## Package 'MM'

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## Description

Two generalizations of the Multiplicative Binomial distribution of Altham (1978).

## Details

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## Author(s)

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## References

P. M. E. Altham 1978. "Two Generalizations of the Binomial Distribution". Applied Statistics 27:162-167
P. M. E. Altham and Robin K. S. Hankin 2012. "Multivariate Generalizations of the Multiplicative Binomial Distribution: Introducing the MM Package", Journal of Statistical Software, 46(12), 1-23. http://www.jstatsoft.org/v46/i12/

## Examples

```
data(voting)
Lindsey(voting, voting_tally)
jj <- paras(3)
```

rMM (10, 4, jj)
danaher $\quad$ Dataset due to Danaher

## Description

Dataset due to Danaher; also an analysis ab initio

## Usage

data(danaher)

## Format

- danaher is a matrix (of class Oarray) that represents Danaher and Hardie's Table 1


## Details

Since bacon is often eaten with eggs, it is reasonable to expect that it is purchased with eggs.
Danaher and Hardie use a dataset obtained from a sample of 548 households over four consecutive store trips. They considered only grocery shopping trips with a total basket value of at least five dollars. For each household, they counted the total number of bacon purchases in their four eligible shopping trips, and the total number of egg purchases for the same trips.

Object danaher is a five-by-five matrix of class Oarray with entry $(i, j)$ indicating the number of shoppers buying bacon on $i$ occasions and eggs on $j$ occasions (note the zero offset). Thus danaher $[1,2]=16$ indicates that 16 shoppers bought bacon on 1 occasion and eggs on 2 occasions.

## References

P. J. Danaher and B. G. S. Hardie 2005. "Bacon with your eggs? Applications of a new bivariate beta-binomial distribution". The American Statistician, 59(4):282

## See Also

```
optimizer
```


## Examples

```
data(danaher)
Lindsey_MB(danaher)
# Dataset from table 3 follows; see also the example at Lindsey.Rd
mags <-
c(2463, 35, 44, 14, 16, 7, 262, 20, 2, 2, 0, 0, 0, 2, 17, 2,
0, 2, 0, 0, 3, 8, 0, 0, 1, 0, 0, 4, 8, 0, 1, 1, 0, 0, 3, 3,
0, 0, 0, 0, 0, 1, 52, 2, 1, 0, 2, 0, 22)
```

```
dim(mags) <- c(7,7)
mags <- as.Oarray(mags,offset=0)
dimnames(mags) <-
list(AA=as.character(0:6),Sig=as.character(0:6)) # messy kludge in Lindsey_MB()
summary(Lindsey_MB(mags))
```

    Extract. paras Extract or Replace parameters of a paras object
    
## Description

Methods for "[" and "[<-", i.e., extraction or subsetting of paras objects.

## Arguments

| $x$ | Object of class paras |
| :--- | :--- |
| $i$ | Elements to extract or replace |
| value | Replacement value |

## Value

Always returns an object of class paras.

## Methods

- $\mathrm{x}[\mathrm{i}]$
- $x[i]<-$ value
- $x[i, j]$
- $x[i, j]<-$ value


## Note

These methods are included for completeness; it's not clear to me that they are likely to be used by anyone. It might be better to always use constructions like $x<-\operatorname{paras}(4) ; p(x)[2]<-0.1$ instead; YMMV.

## Author(s)

Robin K. S. Hankin

## Examples

```
x <- paras(4)
x[2] <- 0.1
x[1,2] <- 0.12
x
```

gunter Convert from multiple multivariate observations to tabular form

## Description

Convert from a matrix with rows corresponding to multivariate observations, to a tabular form listing every possible combination together with the number of times that combination was observed.

## Usage

gunter (obs)
\#\# S3 method for class 'gunter'
print(x, ...)

