

# Package ‘IndTestPP’

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**Type** Package

**Title** Tests of Independence Between Point Processes in Time

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**Imports** parallel, stats

**Description** Several parametric and non-parametric tests and measures to check independence between two or more (homogeneous or nonhomogeneous) point processes in time are provided. Tools for simulating point processes in one dimension with different types of dependence are also implemented.

**License** GPL

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## R topics documented:

IndTestPP-package . . . . .	2
BarTxTn . . . . .	3
ComplPos.fun . . . . .	4
CondTest.fun . . . . .	5
CPSPpoints.fun . . . . .	7
CSPPOtEvents.fun . . . . .	8
depchi.fun . . . . .	10
DepMarkedNHPP.fun . . . . .	12
DepNHCPSP.fun . . . . .	13
DepNHNeyScot.fun . . . . .	15
DepNHPPqueue.fun . . . . .	17
DistObs.fun . . . . .	19
DistShift.fun . . . . .	20
DistSim.fun . . . . .	22

DutilleulPlot.fun . . . . .	24
IndNHNeyScot.fun . . . . .	25
IndNHPP.fun . . . . .	27
nearestdist.fun . . . . .	28
NHD.fun . . . . .	29
NHF.fun . . . . .	31
NHJ.fun . . . . .	33
NHK.fun . . . . .	37
simHPc.fun . . . . .	40
simNHPc.fun . . . . .	41
SpecGap.fun . . . . .	42
TestIndLS.fun . . . . .	43
TestIndNH.fun . . . . .	45
uniongentri.fun . . . . .	48

<b>Index</b>	<b>50</b>
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IndTestPP-package	<i>Tests of Independence Between Point Processes in Time</i>
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## Description

It provides several parametric and non-parametric tests to check the independence between two or more homogeneous and nonhomogeneous point processes. In addition, it provides tools for simulating point processes in one dimension, usually time, with different types of dependence, in particular: common Poisson shock processes, a network of queues with exponential arrivals, multivariate Neyman-Scott process with dependent cluster centers, and a Poisson process with dependent marks.

## Details

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## Author(s)

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BarTxTn	<i>Barcelona temperature data</i>
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### Description

Barcelona daily temperature series during the summer months (May, June, July, August and September) from 1951 to 2004.

### Usage

```
data(BarTxTn)
```

### Details

Variables

dia: Position of the day in the year, from 121 (1st of May) to 253 (30th of September).

mes: Month of the year, from 5 to 9.

ano: Year, from 1951 to 2004.

diames: Position of the day in the month, from 1 to 30 or 31.

Tx: Daily maximum temperature.

Tn: Daily minimum temperature.

Txm31: Local maximum temperature signal. Lowess of Tx with a centered window of 31 days.

Txm15: Local maximum temperature signal. Lowess of Tx with a centered window of 15 days.

Tnm31: Local minimum temperature signal. Lowess of Tn with a centered window of 31 days.

Tnm15: Local minimum temperature signal. Lowess of Tn with a centered window of 15 days.

TTx: Long term maximum temperature signal. Lowess of Tx with a centered 40% window.

TTn: Long term minimum temperature signal. Lowess of Tn with a centered 40% window.

### References

Cebrian, A.C., Abaurrea, J. and Asin, J. (2015). NHPoisson: An R Package for Fitting and Validating Nonhomogeneous Poisson Processes. *Journal of Statistical Software*, 64(6), 1-24.

### Examples

```
data(BarTxTn)
```

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ComplPos.fun	<i>Changes the format of a vector of the occurrence times in a point process</i>
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### Description

It changes the format of the vector of the occurrence points in a point process. It builds a vector of length T, the length of the observation period, which takes value 0 at the non occurrence times and the position value, or 1, at the occurrence times.

### Usage

```
ComplPos.fun(pos, T, type='Pos')
```

### Arguments

pos	Numeric vector. Occurrence times
T	Integer. Length of the observation period
type	Character string, 'Pos' or 'Bin'. Type of the new format

### Details

This function changes the format of the occurrence points in a point process. The new format is useful when several point processes, in the same observation period, must be specified; for example, in function [NHJ.fun](#) or [NHD.fun](#), where the occurrence times of different point processes must be introduced as a matrix. Since the number of occurrences in each process can be different, in the new format, occurrences in each process are specified as a vector of length T, which takes value 0 at non occurrence times and the time position (if type='Pos') or 1 (if type='Bin') at the occurrence times.

### Value

Npos	Numeric vector of length T containing the occurrence times in the new format.
------	---

### See Also

[NHD.fun](#), [NHF.fun](#), [NHK.fun](#), [NHJ.fun](#)

### Examples

```
pos<-c(4, 15, 18, 34, 36, 67, 98)
Npos<-ComplPos.fun(pos, T=100)
```

CondTest.fun

*Conditional Test of independence between two Poisson process***Description**

It calculates a test of independence between two Poisson process (PP), based on the analysis of the occurrences in the second process, given that there is an occurrence in the first one. Two different approaches to calculate the p-value are implemented.

It calls the auxiliary function `calcNmu`, not intended for the users.

**Usage**

```
CondTest.fun(pos1, pos2, lambda2, r, changer = TRUE, type = "All", plotRes = FALSE, ...)
```

**Arguments**

<code>pos1</code>	Numeric vector. Occurrence points in the first PP, $N_1$
<code>pos2</code>	Numeric vector. Occurrence points in the second PP, $N_2$
<code>lambda2</code>	Numeric vector. Intensity in each time in $N_2$
<code>r</code>	Numerical value. The radius of the intervals centered on the occurrence times in $N_1$
<code>changer</code>	Optional. Logical flag. If it is TRUE, when the defined intervals overlap, their lengths are changed to obtain not overlapped intervals. The two overlapped intervals are shortened by half of the overlapped period. In general, the resulting intervals are not centered .
<code>type</code>	Optional. Label 'Poisson', 'Normal' or 'All'. Approach to be used to calculate test p-values.
<code>plotRes</code>	Logical flag. If it is TRUE, residuals are plotted.
<code>...</code>	Further arguments to pass to the residual <code>plot</code> .

**Details**

The underlying idea of the tests is to analyze the behaviour of the second PP  $N_2$ , given that a point has occurred in the first one,  $N_1$ . Under independence between  $N_1$  and  $N_2$ ,  $N_2$  should be a PP with intensity `lambda2`.

For the analysis, intervals of length  $2r$  centered on each point in  $N_1$  are defined. To analyze the behaviour of  $N_2$ , two approaches are implemented, both based on the idea that the number of points in each interval should be a Poisson of mean  $\mu_i$  equal to the integral of `lambda2` in the interval.

'Poisson' option: under the null, and if the intervals are independent (that is if they do not overlap) the number of points in all them should be a Poisson of mean  $\mu$ , equal to the sum of all the  $\mu_i$ . The p-values is calculated as  $2 * \min((P(X < po) + P(X = po)/2), (P(X > po) + P(X = po)/2))$ , where  $X$  is a r.v with distribution  $\text{Poisson}(\mu)$  and  $po$  is the sum of the observed number of points in all the intervals. Obviously, since the p-value is calculated from a discrete distribution, it will not be uniformly distributed, and the size of the test  $P(\text{reject } H_0 \text{ when it is true})$  will not be 0.05. The size

depends on the value of the Poisson mean, only when it  $X$  can be approximated by a Normal, the size will be 0.05

'Normal' option: under the null, the variables  $(N_i - \mu_i)/(\mu_i * 0.5)$  must be zero mean and variance one variables but they are not identically distributed, and consequently some approximation has to be used to calculate the p-value. Since under the null, each  $N_i$  is a Poisson ( $\mu_i$ ), if the means  $\mu_i$  are high enough, each variable  $(N_i - \mu_i)/(\mu_i * 0.5)$  can be approximated by a  $N(0,1)$  distribution. If  $\mu_i$  are not high enough, still, the mean of the variables  $(N_i - \mu_i)/(\mu_i * 0.5)$  can be approximated by a Normal distribution using the Central limit theorem under the Lindeberg condition for r.v which are independent but not identically distributed. To apply this approximation a big sample size (number of points in  $N_1$ ) is needed. Empirically, it has been seen that the smaller the values of  $\mu_i$ , a bigger sample size is required to have a valid normal approximation. As a reference, with  $\mu_i$  values between 0 and 1, a sample size of 50 is satisfactory

Hence, if the processes are independent, the variables  $(N_i - \mu_i)/(\mu_i * 0.5)$  must have zero mean and variance one, and  $Nn$  the sample mean of  $(N_i - \mu_i)/(\mu_i * 0.5)$  can be approximated by a  $N(0, 1/n * 0.5)$  distribution, so that the p-value is calculated as  $2 * P(Nn > abs(Nn0)) = 2 * P(Z > abs(Nn0) * n * 0.5)$ .

### Value

Ni	Number of occurrences in each interval
mui	Theoretical mean of the number of occurrences in each interval under the independence assumption
Res	Residual vector obtained from each interval
pvP	P-value obtained from the 'Poisson' approach
PvN	P-value obtained from the 'Normal' approach
linf	Lower limit of each interval
lsup	Upper limit of each interval

### References

Billingsley, P. (1995). Probability and Measure. 3rd Ed. John Wiley and sons.

### See Also

[TestIndNH.fun](#), [DutilleulPlot.fun](#)

### Examples

```
#Two dependent Poisson processes from a NHCPSP
set.seed(123)
lambdao1<-runif(1000)/10
set.seed(124)
lambdao2<-runif(1000)/10
set.seed(125)
lambda12<-runif(1000)/20
aux<-DepNHCPSP.fun(lambda11=lambdao1, lambda12=lambdao2, lambda12=lambda12,
fixed.seed=123)
```

```
zz<-CondTest.fun(pos1=aux$posNH1, pos2=aux$posNH2, lambda2=aux$lambda2, r=3)
zz$pVP
zz$pVN
```

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CPSPpoints.fun	<i>Calculates the occurrence times of the three indicator processes of a bivariate Common Poisson shock process</i>
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### Description

This function calculates the occurrence times of the points in the three indicator processes of a bivariate Common Poisson shock process (CPSP), using as input information, the two marginal processes  $N_1$  and  $N_2$ .

### Usage

```
CPSPpoints.fun(X, Y, date = NULL)
```

### Arguments

X	Binary vector. The first CPSP marginal process; occurrence points are marked with 1 and the other with 0.
Y	Binary vector. The second CPSP marginal process; occurrence points are marked with 1 and the other with 0.
date	Optional. A vector or matrix indicating the date of each observation.

### Details

A bivariate CPSP  $N$  is usually specified by its two marginal, and possibly dependent, processes  $N_1$  and  $N_2$ , which are the observed processes. However,  $N$  can be decomposed into three independent indicator processes:  $N_{(1)}$ ,  $N_{(2)}$  and  $N_{(12)}$ , which are the processes of the points occurring only in the first marginal process, only in the second and in both of them (simultaneous points). The union of  $N_{(1)}$  and  $N_{(12)}$ , and  $N_{(2)}$  and  $N_{(12)}$  gives respectively the two marginal processes.

### Value

A list with components

PxX	Vector of the occurrence points in $N_{(1)}$ .
PxY	Vector of the occurrence points in $N_{(2)}$ .
PxXY	Vector of the occurrence points in $N_{(12)}$ .
X	Input argument.
Y	Input argument.
date	Input argument.

## References

Abaurrea, J. Asin, J. and Cebrian, A.C. (2015). A Bootstrap Test of Independence Between Three Temporal Nonhomogeneous Poisson Processes and its Application to Heat Wave Modeling. *Environmental and Ecological Statistics*, 22(1), 127-144.

## See Also

[CPSPPOtEvents.fun](#)

## Examples

```
set.seed(123)
X<-as.numeric(runif(100)<0.10)
set.seed(124)
Y<-as.numeric(runif(100)<0.15)

aux<-CPSPpoints.fun(X=X,Y=Y)
```

---

CPSPPOtEvents.fun	<i>Calculates the occurrence times of the three indicator processes of the bivariate Common Poisson shock process resulting from applying a POT approach</i>
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---

## Description

This function calculates the occurrence times and other characteristics (length, maximum and mean intensity) of the extreme events of the three indicator processes of a bivariate Common Poisson shock process (CPSP) obtained from a POT approach.

## Usage

```
CPSPPOtEvents.fun(X, Y, thresX, thresY, date = NULL)
```

## Arguments

X	Numeric vector. Series $(x_i)$ whose threshold exceedances define the first CPSP marginal process.
Y	Numeric vector. Series $(y_i)$ whose threshold exceedances define the second CPSP marginal process.
thresX	Numeric value. Threshold used to define the extreme events in $(x_i)$ .
thresY	Numeric value. Threshold used to define the extreme events in $(y_i)$ .
date	Optional. A vector or matrix indicating the date of each observation.

