SuppDists has several functions for evaluating the density, distribution function, quantile function and generating random numbers from the inverse Gaussian distribution.

## Examples

```
idata <- data.frame(x2 = runif(nn <- 1000))
idata <- transform(idata, mymu = exp(2 + 1 * x2),
    Lambda = exp(2 + 1 * x2))
idata <- transform(idata, y = rinv.gaussian(nn, mu = mymu, lambda = Lambda))
fit1 <- vglm(y ~ x2, inv.gaussianff, data = idata, trace = TRUE)
rrig <- rrvglm(y ~ x2, inv.gaussianff, data = idata, trace = TRUE)
coef(fit1, matrix = TRUE)
coef(rrig, matrix = TRUE)
Coef(rrig)
summary(fit1)
```

Inv.lomax The Inverse Lomax Distribution

## Description

Density, distribution function, quantile function and random generation for the inverse Lomax distribution with shape parameter $p$ and scale parameter scale.

## Usage

dinv.lomax (x, scale $=1$, shape2.p, log = FALSE)
pinv.lomax (q, scale = 1, shape2.p, lower.tail = TRUE, log.p = FALSE)
qinv.lomax (p, scale = 1, shape2.p, lower.tail = TRUE, log.p = FALSE)
rinv.lomax (n, scale = 1, shape2.p)

## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$\mathrm{n} \quad$ number of observations. If length $(\mathrm{n})>1$, the length is taken to be the number required.
shape2.p shape parameter.
scale scale parameter.
log Logical. If log = TRUE then the logarithm of the density is returned.
lower.tail, log.p
Same meaning as in pnorm or qnorm.

## Details

See inv.lomax, which is the VGAM family function for estimating the parameters by maximum likelihood estimation.

## Value

dinv.lomax gives the density, pinv.lomax gives the distribution function, qinv. lomax gives the quantile function, and rinv.lomax generates random deviates.

## Note

The inverse Lomax distribution is a special case of the 4-parameter generalized beta II distribution.

## Author(s)

T. W. Yee

## References

Kleiber, C. and Kotz, S. (2003). Statistical Size Distributions in Economics and Actuarial Sciences, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

inv.lomax, genbetaII.

## Examples

```
idata <- data.frame(y = rinv.lomax(n = 1000, exp(2), exp(1)))
fit <- vglm(y ~ 1, inv.lomax, data = idata, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit)
```

inv.lomax Inverse Lomax Distribution Family Function

## Description

Maximum likelihood estimation of the 2-parameter inverse Lomax distribution.

## Usage

inv.lomax(lscale = "loglink", lshape2.p = "loglink", iscale = NULL, ishape2.p $=$ NULL, imethod $=1$, gscale $=\exp (-5: 5)$, gshape2.p $=\exp (-5: 5)$, probs.y $=c(0.25,0.5,0.75)$, zero $=" s h a p e 2 . p ")$

## Arguments

lscale, lshape2.p
Parameter link functions applied to the (positive) parameters $b$, and $p$. See Links for more choices.
iscale, ishape2.p, imethod, zero
See CommonVGAMffArguments for information. For imethod $=2$ a good initial value for ishape2. p is needed to obtain a good estimate for the other parameter.
gscale, gshape2.p
See CommonVGAMffArguments for information.
probs.y See CommonVGAMffArguments for information.

## Details

The 2-parameter inverse Lomax distribution is the 4-parameter generalized beta II distribution with shape parameters $a=q=1$. It is also the 3-parameter Dagum distribution with shape parameter $a=1$, as well as the beta distribution of the second kind with $q=1$. More details can be found in Kleiber and Kotz (2003).

The inverse Lomax distribution has density

$$
f(y)=p y^{p-1} /\left[b^{p}\{1+y / b\}^{p+1}\right]
$$

for $b>0, p>0, y \geq 0$. Here, $b$ is the scale parameter scale, and $p$ is a shape parameter. The mean does not seem to exist; the median is returned as the fitted values. This family function handles multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

See the notes in genbetaII.

## Author(s)

T. W. Yee

## References

Kleiber, C. and Kotz, S. (2003). Statistical Size Distributions in Economics and Actuarial Sciences, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

inv.lomax, genbetaII, betaII, dagum, sinmad, fisk, lomax, paralogistic, inv. paralogistic, simulate.vlm.

## Examples

```
idata <- data.frame(y = rinv.lomax(n = 2000, scale = exp(2), exp(1)))
fit <- vglm(y ~ 1, inv.lomax, data = idata, trace = TRUE)
fit <- vglm(y ~ 1, inv.lomax(iscale = exp(3)), data = idata,
    trace = TRUE, epsilon = 1e-8, crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```


## Description

Density, distribution function, quantile function and random generation for the inverse paralogistic distribution with shape parameters a and $p$, and scale parameter scale.

## Usage

dinv.paralogistic(x, scale = 1, shape1.a, log = FALSE)
pinv.paralogistic (q, scale $=1$, shape1.a, lower.tail = TRUE, log.p = FALSE)
qinv.paralogistic ( $p$, scale $=1$, shape1.a, lower.tail $=$ TRUE, log.p = FALSE)
rinv.paralogistic( $n$, scale $=1$, shape1.a)

## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$\mathrm{n} \quad$ number of observations. If length $(\mathrm{n})>1$, the length is taken to be the number required.
shape1.a shape parameter.
scale scale parameter.
log Logical. If log = TRUE then the logarithm of the density is returned.
lower.tail, log.p
Same meaning as in pnorm or qnorm.

## Details

See inv.paralogistic, which is the VGAM family function for estimating the parameters by maximum likelihood estimation.

## Value

dinv. paralogistic gives the density, pinv. paralogistic gives the distribution function, qinv. paralogistic gives the quantile function, and rinv. paralogistic generates random deviates.

## Note

The inverse paralogistic distribution is a special case of the 4-parameter generalized beta II distribution.

## Author(s)

T. W. Yee

## References

Kleiber, C. and Kotz, S. (2003). Statistical Size Distributions in Economics and Actuarial Sciences, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

inv.paralogistic, genbetaII.

## Examples

```
idata <- data.frame(y = rinv.paralogistic(n = 3000, exp(1), scale = exp(2)))
fit <- vglm(y ~ 1, inv.paralogistic(lss = FALSE, ishape1.a = 2.1),
            data = idata, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit)
```

```
inv.paralogistic Inverse Paralogistic Distribution Family Function
```


## Description

Maximum likelihood estimation of the 2-parameter inverse paralogistic distribution.

## Usage

inv.paralogistic(lscale = "loglink", lshape1.a = "loglink", iscale = NULL, ishape1.a $=$ NULL, imethod $=1$, lss $=$ TRUE, gscale $=\exp (-5: 5)$, gshape1.a $=\operatorname{seq}(0.75,4$, by $=0.25)$, probs.y $=c(0.25,0.5,0.75)$, zero = "shape")

## Arguments

lss See CommonVGAMffArguments for important information.
lshape1.a, lscale
Parameter link functions applied to the (positive) parameters a and scale. See Links for more choices.
iscale, ishape1.a, imethod, zero
See CommonVGAMffArguments for information. For imethod $=2$ a good initial value for ishape1. a is needed to obtain a good estimate for the other parameter.
gscale, gshape1.a
See CommonVGAMffArguments for information.
probs.y See CommonVGAMffArguments for information.

## Details

The 2-parameter inverse paralogistic distribution is the 4-parameter generalized beta II distribution with shape parameter $q=1$ and $a=p$. It is the 3-parameter Dagum distribution with $a=p$. More details can be found in Kleiber and Kotz (2003).
The inverse paralogistic distribution has density

$$
f(y)=a^{2} y^{a^{2}-1} /\left[b^{a^{2}}\left\{1+(y / b)^{a}\right\}^{a+1}\right]
$$

for $a>0, b>0, y \geq 0$. Here, $b$ is the scale parameter scale, and $a$ is the shape parameter. The mean is

$$
E(Y)=b \Gamma(a+1 / a) \Gamma(1-1 / a) / \Gamma(a)
$$

provided $a>1$; these are returned as the fitted values. This family function handles multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

See the notes in genbetaII.

## Author(s)

T. W. Yee

## References

Kleiber, C. and Kotz, S. (2003). Statistical Size Distributions in Economics and Actuarial Sciences, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

Inv. paralogistic, genbetaII, betaII, dagum, sinmad, fisk, inv.lomax, lomax, paralogistic, simulate.vlm.

## Examples

```
idata <- data.frame(y = rinv.paralogistic(n = 3000, exp(1), scale = exp(2)))
fit <- vglm(y ~ 1, inv.paralogistic(lss = FALSE), data = idata, trace = TRUE)
fit <- vglm(y ~ 1, inv.paralogistic(imethod = 2, ishape1.a = 4),
    data = idata, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```


## Description

Checks to see if a fitted object suffers from some known bug.

## Usage

is.buggy (object, ...)
is.buggy.vlm(object, each.term = FALSE, ...)

## Arguments

object A fitted VGAM object, e.g., from vgam.
each. term Logical. If TRUE then a logical is returned for each term.
... Unused for now.

## Details

It is known that vgam with s terms do not correctly handle constraint matrices (cmat, say) when crossprod (cmat) is not diagonal. This function detects whether this is so or not. Note that probably all VGAM family functions have defaults where all crossprod(cmat)s are diagonal, therefore do not suffer from this bug. It is more likely to occur if the user inputs constraint matrices using the constraints argument (and setting zero = NULL if necessary).
Second-generation VGAMs based on sm. ps are a modern alternative to using s. It does not suffer from this bug. However, G2-VGAMs require a reasonably large sample size in order to work more reliably.

## Value

The default is a single logical (TRUE if any term is TRUE), otherwise a vector of such with each element corresponding to a term. If the value is TRUE then I suggest replacing the VGAM by a similar model fitted by vglm and using regression splines, e.g., bs, ns.

## Note

When the bug is fixed this function may be withdrawn, otherwise always return FALSEs!

## Author(s)

T. W. Yee

## See Also

vgam. vglm, s, sm.ps, bs, ns.

## Examples

```
fit1 <- vgam(cbind(agaaus, kniexc) ~ s(altitude, df = c(3, 4)),
    binomialff(multiple.responses = TRUE), data = hunua)
is.buggy(fit1) # Okay
is.buggy(fit1, each.term = TRUE) # No terms are buggy
fit2 <- vgam(cbind(agaaus, kniexc) ~ s(altitude, df = c(3, 4)),
            binomialff(multiple.responses = TRUE), data = hunua,
            constraints =
            list("(Intercept)" = diag(2),
                "s(altitude, df = c(3, 4))" = matrix(c(1, 1, 0, 1), 2, 2)))
is.buggy(fit2) # TRUE
is.buggy(fit2, each.term = TRUE)
constraints(fit2)
# fit2b is an approximate alternative to fit2:
fit2b <- vglm(cbind(agaaus, kniexc) ~ bs(altitude, df = 3) + bs(altitude, df = 4),
        binomialff(multiple.responses = TRUE), data = hunua,
        constraints =
        list("(Intercept)" = diag(2),
            "bs(altitude, df = 3)" = rbind(1, 1),
            "bs(altitude, df = 4)" = rbind(0, 1)))
is.buggy(fit2b) # Okay
is.buggy(fit2b, each.term = TRUE)
constraints(fit2b)
```

is.crossing Quantile Crossing Detection

## Description

Returns a logical from testing whether an object such as an extlogF1() VGLM object has crossing quantiles.

## Usage

is.crossing.vglm(object, ...)

## Arguments

object an object such as a vglm object with family function extlogF1.
... additional optional arguments. Currently unused.

## Details

This function was specifically written for a vglm with family function extlogF1. It examines the fitted quantiles to see if any cross. Note that if one uses regression splines such as bs and ns then it is possible that they cross at values of the covariate space that are not represented by actual data. One could use linear interpolation between fitted values to get around this problem.

## Value

A logical. If TRUE then one can try fit a similar model by combining columns of the constraint matrices so that crossing no longer holds; see fix.crossing. For LMS-Box-Cox type quantile regression models it is impossible for the quantiles to cross, by definition, hence FALSE is returned; see lms.bcn.

## See Also

extlogF1, fix.crossing, lms.bcn. vglm.

## Examples

```
## Not run: ooo <- with(bmi.nz, order(age))
bmi.nz <- bmi.nz[ooo, ] # Sort by age
with(bmi.nz, plot(age, BMI, col = "blue"))
mytau <- c(50, 93, 95, 97) / 100 # Some quantiles are quite close
fit1 <- vglm(BMI ~ ns(age, 7), extlogF1(mytau), bmi.nz, trace = TRUE)
plot(BMI ~ age, bmi.nz, col = "blue", las = 1,
    main = "Partially parallel (darkgreen) & nonparallel quantiles",
    sub = "Crossing quantiles are orange")
is.crossing(fit1)
matlines(with(bmi.nz, age), fitted(fit1), lty = 1, col = "orange")
## End(Not run)
```

```
is.parallel Parallelism Constraint Matrices
```


## Description

Returns a logical vector from a test of whether an object such as a matrix or VGLM object corresponds to a parallelism assumption.

## Usage

```
is.parallel.matrix(object, ...)
is.parallel.vglm(object, type = c("term", "lm"), ...)
```


## Arguments

object an object such as a constraint matrix or a vglm object.
type passed into constraints.
... additional optional arguments. Currently unused.

## Details

These functions may be useful for categorical models such as propodds, cumulative, acat, cratio, sratio, multinomial.

Value
A vector of logicals, testing whether each constraint matrix is a one-column matrix of ones. Note that parallelism can still be thought of as holding if the constraint matrix has a non-zero but constant values, however, this is currently not implemented. No checking is done that the constraint matrices have the same number of rows.

## See Also

```
constraints, vglm.
```


## Examples

```
## Not run: require("VGAMdata")
fit <- vglm(educ ~ sm.bs(age) * sex + ethnicity,
    cumulative(parallel = TRUE), head(xs.nz, 200))
is.parallel(fit)
is.parallel(fit, type = "lm") # For each column of the LM matrix
## End(Not run)
```

```
is.smart Test For a Smart Object
```


## Description

Tests an object to see if it is smart.

## Usage

```
is.smart(object)
```


## Arguments

object a function or a fitted model.

## Details

If object is a function then this function looks to see whether object has the logical attribute "smart". If so then this is returned, else FALSE.
If object is a fitted model then this function looks to see whether object@smart. prediction or object\$smart.prediction exists. If it does and it is not equal to list(smart.arg=FALSE) then a TRUE is returned, else FALSE. The reason for this is because, e.g., lm( . . , smart=FALSE) and vglm(... , smart=FALSE), will return such a specific list.
Writers of smart functions manually have to assign this attribute to their smart function after it has been written.

## Value

Returns TRUE or FALSE, according to whether the object is smart or not.

## Examples

```
is.smart(sm.min1) # TRUE
is.smart(sm.poly) # TRUE
library(splines)
is.smart(sm.bs) # TRUE
is.smart(sm.ns) # TRUE
is.smart(tan) # FALSE
## Not run:
udata <- data.frame(x2 = rnorm(9))
fit1 <- vglm(rnorm(9) ~ x2, uninormal, data = udata)
is.smart(fit1) # TRUE
fit2 <- vglm(rnorm(9) ~ x2, uninormal, data = udata, smart = FALSE)
is.smart(fit2) # FALSE
fit2@smart.prediction
## End(Not run)
```

is.zero Zero Constraint Matrices

## Description

Returns a logical vector from a test of whether an object such as a matrix or VGLM object corresponds to a 'zero' assumption.

## Usage

is.zero.matrix(object, ...)
is.zero.vglm(object, ...)

## Arguments

object an object such as a coefficient matrix of a vglm object, or a vglm object.
... additional optional arguments. Currently unused.

## Details

These functions test the effect of the zero argument on a vglm object or the coefficient matrix of a vglm object. The latter is obtained by coef (vglmObject, matrix = TRUE).

## Value

A vector of logicals, testing whether each linear/additive predictor has the zero argument applied to it. It is TRUE if that linear/additive predictor is intercept-only, i.e., all other regression coefficients are set to zero.
No checking is done for the intercept term at all, i.e., that it was estimated in the first place.

## See Also

constraints, vglm.

## Examples

```
coalminers <- transform(coalminers, Age = (age - 42) / 5)
fit <- vglm(cbind(nBnW,nBW,BnW,BW) ~ Age, binom2.or(zero = NULL), coalminers)
is.zero(fit)
is.zero(coef(fit, matrix = TRUE))
```

```
kendall.tau Kendall's Tau Statistic
```


## Description

Computes Kendall's Tau, which is a rank-based correlation measure, between two vectors.

## Usage

kendall.tau(x, y, exact $=$ FALSE, max. $\mathrm{n}=3000$ )

## Arguments

$x, y \quad$ Numeric vectors. Must be of equal length. Ideally their values are continuous and not too discrete. Let length ( x ) be $N$, say.
exact Logical. If TRUE then the exact value is computed.
max.n Numeric. If exact = FALSE and length $(x)$ is more than max. $n$ then a random sample of max. n pairs are chosen.

## Details

Kendall's tau is a measure of dependency in a bivariate distribution. Loosely, two random variables are concordant if large values of one random variable are associated with large values of the other random variable. Similarly, two random variables are disconcordant if large values of one random variable are associated with small values of the other random variable. More formally, if $(x[i]-x[j]) *(y[i]-y[j])>0$ then that comparison is concordant $(i \neq j)$. And if $(x[i]$ $-x[j]) *(y[i]-y[j])<0$ then that comparison is disconcordant $(i \neq j)$. Out of choose $(N, 2)$ comparisons, let $c$ and $d$ be the number of concordant and disconcordant pairs. Then Kendall's tau can be estimated by $(c-d) /(c+d)$. If there are ties then half the ties are deemed concordant and half disconcordant so that $(c-d) /(c+d+t)$ is used.

## Value

Kendall's tau, which lies between -1 and 1.

## Warning

If length $(\mathrm{x})$ is large then the cost is $O\left(N^{2}\right)$, which is expensive! Under these circumstances it is not advisable to set exact = TRUE or max. $n$ to a very large number.

## See Also

binormalcop, cor.

## Examples

```
N <- 5000; x <- 1:N; y <- runif(N)
true.rho <- -0.8
ymat <- rbinorm(N, cov12 = true.rho) # Bivariate normal, aka N_2
x <- ymat[, 1]
y <- ymat[, 2]
## Not run: plot(x, y, col = "blue")
kendall.tau(x, y) # A random sample is taken here
kendall.tau(x, y) # A random sample is taken here
kendall.tau(x, y, exact = TRUE) # Costly if length(x) is large
kendall.tau(x, y, max.n = N) # Same as exact = TRUE
(rhohat <- sin(kendall.tau(x, y) * pi / 2)) # This formula holds for N_2 actually
true.rho # rhohat should be near this value
```

Kumar The Kumaraswamy Distribution

## Description

Density, distribution function, quantile function and random generation for the Kumaraswamy distribution.

## Usage

dkumar (x, shape1, shape2, log = FALSE)
pkumar (q, shape1, shape2, lower.tail = TRUE, log.p = FALSE)
qkumar ( $p$, shape1, shape2, lower.tail = TRUE, log.p = FALSE)
rkumar ( n , shape1, shape2)

## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$n \quad$ number of observations. If length $(n)>1$ then the length is taken to be the number required.

```
shape1, shape2 positive shape parameters.
log Logical. If log = TRUE then the logarithm of the density is returned.
lower.tail, log.p
    Same meaning as in pnorm or qnorm.
```


## Details

See kumar, the VGAM family function for estimating the parameters, for the formula of the probability density function and other details.

## Value

dkumar gives the density, pkumar gives the distribution function, qkumar gives the quantile function, and rkumar generates random deviates.

## Author(s)

T. W. Yee and Kai Huang

## See Also

kumar.

## Examples

```
## Not run:
shape1 <- 2; shape2 <- 2; nn <- 201; # shape1 <- shape2 <- 0.5;
x <- seq(-0.05, 1.05, len = nn)
plot(x, dkumar(x, shape1, shape2), type = "l", las = 1, ylim = c(0,1.5),
    ylab = paste("fkumar(shape1 = ", shape1, ", shape2 = ", shape2, ")"),
    col = "blue", cex.main = 0.8,
    main = "Blue is density, orange is cumulative distribution function",
    sub = "Purple lines are the 10,20,...,90 percentiles")
lines(x, pkumar(x, shape1, shape2), col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qkumar(probs, shape1, shape2)
lines(Q, dkumar(Q, shape1, shape2), col = "purple", lty = 3, type = "h")
lines(Q, pkumar(Q, shape1, shape2), col = "purple", lty = 3, type = "h")
abline(h = probs, col = "purple", lty = 3)
max(abs(pkumar(Q, shape1, shape2) - probs)) # Should be 0
## End(Not run)
```

kumar Kumaraswamy Regression Family Function

## Description

Estimates the two parameters of the Kumaraswamy distribution by maximum likelihood estimation.

## Usage

```
kumar(lshape1 = "loglink", lshape2 = "loglink",
    ishape1 = NULL, ishape2 = NULL, gshape1 = exp(2*ppoints(5) - 1),
    tol12 = 1.0e-4, zero = NULL)
```


## Arguments

1shape1, lshape2
Link function for the two positive shape parameters, respectively, called $a$ and $b$ below. See Links for more choices.
ishape1, ishape2
Numeric. Optional initial values for the two positive shape parameters.
tol12 Numeric and positive. Tolerance for testing whether the second shape parameter is either 1 or 2 . If so then the working weights need to handle these singularities.
gshape1 Values for a grid search for the first shape parameter. See CommonVGAMffArguments for more information.
zero See CommonVGAMffArguments.

## Details

The Kumaraswamy distribution has density function

$$
f(y ; a=\text { shape } 1, b=\text { shape } 2)=a b y^{a-1}\left(1-y^{a}\right)^{b-1}
$$

where $0<y<1$ and the two shape parameters, $a$ and $b$, are positive. The mean is $b \times \operatorname{Beta}(1+$ $1 / a, b)$ (returned as the fitted values) and the variance is $b \times \operatorname{Beta}(1+2 / a, b)-(b \times \operatorname{Beta}(1+$ $1 / a, b))^{2}$. Applications of the Kumaraswamy distribution include the storage volume of a water reservoir. Fisher scoring is implemented. Handles multiple responses (matrix input).

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Author(s)

T. W. Yee

## References

Kumaraswamy, P. (1980). A generalized probability density function for double-bounded random processes. Journal of Hydrology, 46, 79-88.
Jones, M. C. (2009). Kumaraswamy's distribution: A beta-type distribution with some tractability advantages. Statistical Methodology, 6, 70-81.

## See Also

```
dkumar, betaff, simulate.vlm.
```


## Examples

```
shape1 <- exp(1); shape2 <- exp(2)
kdata <- data.frame(y = rkumar( }\textrm{n}=1000\mathrm{ , shape1, shape2))
fit <- vglm(y ~ 1, kumar, data = kdata, trace = TRUE)
c(with(kdata, mean(y)), head(fitted(fit), 1))
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```


## lake0 Annual catches on Lake Otamangakau from October 1974 to October 1989

## Description

Rainbow and brown trout catches by a Mr Swainson at Lake Otamangakau in the central North Island of New Zealand during the 1970s and 1980s.

## Usage

data(lake0)

## Format

A data frame with 15 observations on the following 5 variables.
year a numeric vector, the season began on 1 October of the year and ended 12 months later.
total.fish a numeric vector, the total number of fish caught during the season. Simply the sum of brown and rainbow trout.
brown a numeric vector, the number of brown trout (Salmo trutta) caught.
rainbow a numeric vector, the number of rainbow trout (Oncorhynchus mykiss) caught.
visits a numeric vector, the number of visits during the season that the angler made to the lake. It is necessary to assume that the visits were of an equal time length in order to interpret the usual Poisson regressions.

## Details

The data was extracted from the season summaries at Lake Otamangakau by Anthony Swainson for the seasons 1974-75 to 1988-89.
Mr Swainson was one of a small group of regular fly fishing anglers and kept a diary of his catches. Lake Otamangakau is a lake of area 1.8 squared km and has a maximum depth of about 12 m , and is located in the central North Island of New Zealand. It is trout-infested and known for its trophy-sized fish.
See also trap0.

## Source

Table 7.2 of the reference below. Thanks to Dr Michel Dedual for a copy of the report and for help reading the final year's data. The report is available from TWY on request.

## References

Dedual, M. and MacLean, G. and Rowe, D. and Cudby, E., The Trout Population and Fishery of Lake Otamangakau-Interim Report. National Institute of Water and Atmospheric Research, Hamilton, New Zealand. Consultancy Report Project No. ELE70207, (Dec 1996).

## Examples

data(lake0)
lake0
summary (lake0)
lambertW The Lambert W Function

## Description

Computes the Lambert $W$ function for real values.

## Usage

lambertW(x, tolerance $=1 \mathrm{e}-10$, maxit $=50$ )

## Arguments

x
tolerance Accuracy desired.
maxit Maximum number of iterations of third-order Halley's method.

## Details

The Lambert $W$ function is the root of the equation $W(z) \exp (W(z))=z$ for complex $z$. If $z$ is real and $-1 / e<z<0$ then it has two possible real values, and currently only the upper branch (often called $W_{0}$ ) is computed so that a value that is $\geq-1$ is returned.

## Value

This function returns the principal branch of the $W$ function for real $z$. It returns $W(z) \geq-1$, and NA for $z<-1 / e$.

## Note

If convergence does not occur then increase the value of maxit and/or tolerance.
Yet to do: add an argument lbranch = TRUE to return the lower branch (often called $W_{-1}$ ) for real $-1 / e \leq z<0$; this would give $W(z) \leq-1$.

## Author(s)

T. W. Yee

## References

Corless, R. M. and Gonnet, G. H. and Hare, D. E. G. and Jeffrey, D. J. and Knuth, D. E. (1996). On the Lambert $W$ function. Advances in Computational Mathematics, 5(4), 329-359.

## See Also

log, exp, bell. There is also a package called LambertW.

## Examples

```
    ## Not run:
curve(lambertW, -exp(-1), 3, xlim = c(-1, 3), ylim = c(-2, 1),
    las = 1, col = "orange", n = 1001)
abline(v = -exp(-1), h = -1, lwd = 2, lty = "dotted", col = "gray")
abline(h = 0, v = 0, lty = "dashed", col = "blue")
## End(Not run)
```

laplace Laplace Regression Family Function

## Description

Maximum likelihood estimation of the 2-parameter classical Laplace distribution.

## Usage

```
laplace(llocation = "identitylink", lscale = "loglink",
    ilocation = NULL, iscale = NULL, imethod = 1, zero = "scale")
```


## Arguments

```
llocation, lscale
```

Character. Parameter link functions for location parameter $a$ and scale parameter b. See Links for more choices.
ilocation, iscale
Optional initial values. If given, it must be numeric and values are recycled to the appropriate length. The default is to choose the value internally.
imethod Initialization method. Either the value 1 or 2.
zero See CommonVGAMffArguments for information.

## Details

The Laplace distribution is often known as the double-exponential distribution and, for modelling, has heavier tail than the normal distribution. The Laplace density function is

$$
f(y)=\frac{1}{2 b} \exp \left(-\frac{|y-a|}{b}\right)
$$

where $-\infty<y<\infty,-\infty<a<\infty$ and $b>0$. Its mean is $a$ and its variance is $2 b^{2}$. This parameterization is called the classical Laplace distribution by Kotz et al. (2001), and the density is symmetric about $a$.
For $\mathrm{y} \sim 1$ (where y is the response) the maximum likelihood estimate (MLE) for the location parameter is the sample median, and the MLE for $b$ is mean(abs( $y$-location)) (replace location by its MLE if unknown).

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Warning

This family function has not been fully tested. The MLE regularity conditions do not hold for this distribution, therefore misleading inferences may result, e.g., in the summary and vcov of the object. Hence this family function might be withdrawn from VGAM in the future.

## Note

This family function uses Fisher scoring. Convergence may be slow for non-intercept-only models; half-stepping is frequently required.

## Author(s)

T. W. Yee

## References

Kotz, S., Kozubowski, T. J. and Podgorski, K. (2001). The Laplace distribution and generalizations: a revisit with applications to communications, economics, engineering, and finance, Boston: Birkhauser.

## See Also

rlaplace, alaplace2 (which differs slightly from this parameterization), exponential, median.

## Examples

```
ldata <- data.frame(y = rlaplace(nn <- 100, loc = 2, scale = exp(1)))
fit <- vglm(y ~ 1, laplace, data = ldata, trace = TRUE, crit = "l")
coef(fit, matrix = TRUE)
Coef(fit)
with(ldata, median(y))
ldata <- data.frame(x = runif(nn <- 1001))
ldata <- transform(ldata, y = rlaplace(nn, loc = 2, scale = exp(-1 + 1*x)))
coef(vglm(y ~ x, laplace(iloc = 0.2, imethod = 2, zero = 1), data = ldata,
        trace = TRUE), matrix = TRUE)
```

laplaceUC The Laplace Distribution

## Description

Density, distribution function, quantile function and random generation for the Laplace distribution with location parameter location and scale parameter scale.

## Usage

```
dlaplace(x, location = 0, scale = 1, log = FALSE)
plaplace(q, location = 0, scale = 1, lower.tail = TRUE, log.p = FALSE)
qlaplace(p, location = 0, scale = 1, lower.tail = TRUE, log.p = FALSE)
rlaplace(n, location = 0, scale = 1)
```


## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$n \quad$ number of observations. Same as in runif.
location the location parameter $a$, which is the mean.
scale the scale parameter $b$. Must consist of positive values.
$\log \quad$ Logical. If $\log =$ TRUE then the logarithm of the density is returned.
lower.tail, log.p
Same meaning as in pnorm or qnorm.

## Details

The Laplace distribution is often known as the double-exponential distribution and, for modelling, has heavier tail than the normal distribution. The Laplace density function is

$$
f(y)=\frac{1}{2 b} \exp \left(-\frac{|y-a|}{b}\right)
$$

where $-\infty<y<\infty,-\infty<a<\infty$ and $b>0$. The mean is $a$ and the variance is $2 b^{2}$.
See laplace, the VGAM family function for estimating the two parameters by maximum likelihood estimation, for formulae and details. Apart from n, all the above arguments may be vectors and are recyled to the appropriate length if necessary.

## Value

dlaplace gives the density, plaplace gives the distribution function, qlaplace gives the quantile function, and rlaplace generates random deviates.

## Author(s)

T. W. Yee and Kai Huang

## References

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011). Statistical Distributions, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

## See Also

laplace.

## Examples

```
loc <- 1; b <- 2
y <- rlaplace(n = 100, loc = loc, scale = b)
mean(y) # sample mean
loc # population mean
var(y) # sample variance
2 * b^2 # population variance
## Not run: loc <- 0; b <- 1.5; x <- seq(-5, 5, by = 0.01)
plot(x, dlaplace(x, loc, b), type = "l", col = "blue", ylim = c(0,1),
    main = "Blue is density, orange is cumulative distribution function",
    sub = "Purple are 5,10,\ldots.,95 percentiles", las = 1, ylab = "")
abline(h = 0, col = "blue", lty = 2)
lines(qlaplace(seq(0.05,0.95,by = 0.05), loc, b),
    dlaplace(qlaplace(seq(0.05, 0.95, by = 0.05), loc, b), loc, b),
    col = "purple", lty = 3, type = "h")
lines(x, plaplace(x, loc, b), type = "l", col = "orange")
abline(h = 0, lty = 2)
## End(Not run)
plaplace(qlaplace(seq(0.05, 0.95, by = 0.05), loc, b), loc, b)
```

latvar Latent Variables

## Description

Generic function for the latent variables of a model.

## Usage

latvar(object, ...)
lv(object, ...)

## Arguments

object An object for which the extraction of latent variables is meaningful.
... Other arguments fed into the specific methods function of the model. Sometimes they are fed into the methods function for Coef.

## Details

Latent variables occur in reduced-rank regression models, as well as in quadratic and additive ordination models. For the latter two, latent variable values are often called site scores by ecologists. Latent variables are linear combinations of the explanatory variables.

## Value

The value returned depends specifically on the methods function invoked.

## Warning

latvar and $l v$ are identical, but the latter will be deprecated soon.
Latent variables are not really applicable to vglm/vgam models.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Hastie, T. J. (2003). Reduced-rank vector generalized linear models. Statistical Modelling, 3, 15-41.
Yee, T. W. (2004). A new technique for maximum-likelihood canonical Gaussian ordination. Ecological Monographs, 74, 685-701.
Yee, T. W. (2006). Constrained additive ordination. Ecology, 87, 203-213.

See Also
latvar.qrrvglm, latvar.rrvglm, latvar.cao, lvplot.

## Examples

```
## Not run:
hspider[, 1:6] <- scale(hspider[, 1:6]) # Standardized environmental vars
set.seed(123)
p1 <- cao(cbind(Pardlugu, Pardmont, Pardnigr, Pardpull, Zoraspin) ~
    WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
    family = poissonff, data = hspider, Rank = 1, df1.nl =
    c(Zoraspin = 2.5, 3), Bestof = 3, Crow1positive = TRUE)
var(latvar(p1)) # Scaled to unit variance # Scaled to unit variance
c(latvar(p1)) # Estimated site scores
## End(Not run)
```

leipnik Leipnik Regression Family Function

## Description

Estimates the two parameters of a (transformed) Leipnik distribution by maximum likelihood estimation.

## Usage

```
leipnik(lmu = "logitlink", llambda = logofflink(offset = 1),
```

    imu = NULL, ilambda = NULL)
    
## Arguments

lmu, llambda Link function for the $\mu$ and $\lambda$ parameters. See Links for more choices.
imu, ilambda Numeric. Optional initial values for $\mu$ and $\lambda$.

## Details

The (transformed) Leipnik distribution has density function

$$
f(y ; \mu, \lambda)=\frac{\{y(1-y)\}^{-\frac{1}{2}}}{\operatorname{Beta}\left(\frac{\lambda+1}{2}, \frac{1}{2}\right)}\left[1+\frac{(y-\mu)^{2}}{y(1-y)}\right]^{-\frac{\lambda}{2}}
$$

where $0<y<1$ and $\lambda>-1$. The mean is $\mu$ (returned as the fitted values) and the variance is $1 / \lambda$.
Jorgensen (1997) calls the above the transformed Leipnik distribution, and if $y=(x+1) / 2$ and $\mu=(\theta+1) / 2$, then the distribution of $X$ as a function of $x$ and $\theta$ is known as the the (untransformed) Leipnik distribution. Here, both $x$ and $\theta$ are in $(-1,1)$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Note

Convergence may be slow or fail. Until better initial value estimates are forthcoming try assigning the argument ilambda some numerical value if it fails to converge. Currently, Newton-Raphson is implemented, not Fisher scoring. Currently, this family function probably only really works for intercept-only models, i.e., $\mathrm{y} \sim 1$ in the formula.

## Author(s)

T. W. Yee

## References

Jorgensen, B. (1997). The Theory of Dispersion Models. London: Chapman \& Hall
Johnson, N. L. and Kotz, S. and Balakrishnan, N. (1995). Continuous Univariate Distributions, 2nd edition, Volume 2, New York: Wiley. (pages 612-617).

## See Also

mccullagh89.

## Examples

```
ldata <- data.frame(y = rnorm(n = 2000, mean = 0.5, sd = 0.1)) # Not proper data
fit <- vglm(y ~ 1, leipnik(ilambda = 1), data = ldata, trace = TRUE)
head(fitted(fit))
with(ldata, mean(y))
summary(fit)
coef(fit, matrix = TRUE)
Coef(fit)
sum(weights(fit)) # Sum of the prior weights
sum(weights(fit, type = "work")) # Sum of the working weights
```

    lerch Lerch Phi Function
    
## Description

Computes the Lerch Phi function.

## Usage

$\operatorname{lerch}(x, s, v$, tolerance $=1.0 \mathrm{e}-10$, iter $=100)$

## Arguments

$x, s, v \quad$ Numeric. This function recyles values of $x, s$, and $v$ if necessary.
tolerance $\quad$ Numeric. Accuracy required, must be positive and less than 0.01.
iter Maximum number of iterations allowed to obtain convergence. If iter is too small then a result of NA may occur; if so, try increasing its value.

## Details

Also known as the Lerch transcendent, it can be defined by an integral involving analytical continuation. An alternative definition is the series

$$
\Phi(x, s, v)=\sum_{n=0}^{\infty} \frac{x^{n}}{(n+v)^{s}}
$$

which converges for $|x|<1$ as well as for $|x|=1$ with $s>1$. The series is undefined for integers $v<=0$. Actually, $x$ may be complex but this function only works for real $x$. The algorithm used is based on the relation

$$
\Phi(x, s, v)=x^{m} \Phi(x, s, v+m)+\sum_{n=0}^{m-1} \frac{x^{n}}{(n+v)^{s}}
$$

See the URL below for more information. This function is a wrapper function for the C code described below.

## Value

Returns the value of the function evaluated at the values of $\mathrm{x}, \mathrm{s}, \mathrm{v}$. If the above ranges of $x$ and $v$ are not satisfied, or some numeric problems occur, then this function will return an NA for those values. (The C code returns 6 possible return codes, but this is not passed back up to the R level.)

## Warning

This function has not been thoroughly tested and contains limitations, for example, the zeta function cannot be computed with this function even though $\zeta(s)=\Phi(x=1, s, v=1)$. Several numerical problems can arise, such as lack of convergence, overflow and underflow, especially near singularities. If any problems occur then an NA will be returned. For example, if $|x|=1$ and $s>1$ then convergence may be so slow that changing tolerance and/or iter may be needed to get an answer (that is treated cautiously).

## Note

There are a number of special cases, e.g., the Riemann zeta-function is $\zeta(s)=\Phi(x=1, s, v=1)$. Another example is the Hurwitz zeta function $\zeta(s, v)=\Phi(x=1, s, v=v)$. The special case of $s=1$ corresponds to the hypergeometric 2 F 1 , and this is implemented in the gsl package. The Lerch Phi function should not be confused with the Lerch zeta function though they are quite similar.

## Author(s)

S. V. Aksenov and U. D. Jentschura wrote the C code (called Version 1.00). The R wrapper function was written by T. Yee.

## References

Originally the code was found at http://aksenov.freeshell.org/lerchphi/source/lerchphi.c. Bateman, H. (1953). Higher Transcendental Functions. Volume 1. McGraw-Hill, NY, USA.

## See Also

zeta.

## Examples

```
## Not run:
s <- 2; v <- 1; x <- seq(-1.1, 1.1, length = 201)
plot(x, lerch(x, s = s, v = v), type = "l", col = "blue", las = 1,
    main = paste0("lerch(x, s = ", s,", v = ", v, ")"))
abline(v = 0, h = 1, lty = "dashed", col = "gray")
## End(Not run)
```

leukemia
Acute Myelogenous Leukemia Survival Data

## Description

Survival in patients with Acute Myelogenous Leukemia

## Usage

data(leukemia)

## Format

| time: | survival or censoring time |
| :--- | :--- |
| status: | censoring status |
| $\mathrm{x}:$ | maintenance chemotherapy given? (factor) |

## Note

This data set has been transferred from survival and renamed from aml to leukemia.

## Source

Rupert G. Miller (1997). Survival Analysis. John Wiley \& Sons.

## levy Levy Distribution Family Function

## Description

Estimates the scale parameter of the Levy distribution by maximum likelihood estimation.

## Usage

levy(location = 0, lscale = "loglink", iscale = NULL)

## Arguments

location Location parameter. Must have a known value. Called $a$ below.
lscale Parameter link function for the (positive) scale parameter $b$. See Links for more choices.
iscale Initial value for the $b$ parameter. By default, an initial value is chosen internally.

## Details

The Levy distribution is one of three stable distributions whose density function has a tractable form. The formula for the density is

$$
f(y ; b)=\sqrt{\frac{b}{2 \pi}} \exp \left(\frac{-b}{2(y-a)}\right) /(y-a)^{3 / 2}
$$

where $a<y<\infty$ and $b>0$. Note that if $a$ is very close to $\min (\mathrm{y})$ (where y is the response), then numerical problem will occur. The mean does not exist. The median is returned as the fitted values.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Author(s)

T. W. Yee

## References

Nolan, J. P. (2005). Stable Distributions: Models for Heavy Tailed Data.

## See Also

The Nolan article was at http://academic2.american.edu/~jpnolan/stable/chap1.pdf.

## Examples

```
nn <- 1000; loc1 <- 0; loc2 <- 10
myscale <- 1 # log link ==> 0 is the answer
ldata <- data.frame(y1 = loc1 + myscale/rnorm(nn)^2, # Levy(myscale, a)
    y2 = rlevy(nn, loc = loc2, scale = exp(+2)))
# Cf. Table 1.1 of Nolan for Levy(1,0)
with(ldata, sum(y1 > 1) / length(y1)) # Should be 0.6827
with(ldata, sum(y1 > 2) / length(y1)) # Should be 0.5205
fit1 <- vglm(y1 ~ 1, levy(location = loc1), data = ldata, trace = TRUE)
coef(fit1, matrix = TRUE)
Coef(fit1)
summary(fit1)
head(weights(fit1, type = "work"))
fit2 <- vglm(y2 ~ 1, levy(location = loc2), data = ldata, trace = TRUE)
coef(fit2, matrix = TRUE)
Coef(fit2)
c(median = with(ldata, median(y2)), fitted.median = head(fitted(fit2), 1))
```

lgamma1 Log-gamma Distribution Family Function

## Description

Estimation of the parameter of the standard and nonstandard log-gamma distribution.

## Usage

lgamma1(lshape = "loglink", ishape = NULL)
lgamma3(llocation = "identitylink", lscale = "loglink", lshape = "loglink", ilocation $=$ NULL, iscale $=$ NULL, ishape $=1$, zero = c("scale", "shape"))

## Arguments

llocation, lscale
Parameter link function applied to the location parameter $a$ and the positive scale parameter $b$. See Links for more choices.
lshape Parameter link function applied to the positive shape parameter $k$. See Links for more choices.
ishape Initial value for $k$. If given, it must be positive. If failure to converge occurs, try some other value. The default means an initial value is determined internally.
ilocation, iscale
Initial value for $a$ and $b$. The defaults mean an initial value is determined internally for each.
zero An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set $\{1,2,3\}$. The default value means none are modelled as intercept-only terms. See CommonVGAMffArguments for more information.

## Details

The probability density function of the standard log-gamma distribution is given by

$$
f(y ; k)=\exp [k y-\exp (y)] / \Gamma(k),
$$

for parameter $k>0$ and all real $y$. The mean of $Y$ is digamma(k) (returned as the fitted values) and its variance is trigamma ( k ).
For the non-standard log-gamma distribution, one replaces $y$ by $(y-a) / b$, where $a$ is the location parameter and $b$ is the positive scale parameter. Then the density function is

$$
f(y)=\exp [k(y-a) / b-\exp ((y-a) / b)] /(b \Gamma(k))
$$

The mean and variance of $Y$ are $a+b *$ digamma (k) (returned as the fitted values) and $\mathrm{b}^{\wedge} 2 *$ trigamma (k), respectively.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

The standard log-gamma distribution can be viewed as a generalization of the standard type 1 extreme value density: when $k=1$ the distribution of $-Y$ is the standard type 1 extreme value distribution.
The standard log-gamma distribution is fitted with lgamma1 and the non-standard (3-parameter) log-gamma distribution is fitted with lgamma3.

## Author(s)

T. W. Yee

## References

Kotz, S. and Nadarajah, S. (2000). Extreme Value Distributions: Theory and Applications, pages 48-49, London: Imperial College Press.

Johnson, N. L. and Kotz, S. and Balakrishnan, N. (1995). Continuous Univariate Distributions, 2nd edition, Volume 2, p.89, New York: Wiley.

## See Also

rlgamma, gengamma.stacy, prentice74, gamma1, lgamma.

## Examples

```
ldata <- data.frame(y = rlgamma(100, shape = exp(1)))
fit <- vglm(y ~ 1, lgamma1, data = ldata, trace = TRUE, crit = "coef")
summary(fit)
coef(fit, matrix = TRUE)
Coef(fit)
ldata <- data.frame(x2 = runif(nn <- 5000)) # Another example
ldata <- transform(ldata, loc = -1 + 2 * x2, Scale = exp(1))
ldata <- transform(ldata, y = rlgamma(nn, loc, scale = Scale, shape = exp(0)))
fit2 <- vglm(y ~ x2, lgamma3, data = ldata, trace = TRUE, crit = "c")
coef(fit2, matrix = TRUE)
```

lgammaUC The Log-Gamma Distribution

## Description

Density, distribution function, quantile function and random generation for the log-gamma distribution with location parameter location, scale parameter scale and shape parameter k .

## Usage

```
dlgamma(x, location = 0, scale = 1, shape = 1, log = FALSE)
plgamma(q, location = 0, scale = 1, shape = 1,
    lower.tail = TRUE, log.p = FALSE)
qlgamma(p, location = 0, scale = 1, shape = 1,
    lower.tail = TRUE, log.p = FALSE)
rlgamma(n, location = 0, scale = 1, shape = 1)
```


## Arguments

| $\mathrm{x}, \mathrm{q}$ | vector of quantiles. |
| :--- | :--- |
| p | vector of probabilities. |
| n | number of observations. Same as runif. |
| location | the location parameter $a$. |
| scale | the (positive) scale parameter $b$. |
| shape | the (positive) shape parameter $k$. |
| log | Logical. If log = TRUE then the logarithm of the density is returned. |
| lower.tail, log. p |  |
|  | Same meaning as in pnorm or qnorm. |

## Details

See lgamma1, the VGAM family function for estimating the one parameter standard log-gamma distribution by maximum likelihood estimation, for formulae and other details. Apart from n , all the above arguments may be vectors and are recyled to the appropriate length if necessary.

## Value

dlgamma gives the density, plgamma gives the distribution function, qlgamma gives the quantile function, and rlgamma generates random deviates.

## Note

The VGAM family function lgamma3 is for the three parameter (nonstandard) log-gamma distribution.

## Author(s)

T. W. Yee and Kai Huang

## References

Kotz, S. and Nadarajah, S. (2000). Extreme Value Distributions: Theory and Applications, pages 48-49, London: Imperial College Press.

## See Also

lgamma1, prentice74.

## Examples

```
## Not run: loc <- 1; Scale <- 1.5; shape <- 1.4
x <- seq(-3.2, 5, by = 0.01)
plot(x, dlgamma(x, loc = loc, Scale, shape = shape), type = "l",
    col = "blue", ylim = 0:1,
    main = "Blue is density, orange is cumulative distribution function",
    sub = "Purple are 5,10,\ldots.,95 percentiles", las = 1, ylab = "")
abline(h = 0, col = "blue", lty = 2)
lines(qlgamma(seq(0.05, 0.95, by = 0.05), loc = loc, Scale, shape = shape),
    dlgamma(qlgamma(seq(0.05, 0.95, by = 0.05), loc = loc, scale = Scale,
                    shape = shape),
    loc = loc, Scale, shape = shape), col = "purple", lty = 3, type = "h")
lines(x, plgamma(x, loc = loc, Scale, shape = shape), col = "orange")
abline(h = 0, lty = 2)
## End(Not run)
```


## Description

Density, cumulative distribution function, and random generation for the Lindley distribution.

## Usage

dlind(x, theta, log = FALSE)
plind(q, theta, lower.tail = TRUE, log.p = FALSE)
rlind(n, theta)

## Arguments

$x, q \quad$ vector of quantiles.
$n \quad$ number of observations. Same as in runif.
log Logical. If log = TRUE then the logarithm of the density is returned.
theta positive parameter.
lower.tail, log.p
Same meaning as in pnorm or qnorm.

## Details

See lindley for details.

## Value

dlind gives the density, plind gives the cumulative distribution function, and rlind generates random deviates.

## Author(s)

T. W. Yee and Kai Huang

## See Also

lindley.

## Examples

```
theta <- exp(-1); x <- seq(0.0, 17, length = 700)
dlind(0:10, theta)
## Not run:
plot(x, dlind(x, theta), type = "l", las = 1, col = "blue",
    main = "dlind(x, theta = exp(-1))")
abline(h = 1, col = "grey", lty = "dashed")
## End(Not run)
```

lindley $\quad$ 1-parameter Gamma Distribution

## Description

Estimates the (1-parameter) Lindley distribution by maximum likelihood estimation.

## Usage

lindley(link = "loglink", itheta = NULL, zero = NULL)

## Arguments

link Link function applied to the (positive) parameter. See Links for more choices.
itheta, zero See CommonVGAMffArguments for information.

## Details

The density function is given by

$$
f(y ; \theta)=\theta^{2}(1+y) \exp (-\theta y) /(1+\theta)
$$

for $\theta>0$ and $y>0$. The mean of $Y$ (returned as the fitted values) is $\mu=(\theta+2) /(\theta(\theta+1))$. The variance is $\left(\theta^{2}+4 \theta+2\right) /(\theta(\theta+1))^{2}$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

Note
This VGAM family function can handle multiple responses (inputted as a matrix). Fisher scoring is implemented.

## Author(s)

T. W. Yee

## References

Lindley, D. V. (1958). Fiducial distributions and Bayes' theorem. Journal of the Royal Statistical Society, Series B, Methodological, 20, 102-107.
Ghitany, M. E. and Atieh, B. and Nadarajah, S. (2008). Lindley distribution and its application. Math. Comput. Simul., 78, 493-506.

## See Also

dlind, gammaR, simulate.vlm.

## Examples

```
ldata <- data.frame(y = rlind(n = 1000, theta = exp(3)))
fit <- vglm(y ~ 1, lindley, data = ldata, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

linkfun Link Functions

## Description

Generic function for returning the link functions of a fitted object.

## Usage

linkfun(object, ...)

## Arguments

object An object which has parameter link functions.
... Other arguments fed into the specific methods function of the model.

## Details

Fitted models in the VGAM have parameter link functions. This generic function returns these.

## Value

The value returned depends specifically on the methods function invoked.

## Author(s)

Thomas W. Yee

## See Also

linkfun.vglm, multilogitlink, vglm.

## Examples

```
pneumo <- transform(pneumo, let = log(exposure.time))
fit1 <- vglm(cbind(normal, mild, severe) ~ let, propodds, data = pneumo)
coef(fit1, matrix = TRUE)
linkfun(fit1)
linkfun(fit1, earg = TRUE)
fit2 <- vglm(cbind(normal, mild, severe) ~ let, multinomial, data = pneumo)
coef(fit2, matrix = TRUE)
linkfun(fit2)
linkfun(fit2, earg = TRUE)
```

linkfun.vglm

Link Functions for VGLMs

## Description

Returns the link functions, and parameter names, for vector generalized linear models (VGLMs).

## Usage

linkfun.vglm(object, earg = FALSE, ...)

## Arguments

object Object of class "vglm", i.e., a VGLM object.
earg Logical. Return the extra arguments associated with each link function? If TRUE then a list is returned.
... Arguments that might be used in the future.

## Details

All fitted VGLMs have a link function applied to each parameter. This function returns these, and optionally, the extra arguments associated with them.

## Value

Usually just a (named) character string, with the link functions in order. It is named with the parameter names. If earg = TRUE then a list with the following components.

| link | The default output. |
| :--- | :--- |
| earg | The extra arguments, in order. |

## Note

Presently, the multinomial logit model has only one link function, multilogitlink, so a warning is not issued for that link. For other models, if the number of link functions does not equal $M$ then a warning may be issued.

## Author(s)

Thomas W. Yee

## See Also

linkfun, multilogitlink, vglm.

## Examples

```
fit1 <- vgam(cbind(r1, r2) ~ s(year, df = 3), gev(zero = 2:3), venice)
coef(fit1, matrix = TRUE)
linkfun(fit1)
linkfun(fit1, earg = TRUE)
```

Links
Link functions for VGLM/VGAM/etc. families

## Description

The VGAM package provides a number of (parameter) link functions which are described in general here. Collectively, they offer the user considerable choice and flexibility for modelling data.

## Usage

```
TypicalVGAMlink(theta, someParameter \(=0\), bvalue \(=\) NULL, inverse \(=\) FALSE,
    deriv \(=0\), short \(=\) TRUE, tag = FALSE)
```


## Arguments

theta Numeric or character. This is usually $\theta$ (default) but can sometimes be $\eta$, depending on the other arguments. If theta is character then inverse and deriv are ignored. The name theta should always be the name of the first argument.
someParameter Some parameter, e.g., an offset.
bvalue Boundary value, positive if given. If $0<$ theta then values of theta which are less than or equal to 0 can be replaced by bvalue before computing the link function value. Values of theta which are greater than or equal to 1 can be replaced by 1 minus bvalue before computing the link function value. The value bvalue $=$. Machine $\$$ double.eps is sometimes a reasonable value, or something slightly higher.
inverse Logical. If TRUE and deriv $=0$ then the inverse link value $\theta$ is returned, hence the argument theta is really $\eta$. In all other cases, the argument theta is really $\theta$.
deriv Integer. Either 0, 1, or 2, specifying the order of the derivative. Some link functions handle values up to 3 or 4 .
short, tag Logical. These are used for labelling the blurb slot of a vglmff-class object. These arguments are used only if theta is character, and gives the formula for the link in character form. If tag $=$ TRUE then the result is preceeded by a little more information.

## Details

Almost all VGAM link functions have something similar to the argument list as given above. In this help file we have $\eta=g(\theta)$ where $g$ is the link function, $\theta$ is the parameter and $\eta$ is the linear/additive predictor. The link $g$ must be strictly monotonic and twice-differentiable in its range.

The following is a brief enumeration of all VGAM link functions.
For parameters lying between 0 and 1 (e.g., probabilities): logitlink, probitlink, clogloglink, cauchitlink, foldsqrtlink, logclink, gordlink, pordlink, nbordlink.
For positive parameters (i.e., greater than 0): loglink, negloglink, powerlink.
For parameters greater than 1: logloglink, loglogloglink (greater than $e$ ).
For parameters between -1 and 1 : fisherzlink, rhobitlink.
For parameters between $A$ and $B$ : extlogitlink, logofflink $(B=\infty)$.
For unrestricted parameters (i.e., any value): identitylink, negidentitylink, reciprocallink, negreciprocallink.

## Value

Returns one of: the link function value or its first or second derivative, the inverse link or its first or second derivative, or a character description of the link.

Here are the general details. If inverse $=$ FALSE and deriv $=0$ (default) then the ordinary link function $\eta=g(\theta)$ is returned.

If inverse $=$ TRUE and deriv $=0$ then the inverse link function value is returned, hence theta is really $\eta$ (the only occasion this happens).
If inverse $=$ FALSE and deriv $=1$ then it is $d \eta / d \theta$ as a function of $\theta$. If inverse = FALSE and deriv $=2$ then it is $d^{2} \eta / d \theta^{2}$ as a function of $\theta$.

If inverse $=$ TRUE and deriv $=1$ then it is $d \theta / d \eta$ as a function of $\theta$. If inverse $=$ TRUE and deriv $=2$ then it is $d^{2} \theta / d \eta^{2}$ as a function of $\theta$.
It is only when deriv = 1 that linkfun(theta, deriv = 1 , inverse = TRUE) and linkfun(theta, deriv $=1$, inverse $=$ FALSE) are reciprocals of each other. In particular, linkfun(theta, deriv $=2$, inverse $=$ TRUE) and linkfun(theta, deriv $=2$, inverse $=$ FALSE) are not reciprocals of each other in general.

## Warning

The output of link functions changed at VGAM 0.9-9 (date was around 2015-07). Formerly, linkfun(theta, deriv = 1) is now linkfun(theta, deriv=1, inverse $=$ TRUE), or equivalently, $1 /$ linkfun(theta, deriv = 1 , inverse = TRUE). Also, formerly, linkfun(theta, deriv=2) was $1 /$ linkfun(theta, deriv $=2$, inverse = TRUE). This was a bug. Altogether, these are big changes and the user should beware!

In VGAM 1.0-7 (January 2019) all link function names were made to end in the characters "link", e.g., loglink replaces loge, logitlink replaces logit. For this most of them were renamed. Upward compatability holds for older link function names, however, users should adopt the new names immediately.

## Note

VGAM link functions are generally not compatible with other functions outside the package. In particular, they won't work with glm or any other package for fitting GAMs.
From October 2006 onwards, all VGAM family functions will only contain one default value for each link argument rather than giving a vector of choices. For example, rather than binomialff(link = c("logitlink", "probitlink","clogloglink","cauchitlink","identitylink"), ...) it is now binomialff(link = "logitlink", . . .). No checking will be done to see if the user's choice is reasonable. This means that the user can write his/her own VGAM link function and use it within any VGAM family function. Altogether this provides greater flexibility. The downside is that the user must specify the full name of the link function, by either assigning the link argument the full name as a character string, or just the name itself. See the examples below.
From August 2012 onwards, a major change in link functions occurred. Argument esigma (and the like such as earg) used to be in VGAM prior to version 0.9-0 (released during the 2 nd half of 2012). The major change is that arguments such as offset that used to be passed in via those arguments can done directly through the link function. For example, gev (lshape = "logofflink", eshape $=1$ ist $($ offset $=0.5)$ ) is replaced by gev (lshape $=1$ ogofflink (offset $=0.5)$ ). The @misc slot no longer has link and earg components, but two other components replace these. Functions such as dtheta.deta(), d2theta. deta2(), d3theta.deta3(), eta2theta(), theta2eta() are modified.
From January 2019 onwards, all link function names ended in "link". See above for details.

## Author(s)

T. W. Yee

## References

McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, 2nd ed. London: Chapman \& Hall.

## See Also

TypicalVGAMfamilyFunction, linkfun, vglm, vgam, rrvglm. cqo, cao.

## Examples

```
logitlink("a")
logitlink("a", short = FALSE)
logitlink("a", short = FALSE, tag = TRUE)
logofflink(1:5, offset = 1) # Same as log(1:5 + 1)
powerlink(1:5, power = 2) # Same as (1:5)^2
## Not run: # This is old and no longer works:
logofflink(1:5, earg = list(offset = 1))
powerlink(1:5, earg = list(power = 2))
## End(Not run)
```

```
fit1 <- vgam(agaaus ~ altitude,
    binomialff(link = "clogloglink"), hunua) # best
fit2 <- vgam(agaaus ~ altitude,
    binomialff(link = clogloglink ), hunua) # okay
## Not run:
# This no longer works since "clog" is not a valid VGAM link function:
fit3 <- vgam(agaaus ~ altitude,
    binomialff(link = "clog"), hunua) # not okay
# No matter what the link, the estimated var-cov matrix is the same
y <- rbeta(n = 1000, shape1 = exp(0), shape 2 = exp(1))
fit1 <- vglm(y ~ 1, betaR(lshape1 = "identitylink",
                    lshape2 = "identitylink"),
    trace = TRUE, crit = "coef")
fit2 <- vglm(y ~ 1, betaR(lshape1 = logofflink(offset = 1.1),
                            lshape2 = logofflink(offset = 1.1)), trace=TRUE)
vcov(fit1, untransform = TRUE)
vcov(fit1, untransform = TRUE) -
vcov(fit2, untransform = TRUE) # Should be all 0s
\dontrun{ # This is old:
fit1@misc$earg # Some 'special' parameters
fit2@misc$earg # Some 'special' parameters are here
}
par(mfrow = c(2, 2))
p <- seq(0.05, 0.95, len = 200) # A rather restricted range
x <- seq(-4, 4, len = 200)
plot(p, logitlink(p), type = "l", col = "blue")
plot(x, logitlink(x, inverse = TRUE), type = "l", col = "blue")
plot(p, logitlink(p, deriv=1), type="l", col="blue") # 1 / (p*(1-p))
plot(p, logitlink(p, deriv=2), type="l", col="blue") # (2*p-1)/(p*(1-p))^2
## End(Not run)
```

Lino

## Description

Density, distribution function, quantile function and random generation for the generalized beta distribution, as proposed by Libby and Novick (1982).

## Usage

dlino(x, shape1, shape2, lambda = 1, log = FALSE)
plino(q, shape1, shape2, lambda = 1, lower.tail = TRUE, log.p = FALSE)
qlino( $p$, shape1, shape2, lambda = 1, lower.tail = TRUE, log.p = FALSE)
rlino(n, shape1, shape2, lambda = 1)

## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
n number of observations. Same as in runif.
shape1, shape2, lambda
see lino.
log Logical. If log = TRUE then the logarithm of the density is returned.
lower.tail, log.p
Same meaning as in pnorm or qnorm.

## Details

See lino, the VGAM family function for estimating the parameters, for the formula of the probability density function and other details.

## Value

dlino gives the density, plino gives the distribution function, qlino gives the quantile function, and rlino generates random deviates.

## Author(s)

T. W. Yee and Kai Huang

## See Also

lino.

## Examples

```
## Not run:
lambda <- 0.4; shape1 <- exp(1.3); shape2 <- exp(1.3)
x <- seq(0.0, 1.0, len = 101)
plot(x, dlino(x, shape1 = shape1, shape2 = shape2, lambda = lambda),
    type = "l", col = "blue", las = 1, ylab = "",
    main = "Blue is density, red is cumulative distribution function",
    sub = "Purple lines are the 10,20,\ldots.,90 percentiles")
abline(h = 0, col = "blue", lty = 2)
lines(x, plino(x, shape1 = shape1, shape2 = shape2, l = lambda), col = "red")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qlino(probs, shape1 = shape1, shape2 = shape2, lambda = lambda)
lines(Q, dlino(Q, shape1 = shape1, shape2 = shape2, lambda = lambda),
            col = "purple", lty = 3, type = "h")
plino(Q, shape1 = shape1, shape2 = shape2, l = lambda) - probs # Should be all 0
## End(Not run)
```


## Description

Maximum likelihood estimation of the 3-parameter generalized beta distribution as proposed by Libby and Novick (1982).

## Usage

```
lino(lshape1 = "loglink", lshape2 = "loglink", llambda = "loglink",
```

        ishape1 \(=\) NULL, \(\quad\) ishape \(2=\) NULL, \(\quad\) ilambda \(=1\), zero \(=\) NULL)
    
## Arguments

1shape1, lshape2
Parameter link functions applied to the two (positive) shape parameters $a$ and $b$. See Links for more choices.
llambda Parameter link function applied to the parameter $\lambda$. See Links for more choices. ishape1, ishape2, ilambda

Initial values for the parameters. A NULL value means one is computed internally. The argument ilambda must be numeric, and the default corresponds to a standard beta distribution.
zero Can be an integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. Here, the values must be from the set $\{1,2,3\}$ which correspond to $a, b, \lambda$, respectively. See CommonVGAMffArguments for more information.

## Details

Proposed by Libby and Novick (1982), this distribution has density

$$
f(y ; a, b, \lambda)=\frac{\lambda^{a} y^{a-1}(1-y)^{b-1}}{B(a, b)\{1-(1-\lambda) y\}^{a+b}}
$$

for $a>0, b>0, \lambda>0,0<y<1$. Here $B$ is the beta function (see beta). The mean is a complicated function involving the Gauss hypergeometric function. If $X$ has a lino distribution with parameters shape1, shape2, lambda, then $Y=\lambda X /(1-(1-\lambda) X)$ has a standard beta distribution with parameters shape1, shape2.
Since $\log (\lambda)=0$ corresponds to the standard beta distribution, a summary of the fitted model performs a t-test for whether the data belongs to a standard beta distribution (provided the loglink link for $\lambda$ is used; this is the default).

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

The fitted values, which is usually the mean, have not been implemented yet. Currently the median is returned as the fitted values.

Although Fisher scoring is used, the working weight matrices are positive-definite only in a certain region of the parameter space. Problems with this indicate poor initial values or an ill-conditioned model or insufficient data etc.

This model is can be difficult to fit. A reasonably good value of ilambda seems to be needed so if the self-starting initial values fail, try experimenting with the initial value arguments. Experience suggests ilambda is better a little larger, rather than smaller, compared to the true value.

## Author(s)

T. W. Yee

## References

Libby, D. L. and Novick, M. R. (1982). Multivariate generalized beta distributions with applications to utility assessment. Journal of Educational Statistics, 7, 271-294.

Gupta, A. K. and Nadarajah, S. (2004). Handbook of Beta Distribution and Its Applications, NY: Marcel Dekker, Inc.

## See Also

Lino, genbetaII.

## Examples

```
ldata <- data.frame(y1 = rbeta(n = 1000, exp(0.5), exp(1))) # ~ standard beta
fit <- vglm(y1 ~ 1, lino, data = ldata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
head(fitted(fit))
summary(fit)
# Nonstandard beta distribution
ldata <- transform(ldata, y2 = rlino(n = 1000, shape1 = exp(1),
                                    shape2 = exp(2), lambda = exp(1)))
fit2 <- vglm(y2 ~ 1, lino(lshape1 = "identitylink", lshape2 = "identitylink",
            ilamb = 10), data = ldata, trace = TRUE)
coef(fit2, matrix = TRUE)
```


## Description

Low-iron rat teratology data.

## Usage

data(lirat)

## Format

A data frame with 58 observations on the following 4 variables.
$N$ Litter size.
R Number of dead fetuses.
hb Hemoglobin level.
grp Group number. Group 1 is the untreated (low-iron) group, group 2 received injections on day 7 or day 10 only, group 3 received injections on days 0 and 7 , and group 4 received injections weekly.

## Details

The following description comes from Moore and Tsiatis (1991). The data comes from the experimental setup from Shepard et al. (1980), which is typical of studies of the effects of chemical agents or dietary regimens on fetal development in laboratory rats.
Female rats were put in iron-deficient diets and divided into 4 groups. One group of controls was given weekly injections of iron supplement to bring their iron intake to normal levels, while another group was given only placebo injections. Two other groups were given fewer iron-supplement injections than the controls. The rats were made pregnant, sacrificed 3 weeks later, and the total number of fetuses and the number of dead fetuses in each litter were counted.
For each litter the number of dead fetuses may be considered to be $\operatorname{Binomial}(N, p)$ where $N$ is the litter size and $p$ is the probability of a fetus dying. The parameter $p$ is expected to vary from litter to litter, therefore the total variance of the proportions will be greater than that predicted by a binomial model, even when the covariates for hemoglobin level and experimental group are accounted for.

## Source

Moore, D. F. and Tsiatis, A. (1991) Robust Estimation of the Variance in Moment Methods for Extra-binomial and Extra-Poisson Variation. Biometrics, 47, 383-401.

## References

Shepard, T. H., Mackler, B. and Finch, C. A. (1980). Reproductive studies in the iron-deficient rat. Teratology, 22, 329-334.

## Examples

```
## Not run:
# cf. Figure 3 of Moore and Tsiatis (1991)
plot(R / N ~ hb, data = lirat, pch = as.character(grp), col = grp,
    las = 1, xlab = "Hemoglobin level", ylab = "Proportion Dead")
## End(Not run)
```

lms.bcg
LMS Quantile Regression with a Box-Cox transformation to a Gamma Distribution

## Description

LMS quantile regression with the Box-Cox transformation to the gamma distribution.

## Usage

lms.bcg(percentiles $=c(25,50,75), \quad z e r o=c(" l a m b d a ", ~ " s i g m a ")$, llambda = "identitylink", lmu = "identitylink", lsigma = "loglink", idf.mu = 4, idf.sigma = 2, ilambda = 1, isigma = NULL)

## Arguments

percentiles A numerical vector containing values between 0 and 100 , which are the quantiles. They will be returned as 'fitted values'.
zero See lms.bcn.
llambda, lmu, lsigma
See lms.ben.
idf.mu, idf.sigma
See lms.bcn.
ilambda, isigma
See lms.bcn.

## Details

Given a value of the covariate, this function applies a Box-Cox transformation to the response to best obtain a gamma distribution. The parameters chosen to do this are estimated by maximum likelihood or penalized maximum likelihood. Similar details can be found at lms.bcn.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Warning

This VGAM family function comes with the same warnings as lms.ben. Also, the expected value of the second derivative with respect to lambda may be incorrect (my calculations do not agree with the Lopatatzidis and Green manuscript.)

## Note

Similar notes can be found at lms.bcn.

## Author(s)

Thomas W. Yee

## References

Lopatatzidis A. and Green, P. J. (unpublished manuscript). Semiparametric quantile regression using the gamma distribution.
Yee, T. W. (2004). Quantile regression via vector generalized additive models. Statistics in Medicine, 23, 2295-2315.

## See Also

lms.bcn, lms.yjn, qtplot.lmscreg, deplot.lmscreg, cdf.lmscreg, bmi.nz, amlexponential.

## Examples

```
# This converges, but deplot(fit) and qtplot(fit) do not work
fit0 <- vglm(BMI ~ sm.bs(age, df = 4), lms.bcg, data = bmi.nz, trace = TRUE)
coef(fit0, matrix = TRUE)
## Not run:
par(mfrow = c(1, 1))
plotvgam(fit0, se = TRUE) # Plot mu function (only)
## End(Not run)
# Use a trick: fit0 is used for initial values for fit1.
fit1 <- vgam(BMI ~ s(age, df = c(4, 2)), etastart = predict(fit0),
    lms.bcg(zero = 1), bmi.nz, trace = TRUE)
# Difficult to get a model that converges.
# Here, we prematurely stop iterations because it fails near the solution.
fit2 <- vgam(BMI ~ s(age, df = c(4, 2)), maxit = 4,
    lms.bcg(zero = 1, ilam = 3), bmi.nz, trace = TRUE)
summary(fit1)
head(predict(fit1))
head(fitted(fit1))
head(bmi.nz)
# Person 1 is near the lower quartile of BMI amongst people his age
head(cdf(fit1))
## Not run:
# Quantile plot
par(bty = "l", mar=c(5, 4, 4, 3) + 0.1, xpd = TRUE)
qtplot(fit1, percentiles=c(5, 50, 90, 99), main = "Quantiles",
    xlim = c(15, 90), las = 1, ylab = "BMI", lwd = 2, lcol = 4)
# Density plot
```

```
ygrid <- seq(15, 43, len = 100) # BMI ranges
par(mfrow = c(1, 1), lwd = 2)
(aa <- deplot(fit1, x0 = 20, y = ygrid, xlab = "BMI", col = "black",
    main = "Density functions at Age = 20 (black), 42 (red) and 55 (blue)"))
aa <- deplot(fit1, x0=42, y=ygrid, add=TRUE, llty=2, col="red")
aa <- deplot(fit1, x0=55, y=ygrid, add=TRUE, llty=4, col="blue", Attach=TRUE)
aa@post$deplot # Contains density function values
## End(Not run)
```

lms.ben
LMS Quantile Regression with a Box-Cox Transformation to Normal-
ity

## Description

LMS quantile regression with the Box-Cox transformation to normality.

## Usage

lms.bcn(percentiles $=c(25,50,75), ~ z e r o=c(" l a m b d a ", ~ " s i g m a ")$, llambda = "identitylink", lmu = "identitylink", lsigma = "loglink", idf.mu = 4, idf.sigma = 2, ilambda = 1, isigma $=$ NULL, tol0 $=0.001$ )

## Arguments

percentiles A numerical vector containing values between 0 and 100, which are the quantiles. They will be returned as 'fitted values'.
zero Can be an integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set $\{1,2,3\}$. The default value usually increases the chance of successful convergence. Setting zero $=$ NULL means they all are functions of the covariates. For more information see CommonVGAMfffarguments.
llambda, lmu, lsigma
Parameter link functions applied to the first, second and third linear/additive predictors. See Links for more choices, and CommonVGAMffArguments.
idf.mu Degrees of freedom for the cubic smoothing spline fit applied to get an initial estimate of mu. See vsmooth. spline.
idf.sigma Degrees of freedom for the cubic smoothing spline fit applied to get an initial estimate of sigma. See vsmooth. spline. This argument may be assigned NULL to get an initial value using some other algorithm.
ilambda Initial value for lambda. If necessary, it is recycled to be a vector of length $n$ where $n$ is the number of (independent) observations.
isigma Optional initial value for sigma. If necessary, it is recycled to be a vector of length $n$. The default value, NULL, means an initial value is computed in the @initialize slot of the family function.
tol0 Small positive number, the tolerance for testing if lambda is equal to zero.

## Details

Given a value of the covariate, this function applies a Box-Cox transformation to the response to best obtain normality. The parameters chosen to do this are estimated by maximum likelihood or penalized maximum likelihood.
In more detail, the basic idea behind this method is that, for a fixed value of $x$, a Box-Cox transformation of the response $Y$ is applied to obtain standard normality. The 3 parameters $(\lambda, \mu, \sigma$, which start with the letters "L-M-S" respectively, hence its name) are chosen to maximize a penalized log-likelihood (with vgam). Then the appropriate quantiles of the standard normal distribution are back-transformed onto the original scale to get the desired quantiles. The three parameters may vary as a smooth function of $x$.
The Box-Cox power transformation here of the $Y$, given $x$, is

$$
Z=\left[(Y / \mu(x))^{\lambda(x)}-1\right] /(\sigma(x) \lambda(x))
$$

for $\lambda(x) \neq 0$. (The singularity at $\lambda(x)=0$ is handled by a simple function involving a logarithm.) Then $Z$ is assumed to have a standard normal distribution. The parameter $\sigma(x)$ must be positive, therefore VGAM chooses $\eta(x)^{T}=(\lambda(x), \mu(x), \log (\sigma(x)))$ by default. The parameter $\mu$ is also positive, but while $\log (\mu)$ is available, it is not the default because $\mu$ is more directly interpretable. Given the estimated linear/additive predictors, the $100 \alpha$ percentile can be estimated by inverting the Box-Cox power transformation at the $100 \alpha$ percentile of the standard normal distribution.
Of the three functions, it is often a good idea to allow $\mu(x)$ to be more flexible because the functions $\lambda(x)$ and $\sigma(x)$ usually vary more smoothly with $x$. This is somewhat reflected in the default value for the argument zero, viz. zero $=c(1,3)$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Warning

The computations are not simple, therefore convergence may fail. Set trace $=$ TRUE to monitor convergence if it isn't set already. Convergence failure will occur if, e.g., the response is bimodal at any particular value of $x$. In case of convergence failure, try different starting values. Also, the estimate may diverge quickly near the solution, in which case try prematurely stopping the iterations by assigning maxits to be the iteration number corresponding to the highest likelihood value.
One trick is to fit a simple model and use it to provide initial values for a more complex model; see in the examples below.

## Note

The response must be positive because the Box-Cox transformation cannot handle negative values. In theory, the LMS-Yeo-Johnson-normal method can handle both positive and negative values.
In general, the lambda and sigma functions should be more smoother than the mean function. Having zero $=1$, zero $=3$ or zero $=c(1,3)$ is often a good idea. See the example below.

## Author(s)

Thomas W. Yee

## References

Cole, T. J. and Green, P. J. (1992). Smoothing Reference Centile Curves: The LMS Method and Penalized Likelihood. Statistics in Medicine, 11, 1305-1319.
Green, P. J. and Silverman, B. W. (1994). Nonparametric Regression and Generalized Linear Models: A Roughness Penalty Approach, London: Chapman \& Hall.

Yee, T. W. (2004). Quantile regression via vector generalized additive models. Statistics in Medicine, 23, 2295-2315.

## See Also

lms.bcg, lms.yjn, qtplot. 1 msc eg, deplot. $1 \mathrm{mscreg}, \mathrm{cdf} .1 \mathrm{mscreg}$, eCDF, extlogF1, alaplace1, amlnormal, denorm, CommonVGAMffArguments.

## Examples

```
## Not run: require("VGAMdata")
mysubset <- subset(xs.nz, sex == "M" & ethnicity == "Maori" & study1)
mysubset <- transform(mysubset, BMI = weight / height^2)
BMIdata <- na.omit(mysubset)
BMIdata <- subset(BMIdata, BMI < 80 & age < 65,
        select = c(age, BMI)) # Delete an outlier
summary(BMIdata)
fit <- vgam(BMI ~ s(age, df = c(4, 2)), lms.bcn(zero = 1), data = BMIdata)
par(mfrow = c(1, 2))
plot(fit, scol = "blue", se = TRUE) # The two centered smooths
head(predict(fit))
head(fitted(fit))
head(BMIdata)
head(cdf(fit)) # Person 46 is probably overweight, given his age
100 * colMeans(depvar(fit, drop = TRUE) < fitted(fit)) # Empirical proportions
# Correct for "vgam" objects but not very elegant:
fit@family@linkinv(eta = predict(fit, data.frame(age = 60)),
    extra = list(percentiles = c(10, 50)))
if (FALSE) {
# These work for "vglm" objects:
fit2 <- vglm(BMI ~ bs(age, df = 4), lms.bcn(zero = 3), data = BMIdata)
predict(fit2, percentiles = c(10, 50),
            newdata = data.frame(age = 60), type = "response") # A specific age
head(fitted(fit2, percentiles = c(10, 50))) # Get different percentiles
}
# Convergence problems? Try this trick: fit0 is a simpler model used for fit1
fit0 <- vgam(BMI ~ s(age, df = 4), lms.bcn(zero = c(1, 3)), data = BMIdata)
fit1 <- vgam(BMI ~ s(age, df = c(4, 2)), lms.bcn(zero = 1), data = BMIdata,
    etastart = predict(fit0))
```

```
## End(Not run)
## Not run:
# Quantile plot
par(bty = "l", mar = c(5, 4, 4, 3) + 0.1, xpd = TRUE)
qtplot(fit, percentiles = c(5, 50, 90, 99), main = "Quantiles",
        xlim = c(15, 66), las = 1, ylab = "BMI", lwd = 2, lcol = 4)
# Density plot
ygrid <- seq(15, 43, len = 100) # BMI ranges
par(mfrow = c(1, 1), lwd = 2)
(aa <- deplot(fit, x0 = 20, y = ygrid, xlab = "BMI", col = "black",
    main = "Density functions at Age = 20 (black), 42 (red) and 55 (blue)"))
aa <- deplot(fit, x0 = 42, y = ygrid, add = TRUE, llty = 2, col = "red")
aa <- deplot(fit, x0 = 55, y = ygrid, add = TRUE, llty = 4, col = "blue",
                    Attach = TRUE)
aa@post$deplot # Contains density function values
## End(Not run)
```

Ims.yjn LMS Quantile Regression with a Yeo-Johnson Transformation to Normality

## Description

LMS quantile regression with the Yeo-Johnson transformation to normality. This family function is experimental and the LMS-BCN family function is recommended instead.

## Usage

```
lms.yjn(percentiles = c(25, 50, 75), zero = c("lambda", "sigma"),
    llambda = "identitylink", lsigma = "loglink",
    idf.mu = 4, idf.sigma = 2,
    ilambda = 1, isigma = NULL, rule = c(10, 5),
    yoffset = NULL, diagW = FALSE, iters.diagW = 6)
lms.yjn2(percentiles = c(25, 50, 75), zero = c("lambda", "sigma"),
    llambda = "identitylink", lmu = "identitylink", lsigma = "loglink",
    idf.mu = 4, idf.sigma = 2, ilambda = 1.0,
    isigma = NULL, yoffset = NULL, nsimEIM = 250)
```


## Arguments

percentiles A numerical vector containing values between 0 and 100, which are the quantiles. They will be returned as 'fitted values'.
zero See lms.bcn.
llambda, lmu, lsigma
See lms.bcn.
idf.mu, idf.sigma
See lms.bcn.
ilambda, isigma
rule
See lms.bcn.
Number of abscissae used in the Gaussian integration scheme to work out ele-
ments of the weight matrices. The values given are the possible choices, with
the first value being the default. The larger the value, the more accurate the
approximation is likely to be but involving more computational expense.

## Details

Given a value of the covariate, this function applies a Yeo-Johnson transformation to the response to best obtain normality. The parameters chosen to do this are estimated by maximum likelihood or penalized maximum likelihood. The function lms.yjn2() estimates the expected information matrices using simulation (and is consequently slower) while lms.yjn() uses numerical integration. Try the other if one function fails.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Warning

The computations are not simple, therefore convergence may fail. In that case, try different starting values.

The generic function predict, when applied to a lms.yjn fit, does not add back the yoffset value.
As described above, this family function is experimental and the LMS-BCN family function is recommended instead.

## Note

The response may contain both positive and negative values. In contrast, the LMS-Box-Cox-normal and LMS-Box-Cox-gamma methods only handle a positive response because the Box-Cox transformation cannot handle negative values.
Some other notes can be found at lms.ben.

## Author(s)

Thomas W. Yee

## References

Yeo, I.-K. and Johnson, R. A. (2000). A new family of power transformations to improve normality or symmetry. Biometrika, 87, 954-959.

Yee, T. W. (2004). Quantile regression via vector generalized additive models. Statistics in Medicine, 23, 2295-2315.

Yee, T. W. (2002). An Implementation for Regression Quantile Estimation. Pages 3-14. In: Haerdle, W. and Ronz, B., Proceedings in Computational Statistics COMPSTAT 2002. Heidelberg: Physica-Verlag.

## See Also

lms.bcn, lms.bcg, qtplot.lmscreg, deplot.lmscreg, cdf.lmscreg, bmi.nz, amlnormal.

## Examples

```
fit <- vgam(BMI ~ s(age, df = 4), lms.yjn, bmi.nz, trace = TRUE)
head(predict(fit))
head(fitted(fit))
head(bmi.nz)
# Person 1 is near the lower quartile of BMI amongst people his age
head(cdf(fit))
## Not run:
# Quantile plot
par(bty = "l", mar = c(5, 4, 4, 3) + 0.1, xpd = TRUE)
qtplot(fit, percentiles = c(5, 50, 90, 99), main = "Quantiles",
    xlim = c(15, 90), las = 1, ylab = "BMI", lwd = 2, lcol = 4)
# Density plot
ygrid <- seq(15, 43, len = 100) # BMI ranges
par(mfrow = c(1, 1), lwd = 2)
(aa <- deplot(fit, x0 = 20, y = ygrid, xlab = "BMI", col = "black",
    main = "Density functions at Age = 20 (black), 42 (red) and 55 (blue)"))
aa <- deplot(fit, x0 = 42, y = ygrid, add = TRUE, llty = 2, col = "red")
aa <- deplot(fit, x0 = 55, y = ygrid, add = TRUE, llty = 4, col = "blue",
            Attach = TRUE)
with(aa@post, deplot) # Contains density function values; == a@post$deplot
## End(Not run)
```


## Description

Density, distribution function, quantile function, and random generation for the logarithmic distribution.

## Usage

$d \log (x$, shape, $\log =$ FALSE $)$
$\mathrm{plog}(q$, shape, log. $\mathrm{p}=\mathrm{FALSE})$
$q \log (p$, shape $)$
$r \log (n$, shape $)$

## Arguments

$x, q, p, n \quad$ Same interpretation as in runif.
shape The shape parameter value $c$ described in in logff.
$\log , \log . p \quad$ Logical. If log. $p=$ TRUE then all probabilities $p$ are given as $\log (p)$.

## Details

The details are given in logff.

## Value

dlog gives the density, plog gives the distribution function, qlog gives the quantile function, and rlog generates random deviates.

## Note

Given some response data, the VGAM family function logff estimates the parameter shape. For plog() , if argument $q$ contains large values and/or $q$ is long in length then the memory requirements may be very high. Very large values in $q$ are handled by an approximation by Owen (1965).

## Author(s)

T. W. Yee

## References

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011). Statistical Distributions, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

See Also
logff, Oilog. Otlog.

## Examples

```
dlog(1:20, 0.5)
rlog(20, 0.5)
## Not run: shape <- 0.8; x <- 1:10
plot(x, dlog(x, shape = shape), type = "h", ylim = 0:1,
    sub = "shape=0.8", las = 1, col = "blue", ylab = "shape",
    main = "Logarithmic distribution: blue=density; orange=distribution function")
lines(x + 0.1, plog(x, shape = shape), col = "orange", lty = 3, type = "h")
## End(Not run)
```

log1mexp Logarithms with an Unit Offset and Exponential Term

## Description

Computes $\log (1+\exp (x))$ and $\log (1-\exp (-x))$ accurately.

## Usage

$\log 1 \operatorname{mexp}(x)$
$\log 1 \operatorname{pexp}(x)$

## Arguments

x
A vector of reals (numeric). Complex numbers not allowed since expm1 and $\log 1 \mathrm{p}$ do not handle these.

## Details

Computes $\log (1+\exp (x))$ and $\log (1-\exp (-x))$ accurately. An adjustment is made when $x$ is away from 0 in value.

## Value

$\log 1 \operatorname{mexp}(x)$ gives the value of $\log (1-\exp (-x))$.
$\log 1 \operatorname{pexp}(x)$ gives the value of $\log (1+\exp (x))$.

## Note

If NA or NaN is present in the input, the corresponding output will be NA.

## Author(s)

This is a direct translation of the function in Martin Maechler's (2012) paper by Xiangjie Xue and T. W. Yee.

## References

Maechler, Martin (2012). Accurately Computing $\log (1-\exp (-|a|))$. Assessed from the Rmpfr package.

## See Also

$\log 1 \mathrm{p}$, expm1, exp, log

## Examples

```
x <- c(10, 50, 100, 200, 400, 500, 800, 1000, 1e4, 1e5, 1e20, Inf, NA)
log1pexp(x)
log(1 + exp(x)) # Naive; suffers from overflow
log1mexp(x)
log(1 - exp(-x))
y <- -x
log1pexp(y)
log(1 + exp(y)) # Naive; suffers from inaccuracy
```

    logclink Complementary-log Link Function
    
## Description

Computes the Complementary-log Transformation, Including its Inverse and the First Two Derivatives.

## Usage

logclink(theta, bvalue $=$ NULL, inverse $=$ FALSE, deriv $=0$, short $=$ TRUE, tag $=$ FALSE)

## Arguments



## Details

The complementary-log link function is suitable for parameters that are less than unity. Numerical values of theta close to 1 or out of range result in Inf, -Inf, NA or NaN.

## Value

For deriv $=0$, the $\log$ of theta, i.e., $\log (1-$ theta $)$ when inverse $=F A L S E$, and if inverse $=$ TRUE then $1-\exp$ (theta).
For deriv $=1$, then the function returns $d$ eta $/ d$ theta as a function of theta if inverse $=$ FALSE, else if inverse $=$ TRUE then it returns the reciprocal.
Here, all logarithms are natural logarithms, i.e., to base $e$.

## Note

Numerical instability may occur when theta is close to 1 . One way of overcoming this is to use bvalue.

## Author(s)

Thomas W. Yee

## References

McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, 2nd ed. London: Chapman \& Hall.

## See Also

Links, loglink, clogloglink, logloglink, logofflink.

## Examples

```
## Not run:
logclink(seq(-0.2, 1.1, by = 0.1)) # Has NAs
## End(Not run)
logclink(seq(-0.2,1.1,by=0.1),bvalue=1-.Machine$double.eps) # Has no NAs
```

logF Natural Exponential Family Generalized Hyperbolic Secant Distribu- tion Family Function

## Description

Maximum likelihood estimation of the 2-parameter $\log \mathrm{F}$ distribution.

## Usage

```
logF(lshape1 = "loglink", lshape2 = "loglink",
            ishape1 = NULL, ishape2 = 1, imethod = 1)
```


## Arguments

1shape1, lshape2
Parameter link functions for the shape parameters. Called $\alpha$ and $\beta$ respectively. See Links for more choices.
ishape1, ishape2
Optional initial values for the shape parameters. If given, it must be numeric and values are recycled to the appropriate length. The default is to choose the value internally. See CommonVGAMffArguments for more information.
imethod Initialization method. Either the value 1, 2, or .... See CommonVGAMffArguments for more information.

## Details

The density for this distribution is

$$
f(y ; \alpha, \beta)=\exp (\alpha y) /\left[B(\alpha, \beta)\left(1+e^{y}\right)^{\alpha+\beta}\right]
$$

where $y$ is real, $\alpha>0, \beta>0, B(.,$.$) is the beta function beta.$

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Author(s)

Thomas W. Yee

## References

Jones, M. C. (2008). On a class of distributions with simple exponential tails. Statistica Sinica, 18(3), 1101-1110.

## See Also

dlogF, extlogF1, logff.

## Examples

```
nn <- 1000
ldata <- data.frame(y1 = rnorm(nn, m = +1, sd = exp(2)), # Not proper data
                                    x2 = rnorm(nn, m = -1, sd = exp(2)),
                                    y2 = rnorm(nn, m = -1, sd = exp(2))) # Not proper data
fit1 <- vglm(y1 ~ 1 , logF, data = ldata, trace = TRUE)
fit2 <- vglm(y2 ~ x2, logF, data = ldata, trace = TRUE)
coef(fit2, matrix = TRUE)
summary(fit2)
vcov(fit2)
head(fitted(fit1))
with(ldata, mean(y1))
max(abs(head(fitted(fit1)) - with(ldata, mean(y1))))
```


## logff Logarithmic Distribution

## Description

Estimating the (single) parameter of the logarithmic distribution.

## Usage

logff(lshape = "logitlink", gshape = -expm1(-7 * ppoints(4)), zero = NULL)

## Arguments

$$
\begin{array}{ll}
\text { lshape } & \begin{array}{l}
\text { Parameter link function for the parameter } c \text {, which lies between } 0 \text { and 1. See } \\
\text { Links for more choices and information. Soon logfflink() will hopefully be } \\
\text { available for event-rate data. }
\end{array} \\
\text { gshape, zero } \quad \begin{array}{l}
\text { Details at CommonVGAMffArguments. Practical experience shows that having the } \\
\text { initial value for } c \text { being close to the solution is quite important. }
\end{array}
\end{array}
$$

## Details

The logarithmic distribution is a generalized power series distribution that is based specifically on the logarithmic series (scaled to a probability function). Its probability function is $f(y)=a c^{y} / y$, for $y=1,2,3, \ldots$, where $0<c<1$ (called shape), and $a=-1 / \log (1-c)$. The mean is $a c /(1-c)$ (returned as the fitted values) and variance is $a c(1-a c) /(1-c)^{2}$. When the sample mean is large, the value of $c$ tends to be very close to 1 , hence it could be argued that logitlink is not the best choice.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

The function log computes the natural logarithm. In the VGAM library, a link function with option loglink corresponds to this.

Multiple responses are permitted.
The logarithmic distribution is sometimes confused with the log-series distribution. The latter was used by Fisher et al. for species abundance data and has two parameters.

## Author(s)

T. W. Yee

## References

Johnson N. L., Kemp, A. W. and Kotz S. (2005). Univariate Discrete Distributions, 3rd edition, ch.7. Hoboken, New Jersey: Wiley.
Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011) Statistical Distributions, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

## See Also

rlog, oalog, oilog, otlog, log, loglink, logofflink, explogff, simulate.vlm.

## Examples

```
nn <- 1000
ldata <- data.frame(y = rlog(nn, shape = logitlink(0.2, inv = TRUE)))
fit <- vglm(y ~ 1, logff, data = ldata, trace = TRUE, crit = "c")
coef(fit, matrix = TRUE)
Coef(fit)
## Not run: with(ldata,
    hist(y, prob = TRUE, breaks = seq(0.5, max(y) + 0.5, by = 1),
        border = "blue"))
x <- seq(1, with(ldata, max(y)), by = 1)
with(ldata, lines(x, dlog(x, Coef(fit)[1]), col = "orange",
    type = "h", lwd = 2))
## End(Not run)
# Example: Corbet (1943) butterfly Malaya data
corbet <- data.frame(nindiv = 1:24,
            ofreq = c(118, 74, 44, 24, 29, 22, 20, 19, 20, 15, 12,
                            14, 6, 12, 6, 9, 9, 6, 10, 10, 11, 5, 3, 3))
fit <- vglm(nindiv ~ 1, logff, data = corbet, weights = ofreq)
coef(fit, matrix = TRUE)
shapehat <- Coef(fit)["shape"]
pdf2 <- dlog(x = with(corbet, nindiv), shape = shapehat)
print(with(corbet, cbind(nindiv, ofreq, fitted = pdf2 * sum(ofreq))),
    digits = 1)
```

logistic Logistic Distribution Family Function

## Description

Estimates the location and scale parameters of the logistic distribution by maximum likelihood estimation.

## Usage

```
logistic1(llocation = "identitylink", scale.arg = 1, imethod = 1)
logistic(llocation = "identitylink", lscale = "loglink",
    ilocation = NULL, iscale = NULL, imethod = 1, zero = "scale")
```


## Arguments

llocation, lscale
Parameter link functions applied to the location parameter $l$ and scale parameter $s$. See Links for more choices, and CommonVGAMffarguments for more information.
scale.arg Known positive scale parameter (called $s$ below).
ilocation, iscale
See CommonVGAMffArguments for information.
imethod, zero See CommonVGAMffArguments for information.

## Details

The two-parameter logistic distribution has a density that can be written as

$$
f(y ; l, s)=\frac{\exp [-(y-l) / s]}{s(1+\exp [-(y-l) / s])^{2}}
$$

where $s>0$ is the scale parameter, and $l$ is the location parameter. The response $-\infty<y<\infty$. The mean of $Y$ (which is the fitted value) is $l$ and its variance is $\pi^{2} s^{2} / 3$.
A logistic distribution with scale $=0.65$ (see dlogis) resembles $d t$ with $d f=7$; see logistic1 and studentt.
logistic1 estimates the location parameter only while logistic estimates both parameters. By default, $\eta_{1}=l$ and $\eta_{2}=\log (s)$ for logistic.
logistic can handle multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Note

Fisher scoring is used, and the Fisher information matrix is diagonal.

## Author(s)

T. W. Yee

## References

Johnson, N. L. and Kotz, S. and Balakrishnan, N. (1994). Continuous Univariate Distributions, 2nd edition, Volume 1, New York: Wiley. Chapter 15.
Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011). Statistical Distributions, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.
Castillo, E., Hadi, A. S., Balakrishnan, N. Sarabia, J. S. (2005). Extreme Value and Related Models with Applications in Engineering and Science, Hoboken, NJ, USA: Wiley-Interscience, p. 130.
deCani, J. S. and Stine, R. A. (1986). A Note on Deriving the Information Matrix for a Logistic Distribution, The American Statistician, 40, 220-222.

## See Also

rlogis, CommonVGAMffArguments, logitlink, cumulative, bilogistic, simulate.vlm.

## Examples

```
# Location unknown, scale known
ldata <- data.frame(x2 = runif(nn <- 500))
ldata <- transform(ldata, y1 = rlogis(nn, loc = 1 + 5*x2, scale = exp(2)))
fit1 <- vglm(y1 ~ x2, logistic1(scale = exp(2)), data = ldata, trace = TRUE)
coef(fit1, matrix = TRUE)
# Both location and scale unknown
ldata <- transform(ldata, y2 = rlogis(nn, loc = 1 + 5*x2, scale = exp(0 + 1*x2)))
fit2 <- vglm(cbind(y1, y2) ~ x2, logistic, data = ldata, trace = TRUE)
coef(fit2, matrix = TRUE)
vcov(fit2)
summary(fit2)
```

logitlink Logit Link Function

## Description

Computes the logit transformation, including its inverse and the first two derivatives.

## Usage

logitlink(theta, bvalue = NULL, inverse = FALSE, deriv = 0, short $=$ TRUE, tag $=$ FALSE)
extlogitlink(theta, min = 0, max = 1, bminvalue = NULL, bmaxvalue = NULL, inverse $=$ FALSE, deriv $=0$, short $=$ TRUE, tag $=$ FALSE)

## Arguments

theta Numeric or character. See below for further details.
bvalue, bminvalue, bmaxvalue
See Links. These are boundary values. For extlogitlink, values of theta less than or equal to $A$ or greater than or equal to $B$ can be replaced by bminvalue and bmaxvalue.
$\min , \max \quad$ For extlogitlink, min gives $A$, max gives $B$, and for out of range values, bminvalue and bmaxvalue.
inverse, deriv, short, tag
Details at Links.

## Details

The logit link function is very commonly used for parameters that lie in the unit interval. It is the inverse CDF of the logistic distribution. Numerical values of theta close to 0 or 1 or out of range result in Inf, -Inf, NA or NaN.
The extended logit link function extlogitlink should be used more generally for parameters that lie in the interval $(A, B)$, say. The formula is

$$
\log ((\theta-A) /(B-\theta))
$$

and the default values for $A$ and $B$ correspond to the ordinary logit function. Numerical values of theta close to $A$ or $B$ or out of range result in Inf, -Inf, NA or NaN. However these can be replaced by values bminvalue and bmaxvalue first before computing the link function.

## Value

For logitlink with deriv $=0$, the logit of theta, i.e., $\log ($ theta $/(1-$ theta $)$ ) when inverse $=$ FALSE, and if inverse $=$ TRUE then $\exp ($ theta $) /(1+\exp ($ theta $))$.

For deriv $=1$, then the function returns $d$ eta $/ d$ theta as a function of theta if inverse $=$ FALSE, else if inverse = TRUE then it returns the reciprocal.
Here, all logarithms are natural logarithms, i.e., to base $e$.

## Note

Numerical instability may occur when theta is close to 1 or 0 (for logitlink), or close to $A$ or $B$ for extlogitlink. One way of overcoming this is to use, e.g., bvalue.
In terms of the threshold approach with cumulative probabilities for an ordinal response this link function corresponds to the univariate logistic distribution (see logistic).

## Author(s)

Thomas W. Yee

## References

McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, 2nd ed. London: Chapman \& Hall.

## See Also

Links, logitoffsetlink, probitlink, clogloglink, cauchitlink, logistic1, loglink, Logistic, multilogitlink.

## Examples

```
p <- seq(0.01, 0.99, by = 0.01)
logitlink(p)
max(abs(logitlink(logitlink(p), inverse = TRUE) - p)) # Should be 0
p <- c(seq(-0.02, 0.02, by = 0.01), seq(0.97, 1.02, by = 0.01))
```

```
logitlink(p) # Has NAs
logitlink(p, bvalue = .Machine$double.eps) # Has no NAs
p <- seq(0.9, 2.2, by = 0.1)
extlogitlink(p, min = 1, max = 2,
            bminvalue = 1 + .Machine$double.eps,
            bmaxvalue = 2 - .Machine$double.eps) # Has no NAs
## Not run: par(mfrow = c(2,2), lwd = (mylwd <- 2))
y <- seq(-4, 4, length = 100)
p <- seq(0.01, 0.99, by = 0.01)
for (d in 0:1) {
    myinv <- (d > 0)
    matplot(p, cbind( logitlink(p, deriv = d, inverse = myinv),
                    probitlink(p, deriv = d, inverse = myinv)),
                    type = "n", col = "purple", ylab = "transformation", las = 1,
                main = if (d == 0) "Some probability link functions"
                else "1 / first derivative")
    lines(p, logitlink(p, deriv = d, inverse = myinv), col = "limegreen")
    lines(p, probitlink(p, deriv = d, inverse = myinv), col = "purple")
    lines(p, clogloglink(p, deriv = d, inverse = myinv), col = "chocolate")
    lines(p, cauchitlink(p, deriv = d, inverse = myinv), col = "tan")
    if (d == 0) {
        abline(v = 0.5, h = 0, lty = "dashed")
        legend(0, 4.5, c("logitlink", "probitlink", "clogloglink", "cauchitlink"),
                col = c("limegreen", "purple", "chocolate", "tan"), lwd = mylwd)
    } else
        abline(v = 0.5, lty = "dashed")
}
for (d in 0) {
    matplot(y, cbind(logitlink(y, deriv = d, inverse = TRUE),
                    probitlink(y, deriv = d, inverse = TRUE)), las = 1,
                    type = "n", col = "purple", xlab = "transformation", ylab = "p",
                main = if (d == 0) "Some inverse probability link functions"
                else "First derivative")
    lines(y, logitlink(y, deriv = d, inverse = TRUE), col = "limegreen")
    lines(y, probitlink(y, deriv = d, inverse = TRUE), col = "purple")
    lines(y, clogloglink(y, deriv = d, inverse = TRUE), col = "chocolate")
    lines(y, cauchitlink(y, deriv = d, inverse = TRUE), col = "tan")
    if (d == 0) {
        abline(h = 0.5, v = 0, lty = "dashed")
        legend(-4, 1, c("logitlink", "probitlink", "clogloglink", "cauchitlink"),
                col = c("limegreen", "purple", "chocolate", "tan"), lwd = mylwd)
    }
}
p <- seq(0.21, 0.59, by = 0.01)
plot(p, extlogitlink(p, min = 0.2, max = 0.6),
    type = "l", col = "black", ylab = "transformation", xlim = c(0, 1),
    las = 1, main = "extlogitlink(p, min = 0.2, max = 0.6)")
par(lwd = 1)
```

\#\# End(Not run)
logitoffsetlink Logit-with-an-Offset Link Function

## Description

Computes the logitoffsetlink transformation, including its inverse and the first two derivatives.

## Usage

logitoffsetlink(theta, offset $=0$, inverse $=$ FALSE, deriv $=0$, short $=$ TRUE, tag = FALSE)

## Arguments

theta Numeric or character. See below for further details.
offset The offset value(s), which must be non-negative. It is called $K$ below.
inverse, deriv, short, tag
Details at Links.

## Details

This link function allows for some asymmetry compared to the ordinary logitlink link. The formula is

$$
\log (\theta /(1-\theta)-K)
$$

and the default value for the offset $K$ is corresponds to the ordinary logitlink link. When inverse $=$ TRUE will mean that the value will lie in the interval $(K /(1+K), 1)$.

## Value

For logitoffsetlink with deriv $=0$, the logitoffsetlink of theta, i.e., $\log$ (theta/(1-theta) $-K)$ when inverse $=$ FALSE, and if inverse $=$ TRUE then $(K+\exp ($ theta $)) /(1+\exp ($ theta $)+$ $\mathrm{K})$.
For deriv $=1$, then the function returns $d$ eta $/ d$ theta as a function of theta if inverse $=$ FALSE, else if inverse = TRUE then it returns the reciprocal.
Here, all logarithms are natural logarithms, i.e., to base $e$.

## Note

This function is numerical less stability than logitlink.

## Author(s)

Thomas W. Yee

## References

Komori, O. and Eguchi, S. et al., 2016. An asymmetric logistic model for ecological data. Methods in Ecology and Evolution, 7.

## See Also

Links, logitlink.

## Examples

```
p <- seq(0.05, 0.99, by = 0.01); myoff <- 0.05
logitoffsetlink(p, myoff)
max(abs(logitoffsetlink(logitoffsetlink(p, myoff),
    myoff, inverse = TRUE) - p)) # Should be 0
```

loglaplace Log-Laplace and Logit-Laplace Distribution Family Functions

## Description

Maximum likelihood estimation of the 1-parameter log-Laplace and the 1-parameter logit-Laplace distributions. These may be used for quantile regression for counts and proportions respectively.

## Usage

loglaplace1(tau = NULL, llocation = "loglink",
ilocation $=$ NULL, kappa $=\operatorname{sqrt}(t a u /(1-\operatorname{tau}))$, Scale.arg $=1$,
ishrinkage = 0.95, parallel.locat = FALSE, digt = 4,
idf.mu $=3$, rep0 $=0.5$, minquantile $=0$, maxquantile $=\operatorname{Inf}$,
imethod $=1$, zero = NULL)
logitlaplace1(tau = NULL, llocation = "logitlink",
ilocation $=$ NULL, kappa $=\operatorname{sqrt(tau/(1-tau)),~}$
Scale.arg $=1$, ishrinkage $=0.95$, parallel.locat $=$ FALSE,
digt $=4$, idf.mu $=3$, rep01 $=0.5$, imethod $=1$, zero $=$ NULL)

## Arguments

tau, kappa See alaplace1.
llocation Character. Parameter link functions for location parameter $\xi$. See Links for more choices. However, this argument should be left unchanged with count data because it restricts the quantiles to be positive. With proportions data llocation can be assigned a link such as logitlink, probitlink, clogloglink, etc.
ilocation Optional initial values. If given, it must be numeric and values are recycled to the appropriate length. The default is to choose the value internally.
parallel.locat Logical. Should the quantiles be parallel on the transformed scale (argument llocation)? Assigning this argument to TRUE circumvents the seriously embarrassing quantile crossing problem.

```
imethod Initialization method. Either the value 1, 2, or ....
idf.mu, ishrinkage, Scale.arg, digt, zero
                    See alaplace1.
rep0, rep01 Numeric, positive. Replacement values for 0s and 1s respectively. For count
    data, values of the response whose value is 0 are replaced by rep0; it avoids
    computing \(\log (0)\). For proportions data values of the response whose value is
    0 or 1 are replaced by \(\min (r a n g e y 01[1] / 2, r e p 01 / w[y<=0])\) and \(\max ((1+\)
    rangey01[2])/2,1-rep01/w[y >=1]) respectively; e.g., it avoids computing
    logitlink(0) or logitlink(1). Here, rangey01 is the 2 -vector range (y[(y
    \(>0) \&(y<1)])\) of the response.
minquantile, maxquantile
```

Numeric. The minimum and maximum values possible in the quantiles. These argument are effectively ignored by default since loglink keeps all quantiles positive. However, if llocation = logofflink (offset $=1$ ) then it is possible that the fitted quantiles have value 0 because minquantile $=0$.

## Details

These VGAM family functions implement translations of the asymmetric Laplace distribution (ALD). The resulting variants may be suitable for quantile regression for count data or sample proportions. For example, a log link applied to count data is assumed to follow an ALD. Another example is a logit link applied to proportions data so as to follow an ALD. A positive random variable $Y$ is said to have a log-Laplace distribution if $Y=e^{W}$ where $W$ has an ALD. There are many variants of ALDs and the one used here is described in alaplace1.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.
In the extra slot of the fitted object are some list components which are useful. For example, the sample proportion of values which are less than the fitted quantile curves, which is sum (wprior [y <= location]) / sum(wprior) internally. Here, wprior are the prior weights (called ssize below), y is the response and location is a fitted quantile curve. This definition comes about naturally from the transformed ALD data.

## Warning

The VGAM family function logitlaplace1 will not handle a vector of just 0 s and 1 s as the response; it will only work satisfactorily if the number of trials is large.
See alaplace 1 for other warnings. Care is needed with tau values which are too small, e.g., for count data the sample proportion of zeros must be less than all values in tau. Similarly, this also holds with logitlaplace1, which also requires all tau values to be less than the sample proportion of ones.

## Note

The form of input for logitlaplace1 as response is a vector of proportions (values in $[0,1]$ ) and the number of trials is entered into the weights argument of vglm/vgam. See Example 2 below. See alaplace 1 for other notes in general.

## Author(s)

Thomas W. Yee

## References

Kotz, S., Kozubowski, T. J. and Podgorski, K. (2001). The Laplace distribution and generalizations: a revisit with applications to communications, economics, engineering, and finance, Boston: Birkhauser.
Kozubowski, T. J. and Podgorski, K. (2003). Log-Laplace distributions. International Mathematical Journal, 3, 467-495.
Yee, T. W. (2020). Quantile regression for counts and proportions. In preparation.

## See Also

alaplace1, dloglap.

## Examples

```
# Example 1: quantile regression of counts with regression splines
set.seed(123); my.k <- exp(0)
adata <- data.frame(x2 = sort(runif(n <- 500)))
mymu <- function(x) exp( 1 + 3*sin(2*x) / (x+0.5)^2)
adata <- transform(adata, y = rnbinom(n, mu = mymu(x2), size = my.k))
mytau <- c(0.1, 0.25, 0.5, 0.75, 0.9); mydof = 3
# halfstepping is usual:
fitp <- vglm(y ~ sm.bs(x2, df = mydof), data = adata, trace = TRUE,
        loglaplace1(tau = mytau, parallel.locat = TRUE))
## Not run:
par(las = 1) # Plot on a log1p() scale
mylwd <- 1.5
```

plot(jitter(log1p(y), factor = 1.5) ~ x2, adata, col = "red", pch = "o",
main = "Example 1; darkgreen=truth, blue=estimated", cex = 0.75)
with(adata, matlines(x2, $\log 1 p(f i t t e d(f i t p)), ~ c o l=" b l u e "$,
lty = 1, lwd = mylwd))
finexgrid <- seq(0, 1, len = 201)
for (ii in 1:length(mytau))
lines(finexgrid, col = "darkgreen", lwd = mylwd,
$\log 1 p(q n b i n o m(p=m y t a u[i i], m u=m y m u(f i n e x g r i d), s i=m y . k)))$
\#\# End(Not run)
fitp@extra \# Contains useful information
\# Example 2: sample proportions
set.seed(123); nnn <- 1000; ssize <- 100 \# ssize $=1$ will not work!
adata <- data.frame(x2 = sort(runif(nnn)))
mymu <- function(x) logitlink( $1.0+4 * x$, inv = TRUE)
adata <- transform(adata, ssize = ssize,

```
y2 = rbinom(nnn, size = ssize, prob = mymu(x2)) / ssize)
mytau <- c(0.25, 0.50, 0.75)
fit1 <- vglm(y2 ~ sm.bs(x2, df = 3), data = adata,
    weights = ssize, trace = TRUE,
    logitlaplace1(tau = mytau, lloc = "clogloglink", paral = TRUE))
## Not run:
# Check the solution. Note: this may be like comparing apples with oranges.
plotvgam(fit1, se = TRUE, scol = "red", lcol = "blue",
    main = "Truth = 'darkgreen'")
# Centered approximately !
linkFunctionChar <- as.character(fit1@misc$link)
adata <- transform(adata, trueFunction=
    theta2eta(theta = mymu(x2), link=linkFunctionChar))
with(adata, lines(x2, trueFunction - mean(trueFunction), col = "darkgreen"))
# Plot the data + fitted quantiles (on the original scale)
myylim <- with(adata, range(y2))
plot(y2 ~ x2, adata, col = "blue", ylim = myylim, las = 1,
    pch = ".", cex = 2.5)
with(adata, matplot(x2, fitted(fit1), add = TRUE, lwd = 3, type = "l"))
truecol <- rep(1:3, len = fit1@misc$M) # Add the 'truth'
smallxgrid <- seq(0, 1, len = 501)
for (ii in 1:length(mytau))
    lines(smallxgrid, col = truecol[ii], lwd = 2,
            qbinom(p = mytau[ii], prob = mymu(smallxgrid), size = ssize) / ssize)
# Plot on the eta (== logitlink()/probit()/...) scale
with(adata, matplot(x2, predict(fit1), add = FALSE, lwd = 3, type = "l"))
# Add the 'truth'
for (ii in 1:length(mytau)) {
    true.quant <- qbinom(p = mytau[ii], pr = mymu(smallxgrid), si = ssize) / ssize
    lines(smallxgrid, theta2eta(theta = true.quant, link = linkFunctionChar),
        col = truecol[ii], lwd = 2)
}
## End(Not run)
```

loglapUC The Log-Laplace Distribution

## Description

Density, distribution function, quantile function and random generation for the 3-parameter logLaplace distribution with location parameter location.ald, scale parameter scale.ald (on the $\log$ scale), and asymmetry parameter kappa.

## Usage

```
dloglap(x, location.ald = 0, scale.ald = 1,
        tau \(=0.5\), kappa \(=\operatorname{sqrt}(\) tau \(/(1-\) tau \()), \quad\) log \(=\) FALSE \()\)
ploglap(q, location.ald \(=0\), scale.ald \(=1\),
    tau \(=0.5\), kappa \(=\operatorname{sqrt}(\) tau/(1-tau) \()\), lower.tail \(=\) TRUE, log.p = FALSE)
qloglap(p, location.ald = 0, scale.ald = 1,
    tau \(=0.5\), kappa \(=\operatorname{sqrt}(\) tau/(1-tau) \()\), lower.tail \(=\) TRUE, log.p = FALSE)
rloglap(n, location.ald \(=0\), scale.ald \(=1\),
        tau \(=0.5\), kappa \(=\operatorname{sqrt}(\operatorname{tau} /(1-\operatorname{tau})))\)
```


## Arguments

```
    x,q vector of quantiles.
    p vector of probabilities.
    n number of observations. If length(n)>1 then the length is taken to be the
    number required.
    location.ald, scale.ald
    the location parameter }\xi\mathrm{ and the (positive) scale parameter }\sigma\mathrm{ , on the log scale.
    tau the quantile parameter }\tau\mathrm{ . Must consist of values in (0,1). This argument is used
    to specify kappa and is ignored if kappa is assigned.
    kappa the asymmetry parameter }\kappa\mathrm{ . Must consist of positive values.
    log if TRUE, probabilities p are given as log(p).
    lower.tail, log.p
    Same meaning as in pnorm or qnorm.
```


## Details

A positive random variable $Y$ is said to have a $\log$-Laplace distribution if $\log (Y)$ has an asymmetric Laplace distribution (ALD). There are many variants of ALDs and the one used here is described in alaplace3.

## Value

dloglap gives the density, ploglap gives the distribution function, qloglap gives the quantile function, and rloglap generates random deviates.

## Author(s)

T. W. Yee and Kai Huang

## References

Kozubowski, T. J. and Podgorski, K. (2003). Log-Laplace distributions. International Mathematical Journal, 3, 467-495.

## See Also

dalap, alaplace3, loglaplace1.

## Examples

```
loc <- 0; sigma <- exp(0.5); kappa <- 1
x <- seq(-0.2, 5, by = 0.01)
## Not run:
plot(x, dloglap(x, loc, sigma, kappa = kappa), type = "l", col = "blue",
    main = "Blue is density, red is cumulative distribution function",
    ylim = c(0,1), sub = "Purple are 5,10,\ldots,95 percentiles", las = 1, ylab = "")
abline(h = 0, col = "blue", lty = 2)
lines(qloglap(seq(0.05,0.95,by = 0.05), loc, sigma, kappa = kappa),
    dloglap(qloglap(seq(0.05,0.95,by = 0.05), loc, sigma, kappa = kappa),
                loc, sigma, kappa = kappa), col = "purple", lty = 3, type = "h")
lines(x, ploglap(x, loc, sigma, kappa = kappa), type = "l", col = "red")
abline(h = 0, lty = 2)
## End(Not run)
ploglap(qloglap(seq(0.05,0.95,by = 0.05), loc, sigma, kappa = kappa),
        loc, sigma, kappa = kappa)
```

    logLik.vlm
    Extract Log-likelihood for VGLMs/VGAMs/etc.
    
## Description

Calculates the log-likelihood value or the element-by-element contributions of the log-likelihood.

## Usage

\#\# S3 method for class 'vlm'
logLik(object, summation = TRUE, ...)

## Arguments

object Some VGAM object, for example, having class vglmff-class.
summation Logical, apply sum? If FALSE then a $n$-vector or $n$-row matrix (with the number of responses as the number of columns) is returned. Each element is the contribution to the log-likelihood.
... Currently unused. In the future: other possible arguments fed into logLik in order to compute the log-likelihood.

## Details

By default, this function returns the log-likelihood of the object. Thus this code relies on the loglikelihood being defined, and computed, for the object.

## Value

Returns the log-likelihood of the object. If summation $=$ FALSE then a $n$-vector or $n$-row matrix (with the number of responses as the number of columns) is returned. Each element is the contribution to the log-likelihood. The prior weights are assimulated within the answer.

## Warning

Not all VGAM family functions have had the summation checked.

## Note

Not all VGAM family functions currently have the summation argument implemented.

## Author(s)

T. W. Yee.

## See Also

VGLMs are described in vglm-class; VGAMs are described in vgam-class; RR-VGLMs are described in rrvglm-class; AIC; anova.vglm.

