# Package 'CA3variants' 

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Description Provides four variants of three-way correspondence analysis (ca):three-way symmetrical ca, three-way non-symmetrical ca, three-way ordered symmetrical caand three-way ordered non-symmetrical ca.
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ca3basic

## Description

This function is used in the main function CA3variants when the input parameter is ca3type="CA3". It performs the three-way symmetrical correspondence analysis by TUCKALS3 algorithm.

## Usage

ca3basic(x, p, q, r, test $=10^{\wedge}-6$, ctr $=T$, std $=T$ )

## Arguments

x
p
q
$r$
test
ctr

## std

## Value

$x \quad$ The original three-way contingency table.
xs The weighted three-way contingency table.
xhat Three-way contingency table reconstructed after Tuckals3 by principal components and core array.
nxhat2 The inertia of three-way symmetric correspondence analysis (Three-way Pearson ratio).
prp The proportion of inertia reconstructed using the $\mathrm{p}, \mathrm{q}, \mathrm{r}$ principal components and the core array to the total inertia. To select the model dimensions (number of principal components), we examine the inertia explained by the $\mathrm{p}, \mathrm{q}, \mathrm{r}$ principal components with respect to the overall fit.
a
b
cC
g
iteration
The three-way contingency table.
The number of components of the first mode.
The number of components of the second mode.
The number of components of the third mode.
The treshold used in the algorithm TUCKALS3.
The flag parameter ( T or F ), if F the analysis is not centered.
The flag parameter ( T or F ) if F the analysis is not standardized.

| x | The original three-way contingency table. |
| :---: | :---: |
| xs | The weighted three-way contingency table. |
| xhat | Three-way contingency table reconstructed after Tuckals 3 by principal components and core array. |
| nxhat2 | The inertia of three-way symmetric correspondence analysis (Three-way Pearson ratio). |
| prp | The proportion of inertia reconstructed using the $\mathrm{p}, \mathrm{q}, \mathrm{r}$ principal components and the core array to the total inertia. To select the model dimensions (number of principal components), we examine the inertia explained by the $\mathrm{p}, \mathrm{q}, \mathrm{r}$ principal components with respect to the overall fit. |
| a | The row principal components. |
| b | The column principal coordinates. |
| cc | The tube principal coordinates. |
| g | The core array calculated by using the Tuckals3 algorithm and can be interpreted as generalised singular value table. They help to explain the strength of the association between the three principal components. |
| iteration | The number of iterations that are required for the TUCKALS3 algorithm to converge. |

## Author(s)

Rosaria Lombardo, Eric J Beh.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

```
ca3plot Row isometric biplot or Column isometric biplot
```


## Description

This function is used in the main plot function when the plot type parameter is plottype = "biplot" and the variants of three-way CA are not ordered. It can produce a row or a column biplot.

## Usage

ca3plot(frows, gcols, firstaxis, lastaxis, nseg, inertiapc, thingseg, col1, col2, col3, size1, size2)

## Arguments

frows The row principal or standard coordinates.
gcols The column principal or standard coordinates.
firstaxis The first axis number.
lastaxis The second axis number.
nseg The vectors/arrows number where to project principal (or standard) coordinates.
inertiapc The percentage of the explained inertia by each dimension.
thingseg The principal or standard coordinates used to draw vectors (arrows).
col1 The colour for the row variable labels.
col2 The colour for the column variable labels.
col3 The colour for the vectors (arrows) used in biplots.
size1 The size of the plotted symbol for categories in biplot.
size2 The size of the plotted text for categories in biplot.

## Note

This function depends on the R library plotly.

## Author(s)

Rosaria Lombardo and Eric J. Beh

## References

Beh EJ and Lombardo R 2014 Correspondence Analysis: Theory, Practice and New Strategies. Wiley.
Lombardo R Beh EJ 2016 Variants of Simple Correspondence Analysis. The R Journal, 8 (2), 167-184.

## Description

This function performs the three-way symmetrical (when ca3type = "CA3") and non-symmetrical correspondence analysis (when ca3type = "NSCA3") by using the Tucker3 decomposition and an ordered variant of three-way symmetrical correspondence analysis (when ca3type = "OCA3") by using the Trivariate Moment Decomposition. The non-symmetrical variant considers the three variables asymmetrically related, such that one of the variables is the response to be predicted given the other two variables. It calculates the coordinates and inertia values of the chosen analyses.

## Usage

CA3variants(Xtable, $p=\operatorname{dim}(X t a b l e)[[1]], q=\operatorname{dim}(X t a b l e)[[2]], r=\operatorname{dim}(X t a b l e)[[3]]$, ca3type = "CA3", test $=10^{\wedge}-6$, norder $=3$ )

## Arguments

Xtable The three-way data array. It must be an R object array. When non-symmetrical analysis for one response variable is performed, the response mode is the row variable.
$p$ The number of components for the first mode. By default, $p=\operatorname{dim}(X \operatorname{table})$ [[1]].
$q \quad$ The number of components for the second mode. By default, $q=\operatorname{dim}(X \operatorname{table})[[2]]$.
$r \quad$ The number of components for the third mode. By default, $r=\operatorname{dim}(X \operatorname{table})$ [[3]].
ca3type The specification of the analysis to be performed. If ca3type = "CA3", then a three-way symmetrical correspondence analysis will be performed (default analysis). If ca3type = "NSCA3", then three-way non-symmetrical correspondence analysis will be performed. If ca3type = "OCA3", then ordered three-way symmetrical correspondence analysis will be performed. If ca3type = "ONSCA3", then ordered three-way non-symmetrical correspondence analysis will be performed.
test Threshold used in the algorithm for stopping it after the convergence of the solutions.
norder The input parameter for specifying the number of ordered variable when ca3type = "OCA3".

## Details

This function recall internally many other functions, depending on the setting of the input parameters. After performing three-way symmetric or non-symmetric correspondence analysis, it recall two functions for printing and plotting the results. These two important functions are print. CA3variants and plot.CA3variants.