## Arguments

obs Argument. If a matrix, interpret each row as a multivariate observation (so the rowsums are constant). If an object of class MB, interpret appropriately; if an Oarray, coerce to an MB object
$x \quad$ Object of class gunter to be printed by the print method
... Further arguments, currently ignored

## Value

For matrices and data frames, function gunter () returns an object of class gunter: a list of two elments, the first being a matrix ('obs') with rows being possible observations, and the second ('d') a vector with one entry for each row of matrix obs.

For MB objects and Oarray objects, function gunter() returns an object of class gunter_MB.
The print method returns its argument, invisibly, after printing it coerced to a list.

## Author(s)

Bert Gunter, with tiny alterations by Robin Hankin

## Examples

```
data(wilson)
gunter(non_met)
data(danaher)
gunter(danaher) # object of class gunter_MB
```

Lindsey The Poisson device of Lindsey and Mersch (1992).

## Description

Function Lindsey () returns a maximum likelihood fit of the multiplicative multinomial using the Poisson device of Lindsey and Mersch (1992), and in the context of the multiplicative multinomial by Altham and Lindsey (1998).

Function Lindsey_MB() returns a maximum likelihood fit for the multivariate multiplicative binomial, for the special case of a bivariate distribution. An example of coercing a table to the correct form for use with Lindsey_MB() is given in the examples section below. Also, see danaher for another example.

## Usage

Lindsey(obs, $n=$ NULL, give_fit = FALSE)
Lindsey_MB(a)
\#\# S3 method for class 'Lindsey_output'
print(x, ...)

## Arguments

obs In Lindsey (), an integer matrix with each row corresponding to an observation. All row sums must match
$\mathrm{n} \quad$ Vector with elements corresponding to the rows of obs; default of NULL corresponds to observing each row of obs once
a
In Lindsey_MB(), an object that is coerced to one of class gunter_MB. Typically, the user supplies an Oarray object or an MB object
give_fit Boolean, with default FALSE meaning to return just the fit, coerced to an object of class paras and TRUE meaning to return a list with two elements, the first being a paras object and the second being the fit returned by glm()
$x \quad$ In the print method, object of class Lindsey_output
$\ldots$ In the print method, further arguments, currently ignored

## Details

Uses the device first described by Lindsey in 1992; the 'meat' of which has R idiom
Off <- -rowSums(lfactorial(jj\$tbl))
glm(jj\$d ~ -1 + offset(Off) + (.)^2, data=data, family=poisson)
Function Lindsey (..., give_fit=TRUE) returns an object of class Lindsey_output, which has its own print method (which prints the summary of the fit rather than use the default method).

Function Lindsey (..., give_fit=FALSE) returns an object of class paras, which can then be passed on to functions such as rMM() , which take a paras object.
Function Lindsey_MB() returns an object of class glm.

## Author(s)

## P. M. E. Altham and Robin K. S. Hankin

## References

- J. K. Lindsey and G. Mersch 1992. "Fitting and comparing probability distributions with log linear models", Computational Statistics and Data Analysis, 13(4):373-384
- P. M. E. Altham and J. K. Lindsey, 1998. "Analysis of the human sex ratio using overdispersion models", Applied Statistics, 47:149-157


## See Also

```
gunter, danaher
```


## Examples

```
data(voting)
(o <- Lindsey(voting, voting_tally))
rMM(10,5,o)
data(danaher)
Lindsey_MB(danaher)
## Not run: #(takes a long time)
data(pollen)
Lindsey(pollen)
## End(Not run)
# Example of Lindsey_MB() in use follows.
a <- matrix(c(63,40, 26,7,69,42,19,5,48, 21,16,2,33,11,9,1,21, 8, 9,0,
    7,8,1,0,5,3,1,0,9,2,0,0),byrow=TRUE,ncol=4)
# Alternatively, you can get this from the pscl package as follows:
# library(pscl); data(bioChemists)
# a <- table(subset(bioChemists, fem == 'Men' & art < 8))
dimnames(a) <- list(papers=0:7,children=0:3)
require(Oarray)
a <- as.Oarray(a,offset=0)
# thus a[3,1]==11 means that 11 subjects had 3 papers and 1 child
summary(Lindsey_MB(a))
```