## Details

A CPSP  $N$  can be decomposed into three independent indicator processes:  $N_{(1)}$ ,  $N_{(2)}$  and  $N_{(12)}$ , the processes of the points occurring only in the first marginal process, only in the second and in both of them (simultaneous points). In the CPSP resulting from applying a POT approach, the events in  $N_{(1)}$  are a run of consecutive days where  $(x_i)$  exceeds its extreme threshold but  $(y_i)$  does not exceed its extreme threshold. An extreme event in  $N_{(2)}$  is defined analogously. A simultaneous event, or event in  $N_{(12)}$ , is a run where both series exceed their thresholds.

For the events defined in each indicator process, three magnitudes (length, maximum intensity and mean intensity) are calculated together with the initial point and the point of maximum excess of each event. In the  $N_{(12)}$ , the maximum and the mean intensity of both  $(x_i)$  and  $(y_i)$  are also calculated. The occurrence point of each event is located at the time of the maximum sum of the excesses over the thresholds (where an excess is 0 if the observation does not exceed its corresponding threshold). According to this definition, the occurrence point in  $N_{(1)}$  is the point with maximum intensity in  $(x_i)$  within the run.

The vectors `inddatX`, `inddatY` and `inddatXY`, elements of the output list, mark the observations that should be used in the estimation of each indicator process. The observations in an extreme event which are not the occurrence point are marked with 0 and treated as non observed in the estimation process. The rest are marked with 1 and must be included in the likelihood function.

## Value

A list with components

<code>ImX</code>	Vector of mean excesses (over the threshold) of the extreme events in $N_{(1)}$ .
<code>IxX</code>	Vector of maximum excesses (over the threshold) of the extreme events in $N_{(1)}$ .
<code>LX</code>	Vector of lengths of the extreme events in $N_{(1)}$ .
<code>PxX</code>	Vector of points of maximum excess of the extreme events in $N_{(1)}$ .
<code>PiX</code>	Vector of initial points of the extreme events in $N_{(1)}$ .
<code>inddatX</code>	Index equal to 1 in the observations which should be used in the estimation process of $N_{(1)}$ , and to 0 in the others.
<code>ImY</code>	Vector of mean excesses (over the threshold) of the extreme events in $N_{(2)}$ .
<code>IxY</code>	Vector of maximum excesses (over the threshold) of the extreme events in $N_{(2)}$ .
<code>LY</code>	Vector of lengths of the extreme events in $N_{(2)}$ .
<code>PxY</code>	Vector of points of maximum excess of the extreme events in $N_{(2)}$ .
<code>PiY</code>	Vector of initial points of the extreme events in $N_{(2)}$ .
<code>inddatY</code>	Index equal to 1 in the observations which should be used in the estimation process of $N_{(2)}$ and to 0 in the others.
<code>ImXYx</code>	Vector of mean excesses of the series $(x_i)$ in $N_{(12)}$ .
<code>IxXYx</code>	Vector of maximum excesses the series $(x_i)$ in $N_{(12)}$ .
<code>ImXYy</code>	Vector of mean excesses of the series $(y_i)$ in $N_{(12)}$ .
<code>IxXYy</code>	Vector of maximum excesses the series $(y_i)$ in $N_{(12)}$ .
<code>LXY</code>	Vector of lengths of the extreme events in $N_{(12)}$ .
<code>PxXY</code>	Vector of points of maximum excess of the extreme events in $N_{(12)}$ .

PiXY	Vector of initial points of the extreme events in $N_{(12)}$ .
inddatXY	Index equal to 1 in the observations which should be used in the estimation process of $N_{(12)}$ and to 0 in the others.
X	Input argument.
Y	Input argument.
thresX	Input argument.
thresY	Input argument.
date	Input argument.

## References

Abaurrea, J. Asin, J. and Cebrian, A.C. (2015). A Bootstrap Test of Independence Between Three Temporal Nonhomogeneous Poisson Processes and its Application to Heat Wave Modeling. *Environmental and Ecological Statistics*, 22(1), 127-144.

## See Also

[CSPSPoints.fun](#)

## Examples

```
data(BarTxTn)
dateB<-cbind(BarTxTn$ano,BarTxTn$mes,BarTxTn$diames)
BarBivEv<-CSPSP0Tevents.fun(X=BarTxTn$Tx,Y=BarTxTn$Tn,thresX=318,
thresY=220, date=dateB)
```

---

depchi.fun

*Estimates extremal dependence measures between two variables*

---

## Description

This function estimates the extremal dependence coefficients  $\chi$  and  $\bar{\chi}$  by Coles et al. (1999). It also plots the functions  $\chi(u)$  and  $\bar{\chi}(u)$  against a grid of values in  $[0,1]$  to analyse the extremal dependence of two variables.

## Usage

```
depchi.fun(X, Y, thresval = c(0:99)/100, tit = "", indgraph = TRUE,
bothest = TRUE, xlegend = "topleft")
```

**Arguments**

X	Numeric vector. Values of the first variable.
Y	Numeric vector. Values of the second variable.
thresval	Numeric vector. Grid values where the functions $\chi(u)$ and $\bar{\chi}(u)$ are evaluated.
tit	Character string. A title for the plots.
indgraph	Logical flag. If it is TRUE, plots are carried out in individual windows. If it is FALSE, windows with $2 \times 1$ plots are used.
bothest	Logical flag. If it is TRUE, two estimated coefficients (for X given Y and for Y given X) are displayed in the same plot. Otherwise, only the coefficient for Y given X is plotted.
xlegend	Optional. Label "topleft" or "bottomright". Position where the legend on the graph will be located.

**Details**

The extremal dependence between two variables X and Y is the tendency for one variable to be large, given that the other one is large. The extremal dependence coefficients  $\chi$  and  $\bar{\chi}$  are defined as  $\chi = \lim_{u \rightarrow 1} \chi(u)$  and  $\bar{\chi}(u) = 2 \log(P(U > u) / \log P(U > u, V > u)) - 1$ , where  $\chi(u) = P(U > u | V > u)$  and (U,V) are the transformed uniform marginals of the variables X and Y.

$\chi$  is on the scale [0, 1], with the set (0, 1] corresponding to asymptotic dependence, and the measure  $\bar{\chi}$  falls within the range [-1, 1], with the set [-1, 1) corresponding to asymptotic independence. Thus, the complete pair  $(\chi, \bar{\chi})$  is required as a summary of extremal dependence:  $(\chi > 0, \bar{\chi} = 1)$  signifies asymptotic dependence, in which case the value of  $\chi$  determines a measure of strength of dependence within the class; alternatively,  $(\chi = 0, \bar{\chi} < 1)$  signifies asymptotic independence, in which case the value of  $\bar{\chi}$  determines the strength of dependence within this class. Full details can be found in Coles et al. (1999).

In the  $\chi$  plot, the expected behaviour under independence of X and Y is also plotted.

**Value**

A list with elements

chiX	Estimated $\chi$ function for Y given X evaluated at the threshold grid.
chiY	Estimated $\chi$ function for X given Y evaluated at the threshold grid.
chiBX	Estimated $\bar{\chi}$ function for Y given X evaluated at the threshold grid.
chiBY	Estimated $\bar{\chi}$ function for X given Y evaluated at the threshold grid.
PX	Estimation of the probabilities $P(U < thresval)$
PY	Estimation of the probabilities $P(V < thresval)$
PXY	Estimation of the probabilities $P[(U < thresval) \& (V < thresval)]$
thresval	Input argument

**References**

Coles, S., Heffernan, J. and Tawn, J. (1999) Dependence measures for extreme value analysis. *Extremes*, 2, 339-365.

**See Also**

[TestIndNH.fun](#), [DutilleulPlot.fun](#), [CondTest.fun](#)

**Examples**

```
data(BarTxTn)

aux<-depchi.fun(X=BarTxTn$Tx,Y=BarTxTn$Tn, thresval = c(0:99)/100,
tit = "Barcelona", xlegend = "topleft")
```

---

DepMarkedNHPP.fun	<i>Generates trajectories of dependent point processes using a marked Poisson Process</i>
-------------------	---

---

**Description**

This function generates  $d$  dependent (homogeneous or nonhomogeneous) point processes using a marked PP, where the marks are generated by a Markov chain process defined by a given transition matrix.

**Usage**

```
DepMarkedNHPP.fun(lambdaTot, MarkovM, inival = 1, fixed.seed=NULL)
```

**Arguments**

lambdaTot	Numeric vector. Intensity values of the underlying PP used to generate the dependent processes.
MarkovM	Matrix. Transition probabilities of the $d$ -state Markov chain used to generate the marks of the PP.
inival	Optional. Initial mark value used to generate the series of marks.
fixed.seed	Optional. An integer or NULL. Value used to set the seed in random generation processes; if it is NULL, a random seed is used.

**Details**

Points of the marked PP are generated in continuous time, using the following procedure: First, a trajectory of the underlying PP, which represents the global process of the occurrences in all the processes, is generated. Then, the mark series is generated using a  $d$ -state Markov chain. The mark series takes values in  $1, 2, \dots, d$  and determines in which of the  $d$  processes the point occurs

A transition matrix  $P = (p_{ij})$  with equal rows leads to  $d$  independent point processes, and the more similar the rows of  $P$ , the less dependent the resulting processes. Some dependence measures between the generated processes, such as the spectral gap, are suggested in Abaurrea et al. (2014).

It is noteworthy, that the processes defined by the marks are not Poisson, since the generated marks are dependent observations, see Isham (1980).

**Value**

A list with elements

posNH	Numeric vector of the occurrences times of the underlying PP generated.
mark	Vector of the generated marks, which indicate the process where the point occurs.
lambdaTot	Input argument.
MarkovM	Input argument.

**References**

Abaurrea, J. Asin, J. and Cebrian, A.C. (2014). A Bootstrap Test of Independence Between Three Temporal Nonhomogeneous Poisson Processes and its Application to Heat Wave Modeling. *Environmental and Ecological Statistics*, 22(1), 127-144.

Isham, V. (1980). Dependent thinning of point processes. *J. Appl. Probab.*, 17(4), 987-95.

**See Also**

[DepNHPPqueue.fun](#), [DepNHNeyscot.fun](#), [DepNHCPSF.fun](#), [IndNHPP.fun](#), [SpecGap.fun](#)

**Examples**

```
# Generation of three dependent point processes using a marked PP
set.seed(123)
lambdaTot<-runif(1000)/10

aux<-DepMarkedNHPP.fun(lambdaTot=lambdaTot,
MarkovM=cbind(c(0.3,0.1,0.6), c(0.1, 0.6, 0.3), c(0.6, 0.3,0.1)),fixed.seed=123)
print(cbind(aux$posNH, aux$mark))
```

---

DepNHCPSF.fun	<i>Generates a trajectory of two dependent Poisson processes from a Common Poisson shock process</i>
---------------	--

---

**Description**

This function generates dependent (homogeneous or nonhomogeneous) Poisson processes obtained as the two marginal processes of a bivariate Common Poisson shock process.

**Usage**

```
DepNHCPSF.fun(lambda1, lambda2, lambda12, fixed.seed=NULL)
```

**Arguments**

<code>lambdai1</code>	Numeric vector. Intensity values of $N_{(1)}$ .
<code>lambdai2</code>	Numeric vector. Intensity values of $N_{(2)}$ .
<code>lambdai12</code>	Numeric vector. Intensity values of $N_{(12)}$ .
<code>fixed.seed</code>	Optional. An integer or NULL. Value used to set the seed in random generation processes; if it is NULL, a random seed is used.

**Details**

A bivariate CPSP  $N$  is usually specified by its two marginal, and possibly dependent, processes  $N_1$  and  $N_2$ , which are the observed processes. However,  $N$  can be decomposed into three independent indicator processes:  $N_{(1)}$ ,  $N_{(2)}$  and  $N_{(12)}$ , which are the processes of the points occurring only in the first marginal process, only in the second and in both of them (simultaneous points). The union of  $N_{(1)}$  and  $N_{(12)}$ , and  $N_{(2)}$  and  $N_{(12)}$  gives respectively the two marginal processes.

In a CPSP, the indicator processes are three independent PPs and the CPSP distribution is completely specified by them. The intensity vector of  $N_1$ , is `lambdai1+lambdai12` and the intensity vector of  $N_2$  is `lambdao12+lambdai12`. Conditionally independent marginal processes are obtained if and only if `lambdai12=0`.

The decomposition into indicator processes can be readily applied for data generation, and it reduces to the generation of three independent PPs. Points in each process are generated in continuous time.