## Value

The value of output returned depends on the kind of analysis performed. For a detailed description of the output one can see:
the output value of ca3basic if the input parameter is ca3type="CA3"; the output value of nsca3basic if the input parameter is ca3type="NSCA3"; the output value of oca3basic if the input parameter is ca3type="OCA3" the output value of onsca3basic if the input parameter is ca3type="ONSCA3"

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.
Kroonenberg PM (1994). The TUCKALS line: a suite of programs for three-way data analysis. Computational Statistics and Data Analysis, 18, 73-96.

## Examples

```
data(happy)
CA3variants(happy,p=2,q=2,r=2, ca3type = "CA3")
CA3variants(happy,p=2,q=2,r=2, ca3type = "OCA3")
CA3variants(happy, p=2,q=2,r=2, ca3type = "NSCA3")
CA3variants(happy, p=2,q=2,r=2, ca3type = "ONSCA3")
```


## caplot3d Three dimensional correspondence plot

## Description

This function is used in the plot function plot. CAvariants when the logical parameter is plot3d $=$ TRUE. It produces a 3-dimensional visualization of the association.

## Usage

caplot3d(coordR, coordC, inertiaper, firstaxis = 1, lastaxis = 2, thirdaxis = 3)

## Arguments

$$
\begin{array}{ll}
\text { coordR } & \text { The row principal or standard coordinates. } \\
\text { coordC } & \text { The column principal or standard coordinates. } \\
\text { inertiaper } & \text { The percentage of the total inertia explained inertia by each dimension. } \\
\text { firstaxis } & \text { The first axis number. By default, firstaxis }=1 . \\
\text { lastaxis } & \text { The second axis number. By default, lastaxis }=2 . \\
\text { thirdaxis } & \text { The third axis number. By default, thirdaxis }=3 .
\end{array}
$$

## Note

This function depends on the R library plotly.

## Author(s)

Rosaria Lombardo and Eric J. Beh

## References

Beh EJ and Lombardo R 2014 Correspondence Analysis: Theory, Practice and New Strategies. Wiley.
Lombardo R Beh EJ 2016 Variants of Simple Correspondence Analysis. The R Journal, 8 (2), 167-184.

```
chi3 The partition of the Pearson three-way index
```


## Description

When three categorical variables are symmetrically related, we can analyse the strength of the association using the three-way Pearson mean square contingency coefficient, named the chi-squared index. The function chi3 partitions the Pearson phi-squared statistic when in CA3variants we set the parameter ca3type $=$ "CA3".

## Usage

chi3(f3, digits = 3)

## Arguments

f3 The three-way contingency array given as an input parameter in CA3variants.
digits The number of decimal digits. By default digits=3.

## Value

The partition of the Pearson index into three two-way association terms and one three-way association term. It also shows the explained inertia, the degrees of freedom and p-value of each term of the partition.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons. $\backslash$ Carlier A and Kroonenberg P (1996). Decompositions and biplots in three-way correspondence analysis. Psychometrika, 61, 355-373.

## Examples

```
data(happy)
chi3(f3=happy, digits=3)
```

chi3ordered The partition of the Pearson three-way index.

## Description

When three categorical variables are symmetrically related, we can analyse the strength of the symmetrical association using the three-way Pearson statistic. The function chi3ordered partitions the Pearson phi-squared statistic using orthogonal polynomials when, in CA3variants, we set the parameter ca3type $=$ "OCA3".

## Usage

chi3ordered(f3, digits = 3 )

## Arguments

$\begin{array}{ll}\text { f3 } & \text { The three-way contingency array given as an input parameter in CA3variants. } \\ \text { digits } & \text { The number of decimal digits. By default digits=3. }\end{array}$

## Value

The partition of the Pearson index into three two-way association terms and one three-way association term. It also shows the polynomial componets of inertia, the percentage of explained inertia, the degrees of freedom and $p$-value of each term of the partition.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

## Examples

```
    data(happy)
    chi3ordered(f3=happy, digits=3)
```

    criter The stopping criteria for the Tucker3 algorithm
    
## Description

The function computes the stopping criteria needed for the Tucker3 algorithm.

## Usage

criter (x, xhat)

## Arguments

x
xhat

The three-way contingency table
The reconstruction of the data array using the three component matrices and the core array.

## Value

The criterion used in Tuckals3 is the sum of squares of the differences between the weighted data array and the reconstructed data array.

## Author(s)

Rosaria Lombardo, Eric J Beh.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons. 1

```
    critera Criterion of the Tucker3 algorithm
```


## Description

The function critera is used to define the criterion of the Tucker3 algorithm.

## Usage

critera(aold, anew)

## Arguments

aold $\quad$ The old component of the first mode.

## Author(s)

Rosaria Lombardo, Eric J Beh

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

```
emerson.poly Orthogonal polynomials
```


## Description

This function is called from the function oca3basic when in CA3variants we set ca3type $=$ "OCA3". It allows the analyst to compute the orthogonal polynomials of the ordered categorical variable. The number of the polynomials is equal to the variable category less one. The function computes the polynomial transformation of the ordered categorical variable.

## Usage

emerson.poly(mj, pj)

## Arguments

mj
The ordered scores of an ordered variable. By default mj=NULL, the natural scores $(1,2, \ldots)$ are computed.
pj The marginals, relative frequencies of the ordered variable.

## Value

Describe the value returned

B
The matrix of the orthogonal polynomials without the trivial polynomial.

## Note

Note that the sum of the marginals of the ordered variables should be one.

## Author(s)

Rosaria Lombardo and Eric J Beh.

## References

Beh EJ and Lombardo R 2014 Correspondence Analysis: Theory, Practice and New Strategies. John Wiley \& Sons.
Emerson PL 1968 Numerical construction of orthogonal polynomials from a general recurrence formula. Biometrics, 24 (3), 695-701.
Lombardo R Beh EJ Variants of Simple Correspondence Analysis. The R Journal, 8 (2), 167-184.

## Examples

emerson.poly(c(1,2,3,4,5), as.vector(c(.1,.2,.3,.2,.2)))

## flatten

Flattened table

## Description

The function flattens the three-way table into the concatenation of two-way matrices.

## Usage

flatten(x)

## Arguments

x The three-way contingency table.

## Details

It is utilised by a number of functions: CA3variants, reconst3, newcomp3 and step.g3.

## Value

x
The flattened table of size I,JK where I, J and K are the number of the categories of rows, columns and tubes, respectively.