## Examples

```
zdata <- data.frame(x2 = runif(nn <- 50))
zdata <- transform(zdata, Ps01 = logitlink(-0.5 , inverse = TRUE),
    Ps02 = logitlink( 0.5 , inverse = TRUE),
    lambda1 = loglink(-0.5 + 2*x2, inverse = TRUE),
    lambda2 = loglink( 0.5 + 2*x2, inverse = TRUE))
zdata <- transform(zdata, y1 = rzipois(nn, lambda = lambda1, pstr0 = Ps01),
            y2 = rzipois(nn, lambda = lambda2, pstr0 = Ps02))
with(zdata, table(y1)) # Eyeball the data
with(zdata, table(y2))
fit2 <- vglm(cbind(y1, y2) ~ x2, zipoisson(zero = NULL), data = zdata)
logLik(fit2) # Summed over the two responses
sum(logLik(fit2, sum = FALSE)) # For checking purposes
(ll.matrix <- logLik(fit2, sum = FALSE)) # nn x 2 matrix
colSums(ll.matrix) # log-likelihood for each response
```


## Description

Fits a loglinear model to two binary responses.

## Usage

loglinb2(exchangeable = FALSE, zero = "u12")

## Arguments

exchangeable Logical. If TRUE, the two marginal probabilities are constrained to be equal. Should be set TRUE for ears, eyes, etc. data.
zero Which linear/additive predictors are modelled as intercept-only? A NULL means none of them. See CommonVGAMffArguments for more information.

## Details

The model is

$$
P\left(Y_{1}=y_{1}, Y_{2}=y_{2}\right)=\exp \left(u_{0}+u_{1} y_{1}+u_{2} y_{2}+u_{12} y_{1} y_{2}\right)
$$

where $y_{1}$ and $y_{2}$ are 0 or 1 , and the parameters are $u_{1}, u_{2}, u_{12}$. The normalizing parameter $u_{0}$ can be expressed as a function of the other parameters, viz.,

$$
u_{0}=-\log \left[1+\exp \left(u_{1}\right)+\exp \left(u_{2}\right)+\exp \left(u_{1}+u_{2}+u_{12}\right)\right] .
$$

The linear/additive predictors are $\left(\eta_{1}, \eta_{2}, \eta_{3}\right)^{T}=\left(u_{1}, u_{2}, u_{12}\right)^{T}$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.
When fitted, the fitted. values slot of the object contains the four joint probabilities, labelled as $\left(Y_{1}, Y_{2}\right)=(0,0),(0,1),(1,0),(1,1)$, respectively.

## Note

The response must be a two-column matrix of ones and zeros only. This is more restrictive than binom2.or, which can handle more types of input formats. Note that each of the 4 combinations of the multivariate response need to appear in the data set. After estimation, the response attached to the object is also a two-column matrix; possibly in the future it might change into a four-column matrix.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Wild, C. J. (2001). Discussion to: "Smoothing spline ANOVA for multivariate Bernoulli observations, with application to ophthalmology data (with discussion)" by Gao, F., Wahba, G., Klein, R., Klein, B. Journal of the American Statistical Association, 96, 127-160.

McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, 2nd ed. London: Chapman \& Hall.

## See Also

binom2.or, binom2.rho, loglinb3.

## Examples

```
    coalminers <- transform(coalminers, Age = (age - 42) / 5)
    # Get the n x 4 matrix of counts
    fit0 <- vglm(cbind(nBnW,nBW,BnW,BW) ~ Age, binom2.or, data = coalminers)
    counts <- round(c(weights(fit0, type = "prior")) * depvar(fit0))
    # Create a n x 2 matrix response for loglinb2()
    # bwmat <- matrix(c(0,0, 0,1, 1,0, 1,1), 4, 2, byrow = TRUE)
    bwmat <- cbind(bln = c(0,0,1,1), wheeze = c(0,1,0,1))
    matof1 <- matrix(1, nrow(counts), 1)
    newminers <- data.frame(bln = kronecker(matof1, bwmat[, 1]),
        wheeze = kronecker(matof1, bwmat[, 2]),
        wt = c(t(counts)),
        Age = with(coalminers, rep(age, rep(4, length(age)))))
    newminers <- newminers[with(newminers, wt) > 0,]
    fit <- vglm(cbind(bln,wheeze) ~ Age, loglinb2(zero = NULL),
    weight = wt, data = newminers)
    coef(fit, matrix = TRUE) # Same! (at least for the log odds-ratio)
    summary(fit)
    # Try reconcile this with McCullagh and Nelder (1989), p. }23
    (0.166-0.131) / 0.027458 # 1.275 is approximately 1.25
```

    loglinb3 Loglinear Model for Three Binary Responses
    
## Description

Fits a loglinear model to three binary responses.

## Usage

loglinb3(exchangeable $=$ FALSE, zero = c("u12", "u13", "u23"))

## Arguments

exchangeable Logical. If TRUE, the three marginal probabilities are constrained to be equal.
zero Which linear/additive predictors are modelled as intercept-only? A NULL means none. See CommonVGAMffArguments for further information.

## Details

The model is $P\left(Y_{1}=y_{1}, Y_{2}=y_{2}, Y_{3}=y_{3}\right)=$

$$
\exp \left(u_{0}+u_{1} y_{1}+u_{2} y_{2}+u_{3} y_{3}+u_{12} y_{1} y_{2}+u_{13} y_{1} y_{3}+u_{23} y_{2} y_{3}\right)
$$

where $y_{1}, y_{2}$ and $y_{3}$ are 0 or 1 , and the parameters are $u_{1}, u_{2}, u_{3}, u_{12}, u_{13}, u_{23}$. The normalizing parameter $u_{0}$ can be expressed as a function of the other parameters. Note that a third-order association parameter, $u_{123}$ for the product $y_{1} y_{2} y_{3}$, is assumed to be zero for this family function.
The linear/additive predictors are $\left(\eta_{1}, \eta_{2}, \ldots, \eta_{6}\right)^{T}=\left(u_{1}, u_{2}, u_{3}, u_{12}, u_{13}, u_{23}\right)^{T}$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

When fitted, the fitted.values slot of the object contains the eight joint probabilities, labelled as $\left(Y_{1}, Y_{2}, Y_{3}\right)=(0,0,0),(0,0,1),(0,1,0),(0,1,1),(1,0,0),(1,0,1),(1,1,0),(1,1,1)$, respectively.

## Note

The response must be a 3 -column matrix of ones and zeros only. Note that each of the 8 combinations of the multivariate response need to appear in the data set, therefore data sets will need to be large in order for this family function to work. After estimation, the response attached to the object is also a 3-column matrix; possibly in the future it might change into a 8-column matrix.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Wild, C. J. (2001). Discussion to: "Smoothing spline ANOVA for multivariate Bernoulli observations, with application to ophthalmology data (with discussion)" by Gao, F., Wahba, G., Klein, R., Klein, B. Journal of the American Statistical Association, 96, 127-160.
McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, 2nd ed. London: Chapman \& Hall.

## See Also

loglinb2, hunua.

## Examples

```
lfit <- vglm(cbind(cyadea, beitaw, kniexc) ~ altitude, loglinb3,
            data = hunua, trace = TRUE)
coef(lfit, matrix = TRUE)
head(fitted(lfit))
summary(lfit)
```


## Description

Computes the log transformation, including its inverse and the first two derivatives.

## Usage

loglink(theta, bvalue = NULL, inverse = FALSE, deriv = 0, short = TRUE, tag = FALSE)
negloglink(theta, bvalue $=$ NULL, inverse $=$ FALSE, deriv $=0$, short = TRUE, tag = FALSE)
logneglink(theta, bvalue $=$ NULL, inverse $=$ FALSE, deriv $=0$, short $=$ TRUE, tag $=$ FALSE)

## Arguments

| theta | Numeric or character. See below for further details. |
| :--- | :--- |
| bvalue | See Links. |
| inverse, deriv, short, tag |  |
|  | Details at Links. |

## Details

The log link function is very commonly used for parameters that are positive. Here, all logarithms are natural logarithms, i.e., to base $e$. Numerical values of theta close to 0 or out of range result in Inf, -Inf, NA or NaN.
The function $\log l i n k$ computes $\log (\theta)$ whereas negloglink computes $-\log (\theta)=\log (1 / \theta)$.
The function logneglink computes $\log (-\theta)$, hence is suitable for parameters that are negative, e.g., a trap-shy effect in posbernoulli.b.

## Value

The following concerns loglink. For deriv $=0$, the $\log$ of theta, i.e., $\log$ (theta) when inverse $=$ FALSE, and if inverse $=$ TRUE then $\exp$ (theta). For deriv $=1$, then the function returns $d$ eta $/ d$ theta as a function of theta if inverse = FALSE, else if inverse $=$ TRUE then it returns the reciprocal.

## Note

This function was called loge to avoid conflict with the log function. Numerical instability may occur when theta is close to 0 unless bvalue is used.

## Author(s)

Thomas W. Yee

## References

McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, 2nd ed. London: Chapman \& Hall.

## See Also

Links, explink, logitlink, logclink, logloglink, log, logofflink, lambertW, posbernoulli.b.

## Examples

```
## Not run: loglink(seq(-0.2, 0.5, by = 0.1))
    loglink(seq(-0.2, 0.5, by = 0.1), bvalue = .Machine$double.xmin)
negloglink(seq(-0.2, 0.5, by = 0.1))
negloglink(seq(-0.2, 0.5, by = 0.1), bvalue = .Machine$double.xmin)
## End(Not run)
logneglink(seq(-0.5, -0.2, by = 0.1))
```

logloglink Log-log and Log-log-log Link Functions

## Description

Computes the two transformations, including their inverse and the first two derivatives.

## Usage

logloglink(theta, bvalue = NULL, inverse $=$ FALSE, deriv $=0$, short $=$ TRUE, tag $=$ FALSE)
loglogloglink(theta, bvalue = NULL, inverse = FALSE, deriv = 0, short $=$ TRUE, tag $=$ FALSE)

## Arguments

theta Numeric or character. See below for further details.
bvalue Values of theta which are less than or equal to 1 or $e$ can be replaced by bvalue before computing the link function value. The component name bvalue stands for "boundary value". See Links for more information.
inverse, deriv, short, tag
Details at Links.

## Details

The log-log link function is commonly used for parameters that are greater than unity. Similarly, the log-log-log link function is applicable for parameters that are greater than $e$. Numerical values of theta close to 1 or $e$ or out of range result in Inf, $-\operatorname{Inf}$, NA or NaN. One possible application of loglogloglink() is to the $k$ parameter (also called size) of negbinomial to Poisson-like data but with only a small amount of overdispersion; then $k$ is a large number relative to munb. In such situations a loglink or loglog link may not be sufficient to draw the estimate toward the interior of the parameter space. Using a more stronger link function can help mitigate the Hauck-Donner effect hdeff.

## Value

For $\log \log \operatorname{link}():$ for deriv $=0$, the $\log$ of $\log ($ theta $)$, i.e., $\log (\log ($ theta $)$ ) when inverse $=$ FALSE, and if inverse $=$ TRUE then $\exp (\exp ($ theta $))$.

For $\log \log \log \operatorname{link}():$ for deriv $=0$, the $\log$ of $\log (\log ($ theta $))$, i.e., $\log (\log (\log ($ theta $)))$ when inverse $=$ FALSE, and if inverse $=$ TRUE then $\exp (\exp (\exp ($ theta $))$ ).

For deriv $=1$, then the function returns $d$ theta $/ d$ eta as a function of theta if inverse $=$ FALSE, else if inverse $=$ TRUE then it returns the reciprocal.
Here, all logarithms are natural logarithms, i.e., to base $e$.

## Note

Numerical instability may occur when theta is close to 1 or $e$ unless bvalue is used.

## References

McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, 2nd ed. London: Chapman \& Hall.

## See Also

Links, loglink, logofflink.

## Examples

```
x <- seq(0.8, 1.5, by = 0.1)
logloglink(x) # Has NAs
logloglink(x, bvalue = 1.0 + .Machine$double.eps) # Has no NAs
x <- seq(1.01, 10, len = 100)
logloglink(x)
max(abs(logloglink(logloglink(x), inverse = TRUE) - x)) # Should be 0
```


## lognormal Lognormal Distribution

## Description

Maximum likelihood estimation of the (univariate) lognormal distribution.

## Usage

lognormal(lmeanlog = "identitylink", lsdlog = "loglink", zero = "sdlog")

## Arguments

lmeanlog, lsdlog
Parameter link functions applied to the mean and (positive) $\sigma$ (standard deviation) parameter. Both of these are on the log scale. See Links for more choices.
zero Specifies which linear/additive predictor is modelled as intercept-only. For lognormal(), the values can be from the set $\{1,2\}$ which correspond to mu, sigma, respectively. See CommonVGAMfffarguments for more information.

## Details

A random variable $Y$ has a 2-parameter lognormal distribution if $\log (Y)$ is distributed $N\left(\mu, \sigma^{2}\right)$. The expected value of $Y$, which is

$$
E(Y)=\exp \left(\mu+0.5 \sigma^{2}\right)
$$

and not $\mu$, make up the fitted values. The variance of $Y$ is

$$
\operatorname{Var}(Y)=\left[\exp \left(\sigma^{2}\right)-1\right] \exp \left(2 \mu+\sigma^{2}\right)
$$

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Author(s)

T. W. Yee

## References

Kleiber, C. and Kotz, S. (2003). Statistical Size Distributions in Economics and Actuarial Sciences, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

Lognormal, uninormal, CommonVGAMffArguments, simulate.vlm.

## Examples

```
ldata2 <- data.frame(x2 = runif(nn <- 1000))
ldata2 <- transform(ldata2, y1 = rlnorm(nn, mean = 1 + 2 * x2, sd = exp(-1)),
                        y2 = rlnorm(nn, mean = 1, sd = exp(-1 + x2)))
fit1 <- vglm(y1 ~ x2, lognormal(zero = 2), data = ldata2, trace = TRUE)
fit2 <- vglm(y2 ~ x2, lognormal(zero = 1), data = ldata2, trace = TRUE)
coef(fit1, matrix = TRUE)
coef(fit2, matrix = TRUE)
```

logofflink Log Link Function with an Offset

## Description

Computes the log transformation with an offset, including its inverse and the first two derivatives.

## Usage

logofflink(theta, offset $=0$, inverse $=$ FALSE, deriv $=0$, short $=$ TRUE, tag $=$ FALSE)

## Arguments

$$
\begin{array}{ll}
\text { theta } & \text { Numeric or character. See below for further details. } \\
\text { offset } & \text { Offset value. See Links. } \\
\text { inverse, deriv, short, tag } \\
& \text { Details at Links. }
\end{array}
$$

## Details

The log-offset link function is very commonly used for parameters that are greater than a certain value. In particular, it is defined by $\log$ (theta + offset) where offset is the offset value. For example, if offset $=0.5$ then the value of theta is restricted to be greater than -0.5 .
Numerical values of theta close to -offset or out of range result in Inf, -Inf, NA or NaN.

## Value

For deriv $=0$, the $\log$ of theta+offset, i.e., $\log ($ theta+offset $)$ when inverse $=$ FALSE, and if inverse $=$ TRUE then $\exp$ (theta)-offset.
For deriv $=1$, then the function returns $d$ theta $/ d$ eta as a function of theta if inverse $=$ FALSE, else if inverse = TRUE then it returns the reciprocal.
Here, all logarithms are natural logarithms, i.e., to base $e$.

## Note

The default means this function is identical to loglink.
Numerical instability may occur when theta is close to -offset.

## Author(s)

Thomas W. Yee

## References

McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, 2nd ed. London: Chapman \& Hall.

## See Also

Links, loglink.

## Examples

```
## Not run:
logofflink(seq(-0.2, 0.5, by = 0.1))
logofflink(seq(-0.2, 0.5, by = 0.1), offset = 0.5)
        log(seq(-0.2, 0.5, by = 0.1) + 0.5)
## End(Not run)
```

Lomax The Lomax Distribution

## Description

Density, distribution function, quantile function and random generation for the Lomax distribution with scale parameter scale and shape parameter q.

## Usage

dlomax (x, scale = 1, shape3.q, log = FALSE)
plomax (q, scale $=1$, shape3.q, lower.tail = TRUE, log.p = FALSE)
qlomax (p, scale $=1$, shape3.q, lower.tail = TRUE, log.p = FALSE)
rlomax (n, scale = 1, shape3.q)

## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$\mathrm{n} \quad$ number of observations. If length $(\mathrm{n})>1$, the length is taken to be the number required.
scale scale parameter.
shape3.q shape parameter.
log Logical. If log = TRUE then the logarithm of the density is returned.
lower.tail, log.p
Same meaning as in pnorm or qnorm.

## Details

See lomax, which is the VGAM family function for estimating the parameters by maximum likelihood estimation.

## Value

dlomax gives the density, plomax gives the distribution function, qlomax gives the quantile function, and rlomax generates random deviates.

## Note

The Lomax distribution is a special case of the 4-parameter generalized beta II distribution.

## Author(s)

T. W. Yee and Kai Huang

## References

Kleiber, C. and Kotz, S. (2003). Statistical Size Distributions in Economics and Actuarial Sciences, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

lomax, genbetaII.

## Examples

```
    probs <- seq(0.1, 0.9, by = 0.1)
    max(abs(plomax(qlomax(p = probs, shape3.q = 1),
            shape3.q = 1) - probs)) # Should be 0
    ## Not run: par(mfrow = c(1, 2))
    x <- seq(-0.01, 5, len = 401)
    plot(x, dexp(x), type = "l", col = "black", ylab = "", ylim = c(0, 3),
        main = "Black is standard exponential, others are dlomax(x, shape3.q)")
    lines(x, dlomax(x, shape3.q = 1), col = "orange")
    lines(x, dlomax(x, shape3.q = 2), col = "blue")
    lines(x, dlomax(x, shape3.q = 5), col = "green")
    legend("topright", col = c("orange","blue","green"), lty = rep(1, 3),
        legend = paste("shape3.q =", c(1, 2, 5)))
    plot(x, pexp(x), type = "l", col = "black", ylab = "", las = 1,
        main = "Black is standard exponential, others are plomax(x, shape3.q)")
    lines(x, plomax(x, shape3.q = 1), col = "orange")
    lines(x, plomax(x, shape3.q = 2), col = "blue")
    lines(x, plomax(x, shape3.q = 5), col = "green")
    legend("bottomright", col = c("orange","blue","green"), lty = rep(1, 3),
        legend = paste("shape3.q =", c(1, 2, 5)))
    ## End(Not run)
```

    lomax Lomax Distribution Family Function
    
## Description

Maximum likelihood estimation of the 2-parameter Lomax distribution.

## Usage

lomax(lscale = "loglink", lshape3.q = "loglink", iscale = NULL,
ishape3.q $=$ NULL, imethod $=1$, gscale $=\exp (-5: 5)$,
gshape3.q $=\operatorname{seq}(0.75,4$, by $=0.25)$,
probs.y $=c(0.25,0.5,0.75)$, zero = "shape")

## Arguments

lscale, lshape3.q
Parameter link function applied to the (positive) parameters scale and q. See Links for more choices.
iscale, ishape3.q, imethod
See CommonVGAMffArguments for information. For imethod $=2$ a good initial value for iscale is needed to obtain a good estimate for the other parameter.
gscale, gshape3.q, zero, probs.y
See CommonVGAMffArguments.

## Details

The 2-parameter Lomax distribution is the 4-parameter generalized beta II distribution with shape parameters $a=p=1$. It is probably more widely known as the Pareto (II) distribution. It is also the 3-parameter Singh-Maddala distribution with shape parameter $a=1$, as well as the beta distribution of the second kind with $p=1$. More details can be found in Kleiber and Kotz (2003).
The Lomax distribution has density

$$
f(y)=q /\left[b\{1+y / b\}^{1+q}\right]
$$

for $b>0, q>0, y \geq 0$. Here, $b$ is the scale parameter scale, and $q$ is a shape parameter. The cumulative distribution function is

$$
F(y)=1-[1+(y / b)]^{-q}
$$

The mean is

$$
E(Y)=b /(q-1)
$$

provided $q>1$; these are returned as the fitted values. This family function handles multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

See the notes in genbetaII.

## Author(s)

T. W. Yee

## References

Kleiber, C. and Kotz, S. (2003). Statistical Size Distributions in Economics and Actuarial Sciences, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

Lomax, genbetaII, betaII, dagum, sinmad, fisk, inv.lomax, paralogistic, inv.paralogistic, simulate.vlm.

## Examples

```
ldata <- data.frame(y = rlomax(n = 1000, scale = exp(1), exp(2)))
fit <- vglm(y ~ 1, lomax, data = ldata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

lqnorm Minimizing the L-q norm Family Function

## Description

Minimizes the L-q norm of residuals in a linear model.

## Usage

lqnorm(qpower = 2, link = "identitylink", imethod $=1$, imu $=$ NULL, ishrinkage $=0.95$ )

## Arguments

qpower A single numeric, must be greater than one, called $q$ below. The absolute value of residuals are raised to the power of this argument, and then summed. This quantity is minimized with respect to the regression coefficients.
link Link function applied to the 'mean' $\mu$. See Links for more details.
imethod Must be 1,2 or 3. See CommonVGAMffArguments for more information. Ignored if imu is specified.
imu $\quad$ Numeric, optional initial values used for the fitted values. The default is to use imethod $=1$.
ishrinkage How much shrinkage is used when initializing the fitted values. The value must be between 0 and 1 inclusive, and a value of 0 means the individual response values are used, and a value of 1 means the median or mean is used. This argument is used in conjunction with imethod $=3$.

## Details

This function minimizes the objective function

$$
\sum_{i=1}^{n} w_{i}\left(\left|y_{i}-\mu_{i}\right|\right)^{q}
$$

where $q$ is the argument qpower, $\eta_{i}=g\left(\mu_{i}\right)$ where $g$ is the link function, and $\eta_{i}$ is the vector of linear/additive predictors. The prior weights $w_{i}$ can be inputted using the weights argument of vlm/vglm/vgam etc.; it should be just a vector here since this function handles only a single vector or one-column response.

Numerical problem will occur when $q$ is too close to one. Probably reasonable values range from 1.5 and up, say. The value $q=2$ corresponds to ordinary least squares while $q=1$ corresponds to the MLE of a double exponential (Laplace) distibution. The procedure becomes more sensitive to outliers the larger the value of $q$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

Convergence failure is common, therefore the user is advised to be cautious and monitor convergence!

## Note

This VGAM family function is an initial attempt to provide a more robust alternative for regression and/or offer a little more flexibility than least squares. The @misc slot of the fitted object contains a list component called objectiveFunction which is the value of the objective function at the final iteration.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Wild, C. J. (1996). Vector generalized additive models. Journal of the Royal Statistical Society, Series B, Methodological, 58, 481-493.

## See Also

uninormal.

## Examples

```
set.seed(123)
ldata <- data.frame(x = sort(runif(nn <- 10 )))
realfun <- function(x) 4 + 5*x
ldata <- transform(ldata, y = realfun(x) + rnorm(nn, sd = exp(-1)))
# Make the first observation an outlier
ldata <- transform(ldata, y = c(4*y[1], y[-1]), x = c(-1, x[-1]))
fit <- vglm(y ~ x, lqnorm(qpower = 1.2), data = ldata)
coef(fit, matrix = TRUE)
head(fitted(fit))
fit@misc$qpower
fit@misc$objectiveFunction
## Not run:
# Graphical check
with(ldata, plot(x, y, main = paste("LS = red, lqnorm = blue (qpower = ",
    fit@misc$qpower, "), truth = black", sep = ""), col = "blue"))
lmfit <- lm(y ~ x, data = ldata)
with(ldata, lines(x, fitted(fit), col = "blue"))
with(ldata, lines(x, lmfit$fitted, col = "red"))
with(ldata, lines(x, realfun(x), col = "black"))
## End(Not run)
```

lrt.stat Likelihood Ratio Test Statistics Evaluated at the Null Values

## Description

Generic function that computes likelihood ratio test (LRT) statistics evaluated at the null values (consequently they do not suffer from the Hauck-Donner effect).

## Usage

```
lrt.stat(object, ...)
lrt.stat.vlm(object, values0 \(=0\), subset \(=\) NULL, omit1s \(=\) TRUE,
            all. out \(=\) FALSE, trace \(=\) FALSE,.. .)
```


## Arguments

object, values0, subset
Same as in wald.stat.vlm.
omit1s, all.out, trace
Same as in wald.stat.vlm.
... Ignored for now.

## Details

When summary () is applied to a vglm object a 4-column Wald table is produced. The corresponding p-values are generally viewed as inferior to those from a likelihood ratio test (LRT). For example, the Hauck and Donner (1977) effect (HDE) produces p-values that are biased upwards (see hdeff). Other reasons are that the Wald test is often less accurate (especially in small samples) and is not invariant to parameterization. By default, this function returns p-values based on the LRT by deleting one column at a time from the big VLM matrix and then starting up IRLS to convergence (hopefully). Twice the difference between the log-likelihoods (or equivalently, the difference in the deviances if they are defined) is asymptotically chi-squared with 1 degree of freedom. One might expect the p-values from this function therefore to be more accurate and not suffer from the HDE. Thus this function is a recommended alternative (if it works) to summaryvglm for testing for the significance of a regression coefficient.

## Value

By default, a vector of signed square root of the LRT statistics; these are asymptotically standard normal under the null hypotheses. If all.out = TRUE then a list is returned with the following components: lrt.stat the signed LRT statistics, pvalues the 2 -sided p-values, Lrt.stat2 the usual LRT statistic, values0 the null values.

## Warning

See wald.stat.vlm.

## Author(s)

T. W. Yee.

## See Also

score.stat, wald.stat, summaryvglm, anova.vglm, vglm, lrtest, confintvglm, pchisq, profilevglm, hdeff.

## Examples

```
pneumo <- transform(pneumo, let = log(exposure.time))
fit <- vglm(cbind(normal, mild, severe) ~ let, propodds, data = pneumo)
cbind(coef(summary(fit)),
    "signed LRT stat" = lrt.stat(fit, omit1s = FALSE))
summary(fit, lrt0 = TRUE) # Easy way to get it
```


## lrtest Likelihood Ratio Test of Nested Models

## Description

lrtest is a generic function for carrying out likelihood ratio tests. The default method can be employed for comparing nested VGLMs (see details below).

## Usage

```
lrtest(object, ...)
lrtest_vglm(object, ..., no.warning = FALSE, name = NULL)
```


## Arguments

object a vglm object. See below for details.
... further object specifications passed to methods. See below for details.
no. warning logical; if TRUE then no warning is issued. For example, setting TRUE might be a good idea when testing for linearity of a variable for a "pvgam" object.
name a function for extracting a suitable name/description from a fitted model object. By default the name is queried by calling formula.

## Details

lrtest is intended to be a generic function for comparisons of models via asymptotic likelihood ratio tests. The default method consecutively compares the fitted model object object with the models passed in . . . . Instead of passing the fitted model objects in . . ., several other specifications are possible. The updating mechanism is the same as for waldtest: the models in ... can be specified as integers, characters (both for terms that should be eliminated from the previous model), update formulas or fitted model objects. Except for the last case, the existence of an update method is assumed. See waldtest for details.
Subsequently, an asymptotic likelihood ratio test for each two consecutive models is carried out: Twice the difference in log-likelihoods (as derived by the logLik methods) is compared with a Chi-squared distribution.

## Value

An object of class "VGAManova" which contains a slot with the log-likelihood, degrees of freedom, the difference in degrees of freedom, likelihood ratio Chi-squared statistic and corresponding p value. These are printed by stats:::print.anova(); see anova.

## Warning

Several VGAM family functions implement distributions which do not satisfying the usual regularity conditions needed for the LRT to work. No checking or warning is given for these.

Note
The code was adapted directly from Imtest (written by T. Hothorn, A. Zeileis, G. Millo, D. Mitchell) and made to work for VGLMs and S4. This help file also was adapted from lmtest.

Approximate LRTs might be applied to VGAMs, as produced by vgam, but it is probably better in inference to use vglm with regression splines (bs and ns). This methods function should not be applied to other models such as those produced by rrvglm, by cqo, by cao.

## See Also

lmtest, vglm, lrt.stat.vlm, score.stat.vlm, wald.stat.vlm, anova.vglm.

## Examples

```
set.seed(1)
pneumo <- transform(pneumo, let = log(exposure.time), x3 = runif(nrow(pneumo)))
fit1 <- vglm(cbind(normal, mild, severe) ~ let , propodds, data = pneumo)
fit2 <- vglm(cbind(normal, mild, severe) ~ let + x3, propodds, data = pneumo)
fit3 <- vglm(cbind(normal, mild, severe) ~ let , cumulative, data = pneumo)
# Various equivalent specifications of the LR test for testing x3
(ans1 <- lrtest(fit2, fit1))
ans2 <- lrtest(fit2, 2)
ans3 <- lrtest(fit2, "x3")
ans4 <- lrtest(fit2, . ~ . - x3)
c(all.equal(ans1, ans2), all.equal(ans1, ans3), all.equal(ans1, ans4))
# Doing it manually
(testStatistic <- 2 * (logLik(fit2) - logLik(fit1)))
(mypval <- pchisq(testStatistic, df = length(coef(fit2)) - length(coef(fit1)),
    lower.tail = FALSE))
(ans4 <- lrtest(fit3, fit1)) # Test proportional odds (parallelism) assumption
```

lvplot

Latent Variable Plot

## Description

Generic function for a latent variable plot (also known as an ordination diagram by ecologists).

## Usage

lvplot(object, ...)

## Arguments

object An object for a latent variable plot is meaningful.
... Other arguments fed into the specific methods function of the model. They usually are graphical parameters, and sometimes they are fed into the methods function for Coef.

## Details

Latent variables occur in reduced-rank regression models, as well as in quadratic and additive ordination. For the latter, latent variables are often called the site scores. Latent variable plots were coined by Yee (2004), and have the latent variable as at least one of its axes.

## Value

The value returned depends specifically on the methods function invoked.

## Note

Latent variables are not really applicable to vglm/vgam models.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2004). A new technique for maximum-likelihood canonical Gaussian ordination. Ecological Monographs, 74, 685-701.

Yee, T. W. (2006). Constrained additive ordination. Ecology, 87, 203-213.

## See Also

lvplot. qrrvglm, lvplot.cao, latvar, trplot.

## Examples

```
## Not run:
hspider[,1:6] <- scale(hspider[,1:6]) # Standardized environmental vars
set.seed(123)
p1 <- cao(cbind(Pardlugu, Pardmont, Pardnigr, Pardpull, Zoraspin) ~
    WaterCon + BareSand + FallTwig +
    CoveMoss + CoveHerb + ReflLux,
    family = poissonff, data = hspider, Bestof = 3,
    df1.nl = c(Zoraspin = 2.5, 3), Crow1positive = TRUE)
index <- 1:ncol(depvar(p1))
lvplot(p1, lcol = index, pcol = index, y = TRUE, las = 1)
## End(Not run)
```

lvplot.qrrvglm Latent Variable Plot for QO models

## Description

Produces an ordination diagram (latent variable plot) for quadratic ordination (QO) models. For rank-1 models, the $x$-axis is the first ordination/constrained/canonical axis. For rank- 2 models, the x - and y -axis are the first and second ordination axes respectively.

## Usage

```
lvplot.qrrvglm(object, varI.latvar = FALSE, refResponse = NULL,
    add = FALSE, show.plot = TRUE,
    rug = TRUE, y = FALSE, type = c("fitted.values", "predictors"),
    xlab = paste("Latent Variable", if (Rank == 1) "" else " 1", sep = ""),
    ylab = if (Rank == 1) switch(type, predictors = "Predictors",
    fitted.values = "Fitted values") else "Latent Variable 2",
    pcex = par()$cex, pcol = par()$col, pch = par()$pch,
    llty = par()$lty, lcol = par()$col, llwd = par()$lwd,
    label.arg = FALSE, adj.arg = -0.1,
    ellipse = 0.95, Absolute = FALSE,
    elty = par()$lty, ecol = par()$col, elwd = par()$lwd, egrid = 200,
    chull.arg = FALSE, clty = 2, ccol = par()$col, clwd = par()$lwd,
    cpch = " ",
    C = FALSE, OriginC = c("origin", "mean"),
    Clty = par()$lty, Ccol = par()$col, Clwd = par()$lwd,
    Ccex = par()$cex, Cadj.arg = -0.1, stretchC = 1,
    sites = FALSE, spch = NULL, scol = par()$col, scex = par()$cex,
    sfont = par()$font, check.ok = TRUE, jitter.y = FALSE, ...)
```


## Arguments

| object | A CQO object. |
| :--- | :--- |
| varI.latvar | Logical that is fed into Coef. qrrvglm. |
| refResponse | Integer or character that is fed into Coef. qrrvglm. <br> add <br> show.plot <br> rug |
|  | Logical. Add to an existing plot? If FALSE, a new plot is made. <br> Logical. Plot it? <br> Logical. If TRUE, a rug plot is plotted at the foot of the plot (applies to rank-1 <br> models only). These values are jittered to expose ties. <br> Logical. If TRUE, the responses will be plotted (applies only to rank-1 models <br> and if type = "fitted.values".) |
| type | Either "fitted.values" or "predictors", specifies whether the y-axis is on <br> the response or eta-scales respectively. |
| xlab | Caption for the x-axis. See par. |
| ylab | Caption for the y-axis. See par. <br> pcex <br> Character expansion of the points. Here, for rank-1 models, points are the re- <br> sponse $y$ data. For rank-2 models, points are the optimums. See the cex argu- <br> ment in par. |
| pcol | Color of the points. See the col argument in par. |
| pch | Either an integer specifying a symbol or a single character to be used as the <br> default in plotting points. See par. The pch argument can be of length $M, ~ t h e ~$ |
| number of species. |  |


| llwd | Line width. Rank-1 models only. See the lwd argument of par. |
| :--- | :--- |
| label.arg | Logical. Label the optimums and C? (applies only to rank-2 models only). |
| adj.arg | Justification of text strings for labelling the optimums (applies only to rank-2 |
| models only). See the adj argument of par. |  |
| ellipse | Numerical, of length 0 or 1 (applies only to rank-2 models only). If Absolute |
| is TRUE then ellipse should be assigned a value that is used for the elliptical |  |
| contouring. If Absolute is FALSE then ellipse should be assigned a value |  |
| between 0 and 1, for example, setting ellipse = 0.9 means an ellipse with |  |
| contour =90\% of the maximum will be plotted about each optimum. If ellipse |  |
| is a negative value, then the function checks that the model is an equal-tolerances |  |
| model and varI.latvar =FALSE, and if so, plots circles with radius -ellipse. |  |

$$
\begin{array}{ll}
\text { sites } & \begin{array}{l}
\text { Logical. Add the site scores (aka latent variable values, nu's) to the plot? (ap- } \\
\text { plies only to rank-2 models only). }
\end{array} \\
\text { spch } & \begin{array}{l}
\text { Plotting character of the site scores. The default value of NULL means the row } \\
\text { labels of the data frame are used. They often are the site numbers. See the pch } \\
\text { argument of par. }
\end{array} \\
\text { scol } & \begin{array}{l}
\text { Color of the site scores. See the col argument of par. }
\end{array} \\
\text { scex } & \text { Character expansion of the site scores. See the cex argument of par. } \\
\text { sfont } & \begin{array}{l}
\text { Font used for the site scores. See the font argument of par. }
\end{array} \\
\text { check. ok } & \begin{array}{l}
\text { Logical. Whether a check is performed to see that noRRR }=\sim 1 \text { was used. It } \\
\text { doesn't make sense to have a latent variable plot unless this is so. }
\end{array} \\
\text { jitter.y } & \begin{array}{l}
\text { Logical. If y is plotted, jitter it first? This may be useful for counts and propor- } \\
\text { tions. }
\end{array} \\
\ldots & \begin{array}{l}
\text { Arguments passed into the plot function when setting up the entire plot. Useful } \\
\text { arguments here include xlim and ylim. }
\end{array}
\end{array}
$$

## Details

This function only works for rank-1 and rank-2 QRR-VGLMs with argument noRRR $=\sim 1$.
For unequal-tolerances models, the latent variable axes can be rotated so that at least one of the tolerance matrices is diagonal; see Coef. qrrvglm for details.
Arguments beginning with " $p$ " correspond to the points e.g., $p c e x$ and pcol correspond to the size and color of the points. Such " $p$ " arguments should be vectors of length 1 , or $n$, the number of sites. For the rank-2 model, arguments beginning with " $p$ " correspond to the optimums.

## Value

Returns a matrix of latent variables (site scores) regardless of whether a plot was produced or not.

## Warning

Interpretation of a latent variable plot (CQO diagram) is potentially very misleading in terms of distances if (i) the tolerance matrices of the species are unequal and (ii) the contours of these tolerance matrices are not included in the ordination diagram.

## Note

A species which does not have an optimum will not have an ellipse drawn even if requested, i.e., if its tolerance matrix is not positive-definite.
Plotting $\mathbf{C}$ gives a visual display of the weights (loadings) of each of the variables used in the linear combination defining each latent variable.
The arguments elty, ecol and elwd, may be replaced in the future by llty, lcol and llwd, respectively.
For rank-1 models, a similar function to this one is perspqrrvglm. It plots the fitted values on a more fine grid rather than at the actual site scores here. The result is a collection of smooth bellshaped curves. However, it has the weakness that the plot is more divorced from the data; the user thinks it is the truth without an appreciation of the statistical variability in the estimates.

In the example below, the data comes from an equal-tolerances model. The species' tolerance matrices are all the identity matrix, and the optimums are at $(0,0),(1,1)$ and $(-2,0)$ for species 1,2 , 3 respectively.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2004). A new technique for maximum-likelihood canonical Gaussian ordination. Ecological Monographs, 74, 685-701.

## See Also

lvplot, perspqrrvglm, Coef.qrrvglm, par, cqo.

## Examples

```
set.seed(123); nn <- 200
cdata <- data.frame(x2 = rnorm(nn), # Has mean 0 (needed when I.tol=TRUE)
    x3 = rnorm(nn), # Has mean 0 (needed when I.tol=TRUE)
    x4 = rnorm(nn)) # Has mean 0 (needed when I.tol=TRUE)
cdata <- transform(cdata, latvar1 = x2 + x3 - 2*x4,
    latvar2 = -x2 + x3 + 0*x4)
# Nb. latvar2 is weakly correlated with latvar1
cdata <- transform(cdata,
    lambda1 = exp(6 - 0.5 * (latvar1-0)^2 - 0.5 * (latvar2-0)^2),
    lambda2 = exp(5 - 0.5 * (latvar1-1)^2 - 0.5 * (latvar2-1)^2),
    lambda3 = exp(5 - 0.5 * (latvar1+2)^2 - 0.5 * (latvar2-0)^2))
cdata <- transform(cdata,
    spp1 = rpois(nn, lambda1),
    spp2 = rpois(nn, lambda2),
    spp3 = rpois(nn, lambda3))
set.seed(111)
## Not run:
p2 <- cqo(cbind(spp1, spp2, spp3) ~ x2 + x3 + x4, poissonff,
            data = cdata, Rank = 2, I.tolerances = TRUE,
            Crow1positive = c(TRUE, FALSE)) # deviance = 505.81
if (deviance(p2) > 506) stop("suboptimal fit obtained")
sort(deviance(p2, history = TRUE)) # A history of all the iterations
Coef(p2)
## End(Not run)
## Not run:
lvplot(p2, sites = TRUE, spch = "*", scol = "darkgreen", scex = 1.5,
    chull = TRUE, label = TRUE, Absolute = TRUE, ellipse = 140,
    adj = -0.5, pcol = "blue", pcex = 1.3, las = 1, Ccol = "orange",
    C = TRUE, Cadj = c(-0.3, -0.3, 1), Clwd = 2, Ccex = 1.4,
    main = paste("Contours at Abundance = 140 with",
                "convex hull of the site scores"))
```

```
## End(Not run)
## Not run:
var(latvar(p2)) # A diagonal matrix, i.e., uncorrelated latent vars
var(latvar(p2, varI.latvar = TRUE)) # Identity matrix
Tol(p2)[, , 1:2] # Identity matrix
Tol(p2, varI.latvar = TRUE)[, , 1:2] # A diagonal matrix
## End(Not run)
```

lvplot.rrvglm Latent Variable Plot for RR-VGLMs

## Description

Produces an ordination diagram (also known as a biplot or latent variable plot) for reduced-rank vector generalized linear models (RR-VGLMs). For rank-2 models only, the $x$ - and $y$-axis are the first and second canonical axes respectively.

## Usage

lvplot.rrvglm(object,

$$
A=\text { TRUE, } C=\text { TRUE, scores }=\text { FALSE, show.plot }=\text { TRUE, }
$$

    groups \(=\operatorname{rep}(1, n), \operatorname{gapC}=\operatorname{sqrt}\left(\operatorname{sum}\left(\operatorname{par}() \$ c x y^{\wedge} 2\right)\right)\),
    scaleA = 1,
    xlab = "Latent Variable 1", ylab = "Latent Variable 2",
    Alabels = if (length(object@misc\$predictors.names))
    object@misc\$predictors.names else param.names("LP", M),
    Aadj \(=\operatorname{par}() \$ a d j\), Acex \(=\operatorname{par}() \$ c e x, A c o l=\operatorname{par}() \$ c o l\),
    Apch = NULL,
    Clabels = rownames(Cmat), Cadj = par()\$adj,
    Ccex = par()\$cex, Ccol = par()\$col, Clty = par()\$lty,
    Clwd = par()\$lwd,
    chull.arg = FALSE, ccex = par()\$cex, ccol = par()\$col,
    clty = par()\$lty, clwd = par()\$lwd,
    spch \(=\) NULL, scex \(=\) par()\$cex, scol \(=\operatorname{par}() \$ c o l\),
    slabels = rownames(x2mat), ...)
    
## Arguments

object Object of class "rrvglm".
A Logical. Allow the plotting of A?
C Logical. Allow the plotting of $\mathbf{C}$ ? If TRUE then $\mathbf{C}$ is represented by arrows emenating from the origin.
scores Logical. Allow the plotting of the $n$ scores? The scores are the values of the latent variables for each observation.
show. plot Logical. Plot it? If FALSE, no plot is produced and the matrix of scores ( $n$ latent variable values) is returned. If TRUE, the rank of object need not be 2 .

| groups | A vector whose distinct values indicate which group the observation belongs to. By default, all the observations belong to a single group. Useful for the multinomial logit model (see multinomial. |
| :---: | :---: |
| gapC | The gap between the end of the arrow and the text labelling of $\mathbf{C}$, in latent variable units. |
| scaleA | Numerical value that is multiplied by $\mathbf{A}$, so that $\mathbf{C}$ is divided by this value. |
| xlab | Caption for the x-axis. See par. |
| ylab | Caption for the y-axis. See par. |
| Alabels | Character vector to label A. Must be of length $M$. |
| Aadj | Justification of text strings for labelling A. See the adj argument of par. |
| Acex | Numeric. Character expansion of the labelling of A. See the cex argument of par. |
| Acol | Line color of the arrows representing C. See the col argument of par. |
| Apch | Either an integer specifying a symbol or a single character to be used as the default in plotting points. See par. The pch argument can be of length $M$, the number of species. |
| Clabels | Character vector to label C. Must be of length $p 2$. |
| Cadj | Justification of text strings for labelling C. See the adj argument of par. |
| Ccex | Numeric. Character expansion of the labelling of $\mathbf{C}$. See the cex argument of par. |
| Ccol | Line color of the arrows representing C. See the col argument of par. |
| Clty | Line type of the arrows representing $\mathbf{C}$. See the lty argument of par. |
| Clwd | Line width of the arrows representing $\mathbf{C}$. See the lwd argument of par. |
| chull.arg | Logical. Plot the convex hull of the scores? This is done for each group (see the group argument). |
| ccex | Numeric. Character expansion of the labelling of the convex hull. See the cex argument of par. |
| ccol | Line color of the convex hull. See the col argument of par. |
| clty | Line type of the convex hull. See the lty argument of par. |
| clwd | Line width of the convex hull. See the lwd argument of par. |
| spch | Either an integer specifying a symbol or a single character to be used as the default in plotting points. See par. The spch argument can be of length $M$, the number of species. |
| scex | Numeric. Character expansion of the labelling of the scores. See the cex argument of par. |
| scol | Line color of the arrows representing C. See the col argument of par. |
| slabels | Character vector to label the scores. Must be of length $n$. |
|  | Arguments passed into the plot function when setting up the entire plot. Useful arguments here include xlim and ylim. |

## Details

For RR-VGLMs, a biplot and a latent variable plot coincide. In general, many of the arguments starting with " A " refer to $\mathbf{A}$ (of length $M$ ), " C " to $\mathbf{C}$ (of length $p 2$ ), " c " to the convex hull (of length length(unique(groups))), and " $s$ " to scores (of length $n$ ).

As the result is a biplot, its interpretation is based on the inner product.

## Value

The matrix of scores ( $n$ latent variable values) is returned regardless of whether a plot was produced or not.

## Note

The functions lvplot.rrvglm and biplot.rrvglm are equivalent.
In the example below the predictor variables are centered, which is a good idea.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Hastie, T. J. (2003). Reduced-rank vector generalized linear models. Statistical Modelling, 3, 15-41.

## See Also

lvplot, par, rrvglm, Coef.rrvglm, rrvglm. control.

## Examples

```
nn <- nrow(pneumo) # x1, x2 and x3 are some unrelated covariates
pneumo <- transform(pneumo, slet = scale(log(exposure.time)),
    x1 = rnorm(nn), x2 = rnorm(nn), x3 = rnorm(nn))
fit <- rrvglm(cbind(normal, mild, severe) ~ slet + x1 + x2 + x3,
            multinomial, data = pneumo, Rank = 2,
            Corner = FALSE, Uncorrel = TRUE)
## Not run:
lvplot(fit, chull = TRUE, scores = TRUE, clty = 2, ccol = "blue",
    scol = "red", Ccol = "darkgreen", Clwd = 2, Ccex = 2,
    main = "Biplot of some fictitional data")
## End(Not run)
```

```
machinists Machinists Accidents
```


## Description

A small count data set involving 414 machinists from a three months study, of accidents around the end of WWI.

## Usage

data(machinists)

## Format

A data frame with the following variables.
accidents The number of accidents
ofreq Observed frequency, i.e., the number of machinists with that many accidents

## Details

The data was collected over a period of three months. There were 414 machinists in total. Also, there were data collected over six months, but it is not given here.

## Source

Incidence of Industrial Accidents. Report No. 4 (Industrial Fatigue Research Board), Stationery Office, London, 1919.

## References

Greenwood, M. and Yule, G. U. (1920). An Inquiry into the Nature of Frequency Distributions Representative of Multiple Happenings with Particular Reference to the Occurrence of Multiple Attacks of Disease or of Repeated Accidents. Journal of the Royal Statistical Society, 83, 255-279.

## See Also

negbinomial, poissonff.

## Examples

```
machinists
mean(with(machinists, rep(accidents, times = ofreq)))
    var(with(machinists, rep(accidents, times = ofreq)))
## Not run: barplot(with(machinists, ofreq),
    names.arg = as.character(with(machinists, accidents)),
    main = "Machinists accidents",
    col = "lightblue", las = 1,
    ylab = "Frequency", xlab = "accidents")
## End(Not run)
```

Makeham The Makeham Distribution

## Description

Density, cumulative distribution function, quantile function and random generation for the Makeham distribution.

## Usage

```
dmakeham(x, scale = 1, shape, epsilon = 0, log = FALSE)
pmakeham(q, scale = 1, shape, epsilon = 0, lower.tail = TRUE, log.p = FALSE)
qmakeham(p, scale = 1, shape, epsilon = 0, lower.tail = TRUE, log.p = FALSE)
rmakeham(n, scale = 1, shape, epsilon = 0)
```


## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$n$ number of observations. Same as in runif.
log Logical. If log = TRUE then the logarithm of the density is returned.
lower.tail, log.p Same meaning as in pnorm or qnorm.
scale, shape positive scale and shape parameters.
epsilon another parameter. Must be non-negative. See below.

## Details

See makeham for details. The default value of epsilon $=0$ corresponds to the Gompertz distribution. The function pmakeham uses lambertW.

## Value

dmakeham gives the density, pmakeham gives the cumulative distribution function, qmakeham gives the quantile function, and rmakeham generates random deviates.

## Author(s)

T. W. Yee and Kai Huang

## References

Jodra, P. (2009). A closed-form expression for the quantile function of the Gompertz-Makeham distribution. Mathematics and Computers in Simulation, 79, 3069-3075.

## See Also

makeham, lambertW.

## Examples

```
probs <- seq(0.01, 0.99, by = 0.01)
Shape <- exp(-1); Scale <- exp(1); eps = Epsilon <- exp(-1)
max(abs(pmakeham(qmakeham(p = probs, sca = Scale, Shape, eps = Epsilon),
                sca = Scale, Shape, eps = Epsilon) - probs)) # Should be 0
## Not run: x <- seq(-0.1, 2.0, by = 0.01);
plot(x, dmakeham(x, sca = Scale, Shape, eps = Epsilon), type = "l",
    main = "Blue is density, orange is cumulative distribution function",
    sub = "Purple lines are the 10,20,\ldots.,90 percentiles",
    col = "blue", las = 1, ylab = "")
abline(h = 0, col = "blue", lty = 2)
lines(x, pmakeham(x, sca = Scale, Shape, eps = Epsilon), col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qmakeham(probs, sca = Scale, Shape, eps = Epsilon)
lines(Q, dmakeham(Q, sca = Scale, Shape, eps = Epsilon),
    col = "purple", lty = 3, type = "h")
pmakeham(Q, sca = Scale, Shape, eps = Epsilon) - probs # Should be all zero
abline(h = probs, col = "purple", lty = 3)
## End(Not run)
```

makeham Makeham Regression Family Function

## Description

Maximum likelihood estimation of the 3-parameter Makeham distribution.

## Usage

```
makeham(lscale = "loglink", lshape = "loglink", lepsilon = "loglink",
    iscale = NULL, ishape = NULL, iepsilon = NULL,
    gscale = exp(-5:5),gshape = exp(-5:5), gepsilon = exp(-4:1),
    nsimEIM = 500, oim.mean = TRUE, zero = NULL, nowarning = FALSE)
```


## Arguments

nowarning Logical. Suppress a warning? Ignored for VGAM 0.9-7 and higher.
lshape, lscale, lepsilon
Parameter link functions applied to the shape parameter shape, scale parameter scale, and other parameter epsilon. All parameters are treated as positive here (cf. dmakeham allows epsilon $=0$, etc.). See Links for more choices.
ishape, iscale, iepsilon
Optional initial values. A NULL means a value is computed internally. A value must be given for iepsilon currently, and this is a sensitive parameter!
makeham
gshape, gscale, gepsilon
See CommonVGAMffArguments.
nsimEIM, zero See CommonVGAMffArguments. Argument probs.y is used only when imethod $=2$.
oim.mean To be currently ignored.

## Details

The Makeham distribution, which adds another parameter to the Gompertz distribution, has cumulative distribution function

$$
F(y ; \alpha, \beta, \varepsilon)=1-\exp \left\{-y \varepsilon+\frac{\alpha}{\beta}\left[1-e^{\beta y}\right]\right\}
$$

which leads to a probability density function

$$
f(y ; \alpha, \beta, \varepsilon)=\left[\varepsilon+\alpha e^{\beta y}\right] \exp \left\{-y \varepsilon+\frac{\alpha}{\beta}\left[1-e^{\beta y}\right]\right\}
$$

for $\alpha>0, \beta>0, \varepsilon \geq 0, y>0$. Here, $\beta$ is called the scale parameter scale, and $\alpha$ is called a shape parameter. The moments for this distribution do not appear to be available in closed form.

Simulated Fisher scoring is used and multiple responses are handled.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

A lot of care is needed because this is a rather difficult distribution for parameter estimation, especially when the shape parameter is large relative to the scale parameter. If the self-starting initial values fail then try experimenting with the initial value arguments, especially iepsilon. Successful convergence depends on having very good initial values. More improvements could be made here. Also, monitor convergence by setting trace $=$ TRUE .

A trick is to fit a gompertz distribution and use it for initial values; see below. However, this family function is currently numerically fraught.

## Author(s)

T. W. Yee

## See Also

dmakeham, gompertz, simulate.vlm.

## Examples

```
## Not run: set.seed(123)
mdata <- data.frame(x2 = runif(nn <- 1000))
mdata <- transform(mdata, eta1 = -1,
    ceta1 = 1,
    eeta1 = -2)
mdata <- transform(mdata, shape1 = exp(eta1),
    scale1 = exp(ceta1),
    epsil1 = exp(eeta1))
mdata <- transform(mdata,
        y1 = rmakeham(nn, shape = shape1, scale = scale1, eps = epsil1))
# A trick is to fit a Gompertz distribution first
fit0 <- vglm(y1 ~ 1, gompertz, data = mdata, trace = TRUE)
fit1 <- vglm(y1 ~ 1, makeham, data = mdata,
            etastart = cbind(predict(fit0), log(0.1)), trace = TRUE)
coef(fit1, matrix = TRUE)
summary(fit1)
## End(Not run)
```

    margeff Marginal Effects for Several Categorical Response Models
    
## Description

Marginal effects for the multinomial logit model and cumulative logit/probit/... models and continuation ratio models and stopping ratio models and adjacent categories models: the derivative of the fitted probabilities with respect to each explanatory variable.

## Usage

margeff(object, subset = NULL, ...)

## Arguments

object A vglm object, with one of the following family functions: multinomial, cumulative, cratio, sratio or acat.
subset Numerical or logical vector, denoting the required observation(s). Recycling is used if possible. The default means all observations.
... further arguments passed into the other methods functions.

## Details

Computes the derivative of the fitted probabilities of the categorical response model with respect to each explanatory variable. Formerly one big function, this function now uses S4 dispatch to break up the computations.
The function margeff() is not generic. However, it calls the function margeffS4VGAM() which is. This is based on the class of the VGAMff argument, and it uses the S 4 function setMethod to correctly dispatch to the required methods function. The inheritance is given by the vfamily slot of the VGAM family function.

## Value

A $p$ by $M+1$ by $n$ array, where $p$ is the number of explanatory variables and the (hopefully) nominal response has $M+1$ levels, and there are $n$ observations.

In general, if is.numeric(subset) and length(subset) $==1$ then a $p$ by $M+1$ matrix is returned.

## Warning

Care is needed in interpretation, e.g., the change is not universally accurate for a unit change in each explanatory variable because eventually the 'new' probabilities may become negative or greater than unity. Also, the 'new' probabilities will not sum to one.
This function is not applicable for models with data-dependent terms such as bs and poly. Also the function should not be applied to models with any terms that have generated more than one column of the LM model matrix, such as bs and poly. For such try using numerical methods such as finite-differences. The formula in object should comprise of simple terms of the form $\sim x 2+$ $x 3+x 4$, etc.
Some numerical problems may occur if the fitted values are close to 0 or 1 for the cratio and sratio models. Models with offsets may result in an incorrect answer.

## Note

For multinomial this function should handle any value of refLevel and also any constraint matrices. However, it does not currently handle the xij or form2 arguments, nor vgam objects.
Some other limitations are imposed, e.g., for acat models only a loglink link is allowed.

## Author(s)

T. W. Yee, with some help and motivation from Stasha Rmandic.

## See Also

multinomial, cumulative, propodds, acat, cratio, sratio, vglm.

## Examples

```
# Not a good example for multinomial() because the response is ordinal!!
ii <- 3; hh <- 1/100
pneumo <- transform(pneumo, let = log(exposure.time))
fit <- vglm(cbind(normal, mild, severe) ~ let, multinomial, data = pneumo)
```

```
fit <- vglm(cbind(normal, mild, severe) ~ let,
            cumulative(reverse = TRUE, parallel = TRUE),
    data = pneumo)
fitted(fit)[ii, ]
mynewdata <- with(pneumo, data.frame(let = let[ii] + hh))
(newp <- predict(fit, newdata = mynewdata, type = "response"))
# Compare the difference. Should be the same as hh --> 0.
round(digits = 3, (newp-fitted(fit)[ii, ])/hh) # Finite-difference approxn
round(digits = 3, margeff(fit, subset = ii)["let",])
# Other examples
round(digits = 3, margeff(fit))
round(digits = 3, margeff(fit, subset = 2)["let",])
round(digits = 3, margeff(fit, subset = c(FALSE, TRUE))["let",,]) # recycling
round(digits = 3, margeff(fit, subset = c(2, 4, 6, 8))["let",,])
```

marital.nz New Zealand Marital Data

## Description

Some marital data mainly from a large NZ company collected in the early 1990s.

## Usage

data(marital.nz)

## Format

A data frame with 6053 observations on the following 3 variables.
age a numeric vector, age in years
ethnicity a factor with levels European Maori Other Polynesian. Only Europeans are included in the data set.
mstatus a factor with levels Divorced/Separated, Married/Partnered, Single, Widowed.

## Details

This is a subset of a data set collected from a self-administered questionnaire administered in a large New Zealand workforce observational study conducted during 1992-3. The data were augmented by a second study consisting of retirees. The data can be considered a reasonable representation of the white male New Zealand population in the early 1990s.

## Source

Clinical Trials Research Unit, University of Auckland, New Zealand.

## References

See bmi.nz and chest.nz.

## Examples

summary (marital.nz)
Max Maximums

## Description

Generic function for the maximums (maxima) of a model.

## Usage

Max(object, ...)

## Arguments

object An object for which the computation or extraction of a maximum (or maximums) is meaningful.
... Other arguments fed into the specific methods function of the model. Sometimes they are fed into the methods function for Coef.

## Details

Different models can define a maximum in different ways. Many models have no such notion or definition.

Maximums occur in quadratic and additive ordination, e.g., CQO or CAO. For these models the maximum is the fitted value at the optimum. For quadratic ordination models there is a formula for the optimum but for additive ordination models the optimum must be searched for numerically. If it occurs on the boundary, then the optimum is undefined. For a valid optimum, the fitted value at the optimum is the maximum.

## Value

The value returned depends specifically on the methods function invoked.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2004). A new technique for maximum-likelihood canonical Gaussian ordination. Ecological Monographs, 74, 685-701.
Yee, T. W. (2006). Constrained additive ordination. Ecology, 87, 203-213.

## See Also

Max. $q$ rrvglm, Tol, Opt.

## Examples

```
## Not run:
set.seed(111) # This leads to the global solution
hspider[,1:6] <- scale(hspider[,1:6]) # Standardized environmental vars
p1 <- cqo(cbind(Alopacce, Alopcune, Alopfabr, Arctlute, Arctperi,
            Auloalbi, Pardlugu, Pardmont, Pardnigr, Pardpull,
            Trocterr, Zoraspin) ~
        WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
        poissonff, Bestof = 2, data = hspider, Crow1positive = FALSE)
    Max(p1)
    index <- 1:ncol(depvar(p1))
    persp(p1, col = index, las = 1, llwd = 2)
    abline(h = Max(p1), lty = 2, col = index)
    ## End(Not run)
```

Maxwell
The Maxwell Distribution

## Description

Density, distribution function, quantile function and random generation for the Maxwell distribution.

## Usage

```
dmaxwell(x, rate, log = FALSE)
pmaxwell(q, rate, lower.tail = TRUE, log.p = FALSE)
qmaxwell(p, rate, lower.tail = TRUE, log.p = FALSE)
rmaxwell(n, rate)
```


## Arguments

$\mathrm{x}, \mathrm{q}, \mathrm{p}, \mathrm{n} \quad$ Same as Uniform.
rate the (rate) parameter.
log Logical. If $\log =$ TRUE then the logarithm of the density is returned.
lower.tail, log.p
Same meaning as in pnorm or qnorm.

## Details

See maxwell, the VGAM family function for estimating the (rate) parameter $a$ by maximum likelihood estimation, for the formula of the probability density function.

## Value

dmaxwell gives the density, pmaxwell gives the distribution function, qmaxwell gives the quantile function, and rmaxwell generates random deviates.

## Note

The Maxwell distribution is related to the Rayleigh distribution.

## Author(s)

T. W. Yee and Kai Huang

## References

Balakrishnan, N. and Nevzorov, V. B. (2003). A Primer on Statistical Distributions. Hoboken, New Jersey: Wiley.

## See Also

maxwell, Rayleigh, rayleigh.

## Examples

```
## Not run: rate <- 3; x <- seq(-0.5, 3, length = 100)
plot(x, dmaxwell(x, rate = rate), type = "l", col = "blue", las = 1,
    main = "Blue is density, orange is cumulative distribution function",
    sub = "Purple lines are the 10,20,...,90 percentiles", ylab = "")
abline(h = 0, col = "blue", lty = 2)
lines(x, pmaxwell(x, rate = rate), type = "l", col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qmaxwell(probs, rate = rate)
lines(Q, dmaxwell(Q, rate), col = "purple", lty = 3, type = "h")
lines(Q, pmaxwell(Q, rate), col = "purple", lty = 3, type = "h")
abline(h = probs, col = "purple", lty = 3)
max(abs(pmaxwell(Q, rate) - probs)) # Should be zero
## End(Not run)
```

maxwell Maxwell Regression Family Function

## Description

Estimating the parameter of the Maxwell distribution by maximum likelihood estimation.

## Usage

```
maxwell(link = "loglink", zero = NULL, parallel = FALSE,
    type.fitted = c("mean", "percentiles", "Qlink"),
    percentiles = 50)
```


## Arguments

link Parameter link function applied to $a$, which is called the parameter rate. See Links for more choices and information; a log link is the default because the parameter is positive. More information is at CommonVGAMffArguments.
zero, parallel See CommonVGAMffArguments.
type.fitted, percentiles
See CommonVGAMffArguments for information. Using "Qlink" is for quantilelinks in VGAMextra.

## Details

The Maxwell distribution, which is used in the area of thermodynamics, has a probability density function that can be written

$$
f(y ; a)=\sqrt{2 / \pi} a^{3 / 2} y^{2} \exp \left(-0.5 a y^{2}\right)
$$

for $y>0$ and $a>0$. The mean of $Y$ is $\sqrt{8 /(a \pi)}$ (returned as the fitted values), and its variance is $(3 \pi-8) /(\pi a)$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Note

Fisher-scoring and Newton-Raphson are the same here. A related distribution is the Rayleigh distribution. This VGAM family function handles multiple responses. This VGAM family function can be mimicked by poisson. points(ostatistic $=1.5$, dimension $=2$ ).

## Author(s)

T. W. Yee

## References

von Seggern, D. H. (1993). CRC Standard Curves and Surfaces, Boca Raton, FL, USA: CRC Press.

## See Also

Maxwell, rayleigh, poisson.points.

## Examples

```
mdata <- data.frame(y = rmaxwell(1000, rate = exp(2)))
fit <- vglm(y ~ 1, maxwell, data = mdata, trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit)
```


## Description

Estimates the two parameters of the McCullagh (1989) distribution by maximum likelihood estimation.

## Usage

mccullagh89(litheta = "rhobitlink", lnu = logofflink(offset = 0.5), itheta $=$ NULL, inu $=$ NULL, zero $=$ NULL)

## Arguments

ltheta, lnu Link functions for the $\theta$ and $\nu$ parameters. See Links for general information.
itheta, inu Numeric. Optional initial values for $\theta$ and $\nu$. The default is to internally compute them.
zero See CommonVGAMffArguments for information.

## Details

The McCullagh (1989) distribution has density function

$$
f(y ; \theta, \nu)=\frac{\left\{1-y^{2}\right\}^{\nu-\frac{1}{2}}}{\left(1-2 \theta y+\theta^{2}\right)^{\nu} \operatorname{Beta}\left(\nu+\frac{1}{2}, \frac{1}{2}\right)}
$$

where $-1<y<1$ and $-1<\theta<1$. This distribution is equation (1) in that paper. The parameter $\nu$ satisfies $\nu>-1 / 2$, therefore the default is to use an log-offset link with offset equal to 0.5 , i.e., $\eta_{2}=\log (\nu+0.5)$. The mean is of $Y$ is $\nu \theta /(1+\nu)$, and these are returned as the fitted values.

This distribution is related to the Leipnik distribution (see Johnson et al. (1995)), is related to ultraspherical functions, and under certain conditions, arises as exit distributions for Brownian motion. Fisher scoring is implemented here and it uses a diagonal matrix so the parameters are globally orthogonal in the Fisher information sense. McCullagh (1989) also states that, to some extent, $\theta$ and $\nu$ have the properties of a location parameter and a precision parameter, respectively.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Note

Convergence may be slow or fail unless the initial values are reasonably close. If a failure occurs, try assigning the argument inu and/or itheta. Figure 1 of McCullagh (1989) gives a broad range of densities for different values of $\theta$ and $\nu$, and this could be consulted for obtaining reasonable initial values if all else fails.

## Author(s)

T. W. Yee

## References

McCullagh, P. (1989). Some statistical properties of a family of continuous univariate distributions. Journal of the American Statistical Association, 84, 125-129.
Johnson, N. L. and Kotz, S. and Balakrishnan, N. (1995). Continuous Univariate Distributions, 2nd edition, Volume 2, New York: Wiley. (pages 612-617).

## See Also

leipnik, rhobitlink, logofflink.

## Examples

```
mdata <- data.frame(y = rnorm(n = 1000, sd = 0.2)) # Limit as theta = 0, nu = Inf
fit <- vglm(y ~ 1, mccullagh89, data = mdata, trace = TRUE)
head(fitted(fit))
with(mdata, mean(y))
summary(fit)
coef(fit, matrix = TRUE)
Coef(fit)
```

melbmaxtemp Melbourne Daily Maximum Temperatures

## Description

Melbourne daily maximum temperatures in degrees Celsius over the ten-year period 1981-1990.

## Usage

data(melbmaxtemp)

## Format

A vector with 3650 observations.

## Details

This is a time series data from Melbourne, Australia. It is commonly used to give a difficult quantile regression problem since the data is bimodal. That is, a hot day is likely to be followed by either an equally hot day or one much cooler. However, an independence assumption is typically made.

## References

Hyndman, R. J. and Bashtannyk, D. M. and Grunwald, G. K. (1996). Estimating and visualizing conditional densities. J. Comput. Graph. Statist., 5(4), 315-336.

## See Also

lms.ben.

## Examples

```
summary(melbmaxtemp)
## Not run: par(mfrow = c(1, 1), mar = c(5, 4, 0.2, 0.1) + 0.1, las = 1)
melb <- data.frame(today = melbmaxtemp[-1],
        yesterday = melbmaxtemp[-length(melbmaxtemp)])
plot(today ~ yesterday, data = melb,
    xlab = "Yesterday's Max Temperature",
    ylab = "Today's Max Temperature", cex = 1.4, type = "n")
points(today ~ yesterday, data = melb, pch = 0, cex = 0.50, col = "blue")
abline(a = 0, b = 1, lty = 3)
## End(Not run)
```

meplot Mean Excess Plot

## Description

Mean excess plot (also known as a mean residual life plot), a diagnostic plot for the generalized Pareto distribution (GPD).

## Usage

```
meplot(object, ...)
meplot.default(y, main = "Mean Excess Plot",
        xlab = "Threshold", ylab = "Mean Excess", lty = c(2, 1:2),
        conf = 0.95, col = c("blue", "black", "blue"), type = "l", ...)
    meplot.vlm(object, ...)
```


## Arguments

$y \quad$ A numerical vector. NAs etc. are not allowed.
main, xlab, ylab
Character. Overall title for the plot, and titles for the x - and y -axes.
lty Line type. The second value is for the mean excess value, the first and third values are for the envelope surrounding the confidence interval.
conf Confidence level. The default results in approximate 95 percent confidence intervals for each mean excess value.
col Colour of the three lines.
type Type of plot. The default means lines are joined between the mean excesses and also the upper and lower limits of the confidence intervals.
object An object that inherits class "vlm", usually of class vglm-class or vgam-class.
... Graphical argument passed into plot. See par for an exhaustive list. The arguments xlim and ylim are particularly useful.

## Details

If $Y$ has a GPD with scale parameter $\sigma$ and shape parameter $\xi<1$, and if $y>0$, then

$$
E(Y-u \mid Y>u)=\frac{\sigma+\xi u}{1-\xi}
$$

It is a linear function in $u$, the threshold. Note that $Y-u$ is called the excess and values of $Y$ greater than $u$ are called exceedances. The empirical versions used by these functions is to use sample means to estimate the left hand side of the equation. Values of $u$ in the plot are the values of $y$ itself. If the plot is roughly a straight line then the GPD is a good fit; this plot can be used to select an appropriate threshold value. See gpd for more details. If the plot is flat then the data may be exponential, and if it is curved then it may be Weibull or gamma. There is often a lot of variance/fluctuation at the RHS of the plot due to fewer observations.

The function meplot is generic, and meplot. default and meplot.vlm are some methods functions for mean excess plots.

## Value

A list is returned invisibly with the following components.

| threshold | The x axis values. |
| :--- | :--- |
| meanExcess | The y axis values. Each value is a sample mean minus a value $u$. |
| plusminus | The amount which is added or subtracted from the mean excess to give the <br> confidence interval. The last value is a NA because it is based on one observation. |

## Note

The function is designed for speed and not accuracy, therefore huge data sets with extremely large values may cause failure (the function cumsum is used.) Ties may not be well handled.

## Author(s)

T. W. Yee

## References

Davison, A. C. and Smith, R. L. (1990). Models for exceedances over high thresholds (with discussion). Journal of the Royal Statistical Society, Series B, Methodological, 52, 393-442.
Coles, S. (2001). An Introduction to Statistical Modeling of Extreme Values. London: SpringerVerlag.

## See Also

gpd.

## Examples

```
\#\# Not run: meplot(with(venice90, sealevel), las = 1) -> ii
names(ii)
abline(h = ii\$meanExcess[1], col = "orange", lty = "dashed")
\(\operatorname{par}(m f r o w=c(2,2))\)
for (ii in 1:4)
    meplot \((\operatorname{rgpd}(1000)\), col \(=c("\) orange", "blue", "orange"))
\#\# End(Not run)
```

micmen Michaelis-Menten Model

## Description

Fits a Michaelis-Menten nonlinear regression model.

## Usage

```
micmen(rpar = 0.001, divisor = 10, init1 = NULL, init2 = NULL,
imethod = 1, oim = TRUE, link1 = "identitylink", link2 = "identitylink",
firstDeriv = c("nsimEIM", "rpar"), probs.x = c(0.15, 0.85),
nsimEIM = 500, dispersion = 0, zero = NULL)
```


## Arguments

rpar Numeric. Initial positive ridge parameter. This is used to create positive-definite weight matrices.
divisor Numerical. The divisor used to divide the ridge parameter at each iteration until it is very small but still positive. The value of divisor should be greater than one.
init1, init2 Numerical. Optional initial value for the first and second parameters, respectively. The default is to use a self-starting value.
link1, link2 Parameter link function applied to the first and second parameters, respectively. See Links for more choices.
dispersion Numerical. Dispersion parameter.
firstDeriv Character. Algorithm for computing the first derivatives and working weights. The first is the default.
imethod, probs.x
See CommonVGAMffArguments for information.
nsimEIM, zero See CommonVGAMffArguments for information.
oim Use the OIM? See CommonVGAMffArguments for information.

## Details

The Michaelis-Menten model is given by

$$
E\left(Y_{i}\right)=\left(\theta_{1} u_{i}\right) /\left(\theta_{2}+u_{i}\right)
$$

where $\theta_{1}$ and $\theta_{2}$ are the two parameters.
The relationship between iteratively reweighted least squares and the Gauss-Newton algorithm is given in Wedderburn (1974). However, the algorithm used by this family function is different. Details are given at the Author's web site.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

This function is not (nor could ever be) entirely reliable. Plotting the fitted function and monitoring convergence is recommended.

## Note

The regressor values $u_{i}$ are inputted as the RHS of the form2 argument. It should just be a simple term; no smart prediction is used. It should just a single vector, therefore omit the intercept term. The LHS of the formula form2 is ignored.
To predict the response at new values of $u_{i}$ one must assign the @extra $\$ \times \mathrm{m} 2$ slot in the fitted object these values, e.g., see the example below.
Numerical problems may occur. If so, try setting some initial values for the parameters. In the future, several self-starting initial values will be implemented.

## Author(s)

T. W. Yee

## References

Seber, G. A. F. and Wild, C. J. (1989). Nonlinear Regression, New York: Wiley.
Wedderburn, R. W. M. (1974). Quasi-likelihood functions, generalized linear models, and the Gauss-Newton method. Biometrika, 61, 439-447.
Bates, D. M. and Watts, D. G. (1988). Nonlinear Regression Analysis and Its Applications, New York: Wiley.

## See Also

enzyme.

## Examples

mfit <- vglm(velocity ~ 1, micmen, data = enzyme, trace $=$ TRUE,
crit $=$ "coef", form $2=\sim$ conc -1 )
summary (mfit)
\#\# Not run: plot(velocity ~ conc, enzyme, xlab = "concentration", las = 1,
col = "blue", main = "Michaelis-Menten equation for the enzyme data",
ylim $=c(0, \max ($ velocity $)), x \lim =c(0, \max ($ conc $)))$
points(fitted(mfit) ~ conc, enzyme, col = "red", pch = "+", cex = 1.5)
\# This predicts the response at a finer grid:
newenzyme <- data.frame(conc = seq(0, max(with(enzyme, conc)), len = 200))
mfit@extra\$Xm2 <- newenzyme\$conc \# This assignment is needed for prediction
lines(predict(mfit, newenzyme, "response") ~ conc, newenzyme, col = "red")
\#\# End(Not run)
mills.ratio Mills Ratio

## Description

Computes the Mills ratio.

## Usage

mills.ratio( $x$ )
mills.ratio2(x)

## Arguments

X
Numeric (real).

## Details

The Mills ratio here is dnorm(x) / pnorm(x) (some use (1-pnorm(x)) / dnorm(x)). Some care is needed as x approaches -Inf; when $x$ is very negative then its value approaches $-x$.

## Value

mills.ratio returns the Mills ratio, and mills.ratio2 returns dnorm(x) * dnorm(x)/pnorm(x).

## Author(s)

T. W. Yee

## References

Mills, J. P. (1926). Table of the ratio: area to bounding ordinate, for any portion of normal curve. Biometrika. 18(3/4), 395-400.

## See Also

Normal, tobit, cens. poisson.

## Examples

```
## Not run:
curve(mills.ratio, -5, 5, col = "orange", las = 1)
curve(mills.ratio, -5, 5, col = "orange", las = 1, log = "y")
## End(Not run)
```

mix2exp Mixture of Two Exponential Distributions

## Description

Estimates the three parameters of a mixture of two exponential distributions by maximum likelihood estimation.

## Usage

```
mix2exp(lphi = "logitlink", llambda = "loglink", iphi = 0.5, il1 = NULL,
            il2 = NULL, qmu = c(0.8, 0.2), nsimEIM = 100, zero = "phi")
```


## Arguments

lphi, llambda Link functions for the parameters $\phi$ and $\lambda$. The latter is the rate parameter and note that the mean of an ordinary exponential distribution is $1 / \lambda$. See Links for more choices.
iphi, il1, il2 Initial value for $\phi$, and optional initial value for $\lambda_{1}$ and $\lambda_{2}$. The last two have values that must be positive. The default is to compute initial values internally using the argument qmu.
qmu Vector with two values giving the probabilities relating to the sample quantiles for obtaining initial values for $\lambda_{1}$ and $\lambda_{2}$. The two values are fed in as the probs argument into quantile.
nsimEIM, zero See CommonVGAMffArguments.

## Details

The probability density function can be loosely written as

$$
f(y)=\phi \text { Exponential }\left(\lambda_{1}\right)+(1-\phi) \text { Exponential }\left(\lambda_{2}\right)
$$

where $\phi$ is the probability an observation belongs to the first group, and $y>0$. The parameter $\phi$ satisfies $0<\phi<1$. The mean of $Y$ is $\phi / \lambda_{1}+(1-\phi) / \lambda_{2}$ and this is returned as the fitted values. By default, the three linear/additive predictors are $\left(\operatorname{logit}(\phi), \log \left(\lambda_{1}\right), \log \left(\lambda_{2}\right)\right)^{T}$.
mix2exp

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Warning

This VGAM family function requires care for a successful application. In particular, good initial values are required because of the presence of local solutions. Therefore running this function with several different combinations of arguments such as iphi, il1, il2, qmu is highly recommended. Graphical methods such as hist can be used as an aid.
This VGAM family function is experimental and should be used with care.

## Note

Fitting this model successfully to data can be difficult due to local solutions, uniqueness problems and ill-conditioned data. It pays to fit the model several times with different initial values and check that the best fit looks reasonable. Plotting the results is recommended. This function works better as $\lambda_{1}$ and $\lambda_{2}$ become more different. The default control argument trace $=$ TRUE is to encourage monitoring convergence.

## Author(s)

T. W. Yee

## See Also

rexp, exponential, mix2poisson.

## Examples

```
## Not run: lambda1 <- exp(1); lambda2 <- exp(3)
(phi <- logitlink(-1, inverse = TRUE))
mdata <- data.frame(y1 = rexp(nn <- 1000, lambda1))
mdata <- transform(mdata, y2 = rexp(nn, lambda2))
mdata <- transform(mdata, Y = ifelse(runif(nn) < phi, y1, y2))
fit <- vglm(Y ~ 1, mix2exp, data = mdata, trace = TRUE)
coef(fit, matrix = TRUE)
# Compare the results with the truth
round(rbind('Estimated' = Coef(fit),
    'Truth' = c(phi, lambda1, lambda2)), digits = 2)
with(mdata, hist(Y, prob = TRUE, main = "Orange = estimate, blue = truth"))
abline(v = 1 / Coef(fit)[c(2, 3)], lty = 2, col = "orange", lwd = 2)
abline(v = 1 / c(lambda1, lambda2), lty = 2, col = "blue", lwd = 2)
## End(Not run)
```

```
mix2normal Mixture of Two Univariate Normal Distributions
```


## Description

Estimates the five parameters of a mixture of two univariate normal distributions by maximum likelihood estimation.

## Usage

```
mix2normal(lphi = "logitlink", lmu = "identitylink", lsd = "loglink",
                iphi = 0.5, imu1 = NULL, imu2 = NULL, isd1 = NULL, isd2 = NULL,
                qmu = c(0.2, 0.8), eq.sd = TRUE, nsimEIM = 100, zero = "phi")
```


## Arguments

lphi,lmu,lsd Link functions for the parameters $\phi, \mu$, and $\sigma$. See Links for more choices.
iphi Initial value for $\phi$, whose value must lie between 0 and 1.
imu1, imu2 Optional initial value for $\mu_{1}$ and $\mu_{2}$. The default is to compute initial values internally using the argument qmu.
isd1, isd2 Optional initial value for $\sigma_{1}$ and $\sigma_{2}$. The default is to compute initial values internally based on the argument qmu. Currently these are not great, therefore using these arguments where practical is a good idea.
qmu Vector with two values giving the probabilities relating to the sample quantiles for obtaining initial values for $\mu_{1}$ and $\mu_{2}$. The two values are fed in as the probs argument into quantile.
eq.sd Logical indicating whether the two standard deviations should be constrained to be equal. If TRUE then the appropriate constraint matrices will be used.
nsimEIM See CommonVGAMffArguments.
zero May be an integer vector specifying which linear/additive predictors are modelled as intercept-only. If given, the value or values can be from the set $\{1,2, \ldots, 5\}$. The default is the first one only, meaning $\phi$ is a single parameter even when there are explanatory variables. Set zero $=$ NULL to model all linear/additive predictors as functions of the explanatory variables. See CommonVGAMffArguments for more information.

## Details

The probability density function can be loosely written as

$$
f(y)=\phi N\left(\mu_{1}, \sigma_{1}\right)+(1-\phi) N\left(\mu_{2}, \sigma_{2}\right)
$$

where $\phi$ is the probability an observation belongs to the first group. The parameters $\mu_{1}$ and $\mu_{2}$ are the means, and $\sigma_{1}$ and $\sigma_{2}$ are the standard deviations. The parameter $\phi$ satisfies $0<\phi<1$. The mean of $Y$ is $\phi \mu_{1}+(1-\phi) \mu_{2}$ and this is returned as the fitted values. By default, the five linear/additive predictors are $\left(\operatorname{logit}(\phi), \mu_{1}, \log \left(\sigma_{1}\right), \mu_{2}, \log \left(\sigma_{2}\right)\right)^{T}$. If eq.sd $=$ TRUE then $\sigma_{1}=\sigma_{2}$ is enforced.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

Numerical problems can occur and half-stepping is not uncommon. If failure to converge occurs, try inputting better initial values, e.g., by using iphi, qmu, imu1, imu2, isd1, isd2, etc.
This VGAM family function is experimental and should be used with care.

## Note

Fitting this model successfully to data can be difficult due to numerical problems and ill-conditioned data. It pays to fit the model several times with different initial values and check that the best fit looks reasonable. Plotting the results is recommended. This function works better as $\mu_{1}$ and $\mu_{2}$ become more different.
Convergence can be slow, especially when the two component distributions are not well separated. The default control argument trace = TRUE is to encourage monitoring convergence. Having eq. sd $=$ TRUE often makes the overall optimization problem easier.

## Author(s)

T. W. Yee

## References

McLachlan, G. J. and Peel, D. (2000). Finite Mixture Models. New York: Wiley.
Everitt, B. S. and Hand, D. J. (1981). Finite Mixture Distributions. London: Chapman \& Hall.

## See Also

uninormal, Normal, mix2poisson.

## Examples

```
## Not run: mu1 <- 99; mu2 <- 150; nn <- 1000
sd1 <- sd2 <- exp(3)
(phi <- logitlink(-1, inverse = TRUE))
mdata <- data.frame(y = ifelse(runif(nn) < phi, rnorm(nn, mu1, sd1),
                                    rnorm(nn, mu2, sd2)))
fit <- vglm(y ~ 1, mix2normal(eq.sd = TRUE), data = mdata)
# Compare the results
cfit <- coef(fit)
round(rbind('Estimated' = c(logitlink(cfit[1], inverse = TRUE),
    cfit[2], exp(cfit[3]), cfit[4]),
    'Truth' = c(phi, mu1, sd1, mu2)), digits = 2)
# Plot the results
xx <- with(mdata, seq(min(y), max(y), len = 200))
```

```
plot(xx, (1-phi) * dnorm(xx, mu2, sd2), type = "l", xlab = "y",
    main = "Orange = estimate, blue = truth",
    col = "blue", ylab = "Density")
phi.est <- logitlink(coef(fit)[1], inverse = TRUE)
sd.est <- exp(coef(fit)[3])
lines(xx, phi*dnorm(xx, mu1, sd1), col = "blue")
lines(xx, phi.est * dnorm(xx, Coef(fit)[2], sd.est), col = "orange")
lines(xx, (1-phi.est) * dnorm(xx, Coef(fit)[4], sd.est), col = "orange")
abline(v = Coef(fit)[c(2,4)], lty = 2, col = "orange")
abline(v = c(mu1, mu2), lty = 2, col = "blue")
## End(Not run)
```

mix2poisson

Mixture of Two Poisson Distributions

## Description

Estimates the three parameters of a mixture of two Poisson distributions by maximum likelihood estimation.

## Usage

$$
\begin{aligned}
\text { mix2poisson(lphi } & =\text { "logitlink", llambda }=\text { "loglink", } \\
\text { iphi } & =0.5, \text { il1 }=\text { NULL, il2 }=\text { NULL, } \\
\text { qmu } & =c(0.2,0.8), \text { nsimEIM }=100, \text { zero }=" p h i ")
\end{aligned}
$$

## Arguments

lphi, llambda Link functions for the parameter $\phi$ and $\lambda$. See Links for more choices.
iphi Initial value for $\phi$, whose value must lie between 0 and 1.
il1, il2 Optional initial value for $\lambda_{1}$ and $\lambda_{2}$. These values must be positive. The default is to compute initial values internally using the argument qmu.
qmu Vector with two values giving the probabilities relating to the sample quantiles for obtaining initial values for $\lambda_{1}$ and $\lambda_{2}$. The two values are fed in as the probs argument into quantile.
nsimEIM, zero See CommonVGAMffArguments.

## Details

The probability function can be loosely written as

$$
P(Y=y)=\phi \operatorname{Poisson}\left(\lambda_{1}\right)+(1-\phi) \operatorname{Poisson}\left(\lambda_{2}\right)
$$

where $\phi$ is the probability an observation belongs to the first group, and $y=0,1,2, \ldots$. The parameter $\phi$ satisfies $0<\phi<1$. The mean of $Y$ is $\phi \lambda_{1}+(1-\phi) \lambda_{2}$ and this is returned as the fitted values. By default, the three linear/additive predictors are $\left(\operatorname{logit}(\phi), \log \left(\lambda_{1}\right), \log \left(\lambda_{2}\right)\right)^{T}$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Warning

This VGAM family function requires care for a successful application. In particular, good initial values are required because of the presence of local solutions. Therefore running this function with several different combinations of arguments such as iphi, il1, il2, qmu is highly recommended. Graphical methods such as hist can be used as an aid.

With grouped data (i.e., using the weights argument) one has to use a large value of nsimEIM; see the example below.
This VGAM family function is experimental and should be used with care.

## Note

The response must be integer-valued since dpois is invoked.
Fitting this model successfully to data can be difficult due to local solutions and ill-conditioned data. It pays to fit the model several times with different initial values, and check that the best fit looks reasonable. Plotting the results is recommended. This function works better as $\lambda_{1}$ and $\lambda_{2}$ become more different. The default control argument trace $=$ TRUE is to encourage monitoring convergence.

## Author(s)

T. W. Yee

## See Also

rpois, poissonff, mix2normal.

## Examples

```
## Not run: # Example 1: simulated data
nn <- 1000
mu1 <- exp(2.5) # Also known as lambda1
mu2 <- exp(3)
(phi <- logitlink(-0.5, inverse = TRUE))
mdata <- data.frame(y = rpois(nn, ifelse(runif(nn) < phi, mu1, mu2)))
mfit <- vglm(y ~ 1, mix2poisson, data = mdata)
coef(mfit, matrix = TRUE)
# Compare the results with the truth
round(rbind('Estimated' = Coef(mfit), 'Truth' = c(phi, mu1, mu2)), digits = 2)
ty <- with(mdata, table(y))
plot(names(ty), ty, type = "h", main = "Orange=estimate, blue=truth",
    ylab = "Frequency", xlab = "y")
abline(v = Coef(mfit)[-1], lty = 2, col = "orange", lwd = 2)
```

```
    abline(v = c(mu1, mu2), lty = 2, col = "blue", lwd = 2)
    # Example 2: London Times data (Lange, 1997, p.31)
    ltdata1 <- data.frame(deaths = 0:9,
            freq = c(162, 267, 271, 185, 111, 61, 27, 8, 3, 1))
    ltdata2 <- data.frame(y = with(ltdata1, rep(deaths, freq)))
    # Usually this does not work well unless nsimEIM is large
    Mfit <- vglm(deaths ~ 1, weight = freq, data = ltdata1,
        mix2poisson(iphi = 0.3, il1 = 1, il2 = 2.5, nsimEIM = 5000))
    # This works better in general
    Mfit <- vglm(y ~ 1, mix2poisson(iphi = 0.3, il1 = 1, il2 = 2.5), data = ltdata2)
    coef(Mfit, matrix = TRUE)
    Coef(Mfit)
    ## End(Not run)
```

MNSs
The MNSs Blood Group System

## Description

Estimates the three independent parameters of the the MNSs blood group system.

## Usage

MNSs(link = "logitlink", imS = NULL, ims = NULL, inS = NULL)

## Arguments

link Link function applied to the three parameters. See Links for more choices.
imS, ims, inS Optional initial value for mS , ms and nS respectively. A NULL means they are computed internally.

## Details

There are three independent parameters: $m_{-} S, m_{-} s, n_{\_} S$, say, so that $n_{-} s=1-m_{-} S-m_{-} s-n \_S$. We let the eta vector (transposed) be $\left(g\left(m_{-} S\right), g\left(m_{-} s\right), g\left(n_{-} S\right)\right)$ where $g$ is the link function.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

The input can be a 6-column matrix of counts, where the columns are MS, Ms, MNS, MNs, NS, Ns (in order). Alternatively, the input can be a 6 -column matrix of proportions (so each row adds to 1 ) and the weights argument is used to specify the total number of counts for each row.

## Author(s)

T. W. Yee

## References

Elandt-Johnson, R. C. (1971). Probability Models and Statistical Methods in Genetics, New York: Wiley.

## See Also

```
AA.Aa.aa, AB.Ab.aB.ab, ABO, A1A2A3.
```


## Examples

```
# Order matters only:
y <- cbind(MS = 295, Ms = 107, MNS = 379, MNs = 322, NS = 102, Ns = 214)
fit <- vglm(y ~ 1, MNSs("logitlink", .25, .28, .08), trace = TRUE)
fit <- vglm(y ~ 1, MNSs(link = logitlink), trace = TRUE, crit = "coef")
Coef(fit)
rbind(y, sum(y)*fitted(fit))
sqrt(diag(vcov(fit)))
```

model.framevlm Construct the Model Frame of a VLM Object

## Description

This function returns a data. frame with the variables. It is applied to an object which inherits from class "vlm" (e.g., a fitted model of class "vglm").

## Usage

model.framevlm(object, setupsmart = TRUE, wrapupsmart = TRUE, ...)

## Arguments

object a model object from the VGAM R package that inherits from a vector linear model (VLM), e.g., a model of class "vglm".
... further arguments such as data, na. action, subset. See model.frame for more information on these.
setupsmart, wrapupsmart
Logical. Arguments to determine whether to use smart prediction.

## Details

Since object is an object which inherits from class "vlm" (e.g., a fitted model of class "vglm"), the method will either returned the saved model frame used when fitting the model (if any, selected by argument model $=$ TRUE) or pass the call used when fitting on to the default method.
This code implements smart prediction (see smartpred).

## Value

A data. frame containing the variables used in the object plus those specified in . ...

## References

Chambers, J. M. (1992). Data for models. Chapter 3 of Statistical Models in S eds J. M. Chambers and T. J. Hastie, Wadsworth \& Brooks/Cole.

## See Also

model.frame, model.matrixvlm, predictvglm, smartpred.

## Examples

```
# Illustrates smart prediction
pneumo <- transform(pneumo, let = log(exposure.time))
fit <- vglm(cbind(normal,mild, severe) ~ poly(c(scale(let)), 2),
            multinomial, data = pneumo, trace = TRUE, x = FALSE)
class(fit)
check1 <- head(model.frame(fit))
check1
check2 <- model.frame(fit, data = head(pneumo))
check2
all.equal(unlist(check1), unlist(check2)) # Should be TRUE
q0 <- head(predict(fit))
q1 <- head(predict(fit, newdata = pneumo))
q2 <- predict(fit, newdata = head(pneumo))
all.equal(q0, q1) # Should be TRUE
all.equal(q1, q2) # Should be TRUE
```

model.matrixqrrvglm Construct the Model Matrix of a QRR-VGLM Object

## Description

Creates a model matrix. Two types can be returned: a large one (class "vlm" or one that inherits from this such as "vglm") or a small one (such as returned if it were of class "lm").

## Usage

model.matrixqrrvglm(object, type = c("latvar", "lm", "vlm"), ...)

## Arguments

object an object of a class "qrrvglm", i.e., a cqo object.
type Type of model (or design) matrix returned. The first is the default. The value "latvar" is model matrix mainly comprising of the latent variable values (sometimes called the site scores). The value "lm" is the LM matrix directly corresponding to the formula argument. The value "vlm" is the big VLM model matrix given $C$.
... further arguments passed to or from other methods.

## Details

This function creates one of several design matrices from object. For example, this can be a small LM object or a big VLM object.

When type $=$ "vlm" this function calls fnumat2R() to construct the big model matrix given C. That is, the constrained coefficients are assumed known, so that something like a large Poisson or logistic regression is set up. This is because all responses are fitted simultaneously here. The columns are labelled in the following order and with the following prefixes: " A " for the $A$ matrix (linear in the latent variables), " D " for the $D$ matrix (quadratic in the latent variables), " $\times 1$." for the $B 1$ matrix (usually contains the intercept; see the argument noRRR in qrrvglm. control).

## Value

The design matrix after scaling for a regression model with the specified formula and data. By after scaling, it is meant that it matches the output of coef (qrrvglmObject) rather than the original scaling of the fitted object.

## See Also

model.matrixvlm, cqo, vcovqrrvglm.

## Examples

```
## Not run:
set.seed(1); n <- 40; p <- 3; S <- 4; myrank <- 1
mydata <- rcqo(n, p, S, Rank = myrank, es.opt = TRUE, eq.max = TRUE)
(myform <- attr(mydata, "formula"))
mycqo <- cqo(myform, poissonff, data = mydata,
    I.tol = TRUE, Rank = myrank, Bestof = 5)
model.matrix(mycqo, type = "latvar")
model.matrix(mycqo, type = "lm")
model.matrix(mycqo, type = "vlm")
## End(Not run)
```


## Description

Creates a design matrix. Two types can be returned: a large one (class "vlm" or one that inherits from this such as "vglm") or a small one (such as returned if it were of class "lm").

## Usage

model.matrixvlm(object, type = c("vlm", "lm", "lm2", "bothlmlm2"),
linpred.index = NULL, ...)

## Arguments

object an object of a class that inherits from the vector linear model (VLM).
type Type of design matrix returned. The first is the default. The value "vlm" is the VLM model matrix corresponding to the formula argument. The value " 1 m " is the LM model matrix corresponding to the formula argument. The value "lm2" is the second (LM) model matrix corresponding to the form2 argument. The value "bothlmlm2" means both LM and VLM model matrices.
linpred.index Vector of integers. The index for a linear/additive predictor, it must have values from the set $1: M$. Also, if length(linpred.index) $==1$ then type $=" l m "$ must be assigned, whereas if length(linpred.index) $>1$ then type $=$ "vlm" must be assigned, Then it returns a subset of the VLM matrix corresponding to the linpred.indexth linear/additive predictor(s); this is a LM-type matrix when it is of unit length. Currently some attributes are returned, but these may change in value in the future because of ongoing development work.
.. further arguments passed to or from other methods. These include data (which is a data frame created with model.framevlm), contrasts.arg, and xlev. See model.matrix for more information.

## Details

This function creates a design matrix from object. This can be a small LM object or a big VLM object (default). The latter is constructed from the former and the constraint matrices.
This code implements smart prediction (see smartpred).

## Value

The design matrix for a regression model with the specified formula and data. If type = "bothlmlm2" then a list is returned with components " X " and "Xm2".
Sometimes (especially if $x=$ TRUE when calling vglm) the model matrix has attributes: "assign" ("lm"-type) and "vassign" ("vlm"-type) and "orig.assign.lm" ("lm"-type). These are used internally a lot for bookkeeping, especially regarding the columns of both types of model matrices. In particular, constraint matrices and variable selection relies on this information a lot. The "orig.assign.lm" is the ordinary "assign" attribute for lm and glm objects.
moffset

## References

Chambers, J. M. (1992). Data for models. Chapter 3 of Statistical Models in $S$ eds J. M. Chambers and T. J. Hastie, Wadsworth \& Brooks/Cole.

## See Also

model.matrix, model.framevlm, predictvglm, smartpred, constraints.vlm, add1.vglm, drop1.vglm, step4vglm.

## Examples

\# (I) Illustrates smart prediction ,,,,,,,,,,,,,,,,,,,,,, pneumo <- transform(pneumo, let $=\log ($ exposure.time))
fit <- vglm(cbind(normal, mild, severe) ~
sm.poly(c(sm.scale(let)), 2),
multinomial, data $=$ pneumo, trace $=$ TRUE, $x=$ FALSE)
class(fit)
fit@smart.prediction \# Data-dependent parameters
fit@x \# Not saved on the object
model.matrix(fit)
model.matrix(fit, linpred.index $=1$, type = "lm")
model.matrix(fit, linpred.index $=2$, type = "lm")
(Check1 <- head(model.matrix(fit, type = "lm")))
(Check2 <- model.matrix(fit, data $=$ head(pneumo), type = "lm"))
all.equal(c(Check1), c(Check2)) \# Should be TRUE
q0 <- head(predict(fit))
q1 <- head(predict(fit, newdata = pneumo))
q2 <- predict(fit, newdata $=$ head(pneumo))
all.equal(q0, q1) \# Should be TRUE
all.equal(q1, q2) \# Should be TRUE
\# (II) Attributes ,,,,,,,,,,,,,,,,,,,,,,,,,,,,,, ,, ,, ,, ,,
fit2 <- vglm(cbind(normal, mild, severe) ~ let, \# x = TRUE
multinomial, data $=$ pneumo, trace $=$ TRUE)
fit2@x \# "lm"-type; saved on the object; note the attributes model.matrix(fit2, type = "lm") \# Note the attributes
model.matrix(fit2, type = "vlm") \# Note the attributes
moffset Matrix Offset

## Description

Modify a matrix by shifting successive elements.

## Usage

```
    moffset(mat, roffset \(=0\), coffset \(=0\), postfix \(=" "\),
        rprefix = "Row.", cprefix = "Col.")
```


## Arguments

mat Data frame or matrix. This ought to have at least three rows and three columns. The elements are shifted in the order of c (mat), i.e., going down successive columns, as the columns go from left to right. Wrapping of values is done.
roffset, coffset
Numeric or character. If numeric, the amount of shift (offset) for each row and column. The default is no change to mat. If character, the offset is computed by matching with the row or column names. For example, for the alcoff, put roffset = "6" means that we make an effective day's dataset start from 6:00 am, and this wraps around to include midnight to 05.59 am on the next day.
postfix Character. Modified rows and columns are renamed by pasting this argument to the end of each name. The default is no change.
rprefix, cprefix
Same as rcim.

## Details

This function allows a matrix to be rearranged so that element (roffset +1 , coffset +1 ) becomes the $(1,1)$ element. The elements are assumed to be ordered in the same way as the elements of c (mat),

This function is applicable to, e.g., alcoff, where it is useful to define the effective day as starting at some other hour than midnight, e.g., 6.00am. This is because partying on Friday night continues on into Saturday morning, therefore it is more interpretable to use the effective day when considering a daily effect.
This is a data preprocessing function for rcim and plotrcim0. The differences between Rcim and moffset is that Rcim only reorders the level of the rows and columns so that the data is shifted but not moved. That is, a value in one row stays in that row, and ditto for column. But in moffset values in one column can be moved to a previous column. See the examples below.

## Value

A matrix of the same dimensional as its input.

## Note

The input mat should have row names and column names.

## Author(s)

T. W. Yee, Alfian F. Hadi.

## See Also

Rcim, rcim, plotrcim0, alcoff, crashi.

## Examples

```
moffset(alcoff, 3, 2, "*") # Some day's data is moved to previous day.
Rcim(alcoff, 3 + 1, 2 + 1) # Data does not move as much.
alcoff # Original data
moffset(alcoff, 3, 2, "*") - Rcim(alcoff, 3+1, 2+1) # Note the differences
# An 'effective day' data set:
alcoff.e <- moffset(alcoff, roffset = "6", postfix = "*")
fit.o <- rcim(alcoff) # default baselines are first row and col
fit.e <- rcim(alcoff.e) # default baselines are first row and col
## Not run: par(mfrow = c(2, 2), mar = c(9, 4, 2, 1))
plot(fit.o, rsub = "Not very interpretable", csub = "Not very interpretable")
plot(fit.e, rsub = "More interpretable", csub = "More interpretable")
## End(Not run)
# Some checking
all.equal(moffset(alcoff), alcoff) # Should be no change
moffset(alcoff, 1, 1, "*")
moffset(alcoff, 2, 3, "*")
moffset(alcoff, 1, 0, "*")
moffset(alcoff, 0, 1, "*")
moffset(alcoff, "6", "Mon", "*") # This one is good
# Customise row and column baselines
fit2 <- rcim(Rcim(alcoff.e, rbaseline = "11", cbaseline = "Mon*"))
```

```
multilogitlink
```

Multi-logit Link Function

## Description

Computes the multilogit transformation, including its inverse and the first two derivatives.

## Usage

multilogitlink(theta, refLevel = "(Last)", M = NULL, whitespace = FALSE, bvalue = NULL, inverse = FALSE, deriv = 0, all.derivs = FALSE, short $=$ TRUE, tag $=$ FALSE)

## Arguments

theta Numeric or character. See below for further details.
refLevel, M, whitespace
See multinomial.
bvalue See Links.
all.derivs Logical. This is currently experimental only.
inverse, deriv, short, tag
Details at Links.

## Details

The multilogitlink() link function is a generalization of the logitlink link to $M$ levels/classes. It forms the basis of the multinomial logit model. It is sometimes called the multi-logit link or the multinomial logit link. When its inverse function is computed it returns values which are positive and add to unity.

## Value

For multilogitlink with deriv $=0$, the multilogit of theta, i.e., $\log (\operatorname{theta}[, j] / \operatorname{theta}[, M+1])$ when inverse $=$ FALSE, and if inverse $=$ TRUE then $\exp ($ theta $[, j]) /(1+r o w S u m s(\exp ($ theta $)))$.

For deriv $=1$, then the function returns $d$ eta $/ d$ theta as a function of theta if inverse $=$ FALSE, else if inverse $=$ TRUE then it returns the reciprocal.

Here, all logarithms are natural logarithms, i.e., to base $e$.

## Note

Numerical instability may occur when theta is close to 1 or 0 (for multilogitlink). One way of overcoming this is to use, e.g., bvalue. Currently care. $\exp ()$ is used to avoid NAs being returned if the probability is too close to 1 .

## Author(s)

Thomas W. Yee

## References

McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, 2nd ed. London: Chapman \& Hall.

## See Also

Links, multinomial, logitlink, gaitpoisson.mlm, normal.vcm, CommonVGAMffArguments.

## Examples

```
    pneumo <- transform(pneumo, let = log(exposure.time))
    fit <- vglm(cbind(normal, mild, severe) ~ let,
        multinomial, trace = TRUE, data = pneumo) # For illustration only!
    fitted(fit)
    predict(fit)
    multilogitlink(fitted(fit))
    multilogitlink(fitted(fit)) - predict(fit) # Should be all 0s
    multilogitlink(predict(fit), inverse = TRUE) # rowSums() add to unity
    multilogitlink(predict(fit), inverse = TRUE, refLevel = 1) # For illustration only
    multilogitlink(predict(fit), inverse = TRUE) - fitted(fit) # Should be all 0s
    multilogitlink(fitted(fit), deriv = 1)
    multilogitlink(fitted(fit), deriv = 2)
```

multinomial

Multinomial Logit Model

## Description

Fits a multinomial logit model to a (preferably unordered) factor response.

## Usage

```
multinomial(zero = NULL, parallel = FALSE, nointercept = NULL,
    refLevel = "(Last)", whitespace = FALSE)
```


## Arguments

zero Can be an integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. Any values must be from the set $\{1,2, \ldots, M\}$. The default value means none are modelled as intercept-only terms. See CommonVGAMffArguments for more information.
parallel A logical, or formula specifying which terms have equal/unequal coefficients.
nointercept, whitespace
See CommonVGAMffarguments for more details.
refLevel Either a (1) single positive integer or (2) a value of the factor or (3) a character string. If inputted as an integer then it specifies which column of the response matrix is the reference or baseline level. The default is the last one (the $(M+1)$ th one). If used, this argument will be usually assigned the value 1 . If inputted as a value of a factor then beware of missing values of certain levels of the factor (drop. unused.levels = TRUE or drop. unused.levels = FALSE). See the example below. If inputted as a character string then this should be equal to (A) one of the levels of the factor response, else (B) one of the column names of the matrix response of counts; e.g., vglm(cbind(normal,mild, severe)
$\sim$ let, multinomial (refLevel = "severe"), data = pneumo) if it was (incor-
rectly because the response is ordinal) applied to the pneumo data set. Another
example is vglm(ethnicity $\sim$ age, multinomial (refLevel = "European"), data
$=$ xs.nz) if it was applied to the xs.nz data set.

## Details

In this help file the response $Y$ is assumed to be a factor with unordered values $1,2, \ldots, M+1$, so that $M$ is the number of linear/additive predictors $\eta_{j}$.

The default model can be written

$$
\eta_{j}=\log (P[Y=j] / P[Y=M+1])
$$

where $\eta_{j}$ is the $j$ th linear/additive predictor. Here, $j=1, \ldots, M$, and $\eta_{M+1}$ is 0 by definition. That is, the last level of the factor, or last column of the response matrix, is taken as the reference level or baseline-this is for identifiability of the parameters. The reference or baseline level can be changed with the refLevel argument.
In almost all the literature, the constraint matrices associated with this family of models are known. For example, setting parallel = TRUE will make all constraint matrices (including the intercept) equal to a vector of $M 1$ 's; to suppress the intercepts from being parallel then set parallel = FALSE $\sim 1$. If the constraint matrices are unknown and to be estimated, then this can be achieved by fitting the model as a reduced-rank vector generalized linear model (RR-VGLM; see rrvglm). In particular, a multinomial logit model with unknown constraint matrices is known as a stereotype model (Anderson, 1984), and can be fitted with rrvglm.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Warning

No check is made to verify that the response is nominal.
See CommonVGAMffArguments for more warnings.

## Note

The response should be either a matrix of counts (with row sums that are all positive), or a factor. In both cases, the $y$ slot returned by $\mathrm{vglm} / \mathrm{vgam} / \mathrm{rrvglm}$ is the matrix of sample proportions.

The multinomial logit model is more appropriate for a nominal (unordered) factor response than for an ordinal (ordered) factor response. Models more suited for the latter include those based on cumulative probabilities, e.g., cumulative.
multinomial is prone to numerical difficulties if the groups are separable and/or the fitted probabilities are close to 0 or 1 . The fitted values returned are estimates of the probabilities $P[Y=j]$ for $j=1, \ldots, M+1$. See safeBinaryRegression for the logistic regression case.
Here is an example of the usage of the parallel argument. If there are covariates $\times 2, x 3$ and $x 4$, then parallel $=\operatorname{TRUE} \sim x 2+x 3-1$ and parallel $=$ FALSE $\sim x 4$ are equivalent. This would
constrain the regression coefficients for $x 2$ and $x 3$ to be equal; those of the intercepts and $x 4$ would be different.
In Example 4 below, a conditional logit model is fitted to an artificial data set that explores how cost and travel time affect people's decision about how to travel to work. Walking is the baseline group. The variable cost.car is the difference between the cost of travel to work by car and walking, etc. The variable Time.car is the difference between the travel duration/time to work by car and walking, etc. For other details about the xij argument see vglm. control and fill.

The multinom function in the nnet package uses the first level of the factor as baseline, whereas the last level of the factor is used here. Consequently the estimated regression coefficients differ.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2010). The VGAM package for categorical data analysis. Journal of Statistical Software, 32, 1-34. http://www.jstatsoft.org/v32/i10/.
Yee, T. W. and Hastie, T. J. (2003). Reduced-rank vector generalized linear models. Statistical Modelling, 3, 15-41.
McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, 2nd ed. London: Chapman \& Hall.

Agresti, A. (2013). Categorical Data Analysis, 3rd ed. Hoboken, NJ, USA: Wiley.
Hastie, T. J., Tibshirani, R. J. and Friedman, J. H. (2009). The Elements of Statistical Learning:
Data Mining, Inference and Prediction, 2nd ed. New York, USA: Springer-Verlag.
Simonoff, J. S. (2003). Analyzing Categorical Data, New York, USA: Springer-Verlag.
Anderson, J. A. (1984). Regression and ordered categorical variables. Journal of the Royal Statistical Society, Series B, Methodological, 46, 1-30.
Tutz, G. (2012). Regression for Categorical Data, Cambridge University Press.

## See Also

multilogitlink, margeff, cumulative, acat, cratio, sratio, dirichlet, dirmultinomial, rrvglm, fill1, Multinomial, Gaitpois, gatnbinomial.mlm, iris.

## Examples

```
# Example 1: fit a multinomial logit model to Edgar Anderson's iris data
data(iris)
## Not run: fit <- vglm(Species ~ ., multinomial, iris)
coef(fit, matrix = TRUE)
## End(Not run)
# Example 2a: a simple example
ycounts <- t(rmultinom(10, size = 20, prob = c(0.1, 0.2, 0.8))) # Counts
fit <- vglm(ycounts ~ 1, multinomial)
head(fitted(fit)) # Proportions
```

```
fit@prior.weights # NOT recommended for extraction of prior weights
weights(fit, type = "prior", matrix = FALSE) # The better method
depvar(fit) # Sample proportions; same as fit@y
constraints(fit) # Constraint matrices
# Example 2b: Different reference level used as the baseline
fit2 <- vglm(ycounts ~ 1, multinomial(refLevel = 2))
coef(fit2, matrix = TRUE)
coef(fit , matrix = TRUE) # Easy to reconcile this output with fit2
# Example 3: The response is a factor.
nn <- 10
dframe3 <- data.frame(yfac = gl(3, nn, labels = c("Ctrl", "Trt1", "Trt2")),
            x2 = runif(3 * nn))
myrefLevel <- with(dframe3, yfac[12])
fit3a <- vglm(yfac ~ x2, multinomial(refLevel = myrefLevel), dframe3)
fit3b <- vglm(yfac ~ x2, multinomial(refLevel = 2), dframe3)
coef(fit3a, matrix = TRUE) # "Trt1" is the reference level
coef(fit3b, matrix = TRUE) # "Trt1" is the reference level
margeff(fit3b)
# Example 4: Fit a rank-1 stereotype model
fit4 <- rrvglm(Country ~ Width + Height + HP, multinomial, data = car.all)
coef(fit4) # Contains the C matrix
constraints(fit4)$HP # The A matrix
coef(fit4, matrix = TRUE) # The B matrix
Coef(fit4)@C # The C matrix
concoef(fit4) # Better to get the C matrix this way
Coef(fit4)@A # The A matrix
svd(coef(fit4, matrix = TRUE)[-1, ])$d # This has rank 1; = C %*% t(A)
# Classification (but watch out for NAs in some of the variables):
apply(fitted(fit4), 1, which.max) # Classification
colnames(fitted(fit4))[apply(fitted(fit4), 1, which.max)] # Classification
apply(predict(fit4, car.all, type = "response"), 1, which.max) # Ditto
# Example 5: The use of the xij argument (aka conditional logit model)
set.seed(111)
nn <- 100 # Number of people who travel to work
M <- 3 # There are M+1 models of transport to go to work
ycounts <- matrix(0, nn, M+1)
ycounts[cbind(1:nn, sample(x = M+1, size = nn, replace = TRUE))] = 1
dimnames(ycounts) <- list(NULL, c("bus","train","car","walk"))
gotowork <- data.frame(cost.bus = runif(nn), time.bus = runif(nn),
cost.train= runif(nn), time.train= runif(nn),
cost.car = runif(nn), time.car = runif(nn),
cost.walk = runif(nn), time.walk = runif(nn))
gotowork <- round(gotowork, digits = 2) # For convenience
gotowork <- transform(gotowork,
```

```
Cost.bus = cost.bus - cost.walk,
```

Cost.bus = cost.bus - cost.walk,
Cost.car = cost.car - cost.walk,
Cost.car = cost.car - cost.walk,
Cost.train = cost.train - cost.walk,
Cost.train = cost.train - cost.walk,
Cost = cost.train - cost.walk, \# for labelling

```
Cost = cost.train - cost.walk, # for labelling
```

```
        Time.bus = time.bus - time.walk,
        Time.car = time.car - time.walk,
        Time.train = time.train - time.walk,
        Time = time.train - time.walk) # for labelling
fit <- vglm(ycounts ~ Cost + Time,
    multinomial(parall = TRUE ~ Cost + Time - 1),
    xij = list(Cost ~ Cost.bus + Cost.train + Cost.car,
            Time ~ Time.bus + Time.train + Time.car),
    form2 = ~ Cost + Cost.bus + Cost.train + Cost.car +
                Time + Time.bus + Time.train + Time.car,
    data = gotowork, trace = TRUE)
head(model.matrix(fit, type = "lm")) # LM model matrix
head(model.matrix(fit, type = "vlm")) # Big VLM model matrix
coef(fit)
coef(fit, matrix = TRUE)
constraints(fit)
summary(fit)
max(abs(predict(fit) - predict(fit, new = gotowork))) # Should be 0
```

Nakagami Nakagami Distribution

## Description

Density, cumulative distribution function, quantile function and random generation for the Nakagami distribution.

## Usage

dnaka( $x$, scale $=1$, shape, $\log =$ FALSE)
pnaka(q, scale $=1$, shape, lower.tail $=$ TRUE, log. $\mathrm{p}=$ FALSE)
qnaka( $p$, scale $=1$, shape, ...)
rnaka( n , scale $=1$, shape, Smallno $=1.0 \mathrm{e}-6$ )

## Arguments

| $\mathrm{x}, \mathrm{q}$ | vector of quantiles. |
| :---: | :---: |
| p | vector of probabilities. |
| n | number of observations. Same as in runif. |
| scale, shape | arguments for the parameters of the distribution. See nakagami for more details. For rnaka, arguments shape and scale must be of length 1. |
| Smallno | Numeric, a small value used by the rejection method for determining the upper limit of the distribution. That is, pnaka $(U)>1-$ Smallno where $U$ is the upper limit. |
|  | Arguments that can be passed into uniroot. |
| log | Logical. If log = TRUE then the logarithm of the density is returned. |
| lower.tail, log.p |  |
|  | Same meaning as in pnorm or qnorm. |

## Details

See nakagami for more details.

## Value

dnaka gives the density, pnaka gives the cumulative distribution function, qnaka gives the quantile function, and rnaka generates random deviates.

## Author(s)

T. W. Yee and Kai Huang

## See Also

nakagami.

## Examples

```
    ## Not run: x <- seq(0, 3.2, len = 200)
    plot(x, dgamma(x, shape = 1), type = "n", col = "black", ylab = "",
    ylim = c(0,1.5), main = "dnaka(x, shape = shape)")
lines(x, dnaka(x, shape = 1), col = "orange")
lines(x, dnaka(x, shape = 2), col = "blue")
lines(x, dnaka(x, shape = 3), col = "green")
legend(2, 1.0, col = c("orange","blue","green"), lty = rep(1, len = 3),
        legend = paste("shape =", c(1, 2, 3)))
plot(x, pnorm(x), type = "n", col = "black", ylab = "",
    ylim = 0:1, main = "pnaka(x, shape = shape)")
lines(x, pnaka(x, shape = 1), col = "orange")
lines(x, pnaka(x, shape = 2), col = "blue")
lines(x, pnaka(x, shape = 3), col = "green")
legend(2, 0.6, col = c("orange","blue","green"), lty = rep(1, len = 3),
            legend = paste("shape =", c(1, 2, 3)))
## End(Not run)
probs <- seq(0.1, 0.9, by = 0.1)
pnaka(qnaka(p = probs, shape = 2), shape = 2) - probs # Should be all 0
```

nakagami Nakagami Regression Family Function

## Description

Estimation of the two parameters of the Nakagami distribution by maximum likelihood estimation.

## Usage

nakagami(lscale = "loglink", lshape = "loglink", iscale = 1, ishape = NULL, nowarning = FALSE)

## Arguments

nowarning Logical. Suppress a warning?
lscale, lshape Parameter link functions applied to the scale and shape parameters. Log links ensure they are positive. See Links for more choices and information.
iscale, ishape Optional initial values for the shape and scale parameters. For ishape, a NULL value means it is obtained in the initialize slot based on the value of iscale. For iscale, assigning a NULL means a value is obtained in the initialize slot, however, setting another numerical value is recommended if convergence fails or is too slow.

## Details

The Nakagami distribution, which is useful for modelling wireless systems such as radio links, can be written

$$
f(y)=2(\text { shape } / \text { scale })^{\text {shape }} y^{2 \times \text { shape }-1} \exp \left(- \text { shape } \times y^{2} / \text { scale }\right) / \Gamma(\text { shape })
$$

for $y>0$, shape $>0$, scale $>0$. The mean of $Y$ is $\sqrt{\text { scale } / \text { shape }} \times \Gamma($ shape +0.5$) / \Gamma($ shape $)$ and these are returned as the fitted values. By default, the linear/additive predictors are $\eta_{1}=$ $\log ($ scale $)$ and $\eta_{2}=\log ($ shape $)$. Fisher scoring is implemented.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

The Nakagami distribution is also known as the Nakagami- $m$ distribution, where $m=$ shape here. Special cases: $m=0.5$ is a one-sided Gaussian distribution and $m=1$ is a Rayleigh distribution. The second moment is $E\left(Y^{2}\right)=m$.
If $Y$ has a Nakagami distribution with parameters shape and scale then $Y^{2}$ has a gamma distribution with shape parameter shape and scale parameter scale/shape.

## Author(s)

T. W. Yee

## References

Nakagami, M. (1960). The $m$-distribution: a general formula of intensity distribution of rapid fading, pp.3-36 in: Statistical Methods in Radio Wave Propagation. W. C. Hoffman, Ed., New York: Pergamon.

## See Also

rnaka, gamma2, rayleigh.

## Examples