## Description

Various utilities to coerce and manipulate MB objects

## Usage

```
MB(dep, m, pnames=character(0))
## S3 method for class 'MB'
as.array(x, ...)
## S4 method for signature 'MB'
getM(x)
## S3 method for class 'gunter_MB'
print(x, ...)
```


## Arguments

dep Primary argument to MB() . Typically a matrix with each row being an observation (see 'details' section below for an example). If an object of class Oarray, function MB() coerces to an MB object
m
Vector containing the relative sizes of the various marginal binomial distributions
x
Object of class MB to be converted to an Oarray object
... Further arguments to as.array (), currently ignored
pnames In function $M B()$, a character vector of names for the entries

## Details

Function MB() returns an object of class MB . This is essentially a matrix with one row corresponding to a single observation; repeated rows indicate identical observations as shown below. Observational data is typically in this form. The idea is that the user can coerce to a gunter_MB object, which is then analyzable by Lindsey ().
The multivariate multiplicative binomial distribution is defined by

$$
\prod_{i=1}^{t}\binom{m_{i}}{x_{i} z_{i}} p_{i}^{x_{i}} q_{i}^{z_{i}} \theta_{i}^{x_{i} z_{i}} \prod_{i<j} \phi_{i j}^{x_{i} x_{j}}
$$

Thus if $\theta=\phi=1$ the system reduces to a product of independent binomial distributions with probability $p_{i}$ and size $m_{i}$ for $i=1, \ldots, t$.
There follows a short $R$ transcript showing the MB class in use, with annotation.
The first step is to define an $m$ vector:

```
R> m <- c(2,3,1)
```

This means that $m_{1}=2, m_{2}=3, m_{3}=1$. So $m_{1}=2$ means that $i=1$ corresponds to a binomial distribution with size 2 [that is, the observation is in the set $\{0,1,2\}$ ]; and $m_{2}=3$ means that $i=2$ corresponds to a binomial with size 3 [ie the set $\{0,1,2,3\}$ ].
Now we need some observations:

```
R> a <- matrix(c(1,0,0, 1,0,0, 1,1,1, 2,3,1, 2,0,1),5,3,byrow=T)
R> a
\begin{tabular}{lrrr} 
& {\([, 1]\)} & {\([, 2]\)} & {\([, 3]\)} \\
{\([1]\),} & 1 & 0 & 0 \\
{\([2]\),} & 1 & 0 & 0 \\
{\([3]\),} & 1 & 1 & 1 \\
{\([4]\),} & 2 & 3 & 1 \\
{\([5]\),} & 2 & 0 & 1
\end{tabular}
```

In matrix $a$, the first observation, viz $\mathrm{c}(1,0,0)$ is interpreted as $x_{1}=1, x_{2}=0, x_{3}=0$. Thus, because $x_{i}+z_{i}=m_{i}$, we have $z_{1}=1, z_{2}=3, z_{3}=1$. Now we can create an object of class MB, using function MB():

```
R> mx <- MB(a, m, letters[1:3])
```

The third argument gives names to the observations corresponding to the columns of a. The values of $m_{1}, m_{2}, m_{3}$ may be extracted using getM():
$R>\operatorname{get}(m x)$
a b c
231
R>
The getM() function returns a named vector, with names given as the third argument to MB() .
Now we illustrate the print method:
$R>m x$

|  | a na | $b$ | nb | $c$ | nc |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $[1]$, | 1 | 1 | 0 | 3 | 0 |
| $[2]$, | 1 | 1 | 0 | 3 | 0 |