## Author(s)

Rosaria Lombardo, Eric J Beh.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.
happy Three-way contingency table

## Description

This three-way contingency table represents an historical data set found in Beh and Lombardo (2014).

## Usage

data(happy)

## Format

The format is:
row names [1:3] "H1", "H2", "H3" col names [1:5] "S1", "S2", "S3", "S4", "S5" tube names [1:4] "E1", "E2", "E3", "E4"

## References

Beh EJ and Lombardo R 2014 Correspondence Analysis: Theory, Practice and New Strategies. John Wiley \& Sons.

## Examples

```
happy <-
structure(c(15, 17, 7, 34, 53, 20, 36, 70, 23, 22, 67, 16, 61,
79, 36, 31, 60, 5, 60, 96, 12, 46, 45, 11, 25, 40, 12, 26, 31,
7, 35, 63, 5, 45, 74, 10, 30, 39, 4, 13, 24, 4, 8, 7, 3, 18,
15, 2, 14, 15, 1, 3, 9, 2, 3, 2, 0, 4, 1, 1),.Dim = c(3L, 5L,
4L), .Dimnames = list(c("H1", "H2", "H3"), c("S1", "S2", "S3",
"S4", "S5"), c("E1", "E2", "E3", "E4")))
dim(happy)
```


## init3 Initial components from the Tuckals3 algorithm

## Description

The function is utilised from the function tucker to compute the initial components for each of the three categorical variables.

## Usage

init3(x, p, q, r)

## Arguments

x
$r$
p The number of components of the first mode.
q The number of components of the second mode.
The three-way contingency table.

The number of components of the third mode.

## Value

The initial components for each of the three categorical variables.
a
The initial component derived from the Tucker3 decomposition for the first mode.
b
The initial component derived from the Tucker3 decomposition for the second mode.
cc The initial component derived from the Tucker3 decomposition for the third mode.

X
The three-way contingency table

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.
init3ordered Initial components from the Trivariate Moment Decomposition algorithm

## Description

The function is utilised from the function tuckerordered to compute the initial components for each of the three ordered categorical variables.

## Usage

init3ordered(x, p, q, r, x0)

## Arguments

$x \quad$ The three-way contingency table.
p The number of components of the first mode.
q The number of components of the second mode.
$r \quad$ The number of components of the third mode.
$x 0 \quad$ The original three-way contingency table.

## Value

The initial components for each of the three categorical variables.
a
The initial component derived from the Trivariate Moment Decomposition for the first mode.
b
The initial component derived from the Trivariate Moment Decomposition for the second mode.
cc The initial component derived from the Trivariate Moment Decomposition for the third mode.
$x \quad$ The three-way contingency table.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh E J and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.
init3ordered1 Initial components from the Trivariate Moment Decomposition algorithm

## Description

The function is utilised from the function tuckerORDERED to compute the initial components for the first ordered categorical variables.

## Usage

init3ordered1(x, p, q, r, x0)

## Arguments

$x \quad$ The three-way contingency table.
p The number of components of the first mode.
q The number of components of the second mode.
$r \quad$ The number of components of the third mode.
$x 0 \quad$ The original three-way contingency table.

## Value

The initial components for each of the three categorical variables.
a
b The initial component derived from the Trivariate Moment Decomposition for the second mode.

The initial component derived from the Trivariate Moment Decomposition for the third mode.
The three-way contingency table.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

## init3ordered2 Initial components from the Trivriate Moment Decomposition algorithm

## Description

The function is utilised from the function tuckerordered to compute the initial components for each of the two ordered categorical variables.

## Usage

init3ordered2(x, p, q, r, x0)

## Arguments

$x \quad$ The three-way contingency table.
$\mathrm{p} \quad$ The number of components of the first mode.
q The number of components of the second mode.
$r \quad$ The number of components of the third mode.
$x 0 \quad$ The original three-way contingency table.

## Value

The initial components for each of the three categorical variables.
a
b The initial component derived from the Trivriate Moment Decomposition for the second mode.
cc The initial component derived from the Trivriate Moment Decomposition for the third mode.
$x \quad$ The three-way contingency table.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.
Kron Kronecker product

## Description

The function performs the Kronecker product. Starting from two matrices of dimension IxP and JxQ the resulting matrix will be of dimension IxJ,PxQ.

## Usage

Kron(a, b)

## Arguments

a The first matrix of dimension IxP involved in the kronecker product.
b The second matrix of dimension JxQ involved in the kronecker product.

## Details

This function is utilised from several other functions like CA3variants, newcomp3, step.g3 and reconst3.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

```
    loss1.3 General loss criterion
```


## Description

This function represents the general loss function on which is based Tuckals3 and calculates the difference between two arrays, $x$ and xhat, where $x$ is the three-way contingency table and xhat is the reconstruction of this table by means of components and core array.

## Usage

loss1.3(param, comp.old)

## Arguments

param The matrices of the row, column and tube components derived via the Tucker3 model.
comp.old The matrices of the row, column and tube components derived in the foregoing iteration of the Tuckals3 algorithm.

## Value

The difference between three-way contingency table and its reconstruction from the Tucker3 model.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

```
loss1.3ordered General loss criterion
```


## Description

This function represents the general loss function on which is based the Trivariate Moment Decomposition and calculates the difference between two arrays, x and xhat , where x is the three-way contingency table and xhat is the reconstruction of this table by means of components and core array.

## Usage

loss1.3ordered(param, comp.old)

## Arguments

param The matrices of the row, column and tube components derived via the Trivariate Moment Decomposition model.
comp.old The matrices of the row, column and tube components derived in the foregoing iteration of the Trivriate Moment Decomposition algorithm.

## Value

The difference between three-way contingency table and its reconstruction from the Trivariate Moment Decomposition model.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

## Description

The function computes the difference between two successive components in the iteration of the Tuckals3 algorithm.

## Usage

loss2(param, comp.old)

## Arguments

param The matrices of the row, column and tube components derived via the Tucker3 model.
comp.old The matrices of the row, column and tube components derived in the foregoing iteration of the Tuckals3 algorithm.

## Value

The difference between two successive components in the iteration of the Tuckals3 algorithm.