```
nn <- 1000; shape <- exp(0); Scale <- exp(1)
ndata <- data.frame(y1 = sqrt(rgamma(nn, shape = shape, scale = Scale/shape)))
nfit <- vglm(y1 ~ 1, nakagami, data = ndata, trace = TRUE, crit = "coef")
ndata <- transform(ndata, y2 = rnaka(nn, scale = Scale, shape = shape))
nfit <- vglm(y2 ~ 1, nakagami(iscale = 3), data = ndata, trace = TRUE)
head(fitted(nfit))
with(ndata, mean(y2))
coef(nfit, matrix = TRUE)
(Cfit <- Coef(nfit))
## Not run: sy <- with(ndata, sort(y2))
hist(with(ndata, y2), prob = TRUE, main = "", xlab = "y", ylim = c(0, 0.6),
    col = "lightblue")
lines(dnaka(sy, scale = Cfit["scale"], shape = Cfit["shape"]) ~ sy,
    data = ndata, col = "orange")
## End(Not run)
```

nbcanlink Negative Binomial Canonical Link Function

## Description

Computes the negative binomial canonical link transformation, including its inverse and the first two derivatives.

## Usage

```
nbcanlink(theta, size = NULL, wrt.param = NULL, bvalue = NULL,
    inverse = FALSE, deriv = 0, short = TRUE, tag = FALSE)
```


## Arguments

theta Numeric or character. Typically the mean of a negative binomial distribution (NBD). See below for further details.
size, wrt.param
size contains the $k$ matrix which must be of a conformable dimension as theta. Also, if deriv >0 then wrt. param is either 1 or 2 ( 1 for with respect to the first parameter, and 2 for with respect to the second parameter (size)).
bvalue Details at Links.
inverse, deriv, short, tag
Details at Links.

## Details

The NBD canonical link is $\log (\theta /(\theta+k))$ where $\theta$ is the NBD mean. The canonical link is used for theoretically relating the NBD to GLM class.
This link function was specifically written for negbinomial and negbinomial.size, and should not be used elsewhere (these VGAM family functions have code that specifically handles nbcanlink().)

## Value

For deriv $=0$, the above equation when inverse $=$ FALSE, and if inverse $=$ TRUE then kmatrix $/$ expm1 (-theta) where theta ie really eta. For deriv $=1$, then the function returns $d$ eta $/ d$ theta as a function of theta if inverse $=$ FALSE, else if inverse $=$ TRUE then it returns the reciprocal.

## Warning

This function works with negbinomial but care is needed because it is numerically fraught. In particular, the first linear/additive predictor must have negative values, and finding good initial values may be difficult, leading to it crashing at the start. Hence the NB-C model is sensitive to the initial values and may converge to a local solution. Pages 210 and 309 of Hilbe (2011) notes convergence difficulties (of Newton-Raphson type algorithms), and some of that this applies here. Setting trace $=$ TRUE is a good idea, as is trying various values of imethod in negbinomial.

## Note

While theoretically nice, this function is not recommended in general since its value is always negative (linear predictors ought to be unbounded in general). A loglink link for argument lmu is recommended instead.
Numerical instability may occur when theta is close to 0 or 1 . Values of theta which are less than or equal to 0 can be replaced by bvalue before computing the link function value. See Links.

## Author(s)

Victor Miranda and Thomas W. Yee.

## References

Miranda, V. S. and Yee, T. W. (2018). On mean function modelling for several one-parameter discrete distributions. Manuscript in preparation.
Yee, T. W. (2014). Reduced-rank vector generalized linear models with two linear predictors. Computational Statistics and Data Analysis, 71, 889-902.
Hilbe, J. M. (2011). Negative Binomial Regression, 2nd Edition. Cambridge: Cambridge University Press.

## See Also

```
negbinomial, negbinomial.size.
```


## Examples

```
nbcanlink("mu", short = FALSE)
mymu <- 1:10 # Test some basic operations:
kmatrix <- cbind(runif(length(mymu)))
eta1 <- nbcanlink(mymu, size = kmatrix)
ans2 <- nbcanlink(eta1, size = kmatrix, inverse = TRUE)
max(abs(ans2 - mymu)) # Should be 0
```

```
## Not run: mymu <- seq(0.5, 10, length = 101)
kmatrix <- matrix(10, length(mymu), 1)
plot(nbcanlink(mymu, size = kmatrix) ~ mymu, las = 1,
    type = "l", col = "blue", xlab = expression({mu}))
## End(Not run)
# Estimate the parameters from some simulated data
ndata <- data.frame(x2 = runif(nn <- 100))
ndata <- transform(ndata, eta1 = -1 - 1 * x2, # eta1 < 0
                        size1 = exp(1),
                        size2 = exp(2))
ndata <- transform(ndata,
        mu1 = nbcanlink(eta1, size = size1, inverse = TRUE),
        mu2 = nbcanlink(eta1, size = size2, inverse = TRUE))
ndata <- transform(ndata, y1 = rnbinom(nn, mu = mu1, size = size1),
                y2 = rnbinom(nn, mu = mu2, size = size2))
summary(ndata)
nbcfit <- vglm(cbind(y1, y2) ~ x2,
    negbinomial(lmu = "nbcanlink", imethod = 1), # Try this
    negbinomial(lmu = "nbcanlink", imethod = 2), # Try this
    data = ndata, trace = TRUE)
coef(nbcfit, matrix = TRUE)
summary(nbcfit)
```

nbordlink Negative Binomial-Ordinal Link Function

## Description

Computes the negative binomial-ordinal transformation, including its inverse and the first two derivatives.

## Usage

nbordlink(theta, cutpoint $=$ NULL, $\mathrm{k}=$ NULL,

```
inverse = FALSE, deriv = 0, short = TRUE, tag = FALSE)
```


## Arguments

theta Numeric or character. See below for further details.
cutpoint, $\mathrm{k} \quad$ Here, k is the $k$ parameter associated with the negative binomial distribution; see negbinomial. The cutpoints should be non-negative integers. If nbordlink() is used as the link function in cumulative then one should choose reverse $=$ TRUE, parallel = TRUE.
inverse, deriv, short, tag
Details at Links.

## Details

The negative binomial-ordinal link function (NBOLF) can be applied to a parameter lying in the unit interval. Its purpose is to link cumulative probabilities associated with an ordinal response coming from an underlying negative binomial distribution.
See Links for general information about VGAM link functions.

## Value

See Yee (2018) for details.

## Warning

Prediction may not work on vglm or vgam etc. objects if this link function is used.

## Note

Numerical values of theta too close to 0 or 1 or out of range result in large positive or negative values, or maybe 0 depending on the arguments. Although measures have been taken to handle cases where theta is too close to 1 or 0 , numerical instabilities may still arise.
In terms of the threshold approach with cumulative probabilities for an ordinal response this link function corresponds to the negative binomial distribution (see negbinomial) that has been recorded as an ordinal response using known cutpoints.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2020). Ordinal ordination with normalizing link functions for count data, (in preparation).

## See Also

Links, negbinomial, pordlink, gordlink, nbord2link, cumulative, CommonVGAMffArguments.

## Examples

```
## Not run:
nbordlink("p", cutpoint = 2, k = 1, short = FALSE)
nbordlink("p", cutpoint = 2, k = 1, tag = TRUE)
p <- seq(0.02, 0.98, by = 0.01)
y <- nbordlink(p,cutpoint = 2, k = 1)
y. <- nbordlink(p,cutpoint = 2, k = 1, deriv = 1)
max(abs(nbordlink(y,cutpoint = 2, k = 1, inv = TRUE) - p)) # Should be 0
#\ dontrun{ par(mfrow = c(2, 1), las = 1)
#plot(p, y, type = "l", col = "blue", main = "nbordlink()")
#abline(h = 0, v = 0.5, col = "red", lty = "dashed")
```

```
#
#plot(p, y., type = "l", col = "blue",
# main = "(Reciprocal of) first NBOLF derivative") }
# Another example
nn <- 1000
x2 <- sort(runif(nn))
x3 <- runif(nn)
mymu <- exp( 3 + 1 * x2 - 2 * x3)
k <- 4
y1 <- rnbinom(nn, mu = mymu, size = k)
cutpoints <- c(-Inf, 10, 20, Inf)
cuty <- Cut(y1, breaks = cutpoints)
#\ dontrun{ plot(x2, x3, col = cuty, pch = as.character(cuty)) }
table(cuty) / sum(table(cuty))
fit <- vglm(cuty ~ x2 + x3, trace = TRUE,
    cumulative(reverse = TRUE, multiple.responses = TRUE,
        parallel = TRUE,
        link = nbordlink(cutpoint = cutpoints[2:3], k = k)))
head(depvar(fit))
head(fitted(fit))
head(predict(fit))
coef(fit)
coef(fit, matrix = TRUE)
constraints(fit)
fit@misc
## End(Not run)
```

negbinomial Negative Binomial Distribution Family Function

## Description

Maximum likelihood estimation of the two parameters of a negative binomial distribution.

## Usage

```
negbinomial(zero = "size", parallel = FALSE, deviance.arg = FALSE,
    type.fitted = c("mean", "quantiles"),
    percentiles = c(25, 50, 75),
    mds.min = 1e-3, nsimEIM = 500, cutoff.prob = 0.999,
    eps.trig = 1e-7, max.support = 4000, max.chunk.MB = 30,
    lmu = "loglink", lsize = "loglink",
    imethod = 1, imu = NULL, iprobs.y = NULL,
    gprobs.y = ppoints(6), isize = NULL,
    gsize.mux = exp(c(-30, -20, -15, -10, -6:3)))
polya(zero = "size", type.fitted = c("mean", "prob"),
    mds.min = 1e-3, nsimEIM = 500, cutoff.prob = 0.999,
```

```
    eps.trig = 1e-7, max.support = 4000, max.chunk.MB = 30,
    lprob = "logitlink", lsize = "loglink", imethod = 1, iprob = NULL,
    iprobs.y = NULL, gprobs.y = ppoints(6), isize = NULL,
    gsize.mux = exp(c(-30, -20, -15, -10, -6:3)), imunb = NULL)
polyaR(zero = "size", type.fitted = c("mean", "prob"),
    mds.min = 1e-3, nsimEIM = 500, cutoff.prob = 0.999,
    eps.trig = 1e-7, max.support = 4000, max.chunk.MB = 30,
    lsize = "loglink", lprob = "logitlink", imethod = 1, iprob = NULL,
    iprobs.y = NULL, gprobs.y = ppoints(6), isize = NULL,
    gsize.mux = exp(c(-30, -20, -15, -10, -6:3)), imunb = NULL)
```


## Arguments

zero $\quad$ Can be an integer-valued vector, and if so, then it is usually assigned -2 or 2. Specifies which of the two linear/additive predictors are modelled as an intercept only. By default, the $k$ parameter (after lsize is applied) is modelled as a single unknown number that is estimated. It can be modelled as a function of the explanatory variables by setting zero = NULL; this has been called a NBH model by Hilbe (2011). A negative value means that the value is recycled, so setting -2 means all $k$ are intercept-only. See CommonVGAMffArguments for more information.
lmu, lsize, lprob
Link functions applied to the $\mu, k$ and $p$ parameters. See Links for more choices. Note that the $\mu, k$ and $p$ parameters are the mu , size and prob arguments of rnbinom respectively. Common alternatives for lsize are negloglink and reciprocallink, and logloglink (if $k>1$ ).
imu, imunb, isize, iprob
Optional initial values for the mean and $k$ and $p$. For $k$, if failure to converge occurs then try different values (and/or use imethod). For a $S$-column response, isize can be of length $S$. A value NULL means an initial value for each response is computed internally using a gridsearch based on gsize.mux. The last argument is ignored if used within cqo; see the iKvector argument of qrrvglm. control instead. In the future isize and iprob might be depreciated.
nsimEIM This argument is used for computing the diagonal element of the expected information matrix (EIM) corresponding to $k$ based on the simulated Fisher scoring (SFS) algorithm. See CommonVGAMffArguments for more information and the notes below. SFS is one of two algorithms for computing the EIM elements (so that both algorithms may be used on a given data set). SFS is faster than the exact method when Qmax is large.
cutoff.prob Fed into the $p$ argument of qnbinom in order to obtain an upper limit for the approximate support of the distribution, called Qmax, say. Similarly, the value $1-p$ is fed into the $p$ argument of qnbinom in order to obtain a lower limit for the approximate support of the distribution, called Qmin, say. Hence the approximate support is Qmin:Qmax. This argument should be a numeric and close to 1 but never exactly 1 . Used to specify how many terms of the infinite series for computing the second diagonal element of the EIM are actually used. The closer this argument is to 1 , the more accurate the standard errors of the regression coefficients will be. If this argument is too small, convergence will take
longer.
max. chunk. MB, max. support
max. support is used to describe the eligibility of individual observations to have their EIM computed by the exact method. Here, we are concerned about computing the EIM wrt $k$. The exact method algorithm operates separately on each response variable, and it constructs a large matrix provided that the number of columns is less than max. support. If so, then the computations are done in chunks, so that no more than about max. chunk. MB megabytes of memory is used at a time (actually, it is proportional to this amount). Regarding eligibility of this algorithm, each observation must have the length of the vector, starting from the 1-cutoff. prob quantile and finishing up at the cutoff. prob quantile, less than max. support (as its approximate support). If you have abundant memory then you might try setting max. chunk. MB = Inf, but then the computations might take a very long time. Setting max. chunk. $\mathrm{MB}=0$ or max. support $=0$ will force the EIM to be computed using the SFS algorithm only (this used to be the default method for all the observations). When the fitted values of the model are large and $k$ is small, the computation of the EIM will be costly with respect to time and memory if the exact method is used. Hence the argument max. support limits the cost in terms of time. For intercept-only models max. support is multiplied by a number (such as 10) because only one inner product needs be computed. Note: max. support is an upper bound and limits the number of terms dictated by the eps.trig argument.
mds.min Numeric. Minimum value of the NBD mean divided by size parameter. The closer this ratio is to 0 , the closer the distribution is to a Poisson. Iterations will stop when an estimate of $k$ is so large, relative to the mean, than it is below this threshold (this is treated as a boundary of the parameter space).
eps.trig Numeric. A small positive value used in the computation of the EIMs. It focusses on the denominator of the terms of a series. Each term in the series (that is used to approximate an infinite series) has a value greater than size / sqrt(eps.trig), thus very small terms are ignored. It's a good idea to set a smaller value that will result in more accuracy, but it will require a greater computing time (when $k$ is close to 0 ). And adjustment to max. support may be needed. In particular, the quantity computed by special means is $\psi^{\prime}(k)-$ $E\left[\psi^{\prime}(Y+k)\right]$, which is the difference between two trigamma. functions. It is part of the calculation of the EIM with respect to the size parameter.
gsize.mux Similar to gsigma in CommonVGAMffArguments. However, this grid is multiplied by the initial estimates of the NBD mean parameter. That is, it is on a relative scale rather than on an absolute scale. If the counts are very large in value then convergence fail might occur; if so, then try a smaller value such as gsize.mux $=\exp (-40)$.
type.fitted, percentiles
See CommonVGAMffArguments for more information.
deviance.arg Logical. If TRUE, the deviance is computed after convergence. It only works in the NB-2 model. It is also necessary to set criterion = "coefficients" or half. step $=$ FALSE since one cannot use that criterion properly for the minimization within the IRLS algorithm. It should be set TRUE when used with cqo under the fast algorithm.
imethod An integer with value 1 or 2 etc. which specifies the initialization method for the $\mu$ parameter. If failure to converge occurs try another value and/or else specify a value for iprobs.y and/or else specify a value for isize.
parallel See CommonVGAMffArguments for more information. Setting parallel = TRUE is useful in order to get something similar to quasipoisson or what is known as NB-1. If parallel $=$ TRUE then the parallelism constraint does not apply to any intercept term. You should set zero $=$ NULL too if parallel $=$ TRUE to avoid a conflict.
gprobs.y A vector representing a grid; passed into the probs argument of quantile when imethod $=1$ to obtain an initial value for the mean of each response. Is overwritten by any value of iprobs.y.
iprobs.y Passed into the probs argument of quantile when imethod $=1$ to obtain an initial value for the mean of each response. Overwrites any value of gprobs.y. This argument might be deleted in the future.

## Details

The negative binomial distribution (NBD) can be motivated in several ways, e.g., as a Poisson distribution with a mean that is gamma distributed. There are several common parametrizations of the NBD. The one used by negbinomial() uses the mean $\mu$ and an index parameter $k$, both which are positive. Specifically, the density of a random variable $Y$ is

$$
f(y ; \mu, k)=\binom{y+k-1}{y}\left(\frac{\mu}{\mu+k}\right)^{y}\left(\frac{k}{k+\mu}\right)^{k}
$$

where $y=0,1,2, \ldots$, and $\mu>0$ and $k>0$. Note that the dispersion parameter is $1 / k$, so that as $k$ approaches infinity the NBD approaches a Poisson distribution. The response has variance $\operatorname{Var}(Y)=\mu+\mu^{2} / k$. When fitted, the fitted.values slot of the object contains the estimated value of the $\mu$ parameter, i.e., of the mean $E(Y)$. It is common for some to use $\alpha=1 / k$ as the ancillary or heterogeneity parameter; so common alternatives for lsize are negloglink and reciprocallink.

For polya the density is

$$
f(y ; p, k)=\binom{y+k-1}{y}(1-p)^{y} p^{k}
$$

where $y=0,1,2, \ldots$, and $k>0$ and $0<p<1$.
Family function polyaR() is the same as polya() except the order of the two parameters are switched. The reason is that polyaR() tries to match with rnbinom closely in terms of the argument order, etc. Should the probability parameter be of primary interest, probably, users will prefer using polya() rather than polyaR(). Possibly polyaR() will be decommissioned one day.
The NBD can be coerced into the classical GLM framework with one of the parameters being of interest and the other treated as a nuisance/scale parameter (this is implemented in the MASS library). The VGAM family function negbinomial() treats both parameters on the same footing, and estimates them both by full maximum likelihood estimation.

The parameters $\mu$ and $k$ are independent (diagonal EIM), and the confidence region for $k$ is extremely skewed so that its standard error is often of no practical use. The parameter $1 / k$ has been used as a measure of aggregation. For the NB-C the EIM is not diagonal.

These VGAM family functions handle multiple responses, so that a response matrix can be inputted. The number of columns is the number of species, say, and setting zero $=-2$ means that all species have a $k$ equalling a (different) intercept only.
Conlisk, et al. (2007) show that fitting the NBD to presence-absence data will result in identifiability problems. However, the model is identifiable if the response values include 0,1 and 2 .

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Warning

Poisson regression corresponds to $k$ equalling infinity. If the data is Poisson or close to Poisson, numerical problems may occur. Some corrective measures are taken, e.g., $k$ is effectively capped (relative to the mean) during estimation to some large value and a warning is issued. And setting stepsize $=0.5$ for half stepping is probably a good idea too when the data is extreme.

The NBD is a strictly unimodal distribution. Any data set that does not exhibit a mode (somewhere in the middle) makes the estimation problem difficult. Set trace $=$ TRUE to monitor convergence.
These functions are fragile; the maximum likelihood estimate of the index parameter is fraught (see Lawless, 1987). Other alternatives to negbinomial are to fit a NB-1 or RR-NB (aka NB-P) model; see Yee (2014). Also available are the NB-C, NB-H and NB-G. Assigning values to the isize argument may lead to a local solution, and smaller values are preferred over large values when using this argument.
If one wants to force SFS to be used on all observations, then set max. support $=0$ or max. chunk. MB $=0$. If one wants to force the exact method to be used for all observations, then set max. support $=$ Inf. If the computer has much memory, then trying max. chunk. MB = Inf and max. support = Inf may provide a small speed increase. If SFS is used at all, then the @weights slot of the fitted object will be a matrix; otherwise that slot will be a $0 \times 0$ matrix.

Yet to do: write a family function which uses the methods of moments estimator for $k$.

## Note

These 3 functions implement 2 common parameterizations of the negative binomial (NB). Some people called the NB with integer $k$ the Pascal distribution, whereas if $k$ is real then this is the Polya distribution. I don't. The one matching the details of rnbinom in terms of $p$ and $k$ is polya().

For polya() the code may fail when $p$ is close to 0 or 1 . It is not yet compatible with cqo or cao.
Suppose the response is called ymat. For negbinomial() the diagonal element of the expected information matrix (EIM) for parameter $k$ involves an infinite series; consequently SFS (see nsimEIM) is used as the backup algorithm only. SFS should be better if max (ymat) is large, e.g., max (ymat) > 1000 , or if there are any outliers in ymat. The default algorithm involves a finite series approximation to the support 0 : Inf; the arguments max.memory, min. size and cutoff. prob are pertinent.

Regardless of the algorithm used, convergence problems may occur, especially when the response has large outliers or is large in magnitude. If convergence failure occurs, try using arguments (in recommended decreasing order) max. support, nsimEIM, cutoff.prob, iprobs.y, imethod, isize, zero, max. chunk.MB.

The function negbinomial can be used by the fast algorithm in cqo, however, setting eq. tolerances $=$ TRUE and I. tolerances $=$ FALSE is recommended.
In the first example below (Bliss and Fisher, 1953), from each of 6 McIntosh apple trees in an orchard that had been sprayed, 25 leaves were randomly selected. On each of the leaves, the number of adult female European red mites were counted.

There are two special uses of negbinomial for handling count data. Firstly, when used by rrvglm this results in a continuum of models in between and inclusive of quasi-Poisson and negative binomial regression. This is known as a reduced-rank negative binomial model ( $R R-N B$ ). It fits a negative binomial log-linear regression with variance function $\operatorname{Var}(Y)=\mu+\delta_{1} \mu^{\delta_{2}}$ where $\delta_{1}$ and $\delta_{2}$ are parameters to be estimated by MLE. Confidence intervals are available for $\delta_{2}$, therefore it can be decided upon whether the data are quasi-Poisson or negative binomial, if any.
Secondly, the use of negbinomial with parallel = TRUE inside vglm can result in a model similar to quasipoisson. This is named the NB-1 model. The dispersion parameter is estimated by MLE whereas glm uses the method of moments. In particular, it fits a negative binomial log-linear regression with variance function $\operatorname{Var}(Y)=\phi_{0} \mu$ where $\phi_{0}$ is a parameter to be estimated by MLE. Confidence intervals are available for $\phi_{0}$.

## Author(s)

Thomas W. Yee, and with a lot of help by Victor Miranda to get it going with nbcanlink (NB-C).

## References

Lawless, J. F. (1987). Negative binomial and mixed Poisson regression. The Canadian Journal of Statistics 15, 209-225.

Hilbe, J. M. (2011). Negative Binomial Regression, 2nd Edition. Cambridge: Cambridge University Press.
Bliss, C. and Fisher, R. A. (1953). Fitting the negative binomial distribution to biological data. Biometrics 9, 174-200.
Conlisk, E. and Conlisk, J. and Harte, J. (2007). The impossibility of estimating a negative binomial clustering parameter from presence-absence data: A comment on He and Gaston. The American Naturalist 170, 651-654.
Yee, T. W. (2014). Reduced-rank vector generalized linear models with two linear predictors. Computational Statistics and Data Analysis, 71, 889-902.
Yee, T. W. (2020). The VGAM package for negative binomial regression. Australian $\backslash \& ~ N e w$ Zealand Journal of Statistics, 61, in press.

## See Also

quasipoisson, poissonff, zinegbinomial, negbinomial.size (e.g., NB-G), nbcanlink (NBC), posnegbinomial, inv.binomial, NegBinomial, nbordlink, rrvglm, cao, cqo, CommonVGAMffArguments, simulate.vlm, ppoints,

## Examples

```
# Example 1: apple tree data (Bliss and Fisher, 1953)
appletree <- data.frame(y = 0:7, w = c(70, 38, 17, 10, 9, 3, 2, 1))
```

```
fit <- vglm(y ~ 1, negbinomial(deviance = TRUE), data = appletree,
    weights = w, crit = "coef") # Obtain the deviance
fit <- vglm(y ~ 1, negbinomial(deviance = TRUE), data = appletree,
            weights = w, half.step = FALSE) # Alternative method
summary(fit)
coef(fit, matrix = TRUE)
Coef(fit) # For intercept-only models
deviance(fit) # NB2 only; needs 'crit = "coef"' & 'deviance = TRUE' above
# Example 2: simulated data with multiple responses
## Not run:
ndata <- data.frame(x2 = runif(nn <- 200))
ndata <- transform(ndata, y1 = rnbinom(nn, mu = exp(3+x2), size = exp(1)),
    y2 = rnbinom(nn, mu = exp(2-x2), size = exp(0)))
fit1 <- vglm(cbind(y1, y2) ~ x2, negbinomial, data = ndata, trace = TRUE)
coef(fit1, matrix = TRUE)
## End(Not run)
# Example 3: large counts implies SFS is used
## Not run:
ndata <- transform(ndata, y3 = rnbinom(nn, mu = exp(10+x2), size = exp(1)))
with(ndata, range(y3)) # Large counts
fit2 <- vglm(y3 ~ x2, negbinomial, data = ndata, trace = TRUE)
coef(fit2, matrix = TRUE)
head(fit2@weights) # Non-empty; SFS was used
## End(Not run)
# Example 4: a NB-1 to estimate a negative binomial with Var(Y) = phi0 * mu
nn <- 200 # Number of observations
phi0 <- 10 # Specify this; should be greater than unity
delta0 <- 1 / (phi0 - 1)
mydata <- data.frame(x2 = runif(nn), x3 = runif(nn))
mydata <- transform(mydata, mu = exp(2 + 3 * x2 + 0 * x3))
mydata <- transform(mydata, y3 = rnbinom(nn, mu = mu, size = delta0 * mu))
## Not run:
plot(y3 ~ x2, data = mydata, pch = "+", col = "blue",
    main = paste("Var(Y) = ", phi0, " * mu", sep = ""), las = 1)
## End(Not run)
nb1 <- vglm(y3 ~ x2 + x3, negbinomial(parallel = TRUE, zero = NULL),
    data = mydata, trace = TRUE)
# Extracting out some quantities:
cnb1 <- coef(nb1, matrix = TRUE)
mydiff <- (cnb1["(Intercept)", "loglink(size)"] -
    cnb1["(Intercept)", "loglink(mu)"])
delta0.hat <- exp(mydiff)
(phi.hat <- 1 + 1 / delta0.hat) # MLE of phi
summary(nb1)
# Obtain a 95 percent confidence interval for phi0:
myvec <- rbind(-1, 1, 0, 0)
(se.mydiff <- sqrt(t(myvec) %*% vcov(nb1) %*% myvec))
ci.mydiff <- mydiff + c(-1.96, 1.96) * c(se.mydiff)
```

```
    ci.delta0 <- ci.exp.mydiff <- exp(ci.mydiff)
    (ci.phi0 <- 1 + 1 / rev(ci.delta0)) # The 95 percent conf. interval for phi0
    Confint.nb1(nb1) # Quick way to get it
    summary(glm(y3 ~ x2 + x3, quasipoisson, mydata))$disper # cf. moment estimator
```

negbinomial.size Negative Binomial Distribution Family Function With Known Size

## Description

Maximum likelihood estimation of the mean parameter of a negative binomial distribution with known size parameter.

## Usage

```
negbinomial.size(size = Inf, lmu = "loglink", imu = NULL,
    iprobs.y = 0.35, imethod = 1,
    ishrinkage = 0.95, zero = NULL)
```


## Arguments

size Numeric, positive. Same as argument size of rnbinom. If the response is a matrix then this is recycled to a matrix of the same dimension, by row (matrix with byrow = TRUE).
lmu, imu Same as negbinomial.
iprobs.y, imethod
Same as negbinomial.
zero, ishrinkage
Same as negbinomial.

## Details

This VGAM family function estimates only the mean parameter of the negative binomial distribution. See negbinomial for general information. Setting size $=1$ gives what might be called the NB-G (geometric model; see Hilbe (2011)). The default, size = Inf, corresponds to the Poisson distribution.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Note

If lmu = "nbcanlink" in negbinomial.size() then the size argument here should be assigned and these values are recycled.

## Author(s)

Thomas W. Yee

## References

Hilbe, J. M. (2011). Negative Binomial Regression, 2nd Edition. Cambridge: Cambridge University Press.

Yee, T. W. (2014). Reduced-rank vector generalized linear models with two linear predictors. Computational Statistics and Data Analysis, 71, 889-902.

## See Also

negbinomial, nbcanlink (NB-C model), poissonff, rnbinom, simulate.vlm.

## Examples

```
# Simulated data with various multiple responses
size1 <- exp(1); size2 <- exp(2); size3 <- exp(0); size4 <- Inf
ndata <- data.frame(x2 = runif(nn <- 1000))
ndata <- transform(ndata, eta1 = -1 - 2 * x2, # eta1 must be negative
    size1 = size1)
ndata <- transform(ndata,
    mu1 = nbcanlink(eta1, size = size1, inv = TRUE))
ndata <- transform(ndata,
    y1 = rnbinom(nn, mu = mu1, size = size1), # NB-C
    y2 = rnbinom(nn, mu = exp(2 - x2), size = size2),
    y3 = rnbinom(nn, mu = exp(3 + x2), size = size3), # NB-G
    y4 = rpois(nn, lambda = exp(1 + x2)))
# Also known as NB-C with size known (Hilbe, 2011)
fit1 <- vglm(y1 ~ x2, negbinomial.size(size = size1, lmu = "nbcanlink"),
    data = ndata, trace = TRUE)
coef(fit1, matrix = TRUE)
head(fit1@misc$size) # size saved here
fit2 <- vglm(cbind(y2, y3, y4) ~ x2, data = ndata, trace = TRUE,
    negbinomial.size(size = c(size2, size3, size4)))
coef(fit2, matrix = TRUE)
head(fit2@misc$size) # size saved here
```

normal.vcm Univariate Normal Distribution as a Varying-Coefficient Model

## Description

Maximum likelihood estimation of all the coefficients of a LM where each of the usual regression coefficients is modelled with other explanatory variables via parameter link functions. Thus this is a basic varying-coefficient model.

```
Usage
    normal.vcm(link.list = list("(Default)" = "identitylink"),
                earg.list = list("(Default)" = list()),
        lsd = "loglink", lvar = "loglink",
        esd = list(), evar = list(),
        var.arg = FALSE, imethod = 1,
        icoefficients = NULL, isd = NULL, zero = "sd",
        sd.inflation.factor = 2.5)
```


## Arguments

link.list, earg.list
Link functions and extra arguments applied to the coefficients of the LM, excluding the standard deviation/variance. See CommonVGAMfffarguments for more information. The default is for an identity link to be applied to each of the regression coefficients.
lsd, esd, lvar, evar
Link function and extra argument applied to the standard deviation/variance. See CommonVGAMffArguments for more information. Same as uninormal.
icoefficients Optional initial values for the coefficients. Recycled to length $M-1$ (does not include the standard deviation/variance). Try using this argument if there is a link function that is not programmed explicitly to handle range restrictions in the initialize slot.
var.arg, imethod, isd
Same as, or similar to, uninormal.
zero See CommonVGAMffArguments for more information. The default applies to the last one, viz. the standard deviation/variance parameter.
sd.inflation.factor
Numeric, should be greater than 1. The initial value of the standard deviation is multiplied by this, unless isd is inputted. Experience has shown that it is safer to start off with a larger value rather than a smaller one.

## Details

This function allows all the usual LM regression coefficients to be modelled as functions of other explanatory variables via parameter link functions. For example, we may want some of them to be positive. Or we may want a subset of them to be positive and add to unity. So a class of such models have been named varying-coefficient models (VCMs).
The usual linear model is specified through argument form2. As with all other VGAM family functions, the linear/additive predictors are specified through argument formula.
The multilogitlink link allows a subset of the coefficients to be positive and add to unity. Either none or more than one call to multilogitlink is allowed. The last variable will be used as the baseline/reference group, and therefore excluded from the estimation.
By default, the log of the standard deviation is the last linear/additive predictor. It is recommended that this parameter be estimated as intercept-only, for numerical stability.
Technically, the Fisher information matrix is of unit-rank for all but the last parameter (the standard deviation/variance). Hence an approximation is used that pools over all the observations.

This VGAM family function cannot handle multiple responses. Also, this function will probably not have the full capabilities of the class of varying-coefficient models as described by Hastie and Tibshirani (1993). However, it should be able to manage some simple models, especially involving the following links: identitylink, loglink, logofflink, logloglink, logitlink, probitlink, cauchitlink. clogloglink, rhobitlink, fisherzlink.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

This VGAM family function is fragile. One should monitor convergence, and possibly enter initial values especially when there are non-identity-link functions. If the initial value of the standard deviation/variance is too small then numerical problems may occur. One trick is to fit an interceptonly only model and feed its predict() output into argument etastart of a more complicated model. The use of the zero argument is recommended in order to keep models as simple as possible.

## Note

The standard deviation/variance parameter is best modelled as intercept-only.
Yet to do: allow an argument such as parallel that enables many of the coefficients to be equal. Fix a bug: Coef () does not work for intercept-only models.

## Author(s)

T. W. Yee

## References

Hastie, T. and Tibshirani, R. (1993). Varying-coefficient models. J. Roy. Statist. Soc. Ser. B, 55, 757-796.

## See Also

uninormal, lm.

## Examples

```
ndata <- data.frame(x2 = runif(nn <- 2000))
# Note that coeff1 + coeff2 + coeff5 == 1. So try "multilogitlink".
myoffset <- 10
ndata <- transform(ndata,
    coeff1 = 0.25, # "multilogitlink"
    coeff2 = 0.25, # "multilogitlink"
    coeff3 = exp(-0.5), # "loglink"
# "logofflink" link:
    coeff4 = logofflink(+0.5, offset = myoffset, inverse = TRUE),
    coeff5 = 0.50, # "multilogitlink"
    coeff6 = 1.00, # "identitylink"
```

```
    v2 = runif(nn),
    v3 = runif(nn),
    v4 = runif(nn),
    v5 = rnorm(nn),
    v6 = rnorm(nn))
ndata <- transform(ndata,
    Coeff1 = 0.25 - 0 * x2,
    Coeff2 = 0.25-0 * x2,
    Coeff3 = logitlink(-0.5 - 1 * x2, inverse = TRUE),
    Coeff4 = logloglink( 0.5 - 1 * x2, inverse = TRUE),
    Coeff5 = 0.50-0 * x2,
    Coeff6 = 1.00 + 1* x2)
ndata <- transform(ndata,
        y1 = coeff1 * 1 +
                        coeff2 * v2 +
                        coeff3 * v3 +
                        coeff4 * v4 +
                coeff5 * v5 +
                coeff6 * v6 + rnorm(nn, sd = exp(0)),
        y2 = Coeff1 * 1 +
                        Coeff2 * v2 +
                    Coeff3 * v3 +
                        Coeff4 * v4 +
                        Coeff5 * v5 +
                Coeff6 * v6 + rnorm(nn, sd = exp(0)))
# An intercept-only model
fit1 <- vglm(y1 ~ 1,
    form2 = ~ 1 + v2 + v3 + v4 + v5 + v6,
    normal.vcm(link.list = list("(Intercept)" = "multilogitlink",
                        "v2" = "multilogitlink",
                            "v3" = "loglink",
                            "v4" = "logofflink",
                            "(Default)" = "identitylink",
                            "v5" = "multilogitlink"),
            earg.list = list("(Intercept)" = list(),
                        "v2" = list(),
                            "v4" = list(offset = myoffset),
                            "v3" = list(),
                        "(Default)" = list(),
                        "v5" = list()),
            zero = c(1:2, 6)),
        data = ndata, trace = TRUE)
coef(fit1, matrix = TRUE)
summary(fit1)
# This works only for intercept-only models:
multilogitlink(rbind(coef(fit1, matrix = TRUE)[1, c(1, 2)]), inverse = TRUE)
# A model with covariate x2 for the regression coefficients
fit2 <- vglm(y2 ~ 1 + x2,
    form2 = ~ 1 + v2 + v3 + v4 + v5 + v6,
    normal.vcm(link.list = list("(Intercept)" = "multilogitlink",
                        "v2" = "multilogitlink",
```

```
    "v3" = "logitlink",
    "v4" = "logloglink",
    "(Default)" = "identitylink",
    "v5" = "multilogitlink"),
        earg.list = list("(Intercept)" = list(),
    "v2" = list(),
    "v3" = list(),
    "v4" = list(),
    "(Default)" = list(),
    "v5" = list()),
        zero = c(1:2, 6)),
        data = ndata, trace = TRUE)
    coef(fit2, matrix = TRUE)
    summary(fit2)
```

    nparam. vlm Number of Parameters
    
## Description

Returns the number of parameters in a fitted model object.

## Usage

```
            nparam(object, ...)
            nparam.vlm(object, dpar = TRUE, ...)
            nparam.vgam(object, dpar = TRUE, linear.only = FALSE, ...)
    nparam.rrvglm(object, dpar = TRUE, ...)
nparam.qrrvglm(object, dpar = TRUE, ...)
    nparam.rrvgam(object, dpar = TRUE, ...)
```


## Arguments

object Some VGAM object, for example, having class vglmff-class.
... Other possible arguments fed into the function.
dpar Logical, include any (estimated) dispersion parameters as a parameter?
linear.only Logical, include only the number of linear (parametric) parameters?

## Details

The code was copied from the AIC() methods functions.

## Value

Returns a numeric value with the corresponding number of parameters. For vgam objects, this may be real rather than integer, because the nonlinear degrees of freedom is real-valued.

## Warning

This code has not been double-checked.

## Author(s)

T. W. Yee.

## See Also

VGLMs are described in vglm-class; VGAMs are described in vgam-class; RR-VGLMs are described in rrvglm-class; AICvlm.

## Examples

```
pneumo <- transform(pneumo, let = log(exposure.time))
(fit1 <- vglm(cbind(normal, mild, severe) ~ let, propodds, data = pneumo))
coef(fit1)
coef(fit1, matrix = TRUE)
nparam(fit1)
(fit2 <- vglm(hits ~ 1, poissonff, weights = ofreq, data = V1))
coef(fit2)
coef(fit2, matrix = TRUE)
nparam(fit2)
nparam(fit2, dpar = FALSE)
```

Oalog One-Altered Logarithmic Distribution

## Description

Density, distribution function, quantile function and random generation for the one-altered logarithmic distribution with parameter pobs1.

## Usage

```
doalog(x, shape, pobs1 = 0, log = FALSE)
poalog(q, shape, pobs1 = 0)
qoalog(p, shape, pobs1 = 0)
roalog(n, shape, pobs1 = 0)
```


## Arguments

| $\mathrm{x}, \mathrm{q}, \mathrm{n}, \mathrm{p}$ | Same Unif. |
| :--- | :--- |
| shape, log | Same as Otlog). |
| pobs1 | Probability of (an observed) one, called pobs1. The default value of pobs1 $=0$ <br> corresponds to the response having a 1-truncated logarithmic distribution. |

## Details

The probability function of $Y$ is 1 with probability pobs1, else a 1-truncated logarithmic(shape) distribution.

## Value

doalog gives the density and poalog gives the distribution function, qoalog gives the quantile function, and roalog generates random deviates.

## Note

The argument pobs 1 is recycled to the required length, and must have values which lie in the interval $[0,1]$.

## Author(s)

T. W. Yee

## See Also

Gaitlog, oalog, oilog, Otlog.

## Examples

```
shape <- 0.75; pobs1 <- 0.10; x <- (-1):7
doalog(x, shape = shape, pobs1 = pobs1)
table(roalog(100, shape = shape, pobs1 = pobs1))
## Not run: x <- 0:10
barplot(rbind(doalog(x, shape = shape, pobs1 = pobs1),
            dlog(x, shape = shape)),
        beside = TRUE, col = c("blue", "orange"), cex.main = 0.7, las = 1,
        ylab = "Probability", names.arg = as.character(x),
        main = paste("OAL(shape = ", shape, ", pobs1 = ", pobs1,
            ") [blue] vs", " Logarithmic(shape = ", shape,
            ") [orange] densities", sep = ""))
## End(Not run)
```

oalog

One-Altered Logarithmic Distribution

## Description

Fits a one-altered logarithmic distribution based on a conditional model involving a Bernoulli distribution and a 1-truncated logarithmic distribution.

## Usage

oalog(lpobs1 = "logitlink", lshape = "logitlink", type.fitted = c("mean", "shape", "pobs1", "onempobs1"), ipobs1 = NULL, gshape = ppoints(8), zero = NULL)

## Arguments

lpobs1 Link function for the parameter $p_{1}$ or $\phi$, called pobs1 or phi here. See Links for more choices.
lshape See logff for details.
gshape, type.fitted
See CommonVGAMffArguments and fittedvlm for information.
ipobs1, zero See CommonVGAMffArguments for information.

## Details

The response $Y$ is one with probability $p_{1}$, or $Y$ has a 1-truncated logarithmic distribution with probability $1-p_{1}$. Thus $0<p_{1}<1$, which is modelled as a function of the covariates. The one-altered logarithmic distribution differs from the one-inflated logarithmic distribution in that the former has ones coming from one source, whereas the latter has ones coming from the logarithmic distribution too. The one-inflated logarithmic distribution is implemented in the VGAM package. Some people call the one-altered logarithmic a hurdle model.
The input can be a matrix (multiple responses). By default, the two linear/additive predictors of oalog are $(\operatorname{logit}(\phi), \operatorname{logit}(s))^{T}$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.
The fitted.values slot of the fitted object, which should be extracted by the generic function fitted, returns the mean $\mu$ (default) which is given by

$$
\mu=\phi+(1-\phi) A
$$

where $A$ is the mean of the one-truncated logarithmic distribution. If type.fitted = "pobs1" then $p_{1}$ is returned.

## Note

This family function effectively combines binomialff and otlog into one family function.

## Author(s)

T. W. Yee

## See Also

Gaitlog, Oalog, logff, oilog, CommonVGAMffArguments, simulate.vlm.

## Examples

```
odata <- data.frame(x2 = runif(nn <- 1000))
odata <- transform(odata, pobs1 = logitlink(-1 + 2*x2, inverse = TRUE),
    shape = logitlink(-2 + 3*x2, inverse = TRUE))
odata <- transform(odata, y1 = roalog(nn, shape = shape, pobs1 = pobs1),
    y2 = roalog(nn, shape = shape, pobs1 = pobs1))
with(odata, table(y1))
ofit <- vglm(cbind(y1, y2) ~ x2, oalog, data = odata, trace = TRUE)
coef(ofit, matrix = TRUE)
head(fitted(ofit))
head(predict(ofit))
summary(ofit)
```


## Description

Density, distribution function, quantile function and random generation for the one-altered positivePoisson distribution with parameter pobs1.

## Usage

doapospois(x, lambda, pobs1 $=0, \log =$ FALSE)
poapospois(q, lambda, pobs1 = 0)
qoapospois(p, lambda, pobs1 = 0)
roapospois(n, lambda, pobs1 = 0)

## Arguments

$\mathrm{x}, \mathrm{q}, \mathrm{n}, \mathrm{p} \quad$ Same Unif.
lambda, log Same as Otpospois).
pobs1 Probability of (an observed) one, called pobs1. The default value of pobs1 = 0 corresponds to the response having a 1 -truncated positive-Poisson distribution.

## Details

The probability function of $Y$ is 1 with probability pobs1, else a 1-truncated positive-Poisson(lambda) distribution.

## Value

doapospois gives the density and poapospois gives the distribution function, qoapospois gives the quantile function, and roapospois generates random deviates.

## Note

The argument pobs 1 is recycled to the required length, and must have values which lie in the interval $[0,1]$.

## Author(s)

T. W. Yee

## See Also

oapospoisson, Oipospois, Otpospois.

## Examples

```
lambda <- 3; pobs1 <- 0.30; x <- (-1):7
doapospois(x, lambda = lambda, pobs1 = pobs1)
table(roapospois(100, lambda = lambda, pobs1 = pobs1))
## Not run: x <- 0:10
barplot(rbind(doapospois(x, lambda = lambda, pobs1 = pobs1),
            dpospois(x, lambda = lambda)),
    beside = TRUE, col = c("blue", "orange"), cex.main = 0.7, las = 1,
    ylab = "Probability", names.arg = as.character(x),
    main = paste("OAPP(lambda = ", lambda, ", pobs1 = ", pobs1,
            ") [blue] vs", " PosPoisson(lambda = ", lambda,
            ") [orange] densities", sep = ""))
## End(Not run)
```

oapospoisson One-Altered Positive-Poisson Distribution

## Description

Fits a one-altered positive-Poisson distribution based on a conditional model involving a Bernoulli distribution and a 1-truncated positive-Poisson distribution.

## Usage

oapospoisson(lpobs1 = "logitlink", llambda = "loglink", type.fitted = c("mean", "lambda", "pobs1", "onempobs1"), ipobs1 = NULL, zero = NULL)

## Arguments

lpobs1 Link function for the parameter $p_{1}$ or $\phi$, called pobs1 or phi here. See Links for more choices.
llambda See pospoisson for details.
type.fitted See CommonVGAMffarguments and fittedvlm for information.
ipobs1, zero See CommonVGAMffArguments for information.

## Details

The response $Y$ is one with probability $p_{1}$, or $Y$ has a 1-truncated positive-Poisson distribution with probability $1-p_{1}$. Thus $0<p_{1}<1$, which is modelled as a function of the covariates. The one-altered positive-Poisson distribution differs from the one-inflated positive-Poisson distribution in that the former has ones coming from one source, whereas the latter has ones coming from the positive-Poisson distribution too. The one-inflated positive-Poisson distribution is implemented in the VGAM package. Some people call the one-altered positive-Poisson a hurdle model.
The input can be a matrix (multiple responses). By default, the two linear/additive predictors of oapospoisson are $(\operatorname{logit}(\phi), \log (\lambda))^{T}$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.
The fitted.values slot of the fitted object, which should be extracted by the generic function fitted, returns the mean $\mu$ (default) which is given by

$$
\mu=\phi+(1-\phi) A
$$

where $A$ is the mean of the one-truncated positive-Poisson distribution. If type.fitted = "pobs 1 " then $p_{1}$ is returned.

## Note

This family function effectively combines binomialff and otpospoisson into one family function.

## Author(s)

T. W. Yee

## See Also

Oapospois, pospoisson, oipospoisson, CommonVGAMffArguments, simulate.vlm.

## Examples

```
odata <- data.frame(x2 = runif(nn <- 1000))
odata <- transform(odata, pobs1 = logitlink(-1 + 2*x2, inverse = TRUE),
    lambda = loglink( 1 + 1*x2, inverse = TRUE))
odata <- transform(odata, y1 = roapospois(nn, lambda = lambda, pobs1 = pobs1),
    y2 = roapospois(nn, lambda = lambda, pobs1 = pobs1))
with(odata, table(y1))
ofit <- vglm(cbind(y1, y2) ~ x2, oapospoisson, data = odata, trace = TRUE)
coef(ofit, matrix = TRUE)
head(fitted(ofit))
head(predict(ofit))
summary(ofit)
```

Oazeta One-Altered Logarithmic Distribution

## Description

Density, distribution function, quantile function and random generation for the one-altered zeta distribution with parameter pobs 1 .

## Usage

doazeta(x, shape, pobs1 = 0, log = FALSE)
poazeta(q, shape, pobs1 = 0)
qoazeta(p, shape, pobs1 = 0)
$\operatorname{roazeta}(\mathrm{n}$, shape, pobs1 = 0)

## Arguments

$x, q, n, p \quad$ Same Unif.
shape, log Same as Otzeta).
pobs1 Probability of (an observed) one, called pobs1. The default value of pobs1 = 0 corresponds to the response having a 1 -truncated zeta distribution.

## Details

The probability function of $Y$ is 1 with probability pobs1, else a 1-truncated zeta distribution.

## Value

doazeta gives the density and poazeta gives the distribution function, qoazeta gives the quantile function, and roazeta generates random deviates.

## Note

The argument pobs 1 is recycled to the required length, and must have values which lie in the interval $[0,1]$.

## Author(s)

T. W. Yee

## See Also

```
    oazeta, Oizeta, Otzeta, zeta.
```


## Examples

```
shape <- 1.1; pobs1 <- 0.10; x <- (-1):7
doazeta(x, shape = shape, pobs1 = pobs1)
table(roazeta(100, shape = shape, pobs1 = pobs1))
## Not run: x <- 0:10
barplot(rbind(doazeta(x, shape = shape, pobs1 = pobs1),
                dzeta(x, shape = shape)),
    beside = TRUE, col = c("blue", "orange"), cex.main = 0.7, las = 1,
    ylab = "Probability", names.arg = as.character(x),
    main = paste("OAZ(shape = ", shape, ", pobs1 = ", pobs1,
            ") [blue] vs", " zeta(shape = ", shape,
            ") [orange] densities", sep = ""))
## End(Not run)
```

oazeta
One-Altered Zeta Distribution

## Description

Fits a one-altered zeta distribution based on a conditional model involving a Bernoulli distribution and a 1-truncated zeta distribution.

## Usage

```
oazeta(lpobs1 = "logitlink", lshape = "loglink",
    type.fitted = c("mean", "shape", "pobs1", "onempobs1"),
    gshape = exp((-4:3)/4), ishape = NULL, ipobs1 = NULL, zero = NULL)
```


## Arguments

lpobs1 Link function for the parameter $p_{1}$ or $\phi$, called pobs1 or phi here. See Links for more choices.
lshape See zeta for details.
type.fitted See CommonVGAMffArguments and fittedvlm for information.
gshape, ishape, ipobs1, zero
See CommonVGAMffArguments for information.

## Details

The response $Y$ is one with probability $p_{1}$, or $Y$ has a 1-truncated zeta distribution with probability $1-p_{1}$. Thus $0<p_{1}<1$, which is modelled as a function of the covariates. The one-altered zeta distribution differs from the one-inflated zeta distribution in that the former has ones coming from one source, whereas the latter has ones coming from the zeta distribution too. The one-inflated zeta distribution is implemented in the VGAM package. Some people call the one-altered zeta a hurdle model.
The input can be a matrix (multiple responses). By default, the two linear/additive predictors of oazeta are $(\operatorname{logit}(\phi), \log (\text { shape }))^{T}$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

The fitted.values slot of the fitted object, which should be extracted by the generic function fitted, returns the mean $\mu$ (default) which is given by

$$
\mu=\phi+(1-\phi) A
$$

where $A$ is the mean of the one-truncated zeta distribution. If type.fitted $=$ "pobs 1 " then $p_{1}$ is returned.

## Note

This family function effectively combines binomialff and otzeta into one family function.

## Author(s)

T. W. Yee

## See Also

Oazeta, zetaff, oizeta, otzeta, CommonVGAMffArguments, simulate.vlm.

## Examples

```
odata <- data.frame(x2 = runif(nn <- 1000))
odata <- transform(odata, pobs1 = logitlink(-1 + 2*x2, inverse = TRUE),
    shape = loglink( 1 + 1*x2, inverse = TRUE))
odata <- transform(odata, y1 = roazeta(nn, shape = shape, pobs1 = pobs1),
    y2 = roazeta(nn, shape = shape, pobs1 = pobs1))
with(odata, table(y1))
ofit <- vglm(cbind(y1, y2) ~ x2, oazeta, data = odata, trace = TRUE)
coef(ofit, matrix = TRUE)
head(fitted(ofit))
head(predict(ofit))
summary(ofit)
```

Oilog

One-Inflated Logarithmic Distribution

## Description

Density, distribution function, quantile function and random generation for the one-inflated logarithmic distribution with parameter pstr1.

## Usage

```
doilog(x, shape, pstr1 = 0, log = FALSE)
poilog(q, shape, pstr1 = 0)
qoilog(p, shape, pstr1 = 0)
roilog(n, shape, pstr1 = 0)
```


## Arguments

$\mathrm{x}, \mathrm{q}, \mathrm{p}, \mathrm{n} \quad$ Same as Uniform.
shape $\quad$ Vector of parameters that lie in $(0,1)$.
pstr1 Probability of a structural one (i.e., ignoring the logarithmic distribution), called $\phi$. The default value of $\phi=0$ corresponds to the response having an ordinary logarithmic distribution.
log Same as Uniform.

## Details

The probability function of $Y$ is 1 with probability $\phi$, and $\operatorname{Logarithmic(prob)~with~probability~}$ $1-\phi$. Thus

$$
P(Y=1)=\phi+(1-\phi) P(W=1)
$$

where $W$ is distributed as a Logarithmic(shape) random variable. The VGAM family function oilog estimates $\phi$ by MLE.

## Value

doilog gives the density, poilog gives the distribution function, qoilog gives the quantile function, and roilog generates random deviates.

## Note

The argument pstr1 is recycled to the required length, and usually has values which lie in the interval $[0,1]$.
These functions actually allow for the zero-deflated logarithmic distribution. Here, pstr1 is also permitted to lie in the interval $[-\mathrm{dlog}(1$, shape $) /(1-\mathrm{d} \log (1$, shape $)), 0]$. The resulting probability of a unit count is less than the nominal logarithmic value, and the use of pstr1 to stand for the probability of a structural 1 loses its meaning.
When pstr1 equals $-\mathrm{d} \log (1$, shape $) /(1-\mathrm{d} \log (1$, shape $))$ this corresponds to the 1 -truncated logarithmic distribution.

## Author(s)

T. W. Yee

## See Also

Gaitlog, oilog, rlog, logff, Otlog.

## Examples

```
shape <- 0.5; pstr1 <- 0.3; x <- (-1):7
(ii <- doilog(x, shape, pstr1 = pstr1))
max(abs(poilog(1:200, shape) -
    cumsum(shape^(1:200) / (-(1:200) * log1p(-shape))))) # Should be 0
## Not run: x <- 0:10
par(mfrow = c(2, 1)) # One-Inflated logarithmic
barplot(rbind(doilog(x, shape, pstr1 = pstr1), dlog(x, shape)),
    beside = TRUE, col = c("blue", "orange"),
    main = paste("OILogff(", shape, ", pstr1 = ", pstr1, ") (blue) vs",
            " Logff(", shape, ") (orange)", sep = ""),
    names.arg = as.character(x))
deflat.limit <- -dlog(1, shape) / plog(1, shape, lower.tail = FALSE)
newpstr1 <- round(deflat.limit, 3) + 0.001 # Inside but near the boundary
barplot(rbind(doilog(x, shape, pstr1 = newpstr1),
            dlog(x, shape)),
            beside = TRUE, col = c("blue","orange"),
            main = paste("ODLogff(", shape, ", pstr1 = ", newpstr1, ") (blue) vs",
                        " Logff(", shape, ") (orange)", sep = ""),
            names.arg = as.character(x))
## End(Not run)
```


## oilog One-inflated Logarithmic Distribution Family Function

## Description

Fits a 1-inflated logarithmic distribution.

## Usage

oilog(lpstr1 = "logitlink", lshape = "logitlink",
type.fitted = c("mean", "shape", "pobs1", "pstr1", "onempstr1"), ishape $=$ NULL, gpstr1 = ppoints(8), gshape $=$ ppoints(8), zero = NULL)

## Arguments

lpstr1, lshape Link functions. For lpstr1: the same idea as zipoisson except it applies to a structural 1.
gpstr1, gshape, ishape
For initial values. See CommonVGAMffArguments for information.
type.fitted, zero
See CommonVGAMffArguments for information.

## Details

The 1-inflated logarithmic distribution is a mixture distribution of the logarithmic distribution with some probability of obtaining a (structural) 1 . Thus there are two sources for obtaining the value 1 . This distribution is written here in a way that retains a similar notation to the one-inflated positivePoisson, i.e., the probability $P[Y=1]$ involves another parameter $\phi$. See oipospoisson.
This family function can handle multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Author(s)

Thomas W. Yee

## See Also

Gaitlog, Oilog, logff, Oizeta.

## Examples

```
## Not run: odata <- data.frame(x2 = runif(nn <- 1000)) # Artificial data
odata <- transform(odata, pstr1 = logitlink( }-1+x2\mathrm{ , inverse = TRUE), shape = 0.5)
odata <- transform(odata, y1 = roilog(nn, shape, pstr1 = pstr1))
with(odata, table(y1))
fit1 <- vglm(y1 ~ x2, oilog(zero = "shape"), data = odata, trace = TRUE)
coef(fit1, matrix = TRUE)
## End(Not run)
```

Oiposbinom One-Inflated Positive Binomial Distribution

## Description

Density, distribution function, quantile function and random generation for the one-inflated positive binomial distribution with parameter pstr1.

## Usage

doiposbinom(x, size, prob, pstr1 = 0, log = FALSE)
poiposbinom(q, size, prob, pstr1 = 0)
qoiposbinom(p, size, prob, pstr1 = 0)
roiposbinom(n, size, prob, pstr1 = 0)

## Arguments

| $\mathrm{x}, \mathrm{p}, \mathrm{q}, \mathrm{n}$ | Same as Posbinom. |
| :--- | :--- |
| size, prob | Same as Posbinom. |
| pstr1 | Probability of a structural one (i.e., ignoring the positive binomial distribution), <br> called $\phi$. The default value of $\phi=0$ corresponds to the response having a pos- <br> itive binomial distribution. However, pstr1 can also be negative, in which case <br> it ceases its interpretation as a probability, and this is known as one-deflation. |
| log | Logical. Return the logarithm of the answer? |

## Details

The probability function of $Y$ is 1 with probability $\phi$, and PosBinomial(size, prob) with probability $1-\phi$. Thus

$$
P(Y=1)=\phi+(1-\phi) P(W=1)
$$

where $W$ is distributed as a positive $\operatorname{binomial}($ size, prob) random variable.

## Value

doiposbinom gives the density, poiposbinom gives the distribution function, qoiposbinom gives the quantile function, and roiposbinom generates random deviates.

## Note

The argument pstr1 is recycled to the required length, and usually has values which lie in the interval $[0,1]$.
These functions actually allow for the zero-deflated binomial distribution. Here, pstr1 is also permitted to lie in the interval $[-A, 0]$ for some positive quantity $A$. The resulting probability of a unit value is less than the nominal positive binomial value, and the use of pstr1 to stand for the probability of a structural 1 loses its meaning.
If pstr1 equals $A$ then this corresponds to the 0 - and 1 -truncated binomial distribution.

## Author(s)

T. W. Yee

## See Also

posbinomial, dbinom, binomialff.

## Examples

```
size <- 10; prob <- 0.2; pstr1 <- 0.4; x <- (-1):size
(ii <- doiposbinom(x, size, prob, pstr1 = pstr1))
table(roiposbinom(100, size, prob, pstr1 = pstr1))
round(doiposbinom(x , size, prob, pstr1 = pstr1) * 100) # Should be similar
## Not run: x <- 0:size
par(mfrow = c(2, 1)) # One-Inflated Positive Binomial
```

```
    barplot(rbind(doiposbinom(x, size, prob, pstr1 = pstr1),
        dposbinom(x, size, prob)),
    beside = TRUE, col = c("blue", "orange"),
    main = paste("OIPB(", size, ",", prob, ", pstr1 = ", pstr1, ") (blue) vs",
                            " PosBinomial(", size, ",", prob, ") (orange)", sep = ""),
    names.arg = as.character(x))
# Zero-deflated Pos Binomial
deflat.limit <- -dposbinom(1, size, prob) / (1 - dposbinom(1, size, prob))
deflat.limit <- size * prob / (1 + (size-1) * prob - 1 / (1-prob)^(size-1))
newpstr1 <- round(deflat.limit, 3) + 0.001 # A little from the boundary
barplot(rbind(doiposbinom(x, size, prob, pstr1 = newpstr1),
            dposbinom(x, size, prob)),
        beside = TRUE, col = c("blue","orange"),
        main = paste("ODPB(", size, ",", prob, ", pstr1 = ", newpstr1, ") (blue) vs",
            " PosBinomial(", size, ",", prob, ") (orange)", sep = ""),
        names.arg = as.character(x))
## End(Not run)
```

oiposbinomial One-Inflated Positive Binomial Distribution Family Function

## Description

Fits a one-inflated positive binomial distribution by maximum likelihood estimation.

## Usage

```
oiposbinomial(lpstr1 = "logitlink", lprob = "logitlink",
    type.fitted = c("mean", "prob", "pobs1", "pstr1", "onempstr1"),
    iprob = NULL, gpstr1 = ppoints(9), gprob = ppoints(9),
    multiple.responses = FALSE, zero = NULL)
```


## Arguments

lpstr1, lprob Link functions for the parameter $\phi$ and the positive binomial probability $\mu$ parameter. See Links for more choices. See CommonVGAMffArguments also. For the one-deflated model see below.
type.fitted See CommonVGAMffArguments and fittedvlm.
iprob, gpstr1, gprob
For initial values; see CommonVGAMffArguments.
multiple.responses
Logical. See binomialff and posbinomial.
zero See CommonVGAMffArguments for information.

## Details

These functions are based on

$$
P(Y=y)=\phi+(1-\phi) N \mu(1-\mu)^{N} /\left(1-(1-\mu)^{N}\right),
$$

for $y=1 / N$, and

$$
P(Y=y)=(1-\phi)\binom{N}{N y} \mu^{N y}(1-\mu)^{N(1-y)} /\left(1-(1-\mu)^{N}\right)
$$

for $y=2 / N, \ldots, 1$. That is, the response is a sample proportion out of $N$ trials, and the argument size in roiposbinom is $N$ here. Ideally $N>2$ is needed. The parameter $\phi$ is the probability of a structural one, and it satisfies $0<\phi<1$ (usually). The mean of $Y$ is $E(Y)=\phi+(1-\phi) \mu /(1-$ $\left.(1-\mu)^{N}\right)$ and these are returned as the default fitted values. By default, the two linear/additive predictors for oiposbinomial() are $(\operatorname{logit}(\phi), \operatorname{logit}(\mu))^{T}$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

The response variable should have one of the formats described by binomialff, e.g., a factor or two column matrix or a vector of sample proportions with the weights argument specifying the values of $N$.
To work well, one ideally needs large values of $N$ and $\mu$ much greater than 0 , i.e., the larger $N$ and $\mu$ are, the better. If $N=1$ then the model is unidentifiable since the number of parameters is excessive.
Estimated probabilities of a structural one and an observed one are returned, as in zipoisson.
The one-deflated positive binomial distribution might be fitted by setting lpstr1 = "identitylink", albeit, not entirely reliably. See zipoisson for information that can be applied here.

## Author(s)

T. W. Yee

## See Also

roiposbinom, posbinomial, binomialff, rbinom.

## Examples

```
size <- 10 # Number of trials; N in the notation above
nn <- 200
odata <- data.frame(pstr1 = logitlink( 0, inverse = TRUE), # 0.50
    mubin1 = logitlink(-1, inverse = TRUE), # Mean of usual binomial
    svec = rep(size, length = nn),
    x2 = runif(nn))
odata <- transform(odata,
```

```
    mubin2 = logitlink(-1 + x2, inverse = TRUE))
odata <- transform(odata,
            y1 = roiposbinom(nn, svec, pr = mubin1, pstr1 = pstr1),
            y2 = roiposbinom(nn, svec, pr = mubin2, pstr1 = pstr1))
with(odata, table(y1))
fit1 <- vglm(y1 / svec ~ 1, oiposbinomial, data = odata,
    weights = svec, trace = TRUE, crit = "coef")
fit2 <- vglm(y2 / svec ~ x2, oiposbinomial, data = odata,
    weights = svec, trace = TRUE)
coef(fit1, matrix = TRUE)
Coef(fit1) # Useful for intercept-only models
head(fitted(fit1, type = "pobs1")) # Estimate of P(Y = 1)
head(fitted(fit1))
with(odata, mean(y1)) # Compare this with fitted(fit1)
summary(fit1)
```


## Description

Density, distribution function, quantile function and random generation for the one-inflated positive Poisson distribution with parameter pstr1.

## Usage

doipospois(x, lambda, pstr1 = 0, log = FALSE)
poipospois(q, lambda, pstr1 = 0)
qoipospois(p, lambda, pstr1 = 0)
roipospois(n, lambda, pstr1 = 0)

## Arguments

$\mathrm{x}, \mathrm{p}, \mathrm{q}, \mathrm{n} \quad$ Same as Pospois.
lambda Vector of positive means.
pstr1 Probability of a structural one (i.e., ignoring the positive Poisson distribution), called $\phi$. The default value of $\phi=0$ corresponds to the response having a positive Poisson distribution.
log Logical. Return the logarithm of the answer?

## Details

The probability function of $Y$ is 1 with probability $\phi$, and $\operatorname{PosPoisson}(\lambda)$ with probability $1-\phi$. Thus

$$
P(Y=1)=\phi+(1-\phi) P(W=1)
$$

where $W$ is distributed as a positive Poisson $(\lambda)$ random variate.

## Value

doipospois gives the density, poipospois gives the distribution function, qoipospois gives the quantile function, and roipospois generates random deviates.

## Note

The argument pstr1 is recycled to the required length, and usually has values which lie in the interval $[0,1]$.
These functions actually allow for the zero-deflated Poisson distribution. Here, pstr1 is also permitted to lie in the interval [-lambda / (expm1 (lambda) -lambda), 0]. The resulting probability of a unit count is less than the nominal positive Poisson value, and the use of pstr1 to stand for the probability of a structural 1 loses its meaning.

When pstr1 equals - lambda / (expm1 (lambda) -lambda) this corresponds to the 0 - and 1-truncated Poisson distribution.

## Author(s)

T. W. Yee

## See Also

Pospois, oapospoisson, oipospoisson, otpospoisson, pospoisson, dpois, poissonff.

## Examples

```
lambda <- 3; pstr1 <- 0.2; x <- (-1):7
(ii <- doipospois(x, lambda, pstr1 = pstr1))
table(roipospois(100, lambda, pstr1 = pstr1))
round(doipospois(1:10, lambda, pstr1 = pstr1) * 100) # Should be similar
## Not run: x <- 0:10
par(mfrow = c(2, 1)) # One-Inflated Positive Poisson
barplot(rbind(doipospois(x, lambda, pstr1 = pstr1), dpospois(x, lambda)),
    beside = TRUE, col = c("blue", "orange"),
    main = paste("OIPP(", lambda, ", pstr1 = ", pstr1, ") (blue) vs",
            " PosPoisson(", lambda, ") (orange)", sep = ""),
    names.arg = as.character(x))
deflat.limit <- -lambda / (expm1(lambda) - lambda) # 0-deflated Pos Poisson
newpstr1 <- round(deflat.limit, 3) + 0.001 # Inside and near the boundary
barplot(rbind(doipospois(x, lambda, pstr1 = newpstr1),
                        dpospois(x, lambda)),
    beside = TRUE, col = c("blue","orange"),
    main = paste("ODPP(", lambda, ", pstr1 = ", newpstr1, ") (blue) vs",
                            " PosPoisson(", lambda, ") (orange)", sep = ""),
        names.arg = as.character(x))
## End(Not run)
```


## Description

Fits a 1-inflated positive Poisson distribution.

## Usage

oipospoisson(lpstr1 = "logitlink", llambda = "loglink",
type.fitted = c("mean", "lambda", "pobs1", "pstr1", "onempstr1"), ilambda $=$ NULL, gpstr1 $=(1: 19) / 20$, gprobs.y $=(1: 19) / 20$, imethod $=1$, zero $=$ NULL)

## Arguments

lpstr1, llambda
For lpstr1: the same idea as zipoisson except it applies to a structural 1.
ilambda, gpstr1, gprobs.y, imethod
For initial values. See CommonVGAMffArguments for information.
type.fitted, zero
See CommonVGAMffArguments for information.

## Details

The 1-inflated positive Poisson distribution is a mixture distribution of the positive (0-truncated) Poisson distribution with some probability of obtaining a (structural) 1. Thus there are two sources for obtaining the value 1. It is similar to a zero-inflated Poisson model, except the Poisson is replaced by a positive Poisson and the 0 is replaced by 1 . This distribution is written here in a way that retains a similar notation to the zero-inflated Poisson, i.e., the probability $P[Y=1]$ involves another parameter $\phi$. See zipoisson.
This family function can handle multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Warning

Under- or over-flow may occur if the data is ill-conditioned.

## Author(s)

Thomas W. Yee

## See Also

```
Oipospois, pospoisson, oapospoisson, otpospoisson, zipoisson, poissonff, simulate.vlm.
```


## Examples

```
    set.seed(1)
pdata <- data.frame(x2 = runif(nn <- 1000)) # Artificial data
pdata <- transform(pdata, pstr1 = 0.5, lambda = exp(3 - x2))
pdata <- transform(pdata, y1 = roipospois(nn, lambda, pstr1 = pstr1))
with(pdata, table(y1))
fit1 <- vglm(y1 ~ x2, oipospoisson, data = pdata, trace = TRUE)
coef(fit1, matrix = TRUE)
```


## Oizeta One-Inflated Zeta Distribution

## Description

Density, distribution function, quantile function and random generation for the one-inflated zeta distribution with parameter pstr1.

## Usage

doizeta(x, shape, pstr1 = 0, log = FALSE)
poizeta(q, shape, pstr1 = 0)
qoizeta(p, shape, pstr1 = 0)
roizeta(n, shape, pstr1 = 0)

## Arguments

| $\mathrm{x}, \mathrm{q}, \mathrm{p}, \mathrm{n}$ | Same as Uniform. |
| :--- | :--- |
| shape | Vector of positive shape parameters. |
| pstr1 | Probability of a structural one (i.e., ignoring the zeta distribution), called $\phi$. The <br> default value of $\phi=0$ corresponds to the response having an ordinary zeta <br> distribution. |
| log | Same as Uniform. |

## Details

The probability function of $Y$ is 1 with probability $\phi$, and $Z e t a($ shape $)$ with probability $1-\phi$. Thus

$$
P(Y=1)=\phi+(1-\phi) P(W=1)
$$

where $W$ is distributed as a zeta(shape) random variable.

## Value

doizeta gives the density, poizeta gives the distribution function, qoizeta gives the quantile function, and roizeta generates random deviates.

## Note

The argument pstr 1 is recycled to the required length, and usually has values which lie in the interval $[0,1]$.
These functions actually allow for the zero-deflated zeta distribution. Here, pstr1 is also permitted to lie in the interval $[-\operatorname{dzeta}(1$, shape $) /(1-\operatorname{dzeta}(1$, shape $)), 0]$. The resulting probability of a unit count is less than the nominal zeta value, and the use of pstr1 to stand for the probability of a structural 1 loses its meaning.

When pstr1 equals -dzeta(1, shape) / (1-dzeta(1, shape)) this corresponds to the 1-truncated zeta distribution.

## Author(s)

T. W. Yee

## See Also

Zeta, zetaff. Otzeta,

## Examples

```
shape <- 1.5; pstr1 <- 0.3; x <- (-1):7
(ii <- doizeta(x, shape, pstr1 = pstr1))
max(abs(poizeta(1:200, shape) -
    cumsum(1/(1:200)^(1+shape)) / zeta(shape+1))) # Should be 0
## Not run: x <- 0:10
par(mfrow = c(2, 1)) # One-Inflated zeta
barplot(rbind(doizeta(x, shape, pstr1 = pstr1), dzeta(x, shape)),
    beside = TRUE, col = c("blue", "orange"),
    main = paste("OIZeta(", shape, ", pstr1 = ", pstr1, ") (blue) vs",
            " Zeta(", shape, ") (orange)", sep = ""),
    names.arg = as.character(x))
deflat.limit <- -dzeta(1, shape) / pzeta(1, shape, lower.tail = FALSE)
newpstr1 <- round(deflat.limit, 3) + 0.001 # Inside but near the boundary
barplot(rbind(doizeta(x, shape, pstr1 = newpstr1),
                        dzeta(x, shape)),
    beside = TRUE, col = c("blue","orange"),
    main = paste("ODZeta(", shape, ", pstr1 = ", newpstr1, ") (blue) vs",
                    "Zeta(", shape, ") (orange)", sep = ""),
    names.arg = as.character(x))
## End(Not run)
```


## Description

Fits a 1-inflated zeta distribution.

## Usage

```
oizeta(lpstr1 = "logitlink", lshape = "loglink",
```

    type.fitted = c("mean", "shape", "pobs1", "pstr1", "onempstr1"),
    ishape \(=\) NULL, gpstr1 \(=\operatorname{ppoints}(8)\), gshape \(=\exp ((-3: 3) / 4)\), zero \(=\) NULL)
    
## Arguments

lpstr1, lshape For lpstr1: the same idea as zipoisson except it applies to a structural 1.
gpstr1, gshape, ishape
For initial values. See CommonVGAMffArguments for information.
type.fitted, zero
See CommonVGAMffArguments for information.

## Details

The 1-inflated zeta distribution is a mixture distribution of the zeta distribution with some probability of obtaining a (structural) 1. Thus there are two sources for obtaining the value 1. This distribution is written here in a way that retains a similar notation to the zero-inflated Poisson, i.e., the probability $P[Y=1]$ involves another parameter $\phi$. See zipoisson.
This family function can handle multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Warning

Under- or over-flow may occur if the data is ill-conditioned. Lots of data is needed to estimate the parameters accurately. Usually, probably the shape parameter is best modelled as intercept-only.

## Author(s)

Thomas W. Yee

## See Also

Oizeta, zetaff, oazeta, otzeta, diffzeta, zeta, Oizipf.

## Examples

```
## Not run: odata <- data.frame(x2 = runif(nn <- 1000)) # Artificial data
odata <- transform(odata, pstr1 = logitlink(-1 + x2, inverse = TRUE), shape = exp(-0.5))
odata <- transform(odata, y1 = roizeta(nn, shape, pstr1 = pstr1))
with(odata, table(y1))
fit1 <- vglm(y1 ~ x2, oizeta(zero = "shape"), data = odata, trace = TRUE)
coef(fit1, matrix = TRUE)
## End(Not run)
```

```
Oizipf One-Inflated Zipf Distribution
```


## Description

Density, distribution function, quantile function and random generation for the one-inflated Zipf distribution with parameter pstr1.

## Usage

doizipf(x, N, shape, pstr1 = 0, log = FALSE)
poizipf(q, N, shape, pstr1 = 0)
qoizipf( $p, N$, shape, pstr1 = 0)
roizipf( $\mathrm{n}, \mathrm{N}$, shape, $\operatorname{pstr} 1=0$ )

## Arguments

$\mathrm{x}, \mathrm{q}, \mathrm{p}, \mathrm{n} \quad$ Same as Uniform.
$N$, shape See Zipf.
pstr1 Probability of a structural one (i.e., ignoring the Zipf distribution), called $\phi$. The default value of $\phi=0$ corresponds to the response having an ordinary Zipf distribution.
log Same as Uniform.

## Details

The probability function of $Y$ is 1 with probability $\phi$, and $\operatorname{Zipf}(N, s)$ with probability $1-\phi$. Thus

$$
P(Y=1)=\phi+(1-\phi) P(W=1)
$$

where $W$ is distributed as a $\operatorname{Zipf}(N, s)$ random variable. The VGAM family function oizeta estimates the two parameters of this model by Fisher scoring.

## Value

doizipf gives the density, poizipf gives the distribution function, qoizipf gives the quantile function, and roizipf generates random deviates.

## Note

The argument pstr 1 is recycled to the required length, and usually has values which lie in the interval $[0,1]$.
These functions actually allow for the zero-deflated Zipf distribution. Here, pstr1 is also permitted to lie in the interval $[-\operatorname{dzipf}(1, N, s) /(1-\operatorname{dzipf}(1, N, s)), 0]$. The resulting probability of a unit count is less than the nominal zipf value, and the use of pstr1 to stand for the probability of a structural 1 loses its meaning.
When pstr1 equals $-\operatorname{dzipf}(1, N, s) /(1-\operatorname{dzipf}(1, N, s))$ this corresponds to the 1-truncated zipf distribution.

## Author(s)

T. W. Yee

## See Also

oizeta. Zipf, zipf, Oizeta.

## Examples

```
    N <- 10; shape <- 1.5; pstr1 <- 0.3; x <- (-1):N
    (ii <- doizipf(x, N, shape, pstr1 = pstr1))
    ## Not run: x <- 0:10
    par(mfrow = c(2, 1)) # One-Inflated zipf
    barplot(rbind(doizipf(x, N, shape, pstr1 = pstr1), dzipf(x, N, shape)),
        beside = TRUE, col = c("blue", "orange"),
        main = paste("OIZipf(", N, ", ", shape, ", pstr1 = ", pstr1, ") (blue) vs",
            " Zipf(", N, ", ", shape, ") (orange)", sep = ""),
        names.arg = as.character(x))
deflat.limit <- -dzipf(1, N, shape) / (1 - dzipf(1, N, shape))
newpstr1 <- round(deflat.limit, 3) + 0.001 # Inside but near the boundary
barplot(rbind(doizipf(x, N, shape, pstr1 = newpstr1),
            dzipf(x, N, shape)),
            beside = TRUE, col = c("blue", "orange"),
            main = paste("ODZipf(", N, ", ", shape, ", pstr1 = ", newpstr1, ") (blue) vs",
                            " Zipf(", N, ", ", shape, ") (orange)", sep = ""),
            names.arg = as.character(x))
## End(Not run)
```

oizipf
One-inflated Zipf Distribution Family Function

## Description

Fits a 1-inflated Zipf distribution.

## Usage

oizipf(N = NULL, lpstr1 = "logitlink", lshape = "loglink", type.fitted = c("mean", "shape", "pobs1", "pstr1", "onempstr1"), ishape $=$ NULL, gpstr1 $=$ ppoints(8), gshape $=\exp ((-3: 3) / 4)$, zero $=$ NULL)

## Arguments

N
Same as zipf.
lpstr1, lshape For lpstr1: the same idea as zipoisson except it applies to a structural 1.
gpstr1, gshape, ishape
For initial values. See CommonVGAMffArguments for information.
type.fitted, zero
See CommonVGAMffArguments for information.

## Details

The 1-inflated Zipf distribution is a mixture distribution of the Zipf distribution with some probability of obtaining a (structural) 1. Thus there are two sources for obtaining the value 1. This distribution is written here in a way that retains a similar notation to the zero-inflated Poisson, i.e., the probability $P[Y=1]$ involves another parameter $\phi$. See zipoisson.
This family function can handle multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Warning

Under- or over-flow may occur if the data is ill-conditioned. Lots of data is needed to estimate the parameters accurately. Usually, probably the shape parameter is best modelled as intercept-only.

## Author(s)

Thomas W. Yee

## See Also

Oizipf. zipf, Oizeta.

## Examples

```
## Not run: odata <- data.frame(x2 = runif(nn <- 1000)) # Artificial data
odata <- transform(odata, pstr1 = logitlink(-1 + x2, inverse = TRUE),
                        myN = 10,
                        shape = exp(-0.5))
odata <- transform(odata, y1 = roizipf(nn, N = myN, s = shape, pstr1 = pstr1))
with(odata, table(y1))
fit1 <- vglm(y1 ~ x2, oizipf(zero = "shape"), data = odata, trace = TRUE)
```

```
    coef(fit1, matrix \(=\) TRUE)
```

    \#\# End(Not run)
    olympics 2008 and 2012 Summer Olympic Final Medal Count Data
    
## Description

Final medal count, by country, for the Summer 2008 and 2012 Olympic Games.

## Usage

data(olym08)
data(olym12)

## Format

A data frame with 87 or 85 observations on the following 6 variables.
rank a numeric vector, overall ranking of the countries.
country a factor.
gold a numeric vector, number of gold medals.
silver a numeric vector, number of silver medals.
bronze a numeric vector, number of bronze medals.
totalmedal a numeric vector, total number of medals.

## Details

The events were held during (i) August 8-24, 2008, in Beijing; and (ii) 27 July-12 August, 2012, in London.

## References

The official English website was/is http://en.beijing2008. cn and http://www. london2012.com. Help from Viet Hoang Quoc is gratefully acknowledged.

## See Also

grc.

## Examples

```
    summary(olym08)
    summary(olym12)
    ## maybe str(olym08) ; plot(olym08) ...
    ## Not run: par(mfrow = c(1, 2))
    myylim <- c(0, 55)
    with(head(olym08, n = 8),
    barplot(rbind(gold, silver, bronze),
    col = c("gold", "grey", "brown"), # No "silver" or "bronze"!
        "gold", "grey71", "chocolate4",
    names.arg = country, cex.names = 0.5, ylim = myylim,
    beside = TRUE, main = "2008 Summer Olympic Final Medal Count",
    ylab = "Medal count", las = 1,
    sub = "Top 8 countries; 'gold'=gold, 'grey'=silver, 'brown'=bronze"))
with(head(olym12, n = 8),
barplot(rbind(gold, silver, bronze),
    col = c("gold", "grey", "brown"), # No "silver" or "bronze"!
    names.arg = country, cex.names = 0.5, ylim = myylim,
    beside = TRUE, main = "2012 Summer Olympic Final Medal Count",
    ylab = "Medal count", las = 1,
    sub = "Top 8 countries; 'gold'=gold, 'grey'=silver, 'brown'=bronze"))
## End(Not run)
```

Opt
Optimums

## Description

Generic function for the optimums (or optima) of a model.

## Usage

Opt (object, ...)

## Arguments

object An object for which the computation or extraction of an optimum (or optimums) is meaningful.
... Other arguments fed into the specific methods function of the model. Sometimes they are fed into the methods function for Coef.

## Details

Different models can define an optimum in different ways. Many models have no such notion or definition.
Optimums occur in quadratic and additive ordination, e.g., CQO or CAO. For these models the optimum is the value of the latent variable where the maximum occurs, i.e., where the fitted value achieves its highest value. For quadratic ordination models there is a formula for the optimum but
for additive ordination models the optimum must be searched for numerically. If it occurs on the boundary, then the optimum is undefined. At an optimum, the fitted value of the response is called the maximum.

## Value

The value returned depends specifically on the methods function invoked.

Note
In ordination, the optimum of a species is sometimes called the species score.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2004). A new technique for maximum-likelihood canonical Gaussian ordination. Ecological Monographs, 74, 685-701.

Yee, T. W. (2006). Constrained additive ordination. Ecology, 87, 203-213.

## See Also

Opt.qrrvglm, Max, Tol.

## Examples

```
set.seed(111) # This leads to the global solution
hspider[,1:6] <- scale(hspider[,1:6]) # Standardized environmental vars
p1 <- cqo(cbind(Alopacce, Alopcune, Alopfabr, Arctlute, Arctperi,
            Auloalbi, Pardlugu, Pardmont, Pardnigr, Pardpull,
            Trocterr, Zoraspin) ~
        WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
        family = poissonff, data = hspider, Crow1positive = FALSE)
Opt(p1)
## Not run:
clr <- (1:(ncol(depvar(p1))+1))[-7] # Omits yellow
persp(p1, col = clr, las = 1, main = "Vertical lines at the optimums")
abline(v = Opt(p1), lty = 2, col = clr)
## End(Not run)
```


## Description

Fits a Poisson regression where the response is ordinal (the Poisson counts are grouped between known cutpoints).

## Usage

ordpoisson(cutpoints, countdata = FALSE, NOS = NULL,
Levels = NULL, init.mu = NULL, parallel = FALSE, zero = NULL, link = "loglink")

## Arguments

cutpoints Numeric. The cutpoints, $K_{l}$. These must be non-negative integers. Inf values may be included. See below for further details.
countdata Logical. Is the response (LHS of formula) in count-data format? If not then the response is a matrix or vector with values $1,2, \ldots, L$, say, where $L$ is the number of levels. Such input can be generated with cut with argument labels $=$ FALSE. If countdata $=$ TRUE then the response is expected to be in the same format as depvar(fit) where fit is a fitted model with ordpoisson as the VGAM family function. That is, the response is matrix of counts with $L$ columns (if NOS $=1$ ).
NOS Integer. The number of species, or more generally, the number of response random variates. This argument must be specified when countdata $=$ TRUE . Usually NOS $=1$.
Levels Integer vector, recycled to length NOS if necessary. The number of levels for each response random variate. This argument should agree with cutpoints. This argument must be specified when countdata $=$ TRUE.
init.mu Numeric. Initial values for the means of the Poisson regressions. Recycled to length NOS if necessary. Use this argument if the default initial values fail (the default is to compute an initial value internally).
parallel, zero, link
See poissonff.

## Details

This VGAM family function uses maximum likelihood estimation (Fisher scoring) to fit a Poisson regression to each column of a matrix response. The data, however, is ordinal, and is obtained from known integer cutpoints. Here, $l=1, \ldots, L$ where $L(L \geq 2)$ is the number of levels. In more detail, let $Y^{*}=l$ if $K_{l-1}<Y \leq K_{l}$ where the $K_{l}$ are the cutpoints. We have $K_{0}=-\infty$ and $K_{L}=\infty$. The response for this family function corresponds to $Y^{*}$ but we are really interested in the Poisson regression of $Y$.

If NOS=1 then the argument cutpoints is a vector $\left(K_{1}, K_{2}, \ldots, K_{L}\right)$ where the last value (Inf) is optional. If NOS>1 then the vector should have NOS-1 Inf values separating the cutpoints. For example, if there are NOS $=3$ responses, then something like ordpoisson(cut $=c(0,5,10, \operatorname{Inf}, 20,30, \operatorname{Inf}, 0,10,40, \operatorname{Inf})$ ) is valid.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Warning

The input requires care as little to no checking is done. If fit is the fitted object, have a look at fit@extra and depvar(fit) to check.

## Note

Sometimes there are no observations between two cutpoints. If so, the arguments Levels and NOS need to be specified too. See below for an example.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2020). Ordinal ordination with normalizing link functions for count data, (in preparation).

## See Also

poissonff, polf, ordered.

## Examples

```
set.seed(123) # Example 1
x2 <- runif(n <- 1000); x3 <- runif(n)
mymu <- exp(3 - 1 * x2 + 2 * x3)
y1 <- rpois(n, lambda = mymu)
cutpts <- c(-Inf, 20, 30, Inf)
fcutpts <- cutpts[is.finite(cutpts)] # finite cutpoints
ystar <- cut(y1, breaks = cutpts, labels = FALSE)
## Not run:
plot(x2, x3, col = ystar, pch = as.character(ystar))
## End(Not run)
table(ystar) / sum(table(ystar))
fit <- vglm(ystar ~ x2 + x3, fam = ordpoisson(cutpoi = fcutpts))
head(depvar(fit)) # This can be input if countdata = TRUE
head(fitted(fit))
head(predict(fit))
```

```
coef(fit, matrix = TRUE)
fit@extra
# Example 2: multivariate and there are no obsns between some cutpoints
cutpts2 <- c(-Inf, 0, 9, 10, 20, 70, 200, 201, Inf)
fcutpts2 <- cutpts2[is.finite(cutpts2)] # finite cutpoints
y2 <- rpois(n, lambda = mymu) # Same model as y1
ystar2 <- cut(y2, breaks = cutpts2, labels = FALSE)
table(ystar2) / sum(table(ystar2))
fit <- vglm(cbind(ystar,ystar2) ~ x2 + x3, fam =
    ordpoisson(cutpoi = c(fcutpts,Inf,fcutpts2,Inf),
    Levels = c(length(fcutpts)+1,length(fcutpts2)+1),
    parallel = TRUE), trace = TRUE)
coef(fit, matrix = TRUE)
fit@extra
constraints(fit)
summary(depvar(fit)) # Some columns have all zeros
```

ordsup Ordinal Superiority Measures

## Description

Ordinal superiority measures for the linear model and cumulative link models: the probability that an observation from one distribution falls above an independent observation from the other distribution, adjusted for explanatory variables in a model.

## Usage

ordsup(object, ...)
ordsup.vglm(object, all.vars = FALSE, confint = FALSE, ...)

## Arguments

## object

all.vars Logical. The default is to use explanatory variables which are binary, but all
A vglm fit. Currently it must be one of: cumulative, uninormal. The links for cumulative must be logitlink or probitlink, and parallel = TRUE is also needed. For uninormal the mean must use identitylink and model the sd as intercept-only. variables are used (except the intercept) if set to TRUE.
confint Logical. If TRUE then confintvglm is called to return confidence intervals for $\gamma$ and $\Delta$. By default, Wald intervals are produced, but they can be replaced by profile intervals by setting method = "profile".
$\ldots \quad$ Parameters that can be fed into confintvglm, e.g., level $=0.95$ and method $=$ c("wald","profile").

## Details

Details are given in Agresti and Kateri (2017) and this help file draws directly from this. This function returns two quantities for comparing two groups on an ordinal categorical response variable, while adjusting for other explanatory variables. They are called "ordinal superiority" measures, and the two groups can be compared without supplementary explanatory variables. Let $Y_{1}$ and $Y_{2}$ be independent random variables from groups A and B , say, for a quantitative ordinal categorical scale. Then $\Delta=P\left(Y_{1}>Y_{2}\right)-P\left(Y_{2}>Y_{1}\right)$ summarizes their relative size. A second quantity is $\gamma=P\left(Y_{1}>Y_{2}\right)-0.5 \times P\left(Y_{2}=Y_{1}\right)$. Then $\Delta=2 \times \gamma-1$. whereas $\gamma=(\Delta+1) / 2$. The range of $\gamma$ is $[0,1]$, while the range of $\Delta$ is $[-1,1]$. The examples below are based on that paper. This function is currently implemented for a very limited number of specific models.

## Value

By default, a list with components gamma and Delta, where each is a vector with elements corresponding to binary explanatory variables (i.e., 0 or 1 ), and if no explanatory variables are binary then a NULL is returned. If confint = TRUE then the list contains 4 more components: lower .gamma, upper.gamma, Lower. Delta, Upper. Delta.

## Author(s)

Thomas W. Yee

## References

Agresti, A. and Kateri, M. (2017). Ordinal probability effect measures for group comparisons in multinomial cumulative link models. Biometrics, 73, 214-219.

## See Also

cumulative, propodds, uninormal.

## Examples

```
## Not run:
Mental <- read.table("http://www.stat.ufl.edu/~aa/glm/data/Mental.dat",
                    header = TRUE) # Make take a while to load in
Mental$impair <- ordered(Mental$impair)
pfit3 <- vglm(impair ~ ses + life, data = Mental,
    cumulative(link = "probitlink", reverse = FALSE, parallel = TRUE))
coef(pfit3, matrix = TRUE)
ordsup(pfit3) # The 'ses' variable is binary
# Fit a crude LM
fit7 <- vglm(as.numeric(impair) ~ ses + life, uninormal, data = Mental)
coef(fit7, matrix = TRUE) # 'sd' is estimated by MLE
ordsup(fit7)
ordsup(fit7, all.vars = TRUE) # Some output may not be meaningful
ordsup(fit7, confint = TRUE, method = "profile")
## End(Not run)
```


## Description

Density, distribution function, quantile function, and random generation for the one-truncated logarithmic distribution.

## Usage

dotlog(x, shape, log = FALSE)
potlog(q, shape, log.p = FALSE)
qotlog(p, shape)
rotlog(n, shape)

## Arguments

$x, q \quad$ Vector of quantiles. For the density, it should be a vector with integer values $>1$ in order for the probabilities to be positive.
$p \quad$ vector of probabilities.
n number of observations. Same as in runif.
shape The parameter value $c$ described in in logff. Here it is called shape because $0<c<1$ is the range.
$\log , \log . p \quad$ Logical. If log. $p=$ TRUE then all probabilities $p$ are given as $\log (p)$.

## Details

The one-truncated logarithmic distribution is a logarithmic distribution but with the probability of a one being zero. The other probabilities are scaled to add to unity. Some more details are given in logff.

## Value

dotlog gives the density, potlog gives the distribution function, qotlog gives the quantile function, and rotlog generates random deviates.

## Note

Given some response data, the VGAM family function ot log estimates the parameter shape. Function $\operatorname{pot} \log ()$ suffers from the problems that plog sometimes has.

## Author(s)

T. W. Yee

## See Also

Gaitlog, otlog, rlog, Oilog.

## Examples

```
dotlog(1:20, 0.5)
rotlog(20, 0.5)
## Not run: shape <- 0.8; x <- 1:10
plot(x, dotlog(x, shape = shape), type = "h", ylim = 0:1,
    sub = "shape=0.8", las = 1, col = "blue", ylab = "Probability",
    main = "1-truncated logarithmic distribution: blue=PMF; orange=CDF")
lines(x + 0.1, potlog(x, shape = shape), col = "orange", lty = 3, type = "h")
## End(Not run)
```

otlog One-truncated Logarithmic Distribution

## Description

Estimating the (single) parameter of the 1-truncated logarithmic distribution.

## Usage

otlog(lshape = "logitlink", gshape = ppoints(8), zero = NULL)

## Arguments

lshape, gshape, zero
Same as logff.

## Details

The 1-truncated logarithmic distribution is a logarithmic distribution but with the probability of a one being zero. The other probabilities are scaled to add to unity. Some more details can be found at logff. Multiple responses are permitted.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Author(s)

T. W. Yee

## See Also

Gaitlog, Otlog, logff, oalog, oilog, simulate.vlm.

## Examples

```
odata <- data.frame(y1 = rotlog(n = 1000, shape = logitlink(1/3, inverse = TRUE)))
ofit <- vglm(y1 ~ 1, otlog, data = odata, trace = TRUE, crit = "c")
coef(ofit, matrix = TRUE)
Coef(ofit)
## Not run: with(odata,
    hist(y1, shape = TRUE, breaks = seq(0.5, max(y1) + 0.5, by = 1),
            border = "blue"))
x <- seq(1, with(odata, max(y1)), by = 1)
with(odata, lines(x, dotlog(x, Coef(ofit)[1]), col = "orange", type = "h", lwd = 2))
## End(Not run)
```


## Description

Density, distribution function, quantile function, and random generation for the one-truncated positivePoisson distribution.

## Usage

dotpospois(x, lambda, log = FALSE)
potpospois(q, lambda, log.p = FALSE)
qotpospois(p, lambda)
rotpospois(n, lambda)

## Arguments

$\mathrm{x}, \mathrm{q}, \mathrm{p}, \mathrm{n} \quad$ Same as Pospois.
lambda, log, log.p
Same as Pospois.

## Details

The one-truncated positive-Poisson is a Poisson distribution but with the probability of a one and a zero being 0 . That is, its support is $2,3, \ldots$. The other probabilities are scaled to add to unity. Some more details are given in pospoisson.

## Value

dotpospois gives the density, potpospois gives the distribution function, qotpospois gives the quantile function, and rotpospois generates random deviates.

## Note

Given some response data, the VGAM family function otpospoisson estimates the parameter lambda.

## Author(s)

## T. W. Yee

## See Also

```
otpospoisson, Pospois, Oipospois.
```


## Examples

```
dotpospois(1:20, 0.5)
rotpospois(20, 0.5)
## Not run: lambda <- 4; x <- 1:10
plot(x, dotpospois(x, lambda = lambda), type = "h", ylim = 0:1,
    sub = "lambda=4", las = 1, col = "blue", ylab = "Probability",
    main = "1-truncated positive-Poisson distribution: blue=PMF; orange=CDF")
lines(x + 0.1, potpospois(x, lambda = lambda), col = "orange", lty = 3, type = "h")
## End(Not run)
```

otpospoisson

One-truncated Poisson Distribution

## Description

Estimating the (single) parameter of the 1-truncated positive Poisson distribution.

## Usage

otpospoisson(llambda = "loglink",
type.fitted = c("mean", "lambda", "prob0", "prob1"), ilambda $=$ NULL, imethod $=1$, zero = NULL)

## Arguments

llambda, type.fitted, ilambda Same as pospoisson.
imethod, zero Same as pospoisson.

## Details

The 1-truncated positive Poisson distribution has support on $2,3, \ldots$ It is a Poisson distribution but with the probability of a one or zero being 0 . The other probabilities are scaled to add to unity. Some more details can be found at pospoisson. Multiple responses are permitted.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Author(s)

T. W. Yee

## See Also

Otpospois, oipospoisson, simulate.vlm.

## Examples

```
odata <- data.frame(y1 = rotpospois(n = 1000, lambda = loglink(1, inverse = TRUE)))
ofit <- vglm(y1 ~ 1, otpospoisson, data = odata, trace = TRUE, crit = "c")
coef(ofit, matrix = TRUE)
Coef(ofit)
## Not run: with(odata,
        hist(y1, prob = TRUE, breaks = seq(0.5, max (y1) + 0.5, by = 1),
            border = "blue"))
x <- seq(1, with(odata, max(y1)), by = 1)
with(odata, lines(x, dotpospois(x, Coef(ofit)[1]), col = "orange", type = "h", lwd = 2))
## End(Not run)
```

Otzeta

One-truncated Zeta Distribution

## Description

Density, distribution function, quantile function, and random generation for the one-truncated zeta distribution.

## Usage

dotzeta(x, shape, log = FALSE)
potzeta(q, shape, log.p = FALSE)
qotzeta( $p$, shape)
rotzeta( $n$, shape)

## Arguments

$\mathrm{x}, \mathrm{q}, \mathrm{p}, \mathrm{n}$
shape
log, log.p Same as in runif.

## Details

The one-truncated zeta distribution is a zeta distribution but with the probability of a one being zero. The other probabilities are scaled to add to unity. Some more details are given in zetaff.

## Value

dotzeta gives the density, potzeta gives the distribution function, qotzeta gives the quantile function, and rotzeta generates random deviates.

## Note

Given some response data, the VGAM family function otzeta estimates the parameter shape.

## Author(s)

T. W. Yee

## See Also

Otzeta, zetaff, Oizeta.

## Examples

```
dotzeta(1:20, 0.5)
rotzeta(20, 0.5)
## Not run: shape <- 0.8; x <- 1:10
plot(x, dotzeta(x, shape = shape), type = "h", ylim = 0:1,
    sub = "shape=0.8", las = 1, col = "blue", ylab = "Probability",
    main = "1-truncated zeta distribution: blue=PMF; orange=CDF")
lines(x + 0.1, potzeta(x, shape = shape), col = "orange", lty = 3, type = "h")
## End(Not run)
```

```
otzeta One-truncated Zeta Distribution Family Function
```


## Description

Estimates the parameter of the 1-truncated zeta distribution.

## Usage

otzeta(lshape = "loglink", ishape = NULL, gshape $=\exp ((-4: 3) / 4)$, zero $=$ NULL)

## Arguments

lshape, ishape, gshape, zero
Same as zetaff.

## Details

The 1-truncated zeta distribution is the ordinary zeta distribution but with the probability of one being 0 . Thus the other probabilities are scaled up (i.e., divided by $1-P[Y=1]$ ). The mean is returned by default as the fitted values. More details can be found at zetaff. Multiple responses are handled.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Author(s)

T. W. Yee

## See Also

Otzeta, zetaff, oizeta, diffzeta, zeta, dzeta, hzeta, zipf.

## Examples

```
odata <- data.frame(x2 = runif(nn <- 1000)) # Artificial data
odata <- transform(odata, shape = loglink(-0.25 + x2, inverse = TRUE))
odata <- transform(odata, y1 = rotzeta(nn, shape))
with(odata, table(y1))
ofit <- vglm(y1 ~ x2, otzeta, data = odata, trace = TRUE, crit = "coef")
coef(ofit, matrix = TRUE)
```

```
oxtemp
Oxford Temperature Data
```


## Description

Annual maximum temperatures collected at Oxford, UK.

## Usage

data(oxtemp)

## Format

A data frame with 80 observations on the following 2 variables.
maxtemp Annual maximum temperatures (in degrees Fahrenheit).
year The values 1901 to 1980 .

## Details

The data were collected from 1901 to 1980.

## Source

Unknown.

## Examples

\#\# Not run: fit <- vglm(maxtemp ~ 1, gevff, data = oxtemp, trace $=$ TRUE)

## Description

Density, distribution function, quantile function and random generation for the paralogistic distribution with shape parameter a and scale parameter scale.

## Usage

```
dparalogistic(x, scale = 1, shape1.a, log = FALSE)
pparalogistic(q, scale = 1, shape1.a, lower.tail = TRUE, log.p = FALSE)
qparalogistic(p, scale = 1, shape1.a, lower.tail = TRUE, log.p = FALSE)
rparalogistic(n, scale = 1, shape1.a)
```


## Arguments

| $x, q$ | vector of quantiles. |
| :--- | :--- |
| $p$ | vector of probabilities. |
| $n$ | number of observations. If length $(n)>1$, the length is taken to be the number <br> required. |
| shape1.a | shape parameter. <br> scale |
| scale parameter. |  |
| log | Logical. If log=TRUE then the logarithm of the density is returned. |
| lower.tail, log.p |  |

Same meaning as in pnorm or qnorm.

## Details

See paralogistic, which is the VGAM family function for estimating the parameters by maximum likelihood estimation.

## Value

dparalogistic gives the density, pparalogistic gives the distribution function, qparalogistic gives the quantile function, and rparalogistic generates random deviates.

## Note

The paralogistic distribution is a special case of the 4-parameter generalized beta II distribution.

## Author(s)

T. W. Yee and Kai Huang

## References

Kleiber, C. and Kotz, S. (2003). Statistical Size Distributions in Economics and Actuarial Sciences, Hoboken, NJ, USA: Wiley-Interscience.

## See Also <br> paralogistic, genbetaII.

## Examples

```
pdata <- data.frame(y = rparalogistic(n = 3000, scale = exp(1), exp(2)))
fit <- vglm(y ~ 1, paralogistic(lss = FALSE, ishape1.a = 4.1),
            data = pdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
```

```
paralogistic Paralogistic Distribution Family Function
```


## Description

Maximum likelihood estimation of the 2-parameter paralogistic distribution.

## Usage

paralogistic(lscale = "loglink", lshape1.a = "loglink", iscale = NULL, ishape1.a $=$ NULL, imethod $=1$, lss $=$ TRUE, gscale $=\exp (-5: 5)$, gshape1.a $=\operatorname{seq}(0.75,4$, by $=0.25)$, probs.y $=c(0.25,0.5,0.75)$, zero = "shape")

## Arguments

lss See CommonVGAMffArguments for important information.
lshape1.a, lscale
Parameter link functions applied to the (positive) parameters $a$ and scale. See Links for more choices.
iscale, ishape1.a, imethod, zero
See CommonVGAMffArguments for information. For imethod $=2$ a good initial value for ishape1. a is needed to obtain good estimates for the other parameter.
gscale, gshape1.a
See CommonVGAMffArguments for information.
probs.y See CommonVGAMffArguments for information.

## Details

The 2-parameter paralogistic distribution is the 4-parameter generalized beta II distribution with shape parameter $p=1$ and $a=q$. It is the 3-parameter Singh-Maddala distribution with $a=q$. More details can be found in Kleiber and Kotz (2003).
The 2-parameter paralogistic has density

$$
f(y)=a^{2} y^{a-1} /\left[b^{a}\left\{1+(y / b)^{a}\right\}^{1+a}\right]
$$

for $a>0, b>0, y \geq 0$. Here, $b$ is the scale parameter scale, and $a$ is the shape parameter. The mean is

$$
E(Y)=b \Gamma(1+1 / a) \Gamma(a-1 / a) / \Gamma(a)
$$

provided $a>1$; these are returned as the fitted values. This family function handles multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

See the notes in genbetaII.

## Author(s)

T. W. Yee

## References

Kleiber, C. and Kotz, S. (2003). Statistical Size Distributions in Economics and Actuarial Sciences, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

Paralogistic, sinmad, genbetaII, betaII, dagum, fisk, inv.lomax, lomax, inv. paralogistic.

## Examples

```
pdata <- data.frame(y = rparalogistic(n = 3000, exp(1), scale = exp(1)))
fit <- vglm(y ~ 1, paralogistic(lss = FALSE), data = pdata, trace = TRUE)
fit <- vglm(y ~ 1, paralogistic(ishape1.a = 2.3, iscale = 5),
    data = pdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```


## Description

Density, distribution function, quantile function and random generation for the Pareto(I) distribution with parameters scale and shape.

## Usage

```
dpareto(x, scale = 1, shape, log = FALSE)
ppareto(q, scale = 1, shape, lower.tail = TRUE, log.p = FALSE)
qpareto(p, scale = 1, shape, lower.tail = TRUE, log.p = FALSE)
rpareto(n, scale = 1, shape)
```


## Arguments

```
    x,q vector of quantiles.
    p vector of probabilities.
    n number of observations. Same as in runif.
    scale, shape the \alpha and k parameters.
    log Logical. If log = TRUE then the logarithm of the density is returned.
    lower.tail, log.p
        Same meaning as in pnorm or qnorm.
```


## Details

See paretoff, the VGAM family function for estimating the parameter $k$ by maximum likelihood estimation, for the formula of the probability density function and the range restrictions imposed on the parameters.

## Value

dpareto gives the density, ppareto gives the distribution function, qpareto gives the quantile function, and rpareto generates random deviates.

## Author(s)

T. W. Yee and Kai Huang

## References

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011). Statistical Distributions, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

## See Also

```
paretoff, ParetoIV.
```


## Examples

```
alpha <- 3; k <- exp(1); x <- seq(2.8, 8, len = 300)
## Not run:
plot(x, dpareto(x, scale = alpha, shape = k), type = "l",
    main = "Pareto density split into 10 equal areas")
abline(h = 0, col = "blue", lty = 2)
qvec <- qpareto(seq(0.1, 0.9, by = 0.1), scale = alpha, shape = k)
lines(qvec, dpareto(qvec, scale = alpha, shape = k),
    col = "purple", lty = 3, type = "h")
## End(Not run)
pvec <- seq(0.1, 0.9, by = 0.1)
qvec <- qpareto(pvec, scale = alpha, shape = k)
ppareto(qvec, scale = alpha, shape = k)
qpareto(ppareto(qvec, scale = alpha, shape = k),
        scale = alpha, shape = k) - qvec # Should be 0
```

```
paretoff
```

Pareto and Truncated Pareto Distribution Family Functions

## Description

Estimates one of the parameters of the Pareto(I) distribution by maximum likelihood estimation. Also includes the upper truncated Pareto(I) distribution.

## Usage

paretoff(scale = NULL, lshape = "loglink")
truncpareto(lower, upper, lshape = "loglink", ishape = NULL, imethod = 1)

## Arguments

lshape Parameter link function applied to the parameter $k$. See Links for more choices. A $\log$ link is the default because $k$ is positive.
scale $\quad$ Numeric. The parameter $\alpha$ below. If the user inputs a number then it is assumed known with this value. The default means it is estimated by maximum likelihood estimation, which means $\min (y)$ is used, where $y$ is the response vector.
lower, upper Numeric. Lower and upper limits for the truncated Pareto distribution. Each must be positive and of length 1 . They are called $\alpha$ and $U$ below.
ishape Numeric. Optional initial value for the shape parameter. A NULL means a value is obtained internally. If failure to converge occurs try specifying a value, e.g., 1 or 2.
imethod See CommonVGAMffArguments for information. If failure to converge occurs then try specifying a value for ishape.

## Details

A random variable $Y$ has a Pareto distribution if

$$
P[Y>y]=C / y^{k}
$$

for some positive $k$ and $C$. This model is important in many applications due to the power law probability tail, especially for large values of $y$.
The Pareto distribution, which is used a lot in economics, has a probability density function that can be written

$$
f(y ; \alpha, k)=k \alpha^{k} / y^{k+1}
$$

for $0<\alpha<y$ and $0<k$. The $\alpha$ is called the scale parameter, and it is either assumed known or else $\min (y)$ is used. The parameter $k$ is called the shape parameter. The mean of $Y$ is $\alpha k /(k-1)$ provided $k>1$. Its variance is $\alpha^{2} k /\left((k-1)^{2}(k-2)\right)$ provided $k>2$.
The upper truncated Pareto distribution has a probability density function that can be written

$$
f(y)=k \alpha^{k} /\left[y^{k+1}\left(1-(\alpha / U)^{k}\right)\right]
$$

for $0<\alpha<y<U<\infty$ and $k>0$. Possibly, better names for $k$ are the index and tail parameters. Here, $\alpha$ and $U$ are known. The mean of $Y$ is $k \alpha^{k}\left(U^{1-k}-\alpha^{1-k}\right) /\left[(1-k)\left(1-(\alpha / U)^{k}\right)\right]$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

The usual or unbounded Pareto distribution has two parameters (called $\alpha$ and $k$ here) but the family function paretoff estimates only $k$ using iteratively reweighted least squares. The MLE of the $\alpha$ parameter lies on the boundary and is $\min (y)$ where $y$ is the response. Consequently, using the default argument values, the standard errors are incorrect when one does a summary on the fitted object. If the user inputs a value for alpha then it is assumed known with this value and then summary on the fitted object should be correct. Numerical problems may occur for small $k$, e.g., $k<1$.

## Note

Outside of economics, the Pareto distribution is known as the Bradford distribution.
For paretoff, if the estimate of $k$ is less than or equal to unity then the fitted values will be NAs. Also, paretoff fits the Pareto(I) distribution. See paretoIV for the more general Pareto(IV/III/II) distributions, but there is a slight change in notation: $s=k$ and $b=\alpha$.
In some applications the Pareto law is truncated by a natural upper bound on the probability tail. The upper truncated Pareto distribution has three parameters (called $\alpha, U$ and $k$ here) but the family function truncpareto() estimates only $k$. With known lower and upper limits, the ML estimator of $k$ has the usual properties of MLEs. Aban (2006) discusses other inferential details.

## Author(s)

T. W. Yee

## References

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011). Statistical Distributions, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

Aban, I. B., Meerschaert, M. M. and Panorska, A. K. (2006). Parameter estimation for the truncated Pareto distribution, Journal of the American Statistical Association, 101(473), 270-277.

## See Also

Pareto, Truncpareto, paretoIV, gpd, benini1.

## Examples

```
alpha <- 2; kay <- exp(3)
pdata <- data.frame(y = rpareto(n = 1000, scale = alpha, shape = kay))
fit <- vglm(y ~ 1, paretoff, data = pdata, trace = TRUE)
fit@extra # The estimate of alpha is here
head(fitted(fit))
with(pdata, mean(y))
coef(fit, matrix = TRUE)
summary(fit) # Standard errors are incorrect!!
# Here, alpha is assumed known
fit2 <- vglm(y ~ 1, paretoff(scale = alpha), data = pdata, trace = TRUE)
fit2@extra # alpha stored here
head(fitted(fit2))
coef(fit2, matrix = TRUE)
summary(fit2) # Standard errors are okay
# Upper truncated Pareto distribution
lower <- 2; upper <- 8; kay <- exp(2)
pdata3 <- data.frame(y = rtruncpareto(n = 100, lower = lower,
                                    upper = upper, shape = kay))
fit3 <- vglm(y ~ 1, truncpareto(lower, upper), data = pdata3, trace = TRUE)
coef(fit3, matrix = TRUE)
c(fit3@misc$lower, fit3@misc$upper)
```


## Description

Density, distribution function, quantile function and random generation for the Pareto(IV/III/II) distributions.

## Usage

```
dparetoIV(x, location = 0, scale = 1, inequality = 1, shape = 1, log = FALSE)
pparetoIV(q, location = 0, scale = 1, inequality = 1, shape = 1,
        lower.tail = TRUE, log.p = FALSE)
qparetoIV(p, location = 0, scale = 1, inequality = 1, shape = 1,
        lower.tail = TRUE, log.p = FALSE)
rparetoIV(n, location = 0, scale = 1, inequality = 1, shape = 1)
dparetoIII(x, location = 0, scale = 1, inequality = 1, log = FALSE)
pparetoIII(q, location = 0, scale = 1, inequality = 1,
        lower.tail = TRUE, log.p = FALSE)
    qparetoIII(p, location = 0, scale = 1, inequality = 1,
        lower.tail = TRUE, log.p = FALSE)
rparetoIII(n, location = 0, scale = 1, inequality = 1)
dparetoII(x, location = 0, scale = 1, shape = 1, log = FALSE)
pparetoII(q, location = 0, scale = 1, shape = 1,
        lower.tail = TRUE, log.p = FALSE)
qparetoII(p, location = 0, scale = 1, shape = 1,
        lower.tail = TRUE, log.p = FALSE)
rparetoII(n, location = 0, scale = 1, shape = 1)
dparetoI(x, scale = 1, shape = 1, log = FALSE)
pparetoI(q, scale = 1, shape = 1,
        lower.tail = TRUE, log.p = FALSE)
qparetoI(p, scale = 1, shape = 1,
        lower.tail = TRUE, log.p = FALSE)
rparetoI(n, scale = 1, shape = 1)
```


## Arguments

| $x, q$ | vector of quantiles. |
| :--- | :--- |
| $p$ | vector of probabilities. |
| $n$ | number of observations. Same as in runif. |
| location the location parameter. <br> scale, shape, inequality  <br> the (positive) scale, inequality and shape parameters.  |  |
| log Logical. If log = TRUE then the logarithm of the density is returned. <br> lower.tail, log.  |  |

Same meaning as in pnorm or qnorm.

## Details

For the formulas and other details see paretoIV.

## Value

Functions beginning with the letter $d$ give the density, functions beginning with the letter $p$ give the distribution function, functions beginning with the letter $q$ give the quantile function, and functions beginning with the letter $r$ generates random deviates.

## Note

The functions [dpqr]paretoI are the same as [dpqr]pareto except for a slight change in notation: $s=k$ and $b=\alpha$; see Pareto.

## Author(s)

T. W. Yee and Kai Huang

## References

Brazauskas, V. (2003). Information matrix for Pareto(IV), Burr, and related distributions. Comm. Statist. Theory and Methods 32, 315-325.

Arnold, B. C. (1983). Pareto Distributions. Fairland, Maryland: International Cooperative Publishing House.

## See Also

paretoIV, Pareto.

## Examples

\#\# Not run:
$x<-\operatorname{seq}(-0.2,4$, by $=0.01)$
loc <- 0 ; Scale <- 1 ; ineq <- 1 ; shape <- 1.0
plot(x, dparetoIV(x, loc, Scale, ineq, shape), type = "l", col = "blue",
main = "Blue is density, orange is cumulative distribution function",
sub $=$ "Purple are $5,10, \ldots, 95$ percentiles", ylim $=0: 1$, las $=1, \mathrm{ylab}=" "$ )
abline(h = 0, col = "blue", lty = 2)
$Q<-$ qparetoIV $(\operatorname{seq}(0.05,0.95$, by $=0.05)$, loc, Scale, ineq, shape)

lines(x, pparetoIV(x, loc, Scale, ineq, shape), col = "orange")
abline(h = 0, lty = 2)
\#\# End(Not run)
paretoIV Pareto(IV/III/II) Distribution Family Functions

## Description

Estimates three of the parameters of the Pareto(IV) distribution by maximum likelihood estimation.
Some special cases of this distribution are also handled.