See how the columns are in pairs: the first pair total 2 (because $m_{1}=2$ ), the second pair total 3 (because $m_{2}=3$ ), and the third pair total 1 (because $m_{3}=1$ ). Each pair of columns has only a single degree of freedom, because $m_{i}$ is known.
Also observe how the column names are in pairs. The print method puts these in place. Take the first two columns. These are named 'a' and 'na': this is intented to mean 'a' and 'not a'.
We can now coerce to a gunter_MB:

```
R> (gx <- gunter(mx))
$tbl
    a b c
1000
2100
3200
[snip]
24 2 3 1
$d
    [1] 0 2 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 1 0 0 0 0 0 0 1
$m
a b c
2 }
```

Take the second line of the element tbl of $g x$, as an example. This reads $c(1,0,0)$ corresponding to the observations of $a, b, c$ respectively, and the second line of element d ["d" for "data"], viz 2, shows that this observation occurred twice (and in fact these were the first two lines of a).
Now we can coerce object mx to an array:

```
R> (ax <- as.array(mx))
, , c = 0
```

    b
    a 0123
00000
10020
20000
, , $c=1$
b
a 0123
00100
10000
21100
>
(actually, $a x$ is an Oarray object). The location of an element in ax corresponds to an observation of abc, and the entry corresponds to the number of times that observation was made. For example, $a x[1,2,0]=2$ shows that $c(1,2,0)$ occurred twice (the first two lines of $a)$.
The Lindsey Poisson device is applicable: see help (danaher) for an application to the bivariate case and help(Lindsey) for an example where a table is created from scratch.

## Author(s)

Robin K. S. Hankin

## See Also

MM, Lindsey, danaher

## Examples

```
a <- matrix(c(1,0,0, 1,0,0, 1,1,1, 2,3,1, 2,0,1),5,3,byrow=TRUE)
m<- c(2,3,1)
mx <- MB(a, m, letters[1:3]) # mx is of class 'MB'; column headings
    # mean "a" and "not a".
ax <- as.array(mx)
gx <- gunter(ax)
ax2 <- as.array(gx)
data(danaher)
summary(Lindsey_MB(danaher))
```

MM Various multiplicative multinomial probability utilities

## Description

Various multiplicative multinomial probability utilities for different types of observation

## Usage

MM (y, n=NULL, paras)
MM_allsamesum(y, n=NULL, paras)
MM_differsums(y, n=NULL, paras)
MM_allsamesum_A(y, paras)
MM_differsums_A(y, paras)
MM_single(yrow, paras, givelog=FALSE)
MM_support(paras, ss)

## Arguments

y
yrow
n
ss
givelog
paras Object of class paras

## Details

Consider non-negative integers $y_{1}, \ldots, y_{k}$ with $\sum y_{i}=y$. Then suppose the frequency function of the distribution $Y_{1}, \ldots, Y_{k}$ is

$$
C \cdot\binom{y}{y_{1}, \ldots, y_{k}} \prod_{i=1}^{k} p_{i}^{y_{i}} \prod_{1 \leq i<j \leq k} \theta_{i j}^{y_{i} y_{j}}
$$

where $p_{i}, \ldots, p_{k} \geq 0, \sum p_{i}=1$ correspond to probabilities; and $\theta_{i j}>0$ for $1 \leq i<j \leq k$ are additional parameters.

Here $C$ stands for a normalization constant:

$$
C=C(p, \theta, Y)=\sum_{y_{1}+\cdots+y_{k}=y} \prod_{i=1}^{k} p_{i}^{y_{i}} \prod_{1 \leq i<j \leq k} \theta_{i j}^{y_{i} y_{j}}
$$

which is evaluated numerically. This is expensive.
The usual case is to use function MM().