**Value**

A list with elements

<code>posNH1</code>	A numeric vector which contains the points in $N_1$
<code>posNH2</code>	A numeric vector which contains the points in $N_2$
<code>posNHi1</code>	A numeric vector which contains the points in $N_{(1)}$
<code>posNHi2</code>	A numeric vector which contains the points in $N_{(2)}$
<code>posNHi12</code>	A numeric vector which contains the points in $N_{(12)}$
<code>lambda1</code>	Numeric vector which contains the intensity vector of $N_1$
<code>lambda2</code>	Numeric vector which contains the intensity vector of $N_2$

**References**

Abaurrea, J. Asin, J. and Cebrian, A.C. (2015). Modeling and projecting the occurrence of bivariate extreme heat events using a nonhomogeneous common Poisson shock process. *Stochastic and Environmental Research and risk assessment*, 29(1), 309-322.

**See Also**

[DepNHneyScot.fun](#), [DepNHPPqueue.fun](#), [DepMarkedNHPP.fun](#)

**Examples**

```

set.seed(123)
lambdai1<-runif(200,0,0.1)
set.seed(124)
lambdai2<-runif(200,0,0.07)
set.seed(125)
lambdai12<-runif(200,0,0.15)

aux<-DepNHCPSF.fun(lambdai1=lambdai1, lambdai2=lambdai2, lambdai12=lambdai12,
fixed.seed=123)

aux$posNH1
aux$posNH2

```

---

DepNHNeyScot.fun	<i>Generates trajectories of dependent Neyman-Scott cluster processes</i>
------------------	---

---

**Description**

This function generates dependent (homogeneous or nonhomogeneous) point processes based on Neyman-Scott processes with the same trajectory of cluster centers.

It calls the auxiliary function GenSons.fun (not intended for the users), see Details section.

**Usage**

```

DepNHNeyScot.fun(lambdaParent, d, lambdaNumP = 1, dist = "normal",
sigmaC = 1, minC = -1, maxC = 1, fixed.seed=NULL)

```

**Arguments**

lambdaParent	Numeric vector. Intensity values of the underlying PP used to generate the centers of the clusters of the Neyman-Scott process.
d	Integer. Number of dependent processes to be generated.
lambdaNumP	Optional. Numeric vector. Mean values of the number of sons of each dependent process. If its length is equal to 1, the same value is used to generate all the dependent processes.
dist	Optional. Label "normal" or "uniform". Distribution used to generate the point locations of each cluster.
sigmaC	Optional. Numeric vector. Only used if dist="normal". Standard deviation of the normal distribution. If its length is equal to 1, the same value is used in the d processes.
minC	Optional. Numeric vector. Only used if dist="uniform". Lower limits of the Uniform distribution. If its length is equal to 1, the same value is used in the d processes.

maxC	Optional. Numeric vector. Only used if dist="uniform". Upper limits of the Uniform distribution. If its length is equal to 1, the same value is used in the $d$ processes.
fixed.seed	Optional. An integer or NULL. Value used to set the seed in random generation processes; if it is NULL, a random seed is used.

### Details

A Neyman-Scott process is a Poisson cluster process where the points in each cluster are randomly distributed around the cluster center.

Dependent homogeneous or NH point processes, in continuous time, are obtained by generating Neyman-Scott processes with the same trajectory of cluster centers. First, the Poisson process of the cluster centers is generated. Then, the number of points in each cluster is generated using a Poisson distribution with means which can be different in each process. The point locations around each center can be generated using two distributions  $N(0, \sigma_C)$  or  $\text{Uniform}(\min C, \max C)$ .

Remark that high values of  $\sigma_C$  or the range  $\max C - \min C$  lead to a high variability around the center and to a low dependence between the processes.

### Value

A list with elements

posNH	A list of $d$ vectors, containing the occurrence points of the $d$ dependent processes. The name of the elements of the list are PP1, PP2...PPd
-------	---

### References

Abaurrea, J. Asin, J. and Cebrian, A.C. (2014). A Bootstrap Test of Independence Between Three Temporal Nonhomogeneous Poisson Processes and its Application to Heat Wave Modeling. *Environmental and Ecological Statistics*.

### See Also

[IndNHNeyScot.fun](#), [DepNHPPqueue.fun](#), [DepMarkedNHPP.fun](#), [DepNHCPSP.fun](#)

### Examples

```
# Generation of three dependent Neyman-Scott processes with the same mean number
#of sons per cluster and locations generated by the same normal distribution
```

```
set.seed(123)
lambdaParent<-runif(100,0,0.1)
```

```
DepNHNeyScot.fun(lambdaParent=lambdaParent, d=3, lambdaNumP = 2,
  dist = "normal", sigmaC = 3, fixed.seed=123)
```

---

DepNHPPqueue.fun	<i>Generates trajectories of dependent Poisson processes based on queue simulation</i>
------------------	--

---

### Description

This function generates  $d$  dependent (homogeneous or nonhomogeneous) Poisson processes. using  $d - 1$  queues in tandem.

### Usage

```
DepNHPPqueue.fun(lambda, d, rate = 1, type="NH", T=NULL, nEv , fixed.seed=NULL)
```

### Arguments

lambda	Numeric value or vector. Intensity of the first PP. If its length is 1, homogeneous PPs are generated.
d	Integer. Number of dependent processes to be generated.
rate	Optional. Numeric value or vector. Parameters (inverse of the mean) of the exponential distributions used to generate the time services, see <a href="#">rexp</a> . If its length is 1, the same parameter is used to generate the time services in the $d - 1$ steps.
type	Optional. Labels "H" or "NH". Type of the processes, homogeneous or nonhomogeneous. If type="H", the length of lambda must be 1.
T	Optional. Positive integer. Length of the period where the point are going to be generated. Only used if type="H" .
nEv	Optional. Positive integer. Number of points to be generated in the PPs. Only used if type="H".
fixed.seed	Optional. An integer or NULL. Value used to set the seed in random generation processes; if it is NULL, a random seed is used.

### Details

The generation of dependent homogeneous PPs is based on Burke s theorem, Burke (1956), which states that in a stationary system formed by a tandem of  $M M 1$  queues with input intensity  $\lambda$  and exponential time services, the input and the output processes of the first queue and the output of the second are three dependent PPs with the same intensity  $\lambda$ .

Points are generated in continuous time, using the following procedure: first the points of a PP and their time services are generated, and their corresponding output times obtained. The output times are the occurrence times of the second process. Using them as input times of the following service and generating new time services, the second output times, which are the occurrence times of the third PP, are obtained. The last step is repeated up to obtain  $d$  dependent PPs.

Analogously, the generation of dependent NHPPs is based on a result by Keilson and Servi (1994), which states that the output process from a  $M(t) G 1$  queue is also a PP with a NH intensity equal to

the convolution  $\lambda_{out}(t) = \lambda_{inp}(t) * f(t)$ , where  $\lambda_{inp}(t)$  is the input intensity and  $f(t)$  the density function of the service time, in this case an exponential distribution.

In the homogeneous processes, the argument  $\lambda$  can be an integer or a vector with equal values. In the first case, the argument nEv must be specified; in the second, the length of the vector determines the length of period where the points are generated (as in the nonhomogeneous case).

## Value

A list with elements

posNHs	A list of $d$ vectors, containing the occurrence points in each PP. The name of the elements of the list are PP1, PP2,..., PPd.
lambdaM	A $d$ -column matrix containing the intensity vectors of the $d$ dependent processes.

## References

Abaurrea, J. Asin, J. and Cebrian, A.C. (2014). A Bootstrap Test of Independence Between Three Temporal Nonhomogeneous Poisson Processes and its Application to Heat Wave Modeling. *Environmental and Ecological Statistics*, 22(1), 127-144.

Burke, P. J. (1956). The Output of a Queuing System. *Operations Research*. 4(6), 699-704.

Keilson, J. Servi, L.D. (1994). Networks of nonhomogeneous M G  $\infty$ . *J. Appl. Probab.*, 31, 157-68.

## See Also

[IndNHPP.fun](#), [DepNHNeyscot.fun](#), [DepMarkedNHPP.fun](#), [DepNHCPSP.fun](#)

## Examples

```
#Generation of 3 dependent HPPs, with mean service time equal to 10
aux<-DepNHPPqueue.fun(lambda=0.05, d=3, rate=0.1, type="H", nEv=25,
fixed.seed=123)
aux$posNHs
```

```
#Generation of 3 dependent NHPPs, with mean service time equal to 10
#lambda<-runif(200,0,0.1)
```

```
#aux<-DepNHPPqueue.fun(lambda=lambda, d=3, rate=0.1, type="NH")
#aux$posNHs
```

---

DistObs.fun	<i>Calculates the set of close points and the mean distance for each point in the first process of a set or two or three processes</i>
-------------	--

---

### Description

Given a set or two or three processes, this function calculates the set of close points and the mean distance for each point in the first process.

It calls the functions `calcdist.fun`, not intended for the users, and `uniongentri.fun`.

### Usage

```
DistObs.fun(posx, posy, posz=NULL, info = FALSE,
            PA = FALSE, procName=c('X', 'Y', 'Z'), ...)
```

### Arguments

posx	Numeric vector. Position of the occurrence points in the first process.
posy	Numeric vector. Position of the occurrence points in the second process.
posz	Optional. Numeric vector. Position of the occurrence points in the third process.
info	Optional logical flag. If it is TRUE, information about the generated points is showed on the screen and dotcharts and bivariate charts of the occurrence points of the processes are displayed.
PA	Optional logical flag. If it is TRUE, the close point relation is broadened by including the previous and the following points to the overlapping intervals.
procName	Vector of character strings. Labels for the first, second and third processes.
...	Further arguments to pass to <code>plot</code> if argument <code>info="TRUE"</code>

### Details

Given a set of two or three point processes, for each point  $t_{x_i}$  in the first process of the set, this function calculates its set of close points and the mean distance to its close points. The definition of set of close points can be found in Abaurrea et al. (2015)) and the distances are defined as  $|t_{y_j} - t_{x_i}|$  if there are two processes and as  $|t_{y_j} - t_{x_i}| + |t_{z_k} - t_{x_i}|$  if there are three.

### Value

DistTri	The vector of the means of the distances of points $t_{x_i}, t_{y_i}, t_{z_i}$ for each x-coordinate
---------	--

### References

Abaurrea, J. Asin, J. and Cebrian, A.C. (2014). A Bootstrap Test of Independence Between Three Temporal Nonhomogeneous Poisson Processes and its Application to Heat Wave Modeling. *Environmental and Ecological Statistics*, 22(1), 127-144.

**See Also**

[TestIndNH.fun](#), [DistSim.fun](#), [uniongentri.fun](#)

**Examples**

```
data(BarTxTn)
dateB<-cbind(BarTxTn$ano,BarTxTn$mes,BarTxTn$diames)
BarBivEv<-CPSPPOTevents.fun(X=BarTxTn$Tx,Y=BarTxTn$Tn,thresX=318,
thresY=220, date=dateB)
DistObs.fun(BarBivEv$PxX, BarBivEv$PxY, BarBivEv$PxXY,info = TRUE)
```

---

DistShift.fun	<i>Shifts, conditionally on the first process, the remaining processes in a set of up to three processes and calculates the set of close points and the mean distance for each point in the first process</i>
---------------	---

---

**Description**

Given a set of up to three processes, this function fixes the first one and shifts the other ones a given distance. Then, it calculates the set of close points and the mean distance for each point  $t_{x_i}$  in the first process. It can be used with homogeneous and non homogeneous processes.

**Usage**

```
DistShift.fun(posx, posy, posz=NULL, T, shii1, shii2=NULL, PA = FALSE, info=FALSE, ...)
```

**Arguments**

posx	Numeric vector. Position of the occurrence points in the first process.
posy	Numeric vector. Position of the occurrence points in the second process.
posz	Optional. Numeric vector. Position of the occurrence points in the third process.
T	Numeric value. Length of the observation period of the processes.
shii1	Numeric value. Distance used to shift the points in the second process. It must be a positive value lower than the length of each lambda vector.
shii2	Optional. Numeric value. Distance used to shift the points in the third process. If there is only two processes it must be NULL. It must be a positive value lower than the length of each lambda vector.
PA	Optional. Logical flag. If it is TRUE, the close point relation is broadened by including the previous and the following points to the overlapping intervals.
info	Optional. Logical flag. If it is TRUE, information about the generated points is shown on the screen and dotcharts and bivariate charts of the occurrence points of the three processes are displayed.
...	Further arguments to pass to <a href="#">plot</a> and to <a href="#">dotchart</a> if the argument info=T

## Details

Given a set of three (or two) point processes, this function aims to transform the set of processes by a translation, conditionally on the first process. The idea of this translation is to keep the distribution of the processes but break any dependence between them, without the need of parametric models to describe the univariate marginal patterns. The only information required is the marginal intensities of the processes.