## Usage

```
paretoIV(location = 0, lscale = "loglink", linequality = "loglink",
        lshape = "loglink", iscale = 1, iinequality = 1, ishape = NULL,
        imethod = 1)
paretoIII(location = 0, lscale = "loglink", linequality = "loglink",
            iscale = NULL, iinequality = NULL)
paretoII(location = 0, lscale = "loglink", lshape = "loglink",
            iscale = NULL, ishape = NULL)
```


## Arguments

location Location parameter, called $a$ below. It is assumed known.
lscale, linequality, lshape
Parameter link functions for the scale parameter (called $b$ below), inequality parameter (called $g$ below), and shape parameter (called $s$ below). See Links for more choices. A log link is the default for all because all these parameters are positive.
iscale, iinequality, ishape
Initial values for the parameters. A NULL value means that it is obtained internally. If convergence failure occurs, use these arguments to input some alternative initial values.
imethod Method of initialization for the shape parameter. Currently only values 1 and 2 are available. Try the other value if convergence failure occurs.

## Details

The Pareto(IV) distribution, which is used in actuarial science, economics, finance and telecommunications, has a cumulative distribution function that can be written

$$
F(y)=1-\left[1+((y-a) / b)^{1 / g}\right]^{-s}
$$

for $y>a, b>0, g>0$ and $s>0$. The $a$ is called the location parameter, $b$ the scale parameter, $g$ the inequality parameter, and $s$ the shape parameter.

The location parameter is assumed known otherwise the Pareto(IV) distribution will not be a regular family. This assumption is not too restrictive in modelling because in typical applications this parameter is known, e.g., in insurance and reinsurance it is pre-defined by a contract and can be represented as a deductible or a retention level.
The inequality parameter is so-called because of its interpretation in the economics context. If we choose a unit shape parameter value and a zero location parameter value then the inequality parameter is the Gini index of inequality, provided $g \leq 1$.
The fitted values are currently the median, e.g., qparetoIV is used for paretoIV().
There are a number of special cases of the Pareto(IV) distribution. These include the Pareto(I), Pareto(II), Pareto(III), and Burr family of distributions. Denoting PIV $(a, b, g, s)$ as the Pareto(IV) distribution, the Burr distribution $\operatorname{Burr}(b, g, s)$ is $\operatorname{PIV}(a=0, b, 1 / g, s)$, the Pareto(III) distribution $\operatorname{PIII}(a, b, g)$ is $\operatorname{PIV}(a, b, g, s=1)$, the $\operatorname{Pareto(II)~distribution~} \operatorname{PII}(a, b, s)$ is $\operatorname{PIV}(a, b, g=$ $1, s)$, and the Pareto(I) distribution $P I(b, s)$ is $\operatorname{PIV}(b, b, g=1, s)$. Thus the Burr distribution can be fitted using the negloglink link function and using the default location=0 argument. The

Pareto(I) distribution can be fitted using paretoff but there is a slight change in notation: $s=k$ and $b=\alpha$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

The Pareto(IV) distribution is very general, for example, special cases include the Pareto(I), Pareto(II), Pareto(III), and Burr family of distributions. [Johnson et al. (1994) says on p. 19 that fitting Type IV by ML is very difficult and rarely attempted]. Consequently, reasonably good initial values are recommended, and convergence to a local solution may occur. For this reason setting trace=TRUE is a good idea for monitoring the convergence. Large samples are ideally required to get reasonable results.

## Note

The extra slot of the fitted object has a component called "location" which stores the location parameter value(s).

## Author(s)

T. W. Yee

## References

Johnson N. L., Kotz S., and Balakrishnan N. (1994). Continuous Univariate Distributions, Volume 1, 2nd ed. New York: Wiley.
Brazauskas, V. (2003). Information matrix for Pareto(IV), Burr, and related distributions. Comm. Statist. Theory and Methods 32, 315-325.
Arnold, B. C. (1983). Pareto Distributions. Fairland, Maryland: International Cooperative Publishing House.

## See Also

ParetoIV, paretoff, gpd.

## Examples

```
pdata <- data.frame(y = rparetoIV(2000, scale = exp(1),
    ineq = exp(-0.3), shape = exp(1)))
## Not run: par(mfrow = c(2, 1))
with(pdata, hist(y)); with(pdata, hist(log(y)))
## End(Not run)
fit <- vglm(y ~ 1, paretoIV, data = pdata, trace = TRUE)
head(fitted(fit))
summary(pdata)
coef(fit, matrix = TRUE)
```

```
Coef(fit)
```

summary(fit)

## Perks

## Description

Density, cumulative distribution function, quantile function and random generation for the Perks distribution.

## Usage

dperks(x, scale = 1, shape, log = FALSE)
pperks(q, scale = 1, shape, lower.tail = TRUE, log.p = FALSE)
qperks(p, scale = 1, shape, lower.tail = TRUE, log.p = FALSE)
rperks(n, scale $=1$, shape)

## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$n \quad$ number of observations. Same as in runif.
log Logical. If log = TRUE then the logarithm of the density is returned.
lower.tail, log.p
Same meaning as in pnorm or qnorm.
shape, scale positive shape and scale parameters.

## Details

See perks for details.

## Value

dperks gives the density, pperks gives the cumulative distribution function, qperks gives the quantile function, and rperks generates random deviates.

## Author(s)

T. W. Yee and Kai Huang

## See Also

perks.

## Examples

```
    probs <- seq(0.01, 0.99, by = 0.01)
    Shape <- exp(-1.0); Scale <- exp(1);
    max(abs(pperks(qperks(p = probs, Shape, Scale),
        Shape, Scale) - probs)) # Should be 0
    ## Not run: x <- seq(-0.1, 07, by = 0.01);
plot(x, dperks(x, Shape, Scale), type = "l", col = "blue", las = 1,
        main = "Blue is density, orange is cumulative distribution function",
        sub = "Purple lines are the 10,20,...,90 percentiles",
        ylab = "", ylim = 0:1)
abline(h = 0, col = "blue", lty = 2)
lines(x, pperks(x, Shape, Scale), col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qperks(probs, Shape, Scale)
lines(Q, dperks(Q, Shape, Scale), col = "purple", lty = 3, type = "h")
pperks(Q, Shape, Scale) - probs # Should be all zero
abline(h = probs, col = "purple", lty = 3)
## End(Not run)
```

perks Perks Distribution Family Function

## Description

Maximum likelihood estimation of the 2-parameter Perks distribution.

## Usage

```
perks(lscale = "loglink", lshape = "loglink",
        iscale = NULL, ishape = NULL,
        gscale = exp(-5:5), gshape = exp(-5:5),
        nsimEIM = 500, oim.mean = FALSE, zero = NULL, nowarning = FALSE)
```


## Arguments

nowarning Logical. Suppress a warning? Ignored for VGAM 0.9-7 and higher.
lscale, lshape Parameter link functions applied to the shape parameter shape, scale parameter scale. All parameters are treated as positive here See Links for more choices.
iscale, ishape Optional initial values. A NULL means a value is computed internally.
gscale, gshape See CommonVGAMffArguments.
nsimEIM, zero See CommonVGAMfffArguments.
oim.mean To be currently ignored.

## Details

The Perks distribution has cumulative distribution function

$$
F(y ; \alpha, \beta)=1-\left\{\frac{1+\alpha}{1+\alpha e^{\beta y}}\right\}^{1 / \beta}
$$

which leads to a probability density function

$$
f(y ; \alpha, \beta)=[1+\alpha]^{1 / \beta} \alpha e^{\beta y} /\left(1+\alpha e^{\beta y}\right)^{1+1 / \beta}
$$

for $\alpha>0, \beta>0, y>0$. Here, $\beta$ is called the scale parameter scale, and $\alpha$ is called a shape parameter. The moments for this distribution do not appear to be available in closed form.
Simulated Fisher scoring is used and multiple responses are handled.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

A lot of care is needed because this is a rather difficult distribution for parameter estimation. If the self-starting initial values fail then try experimenting with the initial value arguments, especially iscale. Successful convergence depends on having very good initial values. Also, monitor convergence by setting trace $=$ TRUE .

## Author(s)

T. W. Yee

## References

Perks, W. (1932). On some experiments in the graduation of mortality statistics. Journal of the Institute of Actuaries, 63, 12-40.

Richards, S. J. (2012). A handbook of parametric survival models for actuarial use. Scandinavian Actuarial Journal. 1-25.

## See Also

dperks, simulate.vlm.

## Examples

```
## Not run: set.seed(123)
pdata <- data.frame(x2 = runif(nn <- 1000)) # x2 unused
pdata <- transform(pdata, eta1 = -1,
        ceta1 = 1)
pdata <- transform(pdata, shape1 = exp(eta1),
    scale1 = exp(ceta1))
pdata <- transform(pdata, y1 = rperks(nn, shape = shape1, scale = scale1))
```

```
fit1 <- vglm(y1 ~ 1, perks, data = pdata, trace = TRUE)
coef(fit1, matrix = TRUE)
summary(fit1)
## End(Not run)
```

```
perspqrrvglm Perspective plot for QRR-VGLMs
```


## Description

Produces a perspective plot for a CQO model (QRR-VGLM). It is only applicable for rank-1 or rank-2 models with argument noRRR $=\sim 1$.

## Usage

```
perspqrrvglm(x, varI.latvar = FALSE, refResponse = NULL, show.plot = TRUE,
    xlim = NULL, ylim = NULL, zlim = NULL,
    gridlength = if (Rank == 1) 301 else c(51,51),
    which.species = NULL,
    xlab = if (Rank == 1) "Latent Variable" else "Latent Variable 1",
    ylab = if (Rank == 1) "Expected Value" else "Latent Variable 2",
    zlab = "Expected value", labelSpecies = FALSE,
    stretch = 1.05, main = "", ticktype = "detailed",
    col = if (Rank == 1) par()$col else "white",
    llty = par()$lty, llwd = par()$lwd,
    add1 = FALSE, ...)
```


## Arguments

X
Object of class "qrrvglm", i.e., a constrained quadratic ordination (CQO) object.
varI.latvar Logical that is fed into Coef. qrrvglm.
refResponse Integer or character that is fed into Coef.qrrvglm.
show.plot Logical. Plot it?
xlim, ylim Limits of the $x$ - and y-axis. Both are numeric of length 2. See par.
zlim Limits of the z-axis. Numeric of length 2. Ignored if rank is 1 . See par.
gridlength Numeric. The fitted values are evaluated on a grid, and this argument regulates the fineness of the grid. If Rank $=2$ then the argument is recycled to length 2, and the two numbers are the number of grid points on the $x$ - and $y$-axes respectively.
which.species Numeric or character vector. Indicates which species are to be plotted. The default is to plot all of them. If numeric, it should contain values in the set $\{1,2, \ldots, S\}$ where $S$ is the number of species.
$x l a b, y l a b \quad$ Character caption for the $x$-axis and $y$-axis. By default, a suitable caption is found. See the xlab argument in plot or title.

| zlab | Character caption for the z-axis. Used only if Rank = 2. By default, a suitable <br> caption is found. See the xlab argument in plot or title. <br> Logical. Whether the species should be labelled with their names. Used for <br> Rank = 1 only. The position of the label is just above the species' maximum. |
| :--- | :--- |
| labelSpecies |  |
| stretch | Numeric. A value slightly more than 1, this argument adjusts the height of the <br> y-axis. Used for Rank = 1 only. |
| main | Character, giving the title of the plot. See the main argument in plot or title. |
| ticktype | Tick type. Used only if Rank = 2. See persp for more information. <br> col |
| Color. See persp for more information. |  |
| llty | Line type. Rank-1 models only. See the lty argument of par. |
| llwd | Line width. Rank-1 models only. See the lwd argument of par. |
| add1 | Logical. Add to an existing plot? Used only for rank-1 models. <br> $\ldots$ | | Arguments passed into persp. Useful arguments here include theta and phi, |
| :--- |
| which control the position of the eye. |

## Details

For a rank-1 model, a perspective plot is similar to lvplot. qrrvglm but plots the curves along a fine grid and there is no rugplot to show the site scores.
For a rank-2 model, a perspective plot has the first latent variable as the x -axis, the second latent variable as the $y$-axis, and the expected value (fitted value) as the z-axis. The result of a CQO is that each species has a response surface with elliptical contours. This function will, at each grid point, work out the maximum fitted value over all the species. The resulting response surface is plotted. Thus rare species will be obscured and abundant species will dominate the plot. To view rare species, use the which. species argument to select a subset of the species.
A perspective plot will be performed if noRRR $=\sim 1$, and Rank $=1$ or 2 . Also, all the tolerance matrices of those species to be plotted must be positive-definite.

## Value

For a rank-2 model, a list with the following components.

$$
\begin{array}{ll}
\text { fitted } & \text { A }\left(G_{1} \times G_{2}\right) \text { by } M \text { matrix of fitted values on the grid. Here, } G_{1} \text { and } G_{2} \text { are the } \\
\text { two values of gridlength. } \\
\text { latvar1grid, latvar2grid }
\end{array} \text { The grid points for the x-axis and y-axis. }
$$

For a rank-1 model, the components latvar2grid and max.fitted are NULL.

## Note

Yee (2004) does not refer to perspective plots. Instead, contour plots via lvplot. qrrvglm are used.
For rank-1 models, a similar function to this one is lvplot. qrrvglm. It plots the fitted values at the actual site score values rather than on a fine grid here. The result has the advantage that the user sees the curves as a direct result from a model fitted to data whereas here, it is easy to think that the smooth bell-shaped curves are the truth because the data is more of a distance away.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2004). A new technique for maximum-likelihood canonical Gaussian ordination. Ecological Monographs, 74, 685-701.

## See Also

persp, cqo, Coef. qrrvglm, lvplot.qrrvglm, par, title.

## Examples

> \#\# Not run:
hspider[, 1:6] <- scale(hspider[, 1:6]) \# Good idea when I.tolerances = TRUE set.seed(111)
r1 <- cqo(cbind(Alopacce, Alopcune, Alopfabr, Arctlute, Arctperi,
Auloalbi, Pardmont, Pardnigr, Pardpull, Trocterr) ~ WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux, poissonff, data $=$ hspider, trace $=$ FALSE, I.tolerances $=$ TRUE)
set.seed(111) \# r2 below is an ill-conditioned model
r2 <- cqo(cbind(Alopacce, Alopcune, Alopfabr, Arctlute, Arctperi,
Auloalbi, Pardmont, Pardnigr, Pardpull, Trocterr) ~
WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux, isd.lv = c(2.4, 1.0), Muxfactor = 3.0, trace = FALSE, poissonff, data $=$ hspider, Rank $=2$, eq.tolerances $=$ TRUE)
sort(deviance(r1, history = TRUE)) \# A history of all the fits
sort(deviance(r2, history = TRUE)) \# A history of all the fits
if (deviance (r2) > 857) stop("suboptimal fit obtained")
$\operatorname{persp}(r 1, x \lim =c(-6,5)$, col $=1: 4$, label $=$ TRUE $)$
\# Involves all species
$\operatorname{persp}(r 2, x \lim =c(-6,5), y l i m=c(-4,5)$, theta $=10$, phi $=20, \operatorname{zlim}=c(0,220))$
\# Omit the two dominant species to see what is behind them
persp(r2, xlim $=c(-6,5), y l i m=c(-4,5)$, theta $=10$, phi $=20, z l i m=c(0,220)$, which $=(1: 10)[-c(8,10)])$ \# Use zlim to retain the original z-scale
\#\# End(Not run)
pgamma.deriv

## Description

The first two derivatives of the incomplete gamma integral.

## Usage

pgamma.deriv(q, shape, tmax $=100$ )

## Arguments

q, shape As in pgamma but these must be vectors of positive values only and finite.
tmax Maximum number of iterations allowed in the computation (per q value).

## Details

Write $x=q$ and shape $=a$. The first and second derivatives with respect to $q$ and $a$ are returned. This function is similar in spirit to pgamma; define

$$
P(a, x)=\frac{1}{\Gamma(a)} \int_{0}^{x} t^{a-1} e^{-t} d t
$$

so that $P(a, x)$ is pgamma $(\mathrm{x}, \mathrm{a})$. Currently a 6 -column matrix is returned (in the future this may change and an argument may be supplied so that only what is required by the user is computed.)

The computations use a series expansion for $a \leq x \leq 1$ or or $x<a$, else otherwise a continued fraction expansion. Machine overflow can occur for large values of $x$ when $x$ is much greater than $a$.

## Value

The first 5 columns, running from left to right, are the derivatives with respect to: $x, x^{2}, a, a^{2}, x a$. The 6th column is $P(a, x)$ (but it is not as accurate as calling pgamma directly).

## Note

If convergence does not occur then try increasing the value of tmax.
Yet to do: add more arguments to give greater flexibility in the accuracy desired and to compute only quantities that are required by the user.

## Author(s)

T. W. Yee wrote the wrapper function to the Fortran subroutine written by R. J. Moore. The subroutine was modified to run using double precision. The original code came from http://lib. stat.cmu. edu/apstat/187. but this website has since become stale.

## References

Moore, R. J. (1982). Algorithm AS 187: Derivatives of the Incomplete Gamma Integral. Journal of the Royal Statistical Society, Series C (Applied Statistics), 31(3), 330-335.

## See Also

pgamma.deriv.unscaled, pgamma.

## Examples

```
x <- seq(2, 10, length = 501)
head(ans <- pgamma.deriv(x, 2))
## Not run: par(mfrow = c(2, 3))
for (jay in 1:6)
    plot(x, ans[, jay], type = "l", col = "blue", cex.lab = 1.5,
        cex.axis = 1.5, las = 1, log = "x",
        main = colnames(ans)[jay], xlab = "q", ylab = "")
## End(Not run)
```

pgamma.deriv.unscaled Derivatives of the Incomplete Gamma Integral (Unscaled Version)

## Description

The first two derivatives of the incomplete gamma integral with scaling.

## Usage

pgamma.deriv.unscaled(q, shape)

## Arguments

q, shape As in pgamma and pgamma.deriv but these must be vectors of positive values only and finite.

## Details

Define

$$
G(x, a)=\int_{0}^{x} t^{a-1} e^{-t} d t
$$

so that $G(x, a)$ is pgamma ( $\mathrm{x}, \mathrm{a}) *$ gamma(a). Write $x=q$ and shape $=a$. The 0th and first and second derivatives with respect to $a$ of $G$ are returned. This function is similar in spirit to pgamma. deriv but here there is no gamma function to scale things. Currently a 3-column matrix is returned (in the future this may change and an argument may be supplied so that only what is required by the user is computed.) This function is based on Wingo (1989).

## Value

The 3 columns, running from left to right, are the $0: 2$ th derivatives with respect to $a$.

## Warning

These function seems inaccurate for $q=1$ and $q=2$; see the plot below.

## Author(s)

T. W. Yee.

## References

See truncweibull.

## See Also

pgamma.deriv, pgamma.

## Examples

```
x <- 3; aa <- seq(0.3, 04, by = 0.01)
ans.u <- pgamma.deriv.unscaled(x, aa)
head(ans.u)
## Not run: par(mfrow = c(1, 3))
for (jay in 1:3) {
    plot(aa, ans.u[, jay], type = "l", col = "blue", cex.lab = 1.5,
                cex.axis = 1.5, las = 1, main = colnames(ans.u)[jay],
            log = "", xlab = "shape", ylab = "")
    abline(h = 0, v = 1:2, lty = "dashed", col = "gray") # Inaccurate at 1 and 2
}
## End(Not run)
```

plotdeplot.lmscreg Density Plot for LMS Quantile Regression

## Description

Plots a probability density function associated with a LMS quantile regression.

## Usage

plotdeplot.lmscreg(answer, y.arg, add.arg = FALSE, xlab = "", ylab = "density", xlim = NULL, ylim = NULL, llty.arg = par()\$lty, col.arg = par()\$col, llwd.arg = par()\$lwd, ...)

## Arguments

answer Output from functions of the form deplot. ??? where ??? is the name of the VGAM LMS family function, e.g., lms.yjn. See below for details.
y.arg Numerical vector. The values of the response variable at which to evaluate the density. This should be a grid that is fine enough to ensure the plotted curves are smooth.
add.arg Logical. Add the density to an existing plot?
$x l a b, y l a b \quad$ Caption for the $x$ - and $y$-axes. See par.
xlim, ylim Limits of the $x$ - and y-axes. See par.

| llty.arg | Line type. See the lty argument of par. |
| :--- | :--- |
| col.arg | Line color. See the col argument of par. |
| llwd.arg | Line width. See the lwd argument of par. |
| $\ldots$ | Arguments passed into the plot function when setting up the entire plot. Useful <br> arguments here include main and las. |

## Details

The above graphical parameters offer some flexibility when plotting the quantiles.

## Value

The list answer, which has components

```
newdata The argument newdata above from the argument list of deplot.lmscreg, or a
        one-row data frame constructed out of the x0 argument.
y The argument y.arg above.
density Vector of the density function values evaluated at y.arg.
```


## Note

While the graphical arguments of this function are useful to the user, this function should not be called directly.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2004). Quantile regression via vector generalized additive models. Statistics in Medicine, 23, 2295-2315.

## See Also

deplot.lmscreg.

## Examples

```
fit <- vgam(BMI ~ s(age, df = c(4,2)), lms.bcn(zero = 1), bmi.nz)
## Not run: y = seq(15, 43, by = 0.25)
deplot(fit, x0 = 20, y = y, xlab = "BMI", col = "green", llwd = 2,
    main = "BMI distribution at ages 20 (green), 40 (blue), 60 (orange)")
deplot(fit, x0 = 40, y = y, add = TRUE, col = "blue", llwd = 2)
deplot(fit, x0 = 60, y = y, add = TRUE, col = "orange", llwd = 2) -> aa
names(aa@post$deplot)
aa@post$deplot$newdata
head(aa@post$deplot$y)
head(aa@post$deplot$density)
## End(Not run)
```

```
plotqrrvglm Model Diagnostic Plots for QRR-VGLMs
```


## Description

The residuals of a QRR-VGLM are plotted for model diagnostic purposes.

## Usage

```
plotqrrvglm(object, rtype = c("response", "pearson", "deviance", "working"),
    ask = FALSE,
    main = paste(Rtype, "residuals vs latent variable(s)"),
    xlab = "Latent Variable",
    I.tolerances = object@control$eq.tolerances, ...)
```


## Arguments

| object | An object of class "qrrvglm". |
| :--- | :--- |
| rtype | Character string giving residual type. By default, the first one is chosen. |
| ask | Logical. If TRUE, the user is asked to hit the return key for the next plot. |
| main | Character string giving the title of the plot. |
| xlab | Character string giving the x-axis caption. |
| I. tolerances | Logical. This argument is fed into Coef (object, I.tolerances = I. tolerances). |
| $\ldots$ | Other plotting arguments (see par). |

## Details

Plotting the residuals can be potentially very useful for checking that the model fit is adequate.

## Value

The original object.

## Note

An ordination plot of a QRR-VGLM can be obtained by lvplot.qrrvglm.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2004). A new technique for maximum-likelihood canonical Gaussian ordination. Ecological Monographs, 74, 685-701.

## See Also

lvplot.qrrvglm, cqo.

## Examples

```
## Not run:
# QRR-VGLM on the hunting spiders data
# This is computationally expensive
set.seed(111) # This leads to the global solution
hspider[, 1:6] <- scale(hspider[, 1:6]) # Standardize environ vars
p1 <- cqo(cbind(Alopacce, Alopcune, Alopfabr, Arctlute, Arctperi,
            Auloalbi, Pardlugu, Pardmont, Pardnigr, Pardpull,
            Trocterr, Zoraspin) ~
    WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
    poissonff, data = hspider, Crow1positive = FALSE)
par(mfrow = c(3, 4))
plot(p1, rtype = "response", col = "blue", pch = 4, las = 1, main = "")
## End(Not run)
```

plotqtplot.lmscreg Quantile Plot for LMS Quantile Regression

## Description

Plots the quantiles associated with a LMS quantile regression.

## Usage

plotqtplot.lmscreg(fitted.values, object, newdata $=$ NULL,
percentiles = object@misc\$percentiles, lp = NULL,
add. arg = FALSE, y = if (length(newdata)) FALSE else TRUE,
spline.fit $=$ FALSE, label $=$ TRUE, size.label $=0.06$,
xlab $=$ NULL, ylab = "",
pch = par()\$pch, pcex = par()\$cex, pcol.arg = par()\$col,
xlim = NULL, ylim = NULL,
llty.arg = par()\$lty, lcol.arg = par()\$col, llwd.arg = par()\$lwd,
tcol.arg $=\operatorname{par}() \$ c o l, \operatorname{tadj}=1, \ldots)$

## Arguments

fitted.values Matrix of fitted values.
object A VGAM quantile regression model, i.e., an object produced by modelling functions such as vglm and vgam with a family function beginning with "lms.", e.g., lms.yjn.
newdata Data frame at which predictions are made. By default, the original data are used.

| percentiles | Numerical vector with values between 0 and 100 that specify the percentiles (quantiles). The default is to use the percentiles when fitting the model. For example, the value 50 corresponds to the median. |
| :---: | :---: |
| $1 p$ | Length of percentiles. |
| add.arg | Logical. Add the quantiles to an existing plot? |
| y | Logical. Add the response as points to the plot? |
| spline.fit | Logical. Add a spline curve to the plot? |
| label | Logical. Add the percentiles (as text) to the plot? |
| size.label | Numeric. How much room to leave at the RHS for the label. It is in percent (of the range of the primary variable). |
| xlab | Caption for the x-axis. See par. |
| ylab | Caption for the x -axis. See par. |
| pch | Plotting character. See par. |
| pcex | Character expansion of the points. See par. |
| pcol.arg | Color of the points. See the col argument of par. |
| xlim | Limits of the x -axis. See par. |
| ylim | Limits of the y -axis. See par. |
| llty.arg | Line type. Line type. See the lty argument of par. |
| lcol.arg | Color of the lines. See the col argument of par. |
| llwd.arg | Line width. See the lwd argument of par. |
| tcol.arg | Color of the text (if label is TRUE). See the col argument of par. |
| tadj | Text justification. See the adj argument of par. |
|  | Arguments passed into the plot function when setting up the entire plot. Useful arguments here include main and las. |

## Details

The above graphical parameters offer some flexibility when plotting the quantiles.

## Value

The matrix of fitted values.

## Note

While the graphical arguments of this function are useful to the user, this function should not be called directly.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2004). Quantile regression via vector generalized additive models. Statistics in Medicine, 23, 2295-2315.

## See Also

```
qtplot.lmscreg.
```


## Examples

```
## Not run:
fit <- vgam(BMI ~ s(age, df = c(4,2)), lms.bcn(zero = 1), data = bmi.nz)
qtplot(fit)
qtplot(fit, perc = c(25,50,75,95), lcol = "blue", tcol = "blue", llwd = 2)
## End(Not run)
```

plotrcim0 Main Effects Plot for a Row-Column Interaction Model (RCIM)

## Description

Produces a main effects plot for Row-Column Interaction Models (RCIMs).

## Usage

plotrcim0(object, centered $=$ TRUE, which. plots $=c(1,2)$, hline0 = TRUE, hlty = "dashed", hcol = par()\$col, hlwd = par()\$lwd, rfirst $=1$, cfirst $=1$, rtype = "h", ctype = "h",
rcex.lab = 1, rcex.axis = 1, rtick = FALSE,
ccex.lab = 1, ccex.axis $=1$, ctick = FALSE,
rmain = "Row effects", rsub = "",
rxlab = "", rylab = "Row effects",
cmain = "Column effects", csub = "",
cxlab= "", cylab = "Column effects",
rcol = par()\$col, ccol = par()\$col,
no. warning $=$ FALSE, ...)

## Arguments

object An rcim object. This should be of rank-0, i.e., main effects only and no interactions.
which.plots Numeric, describing which plots are to be plotted. The row effects plot is 1 and the column effects plot is 2 . Set the value 0 , say, for no plots at all.

| centered | Logical. If TRUE then the row and column effects are centered (but not scaled) by scale. If FALSE then the raw effects are used (of which the first are zero by definition). |
| :---: | :---: |
| hline0, hlty, hcol, hlwd |  |
|  | hline0 is logical. If TRUE then a horizontal line is plotted at 0 and the other arguments describe this line. Probably having hline0 $=$ TRUE only makes sense when centered $=$ TRUE . |
| rfirst, cfirst | the level of row that is placed first in the row effects plot, etc. |
| rmain, cmain | Character. rmain is the main label |
| rtype, ctype, rsub, csub |  |
| See the type and sub arguments of plot. |  |
| rxlab, rylab, cxlab, cylab |  |
| Character. For the row effects plot, rxlab is xlab and rylab is ylab; see par. Ditto for cxlab and cylab for the column effects plot. |  |
| rcex.lab, ccex.lab |  |
|  | Numeric. rcex.lab is cex for the row effects plot label, etc. |
| rcex.axis, ccex.axis |  |
| Numeric. rcex.axis is the cex argument for the row effects axis label, etc. |  |
| rtick, ctick | Logical. If rtick = TRUE then add ticks to the row effects plot, etc. |
| rcol, ccol | rcol give a colour for the row effects plot, etc. |
| no.warning | Logical. If TRUE then no warning is issued if the model is not rank- 0 . |
|  | Arguments fed into both plot calls. |

## Details

This function plots the row and column effects of a rank- 0 RCIM. As the result is a main effects plot of a regression analysis, its interpretation when centered $=$ FALSE is relative to the baseline (reference level) of a row and column, and should also be considered in light of the link function used. Many arguments that start with " $r$ " refer to the row effects plot, and " $c$ " for the column effects plot.

## Value

The original object with the post slot assigned additional information from the plot.

## Note

This function should be only used to plot the object of rank-0 RCIM. If the rank is positive then it will issue a warning.
Using an argument ylim will mean the row and column effects are plotted on a common scale; see plot.window.

## Author(s)

T. W. Yee, A. F. Hadi.

## See Also

moffset Rcim, rcim.

## Examples

```
alcoff.e <- moffset(alcoff, "6", "Mon", postfix = "*") # Effective day
fit0 <- rcim(alcoff.e, family = poissonff)
## Not run: par(oma = c(0, 0, 4, 0), mfrow = 1:2) # For all plots below too
ii <- plot(fit0, rcol = "blue", ccol = "orange",
    lwd = 4, ylim = c(-2, 2), # A common ylim
    cylab = "Effective daily effects", rylab = "Hourly effects",
    rxlab = "Hour", cxlab = "Effective day")
    ii@post # Endowed with additional information
    ## End(Not run)
    # Negative binomial example
    ## Not run:
    fit1 <- rcim(alcoff.e, negbinomial, trace = TRUE)
    plot(fit1, ylim = c(-2, 2))
    ## End(Not run)
    # Univariate normal example
    fit2 <- rcim(alcoff.e, uninormal, trace = TRUE)
    ## Not run: plot(fit2, ylim = c(-200, 400))
    # Median-polish example
    ## Not run:
    fit3 <- rcim(alcoff.e, alaplace1(tau = 0.5), maxit = 1000, trace = FALSE)
    plot(fit3, ylim = c(-200, 250))
    ## End(Not run)
    # Zero-inflated Poisson example on "crashp" (no 0s in alcoff)
    ## Not run:
    cbind(rowSums(crashp)) # Easy to see the data
    cbind(colSums(crashp)) # Easy to see the data
    fit4 <- rcim(Rcim(crashp, rbaseline = "5", cbaseline = "Sun"),
    zipoissonff, trace = TRUE)
    plot(fit4, ylim = c(-3, 3))
## End(Not run)
```

plotvgam Default VGAM Plotting

## Description

Component functions of a vgam-class object can be plotted with plotvgam(). These are on the scale of the linear/additive predictor.

## Usage

```
plotvgam(x, newdata = NULL, y = NULL, residuals = NULL,
            rugplot = TRUE, se = FALSE, scale = 0, raw = TRUE,
            offset.arg = 0, deriv.arg = 0, overlay = FALSE,
                type.residuals = c("deviance", "working", "pearson", "response"),
                plot.arg = TRUE, which.term = NULL, which.cf = NULL,
                control = plotvgam.control(...), varxij = 1, ...)
```


## Arguments

x
newdata Data frame. May be used to reconstruct the original data set.
y Unused.
residuals Logical. If TRUE then residuals are plotted. See type.residuals
rugplot Logical. If TRUE then a rug plot is plotted at the foot of each plot. These values are jittered to expose ties.
se
Logical. If TRUE then approximate $\pm 2$ pointwise standard error bands are included in the plot.
scale Numerical. By default, each plot will have its own y-axis scale. However, by specifying a value, each plot's y-axis scale will be at least scale wide.
raw Logical. If TRUE then the smooth functions are those obtained directly by the algorithm, and are plotted without having to premultiply with the constraint matrices. If FALSE then the smooth functions have been premultiply by the constraint matrices. The raw argument is directly fed into predict.vgam().
offset.arg Numerical vector of length $r$. These are added to the component functions. Useful for separating out the functions when overlay is TRUE. If overlay is TRUE and there is one covariate then using the intercept values as the offsets can be a good idea.
deriv.arg Numerical. The order of the derivative. Should be assigned an small integer such as $0,1,2$. Only applying to $s()$ terms, it plots the derivative.
overlay Logical. If TRUE then component functions of the same covariate are overlaid on each other. The functions are centered, so offset.arg can be useful when overlay is TRUE.
type.residuals if residuals is TRUE then the first possible value of this vector, is used to specify the type of residual.
plot.arg Logical. If FALSE then no plot is produced.
which.term Character or integer vector containing all terms to be plotted, e.g., which.term $=c(" s($ age $) ", " s(h e i g h t "))$ or which.term $=c(2,5,9)$. By default, all are plotted.
which.cf An integer-valued vector specifying which linear/additive predictors are to be plotted. The values must be from the set $\{1,2, \ldots, r\}$. By default, all are plotted.
control Other control parameters. See plotvgam. control.
... Other arguments that can be fed into plotvgam. control. This includes line colors, line widths, line types, etc.
varxij Positive integer. Used if xij of vglm. control was used, this chooses which inner argument the component is plotted against. This argument is related to raw = TRUE and terms such as NS (dum1, dum2) and constraint matrices that have more than one column. The default would plot the smooth against dum1 but setting varxij $=2$ could mean plotting the smooth against dum2. See the VGAM website for further information.

## Details

In this help file $M$ is the number of linear/additive predictors, and $r$ is the number of columns of the constraint matrix of interest.

Many of plotvgam()'s options can be found in plotvgam. control, e.g., line types, line widths, colors.

## Value

The original object, but with the preplot slot of the object assigned information regarding the plot.

## Note

While plot(fit) will work if class(fit) is "vgam", it is necessary to use plotvgam(fit) explicitly otherwise.
plotvgam() is quite buggy at the moment.

## Author(s)

Thomas W. Yee

## See Also

vgam, plotvgam. control, predict.vgam, plotvglm, vglm.

## Examples

```
coalminers <- transform(coalminers, Age = (age - 42) / 5)
fit <- vgam(cbind(nBnW, nBW, BnW, BW) ~ s(Age),
    binom2.or(zero = NULL), data = coalminers)
## Not run: par(mfrow = c(1,3))
plot(fit, se = TRUE, ylim = c(-3, 2), las = 1)
plot(fit, se = TRUE, which.cf = 1:2, lcol = "blue", scol = "orange",
    ylim = c(-3, 2))
plot(fit, se = TRUE, which.cf = 1:2, lcol = "blue", scol = "orange",
    overlay = TRUE)
## End(Not run)
```


## Description

Provides default values for many arguments available for plotvgam().

## Usage

```
plotvgam.control(which.cf = NULL,
    xlim = NULL, ylim = NULL, llty = par()$lty,
    slty = "dashed", pcex = par()$cex,
    pch = par()$pch, pcol = par()$col,
    lcol = par()$col, rcol = par()$col,
    scol = par()$col, llwd = par()$lwd, slwd = par()$lwd,
    add.arg = FALSE, one.at.a.time = FALSE,
    .include.dots = TRUE, noxmean = FALSE,
    shade = FALSE, shcol = "gray80", ...)
```


## Arguments

which.cf Integer vector specifying which component functions are to be plotted (for each covariate). Must have values from the set $\{1,2, \ldots, M\}$.
$x \lim \quad$ Range for the $x$-axis.
ylim Range for the $y$-axis.
lly $\quad$ Line type for the fitted functions (lines). Fed into par (lty).
slty Line type for the standard error bands. Fed into par (lty).
pcex Character expansion for the points (residuals). Fed into par (cex).
pch Character used for the points (residuals). Same as par (pch).
pcol Color of the points. Fed into par (col).
lcol Color of the fitted functions (lines). Fed into par (col).
rcol Color of the rug plot. Fed into par (col).
scol Color of the standard error bands. Fed into par (col).
llwd Line width of the fitted functions (lines). Fed into par (lwd).
slwd Line width of the standard error bands. Fed into par (lwd).
add.arg Logical. If TRUE then the plot will be added to an existing plot, otherwise a new plot will be made.
one.at.a.time Logical. If TRUE then the plots are done one at a time, with the user having to hit the return key between the plots.
.include.dots Not to be used by the user.
noxmean Logical. If TRUE then the point at the mean of $x$, which is added when standard errors are specified and it thinks the function is linear, is not added. One might use this argument if ylab is specified.
shade, shcol shade is logical; if TRUE then the pointwise SE band is shaded gray by default. The colour can be adjusted by setting shcol. These arguments are ignored unless se = TRUE and overlay = FALSE; If shade = TRUE then scol is ignored.
... Other arguments that may be fed into par (). In the above, $M$ is the number of linear/additive predictors.

## Details

The most obvious features of plotvgam can be controlled by the above arguments.

## Value

A list with values matching the arguments.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Wild, C. J. (1996). Vector generalized additive models. Journal of the Royal Statistical Society, Series B, Methodological, 58, 481-493.

## See Also

```
plotvgam.
```


## Examples

```
    plotvgam.control(lcol = c("red", "blue"), scol = "darkgreen", se = TRUE)
```

    plotvglm Plots for VGLMs
    
## Description

Currently this function plots the Pearson residuals versus the linear predictors ( $M$ plots) and plots the Pearson residuals versus the hat values ( $M$ plots).

## Usage

plotvglm(x, which = "(All)", ...)

## Arguments

X
which

An object of class "vglm" (see vglm-class) or inherits from that class.
If a subset of the plots is required, specify a subset of the numbers $1:(2 * M)$. The default is to plot them all.
... Arguments fed into the primitive plot functions.

## Details

This function is under development. Currently it plots the Pearson residuals against the predicted values (on the transformed scale) and the hat values. There are $2 M$ plots in total, therefore users should call par to assign, e.g., the mfrow argument. Note: Section 3.7 of Yee (2015) describes the Pearson residuals and hat values for VGLMs.

## Value

Returns the object invisibly.

## Author(s)

T. W. Yee

## See Also

plotvgam, plotvgam. control, vglm.

## Examples

```
## Not run:
ndata <- data.frame(x2 = runif(nn <- 200))
ndata <- transform(ndata, y1 = rnbinom(nn, mu = exp(3+x2), size = exp(1)))
fit1 <- vglm(y1 ~ x2, negbinomial, data = ndata, trace = TRUE)
coef(fit1, matrix = TRUE)
par(mfrow = c(2, 2))
plot(fit1)
# Manually produce the four plots
plot(fit1, which = 1, col = "blue", las = 1, main = "main1")
abline(h = 0, lty = "dashed", col = "gray50")
plot(fit1, which = 2, col = "blue", las = 1, main = "main2")
abline(h = 0, lty = "dashed", col = "gray50")
plot(fit1, which = 3, col = "blue", las = 1, main = "main3")
plot(fit1, which = 4, col = "blue", las = 1, main = "main4")
## End(Not run)
```

pneumo Pneumoconiosis in Coalminers Data

## Description

The pneumo data frame has 8 rows and 4 columns. Exposure time is explanatory, and there are 3 ordinal response variables.

## Usage

data(pneumo)

## Format

This data frame contains the following columns:
exposure.time a numeric vector, in years
normal a numeric vector, counts
mild a numeric vector, counts
severe a numeric vector, counts

## Details

These were collected from coalface workers. In the original data set, the two most severe categories were combined.

## Source

Ashford, J.R., 1959. An approach to the analysis of data for semi-quantal responses in biological assay. Biometrics, 15, 573-581.

## References

McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, 2nd ed. London: Chapman \& Hall.

## See Also

```
    cumulative.
```


## Examples

\# Fit the proportional odds model, p.179, in McCullagh and Nelder (1989)
pneumo <- transform(pneumo, let $=\log ($ exposure.time))
vglm(cbind(normal, mild, severe) ~ let, propodds, data $=$ pneumo)
poisson.points Poisson-points-on-a-plane/volume Distances Distribution

## Description

Estimating the density parameter of the distances from a fixed point to the u-th nearest point, in a plane or volume.

## Usage

```
poisson.points(ostatistic, dimension = 2, link = "loglink",
    idensity = NULL, imethod = 1)
```


## Arguments

ostatistic Order statistic. A single positive value, usually an integer. For example, the value 5 means the response are the distances of the fifth nearest value to that point (usually over many planes or volumes). Non-integers are allowed because the value 1.5 coincides with maxwell when dimension $=2$. Note: if ostatistic $=1$ and dimension $=2$ then this VGAM family function coincides with rayleigh.
dimension $\quad$ The value 2 or $3 ; 2$ meaning a plane and 3 meaning a volume.
link Parameter link function applied to the (positive) density parameter, called $\lambda$ below. See Links for more choices.
idensity Optional initial value for the parameter. A NULL value means a value is obtained internally. Use this argument if convergence failure occurs.
imethod An integer with value 1 or 2 which specifies the initialization method for $\lambda$. If failure to converge occurs try another value and/or else specify a value for idensity.

## Details

Suppose the number of points in any region of area $A$ of the plane is a Poisson random variable with mean $\lambda A$ (i.e., $\lambda$ is the density of the points). Given a fixed point $P$, define $D_{1}, D_{2}, \ldots$ to be the distance to the nearest point to $P$, second nearest to $P$, etc. This VGAM family function estimates $\lambda$ since the probability density function for $D_{u}$ is easily derived, $u=1,2, \ldots$. Here, $u$ corresponds to the argument ostatistic.

Similarly, suppose the number of points in any volume $V$ is a Poisson random variable with mean $\lambda V$ where, once again, $\lambda$ is the density of the points. This VGAM family function estimates $\lambda$ by specifying the argument ostatistic and using dimension $=3$.
The mean of $D_{u}$ is returned as the fitted values. Newton-Raphson is the same as Fisher-scoring.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Warning

Convergence may be slow if the initial values are far from the solution. This often corresponds to the situation when the response values are all close to zero, i.e., there is a high density of points.

Formulae such as the means have not been fully checked.

## Author(s)

T. W. Yee

## See Also

```
poissonff, maxwell, rayleigh.
```


## Examples

```
    pdata <- data.frame(y = rgamma(10, shape = exp(-1))) # Not proper data!
    ostat <- 2
    fit <- vglm(y ~ 1, poisson.points(ostat, 2), data = pdata,
    trace = TRUE, crit = "coef")
    fit <- vglm(y ~ 1, poisson.points(ostat, 3), data = pdata,
    trace = TRUE, crit = "coef") # Slow convergence?
fit <- vglm(y ~ 1, poisson.points(ostat, 3, idensi = 1), data = pdata,
    trace = TRUE, crit = "coef")
    head(fitted(fit))
    with(pdata, mean(y))
    coef(fit, matrix = TRUE)
    Coef(fit)
```

poissonff Poisson Family Function

## Description

Family function for a generalized linear model fitted to Poisson responses.

## Usage

poissonff(link = "loglink", imu = NULL, imethod = 1, parallel = FALSE, zero = NULL, bred = FALSE, earg.link = FALSE, type.fitted = c("mean", "quantiles"), percentiles $=c(25,50,75))$

## Arguments

link Link function applied to the mean or means. See Links for more choices and information.
parallel A logical or formula. Used only if the response is a matrix.
imu, imethod See CommonVGAMffArguments for more information.
zero Can be an integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set $\{1,2, \ldots, M\}$, where $M$ is the number of columns of the matrix response. See CommonVGAMffArguments for more information.
bred, earg.link
Details at CommonVGAMffArguments. Setting bred = TRUE should work for multiple responses and all VGAM link functions; it has been tested for loglink, identity but further testing is required.
type.fitted, percentiles
Details at CommonVGAMffArguments.

## Details

$M$ defined above is the number of linear/additive predictors. With overdispersed data try negbinomial.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, vgam, rrvglm, cqo, and cao.

## Warning

With multiple responses, assigning a known dispersion parameter for each response is not handled well yet. Currently, only a single known dispersion parameter is handled well.

## Note

This function will handle a matrix response automatically.
Regardless of whether the dispersion parameter is to be estimated or not, its value can be seen from the output from the summary () of the object.

## Author(s)

Thomas W. Yee

## References

McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, 2nd ed. London: Chapman \& Hall.

## See Also

Links, hdeff.vglm, negbinomial, genpoisson, gaitpoisson.mix, gaitpoisson.mlm, zipoisson, pospoisson, oipospoisson, otpospoisson, skellam, mix2poisson, cens.poisson, ordpoisson, amlpoisson, inv.binomial, simulate.vlm, loglink, polf, rrvglm, cqo, cao, binomialff, poisson, Poisson, poisson. points, ruge, V1, V2, residualsvglm.

## Examples

```
poissonff()
set.seed(123)
pdata <- data.frame(x2 = rnorm(nn <- 100))
pdata <- transform(pdata, y1 = rpois(nn, exp(1 + x2)),
    y2 = rpois(nn, exp(1 + x2)))
(fit1 <- vglm(cbind(y1, y2) ~ x2, poissonff, data = pdata))
(fit2 <- vglm(y1 ~ x2, poissonff(bred = TRUE), data = pdata))
coef(fit1, matrix = TRUE)
coef(fit2, matrix = TRUE)
nn <- 200
cdata <- data.frame(x2 = rnorm(nn), x3 = rnorm(nn), x4 = rnorm(nn))
cdata <- transform(cdata, lv1 = 0 + x3 - 2*x4)
cdata <- transform(cdata, lambda1 = exp(3-0.5 * (lv1-0)^2),
    lambda2 = exp(2 - 0.5 * (lv1-1)^2),
    lambda3 = exp(2 - 0.5 * ((lv1+4)/2)^2))
```

```
    cdata <- transform(cdata, y1 = rpois(nn, lambda1),
    y2 = rpois(nn, lambda2),
    y3 = rpois(nn, lambda3))
## Not run: lvplot(p1, y = TRUE, lcol = 2:4, pch = 2:4, pcol = 2:4, rug = FALSE)
```

PoissonPoints Poisson Points Distribution

## Description

Density for the PoissonPoints distribution.

## Usage

dpois.points(x, lambda, ostatistic, dimension = 2, log = FALSE)

## Arguments

| X | vector of quantiles. |
| :--- | :--- |
| lambda | the mean density of points. |
| ostatistic | positive values, usually integers. |
| dimension | Either 2 and/or 3. |
| log | Logical; if TRUE, the logarithm is returned. |

## Details

See poisson. points, the VGAM family function for estimating the parameters, for the formula of the probability density function and other details.

## Value

dpois.points gives the density.

## See Also

poisson. points, dpois, Maxwell.

## Examples

```
## Not run: lambda <- 1; xvec <- seq(0, 2, length = 400)
plot(xvec, dpois.points(xvec, lambda, ostat = 1, dimension = 2),
    type = "l", las = 1, col = "blue",
    sub = "First order statistic",
    main = paste("PDF of PoissonPoints distribution with lambda = ",
            lambda, " and on the plane", sep = ""))
## End(Not run)
```


## Description

Density, distribution function and random generation for the Poisson lognormal distribution.

## Usage

```
dpolono(x, meanlog \(=0\), sdlog \(=1\), bigx \(=170, \ldots\) )
ppolono(q, meanlog \(=0\), sdlog \(=1\),
        isOne = 1 - sqrt( .Machine\$double.eps ), ...)
    rpolono( \(n\), meanlog \(=0\), sdlog \(=1\) )
```


## Arguments

x, q
vector of quantiles.
n
number of observations. If length $(n)>1$ then the length is taken to be the number required.
meanlog, sdlog the mean and standard deviation of the normal distribution (on the log scale). They match the arguments in Lognormal.
bigx Numeric. This argument is for handling large values of $x$ and/or when integrate fails. A first order Taylor series approximation [Equation (7) of Bulmer (1974)] is used at values of $x$ that are greater or equal to this argument. For bigx =10, he showed that the approximation has a relative error less than 0.001 for values of meanlog and sdlog "likely to be encountered in practice". The argument can be assigned Inf in which case the approximation is not used.
isOne Used to test whether the cumulative probabilities have effectively reached unity. ... Arguments passed into integrate.

## Details

The Poisson lognormal distribution is similar to the negative binomial in that it can be motivated by a Poisson distribution whose mean parameter comes from a right skewed distribution (gamma for the negative binomial and lognormal for the Poisson lognormal distribution).

## Value

dpolono gives the density, ppolono gives the distribution function, and rpolono generates random deviates.

## Note

By default, dpolono involves numerical integration that is performed using integrate. Consequently, computations are very slow and numerical problems may occur (if so then the use of . . . may be needed). Alternatively, for extreme values of $x$, meanlog, sdlog, etc., the use of bigx = Inf avoids the call to integrate, however the answer may be a little inaccurate.

For the maximum likelihood estimation of the 2 parameters a VGAM family function called polono(), say, has not been written yet.

## Author(s)

T. W. Yee. Some anonymous soul kindly wrote ppolono() and improved the original dpolono().

## References

Bulmer, M. G. (1974). On fitting the Poisson lognormal distribution to species-abundance data. Biometrics, 30, 101-110.

## See Also

lognormal, poissonff, negbinomial.

## Examples

```
meanlog <- 0.5; sdlog <- 0.5; yy <- 0:19
sum(proby <- dpolono(yy, m = meanlog, sd = sdlog)) # Should be 1
max(abs(cumsum(proby) - ppolono(yy, m = meanlog, sd = sdlog))) # Should be 0
## Not run: opar = par(no.readonly = TRUE)
par(mfrow = c(2, 2))
plot(yy, proby, type = "h", col = "blue", ylab = "P[Y=y]", log = "",
    main = paste("Poisson lognormal(m = ", meanlog,
                            ", sdl = ", sdlog, ")", sep = ""))
y <- 0:190 # More extreme values; use the approximation and plot on a log scale
(sum(proby <- dpolono(y, m = meanlog, sd = sdlog, bigx = 100))) # Should be 1
plot(y, proby, type = "h", col = "blue", ylab = "P[Y=y] (log)", log = "y",
    main = paste("Poisson lognormal(m = ", meanlog,
    ", sdl = ", sdlog, ")", sep = "")) # Note the kink at bigx
# Random number generation
table(y <- rpolono(n = 1000, m = meanlog, sd = sdlog))
hist(y, breaks = ((-1):max(y))+0.5, prob = TRUE, border = "blue")
par(opar)
## End(Not run)
```

```
pordlink Poisson-Ordinal Link Function
```


## Description

Computes the Poisson-ordinal transformation, including its inverse and the first two derivatives.

## Usage

pordlink(theta, cutpoint $=$ NULL,
inverse $=$ FALSE, deriv $=0$, short $=$ TRUE, tag $=$ FALSE)

## Arguments

theta Numeric or character. See below for further details.
cutpoint The cutpoints should be non-negative integers. If pordlink() is used as the link function in cumulative then one should choose reverse $=$ TRUE, parallel $=$ TRUE.
inverse, deriv, short, tag
Details at Links.

## Details

The Poisson-ordinal link function (POLF) can be applied to a parameter lying in the unit interval. Its purpose is to link cumulative probabilities associated with an ordinal response coming from an underlying Poisson distribution. If the cutpoint is zero then a complementary log-log link is used.
See Links for general information about VGAM link functions.

## Value

See Yee (2012) for details.

## Warning

Prediction may not work on vglm or vgam etc. objects if this link function is used.

## Note

Numerical values of theta too close to 0 or 1 or out of range result in large positive or negative values, or maybe 0 depending on the arguments. Although measures have been taken to handle cases where theta is too close to 1 or 0 , numerical instabilities may still arise.
In terms of the threshold approach with cumulative probabilities for an ordinal response this link function corresponds to the Poisson distribution (see poissonff) that has been recorded as an ordinal response using known cutpoints.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2020). Ordinal ordination with normalizing link functions for count data, (in preparation).

## See Also

Links, ordpoisson, poissonff, nbordlink, gordlink, cumulative.

## Examples

```
## Not run:
pordlink("p", cutpoint = 2, short = FALSE)
pordlink("p", cutpoint = 2, tag = TRUE)
p <- seq(0.01, 0.99, by = 0.01)
y <- pordlink(p, cutpoint = 2)
y. <- pordlink(p, cutpoint = 2, deriv = 1)
max(abs(pordlink(y, cutpoint = 2, inv = TRUE) - p)) # Should be 0
#\ dontrun{ par(mfrow = c(2, 1), las = 1)
#plot(p, y, type = "l", col = "blue", main = "pordlink()")
#abline(h = 0, v = 0.5, col = "orange", lty = "dashed")
#
#plot(p, y., type = "l", col = "blue",
# main = "(Reciprocal of) first POLF derivative")
#}
```

\# Rutherford and Geiger data
ruge <- data.frame (yy $=\operatorname{rep}(0: 14$,
times $=c(57,203,383,525,532,408,273,139,45,27,10,4,0,1,1)))$
with(ruge, length(yy)) \# 2608 1/8-minute intervals
cutpoint <- 5
ruge <- transform(ruge, yy01 = ifelse(yy <= cutpoint, 0, 1))
fit <- vglm(yy01 ~ 1, binomialff(link=pordlink(cutpoint=cutpoint)), ruge)
coef(fit, matrix = TRUE)
$\exp (c o e f(f i t))$
\# Another example
pdata <- data.frame(x2 = sort(runif(nn <- 1000)))
pdata <- transform(pdata, x3 $=\operatorname{runif(nn))}$
pdata <- transform(pdata, mymu $=\exp (3+1$ * x2 - 2 * x3))
pdata <- transform(pdata, y1 = rpois(nn, lambda $=$ mymu))
cutpoints <- c(-Inf, 10, 20, Inf)
pdata <- transform(pdata, cuty = Cut(y1, breaks = cutpoints))
\# $\backslash$ dontrun\{ with(pdata, plot(x2, x3, col = cuty, pch = as.character(cuty))) \}
with (pdata, table(cuty) / sum(table(cuty)))
fit <- vglm(cuty $\sim$ x2 $+x 3$, data $=$ pdata, trace $=$ TRUE,
cumulative(reverse = TRUE,
parallel = TRUE,
link = pordlink(cutpoint = cutpoints[2:3]),

```
        multiple.responses = TRUE))
    head(depvar(fit))
    head(fitted(fit))
    head(predict(fit))
    coef(fit)
    coef(fit, matrix = TRUE)
    constraints(fit)
    fit@misc$earg
    ## End(Not run)
```

posbernoulli.b Positive Bernoulli Family Function with Behavioural Effects

## Description

Fits a GLM-/GAM-like model to multiple Bernoulli responses where each row in the capture history matrix response has at least one success (capture). Capture history behavioural effects are accommodated.

## Usage

```
    posbernoulli.b(link = "logitlink", drop.b = FALSE ~ 1,
    type.fitted = c("likelihood.cond", "mean.uncond"), I2 = FALSE,
    ipcapture = NULL, iprecapture = NULL,
    p.small = 1e-4, no.warning = FALSE)
```


## Arguments

link, drop.b, ipcapture, iprecapture
See CommonVGAMfffarguments for information about these arguments. By default the parallelism assumption does not apply to the intercept. With an interceptonly model setting drop. $\mathrm{b}=$ TRUE $\sim 1$ results in the $M_{0} / M_{h}$ model.

I2
Logical. This argument is used for terms that are not parallel. If TRUE then the constraint matrix diag(2) (the general default constraint matrix in VGAM) is used, else cbind $(0: 1,1)$. The latter means the first element/column corresponds to the behavioural effect. Consequently it and its standard error etc. can be accessed directly without subtracting two quantities.
type.fitted Details at posbernoulli.tb.
p.small, no.warning

See posbernoulli.t.

## Details

This model (commonly known as $M_{b} / M_{b h}$ in the capture-recapture literature) operates on a capture history matrix response of 0 s and $1 \mathrm{~s}(n \times \tau)$. See posbernoulli.t for details, e.g., common assumptions with other models. Once an animal is captured for the first time, it is marked/tagged
so that its future capture history can be recorded. The effect of the recapture probability is modelled through a second linear/additive predictor. It is well-known that some species of animals are affected by capture, e.g., trap-shy or trap-happy. This VGAM family function does allow the capture history to be modelled via such behavioural effects. So does posbernoulli.tb but posbernoulli.t cannot.
The number of linear/additive predictors is $M=2$, and the default links are $\left(\operatorname{logit} p_{c}, \operatorname{logit} p_{r}\right)^{T}$ where $p_{c}$ is the probability of capture and $p_{r}$ is the probability of recapture. The fitted value returned is of the same dimension as the response matrix, and depends on the capture history: prior to being first captured, it is pcapture. Afterwards, it is precapture.
By default, the constraint matrices for the intercept term and the other covariates are set up so that $p_{r}$ differs from $p_{c}$ by a simple binary effect, on a logit scale. However, this difference (the behavioural effect) is more directly estimated by having I2 $=$ FALSE. Then it allows an estimate of the trap-happy/trap-shy effect; these are positive/negative values respectively. If I2 $=$ FALSE then the (nonstandard) constraint matrix used is cbind ( $0: 1,1$ ), meaning the first element can be interpreted as the behavioural effect.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

The dependent variable is not scaled to row proportions. This is the same as posbernoulli.t and posbernoulli.tb but different from posbinomial and binomialff.

## Author(s)

Thomas W. Yee.

## References

See posbernoulli.t.

## See Also

posbernoulli.t and posbernoulli.tb (including estimating $N$ ), deermice, dposbern, rposbern, posbinomial, aux. posbernoulli.t, prinia.

## Examples

```
# deermice data
    , , , , , , , , , , , , , , , , , , , , , , , , , , , , , , , , , ,
# Fit a M_b model
M.b <- vglm(cbind(y1, y2, y3, y4, y5, y6) ~ 1,
    posbernoulli.b, data = deermice, trace = TRUE)
coef(M.b)["(Intercept):1"] # Behavioural effect on the logit scale
coef(M.b, matrix = TRUE)
constraints(M.b, matrix = TRUE)
summary(M.b, presid = FALSE)
```

```
# Fit a M_bh model
M.bh <- vglm(cbind(y1, y2, y3, y4, y5, y6) ~ sex + weight,
    posbernoulli.b, data = deermice, trace = TRUE)
coef(M.bh, matrix = TRUE)
coef(M.bh)["(Intercept):1"] # Behavioural effect on the logit scale
constraints(M.bh) # (2,1) element of "(Intercept)" is for the behavioural effect
summary(M.bh, presid = FALSE) # Significant positive (trap-happy) behavioural effect
# Approx. 95 percent confidence for the behavioural effect:
SE.M.bh <- coef(summary(M.bh))["(Intercept):1", "Std. Error"]
coef(M.bh)["(Intercept):1"] + c(-1, 1) * 1.96 * SE.M.bh
# Fit a M_h model
M.h <- vglm(cbind(y1, y2, y3, y4, y5, y6) ~ sex + weight,
    posbernoulli.b(drop.b = TRUE ~ sex + weight),
    data = deermice, trace = TRUE)
coef(M.h, matrix = TRUE)
constraints(M.h, matrix = TRUE)
summary(M.h, presid = FALSE)
# Fit a M_0 model
M.0 <- vglm(cbind( y1 + y2 + y3 + y4 + y5 + y6,
            6 - y1 - y2 - y3 - y4 - y5 - y6) ~ 1,
        posbinomial, data = deermice, trace = TRUE)
coef(M.0, matrix = TRUE)
summary(M.0, presid = FALSE)
# Simulated data set ,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,
set.seed(123); nTimePts <- 5; N <- 1000 # N is the popn size
pdata <- rposbern(n = N, nTimePts = nTimePts, pvars = 2, is.popn = TRUE)
nrow(pdata) # Less than N (because some animals were never captured)
# The truth: xcoeffs are c(-2, 1, 2) and cap.effect = +1
M.bh.2 <- vglm(cbind(y1, y2, y3, y4, y5) ~ x2,
                    posbernoulli.b, data = pdata, trace = TRUE)
coef(M.bh.2)
coef(M.bh.2, matrix = TRUE)
constraints(M.bh.2, matrix = TRUE)
summary(M.bh.2, presid = FALSE)
head(depvar(M.bh.2)) # Capture history response matrix
head(M.bh.2@extra$cap.hist1) # Info on its capture history
head(M.bh.2@extra$cap1) # When it was first captured
head(fitted(M.bh.2)) # Depends on capture history
(trap.effect <- coef(M.bh.2)["(Intercept):1"]) # Should be +1
head(model.matrix(M.bh.2, type = "vlm"), 21)
head(pdata)
summary(pdata)
dim(depvar(M.bh.2))
vcov(M.bh.2)
M.bh.2@extra$N.hat # Estimate of the population size; should be about N
M.bh.2@extra$SE.N.hat # SE of the estimate of the population size
```

\# An approximate 95 percent confidence interval:
round(M.bh.2@extra\$N.hat $+c(-1,1) * 1.96$ * M.bh.2@extra\$SE.N.hat, 1)
posbernoulli.t Positive Bernoulli Family Function with Time Effects

## Description

Fits a GLM/GAM-like model to multiple Bernoulli responses where each row in the capture history matrix response has at least one success (capture). Sampling occasion effects are accommodated.

## Usage

posbernoulli.t(link = "logitlink", parallel.t = FALSE $\sim 1$, iprob = NULL, p.small $=1 \mathrm{e}-4$, no.warning $=$ FALSE, type.fitted = c("probs", "onempall0"))

## Arguments

link, iprob, parallel.t
See CommonVGAMffArguments for information. By default, the parallelism assumption does not apply to the intercept. Setting parallel.t $=$ FALSE $\sim-1$, or equivalently parallel.t $=$ FALSE $\sim 0$, results in the $M_{0} / M_{h}$ model.
p.small, no.warning

A small probability value used to give a warning for the Horvitz-Thompson estimator. Any estimated probability value less than p.small will result in a warning, however, setting no. warning $=$ TRUE will suppress this warning if it occurs. This is because the Horvitz-Thompson estimator is the sum of the reciprocal of such probabilities, therefore any probability that is too close to 0 will result in an unstable estimate.
type.fitted See CommonVGAMffArguments for information. The default is to return a matrix of probabilities. If "onempall0" is chosen then the the probability that each animal is captured at least once in the course of the study is returned. The abbreviation stands for one minus the probability of all 0 s , and the quantity appears in the denominator of the usual formula.

## Details

These models (commonly known as $M_{t}$ or $M_{t h}$ (no prefix $h$ means it is an intercept-only model) in the capture-recapture literature) operate on a capture history matrix response of 0 s and $1 \mathrm{~s}(n \times \tau)$. Each column is a sampling occasion where animals are potentially captured (e.g., a field trip), and each row is an individual animal. Capture is a 1 , else a 0 . No removal of animals from the population is made (closed population), e.g., no immigration or emigration. Each row of the response matrix has at least one capture. Once an animal is captured for the first time, it is marked/tagged so that its future capture history can be recorded. Then it is released immediately back into the population to remix. It is released immediately after each recapture too. It is assumed that the animals are
independent and that, for a given animal, each sampling occasion is independent. And animals do not lose their marks/tags, and all marks/tags are correctly recorded.

The number of linear/additive predictors is equal to the number of sampling occasions, i.e., $M=\tau$, say. The default link functions are $\left(\operatorname{logit} p_{1}, \ldots, \operatorname{logit} p_{\tau}\right)^{T}$ where each $p_{j}$ denotes the probability of capture at time point $j$. The fitted value returned is a matrix of probabilities of the same dimension as the response matrix.

A conditional likelihood is maximized here using Fisher scoring. Each sampling occasion has a separate probability that is modelled here. The probabilities can be constrained to be equal by setting parallel.t $=$ FALSE $\sim 0$; then the results are effectively the same as posbinomial except the binomial constants are not included in the log-likelihood. If parallel. $t=$ TRUE $\sim 0$ then each column should have at least one 1 and at least one 0 .

It is well-known that some species of animals are affected by capture, e.g., trap-shy or trap-happy. This VGAM family function does not allow any behavioral effect to be modelled (posbernoulli.b and posbernoulli.tb do) because the denominator of the likelihood function must be free of behavioral effects.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.
Upon fitting the extra slot has a (list) component called $N$. hat which is a point estimate of the population size $N$ (it is the Horvitz-Thompson (1952) estimator). And there is a component called SE.N. hat containing its standard error.

## Note

The weights argument of vglm need not be assigned, and the default is just a matrix of ones.
Fewer numerical problems are likely to occur for parallel.t = TRUE. Data-wise, each sampling occasion may need at least one success (capture) and one failure. Less stringent conditions in the data are needed when parallel.t = TRUE. Ditto when parallelism is applied to the intercept too.

The response matrix is returned unchanged; i.e., not converted into proportions like posbinomial. If the response matrix has column names then these are used in the labelling, else prob1, prob2, etc. are used.
Using AIC() or BIC() to compare posbernoulli.t, posbernoulli.b, posbernoulli.tb models with a posbinomial model requires posbinomial (omit.constant $=$ TRUE) because one needs to remove the normalizing constant from the log-likelihood function. See posbinomial for an example.

## Author(s)

Thomas W. Yee.

## References

Huggins, R. M. (1991). Some practical aspects of a conditional likelihood approach to capture experiments. Biometrics, 47, 725-732.

Huggins, R. M. and Hwang, W.-H. (2011). A review of the use of conditional likelihood in capturerecapture experiments. International Statistical Review, 79, 385-400.
Otis, D. L. and Burnham, K. P. and White, G. C. and Anderson, D. R. (1978). Statistical inference from capture data on closed animal populations, Wildlife Monographs, 62, 3-135.

Yee, T. W. and Stoklosa, J. and Huggins, R. M. (2015). The VGAM package for capture-recapture data using the conditional likelihood. Journal of Statistical Software, 65, 1-33. http://www. jstatsoft.org/v65/i05/.

## See Also

posbernoulli.b, posbernoulli.tb, Select, deermice, Huggins89table1, Huggins89.t1, dposbern, rposbern, posbinomial, AICvlm, BICvlm, prinia.

## Examples

```
M.t <- vglm(cbind(y1, y2, y3, y4, y5, y6) ~ 1, posbernoulli.t,
    data = deermice, trace = TRUE)
coef(M.t, matrix = TRUE)
constraints(M.t, matrix = TRUE)
summary(M.t, presid = FALSE)
M.h.1 <- vglm(Select(deermice, "y") ~ sex + weight, trace = TRUE,
    posbernoulli.t(parallel.t = FALSE ~ -1), data = deermice)
coef(M.h.1, matrix = TRUE)
constraints(M.h.1)
summary(M.h.1, presid = FALSE)
head(depvar(M.h.1)) # Response capture history matrix
dim(depvar(M.h.1))
M.th.2 <- vglm(cbind(y1, y2, y3, y4, y5, y6) ~ sex + weight, trace = TRUE,
                posbernoulli.t(parallel.t = FALSE), data = deermice)
lrtest(M.h.1, M.th.2) # Test the parallelism assumption wrt sex and weight
coef(M.th.2)
coef(M.th.2, matrix = TRUE)
constraints(M.th.2)
summary(M.th.2, presid = FALSE)
head(model.matrix(M.th.2, type = "vlm"), 21)
M.th.2@extra$N.hat # Estimate of the population size; should be about N
M.th.2@extra$SE.N.hat # SE of the estimate of the population size
# An approximate 95 percent confidence interval:
round(M.th.2@extra$N.hat + c(-1, 1) * 1.96 * M.th.2@extra$SE.N.hat, 1)
# Fit a M_h model, effectively the parallel M_t model, using posbinomial()
deermice <- transform(deermice, ysum = y1 + y2 + y3 + y4 + y5 + y6,
                                    tau = 6)
M.h.3 <- vglm(cbind(ysum, tau - ysum) ~ sex + weight,
    posbinomial(omit.constant = TRUE), data = deermice, trace = TRUE)
max(abs(coef(M.h.1) - coef(M.h.3))) # Should be zero
logLik(M.h.3) - logLik(M.h.1) # Difference is due to the binomial constants
```


## Description

Fits a GLM/GAM-like model to multiple Bernoulli responses where each row in the capture history matrix response has at least one success (capture). Sampling occasion effects and behavioural effects are accommodated.

## Usage

```
    posbernoulli.tb(link = "logitlink", parallel.t = FALSE ~ 1,
    parallel.b = FALSE \(\sim 0\), drop.b = FALSE ~ 1,
    type.fitted = c("likelihood.cond", "mean. uncond"),
    imethod \(=1\), iprob \(=\) NULL,
    p.small \(=1 \mathrm{e}-4\), no. warning \(=\) FALSE,
    ridge. constant \(=0.0001\), ridge. power \(=-4\) )
```


## Arguments

link, imethod, iprob
See CommonVGAMffArguments for information.
parallel.t, parallel.b, drop.b
A logical, or formula with a logical as the response. See CommonVGAMffArguments for information. The parallel.-type arguments specify whether the constraint matrices have a parallelism assumption for the temporal and behavioural effects. Argument parallel.t means parallel with respect to time, and matches the same argument name in posbernoulli.t.
Suppose the model is intercept-only. Setting parallel.t $=$ FALSE $\sim 0$ results in the $M_{b}$ model. Setting drop. $\mathrm{b}=\mathrm{FALSE} \sim 0$ results in the $M_{t}$ model because it drops columns off the constraint matrices corresponding to any behavioural effect. Setting parallel.t $=$ FALSE $\sim 0$ and setting parallel. $b=$ FALSE $\sim 0$ results in the $M_{b}$ model. Setting parallel.t = FALSE $\sim 0$, parallel. $\mathrm{b}=$ FALSE $\sim 0$ and drop. $\mathrm{b}=$ FALSE $\sim 0$ results in the $M_{0}$ model. Note the default for parallel.t and parallel.b may be unsuitable for data sets which have a large $\tau$ because of the large number of parameters; it might be too flexible. If it is desired to have the behaviour affect some of the other covariates then set drop.b $=$ TRUE $\sim 0$.
The default model has a different intercept for each sampling occasion, a timeparallelism assumption for all other covariates, and a dummy variable representing a single behavioural effect (also in the intercept).
The most flexible model is to set parallel. $b=$ TRUE $\sim 0$, parallel. $t=$ TRUE $\sim$ 0 and drop. $b=$ TRUE $\sim 0$. This means that all possible temporal and behavioural effects are estimated, for the intercepts and other covariates. Such a model is not recommended; it will contain a lot of paramters.
type.fitted Character, one of the choices for the type of fitted value returned. The default is the first one. Partial matching is okay. For "likelihood.cond": the probability defined by the conditional likelihood. For "mean. uncond": the unconditional mean, which should agree with colMeans applied to the response matrix for intercept-only models.
ridge.constant, ridge. power
Determines the ridge parameters at each IRLS iteration. They are the constant and power (exponent) for the ridge adjustment for the working weight matrices (the capture probability block matrix, hence the first $\tau$ diagonal values). At iteration $a$ of the IRLS algorithm a positive value is added to the first $\tau$ diagonal elements of the working weight matrices to make them positive-definite. This adjustment is the mean of the diagonal elements of wz multipled by $K \times a^{p}$ where $K$ is ridge. constant and $p$ is ridge. power. This is always positive but decays to zero as iterations proceed (provided $p$ is negative etc.).
p.small, no.warning

See posbernoulli.t.

## Details

This model (commonly known as $M_{t b} / M_{t b h}$ in the capture-recapture literature) operates on a response matrix of 0 s and $1 \mathrm{~s}(n \times \tau)$. See posbernoulli.t for information that is in common. It allows time and behavioural effects to be modelled.
Evidently, the expected information matrix (EIM) seems not of full rank (especially in early iterations), so ridge.constant and ridge. power are used to try fix up the problem. The default link functions are $\left(\operatorname{logit} p_{c 1}, \ldots, \operatorname{logit} p_{c \tau}, \operatorname{logit} p_{r 2}, \ldots, \text { logit } p_{r \tau}\right)^{T}$ where the subscript $c$ denotes capture, the subscript $r$ denotes recapture, and it is not possible to recapture the animal at sampling occasion 1 . Thus $M=2 \tau-1$. The parameters are currently prefixed by pcapture and precapture for the capture and recapture probabilities. This VGAM family function may be further modified in the future.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

It is a good idea to apply the parallelism assumption to each sampling occasion except possibly with respect to the intercepts. Also, a simple behavioural effect such as being modelled using the intercept is recommended; if the behavioural effect is not parallel and/or allowed to apply to other covariates then there will probably be too many parameters, and hence, numerical problems. See M_tbh. 1 below.
It is a good idea to monitor convergence. Simpler models such as the $M_{0} / M_{h}$ models are best fitted with posbernoulli.t or posbernoulli.b or posbinomial.

## Author(s)

Thomas W. Yee.

## References

See posbernoulli.t.

## See Also

posbernoulli.b (including N. hat), posbernoulli.t, posbinomial, Select, fill1, Huggins89table1, Huggins89.t1, deermice, prinia.

## Examples

```
## Not run:
# Example 1: simulated data
nTimePts <- 5 # (aka tau == # of sampling occasions)
nnn <- 1000 # Number of animals
pdata <- rposbern(n = nnn, nTimePts = nTimePts, pvars = 2)
dim(pdata); head(pdata)
M_tbh.1 <- vglm(cbind(y1, y2, y3, y4, y5) ~ x2,
    posbernoulli.tb, data = pdata, trace = TRUE)
coef(M_tbh.1) # First element is the behavioural effect
coef(M_tbh.1, matrix = TRUE)
constraints(M_tbh.1, matrix = TRUE)
summary(M_tbh.1, presid = FALSE) # Standard errors are approximate
head(fitted(M_tbh.1))
head(model.matrix(M_tbh.1, type = "vlm"), 21)
dim(depvar(M_tbh.1))
M_tbh.2 <- vglm(cbind(y1, y2, y3, y4, y5) ~ x2,
    posbernoulli.tb(parallel.t = FALSE ~ 0),
    data = pdata, trace = TRUE)
coef(M_tbh.2) # First element is the behavioural effect
coef(M_tbh.2, matrix = TRUE)
constraints(M_tbh.2, matrix = TRUE)
summary(M_tbh.2, presid = FALSE) # Standard errors are approximate
head(fitted(M_tbh.2))
head(model.matrix(M_tbh.2, type = "vlm"), 21)
dim(depvar(M_tbh.2))
# Example 2: deermice subset data
fit1 <- vglm(cbind(y1, y2, y3, y4, y5, y6) ~ sex + weight,
    posbernoulli.t, data = deermice, trace = TRUE)
coef(fit1)
coef(fit1, matrix = TRUE)
constraints(fit1, matrix = TRUE)
summary(fit1, presid = FALSE) # Standard errors are approximate
# fit1 is the same as Fit1 (a M_{th} model):
Fit1 <- vglm(cbind(y1, y2, y3, y4, y5, y6) ~ sex + weight,
    posbernoulli.tb(drop.b = TRUE ~ sex + weight,
                                    parallel.t = TRUE), # No parallelism for the intercept
    data = deermice, trace = TRUE)
constraints(Fit1)
```

```
## End(Not run)
```

```
posbernUC Positive Bernoulli Sequence Model
```


## Description

Density, and random generation for multiple Bernoulli responses where each row in the response matrix has at least one success.

## Usage

rposbern(n, nTimePts = 5, pvars = length (xcoeff), xcoeff $=c(-2,1,2)$,
Xmatrix $=$ NULL, cap.effect $=1$, is.popn $=$ FALSE,
link = "logitlink", earg.link = FALSE)
dposbern(x, prob, prob0 $=$ prob, log $=$ FALSE $)$

## Arguments

$x$ response vector or matrix. Should only have 0 and 1 values, at least two columns, and each row should have at least one 1.
nTimePts Number of sampling occasions. Called $\tau$ in posbernoulli.b and posbernoulli.t.
$\mathrm{n} \quad$ number of observations. Usually a single positive integer, else the length of the vector is used. See argument is.popn.
is.popn Logical. If TRUE then argument n is the population size and what is returned may have substantially less rows than $n$. That is, if an animal has at least one one in its sequence then it is returned, else that animal is not returned because it never was captured.
Xmatrix Optional $\mathbf{X}$ matrix. If given, the $\mathbf{X}$ matrix is not generated internally.
cap.effect Numeric, the capture effect. Added to the linear predictor if captured previously. A positive or negative value corresponds to a trap-happy and trap-shy effect respectively.
pvars Number of other numeric covariates that make up the linear predictor. Labelled $x 1, x 2, \ldots$, where the first is an intercept, and the others are independent standard runif random variates. The first pvars elements of xcoeff are used.
xcoeff The regression coefficients of the linear predictor. These correspond to $\mathrm{x} 1, \mathrm{x} 2$, $\ldots$, and the first is for the intercept. The length of xcoeff must be at least pvars.
link, earg.link
The former is used to generate the probabilities for capture at each occasion. Other details at CommonVGAMffArguments.
prob, prob0 Matrix of probabilities for the numerator and denominators respectively. The default does not correspond to the $M_{b}$ model since the $M_{b}$ model has a denominator which involves the capture history.
log Logical. Return the logarithm of the answer?

## Details

The form of the conditional likelihood is described in posbernoulli.b and/or posbernoulli.t and/or posbernoulli.tb. The denominator is equally shared among the elements of the matrix $x$.

## Value

rposbern returns a data frame with some attributes. The function generates random deviates ( $\tau$ columns labelled $\mathrm{y} 1, \mathrm{y} 2, \ldots$ ) for the response. Some indicator columns are also included (those starting with ch are for previous capture history). The default setting corresponds to a $M_{b h}$ model that has a single trap-happy effect. Covariates $\times 1, \times 2, \ldots$ have the same affect on capture/recapture at every sampling occasion (see the argument parallel.t in, e.g., posbernoulli.tb).

The function dposbern gives the density,

## Note

The $r$-type function is experimental only and does not follow the usual conventions of $r$-type $R$ functions. It may change a lot in the future. The d-type function is more conventional and is less likely to change.

## Author(s)

Thomas W. Yee.

## See Also

posbernoulli.tb, posbernoulli.b, posbernoulli.t.

## Examples

```
rposbern(n = 10)
attributes(pdata <- rposbern(n = 100))
M.bh <- vglm(cbind(y1, y2, y3, y4, y5) ~ x2 + x3, posbernoulli.b(I2 = FALSE),
    data = pdata, trace = TRUE)
constraints(M.bh)
summary(M.bh)
```


## Description

Density, distribution function, quantile function and random generation for the positive-binomial distribution.

## Usage

dposbinom(x, size, prob, log = FALSE)
pposbinom(q, size, prob)
qposbinom(p, size, prob)
rposbinom(n, size, prob)

## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$n \quad$ number of observations. Fed into runif.
size number of trials. It is the $N$ symbol in the formula given in posbinomial and should be positive.
prob probability of success on each trial. Should be in $(0,1)$.
log See dbinom.

## Details

The positive-binomial distribution is a binomial distribution but with the probability of a zero being zero. The other probabilities are scaled to add to unity. The mean therefore is

$$
\mu /\left(1-(1-\mu)^{N}\right)
$$

where $\mu$ is the argument prob above. As $\mu$ increases, the positive-binomial and binomial distributions become more similar. Unlike similar functions for the binomial distribution, a zero value of prob is not permitted here.

## Value

dposbinom gives the density, pposbinom gives the distribution function, qposbinom gives the quantile function, and rposbinom generates random deviates.

## Note

For dposbinom(), if arguments size or prob equal 0 then a NaN is returned.
The family function posbinomial estimates the parameters by maximum likelihood estimation.

## Author(s)

T. W. Yee.

## See Also

posbinomial, dposbern, Gaitbinom.mlm, zabinomial, zibinomial, Binomial.

## Examples

```
prob <- 0.2; size <- 10
table(y <- rposbinom(n = 1000, size, prob))
mean(y) # Sample mean
size * prob / (1 - (1 - prob)^size) # Population mean
(ii <- dposbinom(0:size, size, prob))
cumsum(ii) - pposbinom(0:size, size, prob) # Should be 0s
table(rposbinom(100, size, prob))
table(qposbinom(runif(1000), size, prob))
round(dposbinom(1:10, size, prob) * 1000) # Should be similar
## Not run: barplot(rbind(dposbinom(x = 0:size, size, prob),
                dbinom(x = 0:size, size, prob)),
    beside = TRUE, col = c("blue", "green"),
    main = paste("Positive-binomial(", size, ",",
        prob, ") (blue) vs",
    " Binomial(", size, ",", prob, ") (green)", sep = ""),
    names.arg = as.character(0:size), las = 1)
## End(Not run)
# Simulated data example
nn <- 1000; sizeval1 <- 10; sizeval2 <- 20
pdata <- data.frame(x2 = seq(0, 1, length = nn))
pdata <- transform(pdata, prob1 = logitlink(-2 + 2 * x2, inverse = TRUE),
                                    prob2 = logitlink(-1 + 1 * x2, inverse = TRUE),
                                    sizev1 = rep(sizeval1, len = nn),
                                    sizev2 = rep(sizeval2, len = nn))
pdata <- transform(pdata, y1 = rposbinom(nn, size = sizev1, prob = prob1),
                                    y2 = rposbinom(nn, size = sizev2, prob = prob2))
with(pdata, table(y1))
with(pdata, table(y2))
# Multiple responses
fit2 <- vglm(cbind(y1, y2) ~ x2, posbinomial(multiple.responses = TRUE),
    trace = TRUE, data = pdata, weight = cbind(sizev1, sizev2))
coef(fit2, matrix = TRUE)
```

posbinomial

Positive Binomial Distribution Family Function

## Description

Fits a positive binomial distribution.

## Usage

posbinomial(link = "logitlink", multiple.responses = FALSE, parallel = FALSE, omit.constant $=$ FALSE, p.small $=1 \mathrm{e}-4$, no. warning $=$ FALSE, zero = NULL)

## Arguments

link, multiple.responses, parallel, zero
Details at CommonVGAMffArguments.
omit.constant Logical. If TRUE then the constant (lchoose (size, size * yprop) is omitted from the loglikelihood calculation. If the model is to be compared using AIC() or BIC() (see AICvlm or BICvlm) to the likes of posbernoulli.tb etc. then it is important to set omit. constant = TRUE because all models then will not have any normalizing constants in the likelihood function. Hence they become comparable. This is because the $M_{0}$ Otis et al. (1978) model coincides with posbinomial(). See below for an example. Also see posbernoulli.t regarding estimating the population size (N. hat and SE.N.hat) if the number of trials is the same for all observations.
p.small, no.warning

See posbernoulli.t.

## Details

The positive binomial distribution is the ordinary binomial distribution but with the probability of zero being zero. Thus the other probabilities are scaled up (i.e., divided by $1-P(Y=0)$ ). The fitted values are the ordinary binomial distribution fitted values, i.e., the usual mean.

In the capture-recapture literature this model is called the $M_{0}$ if it is an intercept-only model. Otherwise it is called the $M_{h}$ when there are covariates. It arises from a sum of a sequence of $\tau$ Bernoulli random variates subject to at least one success (capture). Here, each animal has the same probability of capture or recapture, regardless of the $\tau$ sampling occasions. Independence between animals and between sampling occasions etc. is assumed.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

Under- or over-flow may occur if the data is ill-conditioned.

## Note

The input for this family function is the same as binomialff.
If multiple. responses $=$ TRUE then each column of the matrix response should be a count (the number of successes), and the weights argument should be a matrix of the same dimension as the response containing the number of trials. If multiple.responses $=$ FALSE then the response input should be the same as binomialff.
Yet to be done: a quasi. posbinomial() which estimates a dispersion parameter.

## Author(s)

Thomas W. Yee

## References

Otis, D. L. et al. (1978). Statistical inference from capture data on closed animal populations, Wildlife Monographs, 62, 3-135.
Patil, G. P. (1962). Maximum likelihood estimation for generalised power series distributions and its application to a truncated binomial distribution. Biometrika, 49, 227-237.
Pearson, K. (1913). A Monograph on Albinism in Man. Drapers Company Research Memoirs.

## See Also

posbernoulli.b, posbernoulli.t, posbernoulli.tb, gtbinomial, binomialff, AICvlm, BICvlm, simulate.vlm.

## Examples

```
    # Number of albinotic children in families with 5 kids (from Patil, 1962) ,,,,
    albinos <- data.frame(y = c(rep(1, 25), rep(2, 23), rep(3, 10), 4, 5),
        n = rep(5, 60))
    fit1 <- vglm(cbind(y, n-y) ~ 1, posbinomial, albinos, trace = TRUE)
    summary(fit1)
    Coef(fit1) # = MLE of p = 0.3088
    head(fitted(fit1))
    sqrt(vcov(fit1, untransform = TRUE)) # SE = 0.0322
    # Fit a M_0 model (Otis et al. 1978) to the deermice data ,,,,,,,,,,,,,,,,,,,,,,,
    M.0 <- vglm(cbind( y1 + y2 + y3 + y4 + y5 + y6,
            6 - y1 - y2 - y3 - y4 - y5 - y6) ~ 1, trace = TRUE,
            posbinomial(omit.constant = TRUE), data = deermice)
coef(M.0, matrix = TRUE)
Coef(M.0)
constraints(M.0, matrix = TRUE)
summary(M.0)
c( N.hat = M.0@extra$N.hat, # Since tau = 6, i.e., 6 Bernoulli trials per
        SE.N.hat = M.0@extra$SE.N.hat) # observation is the same for each observation
    # Compare it to the M_b using AIC and BIC
    M.b <- vglm(cbind(y1, y2, y3, y4, y5, y6) ~ 1, trace = TRUE,
            posbernoulli.b, data = deermice)
sort(c(M.0 = AIC(M.0), M.b = AIC(M.b))) # Okay since omit.constant = TRUE
sort(c(M.0 = BIC(M.0), M.b = BIC(M.b))) # Okay since omit.constant = TRUE
```

Posgeom
Positive-Geometric Distribution

## Description

Density, distribution function, quantile function and random generation for the positive-geometric distribution.

## Usage

```
dposgeom(x, prob, log = FALSE)
pposgeom(q, prob)
qposgeom(p, prob)
rposgeom(n, prob)
```


## Arguments

$\mathrm{x}, \mathrm{q}$
p
prob vector of probabilities of success (of an ordinary geometric distribution). Short
log
$n \quad$ number of observations. Fed into runif. vectors are recycled.
vector of quantiles.
vector of probabilities.
logical.

## Details

The positive-geometric distribution is a geometric distribution but with the probability of a zero being zero. The other probabilities are scaled to add to unity. The mean therefore is $1 / p r o b$.
As prob decreases, the positive-geometric and geometric distributions become more similar. Like similar functions for the geometric distribution, a zero value of prob is not permitted here.

## Value

dposgeom gives the density, pposgeom gives the distribution function, qposgeom gives the quantile function, and rposgeom generates random deviates.

## Author(s)

T. W. Yee

## See Also

zageometric, zigeometric, rgeom.

## Examples

```
prob <- 0.75; y <- rposgeom(n = 1000, prob)
table(y)
mean(y) # Sample mean
1 / prob # Population mean
(ii <- dposgeom(0:7, prob))
cumsum(ii) - pposgeom(0:7, prob) # Should be 0s
table(rposgeom(100, prob))
table(qposgeom(runif(1000), prob))
round(dposgeom(1:10, prob) * 1000) # Should be similar
```

```
## Not run:
x <- 0:5
barplot(rbind(dposgeom(x, prob), dgeom(x, prob)),
    beside = TRUE, col = c("blue", "orange"),
    main = paste("Positive geometric(", prob, ") (blue) vs",
    " geometric(", prob, ") (orange)", sep = ""),
    names.arg = as.character(x), las = 1, lwd = 2)
## End(Not run)
```

Posnegbin Positive-Negative Binomial Distribution

## Description

Density, distribution function, quantile function and random generation for the positive-negative binomial distribution.

## Usage

dposnegbin(x, size, prob = NULL, munb = NULL, log = FALSE)
pposnegbin(q, size, prob $=$ NULL, munb $=$ NULL, lower.tail = TRUE, log.p = FALSE)
qposnegbin(p, size, prob $=$ NULL, munb $=$ NULL)
rposnegbin(n, size, $\operatorname{prob}=$ NULL, munb $=$ NULL)

## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
n number of observations. Fed into runif.
size, prob, munb, log
Same arguments as that of an ordinary negative binomial distribution (see dnbinom).
Some arguments have been renamed slightly.
Short vectors are recycled. The parameter $1 /$ size is known as a dispersion parameter; as size approaches infinity, the negative binomial distribution approaches a Poisson distribution.
Note that prob must lie in $(0,1)$, otherwise a NaN is returned.
log.p, lower.tail
Same arguments as that of an ordinary negative binomial distribution (see pnbinom).

## Details

The positive-negative binomial distribution is a negative binomial distribution but with the probability of a zero being zero. The other probabilities are scaled to add to unity. The mean therefore is

$$
\mu /(1-p(0))
$$

where $\mu$ the mean of an ordinary negative binomial distribution.

## Value

dposnegbin gives the density, pposnegbin gives the distribution function, qposnegbin gives the quantile function, and rposnegbin generates $n$ random deviates.

## Author(s)

T. W. Yee

## References

Welsh, A. H., Cunningham, R. B., Donnelly, C. F. and Lindenmayer, D. B. (1996). Modelling the abundances of rare species: statistical models for counts with extra zeros. Ecological Modelling, 88, 297-308.

## See Also

```
gatnbinomial.mlm, Gaitnbinom.mlm, posnegbinomial, zanegbinomial, zinegbinomial, rnbinom.
```


## Examples

```
munb <- 5; size <- 4; n <- 1000
table(y <- rposnegbin(n, munb = munb, size = size))
mean(y) # sample mean
munb / (1 - (size / (size + munb))^size) # population mean
munb / pnbinom(0, mu = munb, size = size, lower.tail = FALSE) # same as before
x <- (-1):17
(ii <- dposnegbin(x, munb = munb, size = size))
max(abs(cumsum(ii) - pposnegbin(x, munb = munb, size = size))) # Should be 0
## Not run:
x <- 0:10
barplot(rbind(dposnegbin(x, munb = munb, size = size),
            dnbinom(x, mu = munb, size = size)),
        beside = TRUE, col = c("blue","green"),
        main = paste("dposnegbin(munb = ", munb, ", size = ", size, ") (blue) vs",
                            " dnbinom(mu = ", munb, ", size = ", size, ") (green)", sep = ""),
        names.arg = as.character(x))
## End(Not run)
# Another test for pposnegbin()
nn <- 5000
mytab <- cumsum(table(rposnegbin(nn, munb = munb, size = size))) / nn
myans <- pposnegbin(sort(as.numeric(names(mytab))), munb = munb, size = size)
max(abs(mytab - myans)) # Should be 0
```


## Description

Maximum likelihood estimation of the two parameters of a positive negative binomial distribution.

## Usage

```
posnegbinomial(zero = "size", type.fitted = c("mean", "munb", "prob0"),
    mds.min = 0.001, nsimEIM = 500, cutoff.prob = 0.999,
    eps.trig = 1e-07, max.support = 4000, max.chunk.MB = 30,
    lmunb = "loglink", lsize = "loglink", imethod = 1,
    imunb = NULL, iprobs.y = NULL,
    gprobs.y = ppoints(8), isize = NULL,
    gsize.mux = exp(c(-30, -20, -15, -10, -6:3)))
```


## Arguments

lmunb Link function applied to the munb parameter, which is the mean $\mu_{n b}$ of an ordinary negative binomial distribution. See Links for more choices.
lsize Parameter link function applied to the dispersion parameter, called k. See Links for more choices.
isize Optional initial value for $k$, an index parameter. The value $1 / k$ is known as a dispersion parameter. If failure to converge occurs try different values (and/or use imethod). If necessary this vector is recycled to length equal to the number of responses. A value NULL means an initial value for each response is computed internally using a range of values.
nsimEIM, zero, eps.trig
See CommonVGAMffArguments.
mds.min, iprobs.y, cutoff.prob
Similar to negbinomial.
imunb, max. support
Similar to negbinomial.
max.chunk.MB, gsize.mux
Similar to negbinomial.
imethod, gprobs.y
See negbinomial.
type.fitted See CommonVGAMffArguments for details.

## Details

The positive negative binomial distribution is an ordinary negative binomial distribution but with the probability of a zero response being zero. The other probabilities are scaled to sum to unity.

This family function is based on negbinomial and most details can be found there. To avoid confusion, the parameter munb here corresponds to the mean of an ordinary negative binomial distribution negbinomial. The mean of posnegbinomial is

$$
\mu_{n b} /(1-p(0))
$$

where $p(0)=\left(k /\left(k+\mu_{n b}\right)\right)^{k}$ is the probability an ordinary negative binomial distribution has a zero value.
The parameters munb and $k$ are not independent in the positive negative binomial distribution, whereas they are in the ordinary negative binomial distribution.
This function handles multiple responses, so that a matrix can be used as the response. The number of columns is the number of species, say, and setting zero $=-2$ means that all species have a k equalling a (different) intercept only.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Warning

This family function is fragile; at least two cases will lead to numerical problems. Firstly, the positive-Poisson model corresponds to $k$ equalling infinity. If the data is positive-Poisson or close to positive-Poisson, then the estimated $k$ will diverge to Inf or some very large value. Secondly, if the data is clustered about the value 1 because the munb parameter is close to 0 then numerical problems will also occur. Users should set trace $=$ TRUE to monitor convergence. In the situation when both cases hold, the result returned (which will be untrustworthy) will depend on the initial values.

The negative binomial distribution (NBD) is a strictly unimodal distribution. Any data set that does not exhibit a mode (in the middle) makes the estimation problem difficult. The positive NBD inherits this feature. Set trace $=$ TRUE to monitor convergence .

See the example below of a data set where posbinomial() fails; the so-called solution is extremely poor. This is partly due to a lack of a unimodal shape because the number of counts decreases only. This long tail makes it very difficult to estimate the mean parameter with any certainty. The result too is that the size parameter is numerically fraught.
This VGAM family function inherits the same warnings as negbinomial. And if $k$ is much less than 1 then the estimation may be slow.

## Note

If the estimated $k$ is very large then fitting a pospoisson model is a good idea.
If both munb and $k$ are large then it may be necessary to decrease eps. trig and increase max. support so that the EIMs are positive-definite, e.g., eps.trig $=1 \mathrm{e}-8$ and max. support $=$ Inf.

## Author(s)

Thomas W. Yee

## References

Barry, S. C. and Welsh, A. H. (2002). Generalized additive modelling and zero inflated count data. Ecological Modelling, 157, 179-188.

Williamson, E. and Bretherton, M. H. (1964). Tables of the logarithmic series distribution. Annals of Mathematical Statistics, 35, 284-297.

## See Also

gatnbinomial.mlm, Gaitnbinom.mlm, rposnegbin, pospoisson, negbinomial, zanegbinomial, rnbinom, CommonVGAMffArguments, corbet, logff, simulate.vlm.

## Examples

```
pdata <- data.frame(x2 = runif(nn <- 1000))
pdata <- transform(pdata, y1 = rposnegbin(nn, munb = exp(0+2*x2), size = exp(1)),
    y2 = rposnegbin(nn, munb = exp(1+2*x2), size = exp(3)))
fit <- vglm(cbind(y1, y2) ~ x2, posnegbinomial, data = pdata, trace = TRUE)
coef(fit, matrix = TRUE)
dim(depvar(fit)) # Using dim(fit@y) is not recommended
```

\# Another artificial data example
pdata2 <- data.frame(munb $=\exp (2)$, size $=\exp (3))$; nn <- 1000
pdata2 <- transform(pdata2, y3 = rposnegbin(nn, munb = munb, size = size))
with(pdata2, table(y3))
fit <- vglm(y3 ~ 1, posnegbinomial, data = pdata2, trace = TRUE)
coef(fit, matrix = TRUE)
with(pdata2, mean(y3)) \# Sample mean
head(with(pdata2, munb/(1-(size/(size+munb))^size)), 1) \# Population mean
head(fitted(fit), 3)
head(predict(fit), 3)
\# Example: Corbet (1943) butterfly Malaya data
fit <- vglm(ofreq ~ 1, posnegbinomial, weights = species, data $=$ corbet)
coef(fit, matrix $=$ TRUE)
Coef(fit)
(khat <- Coef(fit)["size"])
pdf2 <- dposnegbin(x = with(corbet, ofreq), mu = fitted(fit), size = khat)
print(with(corbet, cbind(ofreq, species, fitted $=$ pdf2*sum(species))), dig = 1)
\#\# Not run:
with(corbet,
matplot(ofreq, cbind(species, fitted $=$ pdf2*sum(species)), las $=1$,
xlab = "Observed frequency (of individual butterflies)",
type = "b", ylab = "Number of species", col = c("blue", "orange"),
main = "blue 1s = observe; orange $2 \mathrm{~s}=$ fitted"))
\#\# End(Not run)
\#\# Not run:
\# This data (courtesy of Maxim Gerashchenko) causes posbinomial() to fail

```
pnbd.fail <- data.frame(
    y1 = c(1:16, 18:21, 23:28, 33:38, 42, 44, 49:51, 55, 56, 58,
    59, 61:63, 66, 73, 76, 94, 107, 112, 124, 190, 191, 244),
    ofreq = c(130, 80, 38, 23, 22, 11, 21, 14, 6, 7, 9, 9, 9, 4, 4, 5, 1,
            4, 6, 1, 3, 2, 4, 3, 4, 5, 3, 1, 2, 1, 1, 4, 1, 2, 2, 1, 3,
            1, 1, 2, 2, 2, 1, 3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1))
fit.fail <- vglm(y1 ~ 1, weights = ofreq, posnegbinomial,
            trace = TRUE, data = pnbd.fail)
## End(Not run)
```


## Description

Density, distribution function, quantile function and random generation for the univariate positivenormal distribution.

## Usage

```
dposnorm(x, mean = 0, sd = 1, log = FALSE)
pposnorm(q, mean = 0, sd = 1, lower.tail = TRUE, log.p = FALSE)
qposnorm(p, mean = 0, sd = 1, lower.tail = TRUE, log.p = FALSE)
rposnorm(n, mean = 0, sd = 1)
```


## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$n \quad$ number of observations. If length $(n)>1$ then the length is taken to be the number required.
mean, sd, log, lower.tail, log.p
see rnorm.

## Details

See posnormal, the VGAM family function for estimating the parameters, for the formula of the probability density function and other details.

## Value

dposnorm gives the density, pposnorm gives the distribution function, qposnorm gives the quantile function, and rposnorm generates random deviates.

## Author(s)

T. W. Yee

## See Also

posnormal.

## Examples

```
## Not run: m <- 0.8; x <- seq(-1, 4, len = 501)
plot(x, dposnorm(x, m = m), type = "l", las = 1, ylim = 0:1,
    ylab = paste("posnorm(m = ", m, ", sd = 1)"), col = "blue",
    main = "Blue is density, orange is cumulative distribution function",
    sub = "Purple lines are the 10,20,\ldots.,90 percentiles")
abline(h = 0, col = "grey")
lines(x, pposnorm(x, m = m), col = "orange", type = "l")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qposnorm(probs, m = m)
lines(Q, dposnorm(Q, m = m), col = "purple", lty = 3, type = "h")
lines(Q, pposnorm(Q, m = m), col = "purple", lty = 3, type = "h")
abline(h = probs, col = "purple", lty = 3)
max(abs(pposnorm(Q, m = m) - probs)) # Should be 0
## End(Not run)
```

posnormal Positive Normal Distribution Family Function

## Description

Fits a positive (univariate) normal distribution.

## Usage

```
posnormal(lmean = "identitylink", lsd = "loglink",
    eq.mean = FALSE, eq.sd = FALSE,
    gmean = exp((-5:5)/2), gsd = exp((-1:5)/2),
    imean = NULL, isd = NULL, probs.y = 0.10, imethod = 1,
    nsimEIM = NULL, zero = "sd")
```


## Arguments

lmean, lsd Link functions for the mean and standard deviation parameters of the usual univariate normal distribution. They are $\mu$ and $\sigma$ respectively. See Links for more choices.
gmean, gsd, imethod
See CommonVGAMffArguments for more information. gmean and gsd currently operate on a multiplicative scale, on the sample mean and the sample standard deviation, respectively.
imean, isd Optional initial values for $\mu$ and $\sigma$. A NULL means a value is computed internally. See CommonVGAMffArguments for more information.
eq.mean, eq.sd See CommonVGAMffArguments for more information. The fact that these arguments are supported results in default constraint matrices being a permutation of the identity matrix (effectively trivial constraints).
zero, nsimEIM, probs.y
See CommonVGAMffArguments for information.

## Details

The positive normal distribution is the ordinary normal distribution but with the probability of zero or less being zero. The rest of the probability density function is scaled up. Hence the probability density function can be written

$$
f(y)=\frac{1}{\sqrt{2 \pi} \sigma} \exp \left(-\frac{1}{2}(y-\mu)^{2} / \sigma^{2}\right) /[1-\Phi(-\mu / \sigma)]
$$

where $\Phi()$ is the cumulative distribution function of a standard normal (pnorm). Equivalently, this is

$$
f(y)=\frac{1}{\sigma} \frac{\phi((y-\mu) / \sigma)}{1-\Phi(-\mu / \sigma)}
$$

where $\phi()$ is the probability density function of a standard normal distribution (dnorm).
The mean of $Y$ is

$$
E(Y)=\mu+\sigma \frac{\phi(-\mu / \sigma)}{1-\Phi(-\mu / \sigma)}
$$

This family function handles multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

It is recommended that trace $=$ TRUE be used to monitor convergence; sometimes the estimated mean is -Inf and the estimated mean standard deviation is Inf, especially when the sample size is small. Under- or over-flow may occur if the data is ill-conditioned.

Note
The response variable for this family function is the same as uninormal except positive values are required. Reasonably good initial values are needed.
The distribution of the reciprocal of a positive normal random variable is known as an alpha distribution.

## Author(s)

Thomas W. Yee

## See Also

uninormal, tobit.

## Examples

```
pdata <- data.frame(Mean = 1.0, SD = exp(1.0))
pdata <- transform(pdata, y = rposnorm(n <- 1000, m = Mean, sd = SD))
## Not run: with(pdata, hist(y, prob = TRUE, border = "blue",
    main = paste("posnorm(m =", Mean[1], ", sd =", round(SD[1], 2),")")))
## End(Not run)
fit <- vglm(y ~ 1, posnormal, data = pdata, trace = TRUE)
coef(fit, matrix = TRUE)
(Cfit <- Coef(fit))
mygrid <- with(pdata, seq(min(y), max(y), len = 200)) # Add the fit to the histogram
## Not run: lines(mygrid, dposnorm(mygrid, Cfit[1], Cfit[2]), col = "orange")
```


## Pospois Positive-Poisson Distribution

## Description

Density, distribution function, quantile function and random generation for the positive-Poisson distribution.

## Usage

dpospois(x, lambda, log = FALSE)
ppospois(q, lambda)
qpospois(p, lambda)
rpospois(n, lambda)

## Arguments

| $x, q$ | vector of quantiles. |
| :--- | :--- |
| $p$ | vector of probabilities. |
| $n$ | number of observations. Fed into runif. |
| lambda | vector of positive means (of an ordinary Poisson distribution). Short vectors are <br> recycled. |
| log | logical. |

## Details

The positive-Poisson distribution is a Poisson distribution but with the probability of a zero being zero. The other probabilities are scaled to add to unity. The mean therefore is

$$
\lambda /(1-\exp (-\lambda)) .
$$

As $\lambda$ increases, the positive-Poisson and Poisson distributions become more similar. Unlike similar functions for the Poisson distribution, a zero value of lambda returns a NaN .

## Value

dpospois gives the density, ppospois gives the distribution function, qpospois gives the quantile function, and rpospois generates random deviates.

## Note

The family function pospoisson estimates $\lambda$ by maximum likelihood estimation.

## Author(s)

T. W. Yee

## See Also

Gaitpois, pospoisson, zapoisson, zipoisson, rpois.

## Examples

```
lambda <- 2; y = rpospois(n = 1000, lambda)
table(y)
mean(y) # Sample mean
lambda / (1 - exp(-lambda)) # Population mean
(ii <- dpospois(0:7, lambda))
cumsum(ii) - ppospois(0:7, lambda) # Should be 0s
table(rpospois(100, lambda))
table(qpospois(runif(1000), lambda))
round(dpospois(1:10, lambda) * 1000) # Should be similar
## Not run: x <- 0:7
barplot(rbind(dpospois(x, lambda), dpois(x, lambda)),
    beside = TRUE, col = c("blue", "orange"),
    main = paste("Positive Poisson(", lambda, ") (blue) vs",
    " Poisson(", lambda, ") (orange)", sep = ""),
    names.arg = as.character(x), las = 1, lwd = 2)
## End(Not run)
```

pospoisson Positive Poisson Distribution Family Function

## Description

Fits a positive Poisson distribution.

## Usage

pospoisson(link = "loglink", type.fitted = c("mean", "lambda", "prob0"), expected $=$ TRUE, ilambda $=$ NULL, imethod $=1$, zero $=$ NULL, gt. 1 = FALSE)

## Arguments

link Link function for the usual mean (lambda) parameter of an ordinary Poisson distribution. See Links for more choices.
expected Logical. Fisher scoring is used if expected = TRUE, else Newton-Raphson.
ilambda, imethod, zero
See CommonVGAMffArguments for information.
type.fitted See CommonVGAMffArguments for details.
gt. $1 \quad$ Logical. Enforce lambda > 1? The default is to enforce lambda $>0$.

## Details

The positive Poisson distribution is the ordinary Poisson distribution but with the probability of zero being zero. Thus the other probabilities are scaled up (i.e., divided by $1-P[Y=0]$ ). The mean, $\lambda /(1-\exp (-\lambda))$, can be obtained by the extractor function fitted applied to the object.
A related distribution is the zero-inflated Poisson, in which the probability $P[Y=0]$ involves another parameter $\phi$. See zipoisson.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Warning

Under- or over-flow may occur if the data is ill-conditioned.

## Note

This family function can handle multiple responses.
Yet to be done: a quasi. pospoisson which estimates a dispersion parameter.

## Author(s)

Thomas W. Yee

## References

Coleman, J. S. and James, J. (1961). The equilibrium size distribution of freely-forming groups. Sociometry, 24, 36-45.

## See Also

Pospois, gaitpoisson.mix, gaitpoisson.mlm, posnegbinomial, poissonff, otpospoisson, zapoisson, zipoisson, simulate.vlm.

## Examples

```
    # Data from Coleman and James (1961)
    cjdata <- data.frame(y = 1:6, freq = c(1486, 694, 195, 37, 10, 1))
    fit <- vglm(y ~ 1, pospoisson, data = cjdata, weights = freq)
    Coef(fit)
    summary(fit)
    fitted(fit)
    pdata <- data.frame(x2 = runif(nn <- 1000)) # Artificial data
    pdata <- transform(pdata, lambda = exp(1-2 * x2))
    pdata <- transform(pdata, y1 = rpospois(nn, lambda))
    with(pdata, table(y1))
    fit <- vglm(y1 ~ x2, pospoisson, data = pdata, trace = TRUE, crit = "coef")
    coef(fit, matrix = TRUE)
```

powerlink
Power Link Function

## Description

Computes the power transformation, including its inverse and the first two derivatives.

## Usage

```
powerlink(theta, power = 1, inverse = FALSE, deriv = 0,
```

    short = TRUE, tag = FALSE)
    
## Arguments

theta Numeric or character. See below for further details.
power $\quad$ This denotes the power or exponent.
inverse, deriv, short, tag
Details at Links.

## Details

The power link function raises a parameter by a certain value of power. Care is needed because it is very easy to get numerical problems, e.g., if power $=0.5$ and theta is negative.

## Value

For powerlink with deriv = 0, then theta raised to the power of power. And if inverse $=$ TRUE then theta raised to the power of 1 /power.

For deriv $=1$, then the function returns $d$ theta $/ d$ eta as a function of theta if inverse $=$ FALSE, else if inverse $=$ TRUE then it returns the reciprocal.

## Note

Numerical problems may occur for certain combinations of theta and power. Consequently this link function should be used with caution.

## Author(s)

Thomas W. Yee

## See Also

Links, loglink.

## Examples

```
powerlink("a", power = 2, short = FALSE, tag = TRUE)
powerlink(x <- 1:5)
powerlink(x, power = 2)
max(abs(powerlink(powerlink(x, power = 2),
    power = 2, inverse = TRUE) - x) ) \# Should be 0
powerlink(x <- (-5):5, power \(=0.5\) ) \# Has NAs
\# 1/2 = 0.5
pdata <- data.frame( \(y=\operatorname{rbeta}\left(n=1000\right.\), shape \(1=2^{\wedge} 2\), shape \(\left.2=3^{\wedge} 2\right)\) )
fit <- vglm(y ~ 1, betaR(lshape1 = powerlink(power = 0.5), i1 = 3,
    lshape2 \(=\) powerlink(power = 0.5), i2 = 7), data = pdata)
t(coef(fit, matrix = TRUE))
Coef(fit) \# Useful for intercept-only models
vcov(fit, untransform \(=\) TRUE)
```

    prats Pregnant Rats Toxological Experiment Data
    
## Description

A small toxological experiment data. The subjects are fetuses from two randomized groups of pregnant rats, and they were given a placebo or chemical treatment. The number with birth defects were recorded, as well as each litter size.

## Usage

data(prats)

## Format

A data frame with the following variables.
treatment A 0 means control; a 1 means the chemical treatment.
alive, litter.size The number of fetuses alive at 21 days, out of the number of fetuses alive at 4 days (the litter size).

## Details

The data concerns a toxological experiment where the subjects are fetuses from two randomized groups of 16 pregnant rats each, and they were given a placebo or chemical treatment. The number with birth defects and the litter size were recorded. Half the rats were fed a control diet during pregnancy and lactation, and the diet of the other half was treated with a chemical. For each litter the number of pups alive at 4 days and the number of pups that survived the 21 day lactation period, were recorded.

## Source

Weil, C. S. (1970) Selection of the valid number of sampling units and a consideration of their combination in toxicological studies involving reproduction, teratogenesis or carcinogenesis. Food and Cosmetics Toxicology, 8(2), 177-182.

## References

Williams, D. A. (1975). The Analysis of Binary Responses From Toxicological Experiments Involving Reproduction and Teratogenicity. Biometrics, 31(4), 949-952.

## See Also

betabinomial, betabinomialff.

## Examples

## prats

colSums(subset(prats, treatment ==0))
colSums(subset(prats, treatment == 1))
summary (prats)

```
predictqrrvglm Predict Method for a CQO fit
```


## Description

Predicted values based on a constrained quadratic ordination (CQO) object.

## Usage

```
predictqrrvglm(object, newdata = NULL,
    type = c("link", "response", "latvar", "terms"),
    se.fit = FALSE, deriv = 0, dispersion = NULL,
    extra = object@extra, varI.latvar = FALSE, refResponse = NULL, ...)
```


## Arguments

object Object of class inheriting from "qrrvglm".
newdata An optional data frame in which to look for variables with which to predict. If omitted, the fitted linear predictors are used.
type, se.fit, dispersion, extra See predictvglm.
deriv Derivative. Currently only 0 is handled.
varI.latvar, refResponse
Arguments passed into Coef. qrrvglm.
... Currently undocumented.

## Details

Obtains predictions from a fitted CQO object. Currently there are lots of limitations of this function; it is unfinished.

## Value

See predictvglm.

## Note

This function is not robust and has not been checked fully.

## Author(s)

T. W. Yee

## References

Yee, T. W. (2004). A new technique for maximum-likelihood canonical Gaussian ordination. Ecological Monographs, 74, 685-701.

## See Also

```
cqo, calibrate.qrrvglm.
```


## Examples

```
## Not run: set.seed(1234)
hspider[, 1:6] <- scale(hspider[, 1:6]) # Standardize the X vars
p1 <- cqo(cbind(Alopacce, Alopcune, Alopfabr, Arctlute,
    Arctperi, Auloalbi, Pardlugu, Pardmont,
    Pardnigr, Pardpull, Trocterr, Zoraspin) ~
        WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
        poissonff, data = hspider, Crow1positive = FALSE, I.toler = TRUE)
sort(deviance(p1, history = TRUE)) # A history of all the iterations
head(predict(p1))
```

\# The following should be all 0s:
$\max (\operatorname{abs}(\operatorname{predict}(\mathrm{p} 1$, newdata $=$ head(hspider)) - head(predict(p1))))
$\max (\operatorname{abs}(\operatorname{predict}(\mathrm{p} 1$, newdata $=$ head(hspider), type $=" r e s ")-h e a d(f i t t e d(p 1))))$
\#\# End(Not run)

```
predictvglm Predict Method for a VGLM fit
```


## Description

Predicted values based on a vector generalized linear model (VGLM) object.

## Usage

predictvglm(object, newdata $=$ NULL, type = c("link", "response", "terms"), se.fit $=$ FALSE, deriv $=0$, dispersion $=$ NULL, untransform = FALSE,
type.fitted $=$ NULL, percentiles $=$ NULL, ...)

## Arguments

object Object of class inheriting from "vlm", e.g., vglm.
newdata An optional data frame in which to look for variables with which to predict. If omitted, the fitted linear predictors are used.
type The value of this argument can be abbreviated. The type of prediction required. The default is the first one, meaning on the scale of the linear predictors. This should be a $n \times M$ matrix.
The alternative "response" is on the scale of the response variable, and depending on the family function, this may or may not be the mean. Often this is the fitted value, e.g., fitted(vglmObject) (see fittedvlm). Note that the response is output from the @linkinv slot, where the eta argument is the $n \times M$ matrix of linear predictors.
The "terms" option returns a matrix giving the fitted values of each term in the model formula on the linear predictor scale. The terms have been centered.
se.fit logical: return standard errors?
deriv Non-negative integer. Currently this must be zero. Later, this may be implemented for general values.
dispersion Dispersion parameter. This may be inputted at this stage, but the default is to use the dispersion parameter of the fitted model.
type.fitted Some VGAM family functions have an argument by the same name. If so, then one can obtain fitted values by setting type = "response" and choosing a value of type.fitted from what's available. If type.fitted = "quantiles" is available then the percentiles argument can be used to specify what quantile values are requested.
percentiles Used only if type.fitted = "quantiles" is available and is selected.
untransform Logical. Reverses any parameter link function. This argument only works if type $=$ "link", se.fit $=$ FALSE, deriv $=0$. Setting untransform $=$ TRUE does not work for all VGAM family functions; only ones where there is a one-to-one correspondence between a simple link function and a simple parameter might work.
... Arguments passed into predictvlm.

## Details

Obtains predictions and optionally estimates standard errors of those predictions from a fitted vglm object.
This code implements smart prediction (see smartpred).

## Value

If se.fit $=$ FALSE, a vector or matrix of predictions. If se.fit $=$ TRUE, a list with components
fitted.values Predictions
se.fit Estimated standard errors
df Degrees of freedom
sigma The square root of the dispersion parameter

## Warning

This function may change in the future.