- Function $M M()$ returns the $\log$ of the probability of a matrix of rows of independent multinomial observations. It is a wrapper for MM_allsamesum() and MM_differsums(). Recall that optional argument $n$ specifies the number of times that each row is observed. Calls NormC().
- Function MM_allsamesum() gives the log of the probability of observing a matrix where the rowsums are identical. Calls NormC().
- Function MM_differsums() gives the log of the probability of observing a matrix where the rowsums are not necessarily identical. Warning: This function takes a long time to run. Calls NormC(), possibly many times.
- Functions MM_allsamesum_A() and MM_differsums_A() are analogous to functions MM_allsamesum() and MM_differsums() but interpret the matrix $y$ as having rows corresponding to observations; each row is observed once, as in data(pollen). Both call NormC().
- Function MM_single() gives a likelihood function for a paras object with a single multinomial observation (that is, a single line of matrix y). Does not call NormC().
- Function MM_support() gives the support (that is, the log-likelihood) of a paras object; argument ss is the sufficient statistic, as returned by suffstats(). Does not call NormC().
- Function dMM() [documented more fully at rMM .Rd] gives the probability of a single multivariate observation (ie a single row of the matrix argument y). Calls NormC().


## Author(s)

Robin K. S. Hankin

## Examples

```
data(voting)
data(voting)
p <- Lindsey(voting, voting_tally)
```

MM(voting, voting_tally, $p$ ) \#No other value of ' $p$ ' gives a bigger value

```
multinomial Multinomial function
```


## Description

The multinomial function and its logarithm

## Usage

multinomial(x)
lmultinomial(x)

## Arguments

x
Numeric vector

## Details

Function multinomial() returns

$$
\binom{n}{n_{1}, n_{2}, \ldots, n_{k}}=\frac{n!}{n_{1}!n_{2}!\cdots n_{k}!}
$$

where $\sum_{i} n_{i}=n$, and function lmultinomial() returns the natural logarithm of this.

## Note

Uses logarithmic functions to avoid overflow.

## Author(s)

Robin K. S. Hankin

## Examples

$x<-\operatorname{runif}(10)$
exp(lmultinomial(x)) - multinomial(x) \#should be small

NormC Normalizing constant for the multiplicative multinomial

## Description

Calculates the normalizing constant for the multiplicative multinomial using direct numerical summation

## Usage

NormC(Y, paras, log = FALSE)

## Arguments

$Y \quad$ Total number of observations
paras Object of class paras
$\log \quad$ Boolean, with default FALSE meaning to return the value, and TRUE meaning to return the natural logarithm

## Author(s)

Robin K. S. Hankin

## Examples

jj <- paras(3)
theta(jj) <- 2
$\operatorname{NormC}(5, j j)$

## optimizer Maximum likelihood estimator for the MM

## Description

Maximum likelihood estimator for the MM

## Usage

optimizer (y, $\mathrm{n}=\mathrm{NULL}$, start $=$ NULL, method = "nlm", printing = FALSE, give_fit=FALSE, ...)
optimizer_allsamesum(y, $\mathrm{n}=$ NULL, start = NULL, method = "nlm", printing = FALSE, give_fit=FALSE, ...)
optimizer_differsums(y, $\mathrm{n}=$ NULL, start $=$ NULL, method = "nlm", printing = FALSE, give_fit=FALSE, ...)

## Arguments

y Matrix with each row being a possible observation
n Counts of observations corresponding to rows of $y$
start Start value for optimization routine, taken to be an object of class paras. Default value of NULL means to start with Lindsey $(y, n)$, which theoretically should be the maximum likelihood estimate
method String giving which optimization method to use. Default of Nelder means to use optim() with the Nelder-Mead method; the other supported option is nlm
printing Boolean, with TRUE meaning to print information as the optimization progresses and default FALSE meaning to print nothing
give_fit Boolean, with default FALSE meaning to return the maximum likelihood estimate in the form of a paras object, and TRUE meaning to return a two-element list, the first being the output of $\operatorname{nlm}()$ or optim() and the second being the MLE

Further arguments passed to the optimization routine. In particular, note that hessian=TRUE is useful in conjunction with give_fit=TRUE

## Details

Function optimizer () is the user-friendly version: it is a wrapper for optimizer_samesum() and optimizer_differsums(); it dispatches according to whether the rowsums are identical or not.

