

Package ‘GPvecchia’

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Maintainer Marcin Jurek <marcinjurek1988@gmail.com>

Author Matthias Katzfuss [aut],

 Marcin Jurek [aut, cre],

 Daniel Zilber [aut],

 Wenlong Gong [aut],

 Joe Guinness [ctb],

 Jingjie Zhang [ctb],

 Florian Schaefer [ctb]

Description Fast scalable Gaussian process approximations, particularly well suited to spatial (aerial, remote-sensed) and environmental data, described in more detail in Katzfuss and Guinness (2017) <[arXiv:1708.06302](#)>. Package also contains a fast implementation of the incomplete Cholesky decomposition (IC0), based on Schaefer et al. (2019) <[arXiv:1706.02205](#)> and MaxMin ordering proposed in Guinness (2018) <[arXiv:1609.05372](#)>.

License GPL (>= 2)

Imports Rcpp (>= 0.12.16), methods, stats, sparseinv, fields,
Matrix(>= 1.2.14), parallel, GpGp, FNN

LinkingTo Rcpp, RcppArmadillo, BH

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calculate_posterior_VL

Vecchia Laplace extension of GPVecchia for non-Gaussian data

Description

Vecchia Laplace extension of GPVecchia for non-Gaussian data

Usage

```
calculate_posterior_VL(z, vecchia.approx,
  likelihood_model = c("gaussian", "logistic", "poisson", "gamma",
  "beta", "gamma_alt"), covparms, covmodel = "matern",
  likparms = list(alpha = 2, sigma = sqrt(0.1)), max.iter = 50,
  convg = 1e-06, return_all = FALSE, y_init = NA,
  prior_mean = rep(0, length(z)), verbose = FALSE)
```

Arguments

| | |
|-------------------------------|--|
| <code>z</code> | an array of real numbers representing observations |
| <code>vecchia.approx</code> | a vecchia object as generated by <code>vecchia_specify()</code> |
| <code>likelihood_model</code> | text describing likelihood model to be used for observations. Can be "gaussian", "logistic", "poisson", "gamma", or "beta" |
| <code>covparms</code> | covariance parameters as a vector |
| <code>covmodel</code> | type of the model covariance or selected elements of the covariance matrix |
| <code>likparms</code> | likelihood parameters for the <code>likelihood_model</code> , as a list. Default values are <code>sqrt(.1)</code> for Gaussian noise and 2 for the alpha parameter for Gamma data. |
| <code>max_iter</code> | maximum iterations to perform |
| <code>convg</code> | convergence criteria. End iterations if the Newton step is this small |
| <code>return_all</code> | Return additional posterior covariance terms, TRUE or FALSE |
| <code>y_init</code> | Specify initial guess for posterior mode |
| <code>prior_mean</code> | specify the prior latent mean |
| <code>verbose</code> | if TRUE messages about the posterior estimation will be displayed |

Value

multivariate normal posterior parameters calculated by the Vecchia-Laplace approximation

Examples

```
z=rnorm(10); locs=matrix(1:10,ncol=1); vecchia.approx=vecchia_specify(locs,m=5)
calculate_posterior_VL(z,vecchia.approx,"gaussian",covparms=c(1,2,.5))
```

`createU`

create the sparse triangular U matrix for specific parameters

Description

create the sparse triangular U matrix for specific parameters

Usage

```
createU(vecchia.approx, covparms, nuggets, covmodel = "matern")
```

Arguments

| | |
|-----------------------------|--|
| <code>vecchia.approx</code> | object returned by <code>vecchia_specify</code> |
| <code>covparms</code> | vector of covariance parameters |
| <code>nuggets</code> | nugget variances – if a scalar is provided, variance is assumed constant |
| <code>covmodel</code> | covariance model. currently implemented: |

Value

list containing the sparse upper triangular U, plus additional objects required for other functions

Examples

```
z=rnorm(9); locs=matrix(1:9,ncol=1); vecchia.approx=vecchia_specify(locs,m=5)
U.obj=createU(vecchia.approx,covparms=c(1,2,.5),nuggets=.2)
```

getMatCov

extract the required elements from the covariance matrix

Description

This function takes the entire covariance matrix and creates a matrix of covariances based on the vecchia approximatio object

Usage

```
getMatCov(V, covariances, factor = FALSE)
```

Arguments

- | | |
|-------------|---|
| V | the object returned by vecchia_specify |
| covariances | The full covariance matrix or a covariance function |
| factor | True if we are passing a factor of a matrix |

Value

matrix of size n x (m+1) with only those elements that are used by the incomplete Cholesky decomposition

getMatCovFromFactorCpp

Calculate the covariance values required by HV for matrix factors passed as sparse matrices

Description

Calculate the covariance values required by HV for matrix factors passed as sparse matrices

Usage

```
getMatCovFromFactorCpp(F, revNNarray)
```

Arguments

| | |
|------------|--|
| F | factor of a matrix in a sparse format |
| revNNarray | array with the neighbourhood structure |

Value

matrix with covariance values

GPvecchia*GPvecchia: fast, scalable Gaussian process approximations*

Description

The package can be used for parameter inference and prediction for Gaussian and non-Gaussian spatial data using many popular GP approximation methods.

ic0*Incomplete Cholesky decomposition of a sparse matrix passed in the compressed sparse row format*

Description

Incomplete Cholesky decomposition of a sparse matrix passed in the compressed sparse row format

Usage

```
ic0(ptrs, inds, vals)
```

Arguments

| | |
|------|--------------------------------------|
| ptrs | pointers to the beginning of the row |
| inds | indices of nonzero elements in a row |
| vals | nonzero values |

Value

vector of the values of the incomplete Cholesky factor

MaternFun*Calculate Matern covariance function***Description**

Calculate Matern covariance function

Usage

```
MaternFun(distmat, covparms)
```

Arguments

- | | |
|-----------------------|--|
| <code>distmat</code> | A matrix with distances between points |
| <code>covparms</code> | A vector with parameters (marg. variance, range, smoothness) |

Value

A matrix with covariance values corresponding to the distance matrix

order_coordinate*Sorted coordinate ordering***Description**

Return the ordering of locations sorted along one of the coordinates or the sum of multiple coordinates

Usage

```
order_coordinate(locs, coordinate)
```

Arguments

- | | |
|-------------------------|---|
| <code>locs</code> | A matrix of locations. Each row of <code>locs</code> contains a location, which can be a point in Euclidean space R^d , a point in space-time $R^d \times T$, a longitude and latitude (in degrees) giving a point on the sphere, or a longitude, latitude, and time giving a point in the sphere-time domain. |
| <code>coordinate</code> | integer or vector of integers in 1,...,d. If a single integer, coordinates are ordered along that coordinate. If multiple integers, coordinates are ordered according to the sum of specified coordinate values. For example, when d=2, <code>coordinate = c(1, 2)</code> orders from bottom left to top right. |

Value

A vector of indices giving the ordering, i.e. the first element of this vector is the index of the first location.

Examples

```
n <- 100          # Number of locations
d <- 2            # dimension of domain
locs <- matrix( runif(n*d), n, d )
ord1 <- order_coordinate(locs, 1 )
ord12 <- order_coordinate(locs, c(1,2) )
```

| | |
|---------------------|---|
| order_dist_to_point | <i>Distance to specified point ordering</i> |
|---------------------|---|

Description

Return the ordering of locations increasing in their distance to some specified location

Usage

```
order_dist_to_point(locs, loc0, lonlat = FALSE)
```

Arguments

- | | |
|--------|--|
| locs | A matrix of locations. Each row of locs contains a location, which can be a point in Euclidean space R^d , a point in space-time $R^d \times T$, a longitude and latitude (in degrees) giving a point on the sphere, or a longitude, latitude, and time giving a point in the sphere-time domain. |
| loc0 | A vector containing a single location in R^d . |
| lonlat | TRUE/FALSE whether locations are longitudes and latitudes. |

Value

A vector of indices giving the ordering, i.e. the first element of this vector is the index of the location nearest to loc0.

Examples

```
n <- 100          # Number of locations
d <- 2            # dimension of domain
locs <- matrix( runif(n*d), n, d )
loc0 <- c(1/2,1/2)
ord <- order_dist_to_point(locs,loc0)
```

`order_maxmin_exact` *Maximum minimum distance ordering*

Description

Return the indices of an exact maximum-minimum distance ordering. The first point is chosen as the "center" point, minimizing L2 distance. Dimensions d=2 and d=3 handled separately, dimensions d=1 and d>3 handled similarly. Algorithm is exact and scales quasilinearly.

Usage

```
order_maxmin_exact(locs)
```

Arguments

| | |
|-----|-----------------------|
| loc | Observation locations |
|-----|-----------------------|

Value

A vector of indices giving the ordering, i.e. the first element of this vector is the index of the first location.

Examples

```
n=100; locs <- cbind(runif(n),runif(n))
ord <- order_maxmin_exact(locs)
```

`order_maxmin_exact_obs_pred`
Maximum minimum distance ordering for prediction

Description

Return the indices of an exact maximum-minimum distance ordering. The first point is chosen as the "center" point, minimizing L2 distance. Dimensions d=2 and d=3 handled separately, dimensions d=1 and d>3 handled similarly. Algorithm is exact and scales quasilinearly.

Usage

```
order_maxmin_exact_obs_pred(locs, locs_pred)
```

Arguments

| | |
|-----------|-----------------------|
| loc | Observation locations |
| locs_pred | Prediction locations |

Value

A vector of indices giving the ordering, i.e. the first element of this vector is the index of the first location.

Examples

```
n=100; locs <- cbind(runif(n),runif(n))
locs_pred = cbind(runif(n), runif(n))
ord <- order_maxmin_exact_obs_pred(locs, locs_pred)
```

order_middleout

*Middle-out ordering***Description**

Return the ordering of locations increasing in their distance to the average location

Usage

```
order_middleout(locs, lonlat = FALSE)
```

Arguments

- | | |
|--------|--|
| locs | A matrix of locations. Each row of locs contains a location, which can be a point in Euclidean space R^d , a point in space-time $R^d \times T$, a longitude and latitude (in degrees) giving a point on the sphere, or a longitude, latitude, and time giving a point in the sphere-time domain. |
| lonlat | TRUE/FALSE whether locations are longitudes and latitudes. |

Value

A vector of indices giving the ordering, i.e. the first element of this vector is the index of the location nearest the center.

Examples

```
n <- 100          # Number of locations
d <- 2            # dimension of domain
locs <- matrix( runif(n*d), n, d )
ord <- order_middleout(locs)
```

| | |
|-----------------|----------------------------|
| order_outsidein | <i>Outside-in ordering</i> |
|-----------------|----------------------------|

Description

Return the ordering of locations decreasing in their distance to the average location. Reverses middleout.

Usage

```
order_outsidein(locs, lonlat = FALSE)
```

Arguments

| | |
|--------|--|
| locs | A matrix of locations. Each row of locs contains a location, which can be a point in Euclidean space R^d , a point in space-time $R^d \times T$, a longitude and latitude (in degrees) giving a point on the sphere, or a longitude, latitude, and time giving a point in the sphere-time domain. |
| lonlat | TRUE/FALSE whether locations are longitudes and latitudes. |

Value

A vector of indices giving the ordering, i.e. the first element of this vector is the index of the location farthest from the center.

Examples

```
n <- 100          # Number of locations
d <- 2            # dimension of domain
locs <- matrix( runif(n*d), n, d )
ord <- order_outsidein(locs)
```

| | |
|--------|--|
| SelInv | <i>selected inverse of a sparse matrix</i> |
|--------|--|

Description

selected inverse of a sparse matrix

Usage

```
SelInv(cholmat)
```

Arguments

| | |
|---------|---|
| cholmat | cholesky factor L of a positive definite sparseMatrix A |
|---------|---|

Value

sparse inverse of A, with same sparsity pattern as L

Examples

```
A=Matrix::sparseMatrix(1:9,1:9,x=4); L=chol(A)
SclInv(L)
```

V2covmat

compute covariance matrix from V.ord Do not run this function for large n or n.p!!!

Description

compute covariance matrix from V.ord Do not run this function for large n or n.p!!!

Usage

```
V2covmat(preds)
```

Arguments

| | |
|-------|---|
| preds | Object returned by vecchia_prediction() |
|-------|---|

Value

Covariance matrix at all locations in original order

Examples

```
z=rnorm(5)
locs=matrix(1:5,ncol=1)
vecchia_specify=function(z,locs,m=5,locs.pred=(1:5)+.5)
V2covmat(vecchia_prediction(vecchia.approx,covparms=c(1,2,.5),nuggets=.2))
```

| | |
|-------------------------------|--|
| <code>vecchia_estimate</code> | <i>estimate mean and covariance parameters using Vecchia</i> |
|-------------------------------|--|

Description

estimate mean and covariance parameters using Vecchia

Usage

```
vecchia_estimate(data, locs, X, m = 20, covmodel = "matern", theta.ini,
                 output.level = 1, ...)
```

Arguments

| | |
|---------------------------|---|
| <code>data</code> | data vector of length n |
| <code>locs</code> | n x d matrix of spatial locations |
| <code>X</code> | n x p matrix of trend covariates. default is vector of ones (constant trend). set to NULL if data are already detrended |
| <code>m</code> | number of neighbors for vecchia approximation. default is 20 |
| <code>covmodel</code> | covariance model. default is Matern. see vecchia_likelihood for details. |
| <code>theta.ini</code> | initial values of covariance parameters. nugget variance must be last. |
| <code>output.level</code> | passed on to trace in the <code>stats:::optim</code> function |
| <code>...</code> | additional input parameters for vecchia_specify |

Value

object containing detrended data z, trend coefficients beta.hat, covariance parameters theta.hat, and other quantities necessary for prediction

Examples

```
n=10^2; locs=cbind(runif(n),runif(n))
covparms=c(1,.1,.5); nuggets=rep(.1,n)
Sigma=exp(-fields:::rdist(locs)/covparms[2])+diag(nuggets)
z=as.numeric(t(chol(Sigma))%*%rnorm(n));
data=z+1
vecchia.est=vecchia_estimate(data,locs,theta.ini=c(covparms,nuggets[1]))
```

vecchia_laplace_likelihood*Wrapper for VL version of vecchia_likelihood*

Description

Wrapper for VL version of vecchia_likelihood

Usage

```
vecchia_laplace_likelihood(z, vecchia.approx, likelihood_model, covparms,
  likparms = list(alpha = 2, sigma = sqrt(0.1)), covmodel = "matern",
  max.iter = 50, convg = 1e-05, return_all = FALSE, y_init = NA,
  prior_mean = rep(0, length(z)), vecchia.approx.IW = NA)
```

Arguments

| | |
|-------------------|---|
| z | an array of real numbers representing observations |
| vecchia.approx | a vecchia object as generated by vecchia_specify() |
| likelihood_model | text describing likelihood model to be used for observations |
| covparms | covariance parameters as a vector |
| likparms | likelihood parameters for the likelihood_model, as a list |
| covmodel | describes the covariance model, "matern" by default |
| max.iter | maximum iterations to perform |
| convg | convergence criteria. End iterations if the Newton step is this small |
| return_all | Return additional posterior covariance terms |
| y_init | Specify initial guess for posterior mode |
| prior_mean | specify the prior latent mean |
| vecchia.approx.IW | an optional vecchia approximation object, can reduce computation if method is called repeatedly |