## Note

Setting se.fit $=$ TRUE and type $=$ "response" will generate an error.
The arguments type.fitted and percentiles are provided in this function to give more convenience than modifying the extra slot directly.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Hastie, T. J. (2003). Reduced-rank vector generalized linear models. Statistical Modelling, 3, 15-41.

## See Also

predict, vglm, predictvlm, smartpred, calibrate.

## Examples

```
    \# Illustrates smart prediction
    pneumo <- transform(pneumo, let \(=\log (\) exposure.time))
    fit <- vglm(cbind(normal, mild, severe) ~ poly(c(scale(let)), 2),
            propodds, data \(=\) pneumo, trace \(=\) TRUE, x.arg \(=\) FALSE \()\)
    class(fit)
    (q0 <- head(predict(fit)))
    (q1 <- predict(fit, newdata \(=\) head(pneumo)))
    (q2 <- predict(fit, newdata \(=\) head(pneumo)))
    all.equal(q0, q1) \# Should be TRUE
    all.equal(q1, q2) \# Should be TRUE
    head(predict(fit))
    head(predict(fit, untransform = TRUE))
    p0 <- head(predict(fit, type = "response"))
    p1 <- head(predict(fit, type = "response", newdata = pneumo))
    p2 <- head(predict(fit, type = "response", newdata = pneumo))
    p3 <- head(fitted(fit))
    all.equal(p0, p1) \# Should be TRUE
    all.equal(p1, p2) \# Should be TRUE
    all.equal(p2, p3) \# Should be TRUE
    predict(fit, type = "terms", se = TRUE)
```

    prentice74
    Prentice (1974) Log-gamma Distribution
    
## Description

Estimation of a 3-parameter log-gamma distribution described by Prentice (1974).

## Usage

```
prentice74(llocation = "identitylink", lscale = "loglink",
    lshape = "identitylink", ilocation = NULL, iscale = NULL,
    ishape = NULL, imethod = 1,
    glocation.mux = exp((-4:4)/2), gscale.mux = exp((-4:4)/2),
    gshape = qt(ppoints(6), df = 1), probs.y = 0.3,
    zero = c("scale", "shape"))
```


## Arguments

llocation, lscale, lshape
Parameter link function applied to the location parameter $a$, positive scale parameter $b$ and the shape parameter $q$, respectively. See Links for more choices.

```
ilocation, iscale
                            Initial value for }a\mathrm{ and b, respectively. The defaults mean an initial value is determined internally for each.
ishape Initial value for \(q\). If failure to converge occurs, try some other value. The default means an initial value is determined internally.
imethod, zero See CommonVGAMffArguments for information.
glocation.mux, gscale.mux, gshape, probs.y
See CommonVGAMffArguments for information.
```


## Details

The probability density function is given by

$$
f(y ; a, b, q)=|q| \exp \left(w / q^{2}-e^{w}\right) /\left(b \Gamma\left(1 / q^{2}\right)\right)
$$

for shape parameter $q \neq 0$, positive scale parameter $b>0$, location parameter $a$, and all real $y$. Here, $w=(y-a) q / b+\psi\left(1 / q^{2}\right)$ where $\psi$ is the digamma function, digamma. The mean of $Y$ is $a$ (returned as the fitted values). This is a different parameterization compared to lgamma3.
Special cases: $q=0$ is the normal distribution with standard deviation $b, q=-1$ is the extreme value distribution for maximums, $q=1$ is the extreme value distribution for minima (Weibull). If $q>0$ then the distribution is left skew, else $q<0$ is right skew.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

The special case $q=0$ is not handled, therefore estimates of $q$ too close to zero may cause numerical problems.

## Note

The notation used here differs from Prentice (1974): $\alpha=a, \sigma=b$. Fisher scoring is used.

## Author(s)

T. W. Yee

## References

Prentice, R. L. (1974). A log gamma model and its maximum likelihood estimation. Biometrika, 61, 539-544.

## See Also

lgamma3, lgamma, gengamma.stacy.

## Examples

```
    pdata <- data.frame(x2 = runif(nn <- 1000))
    pdata <- transform(pdata, loc = -1 + 2*x2, Scale = exp(1))
    pdata <- transform(pdata, y = rlgamma(nn, loc = loc, scale = Scale, shape = 1))
    fit <- vglm(y ~ x2, prentice74(zero = 2:3), data = pdata, trace = TRUE)
    coef(fit, matrix = TRUE) # Note the coefficients for location
```

    prinia Yellow-bellied Prinia
    
## Description

A data frame with yellow-bellied Prinia.

## Usage

data(prinia)

## Format

A data frame with 151 observations on the following 23 variables.
length a numeric vector, the scaled wing length (zero mean and unit variance).
fat a numeric vector, fat index; originally 1 (no fat) to 4 (very fat) but converted to 0 (no fat) versus 1 otherwise.
cap a numeric vector, number of times the bird was captured or recaptured.
noncap a numeric vector, number of times the bird was not captured.
$\mathbf{y} 01, \mathbf{y} 02, \mathbf{y} 03, \mathbf{y} 04, \mathbf{y} 05, \mathbf{y} 06$ a numeric vector of 0 s and 1 s ; for noncapture and capture resp.
$\mathbf{y 0 7}, \mathbf{y 0 8}, \mathbf{y} 09, \mathbf{y 1 0}, \mathbf{y 1 1}, \mathbf{y 1 2}$ same as above.
$\mathbf{y 1 3}, \mathbf{y 1 4}, \mathbf{y 1 5}, \mathbf{y 1 6}, \mathbf{y 1 7}, \mathbf{y 1 8}, \mathbf{y 1 9}$ same as above.

## Details

The yellow-bellied Prinia Prinia flaviventris is a common bird species located in Southeast Asia. A capture-recapture experiment was conducted at the Mai Po Nature Reserve in Hong Kong during 1991, where captured individuals had their wing lengths measured and fat index recorded. A total of 19 weekly capture occasions were considered, where 151 distinct birds were captured.
More generally, the prinias are a genus of small insectivorous birds, and are sometimes referred to as wren-warblers. They are a little-known group of the tropical and subtropical Old World, the roughly 30 species being divided fairly equally between Africa and Asia.

## Source

Thanks to Paul Yip for permission to make this data available.
Hwang, W.-H. and Huggins, R. M. (2007) Application of semiparametric regression models in the analysis of capture-recapture experiments. Australian and New Zealand Journal of Statistics 49, 191-202.

## Examples

```
head(prinia)
summary(prinia)
rowSums(prinia[, c("cap", "noncap")]) # 19s
# Fit a positive-binomial distribution (M.h) to the data:
fit1 <- vglm(cbind(cap, noncap) ~ length + fat, posbinomial, data = prinia)
# Fit another positive-binomial distribution (M.h) to the data:
# The response input is suitable for posbernoulli.*-type functions.
fit2 <- vglm(cbind(y01, y02, y03, y04, y05, y06, y07, y08, y09, y10,
    y11, y12, y13, y14, y15, y16, y17, y18, y19) ~
    length + fat, posbernoulli.b(drop.b = FALSE ~ 0), data = prinia)
```

    probitlink Probit Link Function
    
## Description

Computes the probit transformation, including its inverse and the first two derivatives.

## Usage

probitlink(theta, bvalue $=$ NULL, inverse $=$ FALSE, deriv $=0$, short $=$ TRUE, tag = FALSE)

## Arguments

theta Numeric or character. See below for further details.
bvalue See Links.
inverse, deriv, short, tag
Details at Links.

## Details

The probit link function is commonly used for parameters that lie in the unit interval. It is the inverse CDF of the standard normal distribution. Numerical values of theta close to 0 or 1 or out of range result in Inf, $-\mathrm{Inf}, \mathrm{NA}$ or NaN .

## Value

For deriv $=0$, the probit of theta, i.e., qnorm(theta) when inverse $=$ FALSE, and if inverse $=$ TRUE then pnorm(theta).

For deriv $=1$, then the function returns $d$ eta $/ d$ theta as a function of theta if inverse $=$ FALSE, else if inverse $=$ TRUE then it returns the reciprocal.

## Note

Numerical instability may occur when theta is close to 1 or 0 . One way of overcoming this is to use bvalue.
In terms of the threshold approach with cumulative probabilities for an ordinal response this link function corresponds to the univariate normal distribution (see uninormal).

## Author(s)

Thomas W. Yee

## References

McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, 2nd ed. London: Chapman \& Hall.

## See Also

Links, logitlink, clogloglink, cauchitlink, Normal.

## Examples

```
    p <- seq(0.01, 0.99, by = 0.01)
    probitlink(p)
    max(abs(probitlink(probitlink(p), inverse = TRUE) - p)) # Should be 0
    p <- c(seq(-0.02, 0.02, by = 0.01), seq(0.97, 1.02, by = 0.01))
    probitlink(p) # Has NAs
    probitlink(p, bvalue = .Machine$double.eps) # Has no NAs
    ## Not run: p <- seq(0.01, 0.99, by = 0.01); par(lwd = (mylwd <- 2))
    plot(p, logitlink(p), type = "l", col = "limegreen", ylab = "transformation",
    las = 1, main = "Some probability link functions")
    lines(p, probitlink(p), col = "purple")
    lines(p, clogloglink(p), col = "chocolate")
    lines(p, cauchitlink(p), col = "tan")
    abline(v = 0.5, h = 0, lty = "dashed")
    legend(0.1, 4, c("logitlink", "probitlink", "clogloglink", "cauchitlink"),
        col = c("limegreen", "purple", "chocolate", "tan"), lwd = mylwd)
    par(lwd = 1)
    ## End(Not run)
```

    profilevglm
        Method for Profiling vglm Objects
    
## Description

Investigates the profile log-likelihood function for a fitted model of class "vglm".

## Usage

profilevglm(object, which = 1:p.vlm, alpha = 0.01, maxsteps $=10$, del $=$ zmax/5, trace $=$ NULL, ...)

## Arguments

object the original fitted model object.
which the original model parameters which should be profiled. This can be a numeric or character vector. By default, all parameters are profiled.
alpha highest significance level allowed for the profiling.
maxsteps maximum number of points to be used for profiling each parameter.
del suggested change on the scale of the profile t-statistics. Default value chosen to allow profiling at about 10 parameter values.
trace logical: should the progress of profiling be reported? The default is to use the trace value from the fitted object; see vglm. control for details.
... further arguments passed to or from other methods.

## Details

This function is called by confintvglm to do the profiling. See also profile.glm for details.

## Value

A list of classes "profile.glm" and "profile" with an element for each parameter being profiled. The elements are data-frames with two variables

```
par.vals a matrix of parameter values for each fitted model.
tau the profile t-statistics.
```


## Author(s)

T. W. Yee adapted this function from profile.glm, written originally by D. M. Bates and W. N. Venables. (For S in 1996.) The help file was also used as a template.

## See Also

vglm, confintvglm, lrt.stat, profile, profile.glm, plot.profile.

## Examples

```
pneumo <- transform(pneumo, let = log(exposure.time))
fit1 <- vglm(cbind(normal, mild, severe) ~ let, propodds,
    trace = TRUE, data = pneumo)
pfit1 <- profile(fit1, trace = FALSE)
confint(fit1, method = "profile", trace = FALSE)
```


## Description

Fits the proportional odds model to a (preferably ordered) factor response.

## Usage

propodds(reverse = TRUE, whitespace = FALSE)

## Arguments

reverse, whitespace
Logical. Fed into arguments of the same name in cumulative.

## Details

The proportional odds model is a special case from the class of cumulative link models. It involves a logit link applied to cumulative probabilities and a strong parallelism assumption. A parallelism assumption means there is less chance of numerical problems because the fitted probabilities will remain between 0 and 1 ; however the parallelism assumption ought to be checked, e.g., via a likelihood ratio test. This VGAM family function is merely a shortcut for cumulative(reverse $=$ reverse, link = "logit", parallel = TRUE). Please see cumulative for more details on this model.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

No check is made to verify that the response is ordinal if the response is a matrix; see ordered.

## Author(s)

Thomas W. Yee

## References

Agresti, A. (2010). Analysis of Ordinal Categorical Data, 2nd ed. Hoboken, NJ, USA: Wiley.
Yee, T. W. (2010). The VGAM package for categorical data analysis. Journal of Statistical Software, 32, 1-34. http://www. jstatsoft.org/v32/i10/.
Yee, T. W. and Wild, C. J. (1996). Vector generalized additive models. Journal of the Royal Statistical Society, Series B, Methodological, 58, 481-493.

## See Also

cumulative, R2latvar.

## Examples

```
# Fit the proportional odds model, p.179, in McCullagh and Nelder (1989)
pneumo <- transform(pneumo, let = log(exposure.time))
(fit <- vglm(cbind(normal, mild, severe) ~ let, propodds, data = pneumo))
depvar(fit) # Sample proportions
weights(fit, type = "prior") # Number of observations
coef(fit, matrix = TRUE)
constraints(fit) # Constraint matrices
summary(fit)
# Check that the model is linear in let -----------------------
fit2 <- vgam(cbind(normal, mild, severe) ~ s(let, df = 2), propodds, data = pneumo)
## Not run: plot(fit2, se = TRUE, lcol = 2, scol = 2)
# Check the proportional odds assumption with a LRT ----------
(fit3 <- vglm(cbind(normal, mild, severe) ~ let,
                            cumulative(parallel = FALSE, reverse = TRUE), data = pneumo))
pchisq(deviance(fit) - deviance(fit3),
    df = df.residual(fit) - df.residual(fit3), lower.tail = FALSE)
lrtest(fit3, fit) # Easier
```

prplot Probability Plots for Categorical Data Analysis

## Description

Plots the fitted probabilities for some very simplified special cases of categorical data analysis models.

## Usage

prplot(object, control = prplot.control(...), ...)
prplot.control(xlab = NULL, ylab = "Probability", main = NULL, xlim = NULL, ylim = NULL, lty = par()\$lty, col = par()\$col, rcol = par()\$col, lwd = par()\$lwd, rlwd = par()\$lwd, las = par()\$las, rug.arg = FALSE, ...)

## Arguments

object Currently only an cumulative object. This includes a propodds object since that VGAM family function is a special case of cumulative.
control List containing some basic graphical parameters.
xlab, ylab, main, xlim, ylim, lty
See par and . . . below.
col, rcol, lwd, rlwd, las, rug.arg
See par and . . . below. Arguments starting with r refer to the rug. Argument rug. arg is logical: add a rug for the distinct values of the explanatory variable?
... Arguments such as xlab which are fed into prplot.control(). Only a small selection of graphical arguments from par are offered.

## Details

For models involving one term in the RHS of the formula this function plots the fitted probabilities against the single explanatory variable.

## Value

The object is returned invisibly with the preplot slot assigned. This is obtained by a call to plotvgam().

## Note

This function is rudimentary.

## See Also

cumulative.

## Examples

```
pneumo <- transform(pneumo, let = log(exposure.time))
fit <- vglm(cbind(normal, mild, severe) ~ let, propodds, data = pneumo)
M <- npred(fit) # Or fit@misc$M
## Not run: prplot(fit)
prplot(fit, lty = 1:M, col = (1:M)+2, rug = TRUE, las = 1,
    ylim = c(0, 1), rlwd = 2)
## End(Not run)
```

put.smart
Adds a List to the End of the List ".smart.prediction"

## Description

Adds a list to the end of the list . smart. prediction in smartpredenv.

## Usage

put.smart(smart)

## Arguments

smart
a list containing parameters needed later for smart prediction.

## Details

put.smart is used in "write" mode within a smart function. It saves parameters at the time of model fitting, which are later used for prediction. The function put.smart is the opposite of get. smart, and both deal with the same contents.

## Value

Nothing is returned.

## Side Effects

The variable.smart.prediction. counter in smartpredenv is incremented beforehand, and .smart. prediction[[.smar is assigned the list smart. If the list . smart. prediction in smartpredenv is not long enough to hold smart, then it is made larger, and the variable .max. smart in smartpredenv is adjusted accordingly.

## See Also

```
get.smart.
```


## Examples

```
print(sm.min1)
```

```
qrrvglm.control Control Function for QRR-VGLMs (CQO)
```


## Description

Algorithmic constants and parameters for a constrained quadratic ordination (CQO), by fitting a quadratic reduced-rank vector generalized linear model (QRR-VGLM), are set using this function. It is the control function for cqo.

## Usage

qrrvglm. control(Rank = 1, Bestof = if (length(Cinit)) 1 else 10, checkwz = TRUE, Cinit = NULL, Crow1positive = TRUE, epsilon = 1.0e-06, EqualTolerances = NULL, eq.tolerances = TRUE, Etamat.colmax = 10, FastAlgorithm = TRUE, GradientFunction = TRUE, Hstep = 0.001, isd.latvar = rep_len(c(2, 1, rep_len(0.5, Rank)), Rank), iKvector = 0.1, iShape = 0.1, ITolerances = NULL, I.tolerances = FALSE, maxitl = 40, imethod = 1, Maxit.optim = 250, MUXfactor = rep_len(7, Rank), noRRR $=\sim 1$, Norrr = NA, optim.maxit $=20$, Parscale $=$ if (I.tolerances) 0.001 else 1.0,

```
sd.Cinit = 0.02, SmallNo = 5.0e-13, trace = TRUE,
Use.Init.Poisson.QO = TRUE, wzepsilon = .Machine$double.eps^0.75, ...)
```


## Arguments

In the following, $R$ is the Rank, $M$ is the number of linear predictors, and $S$ is the number of responses (species). Thus $M=S$ for binomial and Poisson responses, and $M=2 S$ for the negative binomial and 2-parameter gamma distributions.
The numerical rank $R$ of the model, i.e., the number of ordination axes. Must be an element from the set $\left\{1,2, \ldots, \min \left(M, p_{2}\right)\right\}$ where the vector of explanatory variables $x$ is partitioned into ( $x_{1}, x_{2}$ ), which is of dimension $p_{1}+p_{2}$. The variables making up $x_{1}$ are given by the terms in the noRRR argument, and the rest of the terms comprise $x_{2}$.
Bealkof Integer. The best of Bestof models fitted is returned. This argument helps guard against local solutions by (hopefully) finding the global solution from many fits. The argument has value 1 if an initial value for $C$ is inputted using Cinit.
checkwz logical indicating whether the diagonal elements of the working weight matrices should be checked whether they are sufficiently positive, i.e., greater than wzepsilon. If not, any values less than wzepsilon are replaced with this value.
Cinit Optional initial $C$ matrix, which must be a $p_{2}$ by $R$ matrix. The default is to apply. Init. Poisson.QO() to obtain initial values.

Crow1positive Logical vector of length Rank (recycled if necessary): are the elements of the first row of $C$ positive? For example, if Rank is 4, then specifying Crow1positive $=\mathrm{c}($ FALSE, TRUE $)$ will force $C[1,1]$ and $C[1,3]$ to be negative, and $C[1,2]$ and $C[1,4]$ to be positive. This argument allows for a reflection in the ordination axes because the coefficients of the latent variables are unique up to a sign.
epsilon Positive numeric. Used to test for convergence for GLMs fitted in C. Larger values mean a loosening of the convergence criterion. If an error code of 3 is reported, try increasing this value.
eq. tolerances Logical indicating whether each (quadratic) predictor will have equal tolerances. Having eq. tolerances = TRUE can help avoid numerical problems, especially with binary data. Note that the estimated (common) tolerance matrix may or may not be positive-definite. If it is then it can be scaled to the $R$ by $R$ identity matrix, i.e., made equivalent to I.tolerances = TRUE. Setting I. tolerances $=$ TRUE will force a common $R$ by $R$ identity matrix as the tolerance matrix to the data even if it is not appropriate. In general, setting I. tolerances $=$ TRUE is preferred over eq. tolerances = TRUE because, if it works, it is much faster and uses less memory. However, I. tolerances = TRUE requires the environmental variables to be scaled appropriately. See Details for more details.
EqualTolerances
Defunct argument. Use eq. tolerances instead.
Etamat. colmax Positive integer, no smaller than Rank. Controls the amount of memory used by . Init. Poisson. QO(). It is the maximum number of columns allowed for the pseudo-response and its weights. In general, the larger the value, the better the initial value. Used only if Use. Init. Poisson. QO = TRUE.

| FastAlgorithm | Logical. Whether a new fast algorithm is to be used. The fast algorithm results in a large speed increases compared to Yee (2004). Some details of the fast algorithm are found in Appendix A of Yee (2006). Setting FastAlgorithm = FALSE will give an error. |
| :---: | :---: |
| GradientFunction |  |
|  | Logical. Whether optim's argument gr is used or not, i.e., to compute gradient values. Used only if FastAlgorithm is TRUE. The default value is usually faster on most problems. |
| Hstep | Positive value. Used as the step size in the finite difference approximation to the derivatives by optim. |
| isd.latvar | Initial standard deviations for the latent variables (site scores). Numeric, positive and of length $R$ (recycled if necessary). This argument is used only if I. tolerances = TRUE. Used by . Init.Poisson. QO() to obtain initial values for the constrained coefficients $C$ adjusted to a reasonable value. It adjusts the spread of the site scores relative to a common species tolerance of 1 for each ordination axis. A value between 0.5 and 10 is recommended; a value such as 10 means that the range of the environmental space is very large relative to the niche width of the species. The successive values should decrease because the first ordination axis should have the most spread of site scores, followed by the second ordination axis, etc. |
| iKvector, iShape |  |
|  | Numeric, recycled to length $S$ if necessary. Initial values used for estimating the positive $k$ and $\lambda$ parameters of the negative binomial and 2-parameter gamma distributions respectively. For further information see negbinomial and gamma2. These arguments override the ik and ishape arguments in negbinomial and gamma2. |
| I. tolerances | Logical. If TRUE then the (common) tolerance matrix is the $R$ by $R$ identity matrix by definition. Note that having I. tolerances = TRUE implies eq. tolerances = TRUE, but not vice versa. Internally, the quadratic terms will be treated as offsets (in GLM jargon) and so the models can potentially be fitted very efficiently. However, it is a very good idea to center and scale all numerical variables in the $x_{2}$ vector. See Details for more details. The success of I. tolerances = TRUE often depends on suitable values for isd.latvar and/or MUXfactor. |
| ITolerances | Defunct argument. Use I. tolerances instead. |
| maxitl | Maximum number of times the optimizer is called or restarted. Most users should ignore this argument. |
| imethod | Method of initialization. A positive integer 1 or 2 or 3 etc. depending on the VGAM family function. Currently it is used for negbinomial and gamma2 only, and used within the C . |
| Maxit.optim | Positive integer. Number of iterations given to the function optim at each of the optim.maxit iterations. |
| MUXfactor | Multiplication factor for detecting large offset values. Numeric, positive and of length $R$ (recycled if necessary). This argument is used only if I.tolerances $=$ TRUE. Offsets are -0.5 multiplied by the sum of the squares of all $R$ latent variable values. If the latent variable values are too large then this will result in numerical problems. By too large, it is meant that the standard deviation of |

the latent variable values are greater than MUXfactor[r]*isd.latvar[r] for $r=1$ : Rank (this is why centering and scaling all the numerical predictor variables in $x_{2}$ is recommended). A value about 3 or 4 is recommended. If failure to converge occurs, try a slightly lower value.
optim.maxit Positive integer. Number of times optim is invoked. At iteration i, the ith value of Maxit.optim is fed into optim.
noRRR Formula giving terms that are not to be included in the reduced-rank regression (or formation of the latent variables), i.e., those belong to $x_{1}$. Those variables which do not make up the latent variable (reduced-rank regression) correspond to the $B_{1}$ matrix. The default is to omit the intercept term from the latent variables.

Norrr Defunct. Please use noRRR. Use of Norrr will become an error soon.
Parscale Numerical and positive-valued vector of length $C$ (recycled if necessary). Passed into optim( . . . , control = list (parscale = Parscale)) ; the elements of $C$ become C / Parscale. Setting I.tolerances = TRUE results in line searches that are very large, therefore $C$ has to be scaled accordingly to avoid large step sizes. See Details for more information. It's probably best to leave this argument alone.
sd.Cinit Standard deviation of the initial values for the elements of $C$. These are normally distributed with mean zero. This argument is used only if Use. Init. Poisson. QO $=$ FALSE and $C$ is not inputted using Cinit.
trace Logical indicating if output should be produced for each iteration. The default is TRUE because the calculations are numerically intensive, meaning it may take a long time, so that the user might think the computer has locked up if trace $=$ FALSE.

SmallNo Positive numeric between .Machine\$double.eps and 0.0001. Used to avoid under- or over-flow in the IRLS algorithm. Used only if FastAlgorithm is TRUE.
Use.Init.Poisson.Q0
Logical. If TRUE then the function. Init.Poisson. QO() is used to obtain initial values for the canonical coefficients $C$. If FALSE then random numbers are used instead.
wzepsilon Small positive number used to test whether the diagonals of the working weight matrices are sufficiently positive.
... Ignored at present.

## Details

Recall that the central formula for CQO is

$$
\eta=B_{1}^{T} x_{1}+A \nu+\sum_{m=1}^{M}\left(\nu^{T} D_{m} \nu\right) e_{m}
$$

where $x_{1}$ is a vector (usually just a 1 for an intercept), $x_{2}$ is a vector of environmental variables, $\nu=C^{T} x_{2}$ is a $R$-vector of latent variables, $e_{m}$ is a vector of 0 s but with a 1 in the $m$ th position.

QRR-VGLMs are an extension of RR-VGLMs and allow for maximum likelihood solutions to constrained quadratic ordination (CQO) models.
Having I. tolerances = TRUE means all the tolerance matrices are the order- $R$ identity matrix, i.e., it forces bell-shaped curves/surfaces on all species. This results in a more difficult optimization problem (especially for 2-parameter models such as the negative binomial and gamma) because of overflow errors and it appears there are more local solutions. To help avoid the overflow errors, scaling $C$ by the factor Parscale can help enormously. Even better, scaling $C$ by specifying isd.latvar is more understandable to humans. If failure to converge occurs, try adjusting Parscale, or better, setting eq. tolerances = TRUE (and hope that the estimated tolerance matrix is positive-definite). To fit an equal-tolerances model, it is firstly best to try setting I.tolerances $=$ TRUE and varying isd.latvar and/or MUXfactor if it fails to converge. If it still fails to converge after many attempts, try setting eq. tolerances = TRUE, however this will usually be a lot slower because it requires a lot more memory.
With a $R>1$ model, the latent variables are always uncorrelated, i.e., the variance-covariance matrix of the site scores is a diagonal matrix.
If setting eq. tolerances $=$ TRUE is used and the common estimated tolerance matrix is positivedefinite then that model is effectively the same as the I.tolerances $=$ TRUE model (the two are transformations of each other). In general, I. tolerances = TRUE is numerically more unstable and presents a more difficult problem to optimize; the arguments isd.latvar and/or MUXfactor often must be assigned some good value(s) (possibly found by trial and error) in order for convergence to occur. Setting I.tolerances = TRUE forces a bell-shaped curve or surface onto all the species data, therefore this option should be used with deliberation. If unsuitable, the resulting fit may be very misleading. Usually it is a good idea for the user to set eq. tolerances = FALSE to see which species appear to have a bell-shaped curve or surface. Improvements to the fit can often be achieved using transformations, e.g., nitrogen concentration to log nitrogen concentration.

Fitting a CAO model (see cao) first is a good idea for pre-examining the data and checking whether it is appropriate to fit a CQO model.

## Value

A list with components matching the input names.

## Warning

The default value of Bestof is a bare minimum for many datasets, therefore it will be necessary to increase its value to increase the chances of obtaining the global solution.

## Note

When I.tolerances $=$ TRUE it is a good idea to apply scale to all the numerical variables that make up the latent variable, i.e., those of $x_{2}$. This is to make them have mean 0 , and hence avoid large offset values which cause numerical problems.

This function has many arguments that are common with rrvglm. control and vglm. control.
It is usually a good idea to try fitting a model with I. tolerances = TRUE first, and if convergence is unsuccessful, then try eq. tolerances = TRUE and I. tolerances $=$ FALSE. Ordination diagrams with eq. tolerances $=$ TRUE have a natural interpretation, but with eq. tolerances $=$ FALSE they
are more complicated and requires, e.g., contours to be overlaid on the ordination diagram (see lvplot. qrrvglm).

In the example below, an equal-tolerances CQO model is fitted to the hunting spiders data. Because I. tolerances $=$ TRUE, it is a good idea to center all the $x_{2}$ variables first. Upon fitting the model, the actual standard deviation of the site scores are computed. Ideally, the isd.latvar argument should have had this value for the best chances of getting good initial values. For comparison, the model is refitted with that value and it should run more faster and reliably.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2004). A new technique for maximum-likelihood canonical Gaussian ordination. Ecological Monographs, 74, 685-701.

Yee, T. W. (2006). Constrained additive ordination. Ecology, 87, 203-213.

## See Also

cqo, rcqo, Coef.qrrvglm, Coef.qrrvglm-class, optim, binomialff, poissonff, negbinomial, gamma2.

## Examples

```
## Not run: # Poisson CQO with equal tolerances
set.seed(111) # This leads to the global solution
hspider[,1:6] <- scale(hspider[,1:6]) # Good idea when I.tolerances = TRUE
p1 <- cqo(cbind(Alopacce, Alopcune, Alopfabr, Arctlute, Arctperi, Auloalbi,
            Pardlugu, Pardmont, Pardnigr, Pardpull, Trocterr, Zoraspin) ~
        WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
        poissonff, data = hspider, eq.tolerances = TRUE)
sort(deviance(p1, history = TRUE)) # A history of all the iterations
(isd.latvar <- apply(latvar(p1), 2, sd)) # Should be approx isd.latvar
# Refit the model with better initial values
set.seed(111) # This leads to the global solution
p1 <- cqo(cbind(Alopacce, Alopcune, Alopfabr, Arctlute, Arctperi, Auloalbi,
            Pardlugu, Pardmont, Pardnigr, Pardpull, Trocterr, Zoraspin) ~
        WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
        I.tolerances = TRUE, poissonff, data = hspider,
        isd.latvar = isd.latvar) # Note the use of isd.latvar here
sort(deviance(p1, history = TRUE)) # A history of all the iterations
## End(Not run)
```

qtplot.gumbel Quantile Plot for Gumbel Regression

## Description

Plots quantiles associated with a Gumbel model.

## Usage

qtplot.gumbel(object, show.plot = TRUE, y.arg = TRUE, spline.fit = FALSE, label = TRUE, $R=$ object@misc\$R, percentiles = object@misc\$percentiles, add.arg $=$ FALSE, $m p v=$ object@misc\$mpv, xlab $=$ NULL, ylab $=" "$, main $=" "$, pch $=\operatorname{par}() \$ p c h, \operatorname{pcol} . \arg =\operatorname{par}() \$ c o l$, llty.arg = par()\$lty, lcol.arg = par()\$col, llwd.arg = par()\$lwd, tcol.arg $=\operatorname{par}() \$ c o l, \operatorname{tadj}=1, \ldots)$

## Arguments

object A VGAM extremes model of the Gumbel type, produced by modelling functions such as vglm and vgam, and with a family function that is either gumbel or gumbelff.
show.plot Logical. Plot it? If FALSE no plot will be done.
$y$.arg Logical. Add the raw data on to the plot?
spline.fit Logical. Use a spline fit through the fitted percentiles? This can be useful if there are large gaps between some values along the covariate.
label Logical. Label the percentiles?
$R \quad$ See gumbel.
percentiles See gumbel.
add. arg Logical. Add the plot to an existing plot?
mpv See gumbel.
$x l a b \quad$ Caption for the x-axis. See par.
$y l a b \quad$ Caption for the $y$-axis. See par.
main Title of the plot. See title.
pch Plotting character. See par.
pcol.arg Color of the points. See the col argument of par.
llty.arg Line type. Line type. See the lty argument of par.
lcol.arg Color of the lines. See the col argument of par.
llwd.arg Line width. See the lwd argument of par.
tcol.arg Color of the text (if label is TRUE). See the col argument of par.
tadj Text justification. See the adj argument of par.
Arguments passed into the plot function when setting up the entire plot. Useful arguments here include sub and las.

## Details

There should be a single covariate such as time. The quantiles specified by percentiles are plotted.

## Value

The object with a list called qtplot in the post slot of object. (If show. plot = FALSE then just the list is returned.) The list contains components
fitted.values The percentiles of the response, possibly including the MPV.
percentiles The percentiles (small vector of values between 0 and 100.

## Note

Unlike gumbel, one cannot have percentiles = NULL.

## Author(s)

Thomas W. Yee

## See Also

gumbel.

## Examples

```
ymat <- as.matrix(venice[, paste("r", 1:10, sep = "")])
fit1 <- vgam(ymat ~ s(year, df = 3), gumbel(R = 365, mpv = TRUE),
            data = venice, trace = TRUE, na.action = na.pass)
head(fitted(fit1))
## Not run: par(mfrow = c(1, 1), bty = "l", xpd = TRUE, las = 1)
qtplot(fit1, mpv = TRUE, lcol = c(1, 2, 5), tcol = c(1, 2, 5),
    lwd = 2, pcol = "blue", tadj = 0.4, ylab = "Sea level (cm)")
qtplot(fit1, perc = 97, mpv = FALSE, lcol = 3, tcol = 3,
    lwd = 2, tadj = 0.4, add = TRUE) -> saved
head(saved@post$qtplot$fitted)
## End(Not run)
```

    qtplot.1mscreg Quantile Plot for LMS Quantile Regression
    
## Description

Plots quantiles associated with a LMS quantile regression.

## Usage

qtplot.lmscreg(object, newdata $=$ NULL, percentiles = object@misc\$percentiles, show.plot = TRUE, ...)

## Arguments

| object | A VGAM quantile regression model, i.e., an object produced by modelling func- <br> tions such as vglm and vgam with a family function beginning with "lms.", e.g., <br> lms.yjn. |
| :--- | :--- |
| newdata | Optional data frame for computing the quantiles. If missing, the original data is <br> used. |
| percentiles | Numerical vector with values between 0 and 100 that specify the percentiles <br> (quantiles). The default are the percentiles used when the model was fitted. |
| show.plot | Logical. Plot it? If FALSE no plot will be done. |
| $\ldots$ | Graphical parameter that are passed into plotqtplot.lmscreg. |

## Details

The 'primary' variable is defined as the main covariate upon which the regression or smoothing is performed. For example, in medical studies, it is often the age. In VGAM, it is possible to handle more than one covariate, however, the primary variable must be the first term after the intercept.

## Value

A list with the following components.
fitted. values A vector of fitted percentile values.
percentiles The percentiles used.

## Note

plotqtplot. Imscreg does the actual plotting.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2004). Quantile regression via vector generalized additive models. Statistics in Medicine, 23, 2295-2315.

## See Also

plotqtplot.lmscreg, deplot.lmscreg, lms.bcn, lms.bcg, lms.yjn.

## Examples

```
## Not run:
fit <- vgam(BMI ~ s(age, df = c(4, 2)), lms.bcn(zero=1), data = bmi.nz)
qtplot(fit)
qtplot(fit, perc = c(25, 50, 75, 95), lcol = "blue", tcol = "blue", llwd = 2)
## End(Not run)
```

Qvar Quasi-variances Preprocessing Function

## Description

Takes a vglm fit or a variance-covariance matrix, and preprocesses it for rcim and uninormal so that quasi-variances can be computed.

## Usage

Qvar(object, factorname = NULL, which.linpred $=1$, coef.indices $=$ NULL, labels $=$ NULL,
dispersion = NULL, reference. name = "(reference)", estimates = NULL)

## Arguments

object A "vglm" object or a variance-covariance matrix, e.g., vcov(vglm.object). The former is preferred since it contains all the information needed. If a matrix then factorname and/or coef.indices should be specified to identify the factor.
which.linpred A single integer from the set 1:M. Specifies which linear predictor to use. Let the value of which. linpred be called $j$. Then the factor should appear in that linear predictor, hence the $j$ th row of the constraint matrix corresponding to the factor should have at least one nonzero value. Currently the $j$ th row must have exactly one nonzero value because programming it for more than one nonzero value is difficult.
factorname Character. If the vglm object contains more than one factor as explanatory variable then this argument should be the name of the factor of interest. If object is a variance-covariance matrix then this argument should also be specified.
labels Character. Optional, for labelling the variance-covariance matrix.
dispersion Numeric. Optional, passed into $\operatorname{vcov}()$ with the same argument name.
reference. name Character. Label for for the reference level.
coef.indices Optional numeric vector of length at least 3 specifying the indices of the factor from the variance-covariance matrix.
estimates an optional vector of estimated coefficients (redundant if object is a model).

## Details

Suppose a factor with $L$ levels is an explanatory variable in a regression model. By default, R treats the first level as baseline so that its coefficient is set to zero. It estimates the other $L-1$ coefficients, and with its associated standard errors, this is the conventional output. From the complete variance-covariance matrix one can compute $L$ quasi-variances based on all pairwise difference of the coefficients. They are based on an approximation, and can be treated as uncorrelated. In minimizing the relative (not absolute) errors it is not hard to see that the estimation involves a RCIM (rcim) with an exponential link function (explink).

If object is a model, then at least one of factorname or coef.indices must be non-NULL. The value of coef. indices, if non-NULL, determines which rows and columns of the model's variancecovariance matrix to use. If coef. indices contains a zero, an extra row and column are included at the indicated position, to represent the zero variances and covariances associated with a reference level. If coef.indices is NULL, then factorname should be the name of a factor effect in the model, and is used in order to extract the necessary variance-covariance estimates.
Quasi-variances were first implemented in R with qucalc. This implementation draws heavily from that.

## Value

A $L$ by $L$ matrix whose $i-j$ element is the logarithm of the variance of the $i$ th coefficient minus the $j$ th coefficient, for all values of $i$ and $j$. The diagonal elements are abitrary and are set to zero.

The matrix has an attribute that corresponds to the prior weight matrix; it is accessed by uninormal and replaces the usual weights argument. of vglm. This weight matrix has ones on the off-diagonals and some small positive number on the diagonals.

## Warning

Negative quasi-variances may occur (one of them and only one), though they are rare in practice. If so then numerical problems may occur. See qvcalc() for more information.

## Note

This is an adaptation of qvcalc() in qucalc. It should work for all vglm models with one linear predictor, i.e., $M=1$. For $M>1$ the factor should appear only in one of the linear predictors.

It is important to set maxit to be larger than usual for rcim since convergence is slow. Upon successful convergence the $i$ th row effect and the $i$ th column effect should be equal. A simple computation involving the fitted and predicted values allows the quasi-variances to be extracted (see example below).

A function to plot comparison intervals has not been written here.

## Author(s)

T. W. Yee, based heavily on qvcalc() in qvealc written by David Firth.

## References

Firth, D. (2003). Overcoming the reference category problem in the presentation of statistical models. Sociological Methodology 33, 1-18.
Firth, D. and de Menezes, R. X. (2004). Quasi-variances. Biometrika 91, 65-80.
Yee, T. W. and Hadi, A. F. (2014). Row-column interaction models, with an R implementation. Computational Statistics, 29, 1427-1445.

## See Also

rcim, vglm, qvar, uninormal, explink, qvcalc() in qvealc, ships.

## Examples

```
# Example 1
data("ships", package = "MASS")
Shipmodel <- vglm(incidents ~ type + year + period,
    poissonff, offset = log(service),
#
    trace = TRUE, model = TRUE,
    data = ships, subset = (service > 0))
# Easiest form of input
fit1 <- rcim(Qvar(Shipmodel, "type"), uninormal("explink"), maxit = 99)
qvar(fit1) # Easy method to get the quasi-variances
qvar(fit1, se = TRUE) # Easy method to get the quasi-standard errors
(quasiVar <- exp(diag(fitted(fit1))) / 2) # Version 1
(quasiVar <- diag(predict(fit1)[, c(TRUE, FALSE)]) / 2) # Version 2
(quasiSE <- sqrt(quasiVar))
# Another form of input
fit2 <- rcim(Qvar(Shipmodel, coef.ind = c(0, 2:5), reference.name = "typeA"),
    uninormal("explink"), maxit = 99)
## Not run: qvplot(fit2, col = "green", lwd = 3, scol = "blue", slwd = 2, las = 1)
# The variance-covariance matrix is another form of input (not recommended)
fit3 <- rcim(Qvar(cbind(0, rbind(0, vcov(Shipmodel)[2:5, 2:5])),
    labels = c("typeA", "typeB", "typeC", "typeD", "typeE"),
    estimates = c(typeA = 0, coef(Shipmodel)[2:5])),
        uninormal("explink"), maxit = 99)
(QuasiVar <- exp(diag(fitted(fit3))) / 2) # Version 1
(QuasiVar <- diag(predict(fit3)[, c(TRUE, FALSE)]) / 2) # Version 2
(QuasiSE <- sqrt(quasiVar))
## Not run: qvplot(fit3)
# Example 2: a model with M > 1 linear predictors
## Not run: require("VGAMdata")
xs.nz.f <- subset(xs.nz, sex == "F")
xs.nz.f <- subset(xs.nz.f, !is.na(babies) & !is.na(age) & !is.na(ethnicity))
xs.nz.f <- subset(xs.nz.f, ethnicity != "Other")
```

```
    clist <- list("sm.bs(age, df = 4)" = rbind(1, 0),
        "sm.bs(age, df = 3)" = rbind(0, 1),
        "ethnicity" = diag(2),
        "(Intercept)" = diag(2))
    fit1 <- vglm(babies ~ sm.bs(age, df = 4) + sm.bs(age, df = 3) + ethnicity,
        zipoissonff(zero = NULL), xs.nz.f,
        constraints = clist, trace = TRUE)
Fit1 <- rcim(Qvar(fit1, "ethnicity", which.linpred = 1),
        uninormal("explink", imethod = 1), maxit = 99, trace = TRUE)
Fit2 <- rcim(Qvar(fit1, "ethnicity", which.linpred = 2),
        uninormal("explink", imethod = 1), maxit = 99, trace = TRUE)
    ## End(Not run)
    ## Not run: par(mfrow = c(1, 2))
    qvplot(Fit1, scol = "blue", pch = 16, main = expression(eta[1]),
        slwd = 1.5, las = 1, length.arrows = 0.07)
    qvplot(Fit2, scol = "blue", pch = 16, main = expression(eta[2]),
        slwd = 1.5, las = 1, length.arrows = 0.07)
    ## End(Not run)
```

quar
Quasi-variances Extraction Function

## Description

Takes a rcim fit of the appropriate format and returns either the quasi-variances or quasi-standard errors.

## Usage

qvar (object, se $=$ FALSE, ...)

## Arguments

object A rcim object that has family function uninormal with the explink link. See below for an example.
se Logical. If FALSE then the quasi-variances are returned, else the square root of them, called quasi-standard errors.
... Currently unused.

## Details

This simple function is ad hoc and simply is equivalent to computing the quasi-variances by diag (predict(fit1) [, c (TRUE,
/ 2. This function is for convenience only. Serious users of quasi-variances ought to understand why and how this function works.

## Value

A vector of quasi-variances or quasi-standard errors.

## Author(s)

T. W. Yee.

## See Also

rcim, uninormal, explink, Qvar, ships.

## Examples

```
data("ships", package = "MASS")
Shipmodel <- vglm(incidents ~ type + year + period,
    poissonff, offset = log(service),
    data = ships, subset = (service > 0))
# Easiest form of input
fit1 <- rcim(Qvar(Shipmodel, "type"), uninormal("explink"), maxit = 99)
qvar(fit1) # Quasi-variances
qvar(fit1, se = TRUE) # Quasi-standard errors
# Manually compute them:
(quasiVar <- exp(diag(fitted(fit1))) / 2) # Version 1
(quasiVar <- diag(predict(fit1)[, c(TRUE, FALSE)]) / 2) # Version 2
(quasiSE <- sqrt(quasiVar))
## Not run: qvplot(fit1, col = "green", lwd = 3, scol = "blue", slwd = 2, las = 1)
```


## Description

R-squared goodness of fit for latent variable models, such as cumulative link models. Some software such as Stata call the quantity the McKelvey-Zavoina R-squared, which was proposed in their 1975 paper for cumulative probit models.

## Usage

R2latvar (object)

## Arguments

object A cumulative or binomialff fit using vglm. Only a few selected link functions are currently permitted: logitlink, probitlink, clogloglink. For models with more than one linear predictor, a parallelism assumption is needed also, i.e., the constraint matrices must be a 1-column matrix of 1 s (except for the intercept). The model is assumed to have an intercept term.

## Details

Models such as the proportional odds model have a latent variable interpretation (see, e.g., Section 6.2.6 of Agresti (2018), Section 14.4.1.1 of Yee (2015), Section 5.2.2 of McCullagh and Nelder (1989)). It is possible to summarize the predictive power of the model by computing $R^{2}$ on the transformed scale, e.g., on a standard normal distribution for a probitlink link. For more details see Section 6.3.7 of Agresti (2018).

## Value

The $R^{2}$ value. Approximately, that amount is the variability in the latent variable of the model explained by all the explanatory variables. Then taking the positive square-root gives an approximate multiple correlation $R$.

## Author(s)

Thomas W. Yee

## References

Agresti, A. (2018). An Introduction to Categorical Data Analysis, 3rd ed., New York: John Wiley \& Sons.

McKelvey, R. D. and W. Zavoina (1975). A statistical model for the analysis of ordinal level dependent variables. The Journal of Mathematical Sociology, 4, 103-120.

## See Also

vglm, cumulative, propodds, logitlink, probitlink, clogloglink, summary.lm.

## Examples

```
pneumo <- transform(pneumo, let = log(exposure.time))
(fit <- vglm(cbind(normal, mild, severe) ~ let, propodds, data = pneumo))
R2latvar(fit)
```

Rank Rank

## Description

Returns the rank of reduced-rank regression-type models in the VGAM package.

## Usage

```
Rank(object, ...)
Rank.rrvglm(object, ...)
Rank.qrrvglm(object, ...)
Rank.rrvgam(object, ...)
```


## Arguments

object Some VGAM object, for example, having class rrvglm-class. The class vglm-class is not included since this is not based on reduced-rank regression.
$\ldots \quad$ Other possible arguments fed into the function later (used for added flexibility for the future).

## Details

Regression models based on reduced-rank regression have a quantity called the rank, which is 1 or 2 or 3 etc. The smaller the value the more dimension reduction, so that there are fewer parameters. This function was not called rank() to avoid conflict with rank.

## Value

Returns an integer value, provided the rank of the model makes sense.

## Note

This function has not been defined for VGLMs yet. It might refer to the rank of the VL model matrix, but for now this function should not be applied to vglm fits.

## Author(s)

T. W. Yee.

## See Also

RR-VGLMs are described in rrvglm-class; QRR-VGLMs are described in qrrvglm-class.

## Examples

```
pneumo <- transform(pneumo, let = log(exposure.time),
    x3 = runif(nrow(pneumo)))
(fit1 <- rrvglm(cbind(normal, mild, severe) ~ let + x3,
    acat, data = pneumo))
coef(fit1, matrix = TRUE)
constraints(fit1)
Rank(fit1)
```

    Rayleigh The Rayleigh Distribution
    
## Description

Density, distribution function, quantile function and random generation for the Rayleigh distribution with parameter a.

## Usage

```
drayleigh(x, scale \(=1\), log = FALSE)
prayleigh (q, scale \(=1\), lower.tail = TRUE, log. \(\mathrm{p}=\mathrm{FALSE}\) )
qrayleigh(p, scale \(=1\), lower.tail = TRUE, log.p = FALSE)
rrayleigh(n, scale = 1)
```


## Arguments

| $\mathrm{x}, \mathrm{q}$ | vector of quantiles. |
| :--- | :--- |
| p | vector of probabilities. |
| n | number of observations. Fed into runif. |
| scale | the scale parameter $b$. |
| log | Logical. If log = TRUE then the logarithm of the density is returned. |
| lower.tail, log. p |  |
|  | Same meaning as in pnorm or qnorm. |

## Details

See rayleigh, the VGAM family function for estimating the scale parameter $b$ by maximum likelihood estimation, for the formula of the probability density function and range restrictions on the parameter $b$.

## Value

drayleigh gives the density, prayleigh gives the distribution function, qrayleigh gives the quantile function, and rrayleigh generates random deviates.

## Note

The Rayleigh distribution is related to the Maxwell distribution.

## Author(s)

T. W. Yee and Kai Huang

## References

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011). Statistical Distributions, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

## See Also

rayleigh, maxwell.

## Examples

```
## Not run: Scale <- 2; x <- seq(-1, 8, by = 0.1)
plot(x, drayleigh(x, scale = Scale), type = "l", ylim = c(0,1),
    las = 1, ylab = "",
    main = "Rayleigh density divided into 10 equal areas; orange = cdf")
abline(h = 0, col = "blue", lty = 2)
qq <- qrayleigh(seq(0.1, 0.9, by = 0.1), scale = Scale)
lines(qq, drayleigh(qq, scale = Scale), col = "purple", lty = 3, type = "h")
lines(x, prayleigh(x, scale = Scale), col = "orange")
## End(Not run)
```

rayleigh Rayleigh Regression Family Function

## Description

Estimating the parameter of the Rayleigh distribution by maximum likelihood estimation. Rightcensoring is allowed.

## Usage

```
    rayleigh(lscale = "loglink", nrfs = 1/3 + 0.01,
    oim.mean = TRUE, zero = NULL, parallel = FALSE,
    type.fitted = c("mean", "percentiles", "Qlink"),
    percentiles = 50)
cens.rayleigh(lscale = "loglink", oim = TRUE)
```


## Arguments

lscale Parameter link function applied to the scale parameter $b$. See Links for more choices. A log link is the default because $b$ is positive.
nrfs $\quad$ Numeric, of length one, with value in $[0,1]$. Weighting factor between NewtonRaphson and Fisher scoring. The value 0 means pure Newton-Raphson, while 1 means pure Fisher scoring. The default value uses a mixture of the two algorithms, and retaining positive-definite working weights.
oim.mean Logical, used only for intercept-only models. TRUE means the mean of the OIM elements are used as working weights. If TRUE then this argument has top priority for working out the working weights. FALSE means use another algorithm.
oim Logical. For censored data only, TRUE means the Newton-Raphson algorithm, and FALSE means Fisher scoring.
zero, parallel Details at CommonVGAMffArguments.
type.fitted, percentiles
See CommonVGAMffArguments for information. Using "Qlink" is for quantilelinks in VGAMextra.

## Details

The Rayleigh distribution, which is used in physics, has a probability density function that can be written

$$
f(y)=y \exp \left(-0.5(y / b)^{2}\right) / b^{2}
$$

for $y>0$ and $b>0$. The mean of $Y$ is $b \sqrt{\pi / 2}$ (returned as the fitted values) and its variance is $b^{2}(4-\pi) / 2$.
The VGAM family function cens.rayleigh handles right-censored data (the true value is greater than the observed value). To indicate which type of censoring, input extra = list (rightcensored $=v e c 2$ ) where vec 2 is a logical vector the same length as the response. If the component of this list is missing then the logical values are taken to be FALSE. The fitted object has this component stored in the extra slot.
The VGAM family function rayleigh handles multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Warning

The theory behind the argument oim is not fully complete.

## Note

The poisson. points family function is more general so that if ostatistic = 1 and dimension $=$ 2 then it coincides with rayleigh. Other related distributions are the Maxwell and Weibull distributions.

## Author(s)

T. W. Yee

## References

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011). Statistical Distributions, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

## See Also

Rayleigh, genrayleigh, riceff, maxwell, weibullR, poisson. points, simulate.vlm.

## Examples

```
nn <- 1000; Scale <- exp(2)
rdata <- data.frame(ystar = rrayleigh(nn, scale = Scale))
fit <- vglm(ystar ~ 1, rayleigh, data = rdata, trace = TRUE)
head(fitted(fit))
with(rdata, mean(ystar))
coef(fit, matrix = TRUE)
```

```
    Coef(fit)
    # Censored data
    rdata <- transform(rdata, U = runif(nn, 5, 15))
    rdata <- transform(rdata, y = pmin(U, ystar))
    ## Not run: par(mfrow = c(1, 2))
    hist(with(rdata, ystar)); hist(with(rdata, y))
    ## End(Not run)
    extra <- with(rdata, list(rightcensored = ystar > U))
    fit <- vglm(y ~ 1, cens.rayleigh, data = rdata, trace = TRUE,
    extra = extra, crit = "coef")
    table(fit@extra$rightcen)
    coef(fit, matrix = TRUE)
    head(fitted(fit))
```

Rcim Mark the Baseline of Row and Column on a Matrix data

## Description

Rearrange the rows and columns of the input so that the first row and first column are baseline. This function is for rank-zero row-column interaction models (RCIMs; i.e., general main effects models).

## Usage

```
Rcim(mat, rbaseline = 1, cbaseline = 1)
```


## Arguments

mat Matrix, of dimension $r$ by $c$. It is best that it is labelled with row and column names.
rbaseline, cbaseline
Numeric (row number of the matrix mat) or character (matching a row name of mat) that the user wants as the row baseline or reference level. Similarly cbaseline for the column.

## Details

This is a data preprocessing function for rcim. For rank-zero row-column interaction models this function establishes the baseline (or reference) levels of the matrix response with respect to the row and columns-these become the new first row and column.

## Value

Matrix of the same dimension as the input, with rbaseline and cbaseline specifying the first rows and columns. The default is no change in mat.

Note
This function is similar to moffset; see moffset for information about the differences. If numeric, the arguments rbaseline and cbaseline differ from arguments roffset and coffset in moffset by 1 (when elements of the matrix agree).

## Author(s)

Alfian F. Hadi and T. W. Yee.

## See Also

moffset, rcim, plotrcim0.

## Examples

```
(alcoff.e <- moffset(alcoff, roffset = "6", postfix = "*"))
(aa <- Rcim(alcoff, rbaseline = "11", cbaseline = "Sun"))
(bb <- moffset(alcoff, "11", "Sun", postfix = "*"))
aa - bb # Note the difference!
```

rcqo Constrained Quadratic Ordination

## Description

Random generation for constrained quadratic ordination (CQO).

## Usage

$\operatorname{rcqo}(\mathrm{n}, \mathrm{p}, \mathrm{S}, \operatorname{Rank}=1$, family = c("poisson", "negbinomial", "binomial-poisson", "Binomial-negbinomial", "ordinal-poisson", "Ordinal-negbinomial", "gamma2"), eq.maximums = FALSE, eq.tolerances = TRUE, es.optimums = FALSE, lo. abundance $=$ if (eq.maximums) hi.abundance else 10, hi. abundance $=100$, sd.latvar $=$ head(1.5/2^(0:3), Rank), sd.optimums = ifelse(es.optimums, 1.5/Rank, 1) *
ifelse(scale.latvar, sd.latvar, 1), sd.tolerances $=0.25$, Kvector $=1$, Shape $=1$, sqrt.arg = FALSE, log.arg = FALSE, rhox = 0.5, breaks = 4, seed $=$ NULL, optimums1.arg $=$ NULL, Crow1positive $=$ TRUE, xmat $=$ NULL, scale.latvar $=$ TRUE)

## Arguments

| n |  |
| :--- | :--- |
| p | Number of sites. It is denoted by $n$ below. |
| Number of environmental variables, including an intercept term. It is denoted |  |
| by $p$ below. Must be no less than $1+R$ in value. |  |
| Number of species. It is denoted by $S$ below. |  |
| Rank | The rank or the number of latent variables or true dimension of the data on the |
| reduced space. This must be either 1, 2, 3 or 4 . It is denoted by $R$. |  |
| What type of species data is to be returned. The first choice is the default. If |  |
| binomial then a 0 means absence and 1 means presence. If ordinal then the |  |
| breaks argument is passed into the breaks argument of cut. Note that either |  |
| the Poisson or negative binomial distributions are used to generate binomial and |  |
| ordinal data, and that an upper-case choice is used for the negative binomial |  |
| distribution (this makes it easier for the user). If "gamma2" then this is the $2-$ |  |
| parameter gamma distribution. |  |


|  | diagonal in this function. This argument is ignored if eq. tolerances is TRUE, otherwise it is recycled to length $R$ if necessary. |
| :---: | :---: |
| Kvector | A vector of positive $k$ values (recycled to length $S$ if necessary) for the negative binomial distribution (see negbinomial for details). Note that a natural default value does not exist, however the default value here is probably a realistic one, and that for large values of $\mu$ one has $\operatorname{Var}(Y)=\mu^{2} / k$ approximately. |
| Shape | A vector of positive $\lambda$ values (recycled to length $S$ if necessary) for the 2parameter gamma distribution (see gamma2 for details). Note that a natural default value does not exist, however the default value here is probably a realistic one, and that $\operatorname{Var}(Y)=\mu^{2} / \lambda$. |
| sqrt.arg | Logical. Take the square-root of the negative binomial counts? Assigning sqrt. arg = TRUE when family="negbinomial" means that the resulting species data can be considered very crudely to be approximately Poisson distributed. They will not integers in general but much easier (less numerical problems) to estimate using something like cqo(. . . , family="poissonff"). |
| log.arg | Logical. Take the logarithm of the gamma random variates? Assigning log. arg = TRUE when family="gamma2" means that the resulting species data can be considered very crudely to be approximately Gaussian distributed about its (quadratic) mean. |
| rhox | Numeric, less than 1 in absolute value. The correlation between the environmental variables. The correlation matrix is a matrix of 1's along the diagonal and rhox in the off-diagonals. Note that each environmental variable is normally distributed with mean 0 . The standard deviation of each environmental variable is chosen so that the site scores have the determined standard deviation, as given by argument sd. latvar. |
| breaks | If family is assigned an ordinal value then this argument is used to define the cutpoints. It is fed into the breaks argument of cut. |
| seed | If given, it is passed into set. seed. This argument can be used to obtain reproducible results. If set, the value is saved as the "seed" attribute of the returned value. The default will not change the random generator state, and return .Random. seed as "seed" attribute. |
| optimums1.arg | If assigned and Rank $=1$ then these are the explicity optimums. Recycled to length $S$. |
| Crow1positive | See qrrvglm. control for details. |
| xmat | The $n$ by $p-1$ environmental matrix can be inputted. |
| scale.latvar | Logical. If FALSE the argument sd.latvar is ignored and no scaling of the latent variable values is performed. |

## Details

This function generates data coming from a constrained quadratic ordination (CQO) model. In particular, data coming from a species packing model can be generated with this function. The species packing model states that species have equal tolerances, equal maximums, and optimums which are uniformly distributed over the latent variable space. This can be achieved by assigning the arguments es.optimums = TRUE, eq. maximums $=$ TRUE, eq. tolerances $=$ TRUE.

At present, the Poisson and negative binomial abundances are generated first using lo. abundance and hi . abundance, and if family is binomial or ordinal then it is converted into these forms.
In CQO theory the $n$ by $p$ matrix $X$ is partitioned into two parts $X_{1}$ and $X_{2}$. The matrix $X_{2}$ contains the 'real' environmental variables whereas the variables in $X_{1}$ are just for adjustment purposes; they contain the intercept terms and other variables that one wants to adjust for when (primarily) looking at the variables in $X_{2}$. This function has $X_{1}$ only being a matrix of ones, i.e., containing an intercept only.

## Value

A $n$ by $p-1+S$ data frame with components and attributes. In the following the attributes are labelled with double quotes.
$x 2, x 3, x 4, \ldots, x p$
The environmental variables. This makes up the $n$ by $p-1 X_{2}$ matrix. Note that $\times 1$ is not present; it is effectively a vector of ones since it corresponds to an intercept term when cqo is applied to the data.
$y 1, y 2, x 3, \ldots, y S$
The species data. This makes up the $n$ by $S$ matrix $Y$. This will be of the form described by the argument family.
"concoefficients"
The $p-1$ by $R$ matrix of constrained coefficients (or canonical coefficients). These are also known as weights or loadings.
"formula" The formula involving the species and environmental variable names. This can be used directly in the formula argument of cqo.
"log.maximums" The $S$-vector of species' maximums, on a log scale. These are uniformly distributed between $\log (l o$.abundance) and $\log$ (hi. abundance).
"latvar" The $n$ by $R$ matrix of site scores. Each successive column (latent variable) has sample standard deviation equal to successive values of sd.latvar.
"eta" The linear/additive predictor value.
"optimums" The $S$ by $R$ matrix of species' optimums.
"tolerances" The $S$ by $R$ matrix of species' tolerances. These are the square root of the diagonal elements of the tolerance matrices (recall that all tolerance matrices are restricted to being diagonal in this function).

Other attributes are "break", "family", "Rank", "lo. abundance", "hi. abundance", "eq. tolerances", "eq.maximums", "seed" as used.

## Note

This function is under development and is not finished yet. There may be a few bugs.
Yet to do: add an argument that allows absences to be equal to the first level if ordinal data is requested.

## Author(s)

T. W. Yee

## References

Yee, T. W. (2004). A new technique for maximum-likelihood canonical Gaussian ordination. Ecological Monographs, 74, 685-701.

Yee, T. W. (2006). Constrained additive ordination. Ecology, 87, 203-213.
ter Braak, C. J. F. and Prentice, I. C. (1988). A theory of gradient analysis. Advances in Ecological Research, 18, 271-317.

## See Also

cqo, qrrvglm. control, cut, binomialff, poissonff, negbinomial, gamma2.

## Examples

```
## Not run:
# Example 1: Species packing model:
n <- 100; p <- 5; S <- 5
mydata <- rcqo(n, p, S, es.opt = TRUE, eq.max = TRUE)
names(mydata)
(myform <- attr(mydata, "formula"))
fit <- cqo(myform, poissonff, mydata, Bestof = 3) # eq.tol = TRUE
matplot(attr(mydata, "latvar"), mydata[,-(1:(p-1))], col = 1:S)
persp(fit, col = 1:S, add = TRUE)
lvplot(fit, lcol = 1:S, y = TRUE, pcol = 1:S) # The same plot as above
# Compare the fitted model with the 'truth'
concoef(fit) # The fitted model
attr(mydata, "concoefficients") # The 'truth'
c(apply(attr(mydata, "latvar"), 2, sd),
    apply(latvar(fit), 2, sd)) # Both values should be approx equal
# Example 2: negative binomial data fitted using a Poisson model:
n <- 200; p <- 5; S <- 5
mydata <- rcqo(n, p, S, fam = "negbin", sqrt = TRUE)
myform <- attr(mydata, "formula")
fit <- cqo(myform, fam = poissonff, dat = mydata) # I.tol = TRUE,
lvplot(fit, lcol = 1:S, y = TRUE, pcol = 1:S)
# Compare the fitted model with the 'truth'
concoef(fit) # The fitted model
attr(mydata, "concoefficients") # The 'truth'
## End(Not run)
```

rdiric

## Description

Generates Dirichlet random variates.

## Usage

rdiric(n, shape, dimension $=$ NULL, is.matrix.shape $=$ FALSE)

## Arguments

n number of observations. Note it has two meanings, see is.matrix. shape below.
shape the shape parameters. These must be positive. If dimension is specifed, values are recycled if necessary to length dimension.
dimension the dimension of the distribution. If dimension is not numeric then it is taken to be length(shape) (or ncol(shape) if is.matrix. shape $==$ TRUE).
is.matrix.shape
Logical. If TRUE then shape must be a matrix, and then $n$ is no longer the number of rows of the answer but the answer has $n *$ nrow (shape) rows. If FALSE (the default) then shape is a vector and each of the $n$ rows of the answer have shape as its shape parameters.

## Details

This function is based on a relationship between the gamma and Dirichlet distribution. Random gamma variates are generated, and then Dirichlet random variates are formed from these.

## Value

A $n$ by dimension matrix of Dirichlet random variates. Each element is positive, and each row will sum to unity. If shape has names then these will become the column names of the answer.

## Author(s)

Thomas W. Yee

## References

Lange, K. (2002). Mathematical and Statistical Methods for Genetic Analysis, 2nd ed. New York: Springer-Verlag.

## See Also

dirichlet is a VGAM family function for fitting a Dirichlet distribution to data.

## Examples

```
ddata <- data.frame(rdiric(n = 1000, shape = c(y1 = 3, y2 = 1, y3 = 4)))
fit <- vglm(cbind(y1, y2, y3) ~ 1, dirichlet, data = ddata, trace = TRUE)
Coef(fit)
coef(fit, matrix = TRUE)
```


## Description

Maximum likelihood estimation of the rate parameter of a 1-parameter exponential distribution when the observations are upper record values.

## Usage

rec.exp1(lrate = "loglink", irate = NULL, imethod = 1)

## Arguments

lrate Link function applied to the rate parameter. See Links for more choices.
irate $\quad$ Numeric. Optional initial values for the rate. The default value NULL means they are computed internally, with the help of imethod.
imethod Integer, either 1 or 2 or 3 . Initial method, three algorithms are implemented. Choose the another value if convergence fails, or use irate.

## Details

The response must be a vector or one-column matrix with strictly increasing values.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

Note
By default, this family function has the intercept-only MLE as the initial value, therefore convergence may only take one iteration. Fisher scoring is used.

## Author(s)

T. W. Yee

## References

Arnold, B. C. and Balakrishnan, N. and Nagaraja, H. N. (1998). Records, New York: John Wiley \& Sons.

## See Also

exponential.

## Examples

```
rawy <- rexp(n <- 10000, rate = exp(1))
y <- unique(cummax(rawy)) # Keep only the records
length(y) / y[length(y)] # MLE of rate
fit <- vglm(y ~ 1, rec.exp1, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
```

rec.normal Upper Record Values from a Univariate Normal Distribution

## Description

Maximum likelihood estimation of the two parameters of a univariate normal distribution when the observations are upper record values.

## Usage

```
rec.normal(lmean = "identitylink", lsd = "loglink",
imean = NULL, isd = NULL, imethod = 1, zero = NULL)
```


## Arguments

lmean, lsd Link functions applied to the mean and sd parameters. See Links for more choices.
imean, isd Numeric. Optional initial values for the mean and sd. The default value NULL means they are computed internally, with the help of imethod.
imethod Integer, either 1 or 2 or 3 . Initial method, three algorithms are implemented. Choose the another value if convergence fails, or use imean and/or isd.
zero Can be an integer vector, containing the value 1 or 2 . If so, the mean or standard deviation respectively are modelled as an intercept only. Usually, setting zero $=2$ will be used, if used at all. The default value NULL means both linear/additive predictors are modelled as functions of the explanatory variables. See CommonVGAMfffarguments for more information.

## Details

The response must be a vector or one-column matrix with strictly increasing values.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

This family function tries to solve a difficult problem, and the larger the data set the better. Convergence failure can commonly occur, and convergence may be very slow, so set maxit $=200$, trace = TRUE, say. Inputting good initial values are advised.
This family function uses the BFGS quasi-Newton update formula for the working weight matrices. Consequently the estimated variance-covariance matrix may be inaccurate or simply wrong! The standard errors must be therefore treated with caution; these are computed in functions such as $\operatorname{vcov}()$ and summary ().

## Author(s)

T. W. Yee

## References

Arnold, B. C. and Balakrishnan, N. and Nagaraja, H. N. (1998). Records, New York: John Wiley \& Sons.

## See Also

uninormal, double.cens.normal.

## Examples

```
nn <- 10000; mymean <- 100
# First value is reference value or trivial record
Rdata <- data.frame(rawy = c(mymean, rnorm(nn, me = mymean, sd = exp(3))))
# Keep only observations that are records:
rdata <- data.frame(y = unique(cummax(with(Rdata, rawy))))
fit <- vglm(y ~ 1, rec.normal, data = rdata, trace = TRUE, maxit = 200)
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
```

reciprocallink Reciprocal Link Function

## Description

Computes the reciprocal transformation, including its inverse and the first two derivatives.

## Usage

```
    reciprocallink(theta, bvalue = NULL, inverse = FALSE, deriv = 0,
    short = TRUE, tag = FALSE)
negreciprocallink(theta, bvalue = NULL, inverse = FALSE, deriv = 0,
    short = TRUE, tag = FALSE)
```


## Arguments

```
    theta Numeric or character. See below for further details.
    bvalue See Links.
    inverse, deriv, short, tag
        Details at Links.
```


## Details

The reciprocallink link function is a special case of the power link function. Numerical values of theta close to 0 result in Inf, -Inf, NA or NaN.

The negreciprocallink link function computes the negative reciprocal, i.e., $-1 / \theta$.

## Value

For reciprocallink: for deriv $=0$, the reciprocal of theta, i.e., $1 /$ theta when inverse $=$ FALSE, and if inverse $=$ TRUE then $1 /$ theta. For deriv $=1$, then the function returns $d$ theta $/ d$ eta as a function of theta if inverse $=$ FALSE, else if inverse $=$ TRUE then it returns the reciprocal.

## Note

Numerical instability may occur when theta is close to 0 .

## Author(s)

Thomas W. Yee

## References

McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, 2nd ed. London: Chapman \& Hall.

## See Also

identitylink, powerlink.

## Examples

```
        reciprocallink(1:5)
        reciprocallink(1:5, inverse = TRUE, deriv = 2)
negreciprocallink(1:5)
negreciprocallink(1:5, inverse = TRUE, deriv = 2)
x <- (-3):3
reciprocallink(x) # Has Inf
reciprocallink(x, bvalue = .Machine$double.eps) # Has no Inf
```


## Description

Residuals for a vector generalized linear model (VGLM) object.

## Usage

residualsvglm(object,
type = c("working", "pearson", "response", "deviance", "ldot", "stdres"), matrix.arg = TRUE)

## Arguments

object
type
matrix.arg

Object of class "vglm", i.e., a vglm fit.
The value of this argument can be abbreviated. The type of residuals to be returned. The default is the first one: working residuals corresponding to the IRLS algorithm. These should be defined for all models. They are sometimes be added to VGAM plots of estimated component functions (see plotvgam).
Pearson residuals for GLMs, when squared and summed over the data set, total to the Pearson chi-squared statistic. For VGLMs, Pearson residuals involve the working weight matrices and the score vectors. Under certain limiting conditions, Pearson residuals have 0 means and identity matrix as the variancecovariance matrix.
Response residuals are simply the difference between the observed values and the fitted values. Both have to be of the same dimension, hence not all families have response residuals defined.
Deviance residuals are only defined for models with a deviance function. They tend to GLMs mainly. This function returns a NULL for those models whose deviance is undefined.
The choice "ldot" should not be used currently.
Standardized residuals are currently only defined for 2 types of models: (i) GLMs (poissonff, binomialff); (ii) those fitted to a two-way table of counts, e.g., cumulative, acat, multinomial, sratio, cratio. For (ii), they are defined in Section 2.4.5 of Agresti (2018) and are also the output from the "stdres" component of chisq. test. For the test of independence they are a useful type of residual. Their formula is (observed-expected) / sqrt(V), where $V$ is the residual cell variance (also see Agresti, 2007, section 2.4.5). When an independence null hypothesis is true, each standardized residual (corresponding to a cell in the table) has a a large-sample standard normal distribution. Currently this function merely extracts the table of counts from object and then computes the standardized residuals like chisq. test.
Logical, which applies when if the pre-processed answer is a vector or a 1 column matrix. If TRUE then the value returned will be a matrix, else a vector.

## Details

This function returns various kinds of residuals, sometimes depending on the specific type of model having been fitted. Section 3.7 of Yee (2015) gives some details on several types of residuals defined for the VGLM class.

Standardized residuals for GLMs are described in Section 4.5.6 of Agresti (2013) as the ratio of the raw (response) residuals divided by their standard error. They involve the generalized hat matrix evaluated at the final IRLS iteration. When applied to the LM, standardized residuals for GLMs simplify to rstandard. For GLMs they are basically the Pearson residual divided by the square root of 1 minus the leverage.

## Value

If that residual type is undefined or inappropriate then NULL is returned, otherwise a matrix or vector of residuals is returned.

## Warning

This function may change in the future, especially those whose definitions may change.

## References

Agresti, A. (2007). An Introduction to Categorical Data Analysis, 2nd ed., New York: John Wiley \& Sons. Page 38.

Agresti, A. (2013). Categorical Data Analysis, 3rd ed., New York: John Wiley \& Sons.
Agresti, A. (2018). An Introduction to Categorical Data Analysis, 3rd ed., New York: John Wiley \& Sons.

## See Also

resid, vglm, chisq.test, hatvalues.

## Examples

```
pneumo <- transform(pneumo, let = log(exposure.time))
fit <- vglm(cbind(normal, mild, severe) ~ let, propodds, data = pneumo)
resid(fit) # Same as having type = "working" (the default)
resid(fit, type = "response")
resid(fit, type = "pearson")
resid(fit, type = "stdres") # Test for independence
```

rhobitlink
Rhobit Link Function

## Description

Computes the rhobit link transformation, including its inverse and the first two derivatives.

## Usage

rhobitlink(theta, bminvalue = NULL, bmaxvalue = NULL,
inverse $=$ FALSE, deriv $=0$, short $=$ TRUE, tag $=$ FALSE)

## Arguments

theta Numeric or character. See below for further details.
bminvalue, bmaxvalue
Optional boundary values, e.g., values of theta which are less than or equal
to -1 can be replaced by bminvalue before computing the link function value.
And values of theta which are greater than or equal to 1 can be replaced by bmaxvalue before computing the link function value. See Links.
inverse, deriv, short, tag
Details at Links.

## Details

The rhobitlink link function is commonly used for parameters that lie between -1 and 1 . Numerical values of theta close to -1 or 1 or out of range result in Inf, $-\operatorname{Inf}, \mathrm{NA}$ or NaN .

## Value

For deriv $=0$, the rhobit of theta, i.e., $\log ((1+$ theta $) /(1-$ theta $))$ when inverse $=$ FALSE, and if inverse $=$ TRUE then $(\exp ($ theta $)-1) /(\exp ($ theta $)+1)$.
For deriv $=1$, then the function returns $d$ eta $/ d$ theta as a function of theta if inverse $=$ FALSE, else if inverse $=$ TRUE then it returns the reciprocal.

## Note

Numerical instability may occur when theta is close to -1 or 1 . One way of overcoming this is to use bminvalue, etc.
The correlation parameter of a standard bivariate normal distribution lies between -1 and 1 , therefore this function can be used for modelling this parameter as a function of explanatory variables.
The link function rhobitlink is very similar to fisherzlink, e.g., just twice the value of fisherzlink.

## Author(s)

Thomas W. Yee

## See Also

Links, binom2.rho, fisherz.

## Examples

```
theta <- seq (-0.99, 0.99, by = 0.01)
y <- rhobitlink(theta)
## Not run:
plot(theta, y, type = "l", las = 1, ylab = "", main = "rhobitlink(theta)")
abline(v = 0, h = 0, lty = 2)
## End(Not run)
```



```
rhobitlink(x) # Has NAs
rhobitlink(x, bminvalue = -1 + .Machine$double.eps,
            bmaxvalue = 1 - .Machine$double.eps) # Has no NAs
```

Rice The Rice Distribution

## Description

Density, distribution function, quantile function and random generation for the Rician distribution.

## Usage

drice (x, sigma, vee, log = FALSE)
price(q, sigma, vee, lower.tail = TRUE, log.p = FALSE, ...)
qrice( $p$, sigma, vee, lower.tail = TRUE, log.p = FALSE, $\ldots$ )
rrice(n, sigma, vee)

## Arguments

| $x, q$ | vector of quantiles. |
| :--- | :--- |
| $p$ | vector of probabilities. |
| $n$ | number of observations. Same as in runif. |
| vee, sigma | See riceff. |
| $\ldots$ | Other arguments such as lower.tail. |
| lower.tail, log. |  |
|  | Same meaning as in pnorm or qnorm. |
| $l o g$ | Logical. If log = TRUE then the logarithm of the density is returned. |

## Details

See riceff, the VGAM family function for estimating the two parameters, for the formula of the probability density function and other details.
Formulas for price() and qrice() are based on the Marcum-Q function.

## Value

drice gives the density, price gives the distribution function, qrice gives the quantile function, and rrice generates random deviates.

## Author(s)

T. W. Yee and Kai Huang

## See Also

riceff.

## Examples

```
## Not run: x <- seq(0.01, 7, len = 201)
plot(x, drice(x, vee = 0, sigma = 1), type = "n", las = 1,, ylab = "",
        main = "Density of Rice distribution for various values of v")
sigma <- 1; vee <- c(0, 0.5, 1, 2, 4)
for (ii in 1:length(vee))
    lines(x, drice(x, vee = vee[ii], sigma), col = ii)
legend(x = 5, y = 0.6, legend = as.character(vee),
            col = 1:length(vee), lty = 1)
x <- seq(0, 4, by = 0.01); vee <- 1; sigma <- 1
probs <- seq(0.05, 0.95, by = 0.05)
plot(x, drice(x, vee = vee, sigma = sigma), type = "l", col = "blue",
    main = "Blue is density, orange is cumulative distribution function",
        ylim = c(0, 1), sub = "Purple are 5, 10, ..., 95 percentiles",
        las = 1, ylab = "", cex.main = 0.9)
abline(h = 0:1, col = "black", lty = 2)
Q <- qrice(probs, sigma, vee = vee)
lines(Q, drice(qrice(probs, sigma, vee = vee),
            sigma, vee = vee), col = "purple", lty = 3, type = "h")
lines(x, price(x, sigma, vee = vee), type = "l", col = "orange")
lines(Q, drice(Q, sigma, vee = vee), col = "purple", lty = 3, type = "h")
lines(Q, price(Q, sigma, vee = vee), col = "purple", lty = 3, type = "h")
abline(h = probs, col = "purple", lty = 3)
max(abs(price(Q, sigma, vee = vee) - probs)) # Should be 0
## End(Not run)
```

riceff

Rice Distribution Family Function

## Description

Estimates the two parameters of a Rice distribution by maximum likelihood estimation.

## Usage

```
riceff(lsigma = "loglink", lvee = "loglink", isigma = NULL,
    ivee \(=\) NULL, nsimEIM = 100, zero = NULL, nowarning = FALSE)
```


## Arguments

nowarning Logical. Suppress a warning? Ignored for VGAM 0.9-7 and higher.
lvee, lsigma
Link functions for the $v$ and $\sigma$ parameters. See Links for more choices and for general information.
ivee, isigma Optional initial values for the parameters. If convergence failure occurs (this VGAM family function seems to require good initial values) try using these arguments. See CommonVGAMffArguments for more information.
nsimEIM, zero See CommonVGAMffArguments for information.

## Details

The Rician distribution has density function

$$
f(y ; v, \sigma)=\frac{y}{\sigma^{2}} \exp \left(-\left(y^{2}+v^{2}\right) /\left(2 \sigma^{2}\right)\right) I_{0}\left(y v / \sigma^{2}\right)
$$

where $y>0, v>0, \sigma>0$ and $I_{0}$ is the modified Bessel function of the first kind with order zero. When $v=0$ the Rice distribution reduces to a Rayleigh distribution. The mean is $\sigma \sqrt{\pi / 2} \exp (z / 2)\left((1-z) I_{0}(-z / 2)-z I_{1}(-z / 2)\right)$ (returned as the fitted values) where $z=$ $-v^{2} /\left(2 \sigma^{2}\right)$. Simulated Fisher scoring is implemented.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

Convergence problems may occur for data where $v=0$; if so, use rayleigh or possibly use an identity link.
When $v$ is large (greater than 3, say) then the mean is approximately $v$ and the standard deviation is approximately $\sigma$.

## Author(s)

T. W. Yee

## References

Rice, S. O. (1945). Mathematical Analysis of Random Noise. Bell System Technical Journal, 24, 46-156.

## See Also

drice, rayleigh, besselI, simulate.vlm.

## Examples

```
    ## Not run: sigma <- exp(1); vee <- exp(2)
    rdata <- data.frame(y = rrice(n <- 1000, sigma, vee = vee))
    fit <- vglm(y ~ 1, riceff, data = rdata, trace = TRUE, crit = "coef")
    c(with(rdata, mean(y)), fitted(fit)[1])
    coef(fit, matrix = TRUE)
    Coef(fit)
    summary(fit)
    ## End(Not run)
```

    rigff Reciprocal Inverse Gaussian distribution
    
## Description

Estimation of the parameters of a reciprocal inverse Gaussian distribution.

## Usage

rigff(lmu = "identitylink", llambda = "loglink", imu = NULL, ilambda = 1)

## Arguments

lmu, llambda Link functions for mu and lambda. See Links for more choices.
imu, ilambda Initial values for mu and lambda. A NULL means a value is computed internally.

## Details

See Jorgensen (1997) for details.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

This distribution is potentially useful for dispersion modelling.

## Author(s)

T. W. Yee

## References

Jorgensen, B. (1997). The Theory of Dispersion Models. London: Chapman \& Hall

## See Also

```
simplex.
```


## Examples

```
rdata <- data.frame(y = rchisq(n = 100, df = 14)) # Not 'proper' data!!
fit <- vglm(y ~ 1, rigff, data = rdata, trace = TRUE)
fit <- vglm(y ~ 1, rigff, data = rdata, trace = TRUE, eps = 1e-9, crit = "coef")
summary(fit)
```

rlplot.gevff Return Level Plot for GEV Fits

## Description

A return level plot is constructed for a GEV-type model.

## Usage

```
rlplot.gevff(object, show.plot = TRUE,
    probability = c((1:9)/100, (1:9)/10, 0.95, 0.99, 0.995, 0.999),
    add.arg = FALSE, xlab = if(log.arg) "Return Period (log-scale)" else
    "Return Period", ylab = "Return Level",
    main = "Return Level Plot",
    pch = par()$pch, pcol.arg = par()$col, pcex = par()$cex,
    llty.arg = par()$lty, lcol.arg = par()$col, llwd.arg = par()$lwd,
    slty.arg = par()$lty, scol.arg = par()$col, slwd.arg = par()$lwd,
    ylim = NULL, log.arg = TRUE, CI = TRUE, epsilon = 1e-05, ...)
```


## Arguments

object A VGAM extremes model of the GEV-type, produced by vglm with a family function either "gev" or "gevff".
show. plot Logical. Plot it? If FALSE no plot will be done.
probability Numeric vector of probabilities used.
add.arg Logical. Add the plot to an existing plot?
$x l a b \quad$ Caption for the $x$-axis. See par.
ylab Caption for the $y$-axis. See par.
main Title of the plot. See title.
pch Plotting character. See par.
pcol.arg Color of the points. See the col argument of par.
pcex Character expansion of the points. See the cex argument of par.
llty.arg Line type. Line type. See the lty argument of par.
lcol.arg Color of the lines. See the col argument of par.

| llwd.arg |  |
| :---: | :---: |
| slty.arg, scol.arg, slwd.arg |  |
|  | Correponding arguments for the lines used for the confidence intervals. Used only if CI=TRUE. |
| ylim | Limits for the y-axis. Numeric of length 2. |
| log.arg | Logical. If TRUE then log="" otherwise log=" $x$ ". This changes the labelling of the x -axis only. |
| CI | Logical. Add in a 95 percent confidence interval? |
| epsilon | Numeric, close to zero. Used for the finite-difference approximation to the first derivatives with respect to each parameter. If too small, numerical problems will occur. |
|  | Arguments passed into the plot function when setting up the entire plot. Useful arguments here include sub and las. |

## Details

A return level plot plots $z_{p}$ versus $\log \left(y_{p}\right)$. It is linear if the shape parameter $\xi=0$. If $\xi<0$ then the plot is convex with asymptotic limit as $p$ approaches zero at $\mu-\sigma / \xi$. And if $\xi>0$ then the plot is concave and has no finite bound. Here, $G\left(z_{p}\right)=1-p$ where $0<p<1$ ( $p$ corresponds to the argument probability) and $G$ is the cumulative distribution function of the GEV distribution. The quantity $z_{p}$ is known as the return level associated with the return period $1 / p$. For many applications, this means $z_{p}$ is exceeded by the annual maximum in any particular year with probability $p$.
The points in the plot are the actual data.

## Value

In the post slot of the object is a list called rlplot with list components

$$
\text { yp } \quad-\log (\text { probability }) \text {, which is used on the } x \text {-axis. }
$$

zp values which are used for the $y$-axis
lower, upper lower and upper confidence limits for the 95 percent confidence intervals evaluated at the values of probability (if CI=TRUE).

## Note

The confidence intervals are approximate, being based on finite-difference approximations to derivatives.

## Author(s)

T. W. Yee

## References

Coles, S. (2001). An Introduction to Statistical Modeling of Extreme Values. London: SpringerVerlag.

## See Also

gevff.

## Examples

```
gdata <- data.frame(y = rgev(n <- 100, scale = 2, shape = -0.1))
fit <- vglm(y ~ 1, gevff, data = gdata, trace = TRUE)
# Identity link for all parameters:
fit2 <- vglm(y ~ 1, gevff(lshape = identitylink, lscale = identitylink,
    iscale = 10), data = gdata, trace = TRUE)
coef(fit2, matrix = TRUE)
## Not run:
par(mfrow = c(1, 2))
rlplot(fit) -> i1
rlplot(fit2, pcol = "darkorange", lcol = "blue", log.arg = FALSE,
    scol = "darkgreen", slty = "dashed", las = 1) -> i2
range(i2@post$rlplot$upper - i1@post$rlplot$upper) # Should be near 0
range(i2@post$rlplot$lower - i1@post$rlplot$lower) # Should be near 0
## End(Not run)
```

```
rootogram4 Rootograms (S4 generic) for Assessing Goodness of Fit of Probability
```

    Models
    
## Description

A graphical technique for comparing the observed and fitted counts from a probability model, on a square root scale.

## Usage

rootogram4(object, ...)
rootogram4vglm(object, newdata $=$ NULL, breaks $=$ NULL, max $=$ NULL, xlab $=$ NULL, main $=$ NULL, width $=$ NULL, ...)

## Arguments

object an object of class "vglm". zz This includes "vgam" because "vlm" handles both VGLM and VGAM objects.
newdata Data upon which to base the calculations. The default is the one used to fit the model.
breaks numeric. Breaks for the histogram intervals.
$\max$ maximum count displayed.
xlab, main graphical parameters.
width numeric. Widths of the histogram bars.
$\ldots$ any additional arguments to rootogram. default and plot. rootogram in countreg.

## Details

Rootograms are a useful graphical technique for comparing the observed counts with the expected counts given a probability model.
This S 4 implementation is based very heavily on rootogram coming from countreg. This package is primarily written by A. Zeileis and C. Kleiber. That package is currently on R-Forge but not CRAN, and it is based on S3. Since VGAM is written using S4, it was necessary to define an S4 generic function called rootogram4() which dispatches appropriately for S 4 objects.
Currently, only a selected number of VGAM family functions are implemented. Over time, hopefully more and more will be completed.

## Value

See rootogram in countreg; an object of class "rootogram0" inheriting from "data.frame" with about 8 variables.

## Warning

This function is rudimentary and based totally on the implementation in countreg.

## Note

The function names used coming from countreg have been renamed slightly to avoid conflict.

## Author(s)

Package countreg is primarily written by A. Zeileis and C. Kleiber. Function rootogram4() is based very heavily on countreg. T. W. Yee wrote code to unpack variables from many various models and feed them into the appropriate d-type function.

## References

Friendly, M. and Meyer, D. (2016). Discrete Data Analysis with R: Visualization and Modeling Techniques for Categorical and Count Data, Boca Raton, FL, USA: Chapman \& Hall/CRC Press.
Kleiber, C. and Zeileis, A. (2016) "Visualizing Count Data Regressions Using Rootograms." The American Statistician, 70(3), 296-303. doi: 10.1080/00031305.2016.1173590.
Tukey, J. W. (1977) Exploratory Data Analysis, Reading, MA, USA: Addison-Wesley.

## See Also

vglm, vgam, glm, zipoisson, zapoisson, rootogram in countreg.

## Examples

```
## Not run:
data("hspider", package = "VGAM") # Count responses
hs.p <- vglm(Pardlugu ~ CoveHerb, poissonff, data = hspider)
hs.nb <- vglm(Pardlugu ~ CoveHerb, negbinomial, data = hspider)
hs.zip <- vglm(Pardlugu ~ CoveHerb, zipoisson, data = hspider)
hs.zap <- vglm(Pardlugu ~ CoveHerb, zapoisson, data = hspider)
```

```
    opar <- par(mfrow = c(2, 2)) # Plot the rootograms
    rootogram4(hs.p, max = 15, main = "poissonff")
    rootogram4(hs.nb, max = 15, main = "negbinomial")
    rootogram4(hs.zip, max = 15, main = "zipoisson")
    rootogram4(hs.zap, max = 15, main = "zapoisson")
    par(opar)
    ## End(Not run)
```

    rrar
    
## Description

Estimates the parameters of a nested reduced-rank autoregressive model for multiple time series.

## Usage

rrar(Ranks = 1 , coefstart = NULL)

## Arguments

Ranks Vector of integers: the ranks of the model. Each value must be at least one and no more than $M$, where $M$ is the number of response variables in the time series. The length of Ranks is the lag, which is often denoted by the symbol $L$ in the literature.
coefstart Optional numerical vector of initial values for the coefficients. By default, the family function chooses these automatically.

## Details

Full details are given in Ahn and Reinsel (1988). Convergence may be very slow, so setting maxits $=50$, say, may help. If convergence is not obtained, you might like to try inputting different initial values.

Setting trace $=$ TRUE in vglm is useful for monitoring the progress at each iteration.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

This family function should be used within vglm and not with rrvglm because it does not fit into the RR-VGLM framework exactly. Instead, the reduced-rank model is formulated as a VGLM!

A methods function Coef.rrar, say, has yet to be written. It would return the quantities Ak1, C, D, omegahat, Phi, etc. as slots, and then show. Coef.rrar would also need to be written.

## Author(s)

T. W. Yee

## References

Ahn, S. and Reinsel, G. C. (1988). Nested reduced-rank autoregressive models for multiple time series. Journal of the American Statistical Association, 83, 849-856.

## See Also

vglm, grain.us.

## Examples

```
## Not run:
year <- seq(1961 + 1/12, 1972 + 10/12, by = 1/12)
par(mar = c(4, 4, 2, 2) + 0.1, mfrow = c(2, 2))
for (ii in 1:4) {
    plot(year, grain.us[, ii], main = names(grain.us)[ii], las = 1,
            type = "l", xlab = "", ylab = "", col = "blue")
        points(year, grain.us[, ii], pch = "*", col = "blue")
}
apply(grain.us, 2, mean) # mu vector
cgrain <- scale(grain.us, scale = FALSE) # Center the time series only
fit <- vglm(cgrain ~ 1, rrar(Ranks = c(4, 1)), trace = TRUE)
summary(fit)
print(fit@misc$Ak1, digits = 2)
print(fit@misc$Cmatrices, digits = 3)
print(fit@misc$Dmatrices, digits = 3)
print(fit@misc$omegahat, digits = 3)
print(fit@misc$Phimatrices, digits = 2)
par(mar = c(4, 4, 2, 2) + 0.1, mfrow = c(4, 1))
for (ii in 1:4) {
    plot(year, fit@misc$Z[, ii], main = paste("Z", ii, sep = ""),
            type = "l", xlab = "", ylab = "", las = 1, col = "blue")
        points(year, fit@misc$Z[, ii], pch = "*", col = "blue")
}
## End(Not run)
```

rrvglm
Fitting Reduced-Rank Vector Generalized Linear Models (RRVGLMs)

## Description

A reduced-rank vector generalized linear model (RR-VGLM) is fitted. RR-VGLMs are VGLMs but some of the constraint matrices are estimated. In this documentation, $M$ is the number of linear predictors.

## Usage

```
rrvglm(formula, family = stop("argument 'family' needs to be assigned"),
    data = list(), weights = NULL, subset = NULL,
    na.action = na.fail, etastart = NULL, mustart = NULL,
    coefstart = NULL, control = rrvglm.control(...), offset = NULL,
    method = "rrvglm.fit", model = FALSE, x.arg = TRUE, y.arg = TRUE,
    contrasts = NULL, constraints = NULL, extra = NULL,
    qr.arg = FALSE, smart = TRUE, ...)
```


## Arguments

formula, family, weights
See vglm.
data an optional data frame containing the variables in the model. By default the variables are taken from environment (formula), typically the environment from which rrvglm is called.
subset, na.action
See vglm.
etastart, mustart, coefstart
See vglm.
control a list of parameters for controlling the fitting process. See rrvglm. control for details.
offset, model, contrasts
See vglm.
method the method to be used in fitting the model. The default (and presently only) method rrvglm.fit uses iteratively reweighted least squares (IRLS).
$x . a r g, y . a r g \quad$ logical values indicating whether the model matrix and response vector/matrix used in the fitting process should be assigned in the x and y slots. Note the model matrix is the LM model matrix; to get the VGLM model matrix type model.matrix(vglmfit) where vglmfit is a vglm object.
constraints See vglm.
extra, smart, qr.arg
See vglm.
... further arguments passed into rrvglm. control.

## Details

The central formula is given by

$$
\eta=B_{1}^{T} x_{1}+A \nu
$$

where $x_{1}$ is a vector (usually just a 1 for an intercept), $x_{2}$ is another vector of explanatory variables, and $\nu=C^{T} x_{2}$ is an $R$-vector of latent variables. Here, $\eta$ is a vector of linear predictors, e.g., the $m$ th element is $\eta_{m}=\log \left(E\left[Y_{m}\right]\right)$ for the $m$ th Poisson response. The matrices $B_{1}, A$ and $C$ are estimated from the data, i.e., contain the regression coefficients. For ecologists, the central formula represents a constrained linear ordination (CLO) since it is linear in the latent variables. It means that the response is a monotonically increasing or decreasing function of the latent variables.
For identifiability it is common to enforce corner constraints on $A$ : by default, the top $R$ by $R$ submatrix is fixed to be the order $-R$ identity matrix and the remainder of $A$ is estimated.

The underlying algorithm of RR-VGLMs is iteratively reweighted least squares (IRLS) with an optimizing algorithm applied within each IRLS iteration (e.g., alternating algorithm).
In theory, any VGAM family function that works for vglm and vgam should work for rrvglm too. The function that actually does the work is rrvglm.fit; it is vglm.fit with some extra code.

## Value

An object of class "rrvglm", which has the the same slots as a "vglm" object. The only difference is that the some of the constraint matrices are estimates rather than known. But VGAM stores the models the same internally. The slots of "vglm" objects are described in vglm-class.

## Note

The arguments of rrvglm are in general the same as those of vglm but with some extras in rrvglm. control. The smart prediction (smartpred) library is packed with the VGAM library.
In an example below, a rank-1 stereotype model of Anderson (1984) is fitted to some car data. The reduced-rank regression is performed, adjusting for two covariates. Setting a trivial constraint matrix ( $\operatorname{diag}(M)$ ) for the latent variable variables in $x_{2}$ avoids a warning message when it is overwritten by a (common) estimated constraint matrix. It shows that German cars tend to be more expensive than American cars, given a car of fixed weight and width.
If fit <-rrvglm( . . . , data = mydata) then summary (fit) requires corner constraints and no missing values in mydata. Often the estimated variance-covariance matrix of the parameters is not positive-definite; if this occurs, try refitting the model with a different value for Index. corner.

For constrained quadratic ordination (CQO) see cqo for more details about QRR-VGLMs.
With multiple binary responses, one must use binomialff(multiple. responses = TRUE) to indicate that the response is a matrix with one response per column. Otherwise, it is interpreted as a single binary response variable.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Hastie, T. J. (2003). Reduced-rank vector generalized linear models. Statistical Modelling, 3, 15-41.
Yee, T. W. (2004). A new technique for maximum-likelihood canonical Gaussian ordination. Ecological Monographs, 74, 685-701.

Anderson, J. A. (1984). Regression and ordered categorical variables. Journal of the Royal Statistical Society, Series B, Methodological, 46, 1-30.
Yee, T. W. (2014). Reduced-rank vector generalized linear models with two linear predictors. Computational Statistics and Data Analysis, 71, 889-902.

## See Also

rrvglm. control, lvplot.rrvglm(same as biplot.rrvglm), rrvglm-class, grc, cqo, vglmff-class, vglm, vglm-class, smartpred, rrvglm.fit. Special family functions include negbinomial zipoisson and zinegbinomial. (see Yee (2014) and COZIGAM). Methods functions include Coef.rrvglm, calibrate.rrvglm, summary.rrvglm, etc. Data include crashi.

## Examples

```
## Not run:
# Example 1: RR negative binomial with Var(Y) = mu + delta1 * mu^delta2
nn <- 1000 # Number of observations
delta1 <- 3.0 # Specify this
delta2 <- 1.5 # Specify this; should be greater than unity
a21 <- 2 - delta2
mydata <- data.frame(x2 = runif(nn), x3 = runif(nn))
mydata <- transform(mydata, mu = exp(2 + 3 * x2 + 0 * x3))
mydata <- transform(mydata,
                y2 = rnbinom(nn, mu = mu, size = (1/delta1)*mu^a21))
plot(y2 ~ x2, data = mydata, pch = "+", col = 'blue', las = 1,
    main = paste("Var(Y) = mu + ", delta1, " * mu^", delta2, sep = ""))
rrnb2 <- rrvglm(y2 ~ x2 + x3, negbinomial(zero = NULL),
            data = mydata, trace = TRUE)
a21.hat <- (Coef(rrnb2)@A)["loglink(size)", 1]
beta11.hat <- Coef(rrnb2)@B1["(Intercept)", "loglink(mu)"]
beta21.hat <- Coef(rrnb2)@B1["(Intercept)", "loglink(size)"]
(delta1.hat <- exp(a21.hat * beta11.hat - beta21.hat))
(delta2.hat <- 2 - a21.hat)
# exp(a21.hat * predict(rrnb2)[1,1] - predict(rrnb2)[1,2]) # delta1.hat
summary(rrnb2)
# Obtain a 95 percent confidence interval for delta2:
se.a21.hat <- sqrt(vcov(rrnb2)["I(latvar.mat)", "I(latvar.mat)"])
ci.a21 <- a21.hat + c(-1, 1) * 1.96 * se.a21.hat
(ci.delta2 <- 2 - rev(ci.a21)) # The 95 percent confidence interval
Confint.rrnb(rrnb2) # Quick way to get it
# Plot the abundances and fitted values against the latent variable
plot(y2 ~ latvar(rrnb2), data = mydata, col = "blue",
    xlab = "Latent variable", las = 1)
000 <- order(latvar(rrnb2))
lines(fitted(rrnb2)[ooo] ~ latvar(rrnb2)[ooo], col = "orange")
# Example 2: stereotype model (reduced-rank multinomial logit model)
data(car.all)
```

```
scar <- subset(car.all,
    is.element(Country, c("Germany", "USA", "Japan", "Korea")))
fcols <- c(13,14,18:20,22:26,29:31,33,34,36) # These are factors
scar[, -fcols] <- scale(scar[, -fcols]) # Standardize all numerical vars
ones <- matrix(1, 3, 1)
clist <- list("(Intercept)" = diag(3), Width = ones, Weight = ones,
    Disp. = diag(3), Tank = diag(3), Price = diag(3),
    Frt.Leg.Room = diag(3))
set.seed(111)
fit <- rrvglm(Country ~ Width + Weight + Disp. + Tank +
                            Price + Frt.Leg.Room,
            multinomial, data = scar, Rank = 2, trace = TRUE,
            constraints = clist, noRRR = ~ 1 + Width + Weight,
            Uncor = TRUE, Corner = FALSE, Bestof = 2)
fit@misc$deviance # A history of the fits
Coef(fit)
biplot(fit, chull = TRUE, scores = TRUE, clty = 2, Ccex = 2,
    ccol = "blue", scol = "orange", Ccol = "darkgreen", Clwd = 2,
    main = "1=Germany, 2=Japan, 3=Korea, 4=USA")
## End(Not run)
```

```
rrvglm-class Class"rrvglm"
```


## Description

Reduced-rank vector generalized linear models.

## Objects from the Class

Objects can be created by calls to rrvglm.

## Slots

extra: Object of class "list"; the extra argument on entry to vglm. This contains any extra information that might be needed by the family function.
family: Object of class "vglmff". The family function.
iter: Object of class "numeric". The number of IRLS iterations used.
predictors: Object of class "matrix" with $M$ columns which holds the $M$ linear predictors.
assign: Object of class "list", from class "vlm". This named list gives information matching the columns and the (LM) model matrix terms.
call: Object of class "call", from class "vlm". The matched call.
coefficients: Object of class "numeric", from class "vlm". A named vector of coefficients.
constraints: Object of class "list", from class "vlm". A named list of constraint matrices used in the fitting.
contrasts: Object of class "list", from class "vlm". The contrasts used (if any).
control: Object of class "list", from class "vlm". A list of parameters for controlling the fitting process. See vglm. control for details.
criterion: Object of class "list", from class "vlm". List of convergence criterion evaluated at the final IRLS iteration.
df.residual: Object of class "numeric", from class "vlm". The residual degrees of freedom.
df.total: Object of class "numeric", from class "vlm". The total degrees of freedom.
dispersion: Object of class "numeric", from class "vlm". The scaling parameter.
effects: Object of class "numeric", from class "vlm". The effects.
fitted.values: Object of class "matrix", from class "vlm". The fitted values. This is usually the mean but may be quantiles, or the location parameter, e.g., in the Cauchy model.
misc: Object of class "list", from class "vlm". A named list to hold miscellaneous parameters.
model: Object of class "data.frame", from class "vlm". The model frame.
na.action: Object of class "list", from class "vlm". A list holding information about missing values.
offset: Object of class "matrix", from class "vlm". If non-zero, a $M$-column matrix of offsets. post: Object of class "list", from class "vlm" where post-analysis results may be put.
preplot: Object of class "list", from class "vlm" used by plotvgam; the plotting parameters may be put here.
prior.weights: Object of class "matrix", from class "vlm" holding the initially supplied weights. qr: Object of class "list", from class "vlm". QR decomposition at the final iteration.
R: Object of class "matrix", from class "vlm". The $\mathbf{R}$ matrix in the QR decomposition used in the fitting.
rank: Object of class "integer", from class "vlm". Numerical rank of the fitted model.
residuals: Object of class "matrix", from class "vlm". The working residuals at the final IRLS iteration.
ResSS: Object of class "numeric", from class "vlm". Residual sum of squares at the final IRLS iteration with the adjusted dependent vectors and weight matrices.
smart.prediction: Object of class "list", from class "vlm". A list of data-dependent parameters (if any) that are used by smart prediction.
terms: Object of class "list", from class "vlm". The terms object used.
weights: Object of class "matrix", from class "vlm". The weight matrices at the final IRLS iteration. This is in matrix-band form.
$x$ : Object of class "matrix", from class "vlm". The model matrix (LM, not VGLM).
xlevels: Object of class "list", from class "vlm". The levels of the factors, if any, used in fitting.
$y$ : Object of class "matrix", from class "vlm". The response, in matrix form.
Xm2: Object of class "matrix", from class "vlm". See vglm-class).
Ym2: Object of class "matrix", from class "vlm". See vglm-class).
callXm2: Object of class "call", from class "vlm". The matched call for argument form2.

## Extends

Class "vglm", directly. Class "vlm", by class "vglm".

## Methods

biplot signature ( $\mathrm{x}=$ "rrvglm"): biplot.
Coef signature (object = "rrvglm"): more detailed coefficients giving $\mathbf{A}, \boldsymbol{B}_{1}, \mathbf{C}$, etc.
biplot signature(object = "rrvglm"): biplot.
print signature ( $x=$ "rrvglm"): short summary of the object.
summary signature(object = "rrvglm"): a more detailed summary of the object.

## Note

The slots of "rrvglm" objects are currently identical to "vglm" objects.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Hastie, T. J. (2003). Reduced-rank vector generalized linear models. Statistical Modelling, 3, 15-41.

Yee, T. W. and Wild, C. J. (1996). Vector generalized additive models. Journal of the Royal Statistical Society, Series B, Methodological, 58, 481-493.

## See Also

rrvglm, lvplot.rrvglm, vglmff-class.

## Examples

```
## Not run: # Rank-1 stereotype model of Anderson (1984)
pneumo <- transform(pneumo, let = log(exposure.time),
        x3 = runif(nrow(pneumo))) # x3 is unrelated
fit <- rrvglm(cbind(normal, mild, severe) ~ let + x3,
        multinomial, data = pneumo, Rank = 1)
Coef(fit)
## End(Not run)
```

```
rrvglm.control Control Function for rrvglm()
```


## Description

Algorithmic constants and parameters for running rrvglm are set using this function.

## Usage

```
rrvglm.control(Rank = 1, Algorithm = c("alternating", "derivative"),
        Corner = TRUE, Uncorrelated.latvar = FALSE,
        Wmat = NULL, Svd.arg = FALSE,
        Index.corner = if (length(str0))
        head((1:1000)[-str0], Rank) else 1:Rank,
        Ainit = NULL, Alpha \(=0.5\), Bestof \(=1\), Cinit \(=\) NULL,
        Etamat.colmax = 10,
        sd.Ainit \(=0.02\), sd.Cinit \(=0.02\), str0 \(=\) NULL,
        noRRR = ~1, Norrr = NA,
        noWarning = FALSE,
        trace \(=\) FALSE, Use.Init.Poisson.QO = FALSE,
        checkwz = TRUE, Check.rank = TRUE, Check.cm.rank = TRUE,
        wzepsilon = .Machine\$double.eps^0.75, ...)
```


## Arguments

Rank
The numerical rank $R$ of the model. Must be an element from the set $\{1,2, \ldots, \min (M, p 2)\}$. Here, the vector of explanatory variables $\mathbf{x}$ is partitioned into ( $\mathbf{x} 1, \mathbf{x} \mathbf{2}$ ), which is of dimension $p 1+p 2$. The variables making up $\mathbf{x} \mathbf{1}$ are given by the terms in noRRR argument, and the rest of the terms comprise $\mathbf{x} \mathbf{2}$.
Algorithm Character string indicating what algorithm is to be used. The default is the first one.

Corner Logical indicating whether corner constraints are to be used. This is one method for ensuring a unique solution. If TRUE, Index. corner specifies the $R$ rows of the constraint matrices that are use as the corner constraints, i.e., they hold an order- $R$ identity matrix.
Uncorrelated.latvar
Logical indicating whether uncorrelated latent variables are to be used. This is normalization forces the variance-covariance matrix of the latent variables to be diag(Rank), i.e., unit variance and uncorrelated. This constraint does not lead to a unique solution because it can be rotated.
Wmat Yet to be done.
Svd.arg Logical indicating whether a singular value decomposition of the outer product is to computed. This is another normalization which ensures uniqueness. See the argument Alpha below.
Index. corner $\quad$ Specifies the $R$ rows of the constraint matrices that are used for the corner constraints, i.e., they hold an order- $R$ identity matrix.

| Alpha | The exponent in the singular value decomposition that is used in the first part: if the SVD is $U D V^{T}$ then the first and second parts are $U D^{\alpha}$ and $D^{1-\alpha} V^{T}$ respectively. A value of 0.5 is 'symmetrical'. This argument is used only when Svd.arg=TRUE. |
| :---: | :---: |
| Bestof | Integer. The best of Bestof models fitted is returned. This argument helps guard against local solutions by (hopefully) finding the global solution from many fits. The argument works only when the function generates its own initial value for $\mathbf{C}$, i.e., when $\mathbf{C}$ is not passed in as initial values. |
| Ainit, Cinit | Initial $\mathbf{A}$ and $\mathbf{C}$ matrices which may speed up convergence. They must be of the correct dimension. |
| Etamat.colmax | Positive integer, no smaller than Rank. Controls the amount of memory used by .Init.Poisson.QO(). It is the maximum number of columns allowed for the pseudo-response and its weights. In general, the larger the value, the better the initial value. Used only if Use. Init. Poisson. QO=TRUE. |
| str0 | Integer vector specifying which rows of the estimated constraint matrices (A) are to be all zeros. These are called structural zeros. Must not have any common value with Index.corner, and be a subset of the vector 1:M. The default, str0 = NULL, means no structural zero rows at all. |
| sd.Ainit, sd.Cinit |  |
|  | Standard deviation of the initial values for the elements of $\mathbf{A}$ and $\mathbf{C}$. These are normally distributed with mean zero. This argument is used only if Use. Init.Poisson. QO = FALSE. |
| noRRR | Formula giving terms that are not to be included in the reduced-rank regression. That is, noRRR specifes which explanatory variables are in the $x_{1}$ vector of rrvglm, and the rest go into $x_{2}$. The $x_{1}$ variables constitute the $\boldsymbol{B}_{1}$ matrix in Yee and Hastie (2003). Those $x_{2}$ variables which are subject to the reduced-rank regression correspond to the $\boldsymbol{B}_{2}$ matrix. Set noRRR = NULL for the reduced-rank regression to be applied to every explanatory variable including the intercept. |
| Norrr | Defunct. Please use noRRR. Use of Norrr will become an error soon. |
| trace | Logical indicating if output should be produced for each iteration. |
| Use.Init.Poisson.Q0 |  |
|  | Logical indicating whether the . Init.Poisson.QO() should be used to obtain initial values for the $\mathbf{C}$. The function uses a new method that can work well if the data are Poisson counts coming from an equal-tolerances QRR-VGLM (CQO). This option is less realistic for RR-VGLMs compared to QRR-VGLMs. |
| checkwz | logical indicating whether the diagonal elements of the working weight matrices should be checked whether they are sufficiently positive, i.e., greater than wzepsilon. If not, any values less than wzepsilon are replaced with this value. |
| noWarning, Check.rank, Check.cm.rank |  |
|  | Same as vglm. control. Ignored for VGAM 0.9-7 and higher. |
| wzepsilon | Small positive number used to test whether the diagonals of the working weight matrices are sufficiently positive. |
|  | Variables in ...are passed into vglm. control. If the derivative algorithm is used then ... are also passed into rrvglm. optim. control; and if the alternating algorithm is used then ... are also passed into valt. control. <br> In the above, $R$ is the Rank and $M$ is the number of linear predictors. |

## Details

VGAM supports three normalizations to ensure a unique solution. Of these, only corner constraints will work with summary of RR-VGLM objects.

## Value

A list with components matching the input names. Some error checking is done, but not much.

## Note

The arguments in this function begin with an upper case letter to help avoid interference with those of vglm. control.

In the example below a rank-1 stereotype model (Anderson, 1984) is fitted.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Hastie, T. J. (2003). Reduced-rank vector generalized linear models. Statistical Modelling, 3, 15-41.

## See Also

```
rrvglm, rrvglm.optim.control, rrvglm-class, vglm, vglm.control, cqo.
```


## Examples

```
## Not run:
set.seed(111)
pneumo <- transform(pneumo, let = log(exposure.time),
            x3 = runif(nrow(pneumo))) # x3 is random noise
fit <- rrvglm(cbind(normal, mild, severe) ~ let + x3,
            multinomial, data = pneumo, Rank = 1, Index.corner = 2)
constraints(fit)
vcov(fit)
summary(fit)
## End(Not run)
```

```
rrvglm.optim.control Control Function for rrvglm() Calling optim()
```


## Description

Algorithmic constants and parameters for running optim within rrvglm are set using this function.

## Usage

$$
\begin{aligned}
\text { rrvglm.optim.control } & (\text { Fnscale }=1, \text { Maxit }=100, \\
& \text { Switch.optimizer }=3 \text {, Abstol }=- \text { Inf, } \\
& \text { Reltol }=\operatorname{sqrt}(. \text { Machine\$double.eps }), \ldots)
\end{aligned}
$$

## Arguments

| Fnscale | Passed into optim as fnscale. |
| :--- | :--- |
| Maxit | Passed into optim as maxit. |
| Switch.optimizer |  |$\quad$| Iteration number when the "Nelder-Mead" method of optim is switched to the |
| :--- |
|  |
|  |
| quasi-Newton "BFGS" method. Assigning Switch.optimizer a negative num- |
| ber means always BFGS, while assigning Switch.optimizer a value greater |
| than maxits means always use Nelder-Mead. |
| Abstol |
| Reltol |
| $\ldots$ | | Passed into optim as abstol. |
| :--- |
| Passed into optim as reltol. |

## Details

See optim for more details.

## Value

A list with components equal to the arguments.

## Note

The transition between optimization methods may be unstable, so users may have to vary the value of Switch.optimizer.
Practical experience with Switch.optimizer shows that setting it to too large a value may lead to a local solution, whereas setting it to a low value will obtain the global solution. It appears that, if BFGS kicks in too late when the Nelder-Mead algorithm is starting to converge to a local solution, then switching to BFGS will not be sufficient to bypass convergence to that local solution.

## Author(s)

Thomas W. Yee

## See Also

rrvglm.control, optim.

## ruge Rutherford-Geiger Polonium Data

## Description

Decay counts of polonium recorded by Rutherford and Geiger (1910).

## Usage <br> data(ruge)

## Format

This data frame contains the following columns:
counts a numeric vector, counts or frequencies
number a numeric vector, the number of decays

## Details

These are the radioactive decay counts of polonium recorded by Rutherford and Geiger (1910) representing the number of scintillations in $26081 / 8$ minute intervals. For example, there were 57 frequencies of zero counts. The counts can be thought of as being approximately Poisson distributed.

## Source

Rutherford, E. and Geiger, H. (1910) The Probability Variations in the Distribution of alpha Particles, Philosophical Magazine, 20, 698-704.

## Examples

```
lambdahat <- with(ruge, weighted.mean(number, w = counts))
(N <- with(ruge, sum(counts)))
with(ruge, cbind(number, counts,
    fitted = round(N * dpois(number, lam = lambdahat))))
```


## Description

$s$ is used in the definition of (vector) smooth terms within vgam formulas. This corresponds to 1stgeneration VGAMs that use backfitting for their estimation. The effective degrees of freedom is prespecified.

## Usage

$$
s(x, d f=4, \text { spar }=0, \ldots)
$$

## Arguments

X
covariate (abscissae) to be smoothed. Note that x must be a single variable and not a function of a variable. For example, $s(x)$ is fine but $s(\log (x))$ will fail. In this case, let $\log x<-\log (x)$ (in the data frame), say, and then use $s(\log x)$. At this stage bivariate smoothers ( $x$ would be a two-column matrix) are not implemented.
df numerical vector of length $r$. Effective degrees of freedom: must lie between 1 (linear fit) and $n$ (interpolation). Thus one could say that $\mathrm{df}-1$ is the effective nonlinear degrees of freedom (ENDF) of the smooth. Recycling of values will be used if df is not of length $r$. If spar is positive then this argument is ignored. Thus $s()$ means that the effective degrees of freedom is prespecified. If it is known that the component function(s) are more wiggly than usual then try increasing the value of this argument.
spar numerical vector of length $r$. Positive smoothing parameters (after scaling) . Larger values mean more smoothing so that the solution approaches a linear fit for that component function. A zero value means that df is used. Recycling of values will be used if spar is not of length $r$.
... Ignored for now.

## Details

In this help file $M$ is the number of additive predictors and $r$ is the number of component functions to be estimated (so that $r$ is an element from the set $\{1,2, \ldots, M\}$ ). Also, if $n$ is the number of distinct abscissae, then s will fail if $n<7$.
s , which is symbolic and does not perform any smoothing itself, only handles a single covariate. Note that $s$ works in vgam only. It has no effect in vglm (actually, it is similar to the identity function I so that $s(x 2)$ is the same as $x 2$ in the LM model matrix). It differs from the $s()$ of the gam package and the $s$ of the mgev package; they should not be mixed together. Also, terms involving s should be simple additive terms, and not involving interactions and nesting etc. For example, myfactor: $s(x 2)$ is not a good idea.

## Value

A vector with attributes that are (only) used by vgam.

## Note

The vector cubic smoothing spline which $s()$ represents is computationally demanding for large $M$. The cost is approximately $O\left(n M^{3}\right)$ where $n$ is the number of unique abscissae.
Currently a bug relating to the use of $s()$ is that only constraint matrices whose columns are orthogonal are handled correctly. If any $s()$ term has a constraint matrix that does not satisfy this condition then a warning is issued. See is.buggy for more information.