These functions are slow because they need to evaluate NormC() repeatedly, which is expensive.
Function optimizer_samesum() nominally produces the same output as Lindsey(), but is more computationally intensive.

## Author(s)

Robin K. S. Hankin

## See Also

Lindsey

## Examples

```
data(voting)
p1 <- Lindsey(voting,voting_tally)
p2 <- optimizer(voting,voting_tally,start=p1)
theta(p1) - theta(p2) # Should be zero
## Not run:
data(pollen)
p1 <- optimizer(pollen)
p2 <- Lindsey(pollen)
theta(p1) - theta(p2) # Isn't zero...numerical scruff...
```

    \#\# End(Not run)
    paras Manipulate a paras object

## Description

Various utilities to manipulate paras objects. Functions pnames() and pnames<-() operate on MB objects as expected.

## Usage

```
paras(x, p, theta, pnames = character(0))
p(x) <- value
theta(x) <- value
p(x)
theta(x)
pnames(x)
pnames(x) <- value
getVals(x)
## S4 method for signature 'paras'
length(x)
```


## Arguments

x
theta
pnames
value
$\mathrm{p} \quad$ In function paras(), a vector of the first $k-1$ elements of the probabilities
Object of class paras

In function paras(), a $k$ by $k$ matrix with diagonal composed of ones
In function paras(), a character vector of names for the entries
Replacement value

## Details

A paras object contains the parameters needed to specify a multiplicative multinomial distribution.
Suppose $p$ is an object of class paras object. Then $p$ is a list of two elements. The first element, $p$, is a vector of length length $(p)$ and the second is an upper-diagonal matrix square matrix of size length ( $p$ ). The vignette gives further details.
The functions documented here allow the user to inspect and change paras objects.

## Author(s)

Robin K. S. Hankin

## See Also

MM, MB

## Examples

```
    jj <- paras(5)
    pnames(jj) <- letters[1:5]
    p(jj) <- c(0.1, 0.1, 0.3, 0.1)
    theta(jj) <- matrix(1:25,5,5)
    pnames(jj) <- letters[1:5]
    jj
    # OK, we've defined jj, now use it with some other functions:
    dMM(rep(1,5),jj)
    MM_single(1:5,jj)
rMM(2,9,jj)
```

pollen
Pollen data from Mosimann 1962

## Description

Data from Mosimann 1962 detailing forest pollen counts

## Usage

data(pollen)

## Format

A matrix with four columns and 76 rows.

## Details

The rows each sum to 100 ; the values are counts of four different types of pollen. Each row corresponds to a different level in the core; the levels are in sequence with the first row being most recent and the last row being the oldest.

## References

J. E. Mosimann 1962. "On the compound multinomial distribution, the multivariate $\beta$-distribution, and correlations among proportions". Biometrika, volume 49, numbers 1 and 2, pp65-82.

## Examples

```
## Not run:
data(pollen)
Lindsey(pollen)
## End(Not run)
```

```
    powell Dataset due to Powell (1990)
```


## Description

Dataset due to Powell (1990)

## Usage

data(powell)

## Format

A frequency table of counts of association data.

## Source

- W. Powell, M. Coleman and J. McNicol 1990 "The statistical analysis of potato culture data". Plant Cell, Tissue and Organ Culture 23:159-164


## Examples

data(powell)
Lindsey(powell, powell_counts)
rMM Random samples from the multiplicative multinomial

## Description

Density, and random samples drawn from, the multiplicative multinomial

## Usage

rMM(n, $Y$, paras, burnin $=4 * Y$, every $=4 * Y$, start $=$ NULL $)$
dMM(Y, paras)

## Arguments

| n | Number of observations to make |
| :--- | :--- |
| Y | Sum of each observation (for example, 100 for the pollen dataset, 4 for voting) |
| paras | Parameters of the MM distribution; an object of class paras <br> every |
| Each row is recorded every every steps through the Markov chain. Thus every=10 <br> means every tenth row is written to the returned matrix during MH process (and <br> the other nine values are discarded) |  |
| burnin | Number of initial observations to ignore |
| start | Observation to start simulation, with default NULL corresponding to using a ran- <br> dom start vector |

## Details

Function rMM() uses standard Metropolis-Hastings simulation.
Function dMM() is documented here for convenience; see help(MM) for related functionality.