## Author(s)

Rosaria Lombardo and Eric J Beh.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

```
    makeindicator Make an Indicator matrix
```


## Description

From a three-way contingency table (as can be used in CA3variants), it gives the $\mathrm{N} x$ total number of categories (rows+cols+tubs) indicator matrix

## Usage

makeindicator (X)

## Arguments

$X \quad$ The three-way data array. It must be an $R$ object array.

## Value

$\mathrm{Z} \quad$ Output: the N x total number of categories (rows+cols+tubs) indicator matrix

## Author(s)

Rosaria Lombardo, Michel van de Velden, Eric J Beh.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons. 1

## Examples

```
data(happy)
makeindicator(happy)
```

    margI Row marginals of a three-way contingency table
    
## Description

This function computes the row marginals of the three-way contingency table specified by the input parameter.

## Usage

$\operatorname{margI}(m)$

## Arguments

m
The three-way contingency table.

## Value

The row marginals of the considered three-way contingency table.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

```
margJ Column marginals of a three-way contingency table
```


## Description

The function computes the column marginals of the three-way contingency table specified by the input parameter.

## Usage

margJ (m)

## Arguments

$m \quad$ The three-way contingency table.

## Value

The column marginals of the considered three-way contingency table.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

## Description

The function computes the tube marginals of the three-way contingency table specified by the input parameter.

## Usage

margK (m)

## Arguments

m The three-way contingency table.

## Value

The tube marginals of the considered three-way contingency table.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

```
newcomp3 Updated component matrices
```


## Description

The function computes the updated component matrices of the Tucker3 decomposition.

## Usage

newcomp3(param)

## Arguments

param The initial matrices of the row, column and tube components derived via the init3 function.

## Details

It is utilised from the function tucker.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

## Description

The function computes the updated component matrices of the Trivariate Moment Decomposition. It is supposed that the number of the ordered categorical variables is equal to 3 .

## Usage

newcomp3ordered(param)

## Arguments

param The initial matrices of the row, column and tube components derived via the init3 function.

## Details

It is utilised from the function tuckerORDERED.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

## Description

The function computes the updated component matrices of the Trivariate Moment Decomposition. It is supposed that the number of the ordered categorical variables is equal to 1 .

## Usage

newcomp3ordered1 (param)

## Arguments

param The initial matrices of the row, column and tube components derived via the init3 function.

## Details

It is utilised from the function tuckerORDERED.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

```
newcomp3ordered2 Updated component matrices
```


## Description

The function computes the updated component matrices of the Trivariate Moment Decomposition. It is supposed that the number of the ordered categorical variables is equal to 2.

## Usage

newcomp3ordered2(param)

## Arguments

param The initial matrices of the row, column and tube components derived via the init3 function.

## Details

It is utilised from the function tuckerORDERED.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

```
nsca3basic Three-way Non-Symmetrical Correspondence Analysis
```


## Description

This function is used in the main function CA3variants when the input parameter is catype="NSCA3". It decomposes the Marcotorchino index, computes principal axes, coordinates, weights of rows and columns, total inertia (equal to the Marcotorchino index) and the rank of the matrix.

## Usage

nsca3basic(x, p, q, r, test = 10^-6, ctr = T, std = T)

## Arguments

x
p
q
$r$
ctr
std

The three-way contingency table.
The number of components of the first mode.
The number of components of the second mode.
The number of components of the third mode.
The treshold used in the algorithm.
The flag parameter to center the data ( T or F ), if F the data are not centered.
The flag parameter to weight the data ( T or F ), if F the data are not weighted.

Value
nxhat2 The inertia of the three-way non-symmetrical correspondence analysis
x
xs
xhat

The original three-way contingency table.
The weighted three-way contingency table.
The three-way contingency table reconstructed after Tuckals3 by means of the principal components and core array. for one response (the three-way Marcotorchino index).
prp The proportion of inertia reconstructed using the principal components and the core array to the total inertia.
To select the model dimensions (number of principal components), we examine the inertia explained by the $\mathrm{p}, \mathrm{q}, \mathrm{r}$ principal components with respect to the overall fit.
a The row principal components.
b The column principal components.
cc The tube principal components.
g
The core array (generalized singular values) calculated by using the Tuckals3 algorithm.
They help to explain the strength of the association among the three principal components.
iteration The number of iterations that are required for the TUCKALS3 algorithm to converge.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

```
oca3basic Three-way Ordered Symmetrical Correspondence Analysis
```


## Description

This function is used in the main function CAvariants when the input parameter is ca3type="OCA3". It performs the three-way symmetric correspondence analysis by TUCKALS3.

## Usage

oca3basic(x, p, q, r, test = 10^-6, ctr = T, std = T, norder=3)

## Arguments

$x \quad$ The three-way contingency table.
$\mathrm{p} \quad$ The number of components of the first mode.
q The number of components of the second mode.
$r \quad$ The number of components of the third mode.
test The treshold used in the algorithm TUCKALS3.
ctr The flag parameter ( T or F ), if F the analysis is not centered.
std The flag parameter ( T or F ) if F the analysis is not standardized.
norder The number of ordered variables considered.
olive

Value

| x | The original three-way contingency table. |
| :--- | :--- |
| xs | The weighted three-way contingency table. | (Three-way contingency table reconstructed after Tuckals3 by principal compo-

## Author(s)

Rosaria Lombardo, Eric J Beh.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

```
olive Three-way contingency table
```


## Description

This three-way contingency table represents an historical data set found in Agresti (1990).

## Usage <br> data(olive)

## Format

The format is:
row names [1:6] "A", "B", "C", "D", "E", "F" col names [1:3] "NW", "NE", "SW" tube names [1:2] "urban", "rural"

## References

Beh EJ and Lombardo R 2014 Correspondence Analysis: Theory, Practice and New Strategies. John Wiley \& Sons.

## Examples

```
olive <-structure(c(20, 15, 12, 17, 16, 28, 18, 17, 18, 18,
6, 25, 12, 9, 23, 21, 19, 30, 30, 22, 21, 17, 8,
12, 23, 18, 20, 18, 10, 15, 11, 9, 26, 19, 17, 24
), .Dim = c(6L, 3L, 2L), .Dimnames = list(c("A", "B", "C", "D",
"E", "F"), c("NW", "NE", "SW"), c("urban", "rural")))
dim(olive)
data(olive)
```

onsca3basic Three-way Ordered Non-Symmetrical Correspondence Analysis

## Description

This function is used in the main function CAvariants when the input parameter is ca3type="ONSCA3". It performs the three-way symmetric correspondence analysis by TUCKALS3.

## Usage

onsca3basic(x, p, q, r, test = 10^-6, ctr $=T$, std $=T$, norder=3)

## Arguments

x
$\mathrm{p} \quad$ The number of components of the first mode.
q The number of components of the second mode.
$r \quad$ The number of components of the third mode.
test The treshold used in the algorithm TUCKALS3.
ctr $\quad$ The flag parameter ( T or F ), if F the analysis is not centered.
std The flag parameter ( T or F ) if F the analysis is not standardized.
norder The number of ordered variables considered.

## Value

x
xs
xhat

The original three-way contingency table.
The weighted three-way contingency table.
Three-way contingency table reconstructed after Tuckals3 by principal components and core array

| nxhat2 | The inertia of three-way symmetric correspondence analysis (Three-way Pear- <br> son ratio). <br> The proportion of inertia reconstructed using the p, q, r principal components <br> and the core array to the total inertia. To select the model dimensions (number of <br> principal components), we examine the inertia explained by the p, q, r principal <br> components with respect to the overall fit. |
| :--- | :--- |
| a | The row principal components. <br> b |
| The column principal coordinates. |  |
| cc | The tube principal coordinates. <br> The core array calculated by using the Tuckals3 algorithm and can be interpreted <br> as generalised singular value table. They help to explain the strength of the <br> association between the three principal components. |
| iteration | The number of iterations that are required for the TUCKALS3 algorithm to con- <br> verge. |

## Author(s)

Rosaria Lombardo, Eric J Beh.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

```
p.ext
The external product in Tuckals3.
```


## Description

The computation of external product between the principal components.

## Usage

p.ext (x,y)

## Arguments

$\begin{array}{ll}x & x \text { matrix IxS } \\ y & y \text { matrix JxS }\end{array}$

## Value

resultant matrix (IxJ),S with elements xis per yis

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

```
plot.CA3variants Graphical display resulting from CAvariants3
```


## Description

The function plot. CA3variants allows the analyst to graphically display the biplot from their analysis. When the input parameter is biptype = "column-tube", the function displays the columntube interactive biplot. When the input parameter is biptype = "row", the function displays the row interactive biplot. By default, biptype = "column-tube".

## Usage

\#\# S3 method for class 'CA3variants'
plot(x, firstaxis $=1$, lastaxis $=2$, cex $=0.8$,
biptype="column-tube", scaleplot $=1$, plot3d = FALSE, pos=1,
size1=1, size2=2, invproj = FALSE, col1 = "blue", col2 = "red",...)

## Arguments

| x | The output parameters of the main function CA3variants. |
| :---: | :---: |
| firstaxis | The dimension reflected along the horizontal axis. |
| lastaxis | The dimension reflected along the vertical axis. |
| cex | The parameter that specifies the size of character labels of points in graphical displays. By default, it is equal to 0.8 . |
| biptype | The input parameter for specifying what kind of biplot is requested. By default, it is equal to column-tube, but could be row. |
| scaleplot | The scaling parameter for $\operatorname{pos}=1, \operatorname{size}=2, \operatorname{cols}=\mathrm{c}(1,4)$. By default, it is equal to 1 . |
| plot3d | The logical parameter specifies whether a 3D plot is to be included in the output or not. By default, plot3d = FALSE. |
| pos | The input parameter for changing the label position. By default, it is equal to 1. |
| size1 | The input parameter for specifying the size of pointers. By default, it is equal to 1. |
| size2 | The input parameter for specifying the label size. By default, it is equal to 2 . |
| invproj | The logical parameter specifies whether to portray standard coordinates as vectors and principal coordinates as points or vice-versa. By default, invproj = TRUE. |
| col1 | The input parameter for specifying the row label colour. By default, it is blue. |
| col2 | The input parameter for specifying the column label colour. By default, it is red. |
|  | Further arguments passed to or from other methods. |

## Details

It is utilised by the main function CA3variants and uses the secondary graphical function graph2poly.

## Value

Graphical displays of three-way correspondence analysis variants. Interactive plots or biplots are the graphical results of this function.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons

## Examples

```
data(happy)
ris.ca3<-CA3variants(happy, p = 2, q = 2, r = 2, ca3type = "CA3")
plot(ris.ca3,invproj=TRUE, col1="red", col2="green")
ris.nsca3<-CA3variants(happy, ca3type = "NSCA3")
plot(ris.nsca3, plot3d = TRUE)
ris.oca3<-CA3variants(happy, p = 2, q = 2, r = 2, ca3type = "OCA3", norder = 3)
plot(ris.oca3)
ris.onsca3<-CA3variants(happy, p = 2, q = 2, r = 2, ca3type = "ONSCA3", norder = 3)
plot(ris.onsca3)
```

print.CA3variants Print of three-way correspondence analysis results

## Description

This function prints the results of three-way symmetrical or non-symmetrical correspondence analysis. If the input parameter, in CA3variants, is ca3type="CA3", the function prints the results of three-way symmetrical correspondence analysis. If the input parameter, in CA3variants, is ca3type="NSCA3", the function prints the results of three-way non-symmetrical correspondence analysis. If the input parameter, in CA3variants, is ca3type="OCA3", the function prints the results of ordered three-way symmetrical correspondence analysis. If the input parameter, in CA3variants, is ca3type="ONSCA3", the function prints the results of ordered three-way non-symmetrical correspondence analysis. When the input parameter, in print. CA3variants, is digits $=3$, the function prints all the results using three digital numbers.

```
Usage
    ## S3 method for class 'CA3variants'
    print(x, digits = 3,...)
```


## Arguments

$$
\begin{array}{ll}
x & \text { the name of the output of the main function CAvariants. } \\
\text { digits } & \text { The input parameter specifying the digital number. By default, digits }=3 . \\
\ldots & \text { Further arguments passed to or from other methods. }
\end{array}
$$

## Value

The value of output returned depends on the kind of three-way correspondence analysis variant performed. It also gives the number of the iteration of the algorithm to reach the convergence of the solution. Depending on the variant of three-way correspondence analysis performed, it gives the related weighted contingency table, the reconstructed table by the components and core array, the explained inertia, the total inertia, the inertia in percentage, the proportion of explained inertia given the defined number of the components, the row standard and principal coordinates, the interactive column-tube standard and principal coordinates, the inner-product matrix of coordinates, the core array and index partitioning. In detail:

| CA3variants | The output of the kind of three-way correspondence analysis analysis considered. |
| :---: | :---: |
| DataMatrix | The original three-way contingency table. |
| xs | The centred and weighted three-way contingency table when the input parameters are $c t r=T$ and $s t d=T$. |
| xhat | The three-way contingency table approximated (reconstructed) by the three component matrices (of dimension Ixp, Jxq, and Kxr) and the core array. |
| nxhat2 | The sum of squares of the approximated contingency table. |
| prp | The ratio between the inertia of the complete contingency table and the inertia of the approximated contingency table. |
| fi | The principal row coordinates. |
| fic | The standard row coordinates. |
| gjk | The principal colum-tube coordinates. |
| rows | The row marginals of the three-way data table. |
| cols | The column marginals of the three-way data table. |
| tubes | The tube marginals of the three-way data table. |
| flabels | The row category labels. |
| glabels | The column category labels. |
| maxaxes | The maximum dimension to consider. |
| inertia | The total inertia of a variant of three-way correspondence analysis. |
| inertiaRSS | The residual inertia of a variant of three-way correspondence analysis. |
| inertiapc | The percentage contribution of the three components to the total variation. |
| inertiacoltub | The vector of the percentage contributions of the interactively coded colum-tube components to the total inertia, useful for making interactively coded biplots. |
| inertiarow | The vector of the percentage contributions of the row components to the total inertia, useful for making response biplots. |

iproduct The inner product between the standard row coordinates (fi) and the columntube principal coordinates (gjk).
g
The core array (i.e. the generalized singular values) calculated by using the Tuckals3 algorithm.
index3 When ca3type $=$ "CA3" or ca3type $=$ "OCA3", the index3 represents the partition of the Pearson index into three two-way association terms and one threeway association term. It also shows the C statistic of each term, its degrees of freedom and p-value. If ca3type $=$ "NSCA3" or ca3type $=$ "ONSCA3", the index 3 returns the partition of the Marcotorchino index into three two-way association terms and one three-way association term. It also shows the C statistic of each term, its degrees of freedom and p-value.
ca3type The specification of the analysis to be performed. When ca3type = "CA3", then a three-way symmetrical correspondence analysis will be performed (default analysis). If ca3type = "NSCA3", then three-way non-symmetrical correspondence analysis will be performed, where one of the variables is the response to be predicted given the other two variables. These two three-way variants use the Tucker3 method of decomposition. When ca3type = "OCA3" or ca3type = "ONSCA3", then an ordered three-way symmetrical or non-symmetrical correspondence analysis will be performed. Differently, these analysis use a new method of decomposition called Trivariate Moment Decomposition.
iteration The number of iterations that are required for the Tuckals3 algorithm to converge.

## Author(s)

Rosaria Lombardo, Eric J Beh.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons

## Examples

```
data(happy)
ris.ca3<-CA3variants(happy,p=2,q=2,r=2, ca3type = "CA3")
print(ris.ca3)
ris.nsca3<-CA3variants(happy,p=2,q=2,r=2, ca3type = "NSCA3")
print(ris.nsca3)
ris.oca3<-CA3variants(happy, p=2,q=2,r=2, ca3type = "OCA3",norder=3)
print(ris.oca3)
ris.onsca3<-CA3variants(happy,p=2,q=2,r=2, ca3type = "ONSCA3",norder=3)
print(ris.onsca3)
```

```
prod3 Products among arrays
```


## Description

The function calculates the products among arrays.

## Usage

$\operatorname{prod} 3(m, a 1, a 2, a 3)$

## Arguments

m The original three-way contingency table.
a1 The weight matrix related to the rows of the table.
a2 The weight matrix related to the columns of the table.
a3 The weight matrix related to the tubes of the table.

## Details

It is utilised in standtab, rstand3 and rstand3delta in order to weight the contingency table with respect to the three weigth matrices defined in the row, column and tube spaces differently for the three variants of three-way correspondence analysis.

## Value

The three-way contingency table weighted with respect the suitable weight matrices (depending on the analysis variants).

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons

```
    reconst3 Reconstruction of the three-way centred profile table
```


## Description

The function reconstructs the three-way centred profile table using the component matrices from the Tucker3 decomposition and the core array.

## Usage

reconst3(param)

## Arguments

param The matrices of the row, column and tube components derived via the Tucker3 model.

## Value

The three-way reconstructed table of centred profiles.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh E J and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

| rstand3 | Weighted centred three-way table for three-way non-symmetric corre- <br> spondence analysis |
| :--- | :--- |

## Description

The function computes the three-way weighted centred contingency table to perform three-way non-symmetric correspondence analysis with one response and two predictors.

## Usage

```
rstand3(x, std = T, ctr = T)
```


## Arguments

$x$
std
ctr $\quad$ The flag parameter for centering the original table. If $\mathrm{ctr}=\mathrm{F}$ the original array is not centered.

## Value

xs
The original three-way contingency table.
The flag parameter for weighting the original table. If std=F the original contingency table is not weighted.

The weighted array with respect to the three associated metrics. It is used when CA3variants="NSCA" and represents the three-way weighted and centred column profile table.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.
simulabootsimple Generation of parametric bootstrap samples

## Description

This function allows to generate parametric bootstrap samples in order to check the optimal dimension number of three-way correspondence analysis. The boostrap samples have the same marginal proportions and the total number of the original table. The adopted sampling scheme is simple.

## Usage

simulabootsimple(Xtable,nboots=100,resamptype=1)

## Arguments

Xtable The three-way data array. It must be an R object array. When non-symmetrical analysis for one response variable is performed, the response mode is the row variable.
nboots The number of bootstrap samples to generate when boots $=T$. By default nboots $=0$.
resamptype Set value of resamptype according to two methods: resamptype=1 corresponds to multinomial distribution and resamptype=2 to Poisson distribution.