A more modern alternative to using s with vgam is to use sm.os or sm.ps. This does not require backfitting and allows automatic smoothing parameter selection. However, this alternative should only be used when the sample size is reasonably large ( $>500$, say). These are called Generation-2 VGAMs.
Another alternative to using s with vgam is bs and/or ns with vglm. The latter implements halfstepping, which is helpful if convergence is difficult.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Wild, C. J. (1996). Vector generalized additive models. Journal of the Royal Statistical Society, Series B, Methodological, 58, 481-493.

## See Also

vgam, is.buggy, sm.os, sm.ps, vsmooth.spline.

## Examples

```
# Nonparametric logistic regression
fit1 <- vgam(agaaus ~ s(altitude, df = 2), binomialff, data = hunua)
## Not run: plot(fit1, se = TRUE)
# Bivariate logistic model with artificial data
nn <- 300
bdata <- data.frame(x1 = runif(nn), x2 = runif(nn))
bdata <- transform(bdata,
    y1 = rbinom(nn, size = 1, prob = logitlink(sin(2 * x2), inverse = TRUE)),
    y2 = rbinom(nn, size = 1, prob = logitlink(sin(2 * x2), inverse = TRUE)))
fit2 <- vgam(cbind(y1, y2) ~ x1 + s(x2, 3), trace = TRUE,
    binom2.or(exchangeable = TRUE), data = bdata)
coef(fit2, matrix = TRUE) # Hard to interpret
## Not run: plot(fit2, se = TRUE, which.term = 2, scol = "blue")
```


## Description

Estimates the location and scale parameters of a scaled Student $t$ distribution with 2 degrees of freedom, by maximum likelihood estimation.

## Usage

$$
\begin{gathered}
\text { sc.studentt2 (percentile }=50, \text { llocation }=" \text { identitylink", lscale }=\text { "loglink", } \\
\text { ilocation }=\text { NULL, iscale }=\text { NULL, imethod }=1 \text {, zero }=\text { "scale") }
\end{gathered}
$$

## Arguments

percentile A numerical vector containing values between 0 and 100, which are the quantiles and expectiles. They will be returned as 'fitted values'.
llocation, lscale
See Links for more choices, and CommonVGAMfffarguments.
ilocation, iscale, imethod, zero
See CommonVGAMffArguments for details.

## Details

Koenker (1993) solved for the distribution whose quantiles are equal to its expectiles. Its canonical form has mean and mode at 0 , and has a heavy tail (in fact, its variance is infinite).
The standard ("canonical") form of this distribution can be endowed with a location and scale parameter. The standard form has a density that can be written as

$$
f(z)=2 /\left(4+z^{2}\right)^{3 / 2}
$$

for real $y$. Then $z=(y-a) / b$ for location and scale parameters $a$ and $b>0$. The mean of $Y$ is $a$. By default, $\left.\eta_{1}=a\right)$ and $\eta_{2}=\log (b)$. The expectiles/quantiles corresponding to percentile are returned as the fitted values; in particular, percentile $=50$ corresponds to the mean ( 0.5 expectile) and median ( 0.5 quantile).
Note that if $Y$ has a standard dsc. t2 then $Y=\sqrt{2} T_{2}$ where $T_{2}$ has a Student-t distribution with 2 degrees of freedom. The two parameters here can also be estimated using studentt2 by specifying $\mathrm{df}=2$ and making an adjustment for the scale parameter, however, this VGAM family function is more efficient since the EIM is known (Fisher scoring is implemented.)

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Author(s)

T. W. Yee

## References

Koenker, R. (1993). When are expectiles percentiles? (solution) Econometric Theory, 9, 526-527.

## See Also

```
dsc.t2, studentt2.
```


## Examples

```
set.seed(123); nn <- 1000
kdata <- data.frame(x2 = sort(runif(nn)))
kdata <- transform(kdata, mylocat = 1 + 3 * x2,
                                    myscale = 1)
kdata <- transform(kdata, y = rsc.t2(nn, loc = mylocat, scale = myscale))
fit <- vglm(y ~ x2, sc.studentt2(perc = c(1, 50, 99)), data = kdata)
fit2 <- vglm(y ~ x2, studentt2(df = 2), data = kdata) # 'same' as fit
coef(fit, matrix = TRUE)
head(fitted(fit))
head(predict(fit))
# Nice plot of the results
## Not run: plot(y ~ x2, data = kdata, col = "blue", las = 1,
    sub = paste("n =", nn),
    main = "Fitted quantiles/expectiles using the sc.studentt2() distribution")
matplot(with(kdata, x2), fitted(fit), add = TRUE, type = "l", lwd = 3)
legend("bottomright", lty = 1:3, lwd = 3, legend = colnames(fitted(fit)),
            col = 1:3)
## End(Not run)
fit@extra$percentile # Sample quantiles
```

score.stat

Rao's Score Test Statistics Evaluated at the Null Values

## Description

Generic function that computes Rao's score test statistics evaluated at the null values (consequently they do not suffer from the Hauck-Donner effect).

## Usage

score.stat(object, ...)
score.stat.vlm(object, values $0=0$, subset $=$ NULL, omit1s $=$ TRUE, all. out $=$ FALSE, iterate $=$ TRUE, trace $=$ FALSE,.. )

## Arguments

| object, values0, subset |  |
| :--- | :--- |
|  | Same as in wald.stat.vlm. |
| omit1s, all.out,, iterate |  |
|  | Same as in wald.stat.vlm. |
| trace | Same as in wald.stat.vlm. |
| $\ldots$ | Ignored for now. |

## Details

The (Rao) score test (also known as the Lagrange multiplier test in econometrics) is a third general method for hypothesis testing under a likelihood-based framework (the others are the likelihood ratio test and Wald test; see lrt. stat and wald. stat). Asymptotically, the three tests are equivalent. The Wald test is not invariant to parameterization, and the usual Wald test statistics computed at the estimates make it vulnerable to the Hauck-Donner effect (HDE; see hdeff). This function is similar to wald. stat in that one coefficient is set to 0 (by default) and the other coefficients are iterated by IRLS to get their MLE subject to this constraint.

## Value

By default the signed square root of the Rao score statistics are returned. If all. out = TRUE then a list is returned with the following components: score. stat the score statistic, SE0 the standard error of that coefficient, values0 the null values. Approximately, the default score statistics output are standard normal random variates if each null hypothesis is true.

## Warning

See wald.stat.vlm.

## Author(s)

Thomas W. Yee

## See Also

wald.stat, lrt.stat, summaryvglm, summary.glm, anova.vglm, vglm, hdeff.

## Examples

```
set.seed(1)
pneumo <- transform(pneumo, let = log(exposure.time),
                x3 = rnorm(nrow(pneumo)))
(pfit <- vglm(cbind(normal, mild, severe) ~ let + x3,
        propodds, data = pneumo))
score.stat(pfit) # No HDE here; should be similar to the next line:
coef(summary(pfit))[, "z value"] # Wald statistics computed at the MLE
summary(pfit, score0 = TRUE)
```


## Description

Plots the piecewise segmented curve made up of Wald statistics versus estimates, using a colour code for the HDE severity.

## Usage

$$
\begin{aligned}
& \text { seglines(x, y, dy, ddy, lwd }=2 \text {, cex }=2 \text {, plot.it }=\text { TRUE, } \\
& \text { add.legend }=\text { TRUE, position.legend = "topleft", } \\
& \text { lty.table }=c(" \text { "solid", "dashed", "solid", "dashed", } \\
& \text { "solid", "dashed", "solid"), } \\
& \text { col.table = rainbow.sky[-5], pch.table = 7:1, } \\
& \text { severity.table = c("None", "Faint", "Weak", } \\
& \text { "Moderate", "Strong", "Extreme", "Undetermined"), } \\
& \text { tol0 } 0.1, \text { FYI = FALSE, ...) }
\end{aligned}
$$

## Arguments

$x, y, d y$, ddy $\quad$ Same as hdeffsev.
lwd, cex Graphical parameters: line width, and character expansion.
plot.it Logical, plot it? If FALSE then the other graphical arguments are ignored.
add.legend, position.legend
Logical and character; add a legend? The other argument is fed into legend.
severity.table, tol0
Same as hdeffsev.
lty.table, col.table, pch.table
Graphical parameters for the 7 different types of segments. Usually users should not assign anything to these arguments.
FYI, ... Should be ignored.

## Details

This function was written to complement hdeffsev and is rough-and-ready. It plots the Wald statistics as a function of the estimates, and uses a colour-code to indicate the severity of the HauckDonner effect (HDE). This can be obtained from its first two derivatives.

## Value

This function returns the severity of the HDE, possibly invisibly.

## Note

This function is likely to change in the short future because it is experimental and far from complete.

## Author(s)

Thomas W. Yee.

## See Also

hdeff, hdeffsev.

## Examples

```
    deg <- 4 # myfun is a function that approximates the HDE
    myfun <- function(x, deriv = 0) switch(as.character(deriv),
        '0' = x^deg * exp(-x),
    '1' = (deg * x^(deg-1) - x^deg) * exp(-x),
    '2' = (deg * (deg-1) * x^(deg-2) - 2*deg * x^(deg-1) + x^deg) * exp(-x))
## Not run:
curve(myfun, 0, 10, col = "white")
xgrid <- seq(0, 10, length = 101)
seglines(xgrid, myfun(xgrid), myfun(xgrid, deriv = 1),
        myfun(xgrid, deriv = 2), position = "bottom")
## End(Not run)
```

Select

Select Variables for a Formula Response or the RHS of a Formula

## Description

Select variables from a data frame whose names begin with a certain character string.

## Usage

```
Select(data = list(), prefix = "y",
    lhs = NULL, rhs = NULL, rhs2 = NULL, rhs3 = NULL,
    as.character = FALSE, as.formula.arg = FALSE, tilde = TRUE,
    exclude = NULL, sort.arg = TRUE)
```


## Arguments

| data | A data frame or a matrix. |
| :--- | :--- |
| prefix | A vector of character strings, or a logical. If a character then the variables cho- <br> sen from data begin with the value of prefix. If a logical then only TRUE is <br> accepted and all the variables in data are chosen. |
| lhs | A character string. The response of a formula. |
| rhs | A character string. Included as part of the RHS a formula. Set rhs $=" 0 "$ to <br> suppress the intercept. |
| rhs2, rhs3 | Same as rhs but appended to its RHS, i.e., paste $0(r h s, "+", r h s 2, "+", r h s 3)$. <br> If used, rhs should be used first, and then possibly rhs2 and then possibly rhs3. |


| as.character | Logical. Return the answer as a character string? |
| :--- | :--- |
| as.formula.arg Logical. Is the answer a formula? <br> tilde Logical. If as. character and as.formula. arg are both TRUE then include the <br> tilde in the formula? <br> exclude Vector of character strings. Exclude these variables explicitly. <br> sort.arg Logical. Sort the variables? |  |

## Details

This is meant as a utility function to avoid manually: (i) making a cbind call to construct a big matrix response, and (ii) constructing a formula involving a lot of terms. The savings can be made because the variables of interest begin with some prefix, e.g., with the character " $y$ ".

## Value

If as.character $=$ FALSE and as.formula. $\arg =$ FALSE then matrix such as cbind $(\mathrm{y} 1, \mathrm{y} 2, \mathrm{y} 3)$.
If as .character = TRUE and as. formula. arg = FALSE then a character string such as "cbind $(\mathrm{y} 1, \mathrm{y} 2, \mathrm{y} 3)$ ".
If as.character = FALSE and as. formula. arg = TRUE then a formula such as lhs $\sim \mathrm{y} 1+\mathrm{y} 2+\mathrm{y} 3$. If as. character $=$ TRUE and as. formula. arg = TRUE then a character string such as "lhs $\sim y 1+$ $y 2+y 3 "$. See the examples below. By default, if no variables beginning the the value of prefix is found then a NULL is returned. Setting prefix $=">$ is a way of selecting no variables.

## Note

This function is a bit experimental at this stage and may change in the short future. Some of its utility may be better achieved using subset and its select argument, e.g., subset (pdata, TRUE, select $=y 01: y 10)$.
For some models such as posbernoulli.t the order of the variables in the xij argument is crucial, therefore care must be taken with the argument sort. arg. In some instances, it may be good to rename variables y 1 to $\mathrm{y} 01, \mathrm{y} 2$ to y 02 , etc. when there are variables such as y 14 .
Currently subsetcol() and Select() are identical. One of these functions might be withdrawn in the future.

## Author(s)

T. W. Yee.

## See Also

vglm, cbind, subset, formula, fill.

## Examples

```
Pneumo <- pneumo
colnames(Pneumo) <- c("y1", "y2", "y3", "x2") # The "y" variables are response
Pneumo$x1 <- 1; Pneumo$x3 <- 3; Pneumo$x <- 0; Pneumo$x4 <- 4 # Add these
Select(data = Pneumo) # Same as with(Pneumo, cbind(y1, y2, y3))
```

```
Select(Pneumo, "x")
Select(Pneumo, "x", sort = FALSE, as.char = TRUE)
Select(Pneumo, "x", exclude = "x1")
Select(Pneumo, "x", exclude = "x1", as.char = TRUE)
Select(Pneumo, c("x", "y"))
Select(Pneumo, "z") # Now returns a NULL
Select(Pneumo, " ") # Now returns a NULL
Select(Pneumo, prefix = TRUE, as.formula = TRUE)
Select(Pneumo, "x", exclude = c("x3", "x1"), as.formula = TRUE,
    lhs = "cbind(y1, y2, y3)", rhs = "0")
Select(Pneumo, "x", exclude = "x1", as.formula = TRUE, as.char = TRUE,
    lhs = "cbind(y1, y2, y3)", rhs = "0")
# Now a 'real' example:
Huggins89table1 <- transform(Huggins89table1, x3.tij = t01)
tab1 <- subset(Huggins89table1,
                rowSums(Select(Huggins89table1, "y")) > 0)
# Same as
# subset(Huggins89table1, y1 + y2 + y3 + y4 + y5 + y6 + y7 + y8 + y9 + y10 > 0)
# Long way to do it:
fit.th <-
    vglm(cbind(y01, y02, y03, y04, y05, y06, y07, y08, y09, y10) ~ x2 + x3.tij,
        xij = list(x3.tij ~ t01 + t02 + t03 + t04 + t05 + t06 + t07 + t08 +
                            t09 + t10 - 1),
        posbernoulli.t(parallel.t = TRUE ~ x2 + x3.tij),
        data = tab1, trace = TRUE,
        form2 = ~ x2 + x3.tij + t01 + t02 + t03 + t04 + t05 + t06 + t07 + t08 +
            t09 + t10)
# Short way to do it:
Fit.th <- vglm(Select(tab1, "y") ~ x2 + x3.tij,
    xij = list(Select(tab1, "t", as.formula = TRUE,
                                    sort = FALSE, lhs = "x3.tij", rhs = "0")),
    posbernoulli.t(parallel.t = TRUE ~ x2 + x3.tij),
    data = tab1, trace = TRUE,
    form2 = Select(tab1, prefix = TRUE, as.formula = TRUE))
```

seq2binomial

The Two-stage Sequential Binomial Distribution Family Function

## Description

Estimation of the probabilities of a two-stage binomial distribution.

## Usage

```
seq2binomial(lprob1 = "logitlink", lprob2 = "logitlink",
    iprob1 = NULL, iprob2 = NULL,
    parallel = FALSE, zero = NULL)
```


## Arguments

lprob1, lprob2 Parameter link functions applied to the two probabilities, called $p$ and $q$ below. See Links for more choices.
iprob1, iprob2 Optional initial value for the first and second probabilities respectively. A NULL means a value is obtained in the initialize slot.
parallel, zero Details at Links. If parallel = TRUE then the constraint also applies to the intercept.

## Details

This VGAM family function fits the model described by Crowder and Sweeting (1989) which is described as follows. Each of $m$ spores has a probability $p$ of germinating. Of the $y_{1}$ spores that germinate, each has a probability $q$ of bending in a particular direction. Let $y_{2}$ be the number that bend in the specified direction. The probability model for this data is $P\left(y_{1}, y_{2}\right)=$

$$
\binom{m}{y_{1}} p^{y_{1}}(1-p)^{m-y_{1}}\binom{y_{1}}{y_{2}} q^{y_{2}}(1-q)^{y_{1}-y_{2}}
$$

for $0<p<1,0<q<1, y_{1}=1, \ldots, m$ and $y_{2}=1, \ldots, y_{1}$. Here, $p$ is prob1, $q$ is prob2.
Although the Authors refer to this as the bivariate binomial model, I have named it the (two-stage) sequential binomial model. Fisher scoring is used.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

The response must be a two-column matrix of sample proportions corresponding to $y_{1}$ and $y_{2}$. The $m$ values should be inputted with the weights argument of vglm and vgam. The fitted value is a two-column matrix of estimated probabilities $p$ and $q$. A common form of error is when there are no trials for $y_{1}$, e.g., if mvector below has some values which are zero.

## Author(s)

Thomas W. Yee

## References

Crowder, M. and Sweeting, T. (1989). Bayesian inference for a bivariate binomial distribution. Biometrika, 76, 599-603.

## See Also

binomialff, cfibrosis.

## Examples

```
sdata <- data.frame(mvector = round(rnorm(nn <- 100, m = 10, sd = 2)),
    x2 = runif(nn))
sdata <- transform(sdata, prob1 = logitlink(+2 - x2, inverse = TRUE),
    prob2 = logitlink(-2 + x2, inverse = TRUE))
sdata <- transform(sdata, successes1 = rbinom(nn, size = mvector, prob = prob1))
sdata <- transform(sdata, successes2 = rbinom(nn, size = successes1, prob = prob2))
sdata <- transform(sdata, y1 = successes1 / mvector)
sdata <- transform(sdata, y2 = successes2 / successes1)
fit <- vglm(cbind(y1, y2) ~ x2, seq2binomial, weight = mvector,
    data = sdata, trace = TRUE)
coef(fit)
coef(fit, matrix = TRUE)
head(fitted(fit))
head(depvar(fit))
head(weights(fit, type = "prior")) # Same as with(sdata, mvector)
# Number of first successes:
head(depvar(fit)[, 1] * c(weights(fit, type = "prior")))
# Number of second successes:
head(depvar(fit)[, 2] * c(weights(fit, type = "prior")) *
    depvar(fit)[, 1])
```


## Description

Sets up smart prediction in one of two modes: "write" and "read".

## Usage

setup.smart(mode.arg, smart.prediction = NULL, max.smart = 30)

## Arguments

mode.arg mode.arg must be "write" or "read". If in "read" mode then smart. prediction must be assigned the data structure .smart. prediction that was created while fitting. This is stored in object@smart. prediction or object\$smart. prediction where object is the name of the fitted object.
smart.prediction
If in "read" mode then smart. prediction must be assigned the list of data dependent parameters, which is stored on the fitted object. Otherwise, smart. prediction is ignored.
max.smart max.smart is the initial length of the list .smart.prediction. It is not important because . smart. prediction is made larger if needed.

## Details

This function is only required by programmers writing a modelling function such as 1 m and glm, or a prediction functions of such, e.g., predict. 1 m . The function setup.smart operates by mimicking the operations of a first-in first-out stack (better known as a queue).

## Value

Nothing is returned.

## Side Effects

In "write" mode .smart. prediction in smartpredenv is assigned an empty list with max. smart components. In "read" mode . smart. prediction in smartpredenv is assigned smart. prediction. Then .smart. prediction. counter in smartpredenv is assigned the value 0 , and .smart. prediction.mode and .max. smart are written to smartpredenv too.

## See Also

lm, predict.lm.

## Examples

```
## Not run:
setup.smart("write") # Put at the beginning of lm
## End(Not run)
## Not run: # Put at the beginning of predict.lm
setup.smart("read", smart.prediction = object$smart.prediction)
## End(Not run)
```

Simplex Simplex Distribution

## Description

Density function, and random generation for the simplex distribution.

## Usage

dsimplex(x, mu = 0.5, dispersion = 1, log = FALSE)
rsimplex(n, mu $=0.5$, dispersion $=1$ )

## Arguments

$x \quad$ Vector of quantiles. The support of the distribution is the interval $(0,1)$.
mu , dispersion Mean and dispersion parameters. The former lies in the interval $(0,1)$ and the latter is positive.
$n, \log \quad$ Same usage as runif.

## Details

The VGAM family function simplex fits this model; see that online help for more information. For rsimplex() the rejection method is used; it may be very slow if the density is highly peaked, and will fail if the density asymptotes at the boundary.

## Value

dsimplex(x) gives the density function, rsimplex(n) gives $n$ random variates.

## Author(s)

T. W. Yee

## See Also

simplex.

## Examples

```
sigma <- c(4, 2, 1) # Dispersion parameter
mymu <- c(0.1, 0.5, 0.7); xxx <- seq(0, 1, len = 501)
## Not run: par(mfrow = c(3, 3)) # Figure 2.1 of Song (2007)
for (iii in 1:3)
    for (jjj in 1:3) {
        plot(xxx, dsimplex(xxx, mymu[jjj], sigma[iii]),
                type = "l", col = "blue", xlab = "", ylab = "", main =
                paste("mu = ", mymu[jjj], ", sigma = ", sigma[iii], sep = "")) }
## End(Not run)
```

simplex Simplex Distribution Family Function

## Description

The two parameters of the univariate standard simplex distribution are estimated by full maximum likelihood estimation.

## Usage

```
simplex(lmu = "logitlink", lsigma = "loglink", imu = NULL, isigma = NULL,
    imethod = 1, ishrinkage = 0.95, zero = "sigma")
```


## Arguments

lmu, lsigma Link function for mu and sigma. See Links for more choices.
imu, isigma Optional initial values for mu and sigma. A NULL means a value is obtained internally.
imethod, ishrinkage, zero
See CommonVGAMffArguments for information.

## Details

The probability density function can be written

$$
f(y ; \mu, \sigma)=\left[2 \pi \sigma^{2}(y(1-y))^{3}\right]^{-0.5} \exp \left[-0.5(y-\mu)^{2} /\left(\sigma^{2} y(1-y) \mu^{2}(1-\mu)^{2}\right)\right]
$$

for $0<y<1,0<\mu<1$, and $\sigma>0$. The mean of $Y$ is $\mu$ (called mu, and returned as the fitted values).
The second parameter, sigma, of this standard simplex distribution is known as the dispersion parameter. The unit variance function is $V(\mu)=\mu^{3}(1-\mu)^{3}$. Fisher scoring is applied to both parameters.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

This distribution is potentially useful for dispersion modelling. Numerical problems may occur when mu is very close to 0 or 1 .

## Author(s)

T. W. Yee

## References

Jorgensen, B. (1997). The Theory of Dispersion Models. London: Chapman \& Hall
Song, P. X.-K. (2007). Correlated Data Analysis: Modeling, Analytics, and Applications. Springer.

## See Also

dsimplex, dirichlet, rig, binomialff.

## Examples

```
sdata <- data.frame(x2 = runif(nn <- 1000))
sdata <- transform(sdata, eta1 = 1 + 2 * x2,
    eta2 = 1 - 2 * x2)
sdata <- transform(sdata, y = rsimplex(nn, mu = logitlink(eta1, inverse = TRUE),
    dispersion = exp(eta2)))
```

```
(fit <- vglm(y ~ x2, simplex(zero = NULL), data = sdata, trace = TRUE))
coef(fit, matrix = TRUE)
summary(fit)
```

```
simulate.vlm
```

Simulate Responses for VGLMs and VGAMs

## Description

Simulate one or more responses from the distribution corresponding to a fitted model object.

## Usage

```
## S3 method for class 'vlm'
simulate(object, nsim = 1, seed = NULL, ...)
```


## Arguments

object an object representing a fitted model. Usually an object of class vglm-class or vgam-class.
nsim, seed Same as simulate.
... additional optional arguments.

## Details

This is a methods function for simulate and hopefully should behave in a very similar manner. Only VGAM family functions with a simslot slot have been implemented for simulate.

## Value

Similar to simulate. Note that many VGAM family functions can handle multiple responses. This can result in a longer data frame with more rows (nsim multiplied by $n$ rather than the ordinary n ). In the future an argument may be available so that there is always n rows no matter how many responses were inputted.

## Warning

With multiple response and/or multivariate responses, the order of the elements may differ. For some VGAM families, the order is $n \times N \times F$, where $n$ is the sample size, $N$ is nsim and $F$ is ncol (fitted(vglmObject)). For other VGAM families, the order is $n \times F \times N$. An example of each is given below.

## See Also

Currently the VGAM family functions with a simslot slot are: alaplace1, alaplace2, betabinomial, betabinomialff, betaR, betaff, biamhcop, bifrankcop, bilogistic, binomialff, binormal, binormalcop, biclaytoncop, cauchy, cauchy1, chisq, dirichlet, dagum, erlang, exponential, bifgmcop, fisk, gamma1, gamma2, gammaR, gengamma.stacy, geometric, gompertz, gumbelII, hzeta, inv.lomax, inv. paralogistic, kumar, lgamma1, lgamma3, lindley, lino, logff, logistic1, logistic, lognormal, lomax, makeham, negbinomial, negbinomial.size, paralogistic, perks, poissonff, posnegbinomial, posnormal, pospoisson, polya, polyaR, posbinomial, rayleigh, riceff, simplex, sinmad, slash, studentt, studentt2, studentt3, triangle, uninormal, yulesimon, zageometric, zageometricff, zanegbinomial, zanegbinomialff, zapoisson, zapoissonff, zigeometric, zigeometricff, zinegbinomial, zipf, zipoisson, zipoissonff.

See also RNG about random number generation in R, vglm, vgam for model fitting.

## Examples

```
nn <- 10; mysize <- 20; set.seed(123)
bdata <- data.frame(x2 = rnorm(nn))
bdata <- transform(bdata,
    y1 = rbinom(nn, size = mysize, p = logitlink(1+x2, inverse = TRUE)),
    y2 = rbinom(nn, size = mysize, p = logitlink(1+x2, inverse = TRUE)),
    f1 = factor(as.numeric(rbinom(nn, size = 1,
    p = logitlink(1+x2, inverse = TRUE)))))
(fit1 <- vglm(cbind(y1, aaa = mysize - y1) ~ x2, # Matrix response (2-colns)
            binomialff, data = bdata))
(fit2 <- vglm(f1 ~ x2, binomialff, model = TRUE, data = bdata)) # Factor response
set.seed(123); simulate(fit1, nsim = 8)
set.seed(123); c(simulate(fit2, nsim = 3)) # Use c() when model = TRUE
# An n x N x F example
set.seed(123); n <- 100
bdata <- data.frame(x2 = runif(n), x3 = runif(n))
bdata <- transform(bdata, y1 = rnorm(n, 1 + 2 * x2),
    y2 = rnorm(n, 3 + 4 * x2))
fit1 <- vglm(cbind(y1, y2) ~ x2, binormal(eq.sd = TRUE), data = bdata)
nsim <- 1000 # Number of simulations for each observation
my.sims <- simulate(fit1, nsim = nsim)
dim(my.sims) # A data frame
aaa <- array(unlist(my.sims), c(n, nsim, ncol(fitted(fit1)))) # n by N by F
summary(rowMeans(aaa[, , 1]) - fitted(fit1)[, 1]) # Should be all 0s
summary(rowMeans(aaa[, , 2]) - fitted(fit1)[, 2]) # Should be all 0s
# An n x F x N example
n <- 100; set.seed(111); nsim <- 1000
zdata <- data.frame(x2 = runif(n))
zdata <- transform(zdata, lambda1 = loglink(-0.5 + 2 * x2, inverse = TRUE),
                                lambda2 = loglink( 0.5 + 2 * x2, inverse = TRUE),
                                pstr01 = logitlink( 0, inverse = TRUE),
                                pstr02 = logitlink(-1.0, inverse = TRUE))
zdata <- transform(zdata, y1 = rzipois(n, lambda = lambda1, pstr0 = pstr01),
    y2 = rzipois(n, lambda = lambda2, pstr0 = pstr02))
```

```
zip.fit <- vglm(cbind(y1, y2) ~ x2, zipoissonff, data = zdata, crit = "coef")
my.sims <- simulate(zip.fit, nsim = nsim)
dim(my.sims) # A data frame
aaa <- array(unlist(my.sims), c(n, ncol(fitted(zip.fit)), nsim)) # n by F by N
summary(rowMeans(aaa[, 1, ]) - fitted(zip.fit)[, 1]) # Should be all 0s
summary(rowMeans(aaa[, 2, ]) - fitted(zip.fit)[, 2]) # Should be all 0s
```


## Sinmad The Singh-Maddala Distribution

## Description

Density, distribution function, quantile function and random generation for the Singh-Maddala distribution with shape parameters a and q, and scale parameter scale.

## Usage

```
dsinmad(x, scale = 1, shape1.a, shape3.q, log = FALSE)
psinmad(q, scale = 1, shape1.a, shape3.q, lower.tail = TRUE, log.p = FALSE)
qsinmad(p, scale = 1, shape1.a, shape3.q, lower.tail = TRUE, log.p = FALSE)
rsinmad(n, scale = 1, shape1.a, shape3.q)
```


## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$n \quad$ number of observations. If length $(n)>1$, the length is taken to be the number required.
shape1.a, shape3.q
shape parameters.
scale scale parameter.
$\log \quad$ Logical. If log = TRUE then the logarithm of the density is returned.
lower.tail, log.p
Same meaning as in pnorm or qnorm.

## Details

See sinmad, which is the VGAM family function for estimating the parameters by maximum likelihood estimation.

## Value

dsinmad gives the density, psinmad gives the distribution function, qsinmad gives the quantile function, and rsinmad generates random deviates.

## Note

The Singh-Maddala distribution is a special case of the 4-parameter generalized beta II distribution.

## Author(s)

T. W. Yee and Kai Huang

## References

Kleiber, C. and Kotz, S. (2003). Statistical Size Distributions in Economics and Actuarial Sciences, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

sinmad, genbetaII.

## Examples

```
sdata <- data.frame(y = rsinmad(n = 3000, scale = exp(2),
    shape1 = exp(1), shape3 = exp(1)))
fit <- vglm(y ~ 1, sinmad(lss = FALSE, ishape1.a = 2.1), data = sdata,
    trace = TRUE, crit = "coef")
coef(fit, matrix = TRUE)
Coef(fit)
```

sinmad Singh-Maddala Distribution Family Function

## Description

Maximum likelihood estimation of the 3-parameter Singh-Maddala distribution.

## Usage

```
sinmad(lscale = "loglink", lshape1.a = "loglink", lshape3.q = "loglink",
    iscale = NULL, ishape1.a = NULL, ishape3.q = NULL, imethod = 1,
    lss = TRUE, gscale = exp(-5:5), gshape1.a = exp(-5:5),
    gshape3.q = exp(-5:5), probs.y = c(0.25, 0.5, 0.75),
    zero = "shape")
```


## Arguments

lss See CommonVGAMffArguments for important information.
lshape1.a, lscale, lshape3.q
Parameter link functions applied to the (positive) parameters $a$, scale, and $q$. See Links for more choices.
iscale, ishape1.a, ishape3.q, imethod, zero
See CommonVGAMffArguments for information. For imethod $=2$ a good initial value for ishape3. $q$ is needed to obtain good estimates for the other parameters.
gscale, gshape1.a, gshape3.q
See CommonVGAMffArguments for information.
probs.y See CommonVGAMffArguments for information.

## Details

The 3-parameter Singh-Maddala distribution is the 4-parameter generalized beta II distribution with shape parameter $p=1$. It is known under various other names, such as the Burr XII (or just the Burr distribution), Pareto IV, beta-P, and generalized log-logistic distribution. More details can be found in Kleiber and Kotz (2003).
Some distributions which are special cases of the 3-parameter Singh-Maddala are the Lomax ( $a=$ $1)$, Fisk $(q=1)$, and paralogistic $(a=q)$.
The Singh-Maddala distribution has density

$$
f(y)=a q y^{a-1} /\left[b^{a}\left\{1+(y / b)^{a}\right\}^{1+q}\right]
$$

for $a>0, b>0, q>0, y \geq 0$. Here, $b$ is the scale parameter scale, and the others are shape parameters. The cumulative distribution function is

$$
F(y)=1-\left[1+(y / b)^{a}\right]^{-q} .
$$

The mean is

$$
E(Y)=b \Gamma(1+1 / a) \Gamma(q-1 / a) / \Gamma(q)
$$

provided $-a<1<a q$; these are returned as the fitted values. This family function handles multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

See the notes in genbetaII.

## Author(s)

T. W. Yee

## References

Kleiber, C. and Kotz, S. (2003). Statistical Size Distributions in Economics and Actuarial Sciences, Hoboken, NJ, USA: Wiley-Interscience.

## See Also

Sinmad, genbetaII, betaII, dagum, fisk, inv.lomax, lomax, paralogistic, inv.paralogistic, simulate.vlm.

## Examples

```
sdata <- data.frame \((y=r s i n m a d(n=1000\), shape1 \(=\exp (1)\),
                        scale \(=\exp (2)\), shape \(3=\exp (0))\) )
fit <- vglm(y ~ 1, sinmad(lss = FALSE), data = sdata, trace = TRUE)
fit <- vglm(y ~ 1, sinmad(lss = FALSE, ishape1.a \(=\exp (1))\),
            data \(=\) sdata, trace \(=\) TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
summary (fit)
\# Harder problem (has the shape3.q parameter going to infinity)
set.seed(3)
sdata <- data.frame (y1 = rbeta(1000, 6, 6))
\# hist(with(sdata, y1))
if (FALSE) \{
\# These struggle
        fit1 <- vglm(y1 ~ 1, sinmad(lss = FALSE), data = sdata, trace = TRUE)
        fit1 <- vglm(y1 ~ 1, sinmad(lss = FALSE), data = sdata, trace = TRUE,
                    crit = "coef")
        Coef(fit1)
\}
\# Try this remedy:
fit2 <- vglm(y1 ~ 1, data = sdata, trace \(=\) TRUE, stepsize \(=0.05\), maxit \(=99\),
                                    sinmad(lss = FALSE, ishape3.q = 3, lshape3.q = "logloglink"))
coef(fit2, matrix = TRUE)
Coef(fit2)
```


## Description

Density and random generation for the Skellam distribution.

## Usage

dskellam(x, mu1, mu2, log = FALSE)
rskellam(n, mu1, mu2)

## Arguments

| $x$ | vector of quantiles. |
| :--- | :--- |
| $n$ | number of observations. Same as runif. |
| $m u 1$, mu2 | See skellam |
| log | $\cdot$ |
|  | Logical; if TRUE, the logarithm is returned. |

## Details

See skellam, the VGAM family function for estimating the parameters, for the formula of the probability density function and other details.

## Value

dskellam gives the density, and rskellam generates random deviates.

## Warning

Numerical problems may occur for data if $\mu_{1}$ and/or $\mu_{2}$ are large. The normal approximation for this case has not been implemented yet.

## See Also

skellam, dpois.

## Examples

```
## Not run: mu1 <- 1; mu2 <- 2; x <- (-7):7
plot(x, dskellam(x, mu1, mu2), type = "h", las = 1, col = "blue",
    main = paste("Density of Skellam distribution with mu1 = ", mu1,
        " and mu2 = ", mu2, sep = ""))
## End(Not run)
```

skellam
Skellam Distribution Family Function

## Description

Estimates the two parameters of a Skellam distribution by maximum likelihood estimation.

## Usage

```
skellam(lmu1 = "loglink", lmu2 = "loglink", imu1 = NULL, imu2 = NULL,
    nsimEIM = 100, parallel = FALSE, zero = NULL)
```


## Arguments

lmu1, lmu2 Link functions for the $\mu_{1}$ and $\mu_{2}$ parameters. See Links for more choices and for general information.
imu1, imu2 Optional initial values for the parameters. See CommonVGAMffArguments for more information. If convergence failure occurs (this VGAM family function seems to require good initial values) try using these arguments.
nsimEIM, parallel, zero
See CommonVGAMffArguments for information. In particular, setting parallel=TRUE will constrain the two means to be equal.

## Details

The Skellam distribution models the difference between two independent Poisson distributions (with means $\mu_{j}$, say). It has density function

$$
f\left(y ; \mu_{1}, \mu_{2}\right)=\left(\frac{\mu_{1}}{\mu_{2}}\right)^{y / 2} \exp \left(-\mu_{1}-\mu_{2}\right) I_{|y|}\left(2 \sqrt{\mu_{1} \mu_{2}}\right)
$$

where $y$ is an integer, $\mu_{1}>0, \mu_{2}>0$. Here, $I_{v}$ is the modified Bessel function of the first kind with order $v$.

The mean is $\mu_{1}-\mu_{2}$ (returned as the fitted values), and the variance is $\mu_{1}+\mu_{2}$. Simulated Fisher scoring is implemented.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Warning

This VGAM family function seems fragile and very sensitive to the initial values. Use very cautiously!!

## Note

Numerical problems may occur for data if $\mu_{1}$ and/or $\mu_{2}$ are large.

## References

Skellam, J. G. (1946). The frequency distribution of the difference between two Poisson variates belonging to different populations. Journal of the Royal Statistical Society, Series A, 109, 296.

## See Also

dskellam, dpois, poissonff.

## Examples

```
## Not run:
sdata <- data.frame(x2 = runif(nn <- 1000))
sdata <- transform(sdata, mu1 = exp(1 + x2), mu2 = exp(1 + x2))
sdata <- transform(sdata, y = rskellam(nn, mu1, mu2))
fit1 <- vglm(y ~ x2, skellam, data = sdata, trace = TRUE, crit = "coef")
fit2 <- vglm(y ~ x2, skellam(parallel = TRUE), data = sdata, trace = TRUE)
coef(fit1, matrix = TRUE)
coef(fit2, matrix = TRUE)
summary(fit1)
# Likelihood ratio test for equal means:
pchisq(2 * (logLik(fit1) - logLik(fit2)),
    df = df.residual(fit2) - df.residual(fit1), lower.tail = FALSE)
lrtest(fit1, fit2) # Alternative
```

\#\# End(Not run)
skewnorm Skew-Normal Distribution

## Description

Density and random generation for the univariate skew-normal distribution.

## Usage

dskewnorm(x, location $=0$, scale $=1$, shape $=0$, log $=$ FALSE)
rskewnorm(n, location $=0$, scale $=1$, shape $=0$ )

## Arguments

x
n number of observations. Same as runif.
location The location parameter $\xi$. A vector.
scale The scale parameter $\omega$. A positive vector.
shape The shape parameter. It is called $\alpha$ in skewnormal.
log Logical. If log=TRUE then the logarithm of the density is returned.

## Details

See skewnormal, which currently only estimates the shape parameter. More generally here, $Z=$ $\xi+\omega Y$ where $Y$ has a standard skew-normal distribution (see skewnormal), $\xi$ is the location parameter and $\omega$ is the scale parameter.

## Value

dskewnorm gives the density, rskewnorm generates random deviates.

Note
The default values of all three parameters corresponds to the skew-normal being the standard normal distribution.

## Author(s)

T. W. Yee

## References

http://tango.stat.unipd.it/SN.

## See Also

skewnormal.

## Examples

```
## Not run: N <- 200 # Grid resolution
shape <- 7; x <- seq(-4, 4, len = N)
plot(x, dskewnorm(x, shape = shape), type = "l", col = "blue", las = 1,
    ylab = "", lty = 1, lwd = 2)
abline(v = 0, h = 0, col = "grey")
lines(x, dnorm(x), col = "orange", lty = 2, lwd = 2)
legend("topleft", leg = c(paste("Blue = dskewnorm(x, ", shape,")", sep = ""),
            "Orange = standard normal density"), lty = 1:2, lwd = 2,
            col = c("blue", "orange"))
## End(Not run)
```

skewnormal Univariate Skew-Normal Distribution Family Function

## Description

Maximum likelihood estimation of the shape parameter of a univariate skew-normal distribution.

## Usage

skewnormal(lshape = "identitylink", ishape = NULL, nsimEIM = NULL)

## Arguments

lshape, ishape, nsimEIM

> See Links and CommonVGAMffArguments.

## Details

The univariate skew-normal distribution has a density function that can be written

$$
f(y)=2 \phi(y) \Phi(\alpha y)
$$

where $\alpha$ is the shape parameter. Here, $\phi$ is the standard normal density and $\Phi$ its cumulative distribution function. When $\alpha=0$ the result is a standard normal distribution. When $\alpha=1$ it models the distribution of the maximum of two independent standard normal variates. When the absolute value of the shape parameter increases the skewness of the distribution increases. The limit as the shape parameter tends to positive infinity results in the folded normal distribution or half-normal distribution. When the shape parameter changes its sign, the density is reflected about $y=0$.
The mean of the distribution is $\mu=\alpha \sqrt{2 /\left(\pi\left(1+\alpha^{2}\right)\right)}$ and these are returned as the fitted values. The variance of the distribution is $1-\mu^{2}$. The Newton-Raphson algorithm is used unless the nsimEIM argument is used.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

It is well known that the EIM of Azzalini's skew-normal distribution is singular for skewness parameter tending to zero, and thus produces influential problems.

## Note

It is a good idea to use several different initial values to ensure that the global solution is obtained.
This family function will be modified (hopefully soon) to handle a location and scale parameter too.

## Author(s)

Thomas W. Yee

## References

Azzalini, A. A. (1985). A class of distributions which include the normal. Scandinavian Journal of Statistics, 12, 171-178.

Azzalini, A. and Capitanio, A. (1999). Statistical applications of the multivariate skew-normal distribution. Journal of the Royal Statistical Society, Series B, Methodological, 61, 579-602.

## See Also

skewnorm, uninormal, foldnormal.

## Examples

```
sdata <- data.frame(y1 = rskewnorm(nn <- 1000, shape = 5))
fit1 <- vglm(y1 ~ 1, skewnormal, data = sdata, trace = TRUE)
coef(fit1, matrix = TRUE)
head(fitted(fit1), 1)
with(sdata, mean(y1))
## Not run: with(sdata, hist(y1, prob = TRUE))
x <- with(sdata, seq(min(y1), max(y1), len = 200))
with(sdata, lines(x, dskewnorm(x, shape = Coef(fit1)), col = "blue"))
## End(Not run)
sdata <- data.frame(x2 = runif(nn))
sdata <- transform(sdata, y2 = rskewnorm(nn, shape = 1 + 2*x2))
fit2 <- vglm(y2 ~ x2, skewnormal, data = sdata, trace = TRUE, crit = "coef")
summary(fit2)
```


## Slash Slash Distribution

## Description

Density function, distribution function, and random generation for the slash distribution.

## Usage

dslash(x, mu = 0, sigma = 1, log = FALSE, smallno = .Machine\$double.eps*1000)
pslash(q, mu = 0, sigma $=1$, very.negative $=-10000$,
lower.tail $=$ TRUE, log.p = FALSE)
rslash(n, mu = 0, sigma = 1)

## Arguments

$x, q \quad$ vector of quantiles.
mu, sigma the mean and standard deviation of the univariate normal distribution.
log Logical. If TRUE then the logarithm of the density is returned.
very.negative Numeric, of length 1. A large negative value. For (q-mu)/sigma values less than this, the value 0 is returned because integrate tends to fail. A warning is issued. Similarly, if (q-mu)/sigma is greater than abs(very.negative) then 1 is returned with a warning.
smallno See slash.
lower.tail, log.p
Same meaning as in pnorm or qnorm.

## Details

See slash, the VGAM family function for estimating the two parameters by maximum likelihood estimation, for the formula of the probability density function and other details.
Function pslash uses a for () loop and integrate, meaning it's very slow. It may also be inaccurate for extreme values of $q$, and returns with 1 or 0 values when too extreme compared to very.negative.

## Value

dslash gives the density, and pslash gives the distribution function, rslash generates random deviates.

## Note

pslash is very slow.

## Author(s)

Thomas W. Yee and C. S. Chee

## See Also

slash.

## Examples

```
## Not run:
curve(dslash, col = "blue", ylab = "f(x)", -5, 5, ylim = c(0, 0.4), las = 1,
    main = "Standard slash, normal and Cauchy densities", lwd = 2)
curve(dnorm, col = "black", lty = 2, lwd = 2, add = TRUE)
curve(dcauchy, col = "orange", lty = 3, lwd = 2, add = TRUE)
legend("topleft", c("slash", "normal", "Cauchy"), lty = 1:3,
    col = c("blue","black","orange"), lwd = 2)
curve(pslash, col = "blue", -5, 5, ylim = 0:1)
pslash(c(-Inf, -20000, 20000, Inf)) # Gives a warning
## End(Not run)
```

slash

Slash Distribution Family Function

## Description

Estimates the two parameters of the slash distribution by maximum likelihood estimation.

## Usage

```
    slash(lmu = "identitylink", lsigma = "loglink",
    imu = NULL, isigma = NULL, gprobs.y = ppoints(8), nsimEIM = 250,
    zero = NULL, smallno = .Machine$double.eps*1000)
```


## Arguments

lmu, lsigma Parameter link functions applied to the $\mu$ and $\sigma$ parameters, respectively. See Links for more choices.
imu, isigma Initial values. A NULL means an initial value is chosen internally. See CommonVGAMffArguments for more information.
gprobs.y Used to compute the initial values for mu. This argument is fed into the probs argument of quantile to construct a grid, which is used to evaluate the loglikelihood. This must have values between 0 and 1 .
nsimEIM, zero See CommonVGAMffArguments for information.
smallno Small positive number, used to test for the singularity.

## Details

The standard slash distribution is the distribution of the ratio of a standard normal variable to an independent standard uniform $(0,1)$ variable. It is mainly of use in simulation studies. One of its properties is that it has heavy tails, similar to those of the Cauchy.
The general slash distribution can be obtained by replacing the univariate normal variable by a general normal $N(\mu, \sigma)$ random variable. It has a density that can be written as

$$
f(y)=\left\{\begin{array}{cl}
1 /(2 \sigma \sqrt{( } 2 \pi)) & \text { if } y=\mu \\
\left.\left.1-\exp \left(-\left(((y-\mu) / \sigma)^{2}\right) / 2\right)\right) /(\sqrt{( } 2 p i) \sigma((y-\mu) / \sigma)^{2}\right) & \text { ify }=\mu
\end{array}\right.
$$

where $\mu$ and $\sigma$ are the mean and standard deviation of the univariate normal distribution respectively.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

Fisher scoring using simulation is used. Convergence is often quite slow. Numerical problems may occur.

## Author(s)

T. W. Yee and C. S. Chee

## References

Johnson, N. L. and Kotz, S. and Balakrishnan, N. (1994). Continuous Univariate Distributions, 2nd edition, Volume 1, New York: Wiley.
Kafadar, K. (1982). A Biweight Approach to the One-Sample Problem Journal of the American Statistical Association, 77, 416-424.

## See Also

rslash, simulate.vlm.

## Examples

```
## Not run:
sdata <- data.frame(y = rslash(n = 1000, mu = 4, sigma = exp(2)))
fit <- vglm(y ~ 1, slash, data = sdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
summary(fit)
## End(Not run)
```


## sm.os

Defining O'Sullivan Spline Smooths in VGAM Formulas

## Description

This function represents an O-spline smooth term in a vgam formula and confers automatic smoothing parameter selection.

## Usage

```
sm.os(x, ..., niknots = 6, spar = -1, o.order = 2,
    alg.niknots = c("s", ".nknots.smspl")[1], all.knots = FALSE,
    ridge.adj = 1e-5, spillover = 0.01, maxspar = 1e12,
    outer.ok = FALSE, fixspar = FALSE)
```


## Arguments

x

niknots
alg.niknots
all.knots logical. If TRUE then all distinct points in $x$ are used as the interior knots. If FALSE (default) then a subset of $x[]$ is used, specifically $x[j]$ where the niknots indices are quantiles that are evenly spaced with respect to the argument probs-see quantile. If all.knots = FALSE and niknots = NULL then the argument alg. niknots is used to compute niknots.
spar, maxspar spar is a vector of smoothing parameters. Negative values mean that magic will choose initial values in order to do the optimization at each P-IRLS iteration. Positive values mean that they are used as initial values for magic. If fixspar = TRUE then spar should be assigned a vector of positive values (but having values less than maxspar); then the smoothing parameters will be fixed and magic will not be used.

| o.order | The order of the O'Sullivan penalzed spline. Any one value from $1: 4$ is accept- <br> able. The degree of the spline is $2 * 0$. order -1 , so that cubic splines are the <br> default. Setting o.order $=1$ results in a linear spline which is a piecewise linear <br> function. |
| :--- | :--- |
| ridge.adj | small positive number to stabilize linear dependencies among B-spline bases. <br> spillover <br> small and positive proportion of the range used on the outside of the boundary <br> values. This defines the endpoints $a$ and $b$ that cover the data $x_{i}$, i.e., we are <br> interested in the interval $[a, b]$ which contains all the abscissae. The interior <br> knots are strictly inside $(a, b)$. |
| outer.ok | Fed into the argument (by the same name) of splineDesign. <br> fixsparlogical. If TRUE then spar should be a vector with positive values and the <br> smoothing parameters are fixed at those values. If FALSE then spar contains <br> the initial values for the smoothing parameters, and magic is called to determine <br> (hopefully) some good values for the smoothing parameters. |

## Details

This function is currently used by vgam to allow automatic smoothing parameter selection based on O-splines to minimize an UBRE quantity. In contrast, s operates by having a prespecified amount of smoothing, e.g., its df argument. When the sample size is reasonably large this function is recommended over s also because backfitting is not required. This function therefore allows 2 ndgeneration VGAMs to be fitted (called G2-VGAMs, or Penalized-VGAMs).
This function should only be used with vgam. This function uses quantile to choose the knots, whereas sm.ps chooses equally-spaced knots. As Wand and Ormerod (2008) write, in most situations the differences will be minor, but it is possible for problems to arise for either strategy by constructing certain regression functions and predictor variable distributions. Any differences between O -splines and P -splines tend to be at the boundaries. O-splines have natural boundary constraints so that the solution is linear beyond the boundary knots.
Some arguments in decreasing order of precedence are: all.knots, niknots, alg.niknots.
Unlike s, which is symbolic and does not perform any smoothing itself, this function does compute the penalized spline when used by vgam-it creates the appropriate columns of the model matrix. When this function is used within vgam, automatic smoothing parameter selection is implemented by calling magic after the necessary link-ups are done.
By default this function centres the component function. This function is also smart; it can be used for smart prediction (Section 18.6 of Yee (2015)). Automatic smoothing parameter selection is performed using performance-oriented iteration whereby an optimization problem is solved at each IRLS iteration.

This function works better when the sample size is large, e.g., when in the hundreds, say.

## Value

A matrix with attributes that are (only) used by vgam. The number of rows of the matrix is length ( x ). The number of columns is a function of the number of interior knots $K$ and the order of the O-spline $m$ : $K+2 m-1$. In code, this is niknots $+2 *$ o. order -1 , or using sm. ps-like arguments, ps.int + degree -1 (where ps.int should be more generally interpreted as the number of intervals. The formula is the same as sm.ps.). It transpires then that sm.os and sm.ps are very similar.

## Warning

Being introduced into VGAM for the first time, this function (and those associated with it) should be used cautiously. Not all options are fully working or have been tested yet, and there are bound to be some bugs lurking around.

## Note

This function is currently under development and may change in the future.
One might try using this function with vglm so as to fit a regression spline, however, the default value of niknots will probably be too high for most data sets.

## Author(s)

T. W. Yee, with some of the essential R code coming from the appendix of Wand and Ormerod (2008).

## References

Wand, M. P. and Ormerod, J. T. (2008). On semiparametric regression with O'Sullivan penalized splines. Australian and New Zealand Journal of Statistics, 50(2): 179-198.

## See Also

vgam, sm.ps, s, smartpred, is.smart, summarypvgam, smooth.spline, splineDesign, bs, magic.

## Examples

```
sm.os(runif(20))
## Not run:
data("TravelMode", package = "AER") # Need to install "AER" first
air.df <- subset(TravelMode, mode == "air") # Form 4 smaller data frames
bus.df <- subset(TravelMode, mode == "bus")
trn.df <- subset(TravelMode, mode == "train")
car.df <- subset(TravelMode, mode == "car")
TravelMode2 <- data.frame(income = air.df$income,
    wait.air = air.df$wait - car.df$wait,
    wait.trn = trn.df$wait - car.df$wait,
    wait.bus = bus.df$wait - car.df$wait,
    gcost.air = air.df$gcost - car.df$gcost,
    gcost.trn = trn.df$gcost - car.df$gcost,
    gcost.bus = bus.df$gcost - car.df$gcost,
    wait = air.df$wait) # Value is unimportant
TravelMode2$mode <- subset(TravelMode, choice == "yes")$mode # The response
TravelMode2 <- transform(TravelMode2, incom.air = income, incom.trn = 0,
                                    incom.bus = 0)
set.seed(1)
TravelMode2 <- transform(TravelMode2,
                                    junkx2 = runif(nrow(TravelMode2)))
```

tfit2 <-

```
    vgam(mode ~ sm.os(gcost.air, gcost.trn, gcost.bus) + ns(junkx2, 4) +
            sm.os(incom.air, incom.trn, incom.bus) + wait ,
        crit = "coef",
        multinomial(parallel = FALSE ~ 1), data = TravelMode2,
        xij = list(sm.os(gcost.air, gcost.trn, gcost.bus) ~
            sm.os(gcost.air, gcost.trn, gcost.bus) +
            sm.os(gcost.trn, gcost.bus, gcost.air) +
            sm.os(gcost.bus, gcost.air, gcost.trn),
            sm.os(incom.air, incom.trn, incom.bus) ~
            sm.os(incom.air, incom.trn, incom.bus) +
            sm.os(incom.trn, incom.bus, incom.air) +
            sm.os(incom.bus, incom.air, incom.trn),
            wait ~ wait.air + wait.trn + wait.bus),
        form2 = ~ sm.os(gcost.air, gcost.trn, gcost.bus) +
            sm.os(gcost.trn, gcost.bus, gcost.air) +
            sm.os(gcost.bus, gcost.air, gcost.trn) +
            wait +
            sm.os(incom.air, incom.trn, incom.bus) +
            sm.os(incom.trn, incom.bus, incom.air) +
            sm.os(incom.bus, incom.air, incom.trn) +
            junkx2 + ns(junkx2, 4) +
            incom.air + incom.trn + incom.bus +
            gcost.air + gcost.trn + gcost.bus +
            wait.air + wait.trn + wait.bus)
par(mfrow = c(2, 2))
plot(tfit2, se = TRUE, lcol = "orange", scol = "blue", ylim = c(-4, 4))
summary(tfit2)
## End(Not run)
```


## Description

This function represents a P-spline smooth term in a vgam formula and confers automatic smoothing parameter selection.

## Usage

sm.ps(x, ..., ps.int = NULL, spar = -1, degree = 3, p.order = 2, ridge.adj $=1 \mathrm{e}-5$, spillover $=0.01$, maxspar $=1 \mathrm{e} 12$, outer.ok = FALSE, mux = NULL, fixspar = FALSE)

## Arguments

$x, \ldots \quad$ See sm.os.

| ps.int | the number of equally-spaced B-spline intervals. Note that the number of knots is equal to ps.int $+2 *$ degree +1 . The default, signified by NULL, means that the maximum of the value 7 and degree is chosen. This usually means 6 interior knots for big data sets. However, if this is too high compared to the length of $x$, then some adjustment is made. In the case where mux is assigned a numerical value (suggestions: some value between 1 and 2) then ceiling(mux * $\log (\operatorname{leng} t h(u n i q u e(x . i n d e x))))$ is used, where $x$. index is the combined data. No matter what, the above is not guaranteed to work on every data set. This argument may change in the future. See also argument mux. |
| :---: | :---: |
| spar, maxspar | See sm.os. |
| mux | numeric. If given, then this argument multiplies $\log ($ length(unique $(x))$ ) to obtain ps.int. If ps.int is given then this argument is ignored. |
| degree | degree of B-spline basis. Usually this will be 2 or 3 ; and the values 1 or 4 might possibly be used. |
| p.order | order of difference penalty ( 0 is the ridge penalty). |
| ridge.adj, spillover |  |
|  | See sm.os. |
| outer.ok, fixspar |  |
|  | See sm.os. |

## Details

This function can be used by vgam to allow automatic smoothing parameter selection based on P -splines and minimizing an UBRE quantity.

This function should only be used with vgam and is an alternative to sm.os; see that function for some details that also apply here.

## Value

A matrix with attributes that are (only) used by vgam. The number of rows of the matrix is length ( $x$ ) and the number of columns is ps.int + degree -1 . The latter is because the function is centred.

## Warning

See sm.os.

## Note

This function is currently under development and may change in the future. In particular, the default for ps.int is subject to change.

## Author(s)

B. D. Marx wrote the original function. Subsequent edits were made by T. W. Yee and C. Somchit.

## References

Eilers, P. H. C. and Marx, B. D. (1996). Flexible smoothing with B-splines and penalties (with comments and rejoinder). Statistical Science, 11(2): 89-121.

## See Also

sm.os, s, vgam, smartpred, is.smart, summarypvgam, splineDesign, bs, magic.

## Examples

```
sm.ps(runif(20))
sm.ps(runif(20), ps.int = 5)
## Not run:
data("TravelMode", package = "AER") # Need to install "AER" first
air.df <- subset(TravelMode, mode == "air") # Form 4 smaller data frames
bus.df <- subset(TravelMode, mode == "bus")
trn.df <- subset(TravelMode, mode == "train")
car.df <- subset(TravelMode, mode == "car")
TravelMode2 <- data.frame(income = air.df$income,
    wait.air = air.df$wait - car.df$wait,
    wait.trn = trn.df$wait - car.df$wait,
    wait.bus = bus.df$wait - car.df$wait,
    gcost.air = air.df$gcost - car.df$gcost,
    gcost.trn = trn.df$gcost - car.df$gcost,
    gcost.bus = bus.df$gcost - car.df$gcost,
    wait = air.df$wait) # Value is unimportant
TravelMode2$mode <- subset(TravelMode, choice == "yes")$mode # The response
TravelMode2 <- transform(TravelMode2, incom.air = income, incom.trn = 0,
                                    incom.bus = 0)
set.seed(1)
TravelMode2 <- transform(TravelMode2,
                                    junkx2 = runif(nrow(TravelMode2)))
tfit2 <-
    vgam(mode ~ sm.ps(gcost.air, gcost.trn, gcost.bus) + ns(junkx2, 4) +
                sm.ps(incom.air, incom.trn, incom.bus) + wait ,
        crit = "coef",
        multinomial(parallel = FALSE ~ 1), data = TravelMode2,
        xij = list(sm.ps(gcost.air, gcost.trn, gcost.bus) ~
            sm.ps(gcost.air, gcost.trn, gcost.bus) +
            sm.ps(gcost.trn, gcost.bus, gcost.air) +
            sm.ps(gcost.bus, gcost.air, gcost.trn),
            sm.ps(incom.air, incom.trn, incom.bus) ~
            sm.ps(incom.air, incom.trn, incom.bus) +
            sm.ps(incom.trn, incom.bus, incom.air) +
            sm.ps(incom.bus, incom.air, incom.trn),
            wait ~ wait.air + wait.trn + wait.bus),
        form2 = ~ sm.ps(gcost.air, gcost.trn, gcost.bus) +
            sm.ps(gcost.trn, gcost.bus, gcost.air) +
            sm.ps(gcost.bus, gcost.air, gcost.trn) +
            wait +
```

```
sm.ps(incom.air, incom.trn, incom.bus) +
sm.ps(incom.trn, incom.bus, incom.air) +
sm.ps(incom.bus, incom.air, incom.trn) +
junkx2 + ns(junkx2, 4) +
incom.air + incom.trn + incom.bus +
gcost.air + gcost.trn + gcost.bus +
wait.air + wait.trn + wait.bus)
par(mfrow = c(2, 2))
plot(tfit2, se = TRUE, lcol = "orange", scol = "blue", ylim = c(-4, 4))
summary(tfit2)
## End(Not run)
```

smart.expression SExpression for Smart Functions

## Description

smart. expression is an $S$ expression for a smart function to call itself. It is best if you go through it line by line, but most users will not need to know anything about it. It requires the primary argument of the smart function to be called " $x$ ".
The list component match. call must be assigned the value of match. call() in the smart function; this is so that the smart function can call itself later.

## See Also

```
match.call.
```


## Examples

```
print(sm.min2)
```

Determine What Mode the Smart Prediction is In

## Description

Determine which of three modes the smart prediction is currently in.

## Usage

smart.mode.is(mode.arg = NULL)

## Arguments

mode.arg a character string, either "read", "write" or "neutral".

## Details

Smart functions such as bs and poly need to know what mode smart prediction is in. If it is in "write" mode then the parameters are saved to .smart. prediction using put.smart. If in "read" mode then the parameters are read in using get.smart. If in "neutral" mode then the smart function behaves like an ordinary function.

## Value

If mode. arg is given, then either TRUE or FALSE is returned. If mode. arg is not given, then the mode ("neutral", "read" or "write") is returned. Usually, the mode is "neutral".

## See Also

```
put.smart, bs, poly.
```


## Examples

```
print(sm.min1)
smart.mode.is() # Returns "neutral"
smart.mode.is(smart.mode.is()) # Returns TRUE
```

```
smartpred
Smart Prediction
```


## Description

Data-dependent parameters in formula terms can cause problems in when predicting. The smartpred package saves data-dependent parameters on the object so that the bug is fixed. The 1 m and glm functions have been fixed properly. Note that the VGAM package by T. W. Yee automatically comes with smart prediction.

## Usage

```
sm.bs(x, df = NULL, knots = NULL, degree = 3, intercept = FALSE,
            Boundary.knots = range(x))
sm.ns(x, df = NULL, knots = NULL, intercept = FALSE,
            Boundary.knots = range(x))
    sm.poly(x, ..., degree = 1, coefs = NULL, raw = FALSE)
    sm.scale(x, center = TRUE, scale = TRUE)
```


## Arguments

x
The x argument is actually common to them all.
df, knots, intercept, Boundary. knots
See bs and/or ns.
degree, ..., coefs, raw
See poly.
center, scale See scale.

## Details

$R$ version 1.6.0 introduced a partial fix for the prediction problem because it does not work all the time, e.g., for terms such as $I(\operatorname{poly}(x, 3))$, poly $(c(\operatorname{scale}(x)), 3)$, bs (scale $(x), 3)$, scale $(\operatorname{scale}(x))$. See the examples below. Smart prediction, however, will always work.
The basic idea is that the functions in the formula are now smart, and the modelling functions make use of these smart functions. Smart prediction works in two ways: using smart. expression, or using a combination of put. smart and get. smart.

## Value

The usual value returned by bs, ns, poly and scale, When used with functions such as vglm the data-dependent parameters are saved on one slot component called smart. prediction.

## Side Effects

The variables .max.smart, .smart.prediction and .smart.prediction.counter are created while the model is being fitted. They are created in a new environment called smartpredenv. These variables are deleted after the model has been fitted. However, if there is an error in the model fitting function or the fitting model is killed (e.g., by typing control-C) then these variables will be left in smartpredenv. At the beginning of model fitting, these variables are deleted if present in smartpredenv.
During prediction, the variables .smart.prediction and .smart.prediction.counter are reconstructed and read by the smart functions when the model frame is re-evaluated. After prediction, these variables are deleted.

If the modelling function is used with argument smart = FALSE (e.g., vglm( $\ldots$, smart $=$ FALSE $)$ ) then smart prediction will not be used, and the results should match with the original $R$ functions.

## WARNING

The functions bs, ns, poly and scale are now left alone (from 2014-05 onwards) and no longer smart. They work via safe prediction. The smart versions of these functions have been renamed and they begin with "sm.".
The functions predict.bs and predict.ns are not smart. That is because they operate on objects that contain attributes only and do not have list components or slots. The function predict.poly is not smart.

## Author(s)

T. W. Yee and T. J. Hastie

## See Also

get.smart.prediction, get.smart, put.smart, smart.expression, smart.mode.is, setup.smart, wrapup. smart. For vgam in VGAM, sm.ps is important. Commonly used data-dependent functions include scale, poly, bs, ns. In R, the functions bs and ns are in the splines package, and this library is automatically loaded in because it contains compiled code that bs and ns call.
The functions vglm, vgam, rrvglm and cqo in T. W. Yee's VGAM package are examples of modelling functions that employ smart prediction.

## Examples

```
    # Create some data first
    n <- 20
    set.seed(86) # For reproducibility of the random numbers
    ldata <- data.frame(x2 = sort(runif(n)), y = sort(runif(n)))
    library("splines") # To get ns() in R
    # This will work for R 1.6.0 and later
    fit <- lm(y ~ ns(x2, df = 5), data = ldata)
    ## Not run:
    plot(y ~ x2, data = ldata)
    lines(fitted(fit) ~ x2, data = ldata)
    new.ldata <- data.frame(x2 = seq(0, 1, len = n))
    points(predict(fit, new.ldata) ~ x2, new.ldata, type = "b", col = 2, err = -1)
    ## End(Not run)
    # The following fails for R 1.6.x and later. It can be
    # made to work with smart prediction provided
    # ns is changed to sm.ns and scale is changed to sm.scale:
    fit1 <- lm(y ~ ns(scale(x2), df = 5), data = ldata)
    ## Not run:
    plot(y ~ x2, data = ldata, main = "Safe prediction fails")
    lines(fitted(fit1) ~ x2, data = ldata)
    points(predict(fit1, new.ldata) ~ x2, new.ldata, type = "b", col = 2, err = -1)
    ## End(Not run)
    # Fit the above using smart prediction
    ## Not run:
    library("VGAM") # The following requires the VGAM package to be loaded
    fit2 <- vglm(y ~ sm.ns(sm.scale(x2), df = 5), uninormal, data = ldata)
    fit2@smart.prediction
    plot(y ~ x2, data = ldata, main = "Smart prediction")
    lines(fitted(fit2) ~ x2, data = ldata)
    points(predict(fit2, new.ldata, type = "response") ~ x2, data = new.ldata,
        type = "b", col = 2, err = -1)
    ## End(Not run)
```

    specials Special Values or Quantities in a Fitted Object
    
## Description

Return any special values or quantities in a fitted object, and in particular in a VGLM fit

## Usage

specials(object, ...)
specialsvglm(object, ...)

## Arguments

object an object of class "vglm".
... any additional arguments, to future-proof this function.

## Details

This extractor function was motivated by GAIT regression (Yee and Ma, 2020) where the values from three disjoint sets are referred to as special. More generally, S4 methods functions can be written so that specials() will work on any S4 object, where what is called special depends on the methodology at hand.

## Value

Returns any 'special' values or quantities associated with a fitted regression model. This is often something simple such as a list or a vector.

## References

Yee, T. W. and Ma, C. (2020). Generally-altered, -inflated and -truncated regression, with application to heaped and seeped count data. In preparation.

## See Also

```
vglm, vglm-class, inflated, altered, truncated.
```


## Examples

```
abdata <- data.frame(y = 0:7, w = c(182, 41, 12, 2, 2, 0, 0, 1))
fit1 <- vglm(y ~ 1, gaitpoisson.mix(inflate = 1), abdata,
    weight = w, subset = w > 0)
specials(fit1)
```

    sratio Ordinal Regression with Stopping Ratios
    
## Description

Fits a stopping ratio logit/probit/cloglog/cauchit/... regression model to an ordered (preferably) factor response.

## Usage

```
sratio(link = "logitlink", parallel = FALSE, reverse = FALSE,
    zero = NULL, whitespace = FALSE)
```


## Arguments

link Link function applied to the $M$ stopping ratio probabilities. See Links for more choices.
parallel A logical, or formula specifying which terms have equal/unequal coefficients.
reverse Logical. By default, the stopping ratios used are $\eta_{j}=\operatorname{logit}(P[Y=j \mid Y \geq j])$ for $j=1, \ldots, M$. If reverse is TRUE, then $\eta_{j}=\operatorname{logit}(P[Y=j+1 \mid Y \leq j+1])$ will be used.
zero Can be an integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The values must be from the set $\{1,2, \ldots, M\}$. The default value means none are modelled as intercept-only terms.
whitespace See CommonVGAMffArguments for information.

## Details

In this help file the response $Y$ is assumed to be a factor with ordered values $1,2, \ldots, M+1$, so that $M$ is the number of linear/additive predictors $\eta_{j}$.
There are a number of definitions for the continuation ratio in the literature. To make life easier, in the VGAM package, we use continuation ratios (see cratio) and stopping ratios. Continuation ratios deal with quantities such as logitlink ( $P[Y>j \mid Y>=j]$ ).

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Warning

No check is made to verify that the response is ordinal if the response is a matrix; see ordered.

## Note

The response should be either a matrix of counts (with row sums that are all positive), or a factor. In both cases, the y slot returned by vglm/vgam/rrvglm is the matrix of counts.

For a nominal (unordered) factor response, the multinomial logit model (multinomial) is more appropriate.
Here is an example of the usage of the parallel argument. If there are covariates $\times 1$, $x 2$ and $x 3$, then parallel $=$ TRUE $\sim x 1+x 2-1$ and parallel $=$ FALSE $\sim x 3$ are equivalent. This would constrain the regression coefficients for $x 1$ and $\times 2$ to be equal; those of the intercepts and $\times 3$ would be different.

## Author(s)

Thomas W. Yee

## References

Agresti, A. (2013). Categorical Data Analysis, 3rd ed. Hoboken, NJ, USA: Wiley.
Simonoff, J. S. (2003). Analyzing Categorical Data, New York, USA: Springer-Verlag.
McCullagh, P. and Nelder, J. A. (1989). Generalized Linear Models, 2nd ed. London: Chapman \& Hall.
Yee, T. W. (2010). The VGAM package for categorical data analysis. Journal of Statistical Software, 32, 1-34. http://www.jstatsoft.org/v32/i10/.

## See Also

cratio, acat, cumulative, multinomial, margeff, pneumo, logitlink, probitlink, clogloglink, cauchitlink.

## Examples

```
pneumo <- transform(pneumo, let = log(exposure.time))
(fit <- vglm(cbind(normal, mild, severe) ~ let,
    sratio(parallel = TRUE), data = pneumo))
coef(fit, matrix = TRUE)
constraints(fit)
predict(fit)
predict(fit, untransform = TRUE)
```

```
step4
```

Choose a model by AIC in a Stepwise Algorithm

## Description

Select a formula-based model by AIC.

## Usage

step4(object, ...)
step4vglm(object, scope, direction = c("both", "backward", "forward"), trace $=1$, keep $=$ NULL, steps $=1000, \mathrm{k}=2, \ldots$ )

## Arguments

object an object of class "vglm". This is used as the initial model in the stepwise search.
scope See step.
direction See step.
trace, keep See step.
steps, k See step.
$\ldots \quad$ any additional arguments to extractAIC.vglm, drop1.vglm and add1.vglm.

## Details

This function is a direct adaptation of step for vglm-class objects. Since step is not generic, the name step4() was adopted and it is generic, as well as being S4 rather than S3. It is the intent that this function should work as similar as possible to step.
Internally, the methods function for vglm-class objects calls add1.vglm and drop1.vglm repeatedly.

## Value

The results are placed in the post slot of the stepwise-selected model that is returned. There are up to two additional components. There is an "anova" component corresponding to the steps taken in the search, as well as a "keep" component if the keep= argument was supplied in the call.

## Warning

In general, the same warnings in drop1.glm and drop1.vglm apply here.

## See Also

add1.vglm, drop1.vglm, vglm, add1.glm, drop1.glm, backPain2, step, update.

## Examples

```
data("backPain2", package = "VGAM")
summary(backPain2)
fit1 <- vglm(pain ~ x2 + x3 + x4 + x2:x3 + x2:x4 + x3:x4,
    propodds, data = backPain2)
spom1 <- step4(fit1)
summary(spom1)
spom1@post$anova
```

```
studentt Student t Distribution
```


## Description

Estimating the parameters of a Student t distribution.

## Usage

```
studentt (ldf = "logloglink", idf = NULL, tol1 = 0.1, imethod = 1)
studentt2(df = Inf, llocation = "identitylink", lscale = "loglink",
    ilocation = NULL, iscale = NULL, imethod = 1, zero = "scale")
studentt3(llocation = "identitylink", lscale = "loglink", ldf = "logloglink",
    ilocation \(=\) NULL, iscale \(=\) NULL, idf \(=\) NULL,
    imethod = 1, zero = c("scale", "df"))
```


## Arguments

llocation, lscale, ldf
Parameter link functions for each parameter, e.g., for degrees of freedom $\nu$. See Links for more choices. The defaults ensures the parameters are in range. A loglog link keeps the degrees of freedom greater than unity; see below.
ilocation, iscale, idf
Optional initial values. If given, the values must be in range. The default is to compute an initial value internally.
tol1 A positive value, the tolerance for testing whether an initial value is 1 . Best to leave this argument alone.
df Numeric, user-specified degrees of freedom. It may be of length equal to the number of columns of a response matrix.
imethod, zero See CommonVGAMffArguments.

## Details

The Student $t$ density function is

$$
f(y ; \nu)=\frac{\Gamma((\nu+1) / 2)}{\sqrt{\nu \pi} \Gamma(\nu / 2)}\left(1+\frac{y^{2}}{\nu}\right)^{-(\nu+1) / 2}
$$

for all real $y$. Then $E(Y)=0$ if $\nu>1$ (returned as the fitted values), and $\operatorname{Var}(Y)=\nu /(\nu-2)$ for $\nu>2$. When $\nu=1$ then the Student $t$-distribution corresponds to the standard Cauchy distribution, cauchy1. When $\nu=2$ with a scale parameter of $\operatorname{sqrt}(2)$ then the Student $t$-distribution corresponds to the standard (Koenker) distribution, sc.studentt2. The degrees of freedom can be treated as a parameter to be estimated, and as a real and not an integer. The Student $t$ distribution is used for a variety of reasons in statistics, including robust regression.
Let $Y=(T-\mu) / \sigma$ where $\mu$ and $\sigma$ are the location and scale parameters respectively. Then studentt3 estimates the location, scale and degrees of freedom parameters. And studentt2 estimates the location, scale parameters for a user-specified degrees of freedom, df. And studentt estimates the degrees of freedom parameter only. The fitted values are the location parameters. By default the linear/additive predictors are $(\mu, \log (\sigma), \log \log (\nu))^{T}$ or subsets thereof.
In general convergence can be slow, especially when there are covariates.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

studentt3() and studentt2() can handle multiple responses.
Practical experience has shown reasonably good initial values are required. If convergence failure occurs try using arguments such as idf. Local solutions are also possible, especially when the degrees of freedom is close to unity or the scale parameter is close to zero.
A standard normal distribution corresponds to a $t$ distribution with infinite degrees of freedom. Consequently, if the data is close to normal, there may be convergence problems; best to use uninormal instead.

## Author(s)

T. W. Yee

## References

Student (1908). The probable error of a mean. Biometrika, 6, 1-25.
Zhu, D. and Galbraith, J. W. (2010). A generalized asymmetric Student- $t$ distribution with application to financial econometrics. Journal of Econometrics, 157, 297-305.

## See Also

uninormal, cauchy1, logistic, huber2, sc.studentt2, TDist, simulate.vlm.

## Examples

```
tdata <- data.frame(x2 = runif(nn <- 1000))
tdata <- transform(tdata, y1 = rt(nn, df = exp(exp(0.5 - x2))),
    y2 = rt(nn, df = exp(exp(0.5 - x2))))
fit1 <- vglm(y1 ~ x2, studentt, data = tdata, trace = TRUE)
coef(fit1, matrix = TRUE)
fit2 <- vglm(y1 ~ x2, studentt2(df = exp(exp(0.5))), data = tdata)
coef(fit2, matrix = TRUE) # df inputted into studentt2() not quite right
fit3 <- vglm(cbind(y1, y2) ~ x2, studentt3, data = tdata, trace = TRUE)
coef(fit3, matrix = TRUE)
```


## Description

These functions are all methods for class "pvgam" or summary.pvgam objects.

## Usage

```
summarypvgam(object, dispersion = NULL, digits = options()$digits - 2,
    presid = TRUE)
## S3 method for class 'summary.pvgam'
show(x, quote = TRUE, prefix = "", digits = options()$digits -
    2, signif.stars = getOption("show.signif.stars"))
```


## Arguments

```
    object an object of class "pvgam", which is the result of a call to vgam with at least one
    sm.os or sm.ps term.
    x an object of class "summary.pvgam", which is the result of a call to summarypvgam().
    dispersion, digits, presid
            See summaryvglm.
quote, prefix, signif.stars
    See summaryvglm.
```


## Details

This methods function reports a summary more similar to summary.gam from mgev than summary.gam() from gam. It applies to G2-VGAMs using sm. os and O-splines, else sm. ps and P-splines. In particular, the hypothesis test for whether each sm. os or sm.ps term can be deleted follows quite closely to summary.gam. The p-values from this type of test tend to be biased downwards (too small) and corresponds to $p$. type $=5$. It is hoped in the short future that improved $p$-values be implemented, somewhat like the default of summary.gam. This methods function was adapted from summary .gam.

## Value

summarypvgam returns an object of class "summary.pvgam"; see summary.pvgam-class.

## Warning

See sm.os.

## See Also

vgam, summaryvgam, summary.pvgam-class, sm.os, sm.ps, summary.glm, summary.lm, summary.gam from mgev, summaryvgam for G1-VGAMs.

## Examples

```
hfit2 <- vgam(agaaus ~ sm.os(altitude), binomialff, data = hunua)
coef(hfit2, matrix = TRUE)
summary(hfit2)
```

summaryvgam Summarizing Vector Generalized Additive Model Fits

## Description

These functions are all methods for class vgam or summary. vgam objects.

## Usage

```
summaryvgam(object, dispersion = NULL, digits = options()$digits - 2,
                    presid = TRUE, nopredictors = FALSE)
## S3 method for class 'summary.vgam'
show(x, quote = TRUE, prefix = "",
                        digits = options()$digits-2, nopredictors = NULL)
```


## Arguments

object an object of class "vgam", which is the result of a call to vgam with at least one s term.
x
an object of class "summary. vgam", which is the result of a call to summaryvgam().
dispersion, digits, presid
See summaryvglm.
quote, prefix, nopredictors
See summaryvglm.

## Details

This methods function reports a summary more similar to summary.gam() from gam than summary.gam from mgev. It applies to G1-VGAMs using s and vector backfitting. In particular, an approximate score test for linearity is conducted for each s term-see Section 4.3.4 of Yee (2015) for details. The p-values from this type of test tend to be biased upwards (too large).

## Value

summaryvgam returns an object of class "summary.vgam"; see summary.vgam-class.

## See Also

vgam, summary.glm, summary.lm, summary.gam from mgev, summarypvgam for P-VGAMs.

## Examples

```
hfit <- vgam(agaaus ~ s(altitude, df = 2), binomialff, data = hunua)
summary(hfit)
summary(hfit)@anova # Table for (approximate) testing of linearity
```

summaryvglm Summarizing Vector Generalized Linear Model Fits

## Description

These functions are all methods for class vglm or summary.vglm objects.

## Usage

```
summaryvglm(object, correlation = FALSE, dispersion = NULL,
    digits = NULL, presid = TRUE,
    HDEtest = TRUE, hde.NA = TRUE, threshold.hde = 0.001,
    signif.stars = getOption("show.signif.stars"),
    nopredictors = FALSE,
    lrt0.arg = FALSE, score0.arg = FALSE, wald0.arg = FALSE,
    values0 = 0, subset = NULL, omit1s = TRUE,
    ...)
## S3 method for class 'summary.vglm'
show(x, digits = max(3L, getOption("digits") - 3L),
    quote = TRUE, prefix = "", presid = TRUE,
    HDEtest = TRUE, hde.NA = TRUE, threshold.hde = 0.001,
    signif.stars = NULL, nopredictors = NULL,
    top.half.only = FALSE, ...)
```


## Arguments

object an object of class "vglm", usually, a result of a call to vglm.
x
dispersion
correlation logical; if TRUE, the correlation matrix of the estimated parameters is returned and printed.
digits the number of significant digits to use when printing.
signif.stars logical; if TRUE, 'significance stars' are printed for each coefficient.
presid Pearson residuals; print out some summary statistics of these?
HDEtest logical; if TRUE (the default) then a test for the HDE is performed, else all arguments related to the HDE are ignored.
hde.NA logical; if a test for the Hauck-Donner effect is done (for each coefficient) and it is affirmative should that Wald test p-value be replaced by an NA? The default is to do so. Setting hde. NA = FALSE will print the p-value even though it will be biased upwards. Also see argument threshold.hde.
threshold.hde numeric; used if hde. NA = TRUE and is present for some coefficients. Only pvalues greater than this argument will be replaced by an NA, the reason being that small p-values will already be statistically significant. Hence setting threshold. hde $=0$ will print out a NA if the HDE is present.
quote Fed into print().
nopredictors logical; if TRUE the names of the linear predictors are not printed out. The default is that they are.
lrt0.arg, score0.arg, wald0.arg
Logical. If lrt0. arg = TRUE then the other arguments are passed into lrt.stat.vlm and the equivalent of the so-called Wald table is outputted. Similarly, if score0. arg $=$ TRUE then the other arguments are passed into score.stat.vlm and the equivalent of the so-called Wald table is outputted. Similarly, if wald0.arg = TRUE then the other arguments are passed into wald.stat.vlm and the Wald table
corresponding to that is outputted. See details below. Setting any of these will result in further IRLS iterations being performed, therefore may be computationally expensive.
values0, subset, omit1s
These arguments are used if any of the lrt0.arg, score0.arg, wald0.arg arguments are used. They are passed into the appropriate function, such as wald.stat.vlm.
top.half.only logical; if TRUE then only print out the top half of the usual output. Used for P-VGAMs.
prefix Not used.
... Not used.

## Details

Originally, summaryvglm() was written to be very similar to summary.glm, however now there are a quite a few more options available. By default, show. summary.vglm() tries to be smart about formatting the coefficients, standard errors, etc. and additionally gives 'significance stars' if signif.stars is TRUE. The coefficients component of the result gives the estimated coefficients and their estimated standard errors, together with their ratio. This third column is labelled $z$ value regardless of whether the dispersion is estimated or known (or fixed by the family). A fourth column gives the two-tailed p-value corresponding to the z ratio based on a Normal reference distribution.
In general, the $t$ distribution is not used, but the normal distribution is.
Correlations are printed to two decimal places (or symbolically): to see the actual correlations print summary (object)@correlation directly.
The Hauck-Donner effect (HDE) is tested for almost all models; see hdeff.vglm for details. Arguments hde.NA and threshold.hde here are meant to give some control of the output if this aberration of the Wald statistic occurs (so that the p-value is biased upwards). If the HDE is present then using lrt.stat.vlm to get a more accurate p-value is a good alternative as p-values based on the likelihood ratio test (LRT) tend to be more accurate than Wald tests and do not suffer from the HDE. Alternatively, if the HDE is present then using wald0.arg = TRUE will compute Wald statistics that are HDE-free; see wald. stat.
The arguments lrt0.arg and score0.arg enable the so-called Wald table to be replaced by the equivalent LRT and Rao score test table; see lrt.stat.vlm, score.stat. Further IRLS iterations are performed for both of these, hence the computational cost might be significant.
It is possible for programmers to write a methods function to print out extra quantities when summary (vglmObject) is called. The generic function is summaryvglmS4VGAM(), and one can use the S 4 function setMethod to compute the quantities needed. Also needed is the generic function is showsummaryvglmS4VGAM() to actually print the quantities out.

## Value

summaryvglm returns an object of class "summary.vglm"; see summary.vglm-class.

## Warning

Currently the SE column is deleted when lrt0 = TRUE because SEs are not so meaningful with the LRT. In the future an SE column may be inserted (with NA values) so that it has 4-column output
like the other tests. In the meantime, the columns of this matrix should be accessed by name and not number.

## Author(s)

T. W. Yee.

## See Also

vglm, confintvglm, vcovvlm, summary.glm, summary.lm, summary, hdeff.vglm, lrt.stat.vlm, score.stat, wald.stat.

## Examples

```
## For examples see example(glm)
pneumo <- transform(pneumo, let = log(exposure.time))
(afit <- vglm(cbind(normal, mild, severe) ~ let, acat, data = pneumo))
coef(afit, matrix = TRUE)
summary(afit) # Might suffer from the Hauck-Donner effect
coef(summary(afit))
summary(afit, lrt0 = TRUE, score0 = TRUE, wald0 = TRUE)
```


## SURff

Seemingly Unrelated Regressions Family Function

## Description

Fits a system of seemingly unrelated regressions.

## Usage

SURff(mle.normal = FALSE, divisor = c("n", "n-max(pj,pk)", "sqrt((n-pj)*(n-pk))"), parallel = FALSE, Varcov = NULL, matrix.arg = FALSE)

## Arguments

mle.normal
divisor Character, partial matching allowed and the first choice is the default. The divisor for the estimate of the covariances. If " $n$ " then the estimate will be biased. If the others then the estimate will be unbiased for some elements. If mle.normal $=$ TRUE and this argument is not " $n$ " then a warning or an error will result.
parallel See CommonVGAMffArguments. If parallel = TRUE then the constraint applies to the intercept too.

| Varcov | Numeric. This may be assigned a variance-covariance of the errors. If matrix.arg <br> then this is a $M \times M$ matrix. If $!$ matrix. arg then this is a $M \times M$ matrix in <br>  <br> matrix-band format (a vector with at least $M$ and at most $M \star(M+1) / 2$ elements). |
| :--- | :--- |
| matrix.arg | Logical. Of single length. |

## Details

Proposed by Zellner (1962), the basic seemingly unrelated regressions (SUR) model is a set of LMs ( $M>1$ of them) tied together at the error term level. Each LM's model matrix may potentially have its own set of predictor variables.
Zellner's efficient (ZEF) estimator (also known as Zellner's two-stage Aitken estimator) can be obtained by setting maxit = 1 (and possibly divisor = "sqrt" or divisor = "n-max").
The default value of maxit (in vglm. control) probably means iterative GLS (IGLS) estimator is computed because IRLS will probably iterate to convergence. IGLS means, at each iteration, the residuals are used to estimate the error variance-covariance matrix, and then the matrix is used in the GLS. The IGLS estimator is also known as Zellner's iterative Aitken estimator, or IZEF.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Warning

The default convergence criterion may be a little loose. Try setting epsilon $=1 \mathrm{e}-11$, especially with mle. normal = TRUE.

## Note

The fitted object has slot @extra\$ncols.X.1m which is a $M$ vector with the number of parameters for each LM. Also, @misc\$values.divisor is the $M$-vector of divisor values.
Constraint matrices are needed in order to specify which response variables that each term on the RHS of the formula is a regressor for. See the constraints argument of vglm for more information.

## Author(s)

T. W. Yee.

## References

Zellner, A. (1962). An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias. J. Amer. Statist. Assoc., 57(298), 348-368.
Kmenta, J. and Gilbert, R. F. (1968). Small Sample Properties of Alternative Estimators of Seemingly Unrelated Regressions. J. Amer. Statist. Assoc., 63(324), 1180-1200.

## See Also

uninormal, gew.

## Examples

```
# Obtain some of the results of p.1199 of Kmenta and Gilbert (1968)
clist <- list("(Intercept)" = diag(2),
        "capital.g" = rbind(1, 0),
        "value.g" = rbind(1, 0),
        "capital.w" = rbind(0, 1),
        "value.w" = rbind(0, 1))
zef1 <- vglm(cbind(invest.g, invest.w) ~
    capital.g + value.g + capital.w + value.w,
    SURff(divisor = "sqrt"), maxit = 1,
    data = gew, trace = TRUE, constraints = clist)
round(coef(zef1, matrix = TRUE), digits = 4) # ZEF
zef1@extra$ncols.X.lm
zef1@misc$divisor
zef1@misc$values.divisor
round(sqrt(diag(vcov(zef1))), digits = 4) # SEs
nobs(zef1, type = "lm")
df.residual(zef1, type = "lm")
mle1 <- vglm(cbind(invest.g, invest.w) ~
    capital.g + value.g + capital.w + value.w,
    SURff(mle.normal = TRUE),
    epsilon = 1e-11,
    data = gew, trace = TRUE, constraints = clist)
round(coef(mle1, matrix = TRUE), digits = 4) # MLE
round(sqrt(diag(vcov(mle1))), digits = 4) # SEs
```


## Description

Create a survival object, usually used as a response variable in a model formula.

## Usage

SurvS4(time, time2, event, type =, origin = 0)
is.SurvS4(x)

## Arguments

time for right censored data, this is the follow up time. For interval data, the first argument is the starting time for the interval.
x
any R object.

| event | The status indicator, normally $0=$ alive, $1=$ dead. Other choices are TRUE/FALSE <br> (TRUE = death) or $1 / 2(2=$ death $)$. For interval censored data, the status indica- <br> tor is $0=$ right censored, $1=$ event at time, $2=$ left censored, $3=$ interval censored. <br> Although unusual, the event indicator can be omitted, in which case all subjects <br> are assumed to have an event. |
| :--- | :--- |
| time2ending time of the interval for interval censored or counting process data only, <br> Intervals are assumed to be open on the left and closed on the right, (start, end]. <br> For counting process data, event indicates whether an event occurred at the end <br> of the interval. |  |
| character string specifying the type of censoring. Possible values are "right", |  |
| "left", "counting", "interval", or "interval2". The default is "right" |  |
| or "counting" depending on whether the time2 argument is absent or present, |  |
| respectively. |  |
| for counting process data, the hazard function origin. This is most often used in <br> conjunction with a model containing time dependent strata in order to align the <br> subjects properly when they cross over from one strata to another. |  |

## Details

Typical usages are

```
SurvS4(time, event)
SurvS4(time, time2, event, type=, origin=0)
```

In theory it is possible to represent interval censored data without a third column containing the explicit status. Exact, right censored, left censored and interval censored observation would be represented as intervals of ( $a, a$ ), ( $a$, infinity), (-infinity, b), and ( $a, b$ ) respectively; each specifying the interval within which the event is known to have occurred.

If type = "interval2" then the representation given above is assumed, with NA taking the place of infinity. If 'type="interval" event must be given. If event is 0,1 , or 2 , the relevant information is assumed to be contained in time, the value in time 2 is ignored, and the second column of the result will contain a placeholder.

Presently, the only methods allowing interval censored data are the parametric models computed by survreg, so the distinction between open and closed intervals is unimportant. The distinction is important for counting process data and the Cox model.
The function tries to distinguish between the use of $0 / 1$ and $1 / 2$ coding for left and right censored data using if (max(status)==2). If $1 / 2$ coding is used and all the subjects are censored, it will guess wrong. Use $0 / 1$ coding in this case.

## Value

An object of class SurvS4 (formerly Surv). There are methods for print, is.na, and subscripting survival objects. SurvS4 objects are implemented as a matrix of 2 or 3 columns.

In the case of is. SurvS4, a logical value TRUE if $x$ inherits from class "SurvS4", otherwise a FALSE.

## Note

The purpose of having SurvS4 in VGAM is so that the same input can be fed into vglm as functions in survival such as survreg. The class name has been changed from "Surv" to "SurvS4"; see SurvS4-class.
The format $J+$ is interpreted in VGAM as $\geq J$. If type="interval" then these should not be used in VGAM: $\left(\mathrm{L}, \mathrm{U}-\mathrm{]}\right.$ or $\left(\mathrm{L}, \mathrm{U}^{+}\right]$.

## Author(s)

The code and documentation comes from survival. Slight modifications have been made for conversion to S4 by T. W. Yee. Also, for "interval" data, as. character. SurvS4() has been modified to print intervals of the form (start, end] and not [start, end] as previously. (This makes a difference for discrete data, such as for cens.poisson). All VGAM family functions beginning with "cen" require the packaging function Surv to format the input.

## See Also

SurvS4-class, cens.poisson, survreg, leukemia.

## Examples

with(leukemia, SurvS4(time, status))
class(with(leukemia, SurvS4(time, status)))

SurvS4-class Class "SurvS4"

## Description

S4 version of the Surv class.

## Objects from the Class

A virtual Class: No objects may be created from it.

## Extends

Class "Surv", directly. Class "matrix", directly. Class "oldClass", by class "Surv", distance
2. Class "structure", by class "matrix", distance 2. Class "array", by class "matrix", distance
2. Class "vector", by class "matrix", distance 3, with explicit coerce. Class "vector", by class "matrix", distance 4, with explicit coerce.

## Methods

show signature(object = "SurvS4"): ...

## Warning

This code has not been thoroughly tested.

## Note

The purpose of having SurvS4 in VGAM is so that the same input can be fed into vglm as functions in survival such as survreg.

## Author(s)

T. W. Yee.

## References

See survival.

## See Also

SurvS4.

## Examples

showClass("SurvS4")

TIC
Takeuchi's Information Criterion

## Description

Calculates the Takeuchi information criterion for a fitted model object for which a log-likelihood value has been obtained.

## Usage

$$
\begin{aligned}
& \text { TIC(object, ...) } \\
& \text { TICvlm(object, ...) }
\end{aligned}
$$

## Arguments

object A VGAM object having class vglm-class.
$\ldots \quad$ Other possible arguments fed into logLik in order to compute the log-likelihood.

## Details

The following formula is used for VGLMs: $-2 \log$-likelihood $+2 \operatorname{trace}(V K)$, where $V$ is the inverse of the EIM from the fitted model, and $K$ is the outer product of the score vectors. Both $V$ and $K$ are order- $p . V L M$ matrices. One has $V$ equal to $v \operatorname{cov}$ (object), and $K$ is computed by taking the outer product of the output from the deriv slot multiplied by the large VLM matrix and then taking their sum. Hence for the huge majority of models, the penalty is computed at the MLE and is empirical in nature. Theoretically, if the fitted model is the true model then AIC equals TIC.
When there are prior weights the score vectors are divided by the square root of these, because $\left(a_{i} U_{i} / \sqrt{a_{i}}\right)^{2}=a_{i} U_{i}^{2}$.

This code relies on the log-likelihood being defined, and computed, for the object. When comparing fitted objects, the smaller the TIC, the better the fit. The log-likelihood and hence the TIC is only defined up to an additive constant.
Currently any estimated scale parameter (in GLM parlance) is ignored by treating its value as unity. Also, currently this function is written only for vglm objects and not vgam or rrvglm, etc., objects.

## Value

Returns a numeric TIC value.

## Warning

This code has not been double-checked. The general applicability of TIC for the VGLM/VGAM classes has not been developed fully. In particular, TIC should not be run on some VGAM family functions because of violation of certain regularity conditions, etc.

Some authors note that quite large sample sizes are needed for this IC to work reasonably well.

## Note

TIC has not been defined for RR-VGLMs, QRR-VGLMs, etc., yet.
See AICVlm about models such as posbernoulli.tb that require posbinomial(omit.constant = TRUE).

## Author(s)

T. W. Yee.

## References

Takeuchi, K. (1976). Distribution of informational statistics and a criterion of model fitting. (In Japanese). Suri-Kagaku (Mathematic Sciences), 153, 12-18.

Burnham, K. P. and Anderson, D. R. (2002). Model Selection and Multi-Model Inference: A Practical Information-Theoretic Approach, 2nd ed. New York, USA: Springer.

## See Also

VGLMs are described in vglm-class; AIC, AICvlm. BICvlm.

## Examples

```
pneumo <- transform(pneumo, let = log(exposure.time))
(fit1 <- vglm(cbind(normal, mild, severe) ~ let,
    cumulative(parallel = TRUE, reverse = TRUE), data = pneumo))
coef(fit1, matrix = TRUE)
TIC(fit1)
(fit2 <- vglm(cbind(normal, mild, severe) ~ let,
    cumulative(parallel = FALSE, reverse = TRUE), data = pneumo))
coef(fit2, matrix = TRUE)
TIC(fit2)
```

Tikuv

## A Short-tailed Symmetric Distribution

## Description

Density, cumulative distribution function, quantile function and random generation for the shorttailed symmetric distribution of Tiku and Vaughan (1999).

## Usage

```
dtikuv(x, d, mean = 0, sigma = 1, log = FALSE)
ptikuv(q, d, mean = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qtikuv(p, d, mean = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE, ...)
rtikuv(n, d, mean = 0, sigma = 1, Smallno = 1.0e-6)
```


## Arguments

| $x, q$ | vector of quantiles. |
| :--- | :--- |
| $p$ | vector of probabilities. |

$n \quad$ number of observations. Same as in runif.
d, mean, sigma arguments for the parameters of the distribution. See tikuv for more details. For rtikuv, arguments mean and sigma must be of length 1.
Smallno Numeric, a small value used by the rejection method for determining the lower and upper limits of the distribution. That is, ptikuv (L) < Smallno and ptikuv (U) > 1-Smallno where $L$ and $U$ are the lower and upper limits respectively.
... Arguments that can be passed into uniroot.
log Logical. If log = TRUE then the logarithm of the density is returned.
lower.tail, log.p
Same meaning as in pnorm or qnorm.

## Details

See tikuv for more details.

## Value

dtikuv gives the density, ptikuv gives the cumulative distribution function, qtikuv gives the quantile function, and rtikuv generates random deviates.

## Author(s)

T. W. Yee and Kai Huang

## See Also

tikuv.

## Examples

```
## Not run: par(mfrow = c(2, 1))
x <- seq(-5, 5, len = 401)
plot(x, dnorm(x), type = "l", col = "black", ylab = "", las = 1,
        main = "Black is standard normal, others are dtikuv(x, d)")
lines(x, dtikuv(x, d = -10), col = "orange")
lines(x, dtikuv(x, d = -1 ), col = "blue")
lines(x, dtikuv(x, d = 1 ), col = "green")
legend("topleft", col = c("orange","blue","green"), lty = rep(1, len = 3),
            legend = paste("d =", c(-10, -1, 1)))
plot(x, pnorm(x), type = "l", col = "black", ylab = "", las = 1,
        main = "Black is standard normal, others are ptikuv(x, d)")
lines(x, ptikuv(x, d = -10), col = "orange")
lines(x, ptikuv(x, d = -1 ), col = "blue")
lines(x, ptikuv(x, d = 1 ), col = "green")
legend("topleft", col = c("orange","blue","green"), lty = rep(1, len = 3),
            legend = paste("d =", c(-10, -1, 1)))
## End(Not run)
probs <- seq(0.1, 0.9, by = 0.1)
ptikuv(qtikuv(p = probs, d = 1), d = 1) - probs # Should be all 0
```

tikuv Short-tailed Symmetric Distribution Family Function

## Description

Fits the short-tailed symmetric distribution of Tiku and Vaughan (1999).

## Usage

```
tikuv(d, lmean = "identitylink", lsigma = "loglink", isigma = NULL,
    zero = "sigma")
```


## Arguments

d
The $d$ parameter. It must be a single numeric value less than 2 . Then $h=$ $2-d>0$ is another parameter.
lmean, lsigma Link functions for the mean and standard deviation parameters of the usual univariate normal distribution (see Details below). They are $\mu$ and $\sigma$ respectively. See Links for more choices.
isigma Optional initial value for $\sigma$. A NULL means a value is computed internally.
zero A vector specifying which linear/additive predictors are modelled as interceptonly. The values can be from the set $\{1,2\}$, corresponding respectively to $\mu$, $\sigma$. If zero $=$ NULL then all linear/additive predictors are modelled as a linear combination of the explanatory variables. For many data sets having zero $=2$ is a good idea. See CommonVGAMffArguments for information.

## Details

The short-tailed symmetric distribution of Tiku and Vaughan (1999) has a probability density function that can be written

$$
f(y)=\frac{K}{\sqrt{2 \pi} \sigma}\left[1+\frac{1}{2 h}\left(\frac{y-\mu}{\sigma}\right)^{2}\right]^{2} \exp \left(-\frac{1}{2}(y-\mu)^{2} / \sigma^{2}\right)
$$

where $h=2-d>0, K$ is a function of $h,-\infty<y<\infty, \sigma>0$. The mean of $Y$ is $E(Y)=\mu$ and this is returned as the fitted values.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

Under- or over-flow may occur if the data is ill-conditioned, e.g., when $d$ is very close to 2 or approaches -Inf.

## Note

The density function is the product of a univariate normal density and a polynomial in the response $y$. The distribution is bimodal if $d>0$, else is unimodal. A normal distribution arises as the limit as $d$ approaches $-\infty$, i.e., as $h$ approaches $\infty$. Fisher scoring is implemented. After fitting the value of $d$ is stored in @misc with component name $d$.

## Author(s)

Thomas W. Yee

## References

Akkaya, A. D. and Tiku, M. L. (2008). Short-tailed distributions and inliers. Test, 17, 282-296.
Tiku, M. L. and Vaughan, D. C. (1999). A family of short-tailed symmetric distributions. Technical report, McMaster University, Canada.

## See Also

dtikuv, uninormal.

## Examples

```
m <- 1.0; sigma <- exp(0.5)
tdata <- data.frame(y = rtikuv(n = 1000, d = 1, m = m, s = sigma))
tdata <- transform(tdata, sy = sort(y))
fit <- vglm(y ~ 1, tikuv(d = 1), data = tdata, trace = TRUE)
coef(fit, matrix = TRUE)
(Cfit <- Coef(fit))
with(tdata, mean(y))
## Not run: with(tdata, hist(y, prob = TRUE))
lines(dtikuv(sy, d = 1, m = Cfit[1], s = Cfit[2]) ~ sy, data = tdata, col = "orange")
## End(Not run)
```

Tobit
The Tobit Distribution

## Description

Density, distribution function, quantile function and random generation for the Tobit model.

## Usage

```
    dtobit(x, mean = 0, sd = 1, Lower = 0, Upper = Inf, log = FALSE)
    ptobit(q, mean = 0, sd = 1, Lower = 0, Upper = Inf,
        lower.tail = TRUE, log.p = FALSE)
    qtobit(p, mean = 0, sd = 1, Lower = 0, Upper = Inf,
        lower.tail = TRUE, log.p = FALSE)
    rtobit(n, mean = 0, sd = 1, Lower = 0, Upper = Inf)
```


## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$n \quad$ number of observations. If length $(n)>1$ then the length is taken to be the number required.
Lower, Upper vector of lower and upper thresholds.
mean, sd, lower.tail, log, log.p
see rnorm.

## Details

See tobit, the VGAM family function for estimating the parameters, for details. Note that the density at Lower and Upper is the the area to the left and right of those points. Thus there are two spikes (but less in value); see the example below. Consequently, dtobit (Lower) + dtobit (Upper) + the area in between equals unity.

## Value

dtobit gives the density, ptobit gives the distribution function, qtobit gives the quantile function, and rtobit generates random deviates.

## Author(s)

T. W. Yee

## See Also

tobit, rnorm.

## Examples

```
mu <- 0.5; x <- seq(-2, 4, by = 0.01)
Lower <- -1; Upper <- 2.0
integrate(dtobit, lower = Lower, upper = Upper,
            mean = mu, Lower = Lower, Upper = Upper)$value +
dtobit(Lower, mean = mu, Lower = Lower, Upper = Upper) +
dtobit(Upper, mean = mu, Lower = Lower, Upper = Upper) # Adds to unity
## Not run:
plot(x, ptobit(x, m = mu, Lower = Lower, Upper = Upper),
    type = "l", ylim = 0:1, las = 1, col = "orange",
    ylab = paste("ptobit(m = ", mu, ", sd = 1, Lower =", Lower,
                ", Upper =", Upper, ")"),
    main = "Orange is cumulative distribution function; blue is density",
    sub = "Purple lines are the 10,20,\ldots.,90 percentiles")
abline(h = 0)
lines(x, dtobit(x, m = mu, Lower = Lower, Upper = Upper), col = "blue")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qtobit(probs, m = mu, Lower = Lower, Upper = Upper)
lines(Q, ptobit(Q, m = mu, Lower = Lower, Upper = Upper),
        col = "purple", lty = "dashed", type = "h")
lines(Q, dtobit(Q, m = mu, Lower = Lower, Upper = Upper),
        col = "darkgreen", lty = "dashed", type = "h")
abline(h = probs, col = "purple", lty = "dashed")
max(abs(ptobit(Q, m = mu, Lower = Lower, Upper = Upper) - probs)) # Should be 0
endpts <- c(Lower, Upper) # Endpoints have a spike (not quite, actually)
lines(endpts, dtobit(endpts, m = mu, Lower = Lower, Upper = Upper),
        col = "blue", lwd = 3, type = "h")
```

\#\# End(Not run)

```
tobit Tobit Model
```


## Description

Fits a Tobit model.

## Usage

```
tobit(Lower = 0, Upper = Inf, lmu = "identitylink", lsd = "loglink",
    imu = NULL, isd = NULL,
    type.fitted = c("uncensored", "censored", "mean.obs"),
    byrow.arg = FALSE, imethod = 1, zero = "sd")
```


## Arguments

Lower $\quad$ Numeric. It is the value $L$ described below. Any value of the linear model $x_{i}^{T} \beta$ that is less than this lowerbound is assigned this value. Hence this should be the smallest possible value in the response variable. May be a vector (see below for more information).
Upper $\quad$ Numeric. It is the value $U$ described below. Any value of the linear model $x_{i}^{T} \beta$ that is greater than this upperbound is assigned this value. Hence this should be the largest possible value in the response variable. May be a vector (see below for more information).
lmu, lsd Parameter link functions for the mean and standard deviation parameters. See Links for more choices. The standard deviation is a positive quantity, therefore a $\log \operatorname{link}$ is its default.
imu, isd, byrow.arg
See CommonVGAMffArguments for information.
type.fitted Type of fitted value returned. The first choice is default and is the ordinary uncensored or unbounded linear model. If "censored" then the fitted values in the interval $[L, U]$. If "mean. obs" then the mean of the observations is returned; this is a doubly truncated normal distribution augmented by point masses at the truncation points (see dtobit). See CommonVGAMffArguments for more information.
imethod Initialization method. Either 1 or 2 or 3, this specifies some methods for obtaining initial values for the parameters. See CommonVGAMffArguments for information.
zero A vector, e.g., containing the value 1 or 2 . If so, the mean or standard deviation respectively are modelled as an intercept-only. Setting zero = NULL means both linear/additive predictors are modelled as functions of the explanatory variables. See CommonVGAMffArguments for more information.

## Details

The Tobit model can be written

$$
y_{i}^{*}=x_{i}^{T} \beta+\varepsilon_{i}
$$

where the $e_{i} \sim N\left(0, \sigma^{2}\right)$ independently and $i=1, \ldots, n$. However, we measure $y_{i}=y_{i}^{*}$ only if $y_{i}^{*}>L$ and $y_{i}^{*}<U$ for some cutpoints $L$ and $U$. Otherwise we let $y_{i}=L$ or $y_{i}=U$, whatever is closer. The Tobit model is thus a multiple linear regression but with censored responses if it is below or above certain cutpoints.
The defaults for Lower and Upper and lmu correspond to the standard Tobit model. Fisher scoring is used for the standard and nonstandard models. By default, the mean $x_{i}^{T} \beta$ is the first linear/additive predictor, and the $\log$ of the standard deviation is the second linear/additive predictor. The Fisher information matrix for uncensored data is diagonal. The fitted values are the estimates of $x_{i}^{T} \beta$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

If values of the response and Lower and/or Upper are not integers then there is the danger that the value is wrongly interpreted as uncensored. For example, if the first 10 values of the response were runif(10) and Lower was assigned these value then testing $y[1: 10]==$ Lower[1:10] is numerically fraught. Currently, if any y < Lower or $\mathrm{y}>$ Upper then a warning is issued.

## Note

The response can be a matrix. If so, then Lower and Upper are recycled into a matrix with the number of columns equal to the number of responses, and the recycling is done row-wise if byrow.arg = TRUE. The default order is as matrix, which is byrow. arg = FALSE. For example, these are returned in fit4@misc\$Lower and fit4@misc\$Upper below.
If there is no censoring then uninormal is recommended instead. Any value of the response less than Lower or greater than Upper will be assigned the value Lower and Upper respectively, and a warning will be issued. The fitted object has components censoredL and censoredU in the extra slot which specifies whether observations are censored in that direction. The function cens.normal is an alternative to tobit().

When obtaining initial values, if the algorithm would otherwise want to fit an underdetermined system of equations, then it uses the entire data set instead. This might result in rather poor quality initial values, and consequently, monitoring convergence is advised.

## Author(s)

Thomas W. Yee

## References

Tobin, J. (1958). Estimation of relationships for limited dependent variables. Econometrica 26, 24-36.

## See Also

rtobit, cens.normal, uninormal, double.cens.normal, posnormal, CommonVGAMffArguments, mills.ratio, margeff, rnorm.

## Examples

```
# Here, fit1 is a standard Tobit model and fit2 is a nonstandard Tobit model
tdata <- data.frame(x2 = seq(-1, 1, length = (nn <- 100)))
set.seed(1)
Lower <- 1; Upper <- 4 # For the nonstandard Tobit model
tdata <- transform(tdata,
    Lower.vec = rnorm(nn, Lower, 0.5),
    Upper.vec = rnorm(nn, Upper, 0.5))
meanfun1 <- function(x) 0 + 2*x
meanfun2 <- function(x) 2 + 2*x
meanfun3 <- function(x) 2 + 2*x
meanfun4 <- function(x) 3 + 2*x
tdata <- transform(tdata,
    y1 = rtobit(nn, mean = meanfun1(x2)), # Standard Tobit model
    y2 = rtobit(nn, mean = meanfun2(x2), Lower = Lower, Upper = Upper),
    y3 = rtobit(nn, mean = meanfun3(x2), Lower = Lower.vec, Upper = Upper.vec),
    y4 = rtobit(nn, mean = meanfun3(x2), Lower = Lower.vec, Upper = Upper.vec))
with(tdata, table(y1 == 0)) # How many censored values?
with(tdata, table(y2 == Lower | y2 == Upper)) # How many censored values?
with(tdata, table(attr(y2, "cenL")))
with(tdata, table(attr(y2, "cenU")))
fit1 <- vglm(y1 ~ x2, tobit, data = tdata, trace = TRUE)
coef(fit1, matrix = TRUE)
summary(fit1)
fit2 <- vglm(y2 ~ x2, tobit(Lower = Lower, Upper = Upper, type.f = "cens"),
    data = tdata, trace = TRUE)
table(fit2@extra$censoredL)
table(fit2@extra$censoredU)
coef(fit2, matrix = TRUE)
fit3 <- vglm(y3 ~ x2, tobit(Lower = with(tdata, Lower.vec),
                            Upper = with(tdata, Upper.vec), type.f = "cens"),
    data = tdata, trace = TRUE)
table(fit3@extra$censoredL)
table(fit3@extra$censoredU)
coef(fit3, matrix = TRUE)
# fit4 is fit3 but with type.fitted = "uncen".
fit4 <- vglm(cbind(y3, y4) ~ x2,
    tobit(Lower = rep(with(tdata, Lower.vec), each = 2),
    Upper = rep(with(tdata, Upper.vec), each = 2),
    byrow.arg = TRUE),
    data = tdata, crit = "coeff", trace = TRUE)
head(fit4@extra$censoredL) # A matrix
head(fit4@extra$censoredU) # A matrix
```

```
head(fit4@misc$Lower) # A matrix
head(fit4@misc$Upper) # A matrix
coef(fit4, matrix = TRUE)
## Not run: # Plot fit1--fit4
par(mfrow = c(2, 2))
plot(y1 ~ x2, tdata, las = 1, main = "Standard Tobit model",
        col = as.numeric(attr(y1, "cenL")) + 3,
        pch = as.numeric(attr(y1, "cenL")) + 1)
legend(x = "topleft", leg = c("censored", "uncensored"),
            pch = c(2, 1), col = c("blue", "green"))
legend(-1.0, 2.5, c("Truth", "Estimate", "Naive"),
        col = c("purple", "orange", "black"), lwd = 2, lty = c(1, 2, 2))
lines(meanfun1(x2) ~ x2, tdata, col = "purple", lwd = 2)
lines(fitted(fit1) ~ x2, tdata, col = "orange", lwd = 2, lty = 2)
lines(fitted(lm(y1 ~ x2, tdata)) ~ x2, tdata, col = "black",
        lty = 2, lwd = 2) # This is simplest but wrong!
plot(y2 ~ x2, data = tdata, las = 1, main = "Tobit model",
        col = as.numeric(attr(y2, "cenL")) + 3 +
            as.numeric(attr(y2, "cenU")),
        pch = as.numeric(attr(y2, "cenL")) + 1 +
            as.numeric(attr(y2, "cenU")))
legend(x = "topleft", leg = c("censored", "uncensored"),
        pch = c(2, 1), col = c("blue", "green"))
legend(-1.0, 3.5, c("Truth", "Estimate", "Naive"),
        col = c("purple", "orange", "black"), lwd = 2, lty = c(1, 2, 2))
lines(meanfun2(x2) ~ x2, tdata, col = "purple", lwd = 2)
lines(fitted(fit2) ~ x2, tdata, col = "orange", lwd = 2, lty = 2)
lines(fitted(lm(y2 ~ x2, tdata)) ~ x2, tdata, col = "black",
        lty = 2, lwd = 2) # This is simplest but wrong!
plot(y3 ~ x2, data = tdata, las = 1,
        main = "Tobit model with nonconstant censor levels",
        col = as.numeric(attr(y3, "cenL")) + 2 +
            as.numeric(attr(y3, "cenU") * 2),
        pch = as.numeric(attr(y3, "cenL")) + 1 +
            as.numeric(attr(y3, "cenU") * 2))
legend(x = "topleft", leg = c("censoredL", "censoredU", "uncensored"),
            pch = c(2, 3, 1), col = c(3, 4, 2))
legend(-1.0, 3.5, c("Truth", "Estimate", "Naive"),
        col = c("purple", "orange", "black"), lwd = 2, lty = c(1, 2, 2))
lines(meanfun3(x2) ~ x2, tdata, col = "purple", lwd = 2)
lines(fitted(fit3) ~ x2, tdata, col = "orange", lwd = 2, lty = 2)
lines(fitted(lm(y3 ~ x2, tdata)) ~ x2, tdata, col = "black",
        lty = 2, lwd = 2) # This is simplest but wrong!
plot(y3 ~ x2, data = tdata, las = 1,
    main = "Tobit model with nonconstant censor levels",
    col = as.numeric(attr(y3, "cenL")) + 2 +
            as.numeric(attr(y3, "cenU") * 2),
        pch = as.numeric(attr(y3, "cenL")) + 1 +
```

```
        as.numeric(attr(y3, "cenU") * 2))
legend(x = "topleft", leg = c("censoredL", "censoredU", "uncensored"),
    pch = c(2, 3, 1), col = c(3, 4, 2))
legend(-1.0, 3.5, c("Truth", "Estimate", "Naive"),
    col = c("purple", "orange", "black"), lwd = 2, lty = c(1, 2, 2))
lines(meanfun3(x2) ~ x2, data = tdata, col = "purple", lwd = 2)
lines(fitted(fit4)[, 1] ~ x2, tdata, col = "orange", lwd = 2, lty = 2)
lines(fitted(lm(y3 ~ x2, tdata)) ~ x2, data = tdata, col = "black",
    lty = 2, lwd = 2) # This is simplest but wrong!
## End(Not run)
```

Tol Tolerances

## Description

Generic function for the tolerances of a model.

## Usage

Tol(object, ...)

## Arguments

object An object for which the computation or extraction of a tolerance or tolerances is meaningful.
... Other arguments fed into the specific methods function of the model. Sometimes they are fed into the methods function for Coef.

## Details

Different models can define an optimum in different ways. Many models have no such notion or definition.

Tolerances occur in quadratic ordination, i.e., CQO and UQO. They have ecological meaning because a high tolerance for a species means the species can survive over a large environmental range (stenoecous species), whereas a small tolerance means the species' niche is small (eurycous species). Mathematically, the tolerance is like the variance of a normal distribution.

## Value

The value returned depends specifically on the methods function invoked. For a cqo binomial or Poisson fit, this function returns a $R \times R \times S$ array, where $R$ is the rank and $S$ is the number of species. Each tolerance matrix ought to be positive-definite, and for a rank-1 fit, taking the square root of each tolerance matrix results in each species' tolerance (like a standard deviation).

## Warning

There is a direct inverse relationship between the scaling of the latent variables (site scores) and the tolerances. One normalization is for the latent variables to have unit variance. Another normalization is for all the tolerances to be unit. These two normalization cannot simultaneously hold in general. For rank- $R>1$ models it becomes more complicated because the latent variables are also uncorrelated. An important argument when fitting quadratic ordination models is whether eq. tolerances is TRUE or FALSE. See Yee (2004) for details.

## Note

Tolerances are undefined for 'linear' and additive ordination models. They are well-defined for quadratic ordination models.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2004). A new technique for maximum-likelihood canonical Gaussian ordination. Ecological Monographs, 74, 685-701.

Yee, T. W. (2006). Constrained additive ordination. Ecology, 87, 203-213.

## See Also

```
Tol.qrrvglm. Max, Opt, cqo, rcim for UQO.
```


## Examples

```
## Not run:
set.seed(111) # This leads to the global solution
hspider[,1:6] <- scale(hspider[, 1:6]) # Standardized environmental vars
p1 <- cqo(cbind(Alopacce, Alopcune, Alopfabr, Arctlute, Arctperi,
                                    Auloalbi, Pardlugu, Pardmont, Pardnigr, Pardpull,
            Trocterr, Zoraspin) ~
        WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
        poissonff, data = hspider, Crow1positive = FALSE)
Tol(p1)
## End(Not run)
```


## Description

Density, distribution function, quantile function and random generation for the Topp-Leone distribution.

## Usage

dtopple(x, shape, log = FALSE)
ptopple(q, shape, lower.tail = TRUE, log.p = FALSE)
qtopple(p, shape)
rtopple(n, shape)

## Arguments

| $x, q, p, n$ | Same as Uniform. |
| :--- | :--- |
| shape | the (shape) parameter, which lies in $(0,1)$. |
| log | Logical. If log = TRUE then the logarithm of the density is returned. |
| lower.tail, log. $p$ |  |
|  | Same meaning as in pnorm or qnorm. |

## Details

See topple, the VGAM family function for estimating the (shape) parameter $s$ by maximum likelihood estimation, for the formula of the probability density function.

## Value

dtopple gives the density, ptopple gives the distribution function, qtopple gives the quantile function, and rtopple generates random deviates.

## Note

The Topp-Leone distribution is related to the triangle distribution.

## Author(s)

T. W. Yee

## References

Topp, C. W. and F. C. Leone (1955). A family of J-shaped frequency functions. Journal of the American Statistical Association, 50, 209-219.

## See Also

topple, Triangle.

## Examples

```
## Not run: shape <- 0.7; x <- seq(0.02, 0.999, length = 300)
plot(x, dtopple(x, shape = shape), type = "l", col = "blue", las = 1,
    main = "Blue is density, orange is cumulative distribution function",
    sub = "Purple lines are the 10,20,\ldots,90 percentiles", ylab = "")
abline(h = 0, col = "blue", lty = 2)
lines(x, ptopple(x, shape = shape), type = "l", col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qtopple(probs, shape = shape)
lines(Q, dtopple(Q, shape), col = "purple", lty = 3, type = "h")
lines(Q, ptopple(Q, shape), col = "purple", lty = 3, type = "h")
abline(h = probs, col = "purple", lty = 3)
max(abs(ptopple(Q, shape) - probs)) # Should be zero
## End(Not run)
```

```
topple Topp-Leone Distribution Family Function
```


## Description

Estimating the parameter of the Topp-Leone distribution by maximum likelihood estimation.

## Usage

```
topple(lshape = "logitlink", zero = NULL, gshape = ppoints(8),
    parallel = FALSE, type.fitted = c("mean", "percentiles", "Qlink"),
    percentiles = 50)
```


## Arguments

lshape, gshape Details at CommonVGAMffArguments.
zero, parallel Details at CommonVGAMffArguments.
type.fitted, percentiles
See CommonVGAMffArguments for information. Using "Qlink" is for quantilelinks in VGAMextra.

## Details

The Topple distribution has a probability density function that can be written

$$
f(y ; s)=2 s(1-y)[y(2-y)]^{s-1}
$$

for $0<y<1$ and shape parameter $0<s<1$. The mean of $Y$ is $1-4^{s}[\Gamma(1+s)]^{2} / \Gamma(2+2 s)$ (returned as the fitted values).

Value
An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

Fisher-scoring and Newton-Raphson are the same here. A related distribution is the triangle distribution. This VGAM family function handles multiple responses.

## Author(s)

T. W. Yee

## References

Topp, C. W. and F. C. Leone (1955). A family of J-shaped frequency functions. Journal of the American Statistical Association, 50, 209-219.

## See Also

Topple, Triangle.

## Examples

```
tdata <- data.frame(y = rtopple(1000, shape = logitlink(1, inverse = TRUE)))
tfit <- vglm(y ~ 1, topple, data = tdata, trace = TRUE, crit = "coef")
coef(tfit, matrix = TRUE)
Coef(tfit)
```

toxop Toxoplasmosis Data

## Description

Toxoplasmosis data in 34 cities in El Salvador.

## Usage

data(toxop)

## Format

A data frame with 34 observations on the following 4 variables.
rainfall a numeric vector; the amount of rainfall in each city.
ssize a numeric vector; sample size.
cityNo a numeric vector; the city number.
positive a numeric vector; the number of subjects testing positive for the disease.

## Details

See the references for details.

## Source

See the references for details.

## References

Efron, B. (1978). Regression and ANOVA With zero-one data: measures of residual variation. Journal of the American Statistical Association, 73, 113-121.
Efron, B. (1986). Double exponential families and their use in generalized linear regression. Journal of the American Statistical Association, 81, 709-721.

## See Also

double.expbinomial.

## Examples

```
## Not run: with(toxop, plot(rainfall, positive / ssize, col = "blue"))
plot(toxop, col = "blue")
## End(Not run)
```

Triangle The Triangle Distribution

## Description

Density, distribution function, quantile function and random generation for the Triangle distribution with parameter theta.

## Usage

```
dtriangle(x, theta, lower = 0, upper = 1, log = FALSE)
ptriangle(q, theta, lower = 0, upper = 1, lower.tail = TRUE, log.p = FALSE)
qtriangle(p, theta, lower = 0, upper = 1, lower.tail = TRUE, log.p = FALSE)
rtriangle(n, theta, lower = 0, upper = 1)
```


## Arguments

| $x, q$ | vector of quantiles. |
| :--- | :--- |
| $p$ | vector of probabilities. |
| $n$ | number of observations. Same as runif. |
| theta | the theta parameter which lies between lower and upper. |
| lower, upper | lower and upper limits of the distribution. Must be finite. |

log Logical. If log = TRUE then the logarithm of the density is returned.
lower.tail, log.p
Same meaning as in pnorm or qnorm.

## Details

See triangle, the VGAM family function for estimating the parameter $\theta$ by maximum likelihood estimation.

## Value

dtriangle gives the density, ptriangle gives the distribution function, qtriangle gives the quantile function, and rtriangle generates random deviates.

## Author(s)

T. W. Yee and Kai Huang

## See Also

triangle, topple.

## Examples

```
## Not run: x <- seq(-0.1, 1.1, by = 0.01); theta <- 0.75
plot(x, dtriangle(x, theta = theta), type = "l", col = "blue", las = 1,
    main = "Blue is density, orange is cumulative distribution function",
    sub = "Purple lines are the 10,20,\ldots.,90 percentiles",
    ylim = c(0,2), ylab = "")
abline(h = 0, col = "blue", lty = 2)
lines(x, ptriangle(x, theta = theta), col = "orange")
probs <- seq(0.1, 0.9, by = 0.1)
Q <- qtriangle(probs, theta = theta)
lines(Q, dtriangle(Q, theta = theta), col = "purple", lty = 3, type = "h")
ptriangle(Q, theta = theta) - probs # Should be all zero
abline(h = probs, col = "purple", lty = 3)
## End(Not run)
```

triangle
Triangle Distribution Family Function

## Description

Estimating the parameter of the triangle distribution by maximum likelihood estimation.

## Usage

```
triangle(lower = 0, upper = 1,
        link = extlogitlink(min = 0, max = 1), itheta = NULL)
```


## Arguments

lower, upper lower and upper limits of the distribution. Must be finite. Called $A$ and $B$ respectively below.
link Parameter link function applied to the parameter $\theta$, which lies in $(A, B)$. See Links for more choices. The default constrains the estimate to lie in the interval.
itheta Optional initial value for the parameter. The default is to compute the value internally.

## Details

The triangle distribution has a probability density function that consists of two lines joined at $\theta$, which is the location of the mode. The lines intersect the $y=0$ axis at $A$ and $B$. Here, Fisher scoring is used.
On fitting, the extra slot has components called lower and upper which contains the values of the above arguments (recycled to the right length). The fitted values are the mean of the distribution, which is $(A+B+\theta) / 3$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Warning

The MLE regularity conditions do not hold for this distribution (e.g., the first derivative evaluated at the mode does not exist because it is not continuous) so that misleading inferences may result, e.g., in the summary and vcov of the object. Additionally, convergence to the MLE often appears to fail.

## Note

The response must contain values in $(A, B)$. For most data sets (especially small ones) it is very common for half-stepping to occur.
Arguments lower and upper and link must match. For example, setting lower $=0.2$ and upper $=$ 4 and link $=$ extlogitlink( $\min =0.2, \max =4.1$ ) will result in an error. Ideally link $=$ extlogitlink(min $=$ lower, max $=$ upper ) ought to work but it does not (yet)! Minimal error checking is done for this deficiency.

## Author(s)

T. W. Yee

## References

Kotz, S. and van Dorp, J. R. (2004). Beyond Beta: Other Continuous Families of Distributions with Bounded Support and Applications. Chapter 1. World Scientific: Singapore.
Nguyen, H. D. and McLachlan, G. J. (2016). Maximum likelihood estimation of triangular and polygon distributions. Computational Statistics and Data Analysis, 102, 23-36.

## See Also

Triangle, Topple, simulate.vlm.

## Examples

```
# Example 1
tdata <- data.frame(y = rtriangle(n <- 3000, theta = 3/4))
fit <- vglm(y ~ 1, triangle(link = "identitylink"), data = tdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
head(fit@extra$lower)
head(fitted(fit))
with(tdata, mean(y))
# Example 2; Kotz and van Dorp (2004), p.14
rdata <- data.frame(y = c(0.1, 0.25, 0.3, 0.4, 0.45, 0.6, 0.75, 0.8))
fit <- vglm(y ~ 1, triangle(link = "identitylink"), data = rdata, trace = TRUE,
    crit = "coef", maxit = 1000)
Coef(fit) # The MLE is the 3rd order statistic, which is 0.3.
fit <- vglm(y ~ 1, triangle(link = "identitylink"), data = rdata, trace = TRUE,
    crit = "coef", maxit = 1001)
Coef(fit) # The MLE is the 3rd order statistic, which is 0.3.
```


## Description

Density and random generation for the trivariate normal distribution distribution.

## Usage

```
dtrinorm(x1, x2, x3, mean1 = 0, mean2 = 0, mean3 = 0,
    var1 = 1, var2 = 1, var3 = 1,
    cov12 = 0, cov23 = 0, cov13 = 0, log = FALSE)
rtrinorm(n, mean1 = 0, mean2 = 0, mean3 = 0,
    var1 = 1, var2 = 1, var3 = 1,
    cov12 = 0, cov23 = 0, cov13 = 0)
```


## Arguments

$x 1, x 2, x 3 \quad$ vector of quantiles.
mean1, mean2, mean3
vectors of means.
var1, var2, var3
vectors of variances.
$\operatorname{cov} 12, \operatorname{cov} 23, \operatorname{cov} 13$
vectors of covariances.
n number of observations. Same as rnorm.
log Logical. If log = TRUE then the logarithm of the density is returned.

## Details

The default arguments correspond to the standard trivariate normal distribution with correlation parameters equal to 0 , which corresponds to three independent standard normal distributions. Let sd1 (say) be sqrt (var1) and written $\sigma_{1}$, etc. Then the general formula for each correlation coefficient is of the form $\rho_{12}=\operatorname{cov}_{12} /\left(\sigma_{1} \sigma_{2}\right)$, and similarly for the two others. Thus if the var arguments are left alone then the cov can be inputted with $\rho$ s.

## Value

dtrinorm gives the density, rtrinorm generates random deviates ( $n$ by 3 matrix).

## Warning

dtrinorm()'s arguments might change in the future! It's safest to use the full argument names to future-proof possible changes!

## Note

For rtrinorm(), if the $i$ th variance-covariance matrix is not positive-definite then the $i$ th row is all NAs.

## See Also

pnorm, trinormal, uninormal, binormal, rbinorm.

## Examples

```
## Not run: nn <- 1000
tdata <- data.frame(x2 = sort(runif(nn)))
tdata <- transform(tdata, mean1 = 1 + 2 * x2,
    mean2 = 3 + 1 * x2, mean3 = 4,
    var1 = exp( 1), var2 = exp( 1), var3 = exp(1),
    rho12 = rhobit( 1, inverse = TRUE),
    rho23 = rhobit( 1, inverse = TRUE),
    rho13 = rhobit(-1, inverse = TRUE))
ymat <- with(tdata, rtrinorm(nn, mean1, mean2, mean3,
    var1, var2, var3,
    sqrt(var1)*sqrt(var1)*rho12,
    sqrt(var2)*sqrt(var3)*rho23,
    sqrt(var1)*sqrt(var3)*rho13))
pairs(ymat, col = "blue")
## End(Not run)
```


## Description

Maximum likelihood estimation of the nine parameters of a trivariate normal distribution.

## Usage

trinormal(zero = c("sd", "rho"), eq.mean = FALSE, eq.sd = FALSE, eq.cor $=$ FALSE,
lmean1 = "identitylink", lmean2 = "identitylink",
lmean3 = "identitylink",
lsd1 = "loglink", lsd2 = "loglink", lsd3 = "loglink",
lrho12 = "rhobitlink", lrho23 = "rhobitlink", lrho13 = "rhobitlink",
imean1 = NULL, imean2 = NULL, imean3 = NULL,
isd1 = NULL, isd2 = NULL, isd3 = NULL,
irho12 = NULL, irho23 = NULL, irho13 = NULL, imethod = 1)

## Arguments

lmean1, lmean2, lmean3, lsd1, lsd2, lsd3
Link functions applied to the means and standard deviations. See Links for more choices. Being positive quantities, a log link is the default for the standard deviations.
lrho12, lrho23, lrho13
Link functions applied to the correlation parameters. See Links for more choices. By default the correlation parameters are allowed to have a value between -1 and 1 , but that may be problematic when eq. cor = TRUE because they should have a value between -0.5 and 1 .
imean1, imean2, imean3, isd1, isd2, isd3
See CommonVGAMffArguments for more information.
irho12, irho23, irho13, imethod, zero
See CommonVGAMffArguments for more information.
eq.mean, eq.sd, eq.cor
Logical. Constrain the means or the standard deviations or correlation parameters to be equal?

## Details

For the trivariate normal distribution, this fits a linear model (LM) to the means, and by default, the other parameters are intercept-only. The response should be a three-column matrix. The three correlation parameters are prefixed by rho, and the default gives them values between -1 and 1 however, this may be problematic when the correlation parameters are constrained to be equal, etc.. The fitted means are returned as the fitted values, which is in the form of a three-column matrix. Fisher scoring is implemented.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

The default parameterization does not make the estimated variance-covariance matrix positivedefinite. In order for the variance-covariance matrix to be positive-definite the quantity $1-r h o 12^{\wedge} 2$ -rho13^2-rho23^2 +2 * rho12 * rho13 * rho23 must be positive, and if eq.cor $=$ TRUE then this means that the rhos must be between -0.5 and 1 .

## Author(s)

T. W. Yee

## See Also

uninormal, binormal, rtrinorm.

## Examples

```
    set.seed(123); nn <- 1000
    tdata <- data.frame(x2 = runif(nn), x3 = runif(nn))
    tdata <- transform(tdata, y1 = rnorm(nn, 1 + 2 * x2),
        y2 = rnorm(nn, 3 + 4 * x2),
        y3 = rnorm(nn, 4 + 5 * x2))
    fit1 <- vglm(cbind(y1, y2, y3) ~ x2, data = tdata,
            trinormal(eq.sd = TRUE, eq.cor = TRUE), trace = TRUE)
    coef(fit1, matrix = TRUE)
    constraints(fit1)
    summary(fit1)
    ## Not run: # Try this when eq.sd = TRUE, eq.cor = TRUE:
    fit2 <- vglm(cbind(y1, y2, y3) ~ x2, data = tdata, stepsize = 0.25,
        trinormal(eq.sd = TRUE, eq.cor = TRUE,
            lrho12 = extlogitlink(min = -0.5),
            lrho23 = extlogitlink(min = -0.5),
            lrho13 = extlogitlink(min = -0.5)), trace = TRUE)
coef(fit2, matrix = TRUE)
## End(Not run)
```

trplot Trajectory Plot

## Description

Generic function for a trajectory plot.

## Usage

trplot(object, ...)

## Arguments

object An object for which a trajectory plot is meaningful.
$\ldots \quad$ Other arguments fed into the specific methods function of the model. They usually are graphical parameters, and sometimes they are fed into the methods function for Coef.

## Details

Trajectory plots can be defined in different ways for different models. Many models have no such notion or definition.
For quadratic and additive ordination models they plot the fitted values of two species against each other (more than two is theoretically possible, but not implemented in this software yet).

## Value

The value returned depends specifically on the methods function invoked.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2020). On constrained and unconstrained quadratic ordination. Manuscript in preparation.

## See Also

trplot.qrrvglm, perspqrrvglm, lvplot.

## Examples

```
## Not run: set.seed(123)
hspider[, 1:6] <- scale(hspider[, 1:6]) # Standardized environmental vars
p1cqo <- cqo(cbind(Alopacce, Alopcune, Alopfabr, Arctlute, Arctperi,
            Auloalbi, Pardlugu, Pardmont, Pardnigr, Pardpull,
            Trocterr, Zoraspin) ~
        WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
        poissonff, data = hspider, Crow1positive = FALSE)
nos <- ncol(depvar(p1cqo))
clr <- 1:nos # OR (1:(nos+1))[-7] to omit yellow
trplot(p1cqo, which.species = 1:3, log = "xy",
    col = c("blue", "orange", "green"), lwd = 2, label = TRUE) -> ii
legend(0.00005, 0.3, paste(ii$species[, 1], ii$species[, 2], sep = " and "),
```

lwd = 2, lty = 1, col = c("blue", "orange", "green"))
abline(a = 0, b = 1, lty = "dashed", col = "grey")
\#\# End(Not run)
trplot.arrvglm Trajectory plot for QRR-VGLMs

## Description

Produces a trajectory plot for quadratic reduced-rank vector generalized linear models (QRRVGLMs). It is only applicable for rank-1 models with argument noRRR $=\sim 1$.

## Usage

```
trplot.qrrvglm(object, which.species = NULL, add = FALSE, show.plot = TRUE,
    label.sites = FALSE, sitenames = rownames(object@y),
    axes.equal = TRUE, cex = par()$cex,
    col = 1:(nos * (nos - 1)/2), log = "",
    lty = rep_len(par()$lty, nos * (nos - 1)/2),
    lwd = rep_len(par()$lwd, nos * (nos - 1)/2),
    tcol = rep_len(par()$col, nos * (nos - 1)/2),
    xlab = NULL, ylab = NULL,
    main = "", type = "b", check.ok = TRUE, ...)
```


## Arguments

| object | Object of class "qrrvglm", i.e., a CQO object. |
| :--- | :--- |
| which.species | Integer or character vector specifying the species to be plotted. If integer, these <br> are the columns of the response matrix. If character, these must match exactly <br> with the species' names. The default is to use all species. |
| add | Logical. Add to an existing plot? If FALSE (default), a new plot is made. <br> Logical. Plot it? |
| show.plot | Logical. If TRUE, the points on the curves/trajectories are labelled with the <br> sitenames. |
| sitenames | Character vector. The names of the sites. |
| axes.equal | Logical. If TRUE, the x- and y-axes will be on the same scale. <br> Character expansion of the labelling of the site names. Used only if label.sites |
| col | is TRUE. See the cex argument in par. <br> Color of the lines. See the col argument in par. Here, nos is the number of |
| species. |  |
| $l$ Character, specifying which (if any) of the x- and y-axes are to be on a logarith- |  |

\(\left.$$
\begin{array}{ll}\text { tcol } & \begin{array}{l}\text { Color of the text for the site names. See the col argument in par. Used only if } \\
\text { label.sites is TRUE. } \\
\text { Character caption for the x-axis. By default, a suitable caption is found. See the }\end{array} \\
\text { xlab } & \begin{array}{l}\text { xlab argument in plot or title. }\end{array}
$$ <br>
ylab <br>
Character caption for the y-axis. By default, a suitable caption is found. See the <br>

xlab argument in plot or title.\end{array}\right]\)| Character, giving the title of the plot. See the main argument in plot or title. |
| :--- |
| type |
| Character, giving the type of plot. A common option is to use type="l" for |
| lines only. See the type argument of plot. |

## Details

A trajectory plot plots the fitted values of a 'second' species against a 'first' species. The argument which. species must therefore contain at least two species. By default, all of the species that were fitted in object are plotted. With more than a few species the resulting plot will be very congested, and so it is recommended that only a few species be selected for plotting.

In the above, $M$ is the number of species selected for plotting, so there will be $M(M-1) / 2$ curves/trajectories in total.
A trajectory plot will be fitted only if noRRR $=\sim 1$ because otherwise the trajectory will not be a smooth function of the latent variables.

## Value

A list with the following components.
species.names A matrix of characters giving the 'first' and 'second' species. The number of different combinations of species is given by the number of rows. This is useful for creating a legend.
sitenames A character vector of site names, sorted by the latent variable (from low to high).

## Note

Plotting the axes on a log scale is often a good idea. The use of xlim and ylim to control the axis limits is also a good idea, so as to limit the extent of the curves at low abundances or probabilities. Setting label.sites = TRUE is a good idea only if the number of sites is small, otherwise there is too much clutter.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2020). On constrained and unconstrained quadratic ordination. Manuscript in preparation.

## See Also

cqo, par, title.

## Examples

```
## Not run: set.seed(111) # This leads to the global solution
# hspider[,1:6] <- scale(hspider[,1:6]) # Standardize the environmental variables
p1 <- cqo(cbind(Alopacce, Alopcune, Alopfabr, Arctlute, Arctperi, Auloalbi,
            Pardlugu, Pardmont, Pardnigr, Pardpull, Trocterr, Zoraspin) ~
        WaterCon + BareSand + FallTwig + CoveMoss + CoveHerb + ReflLux,
        poissonff, data = hspider, trace = FALSE)
trplot(p1, which.species = 1:3, log = "xy", type = "b", lty = 1,
    main = "Trajectory plot of three hunting spiders species",
    col = c("blue","red","green"), lwd = 2, label = TRUE) -> ii
legend(0.00005, 0.3, lwd = 2, lty = 1, col = c("blue", "red", "green"),
    with(ii, paste(species.names[,1], species.names[,2], sep = " and ")))
abline(a = 0, b = 1, lty = "dashed", col = "grey") # Useful reference line
## End(Not run)
```


## Description

Density, distribution function, quantile function and random generation for the upper truncated Pareto(I) distribution with parameters lower, upper and shape.

## Usage

```
dtruncpareto(x, lower, upper, shape, log = FALSE)
ptruncpareto(q, lower, upper, shape, lower.tail = TRUE, log.p = FALSE)
qtruncpareto(p, lower, upper, shape)
rtruncpareto(n, lower, upper, shape)
```


## Arguments

| $x, q$ | vector of quantiles. |
| :--- | :---: |
| $p$ | vector of probabilities. |
| $n, \log$ | Same meaning as runif. |
| lower, upper, shape |  |

the lower, upper and shape $(k)$ parameters. If necessary, values are recycled.
lower.tail, log.p
Same meaning as in pnorm or qnorm.

## Details

See truncpareto, the VGAM family function for estimating the parameter $k$ by maximum likelihood estimation, for the formula of the probability density function and the range restrictions imposed on the parameters.

## Value

dtruncpareto gives the density, ptruncpareto gives the distribution function, qtruncpareto gives the quantile function, and rtruncpareto generates random deviates.

## Author(s)

T. W. Yee and Kai Huang

## References

Aban, I. B., Meerschaert, M. M. and Panorska, A. K. (2006). Parameter estimation for the truncated Pareto distribution, Journal of the American Statistical Association, 101(473), 270-277.

## See Also

truncpareto.

## Examples

```
    lower <- 3; upper <- 8; kay <- exp(0.5)
## Not run: xx <- seq(lower - 0.5, upper + 0.5, len = 401)
plot(xx, dtruncpareto(xx, low = lower, upp = upper, shape = kay),
    main = "Truncated Pareto density split into 10 equal areas",
    type = "l", ylim = 0:1, xlab = "x")
abline(h = 0, col = "blue", lty = 2)
qq <- qtruncpareto(seq(0.1, 0.9, by = 0.1), low = lower, upp = upper,
                    shape = kay)
lines(qq, dtruncpareto(qq, low = lower, upp = upper, shape = kay),
    col = "purple", lty = 3, type = "h")
lines(xx, ptruncpareto(xx, low = lower, upp = upper, shape = kay),
    col = "orange")
## End(Not run)
pp <- seq(0.1, 0.9, by = 0.1)
qq <- qtruncpareto(pp, lower = lower, upper = upper, shape = kay)
ptruncpareto(qq, lower = lower, upper = upper, shape = kay)
qtruncpareto(ptruncpareto(qq, lower = lower, upper = upper, shape = kay),
        lower = lower, upper = upper, shape = kay) - qq # Should be all 0
```

```
truncweibull Truncated Weibull Distribution Family Function
```


## Description

Maximum likelihood estimation of the 2-parameter Weibull distribution with lower truncation. No observations should be censored.

## Usage

truncweibull(lower.limit $=1 \mathrm{e}-5$, lAlpha = "loglink", lBetaa = "loglink", iAlpha $=$ NULL, $\quad$ iBetaa $=$ NULL, nrfs $=1$, probs.y $=c(0.2,0.5,0.8)$, imethod = 1, zero = "Betaa")

## Arguments

lower.limit Positive lower truncation limits. Recycled to the same dimension as the response, going across rows first. The default, being close to 0 , should mean effectively the same results as weibullR if there are no response values that are smaller.
lAlpha, lBetaa Parameter link functions applied to the (positive) parameters Alpha (called $\alpha$ below) and (positive) Betaa (called $\beta$ below). See Links for more choices.
iAlpha, iBetaa See CommonVGAMfffarguments.
imethod, nrfs, zero, probs.y
Details at weibullR and CommonVGAMffArguments.

## Details

MLE of the two parameters of the Weibull distribution are computed, subject to lower truncation. That is, all response values are greater than lower. limit, element-wise. For a particular observation this is any known positive value. This function is currently based directly on Wingo (1989) and his parameterization is used (it differs from weibullR.) In particular, $\beta=a$ and $\alpha=(1 / b)^{a}$ where $a$ and $b$ are as in weibullR and dweibull.

Upon fitting the extra slot has a component called lower.limit which is of the same dimension as the response. The fitted values are the mean, which are computed using pgamma. deriv and pgamma.deriv.unscaled.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

This function may be converted to the same parameterization as weibullR at any time. Yet to do: one element of the EIM may be wrong (due to two interpretations of a formula; but it seems to work). Convergence is slower than usual and this may imply something is wrong; use argument maxit. In fact, it's probably because pgamma.deriv. unscaled is inaccurate at $q=1$ and $q=2$. Also, convergence should be monitored, especially if the truncation means that a large proportion of the data is lost compared to an ordinary Weibull distribution.

## Note

More improvements need to be made, e.g., initial values are currently based on no truncation. This VGAM family function handles multiple responses.

## Author(s)

T. W. Yee

## References

Wingo, D. R. (1989). The left-truncated Weibull distribution: theory and computation. Statistical Papers, 30(1), 39-48.

## See Also

weibullR, dweibull, pgamma.deriv, pgamma.deriv.unscaled.

## Examples

```
nn <- 5000; prop.lost <- 0.40 # Proportion lost to truncation
wdata <- data.frame(x2 = runif(nn)) # Complete Weibull data
wdata <- transform(wdata,
    Betaa = exp(1)) # > 2 is okay (satisfies regularity conds)
wdata <- transform(wdata, Alpha = exp(0.5 - 1 * x2))
wdata <- transform(wdata, Shape = Betaa,
# aaa = Betaa,
# bbb = 1 / Alpha^(1 / Betaa),
    Scale = 1 / Alpha^(1 / Betaa))
wdata <- transform(wdata, y2 = rweibull(nn, shape = Shape, scale = Scale))
summary(wdata)
lower.limit2 <- with(wdata, quantile(y2, prob = prop.lost)) # Proportion lost
wdata <- subset(wdata, y2 > lower.limit2) # Smaller due to truncation
fit1 <- vglm(y2 ~ x2, maxit = 100, trace = TRUE,
    truncweibull(lower.limit = lower.limit2), data = wdata)
coef(fit1, matrix = TRUE)
summary(fit1)
vcov(fit1)
head(fit1@extra$lower.limit)
```


## Description

University California Berkeley Graduate Admissions: counts cross-classified by acceptance/rejection and gender, for the six largest departments.

## Usage

data(ucberk)

## Format

A data frame with 6 departmental groups with the following 5 columns.
m.deny Counts of men denied admission.
m.admit Counts of men admitted.
w.deny Counts of women denied admission.
w.admit Counts of women admitted.
dept Department (the six largest), called A, codeB,$\ldots$, codeF.

## Details

From Bickel et al. (1975), the data consists of applications for admission to graduate study at the University of California, Berkeley, for the fall 1973 quarter. In the admissions cycle for that quarter, the Graduate Division at Berkeley received approximately 15,000 applications, some of which were later withdrawn or transferred to a different proposed entry quarter by the applicants. Of the applications finally remaining for the fall 1973 cycle 12,763 were sufficiently complete to permit a decision. There were about 101 graduate department and interdepartmental graduate majors. There were 8442 male applicants and 4321 female applicants. About 44 percent of the males and about 35 percent of the females were admitted. The data are well-known for illustrating Simpson's paradox.

## References

Bickel, P. J., Hammel, E. A. and O’Connell, J. W. (1975). Sex bias in graduate admissions: data from Berkeley. Science, 187(4175): 398-404.
Freedman, D., Pisani, R. and Purves, R. (1998). Chapter 2 of Statistics, 3rd. ed., W. W. Norton \& Company.

## Examples

```
summary(ucberk)
```


## Description

Maximum likelihood estimation of the two parameters of a univariate normal distribution.

## Usage

uninormal(lmean = "identitylink", lsd = "loglink", lvar = "loglink", var.arg $=$ FALSE, imethod $=1$, isd $=$ NULL, parallel = FALSE, smallno = 1e-05, zero = if (var.arg) "var" else "sd")

## Arguments

lmean, lsd, lvar
Link functions applied to the mean and standard deviation/variance. See Links for more choices. Being positive quantities, a $\log$ link is the default for the standard deviation and variance (see var.arg).
var.arg Logical. If TRUE then the second parameter is the variance and lsd and esd are ignored, else the standard deviation is used and lvar and evar are ignored.
smallno Numeric, positive but close to 0 . Used specifically for quasi-variances; if the link for the mean is explink then any non-positive value of eta is replaced by this quantity (hopefully, temporarily and only during early iterations).
imethod, parallel, isd, zero
See CommonVGAMffArguments for more information. If lmean = loglink then try imethod $=2$. If parallel $=$ TRUE then the parallelism constraint is not applied to the intercept.

## Details

This fits a linear model (LM) as the first linear/additive predictor. So, by default, this is just the mean. By default, the $\log$ of the standard deviation is the second linear/additive predictor. The Fisher information matrix is diagonal. This VGAM family function can handle multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

gaussianff() was deprecated but has been brought back into VGAM nominally. It should be called Mickey Mouse. It gives a warning and calls uninormal instead (hopefully all the arguments should pass in correctly). Users should avoid calling gaussianff(); use glm with gaussian instead. It is dangerous to treat what is an uninormal fit as a gaussianff() object.

## Note

Yet to do: allow an argument such as eq. sd that enables the standard devations to be the same. Also, this function used to be called normal1() too, but it has been decommissioned.

## Author(s)

T. W. Yee

## References

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011). Statistical Distributions, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

## See Also

posnormal, mix2normal, ordsup, normal.vcm, Qvar, tobit, cens.normal, foldnormal, skewnormal, double.cens.normal, SURff, AR1, huber2, studentt, binormal, trinormal, dnorm, simulate.vlm, hdeff.vglm.

## Examples

```
udata <- data.frame(x2 = rnorm(nn <- 200))
udata <- transform(udata,
    y1 = rnorm(nn, m = 1 - 3*x2, sd = exp(1 + 0.2*x2)),
    y2a = rnorm(nn, m = 1 + 2*x2, sd = exp(1 + 2.0*x2)^0.5),
    y2b = rnorm(nn, m = 1 + 2*x2, sd = exp(1 + 2.0*x2)^0.5))
fit1 <- vglm(y1 ~ x2, uninormal(zero = NULL), data = udata, trace = TRUE)
coef(fit1, matrix = TRUE)
fit2 <- vglm(cbind(y2a, y2b) ~ x2, data = udata, trace = TRUE,
    uninormal(var = TRUE, parallel = TRUE ~ x2,
        zero = NULL))
    coef(fit2, matrix = TRUE)
    # Generate data from N(mu = theta = 10, sigma = theta) and estimate theta.
    theta <- 10
    udata <- data.frame(y3 = rnorm(100, m = theta, sd = theta))
    fit3a <- vglm(y3 ~ 1, uninormal(lsd = "identitylink"), data = udata,
        constraints = list("(Intercept)" = rbind(1, 1)))
    fit3b <- vglm(y3 ~ 1, uninormal(lsd = "identitylink", parallel = TRUE ~ 1,
                                    zero = NULL), data = udata)
    coef(fit3a, matrix = TRUE)
    coef(fit3b, matrix = TRUE) # Same as fit3a
```

    UtilitiesVGAM Utility Functions for the VGAM Package
    
## Description

A set of common utility functions used by VGAM family functions.

## Usage

param.names(string, $S=1$, skip1 = FALSE, sep = "")
$\operatorname{dimm}(M, h b w=M)$
interleave.VGAM(.M, M1, inverse = FALSE)

## Arguments

string Character. Name of the parameter.
M, .M Numeric. The total number of linear/additive predictors, called M. By total, it is meant summed over the number of responses. Often, $M$ is the total number of parameters to be estimated (but this is not the same as the number of regression coefficients, unless the RHS of the formula is an intercept-only). The use of . $M$ is unfortunate, but it is a compromise solution to what is presented in Yee (2015). Ideally, . M should be just M.

M1 Numeric. The number of linear/additive predictors for one response, called $M_{1}$. This argument used to be called $M$, but is now renamed properly.
inverse Logical. Useful for the inverse function of interleave.VGAM().
$S \quad$ Numeric. The number of responses.
skip1, sep The former is logical; should one skip (or omit) " 1 " when $S=1$ ? The latter is the same argument as paste.
hbw Numeric. The half-bandwidth, which measures the number of bands emanating from the central diagonal band.

## Details

See Yee (2015) for some details about some of these functions.

## Value

For param. names(), this function returns the parameter names for $S$ responses, i.e., string is returned unchanged if $S=1$, else paste(string, $1: \mathrm{S}$, sep $="$ ").
For $\operatorname{dimm}()$, this function returns the number of elements to be stored for each of the working weight matrices. They are represented as columns in the matrix wz in e.g., vglm.fit(). See the matrix-band format described in Section 18.3.5 of Yee (2015).

For interleave.VGAM(), this function returns a reordering of the linear/additive predictors depending on the number of responses. The arguments presented in Table 18.5 may not be valid in your version of Yee (2015).

## Author(s)

T. W. Yee. Victor Miranda added the inverse argument to interleave. VGAM().

## References

Yee, T. W. (2015). Vector Generalized Linear and Additive Models: With an Implementation in R. New York, USA: Springer.

## See Also

CommonVGAMffArguments, VGAM-package.

## Examples

```
param.names("shape", 1) # "shape"
param.names("shape", 3) # c("shape1", "shape2", "shape3")
dimm(3, hbw = 1) # Diagonal matrix; the 3 elements need storage.
dimm(3) # A general 3 x 3 symmetrix matrix has 6 unique elements.
dimm(3, hbw = 2) # Tridiagonal matrix; the 3-3 element is 0 and unneeded.
M1 <- 2; ncoly <- 3; M <- ncoly * M1
mynames1 <- param.names("location", ncoly)
mynames2 <- param.names("scale", ncoly)
(parameters.names <- c(mynames1, mynames2)[interleave.VGAM(M, M1 = M1)])
# The following is/was in Yee (2015) and has a poor/deceptive style:
(parameters.names <- c(mynames1, mynames2)[interleave.VGAM(M, M = M1)])
parameters.names[interleave.VGAM(M, M1 = M1, inverse = TRUE)]
```

V1
V1 Flying-Bombs Hits in London

## Description

A small count data set. During WWII V1 flying-bombs were fired from sites in France (Pas-deCalais) and Dutch coasts towards London. The number of hits per square grid around London were recorded.

## Usage

data(V1)

## Format

A data frame with the following variables.
hits Values between 0 and 4, and 7. Actually, the 7 is really imputed from the paper (it was recorded as "5 and over").
ofreq Observed frequency, i.e., the number of grids with that many hits.

## Details

The data concerns 576 square grids each of 0.25 square kms about south London. The area was selected comprising 144 square kms over which the basic probability function of the distribution was very nearly constant. V1s, which were one type of flying-bomb, were a "Vergeltungswaffen" or vengeance weapon fired during the summer of 1944 at London. The V1s were informally called

Buzz Bombs or Doodlebugs, and they were pulse-jet-powered with a warhead of 850 kg of explosives. Over 9500 were launched at London, and many were shot down by artillery and the RAF. Over the period considered the total number of bombs within the area was 537.
It was asserted that the bombs tended to be grouped in clusters. However, a basic Poisson analysis shows this is not the case. Their guidance system being rather primitive, the data is consistent with a Poisson distribution (random).

Compared to Clarke (1946), the more modern analysis of Shaw and Shaw (2019). shows a higher density of hits in south London, hence the distribution is not really uniform over the entire region.

## Source

Clarke, R. D. (1946). An application of the Poisson distribution. Journal of the Institute of Actuaries, 72(3), 481.

## References

Shaw, L. P. and Shaw, L. F. (2019). The flying bomb and the actuary. Significance, 16(5): 12-17.

## See Also

V2, poissonff.

## Examples

## V1

mean(with(V1, rep(hits, times = ofreq))) $\operatorname{var}(w i t h(V 1, r e p(h i t s$, times $=o f r e q)))$ sum(with(V1, rep(hits, times = ofreq)))
\#\# Not run: barplot(with(V1, ofreq), names.arg = as.character(with(V1, hits)), main = "London V1 buzz bomb hits", col = "lightblue", las = 1, ylab = "Frequency", xlab = "Hits")
\#\# End(Not run)

V2
V2 Missile Hits in London

## Description

A small count data set. During WWII V2 missiles were fired from the continent mainly towards London. The number of hits per square grid around London were recorded.

## Usage

data(V2)

## Format

A data frame with the following variables.
hits Values between 0 and 3 .
ofreq Observed frequency, i.e., the number of grids with that many hits.

## Details

The data concerns 408 square grids each of 0.25 square kms about south London (south of the River Thames). They were picked in a rectangular region of 102 square kilometres where the density of hits were roughly uniformly distributed. The data is somewhat comparable to V 1 albeit is a smaller data set.

## Source

Shaw, L. P. and Shaw, L. F. (2019). The flying bomb and the actuary. Significance, 16(5): 12-17.

## See Also

V1, poissonff.

## Examples

V2
mean(with(V2, rep(hits, times = ofreq))) $\operatorname{var}($ with (V2, $\operatorname{rep}(h i t s$, times $=$ ofreq))) sum(with(V2, rep(hits, times $=o f r e q))$ )
\#\# Not run: barplot(with(V2, ofreq),
names.arg $=$ as.character(with(V2, hits)),
main = "London V2 rocket hits",
col = "lightgreen", las = 1,
ylab = "Frequency", xlab = "Hits")
\#\# End(Not run)
vcovvlm
Calculate Variance-Covariance Matrix for a Fitted VLM or RRVGLM or QRR-VGLM Object

## Description

Returns the variance-covariance matrix of the parameters of a fitted vlm-class object or a fitted rrvglm-class object.

## Usage

vcov(object, ...)
vcovvlm(object, dispersion $=$ NULL, untransform $=$ FALSE, complete $=$ TRUE)
vcovqrrvglm(object, ...)

## Arguments

| object | A fitted model object, having class vlm-class or rrvglm-class or qrrvglm-class <br> or a superclass of such. The former includes a vglm object. |
| :--- | :--- |
| dispersion | Numerical. A value may be specified, else it is estimated for quasi-GLMs (e.g., <br> method of moments). For almost all other types of VGLMs it is usually unity. <br> The value is multiplied by the raw variance-covariance matrix. |
| untransform | logical. For intercept-only models with trivial constraints; if set TRUE then the <br> parameter link function is inverted to give the answer for the untransformed/raw <br> parameter. |
| complete | An argument that is currently ignored. Added only so that linearHypothesis <br> can be called. |
| $\ldots$ | Same as vcov. |

## Details

This methods function is based on the QR decomposition of the (large) VLM model matrix and working weight matrices. Currently vcovvlm operates on the fundamental vlm-class objects because pretty well all modelling functions in VGAM inherit from this. Currently vcovrrvglm is not entirely reliable because the elements of the $\mathbf{A}-\mathbf{C}$ part of the matrix sometimes cannot be computed very accurately, so that the entire matrix is not positive-definite.

For "qrrvglm" objects, vcovqrrvglm is currently working with Rank $=1$ objects or when I. tolerances $=$ TRUE. Then the answer is conditional given $\mathbf{C}$. The code is based on model.matrixarrvglm so that the dimnames are the same.

## Value

Same as vcov.

## Note

For some models inflated standard errors can occur, such as parameter estimates near the boundary of the parameter space. Detection for this is available for some models using hdeff.vglm, which tests for an Hauck-Donner effect (HDE) for each regression coefficient. If the HDE is present, using lrt.stat.vlm should return more accurate p-values.

## Author(s)

Thomas W. Yee

## See Also

```
confintvglm, summaryvglm, vcov, hdeff.vglm, lrt.stat.vlm, model.matrixqrrvglm.
```


## Examples

```
ndata <- data.frame(x2 = runif(nn <- 300))
ndata <- transform(ndata, y1 = rnbinom(nn, mu = exp(3+x2), size = exp(1)),
    y2 = rnbinom(nn, mu = exp(2-x2), size = exp(0)))
fit1 <- vglm(cbind(y1, y2) ~ x2, negbinomial, data = ndata, trace = TRUE)
fit2 <- rrvglm(y1 ~ x2, negbinomial(zero = NULL), data = ndata)
coef(fit1, matrix = TRUE)
vcov(fit1)
vcov(fit2)
```

```
venice Venice Maximum Sea Levels Data
```


## Description

Some sea levels data sets recorded at Venice, Italy.

## Usage

data(venice)
data(venice90)

## Format

venice is a data frame with 51 observations on the following 11 variables. It concerns the maximum heights of sea levels between 1931 and 1981.
year a numeric vector.
$\mathbf{r} 1, \mathbf{r 2 , r 3 , r 4 , r 5 , r 6 , r 7 , r 8 , r 9 , r 1 0}$ numeric vectors; $r 1$ is the highest recorded value, $r 2$ is the second highest recorded value, etc.
venice 90 is a data frame with 455 observations on the following 7 variables.
year, month, day, hour numeric vectors; actual time of the recording.
sealevel numeric; sea level.
ohour numeric; number of hours since the midnight of 31 Dec 1939 and 1 Jan 1940.
Year numeric vector; approximate year as a real number. The formula is start.year + ohour / $(365.26 * 24)$ where start.year is 1940 . One can treat Year as continuous whereas year can be treated as both continuous and discrete.

## Details

Sea levels are in cm. For venice90, the value 0 corresponds to a fixed reference point (e.g., the mean sea level in 1897 at an old palace of Venice). Clearly since the relative (perceived) mean sea level has been increasing in trend over time (more than an overall 0.4 m increase by 2010), therefore the value 0 is (now) a very low and unusual measurement.
For venice, in 1935 only the top six values were recorded.

For venice90, this is a subset of a data set provided by Paolo Pirazzoli consisting of hourly sea levels from 1940 to 2009. Values greater than 90 cm were extracted, and then declustered (each cluster provides no more than one value, and each value is at least 24 hours apart). Thus the values are more likely to be independent. Of the original $(2009-1940+1) * 365.26 * 24$ values about 7 percent of these comprise venice90.
Yet to do: check for consistency between the data sets. Some external data sets elsewhere have some extremes recorded at times not exactly on the hour.

## Source

Pirazzoli, P. (1982) Maree estreme a Venezia (periodo 1872-1981). Acqua Aria, 10, 1023-1039.
Thanks to Paolo Pirazzoli and Alberto Tomasin for the venice90 data.

## References

Smith, R. L. (1986). Extreme value theory based on the $r$ largest annual events. Journal of Hydrology, 86, 27-43.
Battistin, D. and Canestrelli, P. (2006). La serie storica delle maree a Venezia, 1872-2004 (in Italian), Comune di Venezia. Istituzione Centro Previsione e Segnalazioni Maree.

## See Also

guplot, gev, gpd.

## Examples

```
## Not run:
matplot(venice[["year"]], venice[, -1], xlab = "Year",
    ylab = "Sea level (cm)", type = "l")
ymat <- as.matrix(venice[, paste("r", 1:10, sep = "")])
fit1 <- vgam(ymat ~ s(year, df = 3), gumbel(R = 365, mpv = TRUE),
            data = venice, trace = TRUE, na.action = na.pass)
head(fitted(fit1))
par(mfrow = c(2, 1), xpd = TRUE)
plot(fit1, se = TRUE, lcol = "blue", llwd = 2, slty = "dashed")
par(mfrow = c(1,1), bty = "l", xpd = TRUE, las = 1)
qtplot(fit1, mpv = TRUE, lcol = c(1, 2, 5), tcol = c(1, 2, 5),
    llwd = 2, pcol = "blue", tadj = 0.1)
plot(sealevel ~ Year, data = venice90, type = "h", col = "blue")
summary(venice90)
dim(venice90)
round(100 * nrow(venice90) / ((2009 - 1940 + 1) * 365.26 * 24), digits = 3)
## End(Not run)
```

vgam Fitting Vector Generalized Additive Models

## Description

Fit a vector generalized additive model (VGAM). Both 1st-generation VGAMs (based on backfitting) and 2nd-generation VGAMs (based on P-splines, with automatic smoothing parameter selection) are implemented. This is a large class of models that includes generalized additive models (GAMs) and vector generalized linear models (VGLMs) as special cases.

## Usage

```
vgam(formula, family = stop("argument 'family' needs to be assigned"),
    data = list(), weights = NULL, subset = NULL,
    na.action = na.fail, etastart = NULL, mustart = NULL,
    coefstart = NULL, control = vgam.control(...), offset = NULL,
    method = "vgam.fit", model = FALSE, x.arg = TRUE, y.arg = TRUE,
    contrasts = NULL, constraints = NULL,
    extra = list(), form2 = NULL, qr.arg = FALSE, smart = TRUE, ...)
```


## Arguments

formula a symbolic description of the model to be fit. The RHS of the formula is applied to each linear/additive predictor, and should include at least one sm. os term or sm.ps term or s term. Mixing both together is not allowed. Different variables in each linear/additive predictor can be chosen by specifying constraint matrices.
family Same as for vglm.
data an optional data frame containing the variables in the model. By default the variables are taken from environment(formula), typically the environment from which vgam is called.
weights, subset, na.action
Same as for vglm. Note that subset may be unreliable and to get around this problem it is best to use subset to create a new smaller data frame and feed in the smaller data frame. See below for an example. This is a bug that needs fixing.
etastart, mustart, coefstart
Same as for vglm.
control a list of parameters for controlling the fitting process. See vgam.control for details.
method the method to be used in fitting the model. The default (and presently only) method vgam.fit uses iteratively reweighted least squares (IRLS).
constraints, model, offset
Same as for vglm.

```
x.arg, y.arg logical values indicating whether the model matrix and response vector/matrix
            used in the fitting process should be assigned in the x and y slots. Note the
            model matrix is the LM model matrix; to get the VGAM model matrix type
                model.matrix(vgamfit) where vgamfit is a vgam object.
contrasts, extra, form2, qr.arg, smart
            Same as for vglm.
... further arguments passed into vgam.control.
```


## Details

A vector generalized additive model (VGAM) is loosely defined as a statistical model that is a function of $M$ additive predictors. The central formula is given by

$$
\eta_{j}=\sum_{k=1}^{p} f_{(j) k}\left(x_{k}\right)
$$

where $x_{k}$ is the $k$ th explanatory variable (almost always $x_{1}=1$ for the intercept term), and $f_{(j) k}$ are smooth functions of $x_{k}$ that are estimated by smoothers. The first term in the summation is just the intercept. Currently two types of smoothers are implemented: s represents the older and more traditional one, called a vector (cubic smoothing spline) smoother and is based on Yee and Wild (1996); it is more similar to the R package gam. The newer one is represented by sm. os and sm.ps, and these are based on O-splines and P-splines-they allow automatic smoothing parameter selection; it is more similar to the R package mgev.
In the above, $j=1, \ldots, M$ where $M$ is finite. If all the functions are constrained to be linear then the resulting model is a vector generalized linear model (VGLM). VGLMs are best fitted with vglm.
Vector (cubic smoothing spline) smoothers are represented by s() (see s). Local regression via lo() is not supported. The results of vgam will differ from the gam() (in the gam) because vgam() uses a different knot selection algorithm. In general, fewer knots are chosen because the computation becomes expensive when the number of additive predictors $M$ is large.
Second-generation VGAMs are based on the O-splines and P-splines. The latter is due to Eilers and Marx (1996). Backfitting is not required, and estimation is performed using IRLS. The function sm. os represents a smart implementation of O-splines. The function sm.ps represents a smart implementation of P-splines. Written G2-VGAMs or P-VGAMs, this methodology should not be used unless the sample size is reasonably large. Usually an UBRE predictive criterion is optimized (at each IRLS iteration) because the scale parameter for VGAMs is usually assumed to be known. This search for optimal smoothing parameters does not always converge, and neither is it totally reliable. G2-VGAMs implicitly set criterion = "coefficients" so that convergence occurs when the change in the regression coefficients between 2 IRLS iterations is sufficiently small. Otherwise the search for the optimal smoothing parameters might cause the log-likelihood to decrease between 2 IRLS iterations. Currently outer iteration is implemented, by default, rather than performance iteration because the latter is more easy to converge to a local solution; see Wood (2004) for details. One can use performance iteration by setting Maxit. outer = 1 in vgam. control.

The underlying algorithm of VGAMs is IRLS. First-generation VGAMs (called G1-VGAMs) are estimated by modified vector backfitting using vector splines. O-splines are used as the basis functions for the vector (smoothing) splines, which are a lower dimensional version of natural B-splines. The function vgam.fit() actually does the work. The smoothing code is based on F. O'Sullivan's BART code.

A closely related methodology based on VGAMs called constrained additive ordination (CAO) first forms a linear combination of the explanatory variables (called latent variables) and then fits a GAM to these. This is implemented in the function cao for a very limited choice of family functions.

## Value

For G1-VGAMs and G2-VGAMs, an object of class "vgam" or "pvgam" respectively (see vgam-class and pvgam-class for further information).

## WARNING

For G1-VGAMs, currently vgam can only handle constraint matrices cmat, say, such that crossprod (cmat) is diagonal. It can be detected by is.buggy. VGAMs with constraint matrices that have nonorthogonal columns should be fitted with sm. os or sm.ps terms instead of s.
See warnings in vglm. control.

## Note

This function can fit a wide variety of statistical models. Some of these are harder to fit than others because of inherent numerical difficulties associated with some of them. Successful model fitting benefits from cumulative experience. Varying the values of arguments in the VGAM family function itself is a good first step if difficulties arise, especially if initial values can be inputted. A second, more general step, is to vary the values of arguments in vgam. control. A third step is to make use of arguments such as etastart, coefstart and mustart.
Some VGAM family functions end in " $f f$ " to avoid interference with other functions, e.g., binomialff, poissonff. This is because VGAM family functions are incompatible with glm (and also gam in the gam library and gam in the mgev library).
The smart prediction (smartpred) library is packed with the VGAM library.
The theory behind the scaling parameter is currently being made more rigorous, but it it should give the same value as the scale parameter for GLMs.

## Author(s)

Thomas W. Yee

## References

Wood, S. N. (2004). Stable and efficient multiple smoothing parameter estimation for generalized additive models. J. Amer. Statist. Assoc., 99(467): 673-686.
Yee, T. W. and Wild, C. J. (1996). Vector generalized additive models. Journal of the Royal Statistical Society, Series B, Methodological, 58, 481-493.
Yee, T. W. (2008). The VGAM Package. R News, 8, 28-39.
Yee, T. W. (2015). Vector Generalized Linear and Additive Models: With an Implementation in R. New York, USA: Springer.
Yee, T. W. (2016). Comments on "Smoothing parameter and model selection for general smooth models" by Wood, S. N. and Pya, N. and Safken, N., J. Amer. Statist. Assoc., 110(516).

## See Also

is.buggy, vgam. control, vgam-class, vglmff-class, plotvgam, summaryvgam, summarypvgam, sm.os, sm.ps, s, magic. vglm, vsmooth.spline, cao.

## Examples

```
# Nonparametric proportional odds model
pneumo <- transform(pneumo, let = log(exposure.time))
vgam(cbind(normal, mild, severe) ~ s(let),
    cumulative(parallel = TRUE), data = pneumo, trace = TRUE)
# Nonparametric logistic regression
hfit <- vgam(agaaus ~ s(altitude, df = 2), binomialff, data = hunua)
## Not run: plot(hfit, se = TRUE)
phfit <- predict(hfit, type = "terms", raw = TRUE, se = TRUE)
names(phfit)
head(phfit$fitted)
head(phfit$se.fit)
phfit$df
phfit$sigma
# Fit two species simultaneously
hfit2 <- vgam(cbind(agaaus, kniexc) ~ s(altitude, df = c(2, 3)),
    binomialff(multiple.responses = TRUE), data = hunua)
coef(hfit2, matrix = TRUE) # Not really interpretable
## Not run:
plot(hfit2, se = TRUE, overlay = TRUE, lcol = 3:4, scol = 3:4)
000 <- with(hunua, order(altitude))
with(hunua, matplot(altitude[ooo], fitted(hfit2)[ooo,], ylim = c(0, 0.8),
    xlab = "Altitude (m)", ylab = "Probability of presence", las = 1,
    main = "Two plant species' response curves", type = "l", lwd = 2))
with(hunua, rug(altitude))
## End(Not run)
# The 'subset' argument does not work here. Use subset() instead.
set.seed(1)
zdata <- data.frame(x2 = runif(nn <- 500))
zdata <- transform(zdata, y = rbinom(nn, 1, 0.5))
zdata <- transform(zdata, subS = runif(nn) < 0.7)
sub.zdata <- subset(zdata, subS) # Use this instead
if (FALSE)
    fit4a <- vgam(cbind(y, y) ~ s(x2, df = 2),
                            binomialff(multiple.responses = TRUE),
                            data = zdata, subset = subS) # This fails!!!
fit4b <- vgam(cbind(y, y) ~ s(x2, df = 2),
        binomialff(multiple.responses = TRUE),
        data = sub.zdata) # This succeeds!!!
fit4c <- vgam(cbind(y, y) ~ sm.os(x2),
        binomialff(multiple.responses = TRUE),
        data = sub.zdata) # This succeeds!!!
## Not run: par(mfrow = c(2, 2))
```

vgam-class

```
plot(fit4b, se = TRUE, shade = TRUE, shcol = "pink")
plot(fit4c, se = TRUE, shade = TRUE, shcol = "pink")
## End(Not run)
```

```
vgam-class Class "vgam"
```


## Description

Vector generalized additive models.

## Objects from the Class

Objects can be created by calls of the form vgam(...).

## Slots

nl.chisq: Object of class "numeric". Nonlinear chi-squared values.
$\mathrm{nl} . \mathrm{df}$ : Object of class "numeric". Nonlinear chi-squared degrees of freedom values.
spar: Object of class "numeric" containing the (scaled) smoothing parameters.
s.xargument: Object of class "character" holding the variable name of any s() terms.
var: Object of class "matrix" holding approximate pointwise standard error information.
Bspline: Object of class "list" holding the scaled (internal and boundary) knots, and the fitted B-spline coefficients. These are used for prediction.
extra: Object of class "list"; the extra argument on entry to vglm. This contains any extra information that might be needed by the family function.
family: Object of class "vglmff". The family function.
iter: Object of class "numeric". The number of IRLS iterations used.
predictors: Object of class "matrix" with $M$ columns which holds the $M$ linear predictors.
assign: Object of class "list", from class "vlm". This named list gives information matching the columns and the (LM) model matrix terms.
call: Object of class "call", from class "vlm". The matched call.
coefficients: Object of class "numeric", from class "vlm". A named vector of coefficients.
constraints: Object of class "list", from class "vlm". A named list of constraint matrices used in the fitting.
contrasts: Object of class "list", from class "vlm". The contrasts used (if any).
control: Object of class "list", from class "vlm". A list of parameters for controlling the fitting process. See vglm. control for details.
criterion: Object of class "list", from class "vlm". List of convergence criterion evaluated at the final IRLS iteration.
df.residual: Object of class "numeric", from class "vlm". The residual degrees of freedom.
df.total: Object of class "numeric", from class "vlm". The total degrees of freedom.
dispersion: Object of class "numeric", from class "vlm". The scaling parameter.
effects: Object of class "numeric", from class "vlm". The effects.
fitted.values: Object of class "matrix", from class "vlm". The fitted values. This is usually the mean but may be quantiles, or the location parameter, e.g., in the Cauchy model.
misc: Object of class "list", from class "vlm". A named list to hold miscellaneous parameters.
model: Object of class "data.frame", from class "vlm". The model frame.
na. action: Object of class "list", from class "vlm". A list holding information about missing values.
offset: Object of class "matrix", from class "vlm". If non-zero, a $M$-column matrix of offsets. post: Object of class "list", from class "vlm" where post-analysis results may be put.
preplot: Object of class "list", from class "vlm" used by plotvgam; the plotting parameters may be put here.
prior.weights: Object of class "matrix", from class "vlm" holding the initially supplied weights.
qr: Object of class "list", from class "vlm". QR decomposition at the final iteration.
R: Object of class "matrix", from class "vlm". The $\mathbf{R}$ matrix in the QR decomposition used in the fitting.
rank: Object of class "integer", from class "vlm". Numerical rank of the fitted model.
residuals: Object of class "matrix", from class "vlm". The working residuals at the final IRLS iteration.

ResSS: Object of class "numeric", from class "vlm". Residual sum of squares at the final IRLS iteration with the adjusted dependent vectors and weight matrices.
smart.prediction: Object of class "list", from class "vlm". A list of data-dependent parameters (if any) that are used by smart prediction.
terms: Object of class "list", from class "vlm". The terms object used.
weights: Object of class "matrix", from class "vlm". The weight matrices at the final IRLS iteration. This is in matrix-band form.
$x$ : Object of class "matrix", from class "vlm". The model matrix (LM, not VGLM).
xlevels: Object of class "list", from class "vlm". The levels of the factors, if any, used in fitting.
$y$ : Object of class "matrix", from class "vlm". The response, in matrix form.
Xm2: Object of class "matrix", from class "vlm". See vglm-class).
Ym2: Object of class "matrix", from class "vlm". See vglm-class).
callXm2: Object of class "call", from class "vlm". The matched call for argument form2.

## Extends

Class "vglm", directly. Class "vlm", by class "vglm".

## Methods

cdf signature (object $=$ "vglm"): cumulative distribution function. Useful for quantile regression and extreme value data models.
deplot signature(object = "vglm"): density plot. Useful for quantile regression models.
deviance signature (object = "vglm"): deviance of the model (where applicable).
plot signature( $\mathrm{x}=$ " vglm"): diagnostic plots.
predict signature(object $=$ "vglm"): extract the additive predictors or predict the additive predictors at a new data frame.
print signature ( $x=$ "vglm" $)$ : short summary of the object.
qtplot signature (object = "vglm"): quantile plot (only applicable to some models).
resid signature (object $=$ "vglm") : residuals. There are various types of these.
residuals signature (object $=$ "vglm"): residuals. Shorthand for resid.
rlplot signature (object = "vglm"): return level plot. Useful for extreme value data models.
summary signature(object = "vglm"): a more detailed summary of the object.

## Note

VGAMs have all the slots that vglm objects have (vglm-class), plus the first few slots described in the section above.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Wild, C. J. (1996). Vector generalized additive models. Journal of the Royal Statistical Society, Series B, Methodological, 58, 481-493.

## See Also

vgam. control, vglm, s, vglm-class, vglmff-class.

## Examples

```
# Fit a nonparametric proportional odds model
pneumo <- transform(pneumo, let = log(exposure.time))
vgam(cbind(normal, mild, severe) ~ s(let),
    cumulative(parallel = TRUE), data = pneumo)
```

```
vgam.control Control Function for vgam()
```


## Description

Algorithmic constants and parameters for running vgam are set using this function.

## Usage

```
vgam.control(all.knots = FALSE, bf.epsilon = 1e-07, bf.maxit = 30,
checkwz=TRUE, Check.rank = TRUE, Check.cm.rank = TRUE,
criterion \(=\) names(.min.criterion.VGAM),
epsilon \(=1 \mathrm{e}-07\), maxit \(=30\), Maxit.outer \(=10\),
noWarning = FALSE,
na.action = na.fail,
nk = NULL, save.weights = FALSE, se.fit = TRUE,
trace \(=\) FALSE, wzepsilon \(=\).Machine\$double.eps^0.75,
xij = NULL, gamma.arg = 1, ...)
```


## Arguments

all.knots logical indicating if all distinct points of the smoothing variables are to be used as knots. By default, all. knots=TRUE for $n \leq 40$, and for $n>40$, the number of knots is approximately $40+(n-40)^{0.25}$. This increases very slowly with $n$ so that the number of knots is approximately between 50 and 60 for large $n$.
bf.epsilon tolerance used by the modified vector backfitting algorithm for testing convergence. Must be a positive number.
bf.maxit maximum number of iterations allowed in the modified vector backfitting algorithm. Must be a positive integer.
checkwz logical indicating whether the diagonal elements of the working weight matrices should be checked whether they are sufficiently positive, i.e., greater than wzepsilon. If not, any values less than wzepsilon are replaced with this value.

Check.rank, Check.cm.rank
See vglm. control.
criterion character variable describing what criterion is to be used to test for convergence. The possibilities are listed in .min.criterion.VGAM, but most family functions only implement a few of these.
epsilon positive convergence tolerance epsilon. Roughly speaking, the Newton-Raphson/Fisher-scoring/local-scoring iterations are assumed to have converged when two successive criterion values are within epsilon of each other.
maxit maximum number of Newton-Raphson/Fisher-scoring/local-scoring iterations allowed.

Maxit.outer maximum number of outer iterations allowed when there are sm.os or sm.ps terms. See vgam for a little information about the default outer iteration. Note

|  | that one can use performance iteration by setting Maxit. outer $=1$; then the smoothing parameters will be automatically chosen at each IRLS iteration (some specific programming allows this). |
| :---: | :---: |
| na.action | how to handle missing values. Unlike the SPLUS gam function, vgam cannot handle NAs when smoothing. |
| nk | vector of length $d$ containing positive integers. where $d$ be the number of $s$ terms in the formula. Recycling is used if necessary. The $i$ th value is the number of B-spline coefficients to be estimated for each component function of the $i$ th s() term. nk differs from the number of knots by some constant. If specified, nk overrides the automatic knot selection procedure. |
| save.weights | logical indicating whether the weights slot of a "vglm" object will be saved on the object. If not, it will be reconstructed when needed, e.g., summary. |
| se.fit | logical indicating whether approximate pointwise standard errors are to be saved on the object. If TRUE, then these can be plotted with plot (.. , se $=$ TRUE). |
| trace | logical indicating if output should be produced for each iteration. |
| wzepsilon | Small positive number used to test whether the diagonals of the working weight matrices are sufficiently positive. |
| noWarning | Same as vglm. control. |
| xij | Same as vglm. control. |
| gamma.arg | Numeric; same as gamma in magic. Inflation factor for optimizing the UBRE/GCV criterion. If given, a suggested value is 1.4 to help avoid overfitting, based on the work of Gu and co-workers (values between 1.2 and 1.4 appeared reasonable, based on simulations). A warning may be given if the value is deemed out-of-range. |
|  | other parameters that may be picked up from control functions that are specific to the VGAM family function. |

## Details

Most of the control parameters are used within vgam. fit and you will have to look at that to understand the full details. Many of the control parameters are used in a similar manner by vglm.fit ( vglm ) because the algorithm (IRLS) is very similar.

Setting save.weights=FALSE is useful for some models because the weights slot of the object is often the largest and so less memory is used to store the object. However, for some VGAM family function, it is necessary to set save.weights=TRUE because the weights slot cannot be reconstructed later.

## Value

A list with components matching the input names. A little error checking is done, but not much. The list is assigned to the control slot of vgam objects.

## Warning

See vglm. control.

Note
vgam does not implement half-stepsizing, therefore parametric models should be fitted with vglm. Also, vgam is slower than vglm too.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Wild, C. J. (1996). Vector generalized additive models. Journal of the Royal Statistical Society, Series B, Methodological, 58, 481-493.

## See Also

vgam, vglm. control, vsmooth.spline, vglm.

## Examples

pneumo <- transform(pneumo, let $=\log ($ exposure.time))
vgam(cbind(normal, mild, severe) ~ s(let, df = 2), multinomial, data $=$ pneumo, trace $=$ TRUE, eps $=1 \mathrm{e}-4$, maxit $=10$ )

## Description

vglm fits vector generalized linear models (VGLMs). This very large class of models includes generalized linear models (GLMs) as a special case.

## Usage

```
vglm(formula, family = stop("argument 'family' needs to be assigned"),
    data = list(), weights = NULL, subset = NULL,
    na.action = na.fail, etastart = NULL, mustart = NULL,
    coefstart = NULL, control = vglm.control(...), offset = NULL,
    method = "vglm.fit", model = FALSE, x.arg = TRUE, y.arg = TRUE,
    contrasts = NULL, constraints = NULL, extra = list(),
    form2 = NULL, qr.arg = TRUE, smart = TRUE, ...)
```


## Arguments

formula a symbolic description of the model to be fit. The RHS of the formula is applied to each linear predictor. The effect of different variables in each linear predictor can be controlled by specifying constraint matrices-see constraints below.
$\left.\begin{array}{ll}\text { family } & \text { a function of class "vglmff" (see vglmff-class) describing what statistical } \\ \text { model is to be fitted. This is called a "VGAM family function". See CommonVGAMffArguments } \\ \text { for general information about many types of arguments found in this type of } \\ \text { function. The argument name "family" is used loosely and for the ease of ex- } \\ \text { isting glm users; there is no concept of a formal "error distribution" for VGLMs. } \\ \text { Possibly the argument name should be better "model" but unfortunately that } \\ \text { name has already been taken. } \\ \text { an optional data frame containing the variables in the model. By default the vari- } \\ \text { ables are taken from environment (formula), typically the environment from } \\ \text { which vglm is called. } \\ \text { data } & \text { an optional vector or matrix of (prior fixed and known) weights to be used in } \\ \text { the fitting process. If the VGAM family function handles multiple responses } \\ \text { (Q > 1 of them, say) then weights can be a matrix with } Q \text { columns. Each } \\ \text { column matches the respective response. If it is a vector (the usually case) then } \\ \text { it is recycled into a matrix with } Q \text { columns. The values of weights must be } \\ \text { positive; try setting a very small value such as 1.0e-8 to effectively delete an } \\ \text { observation. } \\ \text { Currently the weights argument does not support sampling weights from com- } \\ \text { plex sampling designs. And currently sandwich estimators are not computed in } \\ \text { any shape or form. The present weights are multiplied by the corresponding }\end{array}\right]$ log-likelihood contributions and added to form the overall log-likelihood.

| x.arg, y.arg | logical values indicating whether the LM matrix and response vector/matrix used in the fitting process should be assigned in the $x$ and $y$ slots. Note that the model matrix is the LM matrix; to get the VGLM matrix type model.matrix (vglmfit) where vglmfit is a vglm object. |
| :---: | :---: |
| contrasts | an optional list. See the contrasts.arg of model.matrix. default. |
| constraints | an optional list of constraint matrices. The components of the list must be named (labelled) with the term it corresponds to (and it must match in character format exactly-see below). There are two types of input: "lm"-type and "vlm"type. The former is a subset of the latter. The former has a matrix for each term of the LM matrix. The latter has a matrix for each column of the big VLM matrix. After fitting, the constraints extractor function may be applied; it returns the "vlm"-type list of constraint matrices by default. If "lm"-type are returned by constraints then these can be fed into this argument and it should give the same model as before. |
|  | If the constraints argument is used then the family function's zero argument (if it exists) needs to be set to NULL. This avoids what could be a probable contradiction. Sometimes setting other arguments related to constraint matrices to FALSE is also a good idea, e.g., parallel $=$ FALSE, exchangeable $=$ FALSE. |
|  | Properties: each constraint matrix must have $M$ rows, and be of full-column rank. By default, constraint matrices are the $M$ by $M$ identity matrix unless arguments in the family function itself override these values, e.g., parallel (see CommonVGAMffArguments). If constraints is used then it must contain all the terms; an incomplete list is not accepted. |
|  | As mentioned above, the labelling of each constraint matrix must match exactly, e.g., list("s(x2,df=3)"=diag(2)) will fail as as.character ( $\sim s(x 2, d f=3)$ ) produces white spaces: "s(x2,df=3)". Thus list("s(x2,df=3)" $=\operatorname{diag}(2))$ is needed. See Example 6 below. More details are given in Yee (2015; Section 3.3.1.3) which is on p.101. Note that the label for the intercept is "(Intercept)". |
| extra | an optional list with any extra information that might be needed by the VGAM family function. |
| form2 | the second (optional) formula. If argument xij is used (see vglm. control) then form 2 needs to have all terms in the model. Also, some VGAM family functions such as micmen use this argument to input the regressor variable. If given, the slots @Xm2 and @Ym2 may be assigned. Note that smart prediction applies to terms in form2 too. |
| qr.arg | logical value indicating whether the slot qr, which returns the QR decomposition of the VLM model matrix, is returned on the object. |
| smart | logical value indicating whether smart prediction (smartpred) will be used. |
|  | further arguments passed into vglm. control. |

## Details

A vector generalized linear model (VGLM) is loosely defined as a statistical model that is a function of $M$ linear predictors and can be estimated by Fisher scoring. The central formula is given by

$$
\eta_{j}=\beta_{j}^{T} x
$$

where $x$ is a vector of explanatory variables (sometimes just a 1 for an intercept), and $\beta_{j}$ is a vector of regression coefficients to be estimated. Here, $j=1, \ldots, M$, where $M$ is finite. Then one can write $\eta=\left(\eta_{1}, \ldots, \eta_{M}\right)^{T}$ as a vector of linear predictors.
Most users will find vglm similar in flavour to glm. The function vglm.fit actually does the work.

## Value

An object of class "vglm", which has the following slots. Some of these may not be assigned to save space, and will be recreated if necessary later.

| extra | the list extra at the end of fitting. |
| :---: | :---: |
| family | the family function (of class "vglmff"). |
| iter | the number of IRLS iterations used. |
| predictors | a $M$-column matrix of linear predictors. |
| assign | a named list which matches the columns and the (LM) model matrix terms. |
| call | the matched call. |
| coefficients | a named vector of coefficients. |
| constraints | a named list of constraint matrices used in the fitting. |
| contrasts | the contrasts used (if any). |
| control | list of control parameter used in the fitting. |
| criterion | list of convergence criterion evaluated at the final IRLS iteration. |
| df.residual | the residual degrees of freedom. |
| df.total | the total degrees of freedom. |
| dispersion | the scaling parameter. |
| effects | the effects. |

fitted.values the fitted values, as a matrix. This is often the mean but may be quantiles, or the location parameter, e.g., in the Cauchy model.
misc a list to hold miscellaneous parameters.
model the model frame.
na.action
a list holding information about missing values.
offset if non-zero, a $M$-column matrix of offsets.
post a list where post-analysis results may be put.
preplot used by plotvgam, the plotting parameters may be put here.
prior. weights initially supplied weights (the weights argument). Also see weightsvglm.
qr the QR decomposition used in the fitting.
$\mathrm{R} \quad$ the $\mathbf{R}$ matrix in the QR decomposition used in the fitting.
rank numerical rank of the fitted model.
residuals the working residuals at the final IRLS iteration.
ResSS residual sum of squares at the final IRLS iteration with the adjusted dependent vectors and weight matrices.

```
smart.prediction
    a list of data-dependent parameters (if any) that are used by smart prediction.
    terms the terms object used.
    weights the working weight matrices at the final IRLS iteration. This is in matrix-band
        form.
x the model matrix (linear model LM, not VGLM).
xlevels the levels of the factors, if any, used in fitting.
y the response, in matrix form.
```

This slot information is repeated at vglm-class.

## WARNING

See warnings in vglm. control. Also, see warnings under weights above regarding sampling weights from complex sampling designs.

## Note

This function can fit a wide variety of statistical models. Some of these are harder to fit than others because of inherent numerical difficulties associated with some of them. Successful model fitting benefits from cumulative experience. Varying the values of arguments in the VGAM family function itself is a good first step if difficulties arise, especially if initial values can be inputted. A second, more general step, is to vary the values of arguments in vglm. control. A third step is to make use of arguments such as etastart, coefstart and mustart.
Some VGAM family functions end in " $f f$ " to avoid interference with other functions, e.g., binomialff, poissonff. This is because VGAM family functions are incompatible with glm (and also gam in the gam library and gam in the mgev library).

The smart prediction (smartpred) library is incorporated within the VGAM library.
The theory behind the scaling parameter is currently being made more rigorous, but it it should give the same value as the scale parameter for GLMs.

In Example 5 below, the xij argument to illustrate covariates that are specific to a linear predictor. Here, lop/rop are the ocular pressures of the left/right eye (artificial data). Variables leye and reye might be the presence/absence of a particular disease on the LHS/RHS eye respectively. See vglm. control and fill for more details and examples.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2015). Vector Generalized Linear and Additive Models: With an Implementation in R. New York, USA: Springer.
Yee, T. W. and Hastie, T. J. (2003). Reduced-rank vector generalized linear models. Statistical Modelling, 3, 15-41.
Yee, T. W. and Wild, C. J. (1996). Vector generalized additive models. Journal of the Royal Statistical Society, Series B, Methodological, 58, 481-493.

Yee, T. W. (2014). Reduced-rank vector generalized linear models with two linear predictors. Computational Statistics and Data Analysis, 71, 889-902.
Yee, T. W. (2008). The VGAM Package. $R$ News, 8, 28-39.

## See Also

vglm. control, vglm-class, vglmff-class, smartpred, vglm.fit, fill, rrvglm, vgam. Methods functions include add1.vglm, anova.vglm, AICvlm, coefvlm, confintvglm, constraints.vlm, drop1.vglm, fittedvlm, hatvaluesvlm, hdeff.vglm, linkfun.vglm, lrt.stat.vlm, score.stat.vlm, wald.stat.vlm, nobs.vlm, npred.vlm, plotvglm, predictvglm, residualsvglm, step4vglm, summaryvglm, lrtest_vglm, update, etc.

## Examples

```
# Example 1. See help(glm)
print(d.AD <- data.frame(treatment = gl(3, 3),
    outcome = gl(3, 1, 9),
    counts = c(18,17,15, 20,10, 20, 25,13,12)))
vglm.D93 <- vglm(counts ~ outcome + treatment, family = poissonff,
    data = d.AD, trace = TRUE)
summary(vglm.D93)
# Example 2. Multinomial logit model
pneumo <- transform(pneumo, let = log(exposure.time))
vglm(cbind(normal, mild, severe) ~ let, multinomial, data = pneumo)
# Example 3. Proportional odds model
fit3 <- vglm(cbind(normal, mild, severe) ~ let, propodds, data = pneumo)
coef(fit3, matrix = TRUE)
constraints(fit3)
model.matrix(fit3, type = "lm") # LM model matrix
model.matrix(fit3) # Larger VGLM (or VLM) model matrix
# Example 4. Bivariate logistic model
fit4 <- vglm(cbind(nBnW, nBW, BnW, BW) ~ age, binom2.or, coalminers)
coef(fit4, matrix = TRUE)
depvar(fit4) # Response are proportions
weights(fit4, type = "prior")
# Example 5. The use of the xij argument (simple case).
# The constraint matrix for 'op' has one column.
nn <- 1000
eyesdat <- round(data.frame(lop = runif(nn),
    rop = runif(nn),
    op = runif(nn)), digits = 2)
eyesdat <- transform(eyesdat, eta1 = -1 + 2 * lop,
        eta2 = -1 + 2 * lop)
eyesdat <- transform(eyesdat,
```

```
    leye = rbinom(nn, size = 1, prob = logitlink(eta1, inverse = TRUE)),
    reye = rbinom(nn, size = 1, prob = logitlink(eta2, inverse = TRUE)))
head(eyesdat)
fit5 <- vglm(cbind(leye, reye) ~ op,
    binom2.or(exchangeable = TRUE, zero = 3),
    data = eyesdat, trace = TRUE,
    xij = list(op ~ lop + rop + fill(lop)),
    form2 = ~ op + lop + rop + fill(lop))
coef(fit5)
coef(fit5, matrix = TRUE)
constraints(fit5)
fit5@control$xij
head(model.matrix(fit5))
# Example 6. The use of the 'constraints' argument.
as.character(~ bs(year,df=3)) # Get the white spaces right
clist <- list("(Intercept)" = diag(3),
    "bs(year, df = 3)" = rbind(1, 0, 0))
fit1 <- vglm(r1 ~ bs(year,df=3), gev(zero = NULL),
    data = venice, constraints = clist, trace = TRUE)
coef(fit1, matrix = TRUE) # Check
```

```
vglm-class Class "vglm"
```


## Description

Vector generalized linear models.

## Objects from the Class

Objects can be created by calls of the form $\operatorname{vglm}(\ldots)$.

## Slots

In the following, $M$ is the number of linear predictors.
extra: Object of class "list"; the extra argument on entry to vglm. This contains any extra information that might be needed by the family function.
family: Object of class "vglmff". The family function.
iter: Object of class "numeric". The number of IRLS iterations used.
predictors: Object of class "matrix" with $M$ columns which holds the $M$ linear predictors.
assign: Object of class "list", from class "vlm". This named list gives information matching the columns and the (LM) model matrix terms.
call: Object of class "call", from class "vlm". The matched call.
coefficients: Object of class "numeric", from class "vlm". A named vector of coefficients.
constraints: Object of class "list", from class "vlm". A named list of constraint matrices used in the fitting.
contrasts: Object of class "list", from class "vlm". The contrasts used (if any).
control: Object of class "list", from class "vlm". A list of parameters for controlling the fitting process. See vglm. control for details.
criterion: Object of class "list", from class "vlm". List of convergence criterion evaluated at the final IRLS iteration.
df.residual: Object of class "numeric", from class "vlm". The residual degrees of freedom.
df.total: Object of class "numeric", from class "vlm". The total degrees of freedom.
dispersion: Object of class "numeric", from class "vlm". The scaling parameter.
effects: Object of class "numeric", from class "vlm". The effects.
fitted.values: Object of class "matrix", from class "vlm". The fitted values.
misc: Object of class "list", from class "vlm". A named list to hold miscellaneous parameters.
model: Object of class "data.frame", from class "vlm". The model frame.
na. action: Object of class "list", from class "vlm". A list holding information about missing values.
offset: Object of class "matrix", from class "vlm". If non-zero, a $M$-column matrix of offsets.
post: Object of class "list", from class "vlm" where post-analysis results may be put.
preplot: Object of class "list", from class "vlm" used by plotvgam; the plotting parameters may be put here.
prior. weights: Object of class "matrix", from class "vlm" holding the initially supplied weights.
qr: Object of class "list", from class "vlm". QR decomposition at the final iteration.
R: Object of class "matrix", from class "vlm". The $\mathbf{R}$ matrix in the QR decomposition used in the fitting.
rank: Object of class "integer", from class "vlm". Numerical rank of the fitted model.
residuals: Object of class "matrix", from class "vlm". The working residuals at the final IRLS iteration.

ResSS: Object of class "numeric", from class "vlm". Residual sum of squares at the final IRLS iteration with the adjusted dependent vectors and weight matrices.
smart.prediction: Object of class "list", from class "vlm". A list of data-dependent parameters (if any) that are used by smart prediction.
terms: Object of class "list", from class "vlm". The terms object used.
weights: Object of class "matrix", from class "vlm". The weight matrices at the final IRLS iteration. This is in matrix-band form.
$x$ : Object of class "matrix", from class "vlm". The model matrix (LM, not VGLM).
xlevels: Object of class "list", from class "vlm". The levels of the factors, if any, used in fitting.
$y$ : Object of class "matrix", from class "vlm". The response, in matrix form.
Xm2: Object of class "matrix", from class "vlm". See vglm-class).
Ym2: Object of class "matrix", from class "vlm". See vglm-class).
callXm2: Object of class "call", from class "vlm". The matched call for argument form2.

## Extends

Class "vlm", directly.

## Methods

cdf signature (object = "vglm"): cumulative distribution function. Applicable to, e.g., quantile regression and extreme value data models.
deplot signature (object = "vglm"): Applicable to, e.g., quantile regression.
deviance signature (object = "vglm"): deviance of the model (where applicable).
plot signature( $\mathrm{x}=$ "vglm"): diagnostic plots.
predict signature(object $=$ "vglm"): extract the linear predictors or predict the linear predictors at a new data frame.
print signature $(x=$ "vglm"): short summary of the object.
qtplot signature (object $=$ "vglm"): quantile plot (only applicable to some models).
resid signature (object $=$ "vglm") : residuals. There are various types of these.
residuals signature (object $=$ "vglm"): residuals. Shorthand for resid.
rlplot signature(object = "vglm"): return level plot. Useful for extreme value data models.
summary signature(object = "vglm"): a more detailed summary of the object.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Hastie, T. J. (2003). Reduced-rank vector generalized linear models. Statistical Modelling, 3, 15-41.

Yee, T. W. and Wild, C. J. (1996). Vector generalized additive models. Journal of the Royal Statistical Society, Series B, Methodological, 58, 481-493.

## See Also

vglm, vglmff-class, vgam-class.

## Examples

```
# Multinomial logit model
pneumo <- transform(pneumo, let = log(exposure.time))
vglm(cbind(normal, mild, severe) ~ let, multinomial, data = pneumo)
```

```
vglm.control
Control Function for vglm()
```


## Description

Algorithmic constants and parameters for running vglm are set using this function.

## Usage

```
vglm.control(checkwz = TRUE, Check.rank = TRUE, Check.cm.rank = TRUE,
                    criterion = names(.min.criterion.VGAM),
                    epsilon \(=1 \mathrm{e}-07\), half.stepsizing \(=\) TRUE,
                maxit \(=30\), noWarning \(=\) FALSE,
                stepsize = 1, save. weights = FALSE,
                trace \(=\) FALSE, wzepsilon \(=\). Machine\$double.eps^0.75,
                xij = NULL, ...)
```


## Arguments

checkwz logical indicating whether the diagonal elements of the working weight matrices should be checked whether they are sufficiently positive, i.e., greater than wzepsilon. If not, any values less than wzepsilon are replaced with this value.
Check. rank logical indicating whether the rank of the VLM matrix should be checked. If this is not of full column rank then the results are not to be trusted. The default is to give an error message if the VLM matrix is not of full column rank.
Check.cm.rank logical indicating whether the rank of each constraint matrix should be checked. If this is not of full column rank then an error will occur. Under no circumstances should any constraint matrix have a rank less than the number of columns.
criterion character variable describing what criterion is to be used to test for convergence. The possibilities are listed in .min.criterion. VGAM, but most family functions only implement a few of these.
epsilon positive convergence tolerance epsilon. Roughly speaking, the Newton-Raphson/Fisherscoring iterations are assumed to have converged when two successive criterion values are within epsilon of each other.
half.stepsizing
logical indicating if half-stepsizing is allowed. For example, in maximizing a log-likelihood, if the next iteration has a log-likelihood that is less than the current value of the log-likelihood, then a half step will be taken. If the loglikelihood is still less than at the current position, a quarter-step will be taken etc. Eventually a step will be taken so that an improvement is made to the convergence criterion. half. stepsizing is ignored if criterion == "coefficients".
maxit maximum number of (usually Fisher-scoring) iterations allowed. Sometimes Newton-Raphson is used.
noWarning logical indicating whether to suppress a warning if convergence is not obtained within maxit iterations. This is ignored if maxit $=1$ is set.

| stepsize | usual step size to be taken between each Newton-Raphson/Fisher-scoring itera- <br> tion. It should be a value between 0 and 1 , where a value of unity corresponds <br> to an ordinary step. A value of 0.5 means half-steps are taken. Setting a value <br> near zero will cause convergence to be generally slow but may help increase the <br> chances of successful convergence for some family functions. <br> save.weights <br> logical indicating whether the weights slot of a "vglm" object will be saved on <br> the object. If not, it will be reconstructed when needed, e.g., summary. Some <br> family functions have save.weights = TRUE and others have save.weights = <br> FALSE in their control functions. <br> lrace <br> logical indicating if output should be produced for each iteration. Setting trace <br> = TRUE is recommended in general because VGAM fits a very broad variety <br> of models and distributions, and for some of them, convergence is intrinsically <br> more difficult. Monitoring convergence can help check that the solution is rea- <br> sonable or that a problem has occurred. It may suggest better initial values are <br> needed, the making of invalid assumptions, or that the model is inappropriate <br> for the data, etc. <br> wzepsilon <br> small positive number used to test whether the diagonals of the working weight <br> matrices are sufficiently positive. <br> A formula or a list of formulas. Each formula has a RHS giving $M$ terms making <br> up a covariate-dependent term (whose name is the response). That is, it creates a |
| :--- | :--- |
| variable that takes on different values for each linear/additive predictor, e.g., the |  |
| ocular pressure of each eye. The $M$ terms must be unique; use fill, fill2, |  |
| fill3, etc. if necessary. Each formula should have a response which is taken as |  |
| the name of that variable, and the $M$ terms are enumerated in sequential order. |  |

## Details

Most of the control parameters are used within vglm.fit and you will have to look at that to understand the full details.

Setting save. weights = FALSE is useful for some models because the weights slot of the object is the largest and so less memory is used to store the object. However, for some VGAM family function, it is necessary to set save. weights = TRUE because the weights slot cannot be reconstructed later.

## Value

A list with components matching the input names. A little error checking is done, but not much. The list is assigned to the control slot of vglm objects.

## Warning

For some applications the default convergence criterion should be tightened. Setting something like criterion = "coef", epsilon $=1 \mathrm{e}-09$ is one way to achieve this, and also add trace $=$ TRUE to monitor the convergence. Setting maxit to some higher number is usually not needed, and needing to do so suggests something is wrong, e.g., an ill-conditioned model, over-fitting or under-fitting.

## Note

Reiterating from above, setting trace $=$ TRUE is recommended in general.
In Example 2 below there are two covariates that have linear/additive predictor specific values. These are handled using the xij argument.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Hastie, T. J. (2003). Reduced-rank vector generalized linear models. Statistical Modelling, 3, 15-41.

## See Also

vglm, fill1. The author's homepage has further documentation about the xij argument; see also Select.

## Examples

```
# Example 1.
pneumo <- transform(pneumo, let = log(exposure.time))
vglm(cbind(normal, mild, severe) ~ let, multinomial, data = pneumo,
    crit = "coef", step = 0.5, trace = TRUE, epsil = 1e-8, maxit = 40)
# Example 2. The use of the xij argument (simple case).
ymat <- rdiric(n <- 1000, shape = rep(exp(2), len = 4))
mydat <- data.frame(x1 = runif(n), x2 = runif(n), x3 = runif(n),
    x4 = runif(n),
    z1 = runif(n), z2 = runif(n), z3 = runif(n),
    z4 = runif(n))
mydat <- transform(mydat, X = x1, Z = z1)
mydat <- round(mydat, digits = 2)
fit2 <- vglm(ymat ~ X + Z,
    dirichlet(parallel = TRUE), data = mydat, trace = TRUE,
    xij = list(Z ~ z1 + z2 + z3 + z4,
        X ~ x1 + x2 + x3 + x4),
        form2 = ~ Z + z1 + z2 + z3 + z4 +
        x + x1 + x2 + x3 + x4)
head(model.matrix(fit2, type = "lm")) # LM model matrix
head(model.matrix(fit2, type = "vlm")) # Big VLM model matrix
coef(fit2)
```

```
coef(fit2, matrix = TRUE)
max(abs(predict(fit2)-predict(fit2, new = mydat))) # Predicts correctly
summary(fit2)
## Not run:
# plotvgam(fit2, se = TRUE, xlab = "x1", which.term = 1) # Bug!
# plotvgam(fit2, se = TRUE, xlab = "z1", which.term = 2) # Bug!
plotvgam(fit2, xlab = "x1") # Correct
plotvgam(fit2, xlab = "z1") # Correct
## End(Not run)
# Example 3. The use of the xij argument (complex case).
set.seed(123)
coalminers <- transform(coalminers,
                    Age = (age - 42) / 5,
                    dum1 = round(runif(nrow(coalminers)), digits = 2),
                    dum2 = round(runif(nrow(coalminers)), digits = 2),
                    dum3 = round(runif(nrow(coalminers)), digits = 2),
                    dumm = round(runif(nrow(coalminers)), digits = 2))
BS <- function(x, ..., df = 3)
    sm.bs(c(x,...), df = df)[1:length(x),,drop = FALSE]
NS <- function(x, ..., df = 3)
    sm.ns(c(x,\ldots), df = df)[1:length(x),,drop = FALSE]
# Equivalently...
BS <- function(x, ..., df = 3)
    head(sm.bs(c(x,\ldots), df = df), length(x), drop = FALSE)
NS <- function(x, ..., df = 3)
    head(sm.ns(c(x,...), df = df), length(x), drop = FALSE)
fit3 <- vglm(cbind(nBnW,nBW,BnW,BW) ~ Age + NS(dum1, dum2),
                fam = binom2.or(exchangeable = TRUE, zero = 3),
                xij = list(NS(dum1, dum2) ~ NS(dum1, dum2) +
                                    NS(dum2, dum1) +
                                    fill(NS( dum1))),
        form2 = ~ NS(dum1, dum2) + NS(dum2, dum1) + fill(NS(dum1)) +
                        dum1 + dum2 + dum3 + Age + age + dumm,
        data = coalminers, trace = TRUE)
head(model.matrix(fit3, type = "lm")) # LM model matrix
head(model.matrix(fit3, type = "vlm")) # Big VLM model matrix
coef(fit3)
coef(fit3, matrix = TRUE)
## Not run:
plotvgam(fit3, se = TRUE, lcol = "red", scol = "blue", xlab = "dum1")
## End(Not run)
```

vglmff-class Class "vglmff"

## Description

Family functions for the VGAM package

## Objects from the Class

Objects can be created by calls of the form new("vglmff", ...).

## Slots

In the following, $M$ is the number of linear/additive predictors.
blurb: Object of class "character" giving a small description of the model. Important arguments such as parameter link functions can be expressed here.
charfun: Object of class "function" which returns the characteristic function or variance function (usually for some GLMs only). The former uses a dummy variable x. Both use the linear/additive predictors. The function must have arguments function(x, eta, extra = NULL, varfun $=$ FALSE). The eta and extra arguments are used to obtain the parameter values. If varfun $=$ TRUE then the function returns the variance function, else the characteristic function (default). Note that one should check that the infos slot has a list component called charfun which is TRUE before attempting to use this slot. This is an easier way to test that this slot is operable.
constraints: Object of class "expression" which sets up any constraint matrices defined by arguments in the family function. A zero argument is always fed into cm. zero.vgam, whereas other constraints are fed into cm . vgam.
deviance: Object of class "function" returning the deviance of the model. This slot is optional. If present, the function must have arguments function(mu, $\mathrm{y}, \mathrm{w}$, residuals = FALSE, eta, extra $=$ NULL). Deviance residuals are returned if residuals $=$ TRUE .
fini: Object of class "expression" to insert code at a special position in vglm.fit or vgam.fit. This code is evaluated immediately after the fitting.
first: Object of class "expression" to insert code at a special position in vglm or vgam.
infos: Object of class "function" which returns a list with components such as M1. At present only a very few VGAM family functions have this feature implemented. Those that do do not require specifying the $M 1$ argument when used with rcim.
initialize: Object of class "expression" used to perform error checking (especially for the variable $y$ ) and obtain starting values for the model. In general, etastart or mustart are assigned values based on the variables $y, x$ and $w$.
linkinv: Object of class "function" which returns the fitted values, given the linear/additive predictors. The function must have arguments function(eta, extra = NULL).
last: Object of class "expression" to insert code at a special position (at the very end) of vglm.fit() or vgam.fit(). This code is evaluated after the fitting. The list misc is often assigned components in this slot, which becomes the misc slot on the fitted object.
linkfun: Object of class "function" which, given the fitted values, returns the linear/additive predictors. If present, the function must have arguments function(mu, extra = NULL). Most VGAM family functions do not have a linkfun function. They largely are for classical exponential families, i.e., GLMs.
loglikelihood: Object of class "function" returning the log-likelihood of the model. This slot is optional. If present, the function must have arguments function(mu, $y, w, r e s i d u a l s=$ FALSE, eta, extra = NULL). The argument residuals can be ignored because log-likelihood residuals aren't defined.
middle: Object of class "expression" to insert code at a special position in vglm.fit or vgam. fit.
middle2: Object of class "expression" to insert code at a special position in vglm.fit or vgam.fit.
simslot: Object of class "function" to allow simulate to work.
hadof: Object of class "function"; experimental.
summary. dispersion: Object of class "logical" indicating whether the general VGLM formula (based on a residual sum of squares) can be used for computing the scaling/dispersion parameter. It is TRUE for most models except for nonlinear regression models.
vfamily: Object of class "character" giving class information about the family function. Although not developed at this stage, more flexible classes are planned in the future. For example, family functions sratio, cratio, cumulative, and acat all operate on categorical data, therefore will have a special class called "VGAMcat", say. Then if fit was a vglm object, then coef(fit) would print out the vglm coefficients plus "VGAMcat" information as well.
deriv: Object of class "expression" which returns a $M$-column matrix of first derivatives of the log-likelihood function with respect to the linear/additive predictors, i.e., the score vector. In Yee and Wild (1996) this is the $\boldsymbol{d}_{i}$ vector. Thus each row of the matrix returned by this slot is such a vector.
weight: Object of class "expression" which returns the second derivatives of the log-likelihood function with respect to the linear/additive predictors. This can be either the observed or expected information matrix, i.e., Newton-Raphson or Fisher-scoring respectively. In Yee and Wild (1996) this is the $\boldsymbol{W}_{i}$ matrix. Thus each row of the matrix returned by this slot is such a matrix. Like the weights slot of vglm/vgam, it is stored in matrix-band form, whereby the first $M$ columns of the matrix are the diagonals, followed by the upper-diagonal band, followed by the band above that, etc. In this case, there can be up to $M(M+1)$ columns, with the last column corresponding to the $(1, M)$ elements of the weight matrices.
validfitted, validparams: Functions that test that the fitted values and all parameters are within range. These functions can issue a warning if violations are detected.

## Methods

print signature $(x=$ "vglmff" $)$ : short summary of the family function.

## Warning

VGAM family functions are not compatible with glm, nor gam (from either gam or mgcv packages).

## Note

With link functions etc., one must use substitute to embed the options into the code. There are two different forms: eval(substitute(expression(\{...\}), list(...))) for expressions, and eval(substitute( function(...) \{ ...\},list(...))) for functions.
The extra argument in linkinv, linkfun, deviance, loglikelihood, etc. matches with the argument extra in vglm, vgam and rrvglm. This allows input to be fed into all slots of a VGAM family function.

The expression derivative is evaluated immediately prior to weight, so there is provision for reuse of variables etc. Programmers must be careful to choose variable names that do not interfere with vglm.fit, vgam.fit() etc.

Programmers of VGAM family functions are encouraged to keep to previous conventions regarding the naming of arguments, e.g., link is the argument for parameter link functions, zero for allowing some of the linear/additive predictors to be an intercept term only, etc.
In general, Fisher-scoring is recommended over Newton-Raphson where tractable. Although usually slightly slower in convergence, the weight matrices from using the expected information are positive-definite over a larger parameter space.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. and Wild, C. J. (1996). Vector generalized additive models. Journal of the Royal Statistical Society, Series B, Methodological, 58, 481-493.

## See Also

vglm, vgam, rrvglm, rcim.

## Examples

```
cratio()
cratio(link = "clogloglink")
cratio(link = "clogloglink", reverse = TRUE)
```

```
vonmises von Mises Distribution Family Function
```


## Description

Estimates the location and scale parameters of the von Mises distribution by maximum likelihood estimation.

## Usage

vonmises(llocation = extlogitlink(min = 0, max = 2*pi), lscale = "loglink", ilocation $=$ NULL, iscale $=$ NULL, imethod $=1$, zero $=$ NULL)

## Arguments

## llocation, lscale

Parameter link functions applied to the location $a$ parameter and scale parameter $k$, respectively. See Links for more choices. For $k$, a log link is the default because the parameter is positive.
ilocation Initial value for the location $a$ parameter. By default, an initial value is chosen internally using imethod. Assigning a value will override the argument imethod.
iscale Initial value for the scale $k$ parameter. By default, an initial value is chosen internally using imethod. Assigning a value will override the argument imethod.
imethod An integer with value 1 or 2 which specifies the initialization method. If failure to converge occurs try the other value, or else specify a value for ilocation and iscale.
zero An integer-valued vector specifying which linear/additive predictors are modelled as intercepts only. The default is none of them. If used, one can choose one value from the set $\{1,2\}$. See CommonVGAMffArguments for more information.

## Details

The (two-parameter) von Mises is the most commonly used distribution in practice for circular data. It has a density that can be written as

$$
f(y ; a, k)=\frac{\exp [k \cos (y-a)]}{2 \pi I_{0}(k)}
$$

where $0 \leq y<2 \pi, k>0$ is the scale parameter, $a$ is the location parameter, and $I_{0}(k)$ is the modified Bessel function of order 0 evaluated at $k$. The mean of $Y$ (which is the fitted value) is $a$ and the circular variance is $1-I_{1}(k) / I_{0}(k)$ where $I_{1}(k)$ is the modified Bessel function of order 1. By default, $\eta_{1}=\log (a /(2 \pi-a))$ and $\eta_{2}=\log (k)$ for this family function.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Warning

Numerically, the von Mises can be difficult to fit because of a log-likelihood having multiple maximums. The user is therefore encouraged to try different starting values, i.e., make use of ilocation and iscale.

## Note

The response and the fitted values are scaled so that $0 \leq y<2 \pi$. The linear/additive predictors are left alone. Fisher scoring is used.

## Author(s)

T. W. Yee

## References

Forbes, C., Evans, M., Hastings, N. and Peacock, B. (2011). Statistical Distributions, Hoboken, NJ, USA: John Wiley and Sons, Fourth edition.

## See Also

Bessel, cardioid.
CircStats and circular currently have a lot more R functions for circular data than the VGAM package.

## Examples

```
vdata <- data.frame(x2 = runif(nn <- 1000))
vdata <- transform(vdata, y = rnorm(nn, m = 2+x2, sd = exp(0.2))) # Bad data!!
fit <- vglm(y ~ x2, vonmises(zero = 2), data = vdata, trace = TRUE)
coef(fit, matrix = TRUE)
Coef(fit)
with(vdata, range(y)) # Original data
range(depvar(fit)) # Processed data is in [0,2*pi)
```

vplot.profile

## Description

plot and pairs methods for objects of class "profile", but renamed as vplot and vpairs.

## Usage

vplot.profile(x, ...)
vpairs.profile(x, colours = 2:3, ...)

## Arguments

$x \quad$ an object inheriting from class "profile".
colours Colours to be used for the mean curves conditional on x and y respectively. ... arguments passed to or from other methods.

## Details

See profile.glm for details.

## Author(s)

T. W. Yee adapted this function from profile.glm, written originally by D. M. Bates and W. N. Venables. (For S in 1996.)

## See Also

```
profilevglm, confintvglm,lrt.stat, profile.glm, profile.nls.
```


## Examples

```
pneumo <- transform(pneumo, let = log(exposure.time))
fit1 <- vglm(cbind(normal, mild, severe) ~ let, acat,
    trace = TRUE, data = pneumo)
pfit1 <- profile(fit1, trace = FALSE)
## Not run:
vplot.profile(pfit1)
vpairs.profile(pfit1)
## End(Not run)
```

```
vsmooth.spline
```

Vector Cubic Smoothing Spline

## Description

Fits a vector cubic smoothing spline.

## Usage

```
vsmooth.spline(x, y, w = NULL, df = rep(5, M), spar = NULL,
    i.constraint = diag(M),
    x.constraint = diag(M),
    constraints = list("(Intercepts)" = i.constraint,
                        x = x.constraint),
    all.knots = FALSE, var.arg = FALSE, scale.w = TRUE,
    nk = NULL, control.spar = list())
```


## Arguments

x
A vector, matrix or a list. If a list, the $x$ component is used. If a matrix, the first column is used. $x$ may also be a complex vector, in which case the real part is used, and the imaginary part is used for the response. In this help file, $n$ is the number of unique values of $x$.
y
A vector, matrix or a list. If a list, the $y$ component is used. If a matrix, all but the first column is used. In this help file, $M$ is the number of columns of $y$ if there are no constraints on the functions.

W
The weight matrices or the number of observations. If the weight matrices, then this must be a n-row matrix with the elements in matrix-band form (see iam). If a vector, then these are the number of observations. By default, w is the M by M identity matrix, denoted by matrix $(1, n, M)$.
df Numerical vector containing the degrees of freedom for each component function (smooth). If necessary, the vector is recycled to have length equal to the number of component functions to be estimated ( $M$ if there are no constraints), which equals the number of columns of the $x$-constraint matrix. A value of 2 means a linear fit, and each element of df should lie between 2 and $n$. The larger the values of $d f$ the more wiggly the smooths.
spar Numerical vector containing the non-negative smoothing parameters for each component function (smooth). If necessary, the vector is recycled to have length equal to the number of component functions to be estimated ( $M$ if there are no constraints), which equals the number of columns of the x-constraint matrix. A value of zero means the smooth goes through the data and hence is wiggly. A value of Inf may be assigned, meaning the smooth will be linear. By default, the NULL value of spar means $d f$ is used to determine the smoothing parameters.
all.knots Logical. If TRUE then each distinct value of $x$ will be a knot. By default, only a subset of the unique values of $x$ are used; typically, the number of knots is $0\left(n^{\wedge} 0.25\right)$ for $n$ large, but if $n<=40$ then all the unique values of $x$ are used.
i. constraint A M-row constraint matrix for the intercepts. It must be of full column rank. By default, the constraint matrix for the intercepts is the M by M identity matrix, meaning no constraints.
x.constraint A M-row constraint matrix for $x$. It must be of full column rank. By default, the constraint matrix for the intercepts is the M by M identity matrix, meaning no constraints.
constraints An alternative to specifying i.constraint and x.constraint, this is a list with two components corresponding to the intercept and $x$ respectively. They must both be a M-row constraint matrix with full column rank.
var.arg Logical: return the pointwise variances of the fit? Currently, this corresponds only to the nonlinear part of the fit, and may be wrong.
scale.w Logical. By default, the weights w are scaled so that the diagonal elements have mean 1.
nk Number of knots. If used, this argument overrides all.knots, and must lie between 6 and $n+2$ inclusive.
control.spar See smooth.spline.

## Details

The algorithm implemented is detailed in Yee (2000). It involves decomposing the component functions into a linear and nonlinear part, and using B-splines. The cost of the computation is 0 ( n $M^{\wedge} 3$ ).
The argument spar contains scaled smoothing parameters.

## Value

An object of class "vsmooth.spline" (see vsmooth.spline-class).

## WARNING

See vgam for information about an important bug.

## Note

This function is quite similar to smooth.spline but offers less functionality. For example, cross validation is not implemented here. For $M=1$, the results will be generally different, mainly due to the different way the knots are selected.

The vector cubic smoothing spline which $s()$ represents is computationally demanding for large $M$. The cost is approximately $O\left(n M^{3}\right)$ where $n$ is the number of unique abscissae.
Yet to be done: return the unscaled smoothing parameters.

## Author(s)

Thomas W. Yee

## References

Yee, T. W. (2000). Vector Splines and Other Vector Smoothers. Pages 529-534. In: Bethlehem, J. G. and van der Heijde, P. G. M. Proceedings in Computational Statistics COMPSTAT 2000. Heidelberg: Physica-Verlag.

## See Also

vsmooth.spline-class, plot.vsmooth.spline, predict.vsmooth.spline, iam, sm.os, s, smooth.spline.

## Examples

```
nn <- 20; x <- 2 + 5*(nn:1)/nn
x[2:4] <- x[5:7] # Allow duplication
y1 <- sin(x) + rnorm(nn, sd = 0.13)
y2 <- cos(x) + rnorm(nn, sd = 0.13)
y3 <- 1 + sin(x) + rnorm(nn, sd = 0.13) # Run this for constraints
y <- cbind(y1, y2, y3)
ww <- cbind(rep(3, nn), 4, (1:nn)/nn)
(fit <- vsmooth.spline(x, y, w = ww, df = 5))
## Not run:
plot(fit) # The 1st and 3rd functions do not differ by a constant
## End(Not run)
mat <- matrix(c(1,0,1, 0,1,0), 3, 2)
(fit2 <- vsmooth.spline(x, y, w = ww, df = 5, i.constr = mat,
    x.constr = mat))
# The 1st and 3rd functions do differ by a constant:
mycols <- c("orange", "blue", "orange")
## Not run: plot(fit2, lcol = mycols, pcol = mycols, las = 1)
p <- predict(fit, x = model.matrix(fit, type = "lm"), deriv = 0)
```

waitakere

```
    max(abs(depvar(fit) - with(p, y))) # Should be 0; and fit@y is not good
    par(mfrow = c(3, 1))
    ux <- seq(1, 8, len = 100)
    for (dd in 1:3) {
        pp <- predict(fit, x = ux, deriv = dd)
    ## Not run: with(pp, matplot(x, y, type = "l", main = paste("deriv =", dd),
                        lwd = 2, ylab = "", cex.axis = 1.5,
        cex.lab = 1.5, cex.main = 1.5))
    ## End(Not run)
}
```

waitakere Waitakere Ranges Data

## Description

The waitakere data frame has 579 rows and 18 columns. Altitude is explanatory, and there are binary responses (presence/absence $=1 / 0$ respectively) for 17 plant species.

## Usage

data(waitakere)

## Format

This data frame contains the following columns:
agaaus Agathis australis, or Kauri
beitaw Beilschmiedia tawa, or Tawa
corlae Corynocarpus laevigatus
cyadea Cyathea dealbata
cyamed Cyathea medullaris
daccup Dacrydium cupressinum
dacdac Dacrycarpus dacrydioides
eladen Elaecarpus dentatus
hedarb Hedycarya arborea
hohpop Species name unknown
kniexc Knightia excelsa, or Rewarewa
kuneri Kunzea ericoides
lepsco Leptospermum scoparium
metrob Metrosideros robusta
neslan Nestegis lanceolata
rhosap Rhopalostylis sapida
vitluc Vitex lucens, or Puriri
altitude meters above sea level

## Details

These were collected from the Waitakere Ranges, a small forest in northern Auckland, New Zealand. At 579 sites in the forest, the presence/absence of 17 plant species was recorded, as well as the altitude. Each site was of area size $200 \mathrm{~m}^{2}$.

## Source

Dr Neil Mitchell, University of Auckland.

## See Also

hunua.

## Examples

```
fit <- vgam(agaaus ~ s(altitude, df = 2), binomialff, waitakere)
head(predict(fit, waitakere, type = "response"))
## Not run: plot(fit, se = TRUE, lcol = "orange", scol = "blue")
```

```
wald.stat
```

Wald Test Statistics Evaluated at the Null Values

## Description

Generic function that computes Wald test statistics evaluated at the null values (consequently they do not suffer from the Hauck-Donner effect).

## Usage

wald.stat(object, ...)
wald.stat.vlm(object, values $0=0$, subset $=$ NULL, omit1s $=$ TRUE, all.out $=$ FALSE, iterate $=$ TRUE, trace $=$ FALSE , as.summary = FALSE, ...)

## Arguments

object A vglm fit.
values0 Numeric vector. The null values corresponding to the null hypotheses. Recycled if necessary.
subset Same as in hdeff.
omit1s Logical. Does one omit the intercepts? Because the default would be to test that each intercept is equal to 0 , which often does not make sense or is unimportant, the intercepts are not tested by default. If they are tested then each linear predictor must have at least one coefficient (from another variable) to be estimated.
all. out Logical. If TRUE then a list is returned containing various quantities such as the SEs, instead of just the Wald statistics.

| iterate | Logical. If TRUE then IRLS iterations are performed to get MLEs of the other <br> regression coefficients, subject to that one coefficient equal to the appropriate <br> values0 value. If FALSE then the other regression coefficients have values ob- <br> tained at the original fit. It is recommended that a TRUE be used as the answer <br> tends to be more accurate. If the large model matrix only has one column and <br> iterate = TRUE then an error will occur because there are no other regression <br> coefficients to estimate. <br> Logical. If TRUE then some output is produced as the IRLS iterations pro- <br> ceed. The value NULL means to use the trace value of the fitted object; see <br> vglm. control. <br> as. summary <br> Logical. If TRUE then the usual (or unmodified) Wald statistics etc. are extracted <br> from summary (object). These may suffer from the HDE; and the SEs are <br> evaluated at the MLE of the original object so that effectively iterate = FALSE. |
| :--- | :--- |
| If TRUE then most other arguments will be ignored or overwritten. |  |
| This argument may be renamed to as.glm because the Wald statistics will be |  |
| computed in the same way as summary (glm()). |  |

## Details

By default, summaryvglm and most regression modelling functions such as summary.glm compute the standard errors (SEs) of the estimates at the MLE and not at 0 . This make it vulnerable to the Hauck-Donner effect (HDE) (see hdeff). One solution is to compute the SEs at 0 (or more generally, at the values of the argument values0). This function does that. The two variants of Wald statistics are asymptotically equivalent; however in small samples there can be an appreciable difference, and the difference can be large if the estimates are near to the boundary of the parameter space.
None of the tests here are joint, hence the degrees of freedom is always unity. For a factor with more than 2 levels one can use anova.vglm to test for the significance of the factor. If iterate $=$ FALSE then one retains the MLEs of the original fit for the values of the other coefficients, and replaces one coefficient at a time by the value 0 (or whatever specified by values0). One alternative would be to recompute the MLEs of the other coefficients after replacing one of the values; this is the default because iterate = TRUE. Just like the original IRLS iterations, note that the iterations here are not guaranteed to converge.
Almost all VGAM family functions use the EIM and not the OIM; this affects the resulting standard errors. Also, regularity conditions are assumed for the Wald, likelihood ratio and score tests; some VGAM family functions such as alaplace 1 are experimental and do not satisfy such conditions, therefore naive inference is hazardous.
The default output of this function can be seen by setting wald0.arg = TRUE in summaryvglm.

## Value

By default the signed square root of the Wald statistics whose SEs are computed at one each of the null values. If all. out = TRUE then a list is returned with the following components: wald. stat the Wald statistic, SE0 the standard error of that coefficient, values0 the null values. Approximately, the default Wald statistics output are standard normal random variates if each null hypothesis is true.

## Warning

This function has not yet been thoroughly tested. Convergence failure is possible for some models applied to certain data sets; it is a good idea to set trace = TRUE to monitor convergence. For example, for a particular explanatory variable, the estimated regression coefficients of a non-parallel cumulative logit model (see cumulative) are ordered, and perturbing one coefficient might disrupt the order and create numerical problems.

## Author(s)

Thomas W. Yee

## See Also

lrt.stat, score.stat, summaryvglm, summary.glm, anova.vglm, vglm, hdeff, hdeffsev.

## Examples

```
set.seed(1)
pneumo <- transform(pneumo, let = log(exposure.time),
                        x3 = rnorm(nrow(pneumo)))
(fit <- vglm(cbind(normal, mild, severe) ~ let + x3,
            propodds, data = pneumo))
wald.stat(fit) # No HDE here
summary(fit, wald0 = TRUE) # See them here
coef(summary(fit)) # Usual Wald statistics evaluated at the MLE
wald.stat(fit, as.summary = TRUE) # Same as previous line
```


## waldff Wald Distribution Family Function

## Description

Estimates the parameter of the standard Wald distribution by maximum likelihood estimation.

## Usage

waldff(llambda = "loglink", ilambda = NULL)

## Arguments

llambda, ilambda
See CommonVGAMffArguments for information.

## Details

The standard Wald distribution is a special case of the inverse Gaussian distribution with $\mu=1$. It has a density that can be written as

$$
f(y ; \lambda)=\sqrt{\lambda /\left(2 \pi y^{3}\right)} \exp \left(-\lambda(y-1)^{2} /(2 y)\right)
$$

where $y>0$ and $\lambda>0$. The mean of $Y$ is 1 (returned as the fitted values) and its variance is $1 / \lambda$. By default, $\eta=\log (\lambda)$.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

The VGAM family function inv.gaussianff estimates the location parameter $\mu$ too.

## Author(s)

T. W. Yee

## References

Johnson, N. L. and Kotz, S. and Balakrishnan, N. (1994). Continuous Univariate Distributions, 2nd edition, Volume 1, New York: Wiley.

## See Also

inv.gaussianff, rinv.gaussian.

## Examples

```
wdata <- data.frame(y = rinv.gaussian(n = 1000, mu = 1, lambda = exp(1)))
wfit <- vglm(y ~ 1, waldff(ilambda = 0.2), data = wdata, trace = TRUE)
coef(wfit, matrix = TRUE)
Coef(wfit)
summary(wfit)
```

weibull.mean

Weibull Distribution Family Function, Parameterized by the Mean

## Description

Maximum likelihood estimation of the 2-parameter Weibull distribution. The mean is one of the parameters. No observations should be censored.