## Value

Returns a matrix with n rows and length(paras) columns. Each row is an observation.

## Author(s)

Robin K. S. Hankin

## See Also

MM

## Examples

```
data(voting)
rMM(10,4,Lindsey(voting,voting_tally))
p <- paras(3)
theta(p) <- 2
dMM(1:3,p)
```

    skellam Brassica Dataset due to Catcheside
    
## Description

Dataset due to Catcheside, used by Skellam (1948) and subsequently by Altham (1978).

## Usage

data(skellam)

## Format

A frequency table of counts of association data.

## Source

- J. G. Skellam 1948. "A probability distribution derived from the binomial distribution by regarding the probability of success as variable between the sets of trials". Journal of the Royal Statistical Society, series B (Methodological). Volume 10, number 2, pp257-248.
- D. Catcheside 1937. Cytologia, Fujii Jub. Vol.


## Examples

```
data(skellam)
Lindsey(skellam, skellam_counts)
```

```
suffstats Sufficient statistics for the multiplicative multinomial
```


## Description

Calculate, manipulate, and display sufficient statistics of the multiplicative multinomial. Functionality for analysing datasets, and distributions specified by their parameters is given; summary and print methods are also documented here.

## Usage

```
    suffstats(y, n = NULL)
    expected_suffstats(L,Y)
    ## S3 method for class 'suffstats'
    print(x, ...)
    ## S3 method for class 'suffstats'
    summary(object, ...)
    ## S3 method for class 'summary.suffstats'
    print(x, ...)
```


## Arguments

$y, n \quad$ In function suffstats(), argument $y$ is a matrix with each row being a possible observation and $n$ is counts of observations corresponding to rows of $y$ with default NULL interpreted as each row of $y$ being observed once. If $y$ is an object of class gunter, this is interpreted sensibly
L, $Y \quad$ In function expected_suffstats (), argument $L$ is an object of class Lindsey [typically returned by function Lindsey ()], and $Y$ is the known constant sum (ie the rowSums() of the observations)
$x$, object An object of class suffstats or summary. suffstats, to be printed or summarized
$\ldots \quad$ Further arguments to the print or summary methods. Currently ignored

## Details

Function suffstats() returns a list comprising a set of sufficient statistics for the observations $y,[n]$.
This function requires that the rowsums of $y$ are all identical.

## Value

Function suffstats() returns a list of four components:
Y Rowsums of $y$
nobs Number of observations
row_sums Column sums of $y$, counted with multiplicity
cross_prods Matrix of summed squares
Function summary.suffstats() provides a summary of a suffstats object that is a list with two elements: row_sums and cross_prods, normalized with nobs and $Y$ so that the values are comparable with that returned by expected_suffstats(). In particular, the sum of row_sums is the known sum $y$.

## Author(s)

Robin Hankin and P. M. E. Altham

## Examples

```
data(voting)
suffstats(voting, voting_tally)
data(wilson)
wilson <- gunter(non_met)
suffstats(wilson)
L <- Lindsey(wilson)
expected_suffstats(L,5)
summary(suffstats(wilson)) ## matches.
summary(suffstats(rMM(10,5,L))) # should be close.
```


## Description

Four objects:

- sweets is a $2 \times 3 \times 21$ array
- sweets_tally is a length 37 vector
- sweets_array is a $2 \times 3 \times 37$ vector
- sweets_table is a $37 \times 6$ matrix


## Usage

```
data(sweets)
```


## Details

Object sweets is the raw dataset; objects sweets_table and sweets_tally are processed versions which are easier to analyze.