The key idea, see Lotwick and Silverman (1982), is to wrap the processes onto a torus by identifying opposite sides, The first process is fixed, while the others are translated over the torus. In nonhomogenous processes, translation may change their distribution, and to compensate, the intensity must also be translated.

Then, the function calculates the set of close points and the mean distance for each point  $t_{x_i}$  in the first process, in the new shifted set of processes.

This function is mainly used to develop a Lotwick-Silverman type test to check the independence between the processes, see [TestIndLS.fun](#).

For homogenous processes, the intensity vectors in lambda must be constant (that is all the values in a column must be equal).

## Value

DistTri                    Vector of the mean distance to the points in the set of close points, for each  $t_{x_i}$  in the first process.

## References

Cebrian, A.C. Abaurrea, J. Asin, J. (2014). NHPoisson: An R Package for Fitting and Validating Nonhomogeneous Poisson Processes. *Journal of Statistical Software*.

Lotwick, H.W. and Silverman, B.W. (1982). Methods for analysing Spatial processes of several types of points. *J.R. Statist. Soc. B*, 44(3), pp. 406-13

## See Also

[TestIndLS.fun](#), [DistSim.fun](#)

## Examples

```
set.seed(123)
lambdax<-runif(200, 0.01,0.17)
set.seed(124)
lambday<-runif(200, 0.015,0.15)
set.seed(125)
lambdaz<-runif(200, 0.005,0.1)
posx<-simNHPc.fun(lambda=lambdax, fixed.seed=123)$posNH
posy<-simNHPc.fun(lambda=lambday, fixed.seed=123)$posNH
posz<-simNHPc.fun(lambda=lambdaz, fixed.seed=123)$posNH

DistShift.fun(posx=posx, posy=posy, posz=posz, T=200,
shii1=59, shii2=125 )
```

---

DistSim.fun	<i>Generates a set of up to three independent processes and calculates the set of close points and the mean distance for each point in the first process</i>
-------------	--

---

### Description

Given a point process, this function generates a set of one or two more processes and calculates the set of close points and the mean distance for each point  $t_{x_i}$  in the first process. Two types of processes, Poisson processes (PP) and Neyman-Scott cluster processes (NSP), are implemented up to now. Homogeneous and nonhomogeneous processes can be generated in both cases.

If a seed must be fixed in the generation process, function `DistSimfix.fun` has to be used.

### Usage

```
DistSim.fun(posx, NumProcess=2, type = "Poisson", lambdaMarg = NULL,
lambdaParent = NULL, lambdaNumP=NULL, dist = "normal", sigmaC = 1,
minC = -1, maxC = 1, PA = FALSE,info=FALSE,...)
```

```
DistSimfix.fun(posx, NumProcess=2, type = "Poisson", lambdaMarg = NULL,
lambdaParent = NULL,lambdaNumP=NULL, dist = "normal", sigmaC = 1,
minC = -1, maxC = 1, PA = FALSE,info=FALSE,fixed.seed=1,...)
```

### Arguments

<code>posx</code>	Numeric vector. Position of the occurrence points in the first process.
<code>NumProcess</code>	Optional. Integer equal to 2 or 3, the number of processes involved.
<code>type</code>	Optional. Label "Poisson" or "PoissonCluster". Type of point processes to be generated. Up to now, only two types are available: Poisson processes ("Poisson") and Neyman-Scott cluster processes ("PoissonCluster").
<code>lambdaMarg</code>	Two-column matrix. Only used when <code>type="Poisson"</code> . Each column is the intensity $\lambda(t)$ used to generate the PPs.
<code>lambdaParent</code>	Numeric vector. Only used when <code>type="PoissonCluster"</code> . Intensity values of the underlying PP used to generate the centers of the clusters of the NSP.
<code>lambdaNumP</code>	Numeric vector (length $\leq 2$ ). Only used when <code>type="PoissonCluster"</code> . Mean values of the number of sons of each process. If its length is 1 and <code>NumProcess=2</code> , the same value is used for both processes.
<code>dist</code>	Optional. Label "normal" or "uniform". Only used when <code>type="PoissonCluster"</code> . Distribution used to generate the point locations in each cluster.
<code>sigmaC</code>	Optional. Numeric vector. Only used when <code>type="PoissonCluster"</code> and <code>dist="normal"</code> . Standard deviation of the normal distribution. If its length is 1 and <code>NumProcess=2</code> , the same value is used for both processes.
<code>minC</code>	Optional. Numeric vector. Only used when <code>type="PoissonCluster"</code> and <code>dist="uniform"</code> . Lower limits of the Uniform distribution. If its length is 1 and <code>NumProcess=2</code> , the same value is used for both processes.

maxC	Optional. Numeric vector. Only used when <code>type="PoissonCluster"</code> and <code>dist="uniform"</code> . Upper limits of the Uniform distribution. If its length is 1 and <code>NumProcess=2</code> , the same value is used for both processes.
PA	Optional. Logical flag. If it is TRUE, the close point relation is broadened by including the previous and the following points to the overlapping intervals.
info	Optional. Logical flag. If it is TRUE, information about the generated points is shown on the screen and dotcharts and bivariate charts of the occurrence points of the three processes are displayed.
fixed.seed	Optional. Only available in <code>DistSimfix.fun</code> . Integer value used to set the seed in random generation procedures.
...	Further arguments to pass to <code>plot</code> and to <code>dotchart</code> if the argument <code>info=T</code>

### Details

Up to now, generation of two types of processes (Poisson, "Poisson", and Neyman-Scott cluster processes, "PoissonCluster") is available. Generation of NHPPs is done using the inversion approach, see Cebrian et al. (2014) and `simNHPC.fun`. For generation of NSPs, see `IndNHNeyScot.fun`

The only difference between `DistSim.fun` and `DistSimfix.fun` is that the first one uses a random seed while in the second one a seed is set by the argument `fixed.seed`.

This function is mainly used in `TestIndNH.fun`.

The length of the period where the processes are generated is determined by the length of the argument `lambdaParent` or the number of rows of `lambdaMarg`. Homogenous processes are generated if the intensity vectors in `lambdaParent` or in `lambdaMarg` are constant (that is if all the values in the vector are equal).

### Value

`DistTri` Vector of the mean distance to the points in the set of close points, for each  $t_{x_i}$  in the first process.

### References

Abaurrea, J. Asin, J. and Cebrian, A.C. (2014). A Bootstrap Test of Independence Between Three Temporal Nonhomogeneous Poisson Processes and its Application to Heat Wave Modeling. Environmental and Ecological Statistics.

Cebrian, A.C. Abaurrea, J. Asin, J. (2014). NHPoisson: An R Package for Fitting and Validating Nonhomogeneous Poisson Processes. Journal of Statistical Software. To appear.

### See Also

`TestIndNH.fun`, `DistObs.fun`, `IndNHNeyScot.fun`, `simNHPC.fun`

### Examples

```
#Observed process: PP; simulated processes: two independent PPs
```

```

set.seed(123)
lambdax<-runif(200, 0.01,0.15)
set.seed(124)
lambday<-runif(200, 0.005,0.1)
set.seed(125)
lambdaz<-runif(200, 0.005,0.2)
posaux<-simNHPc.fun(lambda=lambdax, fixed.seed=123)$posNH

DistSimfix.fun(posx=posaux, type = "Poisson",
lambdaMarg = cbind(lambday,lambdaz), fixed.seed=123)
#DistSim.fun(posx=posaux, type = "Poisson",
# lambdaMarg = cbind(lambday,lambdaz))

```

---

DutilleulPlot.fun      *Performs a Diggle's randomization testing procedure to check independence between two point processes*

---

## Description

This function applies the Diggle's randomization testing procedure extended by Dutilleul(2011) and performs a plot which checks graphically the independence of two point processes. It is implemented for homogenous and non homogenous processes.

## Usage

```
DutilleulPlot.fun(posx, posy, lambday, nsim = 1000, lenve = c(0.025, 0.975), ...)
```

## Arguments

posx	Numeric vector. Position of the occurrence points of the first point process.
posy	Numeric vector. Position of the occurrence points of the second point process.
lambday	Numeric vector. Intensity values of the second point process
nsim	Optional. Positive integer. Number of simulations performed to obtain the confidence bands.
lenve	Optional. Numeric vector. Lower and the upper percentiles which determine the limits of the confidence band.
...	Further arguments to be passed to <a href="#">plot</a> .

## Details

This is a procedure to check graphically the independence of two point processes. It is based on the comparison of the cumulative relative frequency of the nearest neighbour distances between the points in the two observed processes, with their counterpart in two independent processes with the same intensities. The procedure consists on plotting the cumulative relative frequency of the observed processes and a confidence band calculated from *nsim* simulated independent processes.

**Value**

A list with the elements used in the plot

quantobs        Vector of observed percentiles of the nearest neighbour distances.  
 enve1            Vector of lower limits of the confidence band.  
 enve2            Vector of upper limits of the confidence band.

**References**

Dutilleul, P. (2011), *Spatio-temporal heterogeneity: Concepts and analyses*, Cambridge University Press.

**See Also**

[TestIndNH.fun](#), [CondTest.fun](#), [nearestdist.fun](#)

**Examples**

```
#Two independent NHPPs
set.seed(123)
lambdax<-runif(200, 0.01,0.1)
set.seed(124)
lambday<-runif(200, 0.015,0.15)
posx<-simNHPPc.fun(lambdax, fixed.seed=123)$posNH
posy<-simNHPPc.fun(lambday, fixed.seed=123)$posNH

aux<-DutilleulPlot.fun(posx, posy, lambday, nsim = 100)

#Two dependent NSPs
#set.seed(123)
#lambdaParent<-runif(200)/10
#DepPro<-DepNHPPc.fun(lambdaParent=lambdaParent, d=2, lambdaNumP = 3,
# dist = "normal", sigmaC = 3, fixed.seed=123)
#posx<-DepPro$PP1
#posy<-DepPro$PP2
#aux<-DutilleulPlot.fun(posx, posy, lambday, nsim = 100)
```

---

IndNHNeyScot.fun        *Generates trajectories of independent Neyman-Scott cluster processes*

---

**Description**

This function generates independent (homogeneous or nonhomogeneous) Neyman-Scott cluster processes with independent trajectories of cluster centers with the same intensity.

It calls the auxiliary function `GenSons.fun` (not intended for the users), see `Details` section.

**Usage**

```
IndNHNeyScot.fun(lambdaParent, d, lambdaNumP = 1, dist = "normal",
  sigmaC = 1, minC = -1, maxC = 1, fixed.seed=NULL)
```

**Arguments**

lambdaParent	Numeric vector. Intensity values of the underlying PP used to generate the centers of the clusters of the Neyman-Scott process.
d	Integer. Number of independent processes to be generated.
lambdaNumP	Optional. Numeric vector. Mean values of the number of sons of each dependent process. If its length is equal to 1, the same value is used to generate all the dependent processes.
dist	Optional. Label "normal" or "uniform". Distribution used to generate the point locations of each cluster.
sigmaC	Optional. Numeric vector. Standard deviation of the normal distribution. Only used if dist="normal". If its length is equal to 1, the same value is used in the d processes.
minC	Optional. Numeric vector. Lower limits of the Uniform distribution. Only used if dist="uniform". If its length is equal to 1, the same value is used in the d processes.
maxC	Optional. Numeric vector. Upper limits of the Uniform distribution. Only used if dist="uniform". If its length is equal to 1, the same value is used in the d processes.
fixed.seed	Optional. An integer or NULL. Value used to set the seed in random generation processes; if it is NULL, a random seed is used.

**Details**

A Neyman-Scott process is a Poisson cluster process where the points in each cluster are randomly distributed around the cluster center. For generating each process, an independent trajectory of the Poisson process of the cluster centers is generated first. Then, the number of points in each cluster is generated using a Poisson distribution with mean value  $\mu_{P_i}$  ( $i=1,\dots,d$ ). Finally, the point locations around each center can be generated using two distributions  $N(0, \text{sigmaC})$  or  $\text{Uniform}(\text{minC}, \text{maxC})$ .

The length of the period where the processes are generated is determined by the length of the argument lambdaParent.

Homogenous processes are generated if the intensity vector lambdaParent is constant (that is if all the values are equal).

**Value**

A list with elements

posNH	A list of d vectors, each one containing the time occurrences of one of the dependent processes. The name of the elements of the list are PP1, PP2...PPd
-------	--

## References

Abaurrea, J. Asin, J. and Cebrian, A.C. (2014). A Bootstrap Test of Independence Between Three Temporal Nonhomogeneous Poisson Processes and its Application to Heat Wave Modeling. *Environmental and Ecological Statistics*.

## See Also

[DepNHNeyscot.fun](#), [IndNHPP.fun](#)

## Examples

```
set.seed(123)
lambda<-runif(1000)/10
```

```
IndNHNeyscot.fun(lambdaParent=lambda, d=3, lambdaNumP = c(2,3,2), dist = "normal",
sigmaC = 2, fixed.seed=123)
```

---

IndNHPP.fun	<i>Generates trajectories of independent Poisson processes</i>
-------------	--

---

## Description

This function generates independent Poisson processes (PPs), which can be homogeneous or non-homogeneous depending on the value of the intensity vectors.

## Usage

```
IndNHPP.fun(lambdas, fixed.seed=NULL)
```

## Arguments

lambdas	Matrix where each column contains the intensity vector to generate a Poisson process
fixed.seed	An integer or NULL. If it is an integer, that is the value used to set the seed in random generation processes. If it is NULL, a random seed is used.

## Details

The number of generated processes is determined by the number of columns of the argument lambdas. The length of the period where the processes are generated is determined by the number of rows of lambdas.

Homogenous processes are generated if the corresponding intensity vector is constant (that is if all the rows of the corresponding column are equal).

For the generation algorithm of each PP, see [simNHPC.fun](#).

**Value**

posNHsA list of d vectors, each one containing the time occurrences of the independent NHPP. The name of the elements of the list are PP1, PP2...PPd

**See Also**

[IndNHNeyscot.fun](#), [simNHPc.fun](#)

**Examples**

```
set.seed(123)
lambdas<-cbind(runif(500)/10, rep(0.05,500))

IndNHPP.fun(lambdas=lambdas, fixed.seed=123)
```

---

nearestdist.fun	<i>Calculates the distance to the nearest point in the second process for each point in the first process</i>
-----------------	---

---

**Description**

Given the occurrence points in two point processes, this function calculates for each point in the first process, the distance to the nearest occurrence point in the second process.

**Usage**

```
nearestdist.fun(posx, posy)
```

**Arguments**

posx	Numeric vector. Position of the occurrence points in the first point process.
posy	Numeric vector. Position of the occurrence points in the second point process.

**Details**

The distance between two points  $x_i$  and  $y_i$  in time, is the absolute value of their difference:  $|x_i - y_i|$ . To obtain the vector of nearest points, this function applies to each point in posx, the function `pdist.fun`, which calculates the distance to its nearest point in posy.

**Value**

Vector of distances to the nearest point in the second process for each point in the first process.

**See Also**

[DutilleulPlot.fun](#)

**Examples**

```
posx<-c(3,8,23,54,57,82)
posy<-c(2,8,14,16,29,32,45,55,65)
nearestdist.fun(posx, posy)
```

---

NHD.fun

*It estimates the cross D-function for two sets of point processes*


---

**Description**

This function estimates the cross nearest neighbour distance distribution function,  $D$ , between two sets,  $C$  and  $D$ , of (homogenous or nonhomogeneous) point processes. The  $D$ -function is evaluated in a grid of values  $r$ , and it can be optionally plotted.

It calls the auxiliary functions NHDaux.fun and other functions, not intended for users.

**Usage**

```
NHD.fun(lambdaC, lambdaD, T=NULL, Ptype='inhom', posC, typeC=1, posD, typeD=1,
r = NULL, dplot = TRUE, tit = "D(r)")
```

**Arguments**

lambdaC	A matrix. Each column is the intensity vector of one of the point processes in $C$ . If there is only one process in $C$ , it can be a vector or even a numeric value if the process is homogeneous.
lambdaD	A matrix. Each column is the intensity vector of one of the point process in $D$ . If there is only one process in $D$ , it can be a vector or even a numeric value if the process is homogeneous.
T	Numeric value. Length of the observation period. It only must be specified if all the processes are homogeneous the number of rows in lambdaC and lambdaD is 1.
Ptype	Optional. Label: 'hom' or 'inhom'. The first one indicates that all the point processes in sets $C$ and $D$ are homogeneous. In that case, columns of arguments lambdaC and lambdaD can be a number or a constant vector.
posC	Numeric vector. Time position of the points in all the point processes in $C$ .
typeC	Numeric vector with the same length as posC. Code of the point process in $C$ where points in posC have occurred. See Details.
posD	Numeric vector. Time position of the points in all the point processes in $D$ .
typeD	Numeric vector with the same length as posD. Code of the point process in $D$ where points in posD have occurred.
r	Numeric vector. Values where $D$ -function must be evaluated.
dplot	Optional. A logical flag. If it is TRUE, a plot of the $D$ -function is shown.
tit	Optional. The title to be used in the plot of the $D$ -function.

## Details

This function estimates the cross nearest neighbour distance distribution function  $D$ , between two sets  $C$  and  $D$  of point processes, using the estimator suggested by Cronie and van Lieshout (2015), adapted to time point processes.

The  $D$ -function is the distribution function of the distances from a point in a process in  $C$  to the nearest point in a process in  $D$ . In homogeneous processes, it estimates the probability that at least one point in a process in set  $D$  occurs at a distance lower than  $r$  of a given point in a process in set  $C$ . If the processes are nonhomogenous, the inhomogenous version of the function, adjusted for time varying intensities, is used. It is calculated using the Hanisch estimator, see Van Lieshout (2006)

Small values of cross  $D$ -function suggest few points in processes in  $D$  in the  $r$ -neighbourhood of points of processes in  $C$ . Large values indicate that points in processes in  $D$  are attracted by those of processes in  $C$ .

For inference about independence of the processes,  $K$  and  $J$ -functions should be used.

The occurrence points in all the processes in  $C$  must be part of the input. Since the number of points will be possibly different in each process, a matrix cannot be used. Instead two vectors with the same length are used: the first one contains the occurrence points in all the processes while the second one indicates the point process in  $C$  where the point in the same row in `posC` has occurred. The codes used in `typeC` are the column number where the intensity of that process is in matrix `lambdaC`. The same for set  $D$ .

See [NHJ.fun](#) for details on default values of  $r$  and  $L$ .

## Value

A list with elements

<code>r</code>	Vector of values where $D$ -function is evaluated.
<code>NHDr</code>	Estimations of $D$ -function at values $r$ .
<code>T</code>	Length of the observation period.

## References

Cronie, O. and van Lieshout, M.N.M. (2015). Summary statistics for inhomogeneous marked point processes. *Ann Inst Stat Math*. DOI 10.1007/s10463-015-0515-z

Stoyan et al (2001). On the estimation of distance distribution functions for points processes and random sets. *Image Anal Stereol*, 20, 65-69

Van Lieshout, M.N.M. (2006) A  $J$ -function for marked point patterns. *AISM*, 58, 235-259. DOI 10.1007/s10463-005-0015-7

## See Also

[NHK.fun](#), [NHJ.fun](#), [NHf.fun](#)

**Examples**

```

#Sets C and D with one independent NHPP
set.seed(123)
lambda1<-runif(500, 0.05, 0.1)
set.seed(124)
lambda2<-runif(500, 0.01, 0.2)
pos1<-simNHPC.fun(lambda=lambda1, fixed.seed=123)$posNH
pos2<-simNHPC.fun(lambda=lambda2, fixed.seed=123)$posNH

aux<-NHD.fun(lambdaC=lambda1, lambdaD=lambda2, posC=pos1, typeC=1, posD=pos2, typeD=1)
aux$NHDr

#Sets C and D with two independent NHPPs
#pos3<-simNHPC.fun(lambda=lambda1, fixed.seed=321)$posNH
#pos4<-simNHPC.fun(lambda=lambda2, fixed.seed=321)$posNH

#NHD.fun(lambdaC=cbind(lambda1,lambda2), lambdaD=cbind(lambda1,lambda2), posC=c(pos1,pos2),
# typeC=c(rep(1, length(pos1)), rep(2, length(pos2))), posD=c(pos3, pos4),
# typeD=c(rep(1, length(pos3)), rep(2, length(pos4))))

```

---

NHF.fun

*It estimates the cross F-function for two sets of point processes*


---

**Description**

This function estimates the cross F-function, between two sets,  $C$  and  $D$ , of (homogenous or non-homogeneous) point processes. The F-function is evaluated in a grid of values  $r$ , and it can be optionally plotted.

It calls the auxiliary functions NHFaux.fun and other functions not intended for users.

**Usage**

```
NHF.fun(lambdaD, T=NULL, Ptype='inhom', posD, typeD=1, r=NULL, L=NULL, dplot=TRUE,
tit='F(r)')
```

**Arguments**

lambdaD	A matrix. Each column is the intensity vector of one of the point process in $D$ . If there is only one process in $D$ , it can be a vector or even a numeric value if the process is homogeneous.
T	Numeric value. Length of the observation period. It only must be specified if all the processes are homogeneous the number of rows in lambdaC and lambdaD is 1.

Ptype	Optional. Label: 'hom' or 'inhom'. The first one indicates that all the point processes in sets $C$ and $D$ are homogeneous. In that case, columns of arguments <code>lambdaC</code> and <code>lambdaD</code> can be a number or a constant vector.
posD	Numeric vector. Time position of the points in all the point processes in $D$ .
typed	Numeric vector with the same length as <code>posD</code> . Code of the point process in $D$ where the point in the same row in <code>posD</code> has occurred. The code must be the column number where the intensity of that process is in matrix <code>lambdaD</code> .
r	Numeric vector. Values where F-function must be evaluated.
L	Optional. Numeric vector. Net of values during the observation period used to calculate the F-function. If the value is NULL, a default vector, formed by all the integers in the observation period, is used
dp1ot	Optional. Logical flag. If it is true, the plot of the F-function is performed.
tit	Optional. The title to be used in the plot of the F-function.

### Details

This function estimates the cross F-function for the processes in set  $D$ , using the estimator suggested by Cronie and van Lieshout (2015), adapted to time point processes.

The cross F-function, also known as empty space function, is the distribution function of the distances from an arbitrary point in the space to the nearest point in a process in  $D$ . In homogeneous processes, it estimates the probability that at least one point in processes in  $D$  occurs at a distance lower than  $r$  of an arbitrary point in the space. If the processes are nonhomogeneous, the inhomogeneous version of the function, adjusted for time varying intensities, is used.

See [NHJ.fun](#) for details on default values of  $r$  and  $L$ .

The occurrence points in all the processes in  $D$  must be part of the input. Since the number of points will be possibly different in each process, a matrix cannot be used. Instead two vectors with the same length are used: the first one contains the occurrence points in all the processes while the second one indicates the point process in  $D$  where the point in the same row in `posD` has occurred. The codes used in `typed` are the column number where the intensity of that process is in matrix `lambdaD`.

### Value

A list with elements

r	Values $r$ where the F-function is evaluated.
NHFr	Estimated values of the F-function evaluated at values $r$ .
T	Length of the observation period of the process.
L	Net of values in the observation period used to calculate the F-function.

### References

- Cronie, O. and van Lieshout, M.N.M. (2015). Summary statistics for inhomogeneous marked point processes. *Ann Inst Stat Math*. DOI 10.1007/s10463-015-0515-z
- Stoyan et al (2001). On the estimation of distance distribution functions for points processes and random sets. *Image Anal Stereol*, 20, 65-69

Van Lieshout, M.N.M. (2006) A J-function for marked point patterns. *AISM*, 58, 235-259. DOI 10.1007/s10463-005-0015-7

### See Also

[NHK.fun](#), [NHJ.fun](#), [NHD.fun](#)

### Examples

```
set.seed(123)
lambda1<-runif(500, 0.05, 0.1)
pos1<-simNHPC.fun(lambda=lambda1, fixed.seed=123)$posNH

aux<-NHf.fun(lambdaD=lambda1, posD=pos1, typeD=1)
aux$NHFr

#Set D with two processes
#lambda2<-runif(1000, 0.01, 0.2)
#pos2<-simNHPC.fun(lambda=lambda2, fixed.seed=123)$posNH
#NHf.fun(lambdaD=cbind(lambda1,lambda2), posD=c(pos1,pos2),
# typeD=c(rep(1, length(pos1)), rep(2, length(pos2))) )
```

---

NHJ.fun

*It estimates the cross J-function for two sets of point processes*

---

### Description

This function estimates the cross J-function between two sets,  $C$  and  $D$ , of (homogenous or non-homogeneous) point processes. The J-function is evaluated in a grid of values  $r$ , and it can be optionally plotted. An independence test based on J-function is also available.

It calls the auxiliary functions NHJaux.fun and Jenv, not intended for users.

### Usage

```
NHJ.fun(lambdaC, lambdaD, T=NULL, Ptype='inhom', posC, typeC=1, posD, typeD=1, r = NULL,
L = NULL, test=FALSE, nTrans=100, rTest=20, conf=0.95, dplot = NULL,
tit = rep("J-function", 3), mfrow = c(1, 1), cores=1, fixed.seed=NULL)
```

### Arguments

lambdaC	A matrix. Each column is the intensity vector of one of the point processes in $C$ . If there is only one process in $C$ , it can be a vector or even a numeric value if the process is homogeneous.
lambdaD	A matrix. Each column is the intensity vector of one of the point process in $D$ . If there is only one process in $D$ , it can be a vector or even a numeric value if the process is homogeneous.

T	Numeric value. Length of the observation period. It only must be specified if all the processes are homogeneous, that is if the number of rows in <code>lambdaC</code> and <code>lambdaD</code> is 1.
Ptype	Optional. Label: 'hom' or 'inhom'. The first one indicates that all the point processes in sets $C$ and $D$ are homogeneous. In that case, columns of arguments <code>lambdaC</code> and <code>lambdaD</code> can be a number or a constant vector.
posC	Numeric vector. Time position of the points in all the point processes in $C$ .
typeC	Numeric vector with the same length as <code>posC</code> . Code of the point process in $C$ where the points in <code>posC</code> have occurred. See Details
posD	Numeric vector. Time position of the points in all the point processes in $D$ .
typeD	Numeric vector with the same length as <code>posD</code> . Code of the point process in $D$ where the points in <code>posD</code> have occurred.
r	Optional. Numeric vector. Values where J-function must be evaluated. If it is NULL, a default vector is used, see Details
L	Optional. Numeric vector. Net of values in the observation period used to calculate the J-function. If it is NULL, a default vector is used, see Details.
test	Optional. Logical flag. If it is TRUE, a test to check the independence and a 95% envelope for the J-function are calculated.
nTrans	Optional. Numeric value. Only used if <code>test=TRUE</code> . Number of translations to be performed in the test and envelope calculation.
rTest	Optional. Numeric value. Maximum value of $r$ used to calculate the independence test statistic, see Details.
conf	Optional. Numeric value in (0,1). Confidence level of the envelope for the K-function.
dplot	Optional. Label 'JDF' or 'J'. If it is equal to 'JDF', plots of J, D and F-functions are displayed. If it is 'J', only J-function is plotted.
tit	Optional. A vector with one or three titles to be used in the plots of J, D and F-functions.
mfrow	Optional. Argument to be passed to <code>par</code> for the plot of the J-function.
cores	Optional. Number of cores of the computer to be used in the calculations.
fixed.seed	An integer or NULL. If it is an integer, that is the value used to set the seed in random generation processes. If it is NULL, a random seed is used.

## Details

This function estimates the cross J-function between two sets,  $C$  and  $D$ , of homogeneous or nonhomogeneous time point processes, using a version of the spatial estimator suggested by Cronie and van Lieshout (2015) adapted to time processes. In the case of nonhomogeneous processes, the estimator is only defined for second order intensity-reweighted stationary point processes. J-function measures the interpoint dependence between points in any of the processes in  $D$ , and points in any of the processes in  $C$ , adjusted for time varying intensity in the case of nonhomogeneous processes.

The cross J-function is defined as  $J_{CD}(r) = (1 - D_{CD}(r))/(1 - F_D(r))$ , if  $F_D(r) < 1$  and it is not calculated otherwise. It compares  $D(r)$ , the distribution function of the distances from a point

in any of the processes in set  $C$  to the nearest point in any of the processes in set  $D$ , to  $F(r)$ , the distribution function of the distances from a fixed point in the space to the nearest point in any of the processes in set  $D$ .

If the processes in  $C$  are independent of the processes in  $D$  (conditionally on the marginal structure of the processes) J-funtion is equal to 1, since  $D(r)=F(r)$ . Hence, deviations of  $J(r)$  estimations from 1, suggest dependence between the two sets of processes.

If argument  $r$  is NULL, it is calculated as

```
r1<-max(20, floor(T/20))
r<-seq(1,r1,by=2)
if (length(r)>200) r<-seq(1,r1,length.out=200)
```

If argument  $L$  is NULL, it is calculated as

```
L<-seq(1, T, by=2)
if (length(L)>200) L<-seq(1,T,by = round((T - 1)/199))
```

Testing independence

In order to test the independence hypothesis using this function, a Lotwick-Silverman type test is available. This test provides a nonparametric way to test independence when the study area is rectangular. The key idea is to wrap the processes onto a torus by identifying opposite sides, keeping the processes in set  $C$  fixed and translating the processes in set  $D$  randomly over the torus. In non-homogenous processes, random translation may change their distribution, and to compensate, the intensity must also be translated. The considered translations keep the distribution of the processes in  $D$  but break any dependence between them, without the need of parametric models to describe the univariate marginal patterns. Critical values are estimated by bootstrap methods. Using this approach, not only the p-value of a test to check independence but also an envelope for  $J(r)$  values is calculated. The test is based on the statistic  $max_r(abs(J(r) - 1))$ .

One disadvantage of the proposed test is that it may be quite sensitive to the values of  $r$ . In addition, in time processes, dependence often appears between close observations, and with high  $r$  values it is more difficult that the J-function is able to discriminate between dependent and independent processes. By this reason, the argument  $rTest$  allows us to fix a maximum value of  $r$  and only  $J(r)$  estimations for  $r < rTest$  will be used to calculate the test statistic. The value  $rTest$  is drawn in the plot of the J-function as a vertical grey line, in order to help us to identify an adequate value.

The occurrence points in all the processes in  $C$  must be part of the input. Since the number of points will be possibly different in each process, a matrix cannot be used. Instead two vectors with the same length are used: the first one contains the occrence points in all the processes while the second one indicates the point process in  $C$  where the point in the same row in  $posC$  has occurred. The codes used in  $typeC$  are the column number where the intensity of that process is in matrix  $lambdaC$ . The same for set  $D$ .

## Value

A list with elements

$r$	Vector of values $r$ where J-function is evaluated.
NHJr	Estimated values of function $J(r)$ .
NHDr	Estimated values of function $D(r)$ .

NHFr	Estimated values of function $J(r)$ .
JenvL	Lower limits of the envelope for $J(r)$ .
JenvU	Upper limits of the envelope for $J(r)$ .
JStatOb	Observed value of the statistic used to test the independence assumption.
JStatTr	Sample of the values of the test statistic obtained by random translation.
pv	P-value of the independence test.
T	Length of the observation period of the process.
L	Net of values $L$ used to calculate F-funtion.

## References

- Cronie, O. and van Lieshout, M.N.M. (2015). Summary statistics for inhomogeneous marked point processes. *Ann Inst Stat Math*. DOI 10.1007/s10463-015-0515-z
- Lotwick, H.W. and Silverman, B.W. (1982). Methods for analysing Spatial processes of several types of points. *J.R. Statist. Soc. B*, 44(3), pp. 406-13
- Stoyan et al (2001). On the estimation of distance distribution functions for points processes and random sets. *Image Anal Stereol*, 20, 65-69
- Van Lieshout, M.N.M. (2006) A J-function for marked point patterns. *AISM*, 58, 235-259. DOI 10.1007/s10463-005-0015-7

## See Also

[NHK.fun](#), [NHD.fun](#), [NHf.fun](#)

## Examples

```
set.seed(122)
lambda1<-runif(100, 0.05, 0.1)
set.seed(121)
lambda2<-runif(100, 0.01, 0.2)
pos1<-simNHPC.fun(lambda=lambda1, fixed.seed=123)$posNH
pos2<-simNHPC.fun(lambda=lambda2, fixed.seed=123)$posNH

aux<-NHJ.fun(lambdaC=lambda1, lambdaD=lambda2, posC=pos1, nTrans=50,
  posD=pos2, rTest=7, dplot='J', cores=1, test=TRUE)
aux$pv

#Sets with two processes
#pos3<-simNHPC.fun(lambda=lambda1, fixed.seed=321)$posNH
#pos4<-simNHPC.fun(lambda=lambda2, fixed.seed=321)$posNH
#aux<-NHJ.fun(lambdaC=cbind(lambda1, lambda2), lambdaD=cbind(lambda1, lambda2),
# posC=c(pos1, pos2), typeC=c(rep(1, length(pos1)), rep(2, length(pos2))),
# posD=c(pos3, pos4), typeD=c(rep(1, length(pos3)), rep(2, length(pos4))),
# dplot='J', test=TRUE)
#aux$pv
```

NHK.fun

*It estimates the cross K-function for two sets of point processes***Description**

This function estimates the cross K-function between two sets,  $C$  and  $D$ , of (homogenous or non-homogeneous) time point processes. It is evaluated in a grid of distances  $r$ , and it can be optionally plotted. An independence test based on K-function is also available.

It calls the auxiliary functions NHKaux.fun, NHKaux2.fun, NHKaux3.fun and Kenv, not intended for users.

**Usage**

```
NHK.fun(lambdaC, lambdaD, T=NULL, posC, typeC=1, posD, typeD=1, r=NULL,
        test=TRUE, nTrans=100, conf=0.95, rTest=20, dplot=FALSE,
        tit="K-function", cores=1, fixed.seed=NULL)
```

**Arguments**

lambdaC	A matrix. Each column is the intensity vector of one of the point processes in $C$ . If there is only one process in $C$ , it can be a vector or even a numeric value if the process is homogeneous.
lambdaD	A matrix. Each column is the intensity vector of one of the point process in $D$ . If there is only one process in $D$ , it can be a vector or even a numeric value if the process is homogeneous.
T	Numeric value. Length of the observation period. It only must be specified if all the processes are homogeneous, that is if the number of rows in lambdaC and lambdaD is 1.
posC	Numeric vector. Time position of the points in all the point processes in $C$ .
typeC	Numeric vector with the same length as posC. Code of the point process in $C$ where the points in posC have occurred. See Details.
posD	Numeric vector. Time position of the points in all the point processes in $D$ .
typeD	Numeric vector with the same length as posD. Code of the point process in $D$ where the points in posD have occurred.
r	Optional. Numeric vector. Values where K-function must be evaluated. If it is NULL, a default vector is used, see Details
test	Optional. Logical flag. If it is TRUE, a test to check the independence and a 95% envelope for the K-function are calculated.
nTrans	Optional. Numeric value. Only used if test=TRUE. Number of translations to be performed in the test and envelope calculation.
conf	Optional. Numeric value in (0,1). Confidence level of the envelope for the K-function.
rTest	Optional. Numeric value. Maximum value of $r$ used to calculate the independence test statistic, see Details.

<code>dplot</code>	Optional. A logical flag. If it is true a plot of the K-function is shown.
<code>tit</code>	Optional. Title to be used in the plot of the K-function.
<code>cores</code>	Optional. Number of cores of the computer to be used in the calculations.
<code>fixed.seed</code>	An integer or NULL. If it is an integer, that is the value used to set the seed in random generation processes. If it is NULL, a random seed is used.

## Details

This function estimates the cross K-function between two sets,  $C$  and  $D$ , of (homogenous or non-homogeneous) time point processes, using a version of the spatial estimator suggested by Moller and Waagepetersen (2007), p662, adapted to cross time processes; the edge correction suggested in that paper is also used. In the case of nonhomogeneous processes, the estimator is only defined for second order intensity-reweighted stationary point processes. Homogenous processes are generated if the intensity vector `lambda` is constant (that is if all the values are equal) or its length is 1.

K-function measures the degree of dependence between two point processes (or two sets of point processes) and counts the expected number of points in any of the processes in  $D$ , within a given distance of a point in any of the processes in  $C$ , adjusted for time varying intensity in the case of nonhomogenous processes.

The cross K-function of independent Poisson processes (conditionally on the marginal structure of the processes) is the length of the considered intervals,  $K_{CD}(r) = 2r + 1$ . Then, values  $K_{CD}(r) > 2r + 1$  indicate attraction between the processes, while values lower than  $2r + 1$  indicate repulsion.

If argument `r` is NULL, it is calculated as

```
r1<-max(20, floor(T/20))
r<-seq(1,r1,by=2)
if (length(r)>200) r<-seq(1,r1,length.out=200)
```

### Testing independence

In order to test the independence hypothesis using this function, a Lotwick-Silverman type test is available. This test provides a nonparametric way to test independence when the study area is rectangular. The key idea is to wrap the processes onto a torus by identifying opposite sides, keeping the processes in set  $C$  fixed and translating the processes in set  $D$  randomly over the torus. In non-homogenous processes, random translation may change their distribution, and to compensate, the intensity must also be translated. The considered translations keep the distribution of the processes in  $D$  but break any dependence between them, without the need of parametric models to describe the univariate marginal patterns. Critical values are estimated by bootstrap methods. Using this approach, not only the p-value of a test to check independence but also an envelope for the  $K(r)$  values is calculated. The test is based on the statistic  $\max_r(K(r)/(2r + 1))$ .

One disadvantage of the proposed test is that it may be quite sensitive to the values of  $r$ . In addition, in time processes, dependence often appears between close observations, and with high  $r$  values it is more difficult that the K-function is able to discriminate between dependent and independent processes. By this reason, the argument `rTest` allows us to fix a maximum value of  $r$  and only  $K(r)$  estimations for  $r < rTest$  will be used to calculate the test statistic. The value `rTest` is drawn in the plot of the K-function as a vertical grey line, in order to help us to identify an adequate value.

The occurrence points in all the processes in  $C$  must be part of the input. Since the number of points will be possibly different in each process, a matrix cannot be used. Instead two vectors with the same length are used: the first one contains the occurrence points in all the processes while the

second one indicates the point process in  $C$  where the point in the same row in `posC` has occurred. The codes used in `typeC` are the column number where the intensity of that process is in matrix `lambdaC`. The same for set  $D$ .

### Value

A list with elements

<code>r</code>	Vector of values $r$ where K-function is evaluated.
<code>NHKr</code>	Estimated values of function $K(r)$ .
<code>KenvL</code>	Lower limits of the envelope for $K(r)$ .
<code>KenvU</code>	Upper limits of the envelope for $K(r)$ .
<code>KStatOb</code>	Observed value of the statistic used to test the independence assumption.
<code>KStatTr</code>	Sample of the values of the test statistic obtained by random translation.
<code>pv</code>	P-value of the independence test.
<code>T</code>	Length of the observation period of the process.

### References

Cronie, O. and van Lieshout, M.N.M. (2015). Summary statistics for inhomogeneous marked point processes. *Ann Inst Stat Math*. DOI 10.1007/s10463-015-0515-z

Lotwick, H.W. and Silverman, B.W. (1982). Methods for analysing Spatial processes of several types of points. *J.R. Statist. Soc. B*, 44(3), pp. 406-13

Moller, J. and Waagepetersen, R.P. (2007). Modern statistics for spatial point processes. *Scandinavian Journal of Statistics*, 34(4), 643 684

### See Also

[NHD.fun](#), [NHJ.fun](#), [NHF.fun](#)

### Examples

```
set.seed(122)
lambda1<-runif(100, 0.05, 0.1)
set.seed(121)
lambda2<-runif(100, 0.01, 0.2)
pos1<-simNHPc.fun(lambda=lambda1, fixed.seed=123)$posNH
pos2<-simNHPc.fun(lambda=lambda2, fixed.seed=123)$posNH

aux<-NHK.fun(lambdaC=lambda1, lambdaD=lambda2, posC=pos1, posD=pos2, rTest=7,
  dplot=TRUE,tit="K(r).", nTrans=75, cores=1)
aux$pv

#Sets with two processes
#pos3<-simNHPc.fun(lambda=lambda1, fixed.seed=321)$posNH
```

```
#pos4<-simNHPC.fun(lambda=lambda2, fixed.seed=321)$posNH
#aux<-NHK.fun(lambdaC=cbind(lambda1,lambda2), lambdaD=cbind(lambda1,lambda2), posC=c(pos1,pos2),
# typeC=c(rep(1, length(pos1)), rep(2, length(pos2))), posD=c(pos3, pos4),
# typeD=c(rep(1, length(pos3)), rep(2, length(pos4))), dplot=TRUE)
#aux$pv
```

---

simHPc.fun	<i>Generate a given number of occurrence points in a homogenous Poisson process in continuous time</i>
------------	--

---

### Description

This function generates a given number of occurrence points in a homogenous Poisson process (HPP) in continuous time.

### Usage

```
simHPc.fun(lambda, nEv, fixed.seed=NULL)
```

### Arguments

lambda	Numeric value. Intensity $\lambda$ used to generate the PP.
nEv	Optional. Positive integer. Number of points to be generated in the PPs.
fixed.seed	An integer or NULL. If it is an integer, that is the value used to set the seed in random generation processes. If it is NULL, a random seed is used.

### Details

The points in a HPP are generated using independent exponentials with mean  $\lambda$ .

Points in a HPP can also be generated using [simNHPC.fun](#). The main difference is that here, the number of points to be generated is given, while in the other function, points are generated in a period of a given length T.

### Value

A list with elements

posN	Numeric vector. Occurrence points of the PP.
T	Length of the period where the given number of points are generated.
fixed.seed	Input argument.

### References

Ross, S.M. (2006). *Simulation*. Academic Press.

### See Also

[simNHPC.fun](#), [IndNHPP.fun](#)

**Examples**

```
aux<-simNHPc.fun(lambda=0.01, nEv=50, fixed.seed=123)
aux$posH
```

---

simNHPc.fun	<i>Generate the occurrence points of a Poisson process in continuous time</i>
-------------	---

---

**Description**

This function generates the occurrence points of a homogenous or nonhomogeneous Poisson process (NHPP) with a given intensity  $\lambda(t)$ , in a continuous period of time  $(0, T)$ .

It calls the auxiliary function `buscar` (not intended for the users), see Details section.

**Usage**

```
simNHPc.fun(lambda, fixed.seed=NULL)
```

**Arguments**

<code>lambda</code>	Numeric vector of length T, the observation period length. Intensity $\lambda(t)$ used to generate the PP.
<code>fixed.seed</code>	An integer or NULL. If it is an integer, that is the value used to set the seed in random generation processes. If it is NULL, a random seed is used.

**Details**

The generation of the NHPP points consists in two steps. First, the points of a homogeneous PP of intensity 1 are generated using independent exponentials. Then, the homogeneous occurrence times are transformed into the points of a non homogeneous process with intensity  $\lambda(t)$ . This transformation is performed by the auxiliary function `buscar` (not intended for the user).

The length of the period where the processes are generated is determined by the length of the argument `lambda`.

Homogenous processes are generated if the intensity vector `lambda` is constant (that is if all the values are equal).

**Value**

A list with elements

<code>posNH</code>	Numeric vector. Occurrence points of the PP.
<code>lambda</code>	Input argument.
<code>fixed.seed</code>	Input argument.

## References

Cebrian, A.C., Abaurrea, J. and Asin, J. (2015). NHPPoisson: An R Package for Fitting and Validating Nonhomogeneous Poisson Processes. *Journal of Statistical Software*, 64(6), 1-24.

Ross, S.M. (2006). *Simulation*. Academic Press.

## See Also

[simHPc.fun](#), [IndNHPP.fun](#)

## Examples

```
#Generation of a HPP
aux<-simNHPPc.fun(lambda=rep(0.1,200),fixed.seed=123)
aux$posNH
```

```
#Generation of a NHPP
set.seed(123)
lambdat<-runif(500, 0.01,0.1)
aux<-simNHPPc.fun(lambda=lambdat,fixed.seed=123)
aux$posNH
```

---

SpecGap.fun

*It calculates the stationary distribution of a matrix and its spectral gap*

---

## Description

This function calculates the stationary distribution of the transition matrix of a Markov chain process and its spectral gap.

## Usage

```
SpecGap.fun(P)
```

## Arguments

P                      Matrix. It must be a markovian matrix

## Details

The spectral gap of a matrix  $P$  assesses the convergence speed of  $P$  to a matrix  $P_I$  with all the rows equal to  $(\pi_1, \pi_2, \dots, \pi_k)$ , the stationary distribution of  $P$ . It takes values in  $[0,1]$ .

The spectral gap of a transition matrix can be used as a dependence measure between the multitype processes defined by a marked Poisson process with marks generated by a Markov chain with that transition matrix, see Abaurrea et al (2015) for details.

**Value**

A list with elements

SG Spectral gap value of the matrix.

$\pi$  Vector of the stationary distribution of the matrix.

**References**

Abaurrea, J. Asin, J. and Cebrian, A.C. (2014). A Bootstrap Test of Independence Between Three Temporal Nonhomogeneous Poisson Processes and its Application to Heat Wave Modeling. *Environmental and Ecological Statistics*, 22(1), 127-144.

**See Also**

[DepMarkedNHPP.fun](#)

**Examples**

```
P<-cbind(c(0.7, 0.1, 0.2), c(0.2, 0.7, 0.1), c(0.1, 0.2, 0.7))
SpecGap.fun(P)
```

---

TestIndLS.fun	<i>Calculate a Lotwick-Silverman test to check the independence between up to three homogeneous point processes in time</i>
---------------	---

---

**Description**

This function calculates a test of type Lotwick-Silverman (LS) to check the independence between up to three homogeneous point processes in time. The statistic is based on the close point relation, which adapts the crossed nearest neighbour distance ideas of spatial point processes to the case of point processes in time. The p-value is obtained using a bootstrap based on a type LS approach.

**Usage**

```
TestIndLS.fun(posx, posy, posz=NULL, T, alpha = 0.05, nTrans = 100,
PA = FALSE, cores=1, fixed.seed=NULL)
```

**Arguments**

posx Numeric vector. Position of the occurrence points in the first process.

posy Numeric vector. Position of the occurrence points in the second process.

posz Numeric vector. Position of the occurrence points in the third process. Only used if there are 3 processes in the set.

T Numeric value. Length of the observation period of the processes.

alpha	Optional. Significance level used to obtain a decision (reject-no reject) based on the test p-value.
nTrans	Optional. Positive integer. Number of translations performed to calculate the test.
PA	Optional. Logical flag. If it is TRUE, the close point relation is broadened by including the previous and the following points to the overlapping intervals.
cores	Optional. Number of cores of the computer to be used in the calculations.
fixed.seed	Optional. An integer or NULL. If it is an integer, that is the value used to set the seed in random generation processes. If it is NULL, a random seed is used.

### Details

The underlying idea of the test is to compare, for each point in the first process, the behavior of its set of close points in the observed process vector  $(N_x, N_y, N_z)$ , and in a new process vector with the same characteristics and mutually independent components. In the new vectors, the process  $N_x$  is fixed and second and third processes are obtained by a translation of the original ones. If the observed behavior is significantly different, independence is rejected. The test statistic is the same as the one used in [TestIndNH.fun](#), but the p-value is obtained using a different bootstrap technique, based on a LS approach, see Lotwick and Silverman (1982).

This test provides a nonparametric way to test independence. The key idea is to wrap the processes onto a torus by identifying opposite sides, keeping the first process fixed and translating the other processes randomly over the torus. The translation keeps the distribution of the processes (since they are homogenous) but breaks any dependence between them, without the need of parametric models to describe the univariate marginal patterns, not even the intensity of the processes.

### Value

A list with elements

KSpv	P-value of the independence test.
reject	Binary vale indicating if the test is rejected ( value 1) or not (value 0) at the alpha significance level.
KSest	Statistic of the independence test.

### References

Abaurrea, J. Asin, J. and Cebrian, A.C. (2015). A Bootstrap Test of Independence Between Three Temporal Nonhomogeneous Poisson Processes and its Application to Heat Wave Modeling. *Environmental and Ecological Statistics*.

Lotwick, H.W. and Silverman, B.W. (1982). Methods for analysing Spatial processes of several types of points. *J.R. Statist. Soc. B*, 44(3), pp. 406-13

### See Also

[TestIndNH.fun](#), [CondTest.fun](#), [DutilleulPlot.fun](#), [DistShift.fun](#)

## Examples

```
#Test applied to three independent NHPP
posx<-simHPc.fun(0.1, 20, fixed.seed=123)$posH
posz<-simHPc.fun(0.15,22, fixed.seed=124)$posH
posy<-simHPc.fun(0.1, 18, fixed.seed=125)$posH
T<-max(posx, posy, posz)+10

aux<-TestIndLS.fun(posx, posy, posz,T=T,
cores=1, fixed.seed=321)
aux$KSpv
```

---

TestIndNH.fun	<i>Calculate a bootstrap test to check the independence between up to three nonhomogeneous Poisson or point processes in time</i>
---------------	---

---

## Description

This function calculates the test by Abaurrea et al. (2014). The statistic is based on the close point relation, which adapts the crossed nearest neighbour distance ideas of spatial point processes to the case of (homogeneous or nonhomogeneous) point processes in time. The test can be applied to two or three homogeneous or non homogeneous Poisson processes or to any point process which can be simulated. Currently, it is implemented for Poisson processes and for Neyman-Scott cluster processes. The p-value is obtained using a parametric bootstrap approach.

## Usage

```
TestIndNH.fun(posx, posy, posz=NULL, NumProcess=2, alpha = 0.05,
nsim = 100, PA = FALSE, cores = 1, type = "Poisson",
lambdaMarg = NULL, lambdaParent = NULL, lambdaNumP = NULL,
dist = "normal", sigmaC = 1, minC = -1, maxC = 1, fixed.seed=NULL)
```

## Arguments

posx	Numeric vector. Position of the occurrence points in the first process.
posy	Numeric vector. Position of the occurrence points in the second process.
posz	Numeric vector. Position of the occurrence points in the third process. Only used if NumProcess=3.
NumProcess	Optional. Integer equal to 1 or 2. Number of processes added to the first one.
alpha	Optional. Significance level used to obtain a decision (reject-no reject) based on the test p-value.
nsim	Optional. Positive integer. Number of simulations performed to calculate the test.
PA	Optional. Logical flag. If it is TRUE, the close point relation is broadened by including the previous and the following points to the overlapping intervals.

cores	Optional. Number of cores of the computer to be used in the calculations.
type	Optional. Label "Poisson" or "PoissonCluster". Type of point processes to be generated in the parametric bootstrap. Up to now, only two types are available: Poisson processes ("Poisson") and Neyman-Scott cluster processes ("PoissonCluster").
lambdaMarg	Matrix of dimension $T \times (NumProcess - 1)$ . Only used if $type='Poisson'$ . Each column is the intensity vector to generate the Poisson processes.
lambdaParent	Numeric vector. Only used if $type='PoissonCluster'$ . Intensity vector of the PP used to generate the centers of the clusters of the Neyman-Scott process.
lambdaNumP	Numeric vector with 1 or 2 values. Only used if $type='PoissonCluster'$ . Mean values of the number of sons of the processes to be generated. If its length is equal to 1, the same value is used in both processes.
dist	Optional. Label "normal" or "uniform". Only used if $type='PoissonCluster'$ . Distribution used to generate the point locations of each cluster.
sigmaC	Optional. Numeric vector with 1 or 2 values. Only used if $type='PoissonCluster'$ and $dist='normal'$ . Standard deviation of the normal distribution. If its length is equal to 1, the same value is used in both processes.
minC	Optional. Numeric vector with 1 or 2 values. Only used if $type='PoissonCluster'$ and $dist='uniform'$ . Lower limits of the Uniform distribution. If its length is equal to 1, the same value is used in both processes.
maxC	Optional. Numeric vector with 1 or 2 values. Only used if $type='PoissonCluster'$ and $dist='uniform'$ . Upper limits of the Uniform distribution. If its length is equal to 1, the same value is used in both processes.
fixed.seed	Optional. An integer or NULL. If it is an integer, that is the value used to set the seed in random generation processes. If it is NULL, a random seed is used.

## Details

The underlying idea of the test is to compare, for each point in the first process, the behavior of its set of close points in the observed vector process  $(N_x, N_y, N_z)$ , and in simulated vector processes with the same characteristics and mutually independent components. In the simulated vectors, the process  $N_x$  is fixed and second and third processes with intensities  $\lambda_y$  and  $\lambda_z$  (the intensities of  $N_y$  and  $N_z$ ) are generated. If the observed behavior is significantly different, independence is rejected. The distribution of the statistic is obtained using a Monte Carlo approach if the intensities  $\lambda_y(t)$  and  $\lambda_z(t)$  are known, or a parametric bootstrap if they have been estimated. See Abaurrea et al. (2014) for more details

It is noteworthy that is being assumed that the processes are Poisson or Neyman-Scott cluster processes. Then, if necessary, validation of that assumption should be previously carried out.

The length of the observation period is determined by the length of the intensity vector  $\lambda$ , that is  $lambdaParent$  (if  $type='PoissonCluster'$ ) or the first element of the dimension of  $lambdaMarg$  (if  $type='PoissonC'$ ).

Homogenous processes are generated if the intensity vector  $lambda$  is constant (that is if all the values are equal).

The test [TestIndLS.fun](#), which uses a non parametric bootstrap approach, is available for any homogeneous point process.

**Value**

A list with elements

KSpv	P-value of the independence test.
reject	Binary vale indicating if the test is rejected ( value 1) or not (value 0) at the alpha significance level.
KSest	Statistic of the independence test.

**References**

Abaurrea, J. Asin, J. and Cebrian, A.C. (2014). A Bootstrap Test of Independence Between Three Temporal Nonhomogeneous Poisson Processes and its Application to Heat Wave Modeling. *Environmental and Ecological Statistics*.

**See Also**

[TestIndLS.fun](#), [CondTest.fun](#), [DutilleulPlot.fun](#), [DistSim.fun](#), [DistObs.fun](#), [uniongentry.fun](#)

**Examples**

```
#Test applied to 3 independent NHPP
set.seed(123)
lambdax<-runif(150, 0.01,0.1)
set.seed(124)
lambday<-runif(150, 0.02,0.1)
set.seed(125)
lambdaz<-runif(150, 0.015,0.1)
posx<-simNHPC.fun(lambdax, fixed.seed=123)$posNH
posy<-simNHPC.fun(lambday, fixed.seed=124)$posNH
posz<-simNHPC.fun(lambdaz, fixed.seed=125)$posNH

aux<-TestIndNH.fun(posx, posy, posz, nsim=50, type='Poisson',
NumProcess=3,lambdaMarg=cbind(lambday,lambdaz), fixed.seed=321)
aux$KSpv
```

```
#Test applied to 3 dependent NS cluster processes with 2 cores
#set.seed(123)
#lambdaParent<-runif(500,0,0.1)
#DepPro<-DepNHNeyscot.fun(lambdaParent=lambdaParent, d=3, lambdaNumP = 3,
# dist = "normal", sigmaC = 1, fixed.seed=123,cores=2)
#posx<-DepPro$PP1
#posy<-DepPro$PP2
#posz<-DepPro$PP3
#aux<-TestIndNH.fun(posx, posy, posz, cores=1, type='PoissonCluster',
# NumProcess=3,lambdaParent = lambdaParent, lambdaNumP = 3,
# dist = "normal", sigmaC = 1, fixed.seed=123, nsim=200)
#aux$KSpv
```

---

uniongentri.fun      *Calculates the set of close points of each point in a process*

---

## Description

This function calculates the set of close points of each occurrence point in the first process of a set of up to three processes.

## Usage

```
uniongentri.fun(posx, posy, posz=NULL, info = FALSE, PA = FALSE,
procName=c('X', 'Y', 'Z'), ...)
```

## Arguments

posx	Numeric vector. Position of the occurrence points in the first process.
posy	Numeric vector. Position of the occurrence points in the second process.
posz	Optional. Numeric vector. Position of the occurrence points in the third process. Only used when three processes are involved.
info	Optional. Logical flag. If it is TRUE, information about the generated points is shown on the screen and dotcharts and bivariate charts of the occurrence points in the processes are displayed.
procName	Vector of character strings. Names for the processes.
PA	Optional. Logical flag. If it is TRUE, the close point relation is broadened by including the previous and the following points to the overlapping intervals.
...	Further arguments to be passed to <a href="#">plot</a> if info=T.

## Details

A point in a process is close to a point in another process, if their time intervals overlap; the time interval of a point is the time interval between itself and the previous point. If there are three processes, the set of close points of  $t_{x_k}$ ,  $S_{x_i;xyz}$  is defined as the set of the pairs of points  $(t_{y_j}, t_{z_k})$  such that  $t_{x_i}$  is close to  $t_{y_j}$  and  $t_{y_j}$  is close to  $t_{z_k}$ . If there are two processes,  $S_{x_i;xy}$  is the set of points  $t_{y_j}$  such that  $t_{x_i}$  is close to  $t_{y_j}$ .

The algorithm to calculate the sets of close points (in the case of three processes) is the following, see Abaurrea et al. (2015) for details: First, given two processes, the pairs of close points in those processes are calculated. If the last point occurs in the first process, there is a censored time interval in the second process (the point overlaps a time interval whose occurrence point has not been observed) and that pair is not considered). This step is performed for all the combinations of pairs of processes. The basic close point relation is commutative, and only three different pairs (XY, YZ, XZ) must be considered. This is not the case of the broadened definition, where two more

points (the previous and the following ones) are added to the set; in that case, the six pairs (XY, YX, YZ, ZY, XZ, ZX) must be calculated.

Once all the pairs of close points are obtained, the set of close points for each point  $t_{x_i}$  is obtained by concatenating the adequate pairs of points from all the possible orders of the three processes: XYZ, XZY and YXZ for the basic definition, and the six possible permutations for the broadened definition. The final set of close points of  $t_{x_i}$  is the union of the different pairs from all the possible permutations.

### Value

A list with elements

X	First elements of the 3-tuples of points $(t_{x_i}, t_{y_i}, t_{z_i})$ in the sets of close points
iX	Position i (=1,2,3....) of the point $t_{x_i}$ in the first process
Y	Second elements of the 3-tuples of points $(t_{x_i}, t_{y_i}, t_{z_i})$ in the sets of close points
iY	Position i (=1,2,3....) of the point $t_{y_i}$ in the second process
Z	Third elements of the 3-tuples of points $(t_{x_i}, t_{y_i}, t_{z_i})$ in the sets of close points. It is NULL if posz=NULL.
iZ	Position i (=1,2,3....) of the point $t_{z_i}$ in the third process. It is NULL if posz=NULL.

### References

Abaurrea, J. Asin, J. and Cebrian, A.C. (2014). A Bootstrap Test of Independence Between Three Temporal Nonhomogeneous Poisson Processes and its Application to Heat Wave Modeling. *Environmental and Ecological Statistics*.

### See Also

[TestIndNH.fun](#), [DistSim.fun](#), [DistObs.fun](#)

### Examples

```
set.seed(123)
posx<-sort(runif(20,0,1000))
posy<-sort(runif(25,0,1000))
posz<-sort(runif(40,0,1000))
aux<-uniongentri.fun(posx, posy, posz, info=TRUE)
```

# Index

## \*Topic **Tests of Independence Between Point Processes in Time**

- IndTestPP-package, 2
- BarTxTn, 3
- buscar (simNHPC.fun), 41
- calcdist.fun, 19
- calcdist.fun (DistObs.fun), 19
- calcNmu (CondTest.fun), 5
- ComplPos.fun, 4
- CondTest.fun, 5, 12, 25, 44, 47
- CPSPpoints.fun, 7, 10
- CPSPPOtvents.fun, 8, 8
- depchi.fun, 10
- DepMarkedNHPP.fun, 12, 14, 16, 18, 43
- DepNHCPSP.fun, 13, 13, 16, 18
- DepNHNeyScot.fun, 13, 14, 15, 18, 27
- DepNHPPqueue.fun, 13, 14, 16, 17
- DistObs.fun, 19, 23, 47, 49
- DistShift.fun, 20, 44
- DistSim.fun, 20, 21, 22, 47, 49
- DistSimfix.fun (DistSim.fun), 22
- dotchart, 20, 23
- DutilleulPlot.fun, 6, 12, 24, 28, 44, 47
- fn2 (TestIndNH.fun), 45
- fn2B (TestIndLS.fun), 43
- fn2fix (TestIndNH.fun), 45
- fn3 (DutilleulPlot.fun), 24
- genbiPos.fun (uniongentri.fun), 48
- GenSons.fun (DepNHNeyScot.fun), 15
- gentriPos.fun (uniongentri.fun), 48
- HDFaux (NHD.fun), 29
- IndNHNeyScot.fun, 16, 23, 25, 28
- IndNHPP.fun, 13, 18, 27, 27, 40, 42
- IndTestPP (IndTestPP-package), 2
- IndTestPP-package, 2
- Jenv (NHJ.fun), 33
- Kenv (NHK.fun), 37
- miKS.fun (TestIndNH.fun), 45
- mirank.fun (TestIndNH.fun), 45
- nearestD (NHD.fun), 29
- nearestdist.fun, 25, 28
- NHD.fun, 4, 29, 33, 36, 39
- NHDaux (NHD.fun), 29
- NHDFaux (NHJ.fun), 33
- NHF.fun, 4, 30, 31, 36, 39
- NHFaux (NHF.fun), 31
- NHJ.fun, 4, 30, 32, 33, 33, 39
- NHJaux (NHJ.fun), 33
- NHK.fun, 4, 30, 33, 36, 37
- NHKaux (NHK.fun), 37
- NHKaux2 (NHK.fun), 37
- NHKaux3 (NHK.fun), 37
- nMenr (NHD.fun), 29
- par, 34
- pdist.fun (nearestdist.fun), 28
- plot, 5, 19, 20, 23, 24, 48
- prodN2 (NHD.fun), 29
- rexp, 17
- simHPc.fun, 40, 42
- simNHPC.fun, 23, 27, 28, 40, 41
- SpecGap.fun, 13, 42
- TestIndLS.fun, 21, 43, 46, 47
- TestIndNH.fun, 6, 12, 20, 23, 25, 44, 45, 49
- uniongentri.fun, 19, 20, 47, 48