## Usage

```
weibull.mean(lmean = "loglink", lshape = "loglink", imean = NULL,
    ishape \(=\) NULL, probs.y \(=c(0.2,0.5,0.8)\),
    imethod = 1, zero = "shape")
```


## Arguments

lmean, lshape Parameter link functions applied to the (positive) mean parameter (called $m u$ below) and (positive) shape parameter (called $a$ below). See Links for more choices.
imean, ishape Optional initial values for the mean and shape parameters.
imethod, zero, probs.y
Details at CommonVGAMffArguments.

## Details

See weibullR for most of the details for this family function too. The mean of $Y$ is $b \Gamma(1+1 / a)$ (returned as the fitted values), and this is the first parameter (a loglink link is the default because it is positive). The other parameter is the positive shape paramter $a$, also having a default loglink link.

This VGAM family function currently does not handle censored data. Fisher scoring is used to estimate the two parameters. Although the expected information matrices used here are valid in all regions of the parameter space, the regularity conditions for maximum likelihood estimation are satisfied only if $a>2$ (according to Kleiber and Kotz (2003)). If this is violated then a warning message is issued. One can enforce $a>2$ by choosing lshape = logofflink (offset =-2). Common values of the shape parameter lie between 0.5 and 3.5.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

See weibullR for more details. This VGAM family function handles multiple responses.

## Author(s)

T. W. Yee

## See Also

weibullR, dweibull, truncweibull, gev, lognormal, expexpff, maxwell, rayleigh, gumbelII.

## Examples

```
wdata <- data.frame(x2 = runif(nn <- 1000)) # Complete data
wdata <- transform(wdata, mu = exp(-1 + 1 * x2),
                        x3 = rnorm(nn),
                        shape1 = exp(1),
                        shape2 = exp(2))
wdata <- transform(wdata,
    y1 = rweibull(nn, shape = shape1, scale = mu / gamma(1 + 1/shape1)),
    y2 = rweibull(nn, shape = shape2, scale = mu / gamma(1 + 1/shape2)))
fit <- vglm(cbind(y1, y2) ~ x2 + x3, weibull.mean, data = wdata, trace = TRUE)
coef(fit, matrix = TRUE)
sqrt(diag(vcov(fit))) # SEs
summary(fit, presid = FALSE)
```

weibullR Weibull Distribution Family Function

## Description

Maximum likelihood estimation of the 2-parameter Weibull distribution. No observations should be censored.

## Usage

```
weibullR(lscale = "loglink", lshape = "loglink",
        iscale = NULL, ishape = NULL, lss = TRUE, nrfs = 1,
        probs.y = c(0.2, 0.5, 0.8), imethod = 1, zero = "shape")
```


## Arguments

lshape, lscale Parameter link functions applied to the (positive) shape parameter (called $a$ below) and (positive) scale parameter (called $b$ below). See Links for more choices.
ishape, iscale Optional initial values for the shape and scale parameters.
nrfs $\quad$ Currently this argument is ignored. Numeric, of length one, with value in $[0,1]$. Weighting factor between Newton-Raphson and Fisher scoring. The value 0 means pure Newton-Raphson, while 1 means pure Fisher scoring. The default value uses a mixture of the two algorithms, and retaining positive-definite working weights.
imethod Initialization method used if there are censored observations. Currently only the values 1 and 2 are allowed.
zero, probs.y, lss
Details at CommonVGAMffArguments.

## Details

The Weibull density for a response $Y$ is

$$
f(y ; a, b)=a y^{a-1} \exp \left[-(y / b)^{a}\right] /\left(b^{a}\right)
$$

for $a>0, b>0, y>0$. The cumulative distribution function is

$$
F(y ; a, b)=1-\exp \left[-(y / b)^{a}\right] .
$$

The mean of $Y$ is $b \Gamma(1+1 / a)$ (returned as the fitted values), and the mode is at $b(1-1 / a)^{1 / a}$ when $a>1$. The density is unbounded for $a<1$. The $k$ th moment about the origin is $E\left(Y^{k}\right)=$ $b^{k} \Gamma(1+k / a)$. The hazard function is $a t^{a-1} / b^{a}$.

This VGAM family function currently does not handle censored data. Fisher scoring is used to estimate the two parameters. Although the expected information matrices used here are valid in all regions of the parameter space, the regularity conditions for maximum likelihood estimation are satisfied only if $a>2$ (according to Kleiber and Kotz (2003)). If this is violated then a warning message is issued. One can enforce $a>2$ by choosing lshape = logofflink (offset $=-2$ ). Common values of the shape parameter lie between 0.5 and 3.5.
Summarized in Harper et al. (2011), for inference, there are 4 cases to consider. If $a \leq 1$ then the MLEs are not consistent (and the smallest observation becomes a hyperefficient solution for the location parameter in the 3-parameter case). If $1<a<2$ then MLEs exist but are not asymptotically normal. If $a=2$ then the MLEs exist and are normal and asymptotically efficient but with a slower convergence rate than when $a>2$. If $a>2$ then MLEs have classical asymptotic properties.
The 3-parameter (location is the third parameter) Weibull can be estimated by maximizing a profile log-likelihood (see, e.g., Harper et al. (2011) and Lawless (2003)), else try gev which is a better parameterization.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

This function is under development to handle other censoring situations. The version of this function which will handle censored data will be called cenweibull(). It is currently being written and will use SurvS4 as input. It should be released in later versions of VGAM.

If the shape parameter is less than two then misleading inference may result, e.g., in the summary and vcov of the object.

## Note

Successful convergence depends on having reasonably good initial values. If the initial values chosen by this function are not good, make use the two initial value arguments.
This VGAM family function handles multiple responses.
The Weibull distribution is often an alternative to the lognormal distribution. The inverse Weibull distribution, which is that of $1 / Y$ where $Y$ has a $\operatorname{Weibull}(a, b)$ distribution, is known as the logGompertz distribution.

There are problems implementing the three-parameter Weibull distribution. These are because the classical regularity conditions for the asymptotic properties of the MLEs are not satisfied because the support of the distribution depends on one of the parameters.
Other related distributions are the Maxwell and Rayleigh distributions.

## Author(s)

T. W. Yee

## References

Kleiber, C. and Kotz, S. (2003). Statistical Size Distributions in Economics and Actuarial Sciences, Hoboken, NJ, USA: Wiley-Interscience.

Johnson, N. L. and Kotz, S. and Balakrishnan, N. (1994). Continuous Univariate Distributions, 2nd edition, Volume 1, New York: Wiley.

Lawless, J. F. (2003). Statistical Models and Methods for Lifetime Data, 2nd ed. Hoboken, NJ, USA: John Wiley \& Sons.
Rinne, Horst. (2009). The Weibull Distribution: A Handbook. Boca Raton, FL, USA: CRC Press.
Gupta, R. D. and Kundu, D. (2006). On the comparison of Fisher information of the Weibull and GE distributions, Journal of Statistical Planning and Inference, 136, 3130-3144.

Harper, W. V. and Eschenbach, T. G. and James, T. R. (2011). Concerns about Maximum Likelihood Estimation for the Three-Parameter Weibull Distribution: Case Study of Statistical Software, The American Statistician, 65(1), 44-54.

Smith, R. L. (1985). Maximum likelihood estimation in a class of nonregular cases. Biometrika, 72, 67-90.

Smith, R. L. and Naylor, J. C. (1987). A comparison of maximum likelihood and Bayesian estimators for the three-parameter Weibull distribution. Applied Statistics, 36, 358-369.

## See Also

weibull.mean, dweibull, truncweibull, gev, lognormal, expexpff, maxwell, rayleigh, gumbelII.

## Examples

```
wdata <- data.frame(x2 = runif(nn <- 1000)) # Complete data
wdata <- transform(wdata,
    y1 = rweibull(nn, shape = exp(1), scale = exp(-2 + x2)),
    y2 = rweibull(nn, shape = exp(2), scale = exp( 1-x2)))
fit <- vglm(cbind(y1, y2) ~ x2, weibullR, data = wdata, trace = TRUE)
coef(fit, matrix = TRUE)
vcov(fit)
summary(fit)
```


## Description

Returns either the prior weights or working weights of a VGLM object.

## Usage

weightsvglm(object, type = c("prior", "working"), matrix.arg $=$ TRUE, ignore.slot $=$ FALSE, deriv.arg = FALSE, ...)

## Arguments

object a model object from the VGAM R package that inherits from a vector generalized linear model (VGLM), e.g., a model of class "vglm".
type $\quad$ Character, which type of weight is to be returned? The default is the first one.
matrix.arg Logical, whether the answer is returned as a matrix. If not, it will be a vector.
ignore.slot Logical. If TRUE then object@weights is ignored even if it has been assigned, and the long calculation for object@weights is repeated. This may give a slightly different answer because of the final IRLS step at convergence may or may not assign the latest value of quantities such as the mean and weights.
deriv.arg Logical. If TRUE then a list with components deriv and weights is returned. See below for more details.
... Currently ignored.

## Details

Prior weights are usually inputted with the weights argument in functions such as vglm and vgam. It may refer to frequencies of the individual data or be weight matrices specified beforehand.
Working weights are used by the IRLS algorithm. They correspond to the second derivatives of the log-likelihood function with respect to the linear predictors. The working weights correspond to positive-definite weight matrices and are returned in matrix-band form, e.g., the first $M$ columns correspond to the diagonals, etc.

If one wants to perturb the linear predictors then the fitted. values slots should be assigned to the object before calling this function. The reason is that, for some family functions, the variable mu is used directly as one of the parameter estimates, without recomputing it from eta.

## Value

If type = "working" and deriv = TRUE then a list is returned with the two components described below. Otherwise the prior or working weights are returned depending on the value of type.

# deriv Typically the first derivative of the log-likelihood with respect to the linear predictors. For example, this is the variable deriv.mu in vglm.fit(), or equivalently, the matrix returned in the "deriv" slot of a VGAM family function. <br> weights The working weights. 

## Note

This function is intended to be similar to weights.glm (see glm).

## Author(s)

Thomas W. Yee

## See Also

glm, vglmff-class, vglm.

## Examples

```
pneumo <- transform(pneumo, let = log(exposure.time))
(fit <- vglm(cbind(normal, mild, severe) ~ let,
    cumulative(parallel = TRUE, reverse = TRUE), data = pneumo))
depvar(fit) # These are sample proportions
weights(fit, type = "prior", matrix = FALSE) # Number of observations
# Look at the working residuals
nn <- nrow(model.matrix(fit, type = "lm"))
M <- ncol(predict(fit))
wwt <- weights(fit, type = "working", deriv = TRUE) # In matrix-band format
wz <- m2a(wwt$weights, M = M) # In array format
wzinv <- array(apply(wz, 3, solve), c(M, M, nn))
wresid <- matrix(NA, nn, M) # Working residuals
for (ii in 1:nn)
    wresid[ii, ] <- wzinv[, , ii, drop = TRUE] %*% wwt$deriv[ii, ]
max(abs(c(resid(fit, type = "work")) - c(wresid))) # Should be 0
(zedd <- predict(fit) + wresid) # Adjusted dependent vector
```

wine Bitterness in Wine Data

## Description

This oenological data frame concerns the amount of bitterness in 78 bottles of white wine.

## Usage

data(wine)

## Format

A data frame with 4 rows on the following 7 variables.
temp temperature, with levels cold and warm.
contact whether contact of the juice with the skin was allowed or avoided, for a specified period. Two levels: no or yes.
bitter1, bitter2, bitter3, bitter4, bitter5 numeric vectors, the counts. The order is none to most intense.

## Details

The data set comes from Randall (1989) and concerns a factorial experiment for investigating factors that affect the bitterness of white wines. There are two factors in the experiment: temperature at the time of crushing the grapes and contact of the juice with the skin. Two bottles of wine were fermented for each of the treatment combinations. A panel of 9 judges were selected and trained for the ability to detect bitterness. Thus there were 72 bottles in total. Originally, the bitterness of the wine were taken on a continuous scale in the interval from 0 (none) to 100 (intense) but later they were grouped using equal lengths into five ordered categories $1,2,3,4$ and 5.

## Source

Christensen, R. H. B. (2013) Analysis of ordinal data with cumulative link models-estimation with the R-package ordinal. R Package Version 2013.9-30. https: //CRAN.R-project. org/package= ordinal.

Randall, J. H. (1989). The analysis of sensory data by generalized linear model. Biometrical Journal 31(7), 781-793.
Kosmidis, I. (2014). Improved estimation in cumulative link models. Journal of the Royal Statistical Society, Series B, Methodological, 76(1): 169-196.

## Examples

```
wine
summary(wine)
```

wrapup.smart

Cleans Up After Smart Prediction

## Description

wrapup. smart deletes any variables used by smart prediction. Needed by both the modelling function and the prediction function.

## Usage

wrapup.smart()

## Details

The variables to be deleted are .smart. prediction, .smart.prediction. counter, and .smart. prediction.mode. The function wrapup. smart is useful in $R$ because these variables are held in smartpredenv.

## See Also

```
setup.smart.
```


## Examples

```
## Not run: # Place this inside modelling functions such as lm, glm, vglm.
wrapup.smart() # Put at the end of lm
## End(Not run)
```

```
yeo.johnson
```


## Yeo-Johnson Transformation

## Description

Computes the Yeo-Johnson transformation, which is a normalizing transformation.

## Usage

```
yeo.johnson(y, lambda, derivative = 0,
    epsilon = sqrt(.Machine$double.eps), inverse = FALSE)
```


## Arguments

| y | Numeric, a vector or matrix. |
| :--- | :--- |
| lambda | Numeric. It is recycled to the same length as y if necessary. |
| derivative | Non-negative integer. The default is the ordinary function evaluation, otherwise <br> the derivative with respect to lambda. |
| epsilon | Numeric and positive value. The tolerance given to values of lambda when <br> comparing it to 0 or 2. |
| inverse | Logical. Return the inverse transformation? |

## Details

The Yeo-Johnson transformation can be thought of as an extension of the Box-Cox transformation. It handles both positive and negative values, whereas the Box-Cox transformation only handles positive values. Both can be used to transform the data so as to improve normality. They can be used to perform LMS quantile regression.

## Value

The Yeo-Johnson transformation or its inverse, or its derivatives with respect to lambda, of y.

## Note

If inverse $=$ TRUE then the argument derivative $=0$ is required.

## Author(s)

Thomas W. Yee

## References

Yeo, I.-K. and Johnson, R. A. (2000). A new family of power transformations to improve normality or symmetry. Biometrika, 87, 954-959.
Yee, T. W. (2004). Quantile regression via vector generalized additive models. Statistics in Medicine, 23, 2295-2315.

## See Also

lms.yjn, boxcox.

## Examples

```
y <- seq(-4, 4, len = (nn <- 200))
ltry <- c(0, 0.5, 1, 1.5, 2) # Try these values of lambda
lltry <- length(ltry)
psi <- matrix(as.numeric(NA), nn, lltry)
for (ii in 1:lltry)
    psi[, ii] <- yeo.johnson(y, lambda = ltry[ii])
## Not run:
matplot(y, psi, type = "l", ylim = c(-4, 4), lwd = 2, lty = 1:lltry,
        ylab = "Yeo-Johnson transformation", col = 1:lltry, las = 1,
        main = "Yeo-Johnson transformation with some values of lambda")
abline(v = 0, h = 0)
legend(x = 1, y = -0.5, lty = 1:lltry, legend = as.character(ltry),
            lwd = 2, col = 1:lltry)
## End(Not run)
```

    yip88 Zero-Inflated Poisson Distribution (Yip (1988) algorithm)
    
## Description

Fits a zero-inflated Poisson distribution based on Yip (1988).

## Usage

```
yip88(link = "loglink", n.arg = NULL, imethod = 1)
```


## Arguments

link Link function for the usual $\lambda$ parameter. See Links for more choices.
n.arg The total number of observations in the data set. Needed when the response variable has all the zeros deleted from it, so that the number of zeros can be determined.
imethod Details at CommonVGAMffArguments.

## Details

The method implemented here, Yip (1988), maximizes a conditional likelihood. Consequently, the methodology used here deletes the zeros from the data set, and is thus related to the positive Poisson distribution (where $P(Y=0)=0$ ).
The probability function of $Y$ is 0 with probability $\phi$, and $\operatorname{Poisson}(\lambda)$ with probability $1-\phi$. Thus

$$
P(Y=0)=\phi+(1-\phi) P(W=0)
$$

where $W$ is Poisson $(\lambda)$. The mean, $(1-\phi) \lambda$, can be obtained by the extractor function fitted applied to the object.
This family function treats $\phi$ as a scalar. If you want to model both $\phi$ and $\lambda$ as a function of covariates, try zipoisson.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Warning

Under- or over-flow may occur if the data is ill-conditioned. Yip (1988) only considered $\phi$ being a scalar and not modelled as a function of covariates. To get around this limitation, try zipoisson.
Inference obtained from summary.vglm and summary.vgam may or may not be correct. In particular, the p-values, standard errors and degrees of freedom may need adjustment. Use simulation on artificial data to check that these are reasonable.

## Note

The data may be inputted in two ways. The first is when the response is a vector of positive values, with the argument $n$ in yip88 specifying the total number of observations. The second is simply include all the data in the response. In this case, the zeros are trimmed off during the computation, and the x and y slots of the object, if assigned, will reflect this.

The estimate of $\phi$ is placed in the misc slot as @misc\$pstr0. However, this estimate is computed only for intercept models, i.e., the formula is of the form $\mathrm{y} \sim 1$.

## Author(s)

Thomas W. Yee

## References

Yip, P. (1988). Inference about the mean of a Poisson distribution in the presence of a nuisance parameter. The Australian Journal of Statistics, 30, 299-306.
Angers, J-F. and Biswas, A. (2003). A Bayesian analysis of zero-inflated generalized Poisson model. Computational Statistics \& Data Analysis, 42, 37-46.

## See Also

zipoisson, Zipois, zapoisson, pospoisson, poissonff, dzipois.

## Examples

```
phi <- 0.35; lambda <- 2 # Generate some artificial data
y <- rzipois(n <- 1000, lambda, phi)
table(y)
# Two equivalent ways of fitting the same model
fit1 <- vglm(y ~ 1, yip88(n = length(y)), subset = y > 0)
fit2 <- vglm(y ~ 1, yip88, trace = TRUE, crit = "coef")
(true.mean <- (1-phi) * lambda)
mean(y)
head(fitted(fit1))
fit1@misc$pstr0 # The estimate of phi
# Compare the ZIP with the positive Poisson distribution
pp <- vglm(y ~ 1, pospoisson, subset = y > 0, crit = "c")
coef(pp)
Coef(pp)
coef(fit1) - coef(pp) # Same
head(fitted(fit1) - fitted(pp)) # Different
# Another example (Angers and Biswas, 2003) -------------------------
abdata <- data.frame(y = 0:7, w = c(182, 41, 12, 2, 2, 0, 0, 1))
abdata <- subset(abdata, w > 0)
yy <- with(abdata, rep(y, w))
fit3 <- vglm(yy ~ 1, yip88(n = length(yy)), subset = yy > 0)
fit3@misc$pstr0 # Estimate of phi (they get 0.5154 with SE 0.0707)
coef(fit3, matrix = TRUE)
Coef(fit3) # Estimate of lambda (they get 0.6997 with SE 0.1520)
head(fitted(fit3))
mean(yy) # Compare this with fitted(fit3)
```

Yules Yule-Simon Distribution

## Description

Density, distribution function, quantile function and random generation for the Yule-Simon distribution.

## Usage

dyules(x, shape, log = FALSE)
pyules(q, shape, lower.tail = TRUE, log.p = FALSE)
qyules(p, shape)
ryules( $n$, shape)

## Arguments

| $\mathrm{x}, \mathrm{q}, \mathrm{p}, \mathrm{n}$ Same meaning as in Normal. <br> shape See yulesimon. <br> log, lower.tail, log.p <br>  Same meaning as in pnorm or qnorm. |  |
| :--- | :--- |
|  |  |

## Details

See yulesimon, the VGAM family function for estimating the parameter, for the formula of the probability density function and other details.

## Value

dyules gives the density, pyules gives the distribution function, qyules gives the quantile function, and ryules generates random deviates.

## Note

Numerical problems may occur with qyules() when $p$ is very close to 1 .

## Author(s)

T. W. Yee

## See Also

yulesimon.

## Examples

```
dyules(1:20, 2.1)
ryules(20, 2.1)
round(1000 * dyules(1:8, 2))
table(ryules(1000, 2))
## Not run: x <- 0:6
plot(x, dyules(x, shape = 2.2), type = "h", las = 1, col = "blue")
## End(Not run)
```

yulesimon Yule-Simon Family Function

## Description

Estimating the shape parameter of the Yule-Simon distribution.

## Usage

yulesimon(lshape $=$ "loglink", ishape $=$ NULL, nsimEIM $=200$, zero $=$ NULL)

## Arguments

lshape Link function for the shape parameter, called $\rho$ below. See Links for more choices and for general information.
ishape Optional initial value for the (positive) parameter. See CommonVGAMffArguments for more information. The default is to obtain an initial value internally. Use this argument if the default fails.
nsimEIM, zero See CommonVGAMffArguments for more information.

## Details

The probability function is

$$
f(y ; \rho)=\rho * \operatorname{beta}(y, \rho+1),
$$

where the parameter $\rho>0$, beta is the beta function, and $y=1,2, \ldots$. The function dyules computes this probability function. The mean of $Y$, which is returned as fitted values, is $\rho /(\rho-1)$ provided $\rho>1$. The variance of $Y$ is $\rho^{2} /\left((\rho-1)^{2}(\rho-2)\right)$ provided $\rho>2$.
The distribution was named after Udny Yule and Herbert A. Simon. Simon originally called it the Yule distribution. This family function can handle multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Author(s)

T. W. Yee

## References

Simon, H. A. (1955). On a class of skew distribution functions. Biometrika, 42, 425-440.

## See Also

ryules, simulate.vlm.

## Examples

```
ydata <- data.frame(x2 = runif(nn <- 1000))
ydata <- transform(ydata, y = ryules(nn, shape = exp(1.5 - x2)))
with(ydata, table(y))
fit <- vglm(y ~ x2, yulesimon, data = ydata, trace = TRUE)
coef(fit, matrix = TRUE)
summary(fit)
```

Zabinom
Zero-Altered Binomial Distribution

## Description

Density, distribution function, quantile function and random generation for the zero-altered binomial distribution with parameter pobs0.

## Usage

dzabinom(x, size, prob, pobs0 $=0, \log =$ FALSE)
pzabinom(q, size, prob, pobs0 = 0)
qzabinom(p, size, prob, pobs0 = 0)
rzabinom(n, size, prob, pobs0 = 0)

## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$n \quad$ number of observations. If length $(n)>1$ then the length is taken to be the number required.
size, prob, log Parameters from the ordinary binomial distribution (see dbinom).
pobs0 Probability of (an observed) zero, called pobs 0 . The default value of pobs $0=0$ corresponds to the response having a positive binomial distribution.

## Details

The probability function of $Y$ is 0 with probability pobs 0 , else a positive binomial(size, prob) distribution.

## Value

dzabinom gives the density and pzabinom gives the distribution function, qzabinom gives the quantile function, and rzabinom generates random deviates.

## Note

The argument pobs 0 is recycled to the required length, and must have values which lie in the interval $[0,1]$.

## Author(s)

T. W. Yee

## See Also

zibinomial, rposbinom.

## Examples

```
size <- 10; prob <- 0.15; pobs0 <- 0.05; x <- (-1):7
dzabinom(x, size = size, prob = prob, pobs0 = pobs0)
table(rzabinom(100, size = size, prob = prob, pobs0 = pobs0))
## Not run: x <- 0:10
barplot(rbind(dzabinom(x, size = size, prob = prob, pobs0 = pobs0),
            dbinom(x, size = size, prob = prob)),
        beside = TRUE, col = c("blue", "orange"), cex.main = 0.7, las = 1,
        ylab = "Probability", names.arg = as.character(x),
        main = paste("ZAB(size = ", size, ", prob = ", prob, ", pobs0 = ", pobs0,
            ") [blue] vs", " Binom(size = ", size, ", prob = ", prob,
            ") [orange] densities", sep = ""))
## End(Not run)
```

zabinomial
Zero-Altered Binomial Distribution

## Description

Fits a zero-altered binomial distribution based on a conditional model involving a Bernoulli distribution and a positive-binomial distribution.

## Usage

```
zabinomial(lpobs0 = "logitlink", lprob = "logitlink",
        type.fitted = c("mean", "prob", "pobs0"),
        ipobs0 = NULL, iprob = NULL, imethod = 1, zero = NULL)
zabinomialff(lprob = "logitlink", lonempobs0 = "logitlink",
            type.fitted = c("mean", "prob", "pobs0", "onempobs0"),
            iprob = NULL, ionempobs0 = NULL, imethod = 1, zero = "onempobs0")
```


## Arguments

lprob Parameter link function applied to the probability parameter of the binomial distribution. See Links for more choices.
lpobs0 Link function for the parameter $p_{0}$, called pobs0 here. See Links for more choices.
type.fitted See CommonVGAMffArguments and fittedvlm for information.

```
iprob, ipobs0 See CommonVGAMffArguments.
```

lonempobs0, ionempobs0

Corresponding argument for the other parameterization. See details below.
imethod, zero See CommonVGAMffArguments.

## Details

The response $Y$ is zero with probability $p_{0}$, else $Y$ has a positive-binomial distribution with probability $1-p_{0}$. Thus $0<p_{0}<1$, which may be modelled as a function of the covariates. The zero-altered binomial distribution differs from the zero-inflated binomial distribution in that the former has zeros coming from one source, whereas the latter has zeros coming from the binomial distribution too. The zero-inflated binomial distribution is implemented in zibinomial. Some people call the zero-altered binomial a hurdle model.

The input is currently a vector or one-column matrix. By default, the two linear/additive predictors for zabinomial() are $\left(\operatorname{logit}\left(p_{0}\right), \log (p)\right)^{T}$.

The VGAM family function zabinomialff() has a few changes compared to zabinomial(). These are: (i) the order of the linear/additive predictors is switched so the binomial probability comes first; (ii) argument onempobs 0 is now 1 minus the probability of an observed 0 , i.e., the probability of the positive binomial distribution, i.e., onempobs 0 is 1 -pobs 0 ; (iii) argument zero has a new default so that the onempobs 0 is intercept-only by default. Now zabinomialff() is generally recommended over zabinomial(). Both functions implement Fisher scoring and neither can handle multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.
The fitted.values slot of the fitted object, which should be extracted by the generic function fitted, returns the mean $\mu$ (default) which is given by

$$
\mu=\left(1-p_{0}\right) \mu_{b} /\left[1-\left(1-\mu_{b}\right)^{N}\right]
$$

where $\mu_{b}$ is the usual binomial mean. If type.fitted $="$ pobs $0 "$ then $p_{0}$ is returned.

## Note

The response should be a two-column matrix of counts, with first column giving the number of successes.
Note this family function allows $p_{0}$ to be modelled as functions of the covariates by having zero $=$ NULL. It is a conditional model, not a mixture model.

These family functions effectively combine posbinomial and binomialff into one family function.

## Author(s)

T. W. Yee

## See Also

dzabinom, zibinomial, posbinomial, binomialff, dbinom, CommonVGAMffArguments.

## Examples

```
zdata <- data.frame(x2 = runif(nn <- 1000))
zdata <- transform(zdata, size = 10,
    prob = logitlink(-2 + 3*x2, inverse = TRUE),
    pobs0 = logitlink(-1 + 2*x2, inverse = TRUE))
zdata <- transform(zdata,
            y1 = rzabinom(nn, size = size, prob = prob, pobs0 = pobs0))
with(zdata, table(y1))
zfit <- vglm(cbind(y1, size - y1) ~ x2, zabinomial(zero = NULL),
                        data = zdata, trace = TRUE)
coef(zfit, matrix = TRUE)
head(fitted(zfit))
head(predict(zfit))
summary(zfit)
```

Zageom Zero-Altered Geometric Distribution

## Description

Density, distribution function, quantile function and random generation for the zero-altered geometric distribution with parameter pobs0.

## Usage

dzageom(x, prob, pobs0 $=0,1 \mathrm{l}=\mathrm{FALSE})$
pzageom(q, prob, pobs0 = 0)
qzageom (p, prob, pobs0 = 0)
rzageom(n, prob, pobs0 = 0)

## Arguments

$x, q \quad$ vector of quantiles.
p
$n \quad$ number of observations. If length $(n)>1$ then the length is taken to be the number required.
prob, log Parameters from the ordinary geometric distribution (see dgeom).
pobs0 Probability of (an observed) zero, called pobs0. The default value of pobs0 $=0$ corresponds to the response having a positive geometric distribution.

## Details

The probability function of $Y$ is 0 with probability pobs 0 , else a positive geometric(prob) distribution.

## Value

dzageom gives the density and pzageom gives the distribution function, qzageom gives the quantile function, and rzageom generates random deviates.

## Note

The argument pobs 0 is recycled to the required length, and must have values which lie in the interval $[0,1]$.

## Author(s)

T. W. Yee

## See Also

zageometric, zigeometric, rposgeom.

## Examples

```
prob <- 0.35; pobs0 <- 0.05; x <- (-1):7
dzageom(x, prob = prob, pobs0 = pobs0)
table(rzageom(100, prob = prob, pobs0 = pobs0))
## Not run: x <- 0:10
barplot(rbind(dzageom(x, prob = prob, pobs0 = pobs0),
    dgeom(x, prob = prob)),
    beside = TRUE, col = c("blue", "orange"), cex.main = 0.7, las = 1,
    ylab = "Probability", names.arg = as.character(x),
    main = paste("ZAG(prob = ", prob, ", pobs0 = ", pobs0,
    ") [blue] vs", " Geometric(prob = ", prob,
    ") [orange] densities", sep = ""))
## End(Not run)
```

    zageometric
    Zero-Altered Geometric Distribution

## Description

Fits a zero-altered geometric distribution based on a conditional model involving a Bernoulli distribution and a positive-geometric distribution.

## Usage

```
zageometric(lpobs0 = "logitlink", lprob = "logitlink",
            type.fitted = c("mean", "prob", "pobs0", "onempobs0"),
            imethod = 1, ipobs0 = NULL, iprob = NULL, zero = NULL)
zageometricff(lprob = "logitlink", lonempobs0 = "logitlink",
                type.fitted = c("mean", "prob", "pobs0", "onempobs0"),
                imethod = 1, iprob = NULL, ionempobs0 = NULL, zero = "onempobs0")
```


## Arguments

lpobs0 Link function for the parameter $p_{0}$ or $\phi$, called pobs0 or phi here. See Links for more choices.
lprob Parameter link function applied to the probability of success, called prob or $p$. See Links for more choices.
type.fitted See CommonVGAMffArguments and fittedvlm for information.
ipobs0, iprob Optional initial values for the parameters. If given, they must be in range. For multi-column responses, these are recycled sideways.
lonempobs0, ionempobs0
Corresponding argument for the other parameterization. See details below.
zero, imethod See CommonVGAMffArguments.

## Details

The response $Y$ is zero with probability $p_{0}$, or $Y$ has a positive-geometric distribution with probability $1-p_{0}$. Thus $0<p_{0}<1$, which is modelled as a function of the covariates. The zero-altered geometric distribution differs from the zero-inflated geometric distribution in that the former has zeros coming from one source, whereas the latter has zeros coming from the geometric distribution too. The zero-inflated geometric distribution is implemented in the VGAM package. Some people call the zero-altered geometric a hurdle model.
The input can be a matrix (multiple responses). By default, the two linear/additive predictors of zageometric are $(\operatorname{logit}(\phi), \operatorname{logit}(p))^{T}$.
The VGAM family function zageometricff() has a few changes compared to zageometric (). These are: (i) the order of the linear/additive predictors is switched so the geometric probability comes first; (ii) argument onempobs 0 is now 1 minus the probability of an observed 0 , i.e., the probability of the positive geometric distribution, i.e., onempobs 0 is 1 -pobs 0 ; (iii) argument zero has a new default so that the pobs0 is intercept-only by default. Now zageometricff() is generally recommended over zageometric(). Both functions implement Fisher scoring and can handle multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.
The fitted.values slot of the fitted object, which should be extracted by the generic function fitted, returns the mean $\mu$ (default) which is given by

$$
\mu=(1-\phi) / p
$$

If type.fitted $=$ "pobs 0 " then $p_{0}$ is returned.

## Warning

Convergence for this VGAM family function seems to depend quite strongly on providing good initial values.

Inference obtained from summary.vglm and summary.vgam may or may not be correct. In particular, the p-values, standard errors and degrees of freedom may need adjustment. Use simulation on artificial data to check that these are reasonable.

## Note

Note this family function allows $p_{0}$ to be modelled as functions of the covariates. It is a conditional model, not a mixture model.
This family function effectively combines binomialff and posgeometric() and geometric into one family function. However, posgeometric() is not written because it is trivially related to geometric.

## Author(s)

T. W. Yee

## See Also

dzageom, geometric, zigeometric, dgeom, CommonVGAMffArguments, simulate.vlm.

## Examples

```
zdata <- data.frame(x2 = runif(nn <- 1000))
zdata <- transform(zdata, pobs0 = logitlink(-1 + 2*x2, inverse = TRUE),
    prob = logitlink(-2 + 3*x2, inverse = TRUE))
zdata <- transform(zdata, y1 = rzageom(nn, prob = prob, pobs0 = pobs0),
    y2 = rzageom(nn, prob = prob, pobs0 = pobs0))
with(zdata, table(y1))
fit <- vglm(cbind(y1, y2) ~ x2, zageometric, data = zdata, trace = TRUE)
coef(fit, matrix = TRUE)
head(fitted(fit))
head(predict(fit))
summary(fit)
```

Zanegbin Zero-Altered Negative Binomial Distribution

## Description

Density, distribution function, quantile function and random generation for the zero-altered negative binomial distribution with parameter pobs0.

## Usage

```
dzanegbin(x, size, prob = NULL, munb = NULL, pobs0 = 0, log = FALSE)
pzanegbin(q, size, prob \(=\) NULL, munb \(=\) NULL, pobs0 \(=0\) )
qzanegbin(p, size, prob = NULL, munb = NULL, pobs0 = 0)
rzanegbin(n, size, prob \(=\) NULL, munb \(=\) NULL, pobs0 \(=0\) )
```


## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$n \quad$ number of observations. If length $(n)>1$ then the length is taken to be the number required.
size, prob, munb, log
Parameters from the ordinary negative binomial distribution (see dnbinom). Some arguments have been renamed slightly.
pobs $0 \quad$ Probability of zero, called pobs 0 . The default value of pobs $0=0$ corresponds to the response having a positive negative binomial distribution.

## Details

The probability function of $Y$ is 0 with probability pobs0, else a positive negative binomial ( $\mu_{n b}$, size) distribution.

## Value

dzanegbin gives the density and pzanegbin gives the distribution function, qzanegbin gives the quantile function, and rzanegbin generates random deviates.

## Note

The argument pobs 0 is recycled to the required length, and must have values which lie in the interval $[0,1]$.

## Author(s)

## T. W. Yee

## See Also

gatnbinomial.mlm, Gaitnbinom.mlm, zanegbinomial, rposnegbin.

## Examples

```
munb <- 3; size <- 4; pobs0 <- 0.3; x <- (-1):7
dzanegbin(x, munb = munb, size = size, pobs0 = pobs0)
table(rzanegbin(100, munb = munb, size = size, pobs0 = pobs0))
## Not run: x <- 0:10
barplot(rbind(dzanegbin(x, munb = munb, size = size, pobs0 = pobs0),
```

```
            dnbinom(x, mu = munb, size = size)),
beside = TRUE, col = c("blue", "green"), cex.main = 0.7, las = 1,
ylab = "Probability", names.arg = as.character(x),
main = paste("ZANB(munb = ", munb, ", size = ", size,",
            pobs0 = ", pobs0,
            ") [blue] vs", " NB(mu = ", munb, ", size = ", size,
            ") [green] densities", sep = ""))
## End(Not run)
```

zanegbinomial
Zero-Altered Negative Binomial Distribution

## Description

Fits a zero-altered negative binomial distribution based on a conditional model involving a binomial distribution and a positive-negative binomial distribution.

## Usage

```
zanegbinomial(zero = "size", type.fitted = c("mean", "munb", "pobs0"),
            mds.min = 1e-3, nsimEIM = 500, cutoff.prob = 0.999,
            eps.trig = 1e-7, max.support = 4000, max.chunk. MB = 30,
            lpobs0 = "logitlink", lmunb = "loglink", lsize = "loglink",
            imethod = 1, ipobs0 = NULL,
            imunb = NULL, iprobs.y = NULL, gprobs.y = (0:9)/10,
            isize = NULL, gsize.mux = exp(c(-30, -20, -15, -10, -6:3)))
zanegbinomialff(lmunb = "loglink", lsize = "loglink", lonempobs0 = "logitlink",
                type.fitted = c("mean", "munb", "pobs0", "onempobs0"),
                isize = NULL, ionempobs0 = NULL, zero = c("size",
    "onempobs0"), mds.min = 1e-3, iprobs.y = NULL, gprobs.y = (0:9)/10,
                cutoff.prob = 0.999, eps.trig = 1e-7, max.support = 4000,
            max.chunk.MB = 30, gsize.mux = exp(c(-30, -20, -15, -10, -6:3)),
                imethod = 1, imunb = NULL,
                nsimEIM = 500)
```


## Arguments

lpobs0 Link function for the parameter $p_{0}$, called pobs0 here. See Links for more choices.
lmunb Link function applied to the munb parameter, which is the mean $\mu_{n b}$ of an ordinary negative binomial distribution. See Links for more choices.
lsize Parameter link function applied to the reciprocal of the dispersion parameter, called $k$. That is, as $k$ increases, the variance of the response decreases. See Links for more choices.
type.fitted See CommonVGAMffArguments and fittedvlm for information.
lonempobs0, ionempobs0
Corresponding argument for the other parameterization. See details below.

```
ipobs0, imunb, isize
```

Optional initial values for $p_{0}$ and munb and $k$. If given then it is okay to give one value for each response/species by inputting a vector whose length is the number of columns of the response matrix.
zero Specifies which of the three linear predictors are modelled as intercept-only.
All parameters can be modelled as a function of the explanatory variables by setting zero $=$ NULL (not recommended). A negative value means that the value is recycled, e.g., setting -3 means all $k$ are intercept-only for zanegbinomial. See CommonVGAMffArguments for more information.
nsimEIM, imethod
See CommonVGAMffArguments.
iprobs.y, gsize.mux, gprobs.y
See negbinomial.
cutoff.prob, eps.trig
See negbinomial.
mds.min, max. support, max. chunk. MB
See negbinomial.

## Details

The response $Y$ is zero with probability $p_{0}$, or $Y$ has a positive-negative binomial distribution with probability $1-p_{0}$. Thus $0<p_{0}<1$, which is modelled as a function of the covariates. The zeroaltered negative binomial distribution differs from the zero-inflated negative binomial distribution in that the former has zeros coming from one source, whereas the latter has zeros coming from the negative binomial distribution too. The zero-inflated negative binomial distribution is implemented in the VGAM package. Some people call the zero-altered negative binomial a hurdle model.
For one response/species, by default, the three linear/additive predictors for zanegbinomial() are $\left(\operatorname{logit}\left(p_{0}\right), \log \left(\mu_{n b}\right), \log (k)\right)^{T}$. This vector is recycled for multiple species.
The VGAM family function zanegbinomialff() has a few changes compared to zanegbinomial(). These are: (i) the order of the linear/additive predictors is switched so the negative binomial mean comes first; (ii) argument onempobs 0 is now 1 minus the probability of an observed 0 , i.e., the probability of the positive negative binomial distribution, i.e., onempobs 0 is 1 -pobs 0 ; (iii) argument zero has a new default so that the pobs 0 is intercept-only by default. Now zanegbinomialff() is generally recommended over zanegbinomial(). Both functions implement Fisher scoring and can handle multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.
The fitted.values slot of the fitted object, which should be extracted by the generic function fitted, returns the mean $\mu$ (default) which is given by

$$
\mu=\left(1-p_{0}\right) \mu_{n b} /\left[1-\left(k /\left(k+\mu_{n b}\right)\right)^{k}\right] .
$$

If type.fitted $="$ pobs 0 " then $p_{0}$ is returned.

## Warning

This family function is fragile; it inherits the same difficulties as posnegbinomial. Convergence for this VGAM family function seems to depend quite strongly on providing good initial values.
This VGAM family function is computationally expensive and usually runs slowly; setting trace $=$ TRUE is useful for monitoring convergence.
Inference obtained from summary.vglm and summary. vgam may or may not be correct. In particular, the p-values, standard errors and degrees of freedom may need adjustment. Use simulation on artificial data to check that these are reasonable.

## Note

Note this family function allows $p_{0}$ to be modelled as functions of the covariates provided zero is set correctly. It is a conditional model, not a mixture model. Simulated Fisher scoring is the algorithm.

This family function effectively combines posnegbinomial and binomialff into one family function.
This family function can handle multiple responses, e.g., more than one species.

## Author(s)

T. W. Yee

## References

Welsh, A. H., Cunningham, R. B., Donnelly, C. F. and Lindenmayer, D. B. (1996). Modelling the abundances of rare species: statistical models for counts with extra zeros. Ecological Modelling, 88, 297-308.
Yee, T. W. (2014). Reduced-rank vector generalized linear models with two linear predictors. Computational Statistics and Data Analysis, 71, 889-902.

## See Also

dzanegbin, posnegbinomial, gatnbinomial.mlm, Gaitnbinom.mlm, negbinomial, binomialff, rposnegbin, zinegbinomial, zipoisson, dnbinom, CommonVGAMffArguments, simulate.vlm.

## Examples

```
## Not run:
zdata <- data.frame(x2 = runif(nn <- 2000))
zdata <- transform(zdata, pobs0 = logitlink(-1 + 2*x2, inverse = TRUE))
zdata <- transform(zdata,
    y1 = rzanegbin(nn, munb = exp(0+2*x2), size = exp(1), pobs0 = pobs0),
    y2 = rzanegbin(nn, munb = exp(1+2*x2), size = exp(1), pobs0 = pobs0))
with(zdata, table(y1))
with(zdata, table(y2))
fit <- vglm(cbind(y1, y2) ~ x2, zanegbinomial, data = zdata, trace = TRUE)
coef(fit, matrix = TRUE)
```

```
head(fitted(fit))
head(predict(fit))
## End(Not run)
```

Zapois Zero-Altered Poisson Distribution

## Description

Density, distribution function, quantile function and random generation for the zero-altered Poisson distribution with parameter pobs0.

## Usage

dzapois(x, lambda, pobs0 = 0, log = FALSE)
pzapois(q, lambda, pobs0 = 0)
qzapois(p, lambda, pobs0 = 0)
rzapois(n, lambda, pobs0 = 0)

## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$n \quad$ number of observations. If length $(n)>1$ then the length is taken to be the number required.
lambda Vector of positive means.
pobs0 Probability of zero, called pobs0. The default value of pobs $0=0$ corresponds to the response having a positive Poisson distribution.
log Logical. Return the logarithm of the answer?

## Details

The probability function of $Y$ is 0 with probability pobs 0 , else a positive $\operatorname{Poisson}(\lambda)$.

## Value

dzapois gives the density, pzapois gives the distribution function, qzapois gives the quantile function, and rzapois generates random deviates.

## Note

The argument pobs 0 is recycled to the required length, and must have values which lie in the interval $[0,1]$.

## Author(s)

## T. W. Yee

## See Also

zapoisson, Gaitpois, dzipois.

## Examples

```
lambda <- 3; pobs0 <- 0.2; x <- (-1):7
(ii <- dzapois(x, lambda, pobs0))
max(abs(cumsum(ii) - pzapois(x, lambda, pobs0))) # Should be 0
table(rzapois(100, lambda, pobs0))
table(qzapois(runif(100), lambda, pobs0))
round(dzapois(0:10, lambda, pobs0) * 100) # Should be similar
## Not run: x <- 0:10
barplot(rbind(dzapois(x, lambda, pobs0), dpois(x, lambda)),
    beside = TRUE, col = c("blue", "green"), las = 1,
    main = paste("ZAP(", lambda, ", pobs0 = ", pobs0, ") [blue] vs",
                            " Poisson(", lambda, ") [green] densities", sep = ""),
        names.arg = as.character(x), ylab = "Probability")
## End(Not run)
```

zapoisson

Zero-Altered Poisson Distribution

## Description

Fits a zero-altered Poisson distribution based on a conditional model involving a Bernoulli distribution and a positive-Poisson distribution.

## Usage

```
zapoisson(lpobs0 = "logitlink", llambda = "loglink", type.fitted =
    c("mean", "lambda", "pobs0", "onempobs0"), imethod = 1,
    ipobs0 = NULL, ilambda = NULL, ishrinkage = 0.95, probs.y = 0.35,
    zero = NULL)
zapoissonff(llambda = "loglink", lonempobs0 = "logitlink", type.fitted =
            c("mean", "lambda", "pobs0", "onempobs0"), imethod = 1,
            ilambda = NULL, ionempobs0 = NULL, ishrinkage = 0.95,
            probs.y = 0.35, zero = "onempobs0")
```


## Arguments

lpobs0 Link function for the parameter $p_{0}$, called pobs0 here. See Links for more choices.
llambda Link function for the usual $\lambda$ parameter. See Links for more choices.

| type.fitted | See CommonVGAMffArguments and fittedvlm for information. |
| :--- | :--- |
| lonempobs0 | Corresponding argument for the other parameterization. See details below. |
| imethod, ipobs 0, ionempobs0, ilambda, ishrinkage |  |
|  | See CommonVGAMffArguments for information. |
| probs.y, zero | See CommonVGAMffArguments for information. |

## Details

The response $Y$ is zero with probability $p_{0}$, else $Y$ has a positive-Poisson $(\lambda)$ distribution with probability $1-p_{0}$. Thus $0<p_{0}<1$, which is modelled as a function of the covariates. The zeroaltered Poisson distribution differs from the zero-inflated Poisson distribution in that the former has zeros coming from one source, whereas the latter has zeros coming from the Poisson distribution too. Some people call the zero-altered Poisson a hurdle model.
For one response/species, by default, the two linear/additive predictors for zapoisson() are $\left(\operatorname{logit}\left(p_{0}\right), \log (\lambda)\right)^{T}$.
The VGAM family function zapoissonff() has a few changes compared to zapoisson(). These are: (i) the order of the linear/additive predictors is switched so the Poisson mean comes first; (ii) argument onempobs0 is now 1 minus the probability of an observed 0 , i.e., the probability of the positive Poisson distribution, i.e., onempobs0 is 1 -pobs0; (iii) argument zero has a new default so that the onempobs0 is intercept-only by default. Now zapoissonff() is generally recommended over zapoisson(). Both functions implement Fisher scoring and can handle multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

The fitted.values slot of the fitted object, which should be extracted by the generic function fitted, returns the mean $\mu$ (default) which is given by

$$
\mu=\left(1-p_{0}\right) \lambda /[1-\exp (-\lambda)]
$$

If type.fitted $=$ "pobs $0 "$ then $p_{0}$ is returned.

## Note

There are subtle differences between this family function and zipoisson and yip88. In particular, zipoisson is a mixture model whereas zapoisson() and yip88 are conditional models.

Note this family function allows $p_{0}$ to be modelled as functions of the covariates.
This family function effectively combines pospoisson and binomialff into one family function. This family function can handle multiple responses, e.g., more than one species.

## Author(s)

T. W. Yee

## References

Welsh, A. H., Cunningham, R. B., Donnelly, C. F. and Lindenmayer, D. B. (1996). Modelling the abundances of rare species: statistical models for counts with extra zeros. Ecological Modelling, 88, 297-308.
Angers, J-F. and Biswas, A. (2003). A Bayesian analysis of zero-inflated generalized Poisson model. Computational Statistics \& Data Analysis, 42, 37-46.

Yee, T. W. (2014). Reduced-rank vector generalized linear models with two linear predictors. Computational Statistics and Data Analysis, 71, 889-902.

## See Also

rzapois, zipoisson, gaitpoisson.mix, gaitpoisson.mlm, pospoisson, posnegbinomial, binomialff, rpospois, CommonVGAMffArguments, simulate.vlm.

## Examples

```
zdata <- data.frame(x2 = runif(nn <- 1000))
zdata <- transform(zdata, pobs0 = logitlink( -1 + 1*x2, inverse = TRUE),
    lambda = loglink(-0.5 + 2*x2, inverse = TRUE))
zdata <- transform(zdata, y = rzapois(nn, lambda, pobs0 = pobs0))
with(zdata, table(y))
fit <- vglm(y ~ x2, zapoisson, data = zdata, trace = TRUE)
fit <- vglm(y ~ x2, zapoisson, data = zdata, trace = TRUE, crit = "coef")
head(fitted(fit))
head(predict(fit))
head(predict(fit, untransform = TRUE))
coef(fit, matrix = TRUE)
summary(fit)
# Another example
# Data from Angers and Biswas (2003)
abdata <- data.frame(y = 0:7, w = c(182, 41, 12, 2, 2, 0, 0, 1))
abdata <- subset(abdata, w > 0)
Abdata <- data.frame(yy = with(abdata, rep(y, w)))
fit3 <- vglm(yy ~ 1, zapoisson, data = Abdata, trace = TRUE, crit = "coef")
coef(fit3, matrix = TRUE)
Coef(fit3) # Estimate lambda (they get 0.6997 with SE 0.1520)
head(fitted(fit3), 1)
with(Abdata, mean(yy)) # Compare this with fitted(fit3)
```

zero

The zero Argument in VGAM Family Functions

## Description

The zero argument allows users to conveniently model certain linear/additive predictors as interceptonly.

## Details

Often a certain parameter needs to be modelled simply while other parameters in the model may be more complex, for example, the $\lambda$ parameter in LMS-Box-Cox quantile regression should be modelled more simply compared to its $\mu$ parameter. Another example is the $\xi$ parameter in a GEV distribution which is should be modelled simpler than its $\mu$ parameter. Using the zero argument allows this to be fitted conveniently without having to input all the constraint matrices explicitly.
The zero argument can be assigned an integer vector from the set $\{1: M\}$ where $M$ is the number of linear/additive predictors. Full details about constraint matrices can be found in the references. See CommonVGAMffArguments for more information.

## Value

Nothing is returned. It is simply a convenient argument for constraining certain linear/additive predictors to be an intercept only.

## Warning

The use of other arguments may conflict with the zero argument. For example, using constraints to input constraint matrices may conflict with the zero argument. Another example is the argument parallel. In general users should not assume any particular order of precedence when there is potential conflict of definition. Currently no checking for consistency is made.
The argument zero may be renamed in the future to something better.

## Side Effects

The argument creates the appropriate constraint matrices internally.

## Note

In all VGAM family functions zero = NULL means none of the linear/additive predictors are modelled as intercepts-only. Almost all VGAM family function have zero = NULL as the default, but there are some exceptions, e.g., binom2.or.
Typing something like coef(fit, matrix = TRUE) is a useful way to ensure that the zero argument has worked as expected.

## Author(s)

T. W. Yee

## References

Yee, T. W. and Wild, C. J. (1996). Vector generalized additive models. Journal of the Royal Statistical Society, Series B, Methodological, 58, 481-493.
Yee, T. W. and Hastie, T. J. (2003). Reduced-rank vector generalized linear models. Statistical Modelling, 3, 15-41.

## See Also

CommonVGAMffArguments, constraints.

## Examples

```
    args(multinomial)
    args(binom2.or)
    args(gpd)
    #LMS quantile regression example
    fit <- vglm(BMI ~ sm.bs(age, df = 4), lms.bcg(zero = c(1, 3)),
        data = bmi.nz, trace = TRUE)
    coef(fit, matrix = TRUE)
```

    Zeta The Zeta Distribution
    
## Description

Density, distribution function, quantile function and random generation for the zeta distribution.

## Usage

dzeta(x, shape, log = FALSE)
pzeta(q, shape, lower.tail = TRUE)
qzeta(p, shape)
rzeta(n, shape)

## Arguments

| $\mathrm{x}, \mathrm{q}, \mathrm{p}, \mathrm{n}$ | Same as Poisson. |
| :--- | :--- |
| shape |  |
| lower.tail, log | The positive shape parameter $p$. |
|  | Same meaning as in Normal. |

## Details

The density function of the zeta distribution is given by

$$
y^{-s-1} / \zeta(s+1)
$$

where $s>0, y=1,2, \ldots$, and $\zeta$ is Riemann's zeta function.

## Value

dzeta gives the density, pzeta gives the distribution function, qzeta gives the quantile function, and rzeta generates random deviates.

## Note

qzeta() runs slower and slower as shape approaches 0 and $p$ approaches 1. The VGAM family function zetaff estimates the shape parameter $s$.

## Author(s)

T. W. Yee

## References

Johnson N. L., Kotz S., and Balakrishnan N. (1993). Univariate Discrete Distributions, 2nd ed. New York: Wiley.

## See Also

zeta, zetaff, Oazeta, Oizeta, Otzeta.

## Examples

```
dzeta(1:20, shape = 2)
myshape <- 0.5
max(abs(pzeta(1:200, myshape) -
    cumsum(1/(1:200)^(1+myshape)) / zeta(myshape+1))) # Should be 0
## Not run: plot(1:6, dzeta(1:6, 2), type = "h", las = 1,
            col = "orange", ylab = "Probability",
    main = "zeta probability function; orange: shape = 2; blue: shape = 1")
points(0.10 + 1:6, dzeta(1:6, 1), type = "h", col = "blue")
## End(Not run)
```

zeta Riemann's Zeta Function

## Description

Computes Riemann's zeta function and its first two derivatives. Also can compute the Hurwitz zeta function.

## Usage

zeta( x, deriv $=0$, shift $=1$ )

## Arguments

x
deriv An integer equalling 0 or 1 or 2 , which is the order of the derivative. The default means it is computed ordinarily.
shift Positive and numeric, called $A$ below. Allows for the Hurwitz zeta to be returned. The default corresponds to the Riemann formula.

## Details

The (Riemann) formula for real $s$ is

$$
\sum_{n=1}^{\infty} 1 / n^{s}
$$

While the usual definition involves an infinite series that converges when the real part of the argument is $>1$, more efficient methods have been devised to compute the value. In particular, this function uses Euler-Maclaurin summation. Theoretically, the zeta function can be computed over the whole complex plane because of analytic continuation.
The (Riemann) formula used here for analytic continuation is

$$
\zeta(s)=2^{s} \pi^{s-1} \sin (\pi s / 2) \Gamma(1-s) \zeta(1-s)
$$

This is actually one of several formulas, but this one was discovered by Riemann himself and is called the functional equation.
The Hurwitz zeta function for real $s>0$ is

$$
\sum_{n=0}^{\infty} 1 /(A+n)^{s}
$$

where $0<A$ is known here as the shift. Since $A=1$ by default, this function will therefore return Riemann's zeta function by default. Currently derivatives are unavailable.

## Value

The default is a vector/matrix of computed values of Riemann's zeta function. If shift contains values not equal to 1 , then this is Hurwitz's zeta function.

## Warning

This function has not been fully tested, especially the derivatives. In particular, analytic continuation does not work here for complex $x$ with $\operatorname{Re}(x)<1$ because currently the gamma function does not handle complex arguments.

## Note

Estimation of the parameter of the zeta distribution can be achieved with zetaff.

## Author(s)

T. W. Yee, with the help of Garry J. Tee.

## References

Riemann, B. (1859). Ueber die Anzahl der Primzahlen unter einer gegebenen Grosse. Monatsberichte der Berliner Akademie, November 1859.

Edwards, H. M. (1974). Riemann's Zeta Function. Academic Press: New York.
Markman, B. (1965). The Riemann zeta function. BIT, 5, 138-141.
Abramowitz, M. and Stegun, I. A. (1972). Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, New York: Dover Publications Inc.

## See Also

zetaff, oazeta, oizeta, otzeta, lerch, gamma.

## Examples

```
zeta(2:10)
## Not run:
curve(zeta, -13, 0.8, xlim = c(-12, 10), ylim = c(-1, 4), col = "orange",
    las = 1, main = expression({zeta}(x)))
curve(zeta, 1.2, 12, add = TRUE, col = "orange")
abline(v = 0, h = c(0, 1), lty = "dashed", col = "gray")
curve(zeta, -14, -0.4, col = "orange", main = expression({zeta}(x)))
abline(v = 0, h = 0, lty = "dashed", col = "gray") # Close up plot
x <- seq(0.04, 0.8, len = 100) # Plot of the first derivative
plot(x, zeta(x, deriv = 1), type = "l", las = 1, col = "blue",
        xlim = c(0.04, 3), ylim = c(-6, 0), main = "zeta'(x)")
x <- seq(1.2, 3, len = 100)
lines(x, zeta(x, deriv = 1), col = "blue")
abline(v = 0, h = 0, lty = "dashed", col = "gray")
## End(Not run)
zeta(2) - pi^2 / 6 # Should be 0
zeta(4) - pi^4 / 90 # Should be 0
zeta(6) - pi^6 / 945 # Should be 0
zeta(8) - pi^8 / 9450 # Should be 0
zeta(0, deriv = 1) + 0.5 * log(2*pi) # Should be 0
gamma0 <- 0.5772156649
gamma1 <- -0.07281584548
zeta(0, deriv = 2) -
    gamma1 + 0.5 * (log(2*pi))^2 + pi^2/24 - gamma0^2 / 2 # Should be 0
zeta(0.5, deriv = 1) + 3.92264613 # Should be 0
zeta(2.0, deriv = 1) + 0.93754825431 # Should be 0
```


## zetaff Zeta Distribution Family Function

## Description

Estimates the parameter of the zeta distribution.

## Usage

```
zetaff(lshape = "loglink", ishape = NULL, gshape = exp(-3:4)/4, zero = NULL)
```


## Arguments

lshape, ishape, zero
These arguments apply to the (positive) parameter $p$. See Links for more choices. Choosing loglog constrains $p>1$, but may fail if the maximum likelihood estimate is less than one. See CommonVGAMffarguments for more information.
gshape See CommonVGAMffArguments for more information.

## Details

In this long tailed distribution the response must be a positive integer. The probability function for a response $Y$ is

$$
P(Y=y)=1 /\left[y^{p+1} \zeta(p+1)\right], \quad p>0, \quad y=1,2, \ldots
$$

where $\zeta$ is Riemann's zeta function. The parameter $p$ is positive, therefore a log link is the default. The mean of $Y$ is $\mu=\zeta(p) / \zeta(p+1)$ (provided $p>1$ ) and these are the fitted values. The variance of $Y$ is $\zeta(p-1) / \zeta(p+1)-\mu^{2}$ provided $p>2$.

It appears that good initial values are needed for successful convergence. If convergence is not obtained, try several values ranging from values near 0 to values about 10 or more.
Multiple responses are handled.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Note

The zeta function may be used to compute values of the zeta function.

## Author(s)

T. W. Yee

## References

pp.527- of Chapter 11 of Johnson N. L., Kemp, A. W. and Kotz S. (2005). Univariate Discrete Distributions, 3rd edition, Hoboken, New Jersey: Wiley.

Knight, K. (2000). Mathematical Statistics. Boca Raton, FL, USA: Chapman \& Hall/CRC Press.

## See Also

```
zeta, oazeta, oizeta, otzeta, diffzeta, dzeta, hzeta, zipf.
```


## Examples

```
zdata <- data.frame(y = 1:5, w = c(63, 14, 5, 1, 2)) # Knight, p. }30
fit <- vglm(y ~ 1, zetaff, data = zdata, trace = TRUE, weight = w, crit = "c")
(phat <- Coef(fit)) # 1.682557
with(zdata, cbind(round(dzeta(y, phat) * sum(w), 1), w))
with(zdata, weighted.mean(y, w))
fitted(fit, matrix = FALSE)
predict(fit)
# The following should be zero at the MLE:
with(zdata, mean(log(rep(y, w))) + zeta(1+phat, deriv = 1) / zeta(1+phat))
```


## Zibinom Zero-Inflated Binomial Distribution

## Description

Density, distribution function, quantile function and random generation for the zero-inflated binomial distribution with parameter pstr0.

## Usage

dzibinom(x, size, prob, pstr0 = 0, log = FALSE)
pzibinom(q, size, prob, pstr0 = 0)
qzibinom(p, size, prob, pstr0 = 0)
rzibinom(n, size, prob, pstr0 = 0)

## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
size number of trials. It is the $N$ symbol in the formula given in zibinomial.
prob probability of success on each trial.
$\mathrm{n} \quad$ Same as in runif.
log Same as pbinom.
pstr0 Probability of a structural zero (i.e., ignoring the binomial distribution), called $\phi$. The default value of $\phi=0$ corresponds to the response having an ordinary binomial distribution.

## Details

The probability function of $Y$ is 0 with probability $\phi$, and Binomial(size, prob) with probability $1-\phi$. Thus

$$
P(Y=0)=\phi+(1-\phi) P(W=0)
$$

where $W$ is distributed Binomial(size, prob).

## Value

dzibinom gives the density, pzibinom gives the distribution function, qzibinom gives the quantile function, and rzibinom generates random deviates.

## Note

The argument pstr0 is recycled to the required length, and must have values which lie in the interval $[0,1]$.

These functions actually allow for zero-deflation. That is, the resulting probability of a zero count is less than the nominal value of the parent distribution. See Zipois for more information.

## Author(s)

T. W. Yee

## See Also

zibinomial, Gaitbinom.mlm, Binomial.

## Examples

```
prob <- 0.2; size <- 10; pstr0 <- 0.5
(ii <- dzibinom(0:size, size, prob, pstr0 = pstr0))
max(abs(cumsum(ii) - pzibinom(0:size, size, prob, pstr0 = pstr0))) # Should be 0
table(rzibinom(100, size, prob, pstr0 = pstr0))
table(qzibinom(runif(100), size, prob, pstr0 = pstr0))
round(dzibinom(0:10, size, prob, pstr0 = pstr0) * 100) # Should be similar
## Not run: x <- 0:size
barplot(rbind(dzibinom(x, size, prob, pstr0 = pstr0),
                dbinom(x, size, prob)),
    beside = TRUE, col = c("blue", "green"), ylab = "Probability",
    main = paste("ZIB(", size, ", ", prob, ", pstr0 = ", pstr0, ") (blue) vs",
            " Binomial(", size, ", ", prob, ") (green)", sep=""),
    names.arg = as.character(x), las = 1, lwd = 2)
## End(Not run)
```

zibinomial Zero-Inflated Binomial Distribution Family Function

## Description

Fits a zero-inflated binomial distribution by maximum likelihood estimation.

## Usage

```
zibinomial(lpstr0 = "logitlink", lprob = "logitlink",
    type.fitted = c("mean", "prob", "pobs0", "pstr0", "onempstr0"),
    ipstr0 = NULL, zero = NULL, multiple.responses = FALSE,
    imethod = 1)
zibinomialff(lprob = "logitlink", lonempstr0 = "logitlink",
    type.fitted = c("mean", "prob", "pobs0", "pstr0", "onempstr0"),
    ionempstr0 = NULL, zero = "onempstr0",
    multiple.responses = FALSE, imethod = 1)
```


## Arguments

lpstr0, lprob Link functions for the parameter $\phi$ and the usual binomial probability $\mu$ parameter. See Links for more choices. For the zero-deflated model see below.
type.fitted See CommonVGAMffArguments and fittedvlm.
ipstr0 Optional initial values for $\phi$, whose values must lie between 0 and 1 . The default is to compute an initial value internally. If a vector then recyling is used.
lonempstr0, ionempstr0
Corresponding arguments for the other parameterization. See details below.
multiple.responses
Logical. Currently it must be FALSE to mean the function does not handle multiple responses. This is to remain compatible with the same argument in binomialff.
zero, imethod See CommonVGAMffArguments for information. Argument zero changed its default value for version 0.9-2.

## Details

These functions are based on

$$
P(Y=0)=\phi+(1-\phi)(1-\mu)^{N}
$$

for $y=0$, and

$$
P(Y=y)=(1-\phi)\binom{N}{N y} \mu^{N y}(1-\mu)^{N(1-y)}
$$

for $y=1 / N, 2 / N, \ldots, 1$. That is, the response is a sample proportion out of $N$ trials, and the argument size in rzibinom is $N$ here. The parameter $\phi$ is the probability of a structural zero, and it satisfies $0<\phi<1$. The mean of $Y$ is $E(Y)=(1-\phi) \mu$ and these are returned as the fitted values by default. By default, the two linear/additive predictors for zibinomial() are $(\operatorname{logit}(\phi), \operatorname{logit}(\mu))^{T}$.
The VGAM family function zibinomialff() has a few changes compared to zibinomial(). These are: (i) the order of the linear/additive predictors is switched so the binomial probability comes first; (ii) argument onempstr0 is now 1 minus the probability of a structural zero, i.e., the probability of the parent (binomial) component, i.e., onempstr0 is 1 -pstr0; (iii) argument zero has a new default so that the onempstr0 is intercept-only by default. Now zibinomialff() is generally recommended over zibinomial(). Both functions implement Fisher scoring.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Warning

Numerical problems can occur. Half-stepping is not uncommon. If failure to converge occurs, make use of the argument ipstr0 or ionempstr0, or imethod.

## Note

The response variable must have one of the formats described by binomialff, e.g., a factor or two column matrix or a vector of sample proportions with the weights argument specifying the values of $N$.

To work well, one needs large values of $N$ and $\mu>0$, i.e., the larger $N$ and $\mu$ are, the better. If $N=1$ then the model is unidentifiable since the number of parameters is excessive.
Setting stepsize $=0.5$, say, may aid convergence .
Estimated probabilities of a structural zero and an observed zero are returned, as in zipoisson.
The zero-deflated binomial distribution might be fitted by setting lpstr0 = identitylink, albeit, not entirely reliably. See zipoisson for information that can be applied here. Else try the zeroaltered binomial distribution (see zabinomial).

## Author(s)

T. W. Yee

## References

Welsh, A. H., Lindenmayer, D. B. and Donnelly, C. F. (2013). Fitting and interpreting occupancy models. PLOS One, 8, 1-21.

## See Also

rzibinom, binomialff, posbinomial, gtbinomial, Binomial.

## Examples

```
size <- 10 # Number of trials; N in the notation above
nn <- 200
zdata <- data.frame(pstr0 = logitlink( 0, inverse = TRUE), # 0.50
    mubin = logitlink(-1, inverse = TRUE), # Mean of usual binomial
    sv = rep(size, length = nn))
zdata <- transform(zdata,
    y = rzibinom(nn, size = sv, prob = mubin, pstr0 = pstr0))
with(zdata, table(y))
fit <- vglm(cbind(y, sv - y) ~ 1, zibinomialff, data = zdata, trace = TRUE)
fit <- vglm(cbind(y, sv - y) ~ 1, zibinomialff, data = zdata, trace = TRUE,
        stepsize = 0.5)
```

```
coef(fit, matrix = TRUE)
Coef(fit) # Useful for intercept-only models
head(fitted(fit, type = "pobs0")) # Estimate of P(Y = 0)
head(fitted(fit))
with(zdata, mean(y)) # Compare this with fitted(fit)
summary(fit)
```

Zigeom Zero-Inflated Geometric Distribution

## Description

Density, and random generation for the zero-inflated geometric distribution with parameter pstr0.

## Usage

```
dzigeom(x, prob, pstr0 = 0, log = FALSE)
pzigeom(q, prob, pstr0 = 0)
qzigeom(p, prob, pstr0 = 0)
rzigeom(n, prob, pstr0 = 0)
```


## Arguments

```
x,q vector of quantiles.
p vector of probabilities.
prob see dgeom.
n Same as in runif.
pstr0 Probability of structural zero (ignoring the geometric distribution), called }\phi\mathrm{ . The
        default value corresponds to the response having an ordinary geometric distri-
    bution.
log Logical. Return the logarithm of the answer?
```


## Details

The probability function of $Y$ is 0 with probability $\phi$, and geometric (prob) with probability $1-\phi$. Thus

$$
P(Y=0)=\phi+(1-\phi) P(W=0)
$$

where $W$ is distributed geometric(prob).

## Value

dzigeom gives the density, pzigeom gives the distribution function, qzigeom gives the quantile function, and rzigeom generates random deviates.

## Note

The argument pstr0 is recycled to the required length, and must have values which lie in the interval $[0,1]$.
These functions actually allow for zero-deflation. That is, the resulting probability of a zero count is less than the nominal value of the parent distribution. See Zipois for more information.

## Author(s)

T. W. Yee

## See Also

zigeometric, dgeom.

## Examples

```
prob <- 0.5; pstr0 <- 0.2; x <- (-1):20
(ii <- dzigeom(x, prob, pstr0))
max(abs(cumsum(ii) - pzigeom(x, prob, pstr0))) # Should be 0
table(rzigeom(1000, prob, pstr0))
## Not run: x <- 0:10
barplot(rbind(dzigeom(x, prob, pstr0), dgeom(x, prob)),
    beside = TRUE, col = c("blue","orange"),
    ylab = "P[Y = y]", xlab = "y", las = 1,
    main = paste("zigeometric(", prob, ", pstr0 = ", pstr0,
                ") (blue) vs",
                " geometric(", prob, ") (orange)", sep = ""),
    names.arg = as.character(x))
## End(Not run)
```

zigeometric Zero-Inflated Geometric Distribution Family Function

## Description

Fits a zero-inflated geometric distribution by maximum likelihood estimation.

## Usage

```
zigeometric(lpstr0 = "logitlink", lprob = "logitlink",
    type.fitted = c("mean", "prob", "pobs0", "pstr0", "onempstr0"),
    ipstr0 = NULL, iprob = NULL,
    imethod = 1, bias.red = 0.5, zero = NULL)
zigeometricff(lprob = "logitlink", lonempstr0 = "logitlink",
        type.fitted = c("mean", "prob", "pobs0", "pstr0", "onempstr0"),
        iprob = NULL, ionempstr0 = NULL,
        imethod = 1, bias.red = 0.5, zero = "onempstr0")
```


## Arguments

lpstr0, lprob Link functions for the parameters $\phi$ and $p$ (prob). The usual geometric probability parameter is the latter. The probability of a structural zero is the former. See Links for more choices. For the zero-deflated model see below.
lonempstr0, ionempstr0 Corresponding arguments for the other parameterization. See details below.
bias.red A constant used in the initialization process of pstr0. It should lie between 0 and 1 , with 1 having no effect.
type.fitted See CommonVGAMffArguments and fittedvlm for information.
ipstr0, iprob See CommonVGAMffArguments for information.
zero, imethod See CommonVGAMffArguments for information.

## Details

Function zigeometric() is based on

$$
P(Y=0)=\phi+(1-\phi) p
$$

for $y=0$, and

$$
P(Y=y)=(1-\phi) p(1-p)^{y}
$$

for $y=1,2, \ldots$. The parameter $\phi$ satisfies $0<\phi<1$. The mean of $Y$ is $E(Y)=(1-\phi) p /(1-p)$ and these are returned as the fitted values by default. By default, the two linear/additive predictors are $(\operatorname{logit}(\phi), \operatorname{logit}(p))^{T}$. Multiple responses are handled.
Estimated probabilities of a structural zero and an observed zero can be returned, as in zipoisson; see fittedvlm for information.
The VGAM family function zigeometricff() has a few changes compared to zigeometric (). These are: (i) the order of the linear/additive predictors is switched so the geometric probability comes first; (ii) argument onempstr0 is now 1 minus the probability of a structural zero, i.e., the probability of the parent (geometric) component, i.e., onempstr0 is 1 -pstr0; (iii) argument zero has a new default so that the onempstr0 is intercept-only by default. Now zigeometricff() is generally recommended over zigeometric(). Both functions implement Fisher scoring and can handle multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

The zero-deflated geometric distribution might be fitted by setting lpstr0 = identitylink, albeit, not entirely reliably. See zipoisson for information that can be applied here. Else try the zeroaltered geometric distribution (see zageometric).

## Author(s)

T. W. Yee

## See Also

rzigeom, geometric, zageometric, rgeom, simulate.vlm.

## Examples

```
gdata <- data.frame(x2 = runif(nn <- 1000) - 0.5)
gdata <- transform(gdata, x3 = runif(nn) - 0.5,
    x4 = runif(nn) - 0.5)
gdata <- transform(gdata, eta1 = 1.0 - 1.0 * x2 + 2.0 * x3,
    eta2 = -1.0,
    eta3 = 0.5)
gdata <- transform(gdata, prob1 = logitlink(eta1, inverse = TRUE),
    prob2 = logitlink(eta2, inverse = TRUE),
    prob3 = logitlink(eta3, inverse = TRUE))
gdata <- transform(gdata, y1 = rzigeom(nn, prob1, pstr0 = prob3),
    y2 = rzigeom(nn, prob2, pstr0 = prob3),
    y3 = rzigeom(nn, prob2, pstr0 = prob3))
```

with(gdata, table(y1))
with(gdata, table(y2))
with(gdata, table(y3))
head(gdata)
fit1 <- vglm(y1 ~ x2 + x3 + x4, zigeometric(zero = 1), data = gdata, trace = TRUE)
coef(fit1, matrix = TRUE)
head(fitted(fit1, type = "pstr0"))
fit2 <- vglm(cbind(y2, y3) ~ 1, zigeometric(zero = 1), data = gdata, trace = TRUE)
coef(fit2, matrix = TRUE)
summary (fit2)

## Zinegbin

Zero-Inflated Negative Binomial Distribution

## Description

Density, distribution function, quantile function and random generation for the zero-inflated negative binomial distribution with parameter pstr0.

## Usage

```
dzinegbin(x, size, prob = NULL, munb = NULL, pstr0 = 0, log = FALSE)
pzinegbin(q, size, prob = NULL, munb = NULL, pstr0 = 0)
qzinegbin(p, size, prob = NULL, munb = NULL, pstr0 = 0)
rzinegbin(n, size, prob = NULL, munb = NULL, pstr0 = 0)
```


## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
$\mathrm{n} \quad$ Same as in runif.
size, prob, munb, log
Arguments matching dnbinom. The argument munb corresponds to mu in dnbinom and has been renamed to emphasize the fact that it is the mean of the negative binomial component.
pstr0 Probability of structural zero (i.e., ignoring the negative binomial distribution), called $\phi$.

## Details

The probability function of $Y$ is 0 with probability $\phi$, and a negative binomial distribution with probability $1-\phi$. Thus

$$
P(Y=0)=\phi+(1-\phi) P(W=0)
$$

where $W$ is distributed as a negative binomial distribution (see rnbinom.) See negbinomial, a VGAM family function, for the formula of the probability density function and other details of the negative binomial distribution.

## Value

dzinegbin gives the density, pzinegbin gives the distribution function, qzinegbin gives the quantile function, and rzinegbin generates random deviates.

## Note

The argument pstr0 is recycled to the required length, and must have values which lie in the interval $[0,1]$.
These functions actually allow for zero-deflation. That is, the resulting probability of a zero count is less than the nominal value of the parent distribution. See Zipois for more information.

## Author(s)

T. W. Yee

## See Also

zinegbinomial, rnbinom, rzipois.

## Examples

```
munb <- 3; pstr0 <- 0.2; size <- k <- 10; x <- 0:10
(ii <- dzinegbin(x, pstr0 = pstr0, mu = munb, size = k))
max(abs(cumsum(ii) - pzinegbin(x, pstr0 = pstr0, mu = munb, size = k))) # 0
table(rzinegbin(100, pstr0 = pstr0, mu = munb, size = k))
table(qzinegbin(runif(1000), pstr0 = pstr0, mu = munb, size = k))
```

```
round(dzinegbin(x, pstr0 = pstr0, mu = munb, size = k) * 1000) # Should be similar
## Not run: barplot(rbind(dzinegbin(x, pstr0 = pstr0, mu = munb, size = k),
        dnbinom(x, mu = munb, size = k)), las = 1,
    beside = TRUE, col = c("blue", "green"), ylab = "Probability",
    main = paste("ZINB(mu = ", munb, ", k = ", k, ", pstr0 = ", pstr0,
            ") (blue) vs NB(mu = ", munb,
            ", size = ", k, ") (green)", sep = ""),
        names.arg = as.character(x))
## End(Not run)
```

zinegbinomial

Zero-Inflated Negative Binomial Distribution Family Function

## Description

Fits a zero-inflated negative binomial distribution by full maximum likelihood estimation.

## Usage

```
zinegbinomial(zero = "size",
    type.fitted = c("mean", "munb", "pobs0", "pstr0",
    "onempstr0"),
    mds.min \(=1 \mathrm{e}-3\), nsimEIM \(=500\), cutoff. prob \(=0.999\),
    eps.trig \(=1 \mathrm{e}-7\), max. support \(=4000\), max.chunk. \(\mathrm{MB}=30\),
    lpstr0 = "logitlink", lmunb = "loglink", lsize = "loglink",
    imethod = 1, ipstr0 = NULL, imunb = NULL,
    iprobs.y = NULL, isize = NULL,
    gprobs.y = (0:9)/10,
    gsize.mux \(=\exp (c(-30,-20,-15,-10,-6: 3)))\)
zinegbinomialff(lmunb = "loglink", lsize = "loglink", lonempstr0 = "logitlink",
        type.fitted = c("mean", "munb", "pobs0", "pstr0",
        "onempstr0"), imunb = NULL, isize = NULL, ionempstr0 =
        NULL, zero = c("size", "onempstr0"), imethod = 1,
        iprobs.y = NULL, cutoff.prob = 0.999,
        eps.trig \(=1 \mathrm{e}-7, \quad\) max.support \(=4000\), max.chunk. \(\mathrm{MB}=30\),
        gprobs.y \(=(0: 9) / 10\), gsize.mux \(=\exp ((-12: 6) / 2)\),
        mds.min \(=1 \mathrm{e}-3\), nsimEIM \(=500\) )
```


## Arguments

lpstr0, lmunb, lsize
Link functions for the parameters $\phi$, the mean and $k$; see negbinomial for details, and Links for more choices. For the zero-deflated model see below.
type.fitted See CommonVGAMffArguments and fittedvlm for more information. ipstr0, isize, imunb

Optional initial values for $\phi$ and $k$ and $\mu$. The default is to compute an initial value internally for both. If a vector then recycling is used.
lonempstr0, ionempstr0
Corresponding arguments for the other parameterization. See details below.

imethod | An integer with value 1 or 2 or 3 which specifies the initialization method |
| :--- |
| for the mean parameter. If failure to converge occurs try another value. See |
| CommonVGAMffArguments for more information. |

zero | Specifies which linear/additive predictors are to be modelled as intercept-only. |
| :--- |
| They can be such that their absolute values are either 1 or 2 or 3. The default is |
| the $\phi$ and $k$ parameters (both for each response). See CommonVGAMffArguments |
| for more information. |

nsimEIM
iprobs.y, cutoff.prob, max. support, max.chunk.MB

See negbinomial and/or posnegbinomial for details.
mds.min, eps.trig See negbinomial for details.
gprobs.y, gsize.mux
These arguments relate to grid searching in the initialization process. See negbinomial and/or posnegbinomial for details.

## Details

These functions are based on

$$
P(Y=0)=\phi+(1-\phi)(k /(k+\mu))^{k}
$$

and for $y=1,2, \ldots$,

$$
P(Y=y)=(1-\phi) \operatorname{dnbinom}(y, \mu, k)
$$

The parameter $\phi$ satisfies $0<\phi<1$. The mean of $Y$ is $(1-\phi) \mu$ (returned as the fitted values). By default, the three linear/additive predictors for zinegbinomial() are $(\operatorname{logit}(\phi), \log (\mu), \log (k))^{T}$. See negbinomial, another VGAM family function, for the formula of the probability density function and other details of the negative binomial distribution.

Independent multiple responses are handled. If so then arguments ipstr0 and isize may be vectors with length equal to the number of responses.

The VGAM family function zinegbinomialff() has a few changes compared to zinegbinomial(). These are: (i) the order of the linear/additive predictors is switched so the NB mean comes first; (ii) onempstr0 is now 1 minus the probability of a structural 0 , i.e., the probability of the parent (NB) component, i.e., onempstr0 is 1-pstr0; (iii) argument zero has a new default so that the onempstr0 is intercept-only by default. Now zinegbinomialff() is generally recommended over zinegbinomial(). Both functions implement Fisher scoring and can handle multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, and vgam.

## Warning

This model can be difficult to fit to data, and this family function is fragile. The model is especially difficult to fit reliably when the estimated $k$ parameter is very large (so the model approaches a zero-inflated Poisson distribution) or much less than 1 (and gets more difficult as it approaches 0 ). Numerical problems can also occur, e.g., when the probability of a zero is actually less than, and not more than, the nominal probability of zero. Similarly, numerical problems can occur if there is little or no 0 -inflation, or when the sample size is small. Half-stepping is not uncommon. Successful convergence is sensitive to the initial values, therefore if failure to converge occurs, try using combinations of arguments stepsize (in vglm. control), imethod, imunb, ipstr0, isize, and/or zero if there are explanatory variables. Else try fitting an ordinary negbinomial model or a zipoisson model.

This VGAM family function can be computationally expensive and can run slowly; setting trace $=$ TRUE is useful for monitoring convergence.

## Note

Estimated probabilities of a structural zero and an observed zero can be returned, as in zipoisson; see fittedvlm for more information.

If $k$ is large then the use of VGAM family function zipoisson is probably preferable. This follows because the Poisson is the limiting distribution of a negative binomial as $k$ tends to infinity.
The zero-deflated negative binomial distribution might be fitted by setting lpstr0 = identitylink, albeit, not entirely reliably. See zipoisson for information that can be applied here. Else try the zero-altered negative binomial distribution (see zanegbinomial).

## Author(s)

T. W. Yee

## See Also

Zinegbin, negbinomial, rpois, CommonVGAMffArguments.

## Examples

```
## Not run:
# Example 1
ndata <- data.frame(x2 = runif(nn <- 1000))
ndata <- transform(ndata, pstr0 = logitlink(-0.5 + 1 * x2, inverse = TRUE),
    munb = exp( 3 + 1*x2),
    size = exp( 0 + 2 * x2))
ndata <- transform(ndata,
            y1 = rzinegbin(nn, mu = munb, size = size, pstr0 = pstr0))
with(ndata, table(y1)["0"] / sum(table(y1)))
nfit <- vglm(y1 ~ x2, zinegbinomial(zero = NULL), data = ndata)
coef(nfit, matrix = TRUE)
summary(nfit)
head(cbind(fitted(nfit), with(ndata, (1 - pstr0) * munb)))
round(vcov(nfit), 3)
```

```
# Example 2: RR-ZINB could also be called a COZIVGLM-ZINB-2
ndata <- data.frame(x2 = runif(nn <- 2000))
ndata <- transform(ndata, x3 = runif(nn))
ndata <- transform(ndata, eta1 = 3 + 1 * x2 + 2 * x3)
ndata <- transform(ndata, pstr0 = logitlink(-1.5 + 0.5 * eta1, inverse = TRUE),
    munb = exp(eta1),
    size = exp(4))
ndata <- transform(ndata,
    y1 = rzinegbin(nn, pstr0 = pstr0, mu = munb, size = size))
with(ndata, table(y1)["0"] / sum(table(y1)))
rrzinb <- rrvglm(y1 ~ x2 + x3, zinegbinomial(zero = NULL), data = ndata,
    Index.corner = 2, str0 = 3, trace = TRUE)
coef(rrzinb, matrix = TRUE)
Coef(rrzinb)
## End(Not run)
```

zipebcom Exchangeable Bivariate cloglog Odds-ratio Model From a Zero-
inflated Poisson Distribution

## Description

Fits an exchangeable bivariate odds-ratio model to two binary responses with a complementary log-log link. The data are assumed to come from a zero-inflated Poisson distribution that has been converted to presence/absence.

## Usage

```
zipebcom(lmu12 = "clogloglink", lphi12 = "logitlink", loratio = "loglink",
    imu12 = NULL, iphi12 = NULL, ioratio = NULL,
    zero = c("phi12", "oratio"), tol = 0.001, addRidge = 0.001)
```


## Arguments

lmu12, imu12 Link function, extra argument and optional initial values for the first (and second) marginal probabilities. Argument lmu12 should be left alone. Argument imu12 may be of length 2 (one element for each response).
lphi 12 Link function applied to the $\phi$ parameter of the zero-inflated Poisson distribution (see zipoisson). See Links for more choices.
loratio Link function applied to the odds ratio. See Links for more choices.
iphi12, ioratio
Optional initial values for $\phi$ and the odds ratio. See CommonVGAMffArguments for more details. In general, good initial values (especially for iphi12) are often required, therefore use these arguments if convergence failure occurs. If inputted, the value of iphi12 cannot be more than the sample proportions of zeros in either response.

| zero | Which linear/additive predictor is modelled as an intercept only? A NULL means <br> none. The default has both $\phi$ and the odds ratio as not being modelled as a <br> function of the explanatory variables (apart from an intercept). |
| :--- | :--- |
| tol | Tolerance for testing independence. Should be some small positive numerical <br> value. |
| addRidge | Some small positive numerical value. The first two diagonal elements of the <br> working weight matrices are multiplied by 1+addRidge to make it diagonally <br> dominant, therefore positive-definite. |

## Details

This VGAM family function fits an exchangeable bivariate odds ratio model (binom2.or) with a clogloglink link. The data are assumed to come from a zero-inflated Poisson (ZIP) distribution that has been converted to presence/absence. Explicitly, the default model is

$$
\operatorname{clog} \log \left[P\left(Y_{j}=1\right) /(1-\phi)\right]=\eta_{1}, \quad j=1,2
$$

for the (exchangeable) marginals, and

$$
\operatorname{logit}[\phi]=\eta_{2}
$$

for the mixing parameter, and

$$
\log \left[P\left(Y_{00}=1\right) P\left(Y_{11}=1\right) /\left(P\left(Y_{01}=1\right) P\left(Y_{10}=1\right)\right)\right]=\eta_{3}
$$

specifies the dependency between the two responses. Here, the responses equal 1 for a success and a 0 for a failure, and the odds ratio is often written $\psi=p_{00} p_{11} /\left(p_{10} p_{01}\right)$. We have $p_{10}=p_{01}$ because of the exchangeability.
The second linear/additive predictor models the $\phi$ parameter (see zipoisson). The third linear/additive predictor is the same as binom2. or, viz., the log odds ratio.
Suppose a dataset 1 comes from a Poisson distribution that has been converted to presence/absence, and that both marginal probabilities are the same (exchangeable). Then binom2.or ("clogloglink", exch=TRUE) is appropriate. Now suppose a dataset 2 comes from a zero-inflated Poisson distribution. The first linear/additive predictor of zipebcom() applied to dataset2 is the same as that of binom2.or("clogloglink", exch=TRUE) applied to dataset1. That is, the $\phi$ has been taken care of by zipebcom() so that it is just like the simpler binom2.or.
Note that, for $\eta_{1}$, mu12 = prob12 / (1-phi12) where prob12 is the probability of a 1 under the ZIP model. Here, mu12 correspond to mu1 and mu2 in the binom2. or-Poisson model.
If $\phi=0$ then zipebcom() should be equivalent to binom2.or("clogloglink", exch=TRUE). Full details are given in Yee and Dirnbock (2009).
The leading $2 \times 2$ submatrix of the expected information matrix (EIM) is of rank-1, not 2 ! This is due to the fact that the parameters corresponding to the first two linear/additive predictors are unidentifiable. The quick fix around this problem is to use the addRidge adjustment. The model is fitted by maximum likelihood estimation since the full likelihood is specified. Fisher scoring is implemented.
The default models $\eta_{2}$ and $\eta_{3}$ as single parameters only, but this can be circumvented by setting zero=NULL in order to model the $\phi$ and odds ratio as a function of all the explanatory variables.

Value
An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.
When fitted, the fitted.values slot of the object contains the four joint probabilities, labelled as $\left(Y_{1}, Y_{2}\right)=(0,0),(0,1),(1,0),(1,1)$, respectively. These estimated probabilities should be extracted with the fitted generic function.

## Warning

The fact that the EIM is not of full rank may mean the model is naturally ill-conditioned. Not sure whether there are any negative consequences wrt theory. For now it is certainly safer to fit binom2. or to bivariate binary responses.

## Note

The " 12 " in the argument names reinforce the user about the exchangeability assumption. The name of this VGAM family function stands for zero-inflated Poisson exchangeable bivariate complementary log-log odds-ratio model or ZIP-EBCOM.
See binom2. or for details that are pertinent to this VGAM family function too. Even better initial values are usually needed here.
The xij (see vglm. control) argument enables environmental variables with different values at the two time points to be entered into an exchangeable binom2. or model. See the author's webpage for sample code.

## References

Yee, T. W. and Dirnbock, T. (2009). Models for analysing species' presence/absence data at two time points. Journal of Theoretical Biology, 259(4), 684-694.

## See Also

binom2.or, zipoisson, clogloglink, CommonVGAMffArguments.

## Examples

```
zdata <- data.frame(x2 = seq(0, 1, len = (nsites <- 2000)))
zdata <- transform(zdata, eta1 = -3 + 5 * x2,
            phi1 = logitlink(-1, inverse = TRUE),
            oratio = exp(2))
zdata <- transform(zdata, mu12 = clogloglink(eta1, inverse = TRUE) * (1-phi1))
tmat <- with(zdata, rbinom2.or(nsites, mu1 = mu12, oratio = oratio, exch = TRUE))
zdata <- transform(zdata, ybin1 = tmat[, 1], ybin2 = tmat[, 2])
with(zdata, table(ybin1, ybin2)) / nsites # For interest only
## Not run:
# Various plots of the data, for interest only
par(mfrow = c(2, 2))
plot(jitter(ybin1) ~ x2, data = zdata, col = "blue")
```

```
plot(jitter(ybin2) ~ jitter(ybin1), data = zdata, col = "blue")
plot(mu12 ~ x2, data = zdata, col = "blue", type = "l", ylim = 0:1,
    ylab = "Probability", main = "Marginal probability and phi")
with(zdata, abline(h = phi1[1], col = "red", lty = "dashed"))
tmat2 <- with(zdata, dbinom2.or(mu1 = mu12, oratio = oratio, exch = TRUE))
with(zdata, matplot(x2, tmat2, col = 1:4, type = "l", ylim = 0:1,
    ylab = "Probability", main = "Joint probabilities"))
## End(Not run)
# Now fit the model to the data.
fit <- vglm(cbind(ybin1, ybin2) ~ x2, zipebcom, data = zdata, trace = TRUE)
coef(fit, matrix = TRUE)
summary(fit)
vcov(fit)
```

Zipf
The Zipf Distribution

## Description

Density, distribution function, quantile function and random generation for the Zipf distribution.

## Usage

dzipf(x, N, shape, log = FALSE)
pzipf(q, N, shape, log.p = FALSE)
qzipf(p, N, shape)
rzipf(n, N, shape)

## Arguments

| $x, q, p, n$ | Same as Poisson. |
| :--- | :--- |
| $N$, shape | the number of elements, and the exponent characterizing the distribution. See <br> zipf for more details. |
| log, log.p | Same meaning as in Normal. |

## Details

This is a finite version of the zeta distribution. See zetaff for more details. In general, these functions runs slower and slower as N increases.

## Value

dzipf gives the density, pzipf gives the cumulative distribution function, qzipf gives the quantile function, and rzipf generates random deviates.

## Author(s)

T. W. Yee

## See Also

```
zipf,Zipfmb.
```


## Examples

```
N <- 10; shape <- 0.5; y <- 1:N
proby <- dzipf(y, N = N, shape = shape)
## Not run: plot(proby ~ y, type = "h", col = "blue", ylab = "Probability",
    ylim = c(0, 0.2), main = paste("Zipf(N = ",N,", shape = ",shape,")", sep = ""),
    lwd = 2, las = 1)
## End(Not run)
sum(proby) # Should be 1
max(abs(cumsum(proby) - pzipf(y, N = N, shape = shape))) # Should be 0
```

```
    zipf Zipf Distribution Family Function
```


## Description

Estimates the parameter of the Zipf distribution.

## Usage

```
zipf(N = NULL, lshape = "loglink", ishape = NULL)
```


## Arguments

$N \quad$ Number of elements, an integer satisfying $1<N<I n f$. The default is to use the maximum value of the response. If given, $N$ must be no less that the largest response value. If $N=\operatorname{Inf}$ and $s>1$ then this is the zeta distribution (use zetaff instead).
lshape Parameter link function applied to the (positive) shape parameter $s$. See Links for more choices.
ishape Optional initial value for the parameter $s$. The default is to choose an initial value internally. If converge failure occurs use this argument to input a value.

## Details

The probability function for a response $Y$ is

$$
P(Y=y)=y^{-s} / \sum_{i=1}^{N} i^{-s}, s>0, y=1,2, \ldots, N
$$

where $s$ is the exponent characterizing the distribution. The mean of $Y$, which are returned as the fitted values, is $\mu=H_{N, s-1} / H_{N, s}$ where $H_{n, m}=\sum_{i=1}^{n} i^{-m}$ is the $n$th generalized harmonic number.

Zipf's law is an experimental law which is often applied to the study of the frequency of words in a corpus of natural language utterances. It states that the frequency of any word is inversely proportional to its rank in the frequency table. For example, "the" and "of" are first two most common words, and Zipf's law states that "the" is twice as common as "of". Many other natural phenomena conform to Zipf's law.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm and vgam.

## Note

Upon convergence, the $N$ is stored as @misc $\$ N$.

## Author(s)

T. W. Yee

## References

pp.526- of Chapter 11 of Johnson N. L., Kemp, A. W. and Kotz S. (2005). Univariate Discrete Distributions, 3rd edition, Hoboken, New Jersey, USA: Wiley.

## See Also

dzipf, zetaff, simulate.vlm.

## Examples

```
zdata <- data.frame(y = 1:5, ofreq = c(63, 14, 5, 1, 2))
zfit <- vglm(y ~ 1, zipf, data = zdata, trace = TRUE, weight = ofreq)
zfit <- vglm(y ~ 1, zipf(lshape = "identitylink", ishape = 3.4), data = zdata,
    trace = TRUE, weight = ofreq, crit = "coef")
zfit@misc$N
(shape.hat <- Coef(zfit))
with(zdata, weighted.mean(y, ofreq))
fitted(zfit, matrix = FALSE)
```

Zipfmb The Zipf-Mandelbrot Distribution

## Description

Density, distribution function, quantile function and random generation for the Mandelbrot distribution.

## Usage

dzipfmb (x, shape, start $=1$, log = FALSE)
pzipfmb(q, shape, start $=1$, lower.tail $=$ TRUE, log. $\mathrm{p}=$ FALSE)
qzipfmb ( $p$, shape, start = 1)
rzipfmb (n, shape, start = 1)

## Arguments

$x \quad$ vector of (non-negative integer) quantiles.
$q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
n number of random values to return.
shape vector of positive shape parameter.
start integer, the minimum value of the support of the distribution.
$\log , \log . \mathrm{p} \quad \operatorname{logical}$; if TRUE, probabilities p are given as $\log (\mathrm{p})$
lower.tail logical; if TRUE (default), probabilities are $\mathrm{P}[\mathrm{X}<=\mathrm{x}]$, otherwise, $\mathrm{P}[\mathrm{X}>\mathrm{x}]$.

## Details

The probability mass function of the zipf-mandelbrot distribution is given by

$$
\operatorname{Pr}(Y=y ; s)=\frac{s \Gamma\left(y_{\min }\right)}{\Gamma\left(y_{\min }-s\right)} \cdot \frac{\Gamma(y-s)}{\Gamma(y+1)}
$$

where $0 \leq b<1$ and the starting value start being by default 1 .

## Value

dzipfmb gives the density, pzipfmb gives the distribution function, qzipfmb gives the quantile function, and rzipfmb generates random deviates.

## Author(s)

M. Chou, with edits by T. W. Yee.

## References

Mandelbrot, B. (1961). On the theory of word frequencies and on related Markovian models of discourse. In R. Jakobson, Structure of Language and its Mathematical Aspects, pp. 190-219, Providence, RI, USA. American Mathematical Society.

Moreno-Sanchez, I. and Font-Clos, F. and Corral, A. (2016). Large-Scale Analysis of Zipf's Law in English Texts. PLos ONE, 11(1), 1-19.

## See Also

> Zipf.

## Examples

```
aa <- 1:10
(pp <- pzipfmb(aa, shape = 0.5, start = 1))
cumsum(dzipfmb(aa, shape = 0.5, start = 1)) # Should be same
qzipfmb(pp, shape = 0.5, start = 1) - aa # Should be all 0s
rdiffzeta(30, 0.5)
## Not run:
x <- 1:10
plot(x, dzipfmb(x, shape = 0.5), type = "h", ylim = 0:1,
    sub = "shape=0.5", las = 1, col = "blue", ylab = "Probability",
    main = "Zipf-Mandelbrot distribution: blue=PMF; orange=CDF")
lines(x + 0.1, pzipfmb(x, shape = 0.8), col = "orange", lty = 3, type = "h")
## End(Not run)
```

Zipois

Zero-Inflated Poisson Distribution

## Description

Density, distribution function, quantile function and random generation for the zero-inflated and zero-deflated Poisson distribution with parameter pstr0.

## Usage

dzipois(x, lambda, pstr0 = 0, log = FALSE)
pzipois(q, lambda, pstr0 = 0)
qzipois(p, lambda, pstr0 = 0)
rzipois(n, lambda, pstr0 = 0)

## Arguments

$x, q \quad$ vector of quantiles.
$p \quad$ vector of probabilities.
n number of observations. Must be a single positive integer.
lambda Vector of positive means.
pstr0 Probability of a structural zero (i.e., ignoring the Poisson distribution), called $\phi$. The default value of $\phi=0$ corresponds to the response having an ordinary Poisson distribution. This argument may be negative to allow for 0 -deflation, hence its interpretation as a probability ceases.
log Logical. Return the logarithm of the answer?

## Details

The probability function of $Y$ is 0 with probability $\phi$, and $\operatorname{Poisson}(\lambda)$ with probability $1-\phi$. Thus

$$
P(Y=0)=\phi+(1-\phi) P(W=0)
$$

where $W$ is distributed $\operatorname{Poisson}(\lambda)$.

## Value

dzipois gives the density, pzipois gives the distribution function, qzipois gives the quantile function, and rzipois generates random deviates.

## Note

The argument pstr0 is recycled to the required length, and must have values which lie in the interval $[0,1]$.
These functions actually allow for the zero-deflated Poisson distribution. Here, pstr0 is also permitted to lie in the interval [-1/expm1 (lambda), 0]. The resulting probability of a zero count is less than the nominal Poisson value, and the use of pstr0 to stand for the probability of a structural zero loses its meaning. When pstr0 equals -1 /expm1 (lambda) this corresponds to the positive-Poisson distribution (e.g., see dpospois).

## Author(s)

T. W. Yee

## See Also

zipoisson, Gaitpois, dpois, rzinegbin.

## Examples

```
lambda <- 3; pstr0 <- 0.2; x <- (-1):7
(ii <- dzipois(x, lambda, pstr0 = pstr0))
max(abs(cumsum(ii) - pzipois(x, lambda, pstr0 = pstr0))) # Should be 0
table(rzipois(100, lambda, pstr0 = pstr0))
```

```
table(qzipois(runif(100), lambda, pstr0))
round(dzipois(0:10, lambda, pstr0 = pstr0) * 100) # Should be similar
## Not run: x <- 0:10
par(mfrow = c(2, 1)) # Zero-inflated Poisson
barplot(rbind(dzipois(x, lambda, pstr0 = pstr0), dpois(x, lambda)),
    beside = TRUE, col = c("blue", "orange"),
    main = paste("ZIP(", lambda, ", pstr0 = ", pstr0, ") (blue) vs",
            " Poisson(", lambda, ") (orange)", sep = ""),
    names.arg = as.character(x))
deflat.limit <- -1 / expm1(lambda) # Zero-deflated Poisson
newpstr0 <- round(deflat.limit / 1.5, 3)
barplot(rbind(dzipois(x, lambda, pstr0 = newpstr0),
    dpois(x, lambda)),
    beside = TRUE, col = c("blue","orange"),
    main = paste("ZDP(", lambda, ", pstr0 = ", newpstr0, ") (blue) vs",
    " Poisson(", lambda, ") (orange)", sep = ""),
    names.arg = as.character(x))
## End(Not run)
```

zipoisson

## Description

Fits a zero-inflated or zero-deflated Poisson distribution by full maximum likelihood estimation.

## Usage

```
zipoisson(lpstr0 = "logitlink", llambda = "loglink", type.fitted =
    c("mean", "lambda", "pobs0", "pstr0", "onempstr0"), ipstr0 =
    NULL, ilambda = NULL, gpstr0 = NULL, imethod = 1,
    ishrinkage = 0.95, probs.y = 0.35, parallel = FALSE, zero = NULL)
zipoissonff(llambda = "loglink", lonempstr0 = "logitlink", type.fitted =
        c("mean", "lambda", "pobs0", "pstr0", "onempstr0"),
        ilambda = NULL, ionempstr0 = NULL, gonempstr0 = NULL,
        imethod = 1, ishrinkage = 0.95, probs.y = 0.35, zero =
        "onempstr0")
```


## Arguments

lpstr0, llambda
Link function for the parameter $\phi$ and the usual $\lambda$ parameter. See Links for more choices; see CommonVGAMffArguments for more information. For the zerodeflated model see below.
ipstr0, ilambda
Optional initial values for $\phi$, whose values must lie between 0 and 1. Optional initial values for $\lambda$, whose values must be positive. The defaults are to compute an initial value internally for each. If a vector then recycling is used.
lonempstr0, ionempstr0
Corresponding arguments for the other parameterization. See details below.
type.fitted Character. The type of fitted value to be returned. The first choice (the expected value) is the default. The estimated probability of an observed 0 is an alternative, else the estimated probability of a structural 0 , or one minus the estimated probability of a structural 0 . See CommonVGAMffArguments and fittedvlm for more information.
imethod An integer with value 1 or 2 which specifies the initialization method for $\lambda$. If failure to converge occurs try another value and/or else specify a value for ishrinkage and/or else specify a value for ipstr0. See CommonVGAMffArguments for more information.
ishrinkage How much shrinkage is used when initializing $\lambda$. The value must be between 0 and 1 inclusive, and a value of 0 means the individual response values are used, and a value of 1 means the median or mean is used. This argument is used in conjunction with imethod. See CommonVGAMffArguments for more information.
zero Specifies which linear/additive predictors are to be modelled as intercept-only. If given, the value can be either 1 or 2 , and the default is none of them. Setting zero $=1$ makes $\phi$ a single parameter. See CommonVGAMffArguments for more information.
gpstr0, gonempstr0, probs.y Details at CommonVGAMffArguments.
parallel Details at CommonVGAMffArguments, but unlikely to be practically used actually.

## Details

These models are a mixture of a Poisson distribution and the value 0 ; it has value 0 with probability $\phi$ else is Poisson $(\lambda)$ distributed. Thus there are two sources for zero values, and $\phi$ is the probability of a structural zero. The model for zipoisson() can be written

$$
P(Y=0)=\phi+(1-\phi) \exp (-\lambda)
$$

and for $y=1,2, \ldots$,

$$
P(Y=y)=(1-\phi) \exp (-\lambda) \lambda^{y} / y!
$$

Here, the parameter $\phi$ satisfies $0<\phi<1$. The mean of $Y$ is $(1-\phi) \lambda$ and these are returned as the fitted values, by default. The variance of $Y$ is $(1-\phi) \lambda(1+\phi \lambda)$. By default, the two linear/additive predictors of zipoisson() are $(\operatorname{logit}(\phi), \log (\lambda))^{T}$.
The VGAM family function zipoissonff() has a few changes compared to zipoisson(). These are: (i) the order of the linear/additive predictors is switched so the Poisson mean comes first; (ii) onempstr0 is now 1 minus the probability of a structural 0 , i.e., the probability of the parent (Poisson) component, i.e., onempstr0 is 1-pstr0; (iii) argument zero has a new default so that the onempstr0 is intercept-only by default. Now zipoissonff() is generally recommended over zipoisson() (and definitely recommended over yip88). Both functions implement Fisher scoring and can handle multiple responses.

## Value

An object of class "vglmff" (see vglmff-class). The object is used by modelling functions such as vglm, rrvglm and vgam.

## Warning

Numerical problems can occur, e.g., when the probability of zero is actually less than, not more than, the nominal probability of zero. For example, in the Angers and Biswas (2003) data below, replacing 182 by 1 results in nonconvergence. Half-stepping is not uncommon. If failure to converge occurs, try using combinations of imethod, ishrinkage, ipstr0, and/or zipoisson(zero = 1) if there are explanatory variables. The default for zipoissonff() is to model the structural zero probability as an intercept-only.

## Note

This family function can be used to estimate the 0 -deflated model, hence pstr0 is not to be interpreted as a probability. One should set, e.g., lpstr0 = "identitylink". Likewise, the functions in Zipois can handle the zero-deflated Poisson distribution too. Although the iterations might fall outside the parameter space, the validparams slot should keep them inside. A (somewhat) similar alternative for zero-deflation is to try the zero-altered Poisson model (see zapoisson).

The use of this VGAM family function with rrvglm can result in a so-called COZIGAM or COZIGLM. That is, a reduced-rank zero-inflated Poisson model (RR-ZIP) is a constrained zeroinflated generalized linear model. See COZIGAM. A RR-ZINB model can also be fitted easily; see zinegbinomial. Jargon-wise, a COZIGLM might be better described as a COZIVGLM-ZIP.

## Author(s)

T. W. Yee

## References

Thas, O. and Rayner, J. C. W. (2005). Smooth tests for the zero-inflated Poisson distribution. Biometrics, 61, 808-815.

Data: Angers, J-F. and Biswas, A. (2003). A Bayesian analysis of zero-inflated generalized Poisson model. Computational Statistics \& Data Analysis, 42, 37-46.
Cameron, A. C. and Trivedi, P. K. (1998). Regression Analysis of Count Data. Cambridge University Press: Cambridge.

Yee, T. W. (2014). Reduced-rank vector generalized linear models with two linear predictors. Computational Statistics and Data Analysis, 71, 889-902.

## See Also

gaitpoisson.mix, gaitpoisson.mlm, zapoisson, Zipois, yip88, rrvglm, zipebcom, rpois, simulate.vlm, hdeff.vglm.

## Examples

```
# Example 1: simulated ZIP data
zdata <- data.frame(x2 = runif(nn <- 1000))
zdata <- transform(zdata, pstr01 = logitlink(-0.5 + 1*x2, inverse = TRUE),
    pstr02 = logitlink( 0.5 - 1*x2, inverse = TRUE),
    Ps01 = logitlink(-0.5 , inverse = TRUE),
    Ps02 = logitlink( 0.5 , inverse = TRUE),
    lambda1 = loglink(-0.5 + 2*x2, inverse = TRUE),
    lambda2 = loglink( 0.5 + 2*x2, inverse = TRUE))
zdata <- transform(zdata, y1 = rzipois(nn, lambda = lambda1, pstr0 = Ps01),
    y2 = rzipois(nn, lambda = lambda2, pstr0 = Ps02))
with(zdata, table(y1)) # Eyeball the data
with(zdata, table(y2))
fit1 <- vglm(y1 ~ x2, zipoisson(zero = 1), data = zdata, crit = "coef")
fit2 <- vglm(y2 ~ x2, zipoisson(zero = 1), data = zdata, crit = "coef")
coef(fit1, matrix = TRUE) # These should agree with the above values
coef(fit2, matrix = TRUE) # These should agree with the above values
# Fit all two simultaneously, using a different parameterization:
fit12 <- vglm(cbind(y1, y2) ~ x2, zipoissonff, data = zdata, crit = "coef")
coef(fit12, matrix = TRUE) # These should agree with the above values
# For the first observation compute the probability that y1 is
# due to a structural zero.
(fitted(fit1, type = "pstr0") / fitted(fit1, type = "pobs0"))[1]
# Example 2: McKendrick (1926). Data from 223 Indian village households
cholera <- data.frame(ncases = 0:4, # Number of cholera cases,
wfreq = c(168, 32, 16, 6, 1)) # Frequencies
fit <- vglm(ncases ~ 1, zipoisson, wei = wfreq, cholera, trace = TRUE)
coef(fit, matrix = TRUE)
with(cholera, cbind(actual = wfreq,
    fitted = round(dzipois(ncases, lambda = Coef(fit)[2],
                                    pstr0 = Coef(fit)[1]) *
                                    sum(wfreq), digits = 2)))
# Example 3: data from Angers and Biswas (2003)
abdata <- data.frame(y = 0:7, w = c(182, 41, 12, 2, 2, 0, 0, 1))
abdata <- subset(abdata, w > 0)
fit <- vglm(y ~ 1, zipoisson(lpstr0 = probitlink, ipstr0 = 0.8),
    data = abdata, weight = w, trace = TRUE)
fitted(fit, type = "pobs0") # Estimate of P(Y = 0)
coef(fit, matrix = TRUE)
Coef(fit) # Estimate of pstr0 and lambda
fitted(fit)
with(abdata, weighted.mean(y, w)) # Compare this with fitted(fit)
summary(fit)
# Example 4: zero-deflated model for intercept-only data
zdata <- transform(zdata, lambda3 = loglink(0.0, inverse = TRUE))
```

```
zdata <- transform(zdata, deflat.limit = -1 / expm1(lambda3)) # Boundary
# The 'pstr0' parameter is negative and in parameter space:
zdata <- transform(zdata, usepstr0 = deflat.limit / 2) # Not too near the boundary
zdata <- transform(zdata, y3 = rzipois(nn, lambda3, pstr0 = usepstr0))
head(zdata)
with(zdata, table(y3)) # A lot of deflation
fit3 <- vglm(y3 ~ 1, zipoisson(zero = -1, lpstr0 = "identitylink"),
    data = zdata, trace = TRUE, crit = "coef")
coef(fit3, matrix = TRUE)
# Check how accurate it was:
zdata[1, "usepstr0"] # Answer
coef(fit3)[1] # Estimate
Coef(fit3)
vcov(fit3) # Is positive-definite
# Example 5: This RR-ZIP is known as a COZIGAM or COZIVGLM-ZIP
set.seed(123)
rrzip <- rrvglm(Alopacce ~ sm.bs(WaterCon, df = 3), zipoisson(zero = NULL),
    data = hspider, trace = TRUE, Index.corner = 2)
coef(rrzip, matrix = TRUE)
Coef(rrzip)
summary(rrzip)
## Not run: plotvgam(rrzip, lcol = "blue")
```

Zoabeta The Zero/One-Inflated Beta Distribution

## Description

Density, distribution function, and random generation for the zero/one-inflated beta distribution.

## Usage

dzoabeta(x, shape1, shape2, pobs0 $=0$, pobs1 $=0$, log $=$ FALSE, tol $=$.Machine\$double.eps)
pzoabeta(q, shape1, shape2, pobs0 $=0$, pobs1 $=0$, lower.tail $=$ TRUE, log.p $=$ FALSE, tol $=$.Machine\$double.eps)
qzoabeta(p, shape1, shape2, pobs0 = 0, pobs1 = 0, lower.tail = TRUE, log.p = FALSE, tol = .Machine\$double.eps)
rzoabeta(n, shape1, shape2, pobs0 $=0$, pobs1 = 0,
tol = .Machine\$double.eps)

## Arguments

$x, q, p, n \quad$ Same as Beta.
pobs0, pobs1 vector of probabilities that 0 and 1 are observed ( $\omega_{0}$ and $\omega_{1}$ ).
shape1, shape2 Same as Beta. They are called $a$ and $b$ in beta respectively.
lower.tail, log, log.p
Same as Beta.
tol Numeric, tolerance for testing equality with 0 and 1.

## Details

This distribution is a mixture of a discrete distribution with a continuous distribution. The cumulative distribution function of $Y$ is

$$
F(y)=\left(1-\omega_{0}-\omega_{1}\right) B(y)+\omega_{0} \times I[0 \leq y]+\omega_{1} \times I[1 \leq y]
$$

where $B(y)$ is the cumulative distribution function of the beta distribution with the same shape parameters (pbeta), $\omega_{0}$ is the inflated probability at 0 and $\omega_{1}$ is the inflated probability at 1 . The default values of $\omega_{j}$ mean that these functions behave like the ordinary Beta when only the essential arguments are inputted.

## Value

dzoabeta gives the density, pzoabeta gives the distribution function, qzoabeta gives the quantile, and rzoabeta generates random deviates.

## Author(s)

Xiangjie Xue and T. W. Yee

## See Also

zoabetaR, beta, betaR, Betabinom.

## Examples

```
## Not run:
N <- 1000; y <- rzoabeta(N, 2, 3, 0.2, 0.2)
hist(y, probability = TRUE, border = "blue", las = 1,
    main = "Blue = 0- and 1-altered; orange = ordinary beta")
sum(y == 0) / N # Proportion of 0s
sum(y == 1) / N # Proportion of 1s
Ngrid <- 1000
lines(seq(0, 1, length = Ngrid),
    dbeta(seq(0, 1, length = Ngrid), 2, 3), col = "orange")
lines(seq(0, 1, length = Ngrid), col = "blue",
            dzoabeta(seq(0, 1, length = Ngrid), 2 , 3, 0.2, 0.2))
## End(Not run)
```

zoabetaR Zero- and One-Inflated Beta Distribution Family Function

## Description

Estimation of the shape parameters of the two-parameter beta distribution plus the probabilities of a 0 and/or a 1 .

## Usage

```
zoabetaR(lshape1 = "loglink", lshape2 = "loglink", lpobs0 = "logitlink",
    lpobs1 = "logitlink", ishape1 = NULL, ishape2 = NULL, trim = 0.05,
    type.fitted = c("mean", "pobs0", "pobs1", "beta.mean"),
    parallel.shape = FALSE, parallel.pobs = FALSE, zero = NULL)
```


## Arguments

lshape1, lshape2, lpobs0, lpobs1
Details at CommonVGAMffArguments. See Links for more choices.
ishape1, ishape2
Details at CommonVGAMffArguments.
trim, zero Same as betaR.
parallel.shape, parallel.pobs
See CommonVGAMfffarguments for more information.
type.fitted The choice "beta.mean" mean to return the mean of the beta distribution; the 0 s and 1 s are ignored. See CommonVGAMffArguments for more information.

## Details

The standard 2-parameter beta distribution has a support on $(0,1)$, however, many datasets have 0 and/or 1 values too. This family function handles 0 s and 1 s (at least one of them must be present) in the data set by modelling the probability of a 0 by a logistic regression (default link is the logit), and similarly for the probability of a 1 . The remaining proportion, 1 -pobs 0 -pobs 1 , of the data comes from a standard beta distribution. This family function therefore extends betaR. One has $M=3$ or $M=4$ per response. Multiple responses are allowed.

## Value

Similar to betaR.

## Author(s)

Thomas W. Yee and Xiangjie Xue.

## See Also

Zoabeta, betaR, betaff, Beta, zipoisson.

## Examples

```
nn <- 1000; set.seed(1)
bdata <- data.frame(x2 = runif(nn))
bdata <- transform(bdata,
    pobs0 = logitlink(-2 + x2, inverse = TRUE),
    pobs1 = logitlink(-2 + x2, inverse = TRUE))
bdata <- transform(bdata,
    y1 = rzoabeta(nn, shape1 = exp(1 + x2), shape2 = exp(2 - x2),
            pobs0 = pobs0, pobs1 = pobs1))
```

summary (bdata)
fit1 <- vglm(y1 ~ x2, zoabetaR(parallel.pobs = TRUE),
data $=$ bdata, trace $=$ TRUE)
coef(fit1, matrix $=$ TRUE)
summary(fit1)

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