The father of a certain family brings home nine sweets of type mm and nine sweets of type jbeach day for 21 days to his children, AMH, ZJH, and AGH.
The children share the sweets amongst themselves in such a way that each child receives exactly 6 sweets.

- Array sweets has dimension $c(2,3,21): 2$ types of sweets, 3 children, and 21 days. Thus sweets[, , 1] shows that on the first day, AMH chose 0 sweets of type mm and 6 sweets of type jb; child ZJH chose 3 of each, and child AGH chose 6 sweets of type mm and 0 sweets of type jb.
Observe the constant marginal totals: the kids have the same overall number of sweets each, and there are a fixed number of each kind of sweet.
- Array sweets_array has dimension c ( $2,3,37$ ): 2 sweets, 3 children, and 37 possible ways of arranging a matrix with the specified marginal totals. This can be produced by allboards() of the aylmer package.
- sweets_table is a dataframe with six columns, one for each combination of child and sweet, and 37 rows, each row showing a permissible arrangement. All possibilities are present. The six entries of sweets $[,, 1]$ correspond to the six elements of sweets_table[1, ]; the column names are mnemonics.
- sweets_tally shows how often each of the arrangements in sweets_tally was observed (that is, it's a table of the 21 observations in sweets)


## Source

The Hankin family

## Examples

```
data(sweets)
# show correspondence between sweets_table and sweets_tally:
cbind(sweets_table, sweets_tally)
# Sum the data, by sweet and child and test:
fisher.test(apply(sweets,1:2,sum))
# Not significant!
```

[^0]```
    jj1 <- apply(sweets_array,3,tcrossprod)
    jj2 <- apply(sweets_array,3, crossprod)
    dim(jj1) <- c(2,2,37)
    dim(jj2) <- c(3,3,37)
    theta_xy <- jj1[1,2,]
        phi_ab <- jj2[1,2,]
        phi_ac <- jj2[1,3,]
        phi_bc <- jj2[2,3,]
    # Now the offset:
    Off <- apply(sweets_array,3,function(x){-sum(lfactorial(x))})
    # Now the formula:
    f <- formula(sweets_tally~ -1 + theta_xy + phi_ab + phi_ac + phi_bc)
    # Now the Lindsey Poisson device:
    out <- glm(formula=f, offset=Off, family=poisson)
    summary(out)
# See how the residual deviance is comparable with the degrees of freedom
```

voting Synthetic dataset of voting behaviour due to Altham

## Description

Synthetic dataset of voting behaviour due to Altham

## Usage

data(voting)

## Format

voting is a three-column matrix with each row being a configuration of voting in a household with four members, and three choices. Vector voting_tally is a list of how many households voted, and Nvoting_tally is a more extreme dataset of the same type, used to uncover bugs in Lindsey ().

## Source

Supplied by P. M. E. Altham

## Examples

```
data(voting)
Lindsey(voting,voting_tally)
```


## Description

Dataset due to Wilson

## Usage

data(wilson)

## Format

Two objects, met_area and non_met, which have three columns and either 17 or 18 rows. Each row corresponds to a neighborhood of five households, each of which votes for one of three choices: US, S, or VS. Each column corresponds to one of these choices. The rowsums are constant because there are exactly five households in each neighborhood.

## Source

- J. R. Wilson 1989. "Chi-square tests for Overdispersion with Multiparameter Estimates", Journal of the Royal Statistical Society. Series C (Applied Statistics), 38(3):441-453
- S. S. Brier 1980. "Analysis of Contingency Tables Under Cluster Sampling", Biometrika 67(3):591-596


## Examples

data(wilson)
Lindsey (non_met)

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[^0]:    \# Now test for overdispersion.
    \# First set up the regressors:

