

# Package ‘DELTD’

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**Type** Package

**Title** Kernel Density Estimation using Lifetime Distributions

**Version** 2.6.5

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**Description** A collection of asymmetrical kernels belong to lifetime distributions for kernel density estimation is presented.

Mean Squared Errors (MSE) are calculated for estimated curves. For this purpose, R functions allow the distribution to be Gamma, Exponential or Weibull.

For details see Chen (2000), Jin and Kawczak (2003) and Salha et al. (2014) <doi:10.12988/pms.2014.4616>.

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DELTD-packageDELTD

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## Description

A collection of asymmetrical kernels belong to lifetime distributions for kernel density estimation is presented. i.e. `plot.BS`, `plot.Erlang`, `plot.Gamma`, `plot.LN` and for plotting all densities at same time `dencomb`. Estimated values can also be observed by using `BS`, `Gamma`, `Erlang` and `LN`. For calculating mean squared error by using different kernels functions are `mseBS`, `mseEr`, `mseGamma` and `mseLN`.

## Details

Kernel Density Estimation using Lifetime Distributions

## Author(s)

Javarria Ahmad Khan, Atif Akbar.

## References

- Jin, X.; Kawczak, J. 2003. Birnbaum-Saunders & Lognormal kernel estimators for modeling durations in high frequency financial data. *Annals of Economics and Finance* **4**, 103–124.
- Salha, R. B.; Ahmed, E. S.; Alhoubi, I. M. 2014. Hazard rate function estimation using Erlang Kernel. *Pure Mathematical Sciences* **3** (4), 141–152.
- Chen, S. X. 2000. Probability density function estimation using Gamma kernels. *Annals of the Institute of Statistical Mathematics* **52** (3), 471–480.

## See Also

Useful links:

- <https://CRAN.R-project.org/package=DELTD>

BS

*Estimated Density Values by Birnbaum-Saunders kernel***Description**

Estimated Values by using Birnbaum-Saunders Kernel.

**Usage**

```
BS(y, k, h)
```

**Arguments**

|   |                                      |
|---|--------------------------------------|
| y | a numeric vector of positive values. |
| k | grid points                          |
| h | the bandwidth                        |

**Details**

The Birnbaum-Saunders kernel is developed by Jin and Kawczak (2003). They claimed that performance of their developed kernel is better near the boundary points in terms of boundary reduction.

$$K_{BS(h^{\frac{1}{2}},x)}(y) = \frac{1}{2\sqrt{2}\pi h} \left( \sqrt{\frac{1}{xy}} + \sqrt{\frac{x}{y^3}} \right) \exp \left( -\frac{1}{2h} \left( \frac{y}{x} - 2 + \frac{x}{y} \right) \right)$$

**Value**

|   |                             |
|---|-----------------------------|
| x | grid points                 |
| y | estimated values of density |

**Author(s)**

Javaria Ahmad Khan, Atif Akbar.

**References**

Jin, X.; Kawczak, J. 2003. Birnbaum-Saunders & Lognormal kernel estimators for modeling durations in high frequency financial data. *Annals of Economics and Finance* **4**, 103–124.

**See Also**

For further kernels see [Erlang](#), [Gamma](#) and [LN](#). To plot the density by using BS kernel [plot.BS](#) and to calculate MSE by using Birnbaum-Saunders Kernel [mseBS](#).

**Examples**

```
y <- rexp(100,1)
h <- 0.79 * IQR(y) * length(y) ^ (-1/5)
BS(y,200,h)
```

dencomb

*Plot the Densities for Comparison***Description**

Plot the Estimated Densities with Real Density for Comparison .

**Usage**

```
dencomb(y, k, h, comb)
```

**Arguments**

- |      |   |
|------|---|
| y    | a numeric vector of positive values.  |
| k    | grid points.  |
| h    | the bandwidth   |
| comb | mention the combination which kernel estimated densities are to be compared. If Lognormal and Birnbaum-Saunders kernel densities are to be compared along with real density then use "TLB". If Lognormal and Erlang then use "TLE". If Lognormal and Gamma to be compared then use "TLG". For Birnbaum-Saunders and Erlang use "TBE". For Birnbaum-Saunders and Gamma then use "TBG". For Erlang and Gamma use "TEG". For Lognormal, Birnbaum-Saunders and Erlang use "TLBE". For Lognormal, Birnbaum-Saunders and Gamma use "TLBG". For Birnbaum-Saunders, Erlang and Gamma use "TBEG". To compare all densities in one graph use "TLBEG". |

**Value**

Plot of Estimated Densities with Real Data Density.

**Author(s)**

Javaria Ahmad Khan, Atif Akbar.

**References**

- Jin, X.; Kawczak, J. 2003. Birnbaum-Saunders & Lognormal kernel estimators for modeling durations in high frequency financial data. *Annals of Economics and Finance* **4**, 103–124.
- Salha, R. B.; Ahmed, E. S.; Alhoubi, I. M. 2014. Hazard rate function estimation using Erlang Kernel. *Pure Mathematical Sciences* **3** (4), 141–152.
- Chen, S. X. 2000. Probability density function estimation using Gamma kernels. *Annals of the Institute of Statistical Mathematics* **52** (3), 471-480.

**See Also**

For individual densities of each kernels see [plot.Erlang](#), [plot.BS](#), [plot.LN](#), and [plot.Gamma](#)

## Examples

```
## Not run:
y <- rexp(10,1)
h <- 0.79 * IQR(y) * length(y) ^ (-1/5)
dencomb(y,20,h,"TLB")
## End(Not run)
```

Erlang

*Estimated Density Values by Erlang kernel*

## Description

Estimated values for density by using Erlang Kernel.

## Usage

```
Erlang(y, k, h)
```

## Arguments

- |   |                                      |
|---|--------------------------------------|
| y | a numeric vector of positive values. |
| k | grid points.                         |
| h | the bandwidth                        |

## Details

Erlang kernel is developed by Salha et al. (2014). They developed this asymmetrical kernel with its hazard function and also proved its asymptotic normality.

$$K_{E(x,\frac{1}{h})}(y) = \frac{1}{\Gamma(1 + \frac{1}{h})} \left[ \frac{1}{x} \left(1 + \frac{1}{h}\right) \right]^{\frac{h+1}{h}} y^{\frac{1}{h}} \exp \left( -\frac{y}{x} \left(1 + \frac{1}{h}\right) \right)$$

## Value

- |   |                             |
|---|-----------------------------|
| x | grid points                 |
| y | estimated values of density |

## Author(s)

Javaria Ahmad Khan, Atif Akbar.

## References

- Salha, R. B.; Ahmed, E. S.; Alhoubi, I. M. 2014. Hazard rate function estimation using Erlang Kernel. *Pure Mathematical Sciences* **3** (4), 141–152.

**See Also**

For further MSE by using other kernels see [BS](#), [Gamma](#) and [LN](#). For plotting these estimated values [plot.Erlang](#) and for calculating MSE by using Erlang Kernel [mseEr](#).

**Examples**

```
y <- rexp(100,1)
h <- 0.79 * IQR(y) * length(y) ^ (-1/5)
Erlang(y,200,h)
```

Gamma

*Estimated Density Values by Gamma kernel***Description**

Estimated Kernel density values by using Gamma Kernel.

**Usage**

```
Gamma(y, k, h)
```

**Arguments**

- |   |                                      |
|---|--------------------------------------|
| y | a numeric vector of positive values. |
| k | grid points.                         |
| h | the bandwidth                        |

**Details**

The Gamma kernel is developed by Chen (2000). He was first to introduce asymmetrical kernels to control boundary Bias. Gamma Kernel is

$$K_{\text{Gam}}(\frac{x}{h+1}, h)(y) = \frac{y^{\frac{x}{h}} \exp(-\frac{y}{h})}{\Gamma(\frac{x}{h+1}) h^{\frac{x}{h+1}}}$$

**Value**

- |   |                             |
|---|-----------------------------|
| x | grid points                 |
| y | estimated values of density |

**Author(s)**

Javarria Ahmad Khan, Atif Akbar.

**References**

- Chen, S. X. 2000. Probability density function estimation using Gamma kernels. *Annals of the Institute of Statistical Mathematics* **52** (3), 471-480.

## See Also

For further kernels see [Erlang](#), [BS](#) and [LN](#). To plot its density see [plot.Gamma](#) and to calculate MSE by using Gamma Kernel [mseGamma](#).

## Examples

```
y <- rexp(100,1)
h <- 0.79 * IQR(y) * length(y) ^ (-1/5)
Gamma(y,200,h)
```

LN

*Estimated Density Values by Lognormal kernel*

## Description

Estimated Values of Density estimation by using Lognormal Kernel.

## Usage

```
LN(y, k, h)
```

## Arguments

- |   |                                      |
|---|--------------------------------------|
| y | a numeric vector of positive values. |
| k | grid points.                         |
| h | the bandwidth                        |

## Details

The Lognomal kernel is also developed by Jin and Kawczak (2003). For this too, they claimed that performance of their developed kernel is better near the boundary points in terms of boundary reduction. Lognormal Kernel is

$$K_{LN(\ln(x),4\ln(1+h))} = \frac{1}{\sqrt{(8\pi \ln(1+h))y}} \exp \left[ -\frac{(\ln(y) - \ln(x))^2}{(8\ln(1+h))} \right]$$

## Value

- |   |                             |
|---|-----------------------------|
| x | grid points                 |
| y | estimated values of density |

## Author(s)

Javaria Ahmad Khan, Atif Akbar.

## References

Jin, X.; Kawczak, J. 2003. Birnbaum-Saunders & Lognormal kernel estimators for modeling durations in high frequency financial data. *Annals of Economics and Finance* **4**, 103–124.

## See Also

For further kernels see [Erlang](#), [Gamma](#) and [BS](#). To plot its density see [plot.LN](#) and to calculate MSE by using Lognormal Kernel [mseLN](#).

## Examples

```
y <- rexp(100,1)
h <- 0.79 * IQR(y) * length(y) ^ (-1/5)
LN(y,200,h)
```

**mseBS**

*Calculate Mean Squared Error( MSE) when Birnbaum-Saunders kernel is used*

## Description

Calculate MSE by using Birnbaum-Saunders Kernel.

## Usage

```
mseBS(y, k, h, type)
```

## Arguments

|             |   |
|-------------|---|
| <b>y</b>    | a numeric vector of positive values.  |
| <b>k</b>    | gird points.  |
| <b>h</b>    | the bandwidth   |
| <b>type</b> | mention distribution of vector.If exponential distribution then use "Exp". if use gamma distribution then use "Gamma".If Weibull distribution then use "Weibull". |

## Value

MSE

## Author(s)

Javarria Ahmad Khan, Atif Akbar.

## References

Jin, X.; Kawczak, J. 2003. Birnbaum-Saunders & Lognormal kernel estimators for modeling durations in high frequency financial data. *Annals of Economics and Finance* **4**, 103–124.

**See Also**

For further MSE by using other kernels see [mseLN](#), [mseEr](#) and [mseGamma](#). For density estimation by using Birnbaum-Saunders Kernel [plot.BS](#) and to examine estimated values with grid points see [BS](#).

**Examples**

```
y <- rexp(100,1)
h <- 0.79 * IQR(y) * length(y) ^ (-1/5)
mseBS(y, 200, h, "Exp")
```

**mseEr***Calculate Mean Squared Error( MSE) when Erlang kernel is used***Description**

Calculate MSE by using Erlang Kernel.

**Usage**

```
mseEr(y, k, h, type)
```

**Arguments**

|      |   |
|------|---|
| y    | a numeric vector of positive values.  |
| k    | grid points.  |
| h    | the bandwidth   |
| type | mention distribution of vector.If exponential distribution then use "Exp". if use gamma distribution then use "Gamma".If Weibull distribution then use "Weibull". |

**Value**

MSE

**Author(s)**

Javaria Ahmad Khan, Atif Akbar.

**References**

Salha, R. B.; Ahmed, E. S.; Alhoubi, I. M. 2014. Hazard rate function estimation using Erlang Kernel. *Pure Mathematical Sciences* **3** (4), 141–152.

**See Also**

For further MSE by using other kernels see [mseBS](#), [mseLN](#) and [mseGamma](#). For estimated values for density estimation [Erlang](#) and for density estimation by using Erlang Kernel [plot.Erlang](#).

## Examples

```
y <- rexp(100,1)
h <- 0.79 * IQR(y) * length(y) ^ (-1/5)
mseEr(y,200,h,"Exp")
```

**mseGamma**

*Calculate Mean Squared Error( MSE) when Gamma kernel is used*

## Description

Calculate MSE by using Gamma Kernel.

## Usage

```
mseGamma(y, k, h, type)
```

## Arguments

|      |   |
|------|---|
| y    | a numeric vector of positive values.  |
| k    | gird points.  |
| h    | the bandwidth   |
| type | mention distribution of vector.If exponential distribution then use "Exp". If use gamma distribution then use "Gamma".If Weibull distribution then use "Weibull". |

## Value

MSE

## Author(s)

Javaria Ahmad Khan, Atif Akbar.

## References

Chen, S. X. 2000. Probability density function estimation using Gamma kernels. *Annals of the Institute of Statistical Mathematics* **52** (3), 471-480.

## See Also

For further MSE by using other kernels see [mseBS](#), [mseEr](#) and [mseLN](#). For density estimation by using Gamma Kernel [plot.Gamma](#) and for estimated values of density [Gamma](#).

## Examples

```
y <- rexp(100,1)
h <- 0.79 * IQR(y) * length(y) ^ (-1/5)
mseGamma(y,200,h,"Exp")
```

**mseLN***Calculate Mean Squared Error( MSE) when Lognormal Kernel is used***Description**

Calculate MSE by using Lognormal Kernel.

**Usage**

```
mseLN(y, k, h, type)
```

**Arguments**

|      |   |
|------|---|
| y    | a numeric vector of positive values.  |
| k    | grid points.  |
| h    | the bandwidth   |
| type | mention distribution of vector.If exponential distribution then use "Exp". if use gamma distribution then use "Gamma".If Weibull distribution then use "Weibull". |

**Value**

MSE

**Author(s)**

Javaria Ahmad Khan, Atif Akbar.

**References**

Jin, X.; Kawczak, J. 2003. Birnbaum-Saunders & Lognormal kernel estimators for modeling durations in high frequency financial data. *Annals of Economics and Finance* **4**, 103–124.

**See Also**

For further MSE by using other kernels see [mseBS](#), [mseEr](#) and [mseGamma](#). For estimated values and for density estimation by using Lognormal Kernel see [LN](#) and [plot.LN](#), respectively.

**Examples**

```
y <- rexp(100,1)
h <- 0.79 * IQR(y) * length(y) ^ (-1/5)
mseLN(y,200,h,"Exp")
```

**plot.BS***Density Plot by Birnbaum-Saunders kernel***Description**

Plot Kernel density by using Birnbaum-Saunders Kernel.

**Usage**

```
## S3 method for class 'BS'
plot(x, ...)
```

**Arguments**

|     |   |
|-----|---|
| x   | An object of class "BS"                   |
| ... | Not presently used in this implementation |

**Value**

Nothing

**Author(s)**

Javarria Ahmad Khan, Atif Akbar.

**References**

Jin, X.; Kawczak, J. 2003. Birnbaum-Saunders & Lognormal kernel estimators for modeling durations in high frequency financial data. *Annals of Economics and Finance* **4**, 103–124.

**See Also**

For further kernels see [plot.Erlang](#), [plot.Gamma](#) and [plot.LN](#). For estimated values [BS](#) and for MSE [mseBS](#).

**Examples**

```
y <- rexp(100,1)
h <- 0.79 * IQR(y) * length(y) ^ (-1/5)
den <- BS(y,200,h)
plot(den, type = "s", ylab = "Density Function", lty = 1, xlab = "Time")
## To add true density along with estimated
d1<-density(y,bw=h)
lines(d1,type="p",col="green")
```

**plot.Erlang***Density Plot by Erlang kernel***Description**

Plot Kernel density by using Erlang Kernel.

**Usage**

```
## S3 method for class 'Erlang'
plot(x, ...)
```

**Arguments**

|     |   |
|-----|---|
| x   | An object of class "Erlang"               |
| ... | Not presently used in this implementation |

**Value**

Nothing

**Author(s)**

Javaria Ahmad Khan, Atif Akbar.

**References**

Salha, R. B.; Ahmed, E. S.; Alhoubi, I. M. 2014. Hazard rate function estimation ksing Erlang Kernel. *Pure Mathematical Sciences* **3** (4), 141–152.

**See Also**

For further MSE by using other kernels see [plot.BS](#), [plot.Gamma](#) and [plot.LN](#). For estimated values [Erlang](#) and for calculating MSE by using Erlang Kernel [mseEr](#).

**Examples**

```
y <- rexp(23,1)
h <- 0.79 * IQR(y) * length(y) ^ (-1/5)
ans <- Erlang(y,90,h)
plot(ans, type = "s", ylab = "Density Function", lty = 1, xlab = "Time")
## To add true density along with estimated
d1<-density(y,bw=h)
lines(d1,type="p",col="red")
legend("topright", c("Real Density", "Density by Erlang Kernel"), col=c("red", "black"), lty=c(1,2))
```

**plot.Gamma***Density Plot by Gamma kernel***Description**

Plot density by using Gamma Kernel.

**Usage**

```
## S3 method for class 'Gamma'
plot(x, ...)
```

**Arguments**

|                  |   |
|------------------|---|
| <code>x</code>   | an object of class "BS"                   |
| <code>...</code> | Not presently used in this implementation |

**Value**

nothing

**Author(s)**

Javarria Ahmad Khan, Atif Akbar.

**References**

Chen, S. X. 2000. Probability density function estimation using Gamma kernels. *Annals of the Institute of Statistical Mathematics* **52** (3), 471-480.

**See Also**

For further kernels see [plot.Erlang](#), [plot.BS](#) and [plot.LN](#). To calculate its estimated values see [Gamma](#) and for MSE by using Gamma Kernel [mseGamma](#).

**Examples**

```
y <- rexp(100,1)
h <- 0.79 * IQR(y) * length(y) ^ (-1/5)
den <- Gamma(y,200,h)
plot(den, type = "s", ylab = "Density Function", lty = 1, xlab = "Time")
```

---

plot.LN                    *Density Plot by Lognormal kernel*

---

### Description

Plot Kernel density by using Lognormal Kernel.

### Usage

```
## S3 method for class 'LN'  
plot(x, ...)
```

### Arguments

|     |   |
|-----|---|
| x   | An object of class "LN"                   |
| ... | Not presently used in this implementation |

### Value

Nothing

### Author(s)

Javaria Ahmad Khan, Atif Akbar.

### References

Jin, X.; Kawczak, J. 2003. Birnbaum-Saunders & Lognormal kernel estimators for modeling durations in high frequency financial data. *Annals of Economics and Finance* **4**, 103–124.

### See Also

For further kernels see [plot.Erlang](#), [plot.Gamma](#) and [plot.BS](#). To calculate MSE by using Lognormal Kernel [mseLN](#) and for estimated values for density estimation see [LN](#).

### Examples

```
y <- rexp(23,1)  
h <- 0.79 * IQR(y) * length(y) ^ (-1/5)  
den <- LN(y,90,h)  
plot(den, type = "s", ylab = "Density Function", lty = 1, xlab = "Time")  
## To add true density along with estimated  
d1 <- density(y,bw=h)  
lines(d1,type="p",col="red")
```

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