Value

(multivariate normal) loglikelihood implied by the Vecchia approximation

Examples

```
z=rnorm(10); locs=matrix(1:10,ncol=1); vecchia.approx=vecchia_specify(locs,m=5)
vecchia_laplace_likelihood(z,vecchia.approx,"gaussian",covparms=c(1,2,.5))
```

vecchia_laplace_prediction*Wrapper for VL version of vecchia_prediction***Description**

Wrapper for VL version of vecchia_prediction

Usage

```
vecchia_laplace_prediction(vl_posterior, vecchia.approx, covparms,
  pred.mean = 0, var.exact = FALSE, covmodel = "matern",
  return.values = "all")
```

Arguments

- `vl_posterior` a posterior estimate object produced by calculate_posterior_VL
- `vecchia.approx` a vecchia object as generated by vecchia_specify()
- `covparms` covariance parameters as a vector
- `pred.mean` provides the prior latent mean for the prediction locations
- `var.exact` should prediction variances be computed exactly, or is a (faster) approximation acceptable
- `covmodel` covariance model, 'matern' by default.
- `return.values` either 'mean' only, 'meanvar', 'meanmat', or 'all'

Value

(multivariate normal) loglikelihood implied by the Vecchia approximation

Examples

```
z=rnorm(10); locs=matrix(1:10,ncol=1); vecchia.approx=vecchia_specify(locs,m=5)
vl_posterior = calculate_posterior_VL(z,vecchia.approx,"gaussian",covparms=c(1,2,.5))
locs.pred=matrix(1:10+.5,ncol=1)
vecchia.approx.pred = vecchia_specify(locs, m=5, locs.pred=locs.pred )
vecchia_laplace_prediction(vl_posterior,vecchia.approx.pred,covparms=c(1,2,.5))
```

`vecchia_likelihood` *evaluation of the likelihood*

Description

evaluation of the likelihood

Usage

```
vecchia_likelihood(z, vecchia.approx, covparms, nuggets,
covmodel = "matern")
```

Arguments

| | |
|-----------------------------|--|
| <code>z</code> | the observed data |
| <code>vecchia.approx</code> | a vecchia object as generated by <code>vecchia_specify()</code> |
| <code>covparms</code> | covariance parameters as a vector |
| <code>nuggets</code> | either a single (constant) nugget or a vector of nugget terms for the observations |
| <code>covmodel</code> | covariance model, 'matern' by default |

Value

(multivariate normal) loglikelihood implied by the Vecchia approximation

Examples

```
z=rnorm(5); locs=matrix(1:5,ncol=1); vecchia.approx=vecchia_specify(locs,m=3)
vecchia_likelihood(z,vecchia.approx,covparms=c(1,.2,.5),nuggets=.2)
```

`vecchia_lincomb` *linear combination of predictions compute the distribution of a linear combination Hy*

Description

linear combination of predictions compute the distribution of a linear combination Hy

Usage

```
vecchia_lincomb(H, U.obj, V.ord, cov.mat = FALSE)
```

Arguments

| | |
|---------|---|
| H | sparse matrix with n.all columns specifying the linear combination |
| U.obj | U matrix is the full joint approximated cholesky matrix |
| V.ord | ordered V matrix from vecchia_prediction() or U2V() |
| cov.mat | logical TRUE or FALSE – should the entire covariance matrix be returned (only do if H has a small number of rows) |

Value

Variance of linear combination of predictions.

Examples

```
n=5; z=rnorm(n); locs=matrix(1:n,ncol=1); n.p=5
vecchia.approx = vecchia_specify(locs,m=3,locs.pred=locs+.5)
preds=vecchia_prediction(z,vecchia.approx,covparms=c(1,2,.5),nuggets=.2)
H=Matrix::sparseMatrix(i=rep(1,n.p),j=n+(1:n.p),x=1/n.p)
vecchia_lincomb(H,vecchia.approx,preds$V.ord,cov.mat=TRUE)
```

vecchia_pred

make spatial predictions using Vecchia based on estimated parameters

Description

make spatial predictions using Vecchia based on estimated parameters

Usage

```
vecchia_pred(vecchia.est, locs.pred, X.pred, m = 30, ...)
```

Arguments

| | |
|-------------|--|
| vecchia.est | object returned by vecchia_estimate |
| locs.pred | n.p x d matrix of prediction locations |
| X.pred | n.p x p matrix of trend covariates at prediction locations. does not need to be specified if constant or no trend was used in vecchia_estimate |
| m | number of neighbors for vecchia approximation. default is 30. |
| ... | additional input parameters for vecchia_specify |

Value

object containing prediction means mean.pred and variances var.pred

Examples

```
n=10^2; locs=cbind(runif(n),runif(n))
covparms=c(1,.1,.5); nuggets=rep(.1,n)
Sigma=exp(-fields::rdist(locs)/covparms[2])+diag(nuggets)
z=as.numeric(t(chol(Sigma))%*%rnorm(n));
data=z+
vecchia.est=vecchia_estimate(data,locs,theta.ini=c(covparms,nuggets[1]))
n.p=30^2; grid.oneside=seq(0,1,length=round(sqrt(n.p)))
locs.pred=as.matrix(expand.grid(grid.oneside,grid.oneside))
vecchia.pred=vecchia_pred(vecchia.est,locs.pred)
```

vecchia_prediction *Vecchia prediction*

Description

Vecchia prediction

Usage

```
vecchia_prediction(z, vecchia.approx, covparms, nuggets, var.exact,
covmodel = "matern", return.values = "all")
```

Arguments

| | |
|-----------------------------|--|
| <code>z</code> | observed data |
| <code>vecchia.approx</code> | a vecchia object as generated by <code>vecchia_specify()</code> |
| <code>covparms</code> | covariance parameters as a vector |
| <code>nuggets</code> | nugget |
| <code>var.exact</code> | should prediction variances be computed exactly, or is a (faster) approximation acceptable |
| <code>covmodel</code> | covariance model, 'matern' by default. |
| <code>return.values</code> | either 'mean' only, 'meanvar', 'meanmat', or 'all' |

Value

posterior mean and variances at observed and unobserved locations; V matrix

Examples

```
z=rnorm(5); locs=matrix(1:5,ncol=1); vecchia.approx=vecchia_specify(locs,m=3,locs.pred=locs+.5)
vecchia_prediction(z,vecchia.approx,covparms=c(1,2,.5),nuggets=.2)
```

| | |
|------------------------------|--|
| <code>vecchia_specify</code> | <i>specify a general vecchia approximation</i> |
|------------------------------|--|

Description

specify the vecchia approximation for later use in likelihood evaluation or prediction. This function does not depend on parameter values, and only has to be run once before repeated likelihood evaluations.

Usage

```
vecchia_specify(locs, m = -1, ordering, cond.yz, locs.pred,
                 ordering.pred, pred.cond, conditioning, mra.options = NULL,
                 verbose = FALSE)
```

Arguments

| | |
|----------------------------|---|
| <code>locs</code> | nxd matrix of observed locs |
| <code>m</code> | Number of nearby points to condition on |
| <code>ordering</code> | options are 'coord' or 'maxmin' |
| <code>cond.yz</code> | options are 'y', 'z', 'SGV', 'SGVT', 'RVP', 'LK', and 'zy' |
| <code>locs.pred</code> | nxd matrix of locations at which to make predictions |
| <code>ordering.pred</code> | options are 'obspred' or 'general' |
| <code>pred.cond</code> | prediction conditioning, options are 'general' or 'independent' |
| <code>conditioning</code> | conditioning on 'NN' (nearest neighbor) or 'firstm' (fixed set for low rank) or 'mra' |
| <code>mra.options</code> | Settings for number of levels and neighbors per level |
| <code>verbose</code> | Provide more detail when using MRA calculations. Default is false. |

Value

An object that specifies the vecchia approximation for later use in likelihood evaluation or prediction.

Examples

```
locs=matrix(1:5,ncol=1); vecchia_specify(locs,m=2)
```

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