## Author(s)

Michel van de Velden, Rosaria Lombardo and Eric J Beh.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. Wiley.
simulabootstrat Generation of parametric bootstrap samples

## Description

This function allows to generate parametric bootstrap samples in order to check the optimal dimension number of three-way correspondence analysis. The boostrap samples have the same marginal proportions and total number of the original table. Te adopted sampling scheme is stratified.

## Usage

simulabootstrat(Xtable,nboots=100, resamptype=1)

## Arguments

Xtable The three-way data array. It must be an $R$ object array. When non-symmetrical analysis for one response variable is performed, the response mode is the row variable.
nboots The number of bootstrap samples to generate when boots $=T$. By default nboots $=0$.
resamptype Set value of resamptype according to two methods: resamptype=1 corresponds to multinomial distribution and resamptype=2 to Poisson distribution.

## Author(s)

Rosaria Lombardo, Michel van de Velden, Eric J Beh.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. Wiley.

```
standtab Three-way centred column profile table for the three-way symmetric correspondence analysis
```


## Description

The function computes the three-way centred column profile table to perform three-way symmetric correspondence analysis.

## Usage

$\operatorname{standtab}(\mathrm{x}, \mathrm{std}=\mathrm{T}, \operatorname{ctr}=\mathrm{T})$

## Arguments

$x \quad$ The original three-way contingency table.
std The flag parameter for weighting the original table. If F the original contingency table is not weighted.
ctr The flag parameter for centering the original table. If F the original array is not centered.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.
step.g3 The core array derived via the Tucker 3 model.

## Description

The Tucker3 model involves the computation of principal components, which are derived for each of the three categorical variables, and of the core array which is akin to the generalised correlations between these components. The function step.g3 computes the core array.

## Usage

step.g3(param)

## Arguments

param The weighted three-way table and the matrices of the row, column and tube components derived via the Tuckals3 algorithm.

## Value

The core matrix whose the general element can be interpreted as a generalized singular value.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.
step.g3ordered The core array derived via the Trivariate Moment Decomposition model.

## Description

The Trivariate Moment Decomposition model involves the computation of principal polynomial components, which are derived for each of the three categorical variables, and of the polynomial core array which is akin to the generalised correlations between these components. The function step. g3ordered computes the core array.

## Usage

step.g3ordered(param)

## Arguments

param The weighted three-way table and the matrices of the row, column and tube components derived via the Trivariate Moment Decomposition algorithm.

## Value

The core matrix whose the general element can be interpreted as a generalized singular value.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

## Description

The function computes the component matrices from the Tuckals3 algorithm.

## Usage

stepi3(param)

## Arguments

param The weighted contingency table and the matrices of the row, column and tube components derived via the Tucker3 model.

## Details

The functions newcomp3, stepi3, init3 and step.g3 compute the component matrices and core array in the iterative steps of Tuckals3. They are all utilised from the function tucker.

## Value

Component matrices from the Tucker3 decomposition.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

```
stepi3ordered Component matrices from the Trivariate Moment Decomposition de-
    composition
```


## Description

The function computes the polynomial component matrices from the Emerson's recurrence formula for the ordered categorical variables of the analysis.

## Usage

stepi3ordered(param)

## Arguments

param The weighted contingency table and the matrices of the row, column and tube components derived via the Trivariate Moment Decomposition model.

## Details

The functions newcomp3ordered, stepi3ordered, init3ordered and step.g3ordered compute the polynomial component matrices and core array in the Trivariate Moment Decomposition. They are all utilised from the function tuckerORDERED.

## Value

Component matrices from the Trivariate Moment Decomposition decomposition.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

## Description

When the association among three categorical variables is asymmetric such that one variable is a logical response variable to the other variables, we recommend calculating the non-symmetrical three-way measure of predictability such as the Marcotorchino index (Marcotorchino, 1985). The function tau3 partitions the Marcotorchino statistic when, in CA3variants, we set the parameter ca3type = "NSCA3".

## Usage

tau3(f3, digits = 3)

## Arguments

f3
Three-way contingency array given as an input parameter in CA3variants.
digits Number of decimal digits. By default digits=3.

## Value

Z

CM the C statistic of the Marcotorchino index.
devt The denominator of the Marcotorchino index.

## Author(s)

Rosaria Lombardo, Eric J Beh, Ida Camminatiello.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

## Examples

```
data(happy)
tau3(happy,digits=3)
```

```
tau3ordered
```

The partition of the Marcotorchino three-way index.

## Description

When three categorical variables are symmetrically related, we can analyse the strength of the association using the three-way Marcotorchino index. The function chi3 partitions the Marcotorchino statistic using orthogonal polynomials when, in CA3variants, we set the parameter ca3type $=$ "ONSCA3".

## Usage

tau3ordered(f3, digits = 3)

## Arguments

f3 The three-way contingency array given as an input parameter in CA3variants.
digits The number of decimal digits. By default digits=3.

## Value

The partition of the Marcotorchino index into three two-way non-symmetrical association terms and one three-way association term. It also shows the polynomial componets of inertia, the percentage of explained inertia, the degrees of freedom and $p$-value of each term of the partition.

## Author(s)

Rosaria Lombardo, Eric J Beh.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.

## Examples

data(olive)
tau3ordered(f3=olive, digits=3)
threewayboot Generation of non-parametric bootstrap samples

## Description

This function allows to generate non-parametric bootstrap samples in order to check the optimal dimension number of three-way correspondence analysis. The boostrap samples have the same marginal proportions and the total number of the original table. Do nboots bootstrap on the indicator matrix X (observations x (rows+cols+tubs) categories). From a three-way contingency table, it makes the indicator using makeindicator. The output is a list of three-way tables.

## Usage

threewayboot (Xdata, nboots=100)

## Arguments

Xdata The three-way contingency array. It must be an $R$ object array.
nboots The number of bootstrap samples to generate when boots $=T$. By default nboots $=0$.

## Author(s)

Rosaria Lombardo, Michel van de Velden, Eric J Beh.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.
tucker
tucker Tucker3 decomposition of the three-way table.

## Description

The Tucker3 model, originally proposed by psychologist Ledyard R. Tucker, involves the computation of principal components, which are derived for each of the three categorical variables, and of the core array which is akin to the generalised correlations between these components. The function represents the heart of the Tuckals3 algorithm to perform the Tucker3 decomposition of the three-way array $x$.

## Usage

tucker (x, p, q, r, test = 10^-6)

## Arguments

$x \quad$ The three-way contingency table.
$p \quad$ The number of components of the first mode.
$q \quad$ The number of components of the second mode.
$r \quad$ The number of components of the third mode.
test The treshold used in the algorithm.

## Details

The function tucker is utilised from the functions ca3basic, nsca3basic and oca3basic.

## Value

a The final component derived from the Tucker3 decomposition for the first mode.
b The final component derived from the Tucker3 decomposition for the second mode.
cc The final component derived from the Tucker3 decomposition for the third mode.
g The core array.
$x \quad$ The three-way contingency table.
cont The number of iterations that are required for the Tucker3 algorithm to converge.

## Author(s)

Rosaria Lombardo, Eric J Beh.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.
tuckerORDERED Trivariate moment decomposition of the three-way table.

## Description

The Trivariate moment decomposition (TMD) represents the heart of a new algorithm to perform the decomposition of the three-way ordered contingency tables. It is based on the orthogonal polynomials (Emerson 1968) computed for each categorical ordered variable.

## Usage

tuckerORDERED(x, p, q, r, test = 10^-6,xi, norder=3)

## Arguments

$x \quad$ The three-way contingency table.
$p \quad$ The number of components of the first mode.
q The number of components of the second mode.
$r \quad$ The number of components of the third mode.
test The treshold used in the algorithm.
xi The original three-way contingency table.
norder The number of ordered variables.

## Details

The function tuckerORDERED is utilised from the function oca3basic.

## Value

a The final component derived from the TMD decomposition for the first mode.
b The final component derived from the TMD decomposition for the second mode.
cc The final component derived from the TMD decomposition for the third mode.
g The core array.
$x \quad$ The three-way contingency table.
cont The number of iterations that are required for the TMD algorithm to converge. If all variables are ordered, the convergence is reached in one step, differently if we have mixed variables. Indeed, the decmposition will become hybrid, a mix of TMD algorithm and Tuckals3 algorithm.

## Author(s)

Rosaria Lombardo, Eric J Beh.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons.
Emerson PL 1968 Numerical construction of orthogonal polynomials from a general recurrence formula. Biometrics, 24 (3), 695-701.
Lombardo R Beh EJ Variants of Simple Correspondence Analysis. The R Journal, 8 (2), 167-184.
Lombardo R Beh EJ and Kroonenberg PM (2016) Modelling Trends in Ordered Correspondence Analysis Using Orthogonal Polynomials. Psychometrika, 325-349.

Dimension selection for three-dimensional correspondence biplot.

## Description

This function allows to select the optimal dimension number for correspondence biplot, given the set of possible dimension combination of the original data. For exploring, it is possible to check the optimal dimension of boostrap samples which have the same marginal proportions and the total number of the original table. When the input parameter boots $=T$, it does bootstrap sampling. There are three kinds of possible bootstrap sampling. When boottype = "bootnp" it performs a non parametric bootstrap sampling. When boottype = "bootpsimple" it performs a parametric (for resamptype $=1$ is multinomial or for resamptype $=2$ is poisson) simple bootstrap sampling. When boottype = "bootpstrat", it performs a parametric stratified bootstrap sampling.

## Usage

tunelocal(Xdata, ca3type = "CA3", norder = 3, digits = 3, boots = FALSE, nboots $=0$, boottype= "bootpsimple", resamptype $=1$ )

## Arguments

| Xdata | The three-way data array. It must be an R object array. When non-symmetrical <br> analysis for one response variable is performed, the response mode is the row <br> variable. |
| :--- | :--- |
| ca3type | The specification of the analysis to be performed. If ca3type = "CA3", then <br> a three-way (symmetrical) correspondence analysis will be performed (default <br> analysis). If ca3type = "NSCA3", then three-way non-symmetrical correspon- <br> dence analysis will be performed. If ca3type = "OCA3", then ordered three- <br> way symmetrical correspondence analysis will be performed. If ca3type $=$ <br> "ONSCA3", then ordered three-way non-symmetrical correspondence analysis <br> will be performed. |
| norder | The input parameter for specifying the number of ordered variable when ca3type <br> $=$ <br> digits$\quad$The input parameter specifying the digital number. By default, digits = 3. |
| boots | The flag parameter to perform the search of optimal dimensions using bootstrap <br> samples. By defaults, boots = FALSE. |


| nboots | The number of bootstrap samples to generate when boots = TRUE. By default <br> nboots $=0$. |
| :--- | :--- |
| boottype | The specification of the kind of bootstrap sampling to be performed. If boottype <br> $=$ <br> "bootpsimple", then a parametric bootstrap using a simple sampling scheme |
|  | will be performed (default sampling). If boottype $=$ "bootpstrat", then a <br> parametric bootstrap using a stratified sampling scheme will be performed. If <br> boottype = "bootnp", then a non-parametric bootstrap using a simple sampling <br> scheme will be performed. |
| resamptype | When the kind of bootstrap is parametric you can set the data distribution using <br> the input parameter resamptype according to two distribution: resamptype=1 <br> corresponds to multinomial distribution and resamptype=2 to Poisson distribu- <br> tion. |

## Value

XG The list of tables on which is performed the three-way CA variant. It consists of the original array and (when boots $=T$ ) bootstrapped arrays.
output1 Chi-square criterion and df of models on the convex hull when using the original array.
output2 Chi-square criterion and df of models on the convex hull when using bootstrapped arrays.
output3 Badness of fit criterion and df of models on the convex hull when using the original array.
output4 Badness of fit criterion and df of models on the convex hull when using bootstrapped arrays.
output5 Goodness of fit criterion and df of models on the convex hull when using the original array.
output6 Goodness of fit criterion and df of models on the convex hull when using bootstrapped arrays.

## Author(s)

Rosaria Lombardo, Michel van de Velden, Eric J Beh.

## References

Beh EJ and Lombardo R (2014). Correspondence Analysis, Theory, Practice and New Strategies. John Wiley \& Sons. $\backslash$ Wilderjans T F, Ceulemans E, and Meers K (2013). CHull: A generic convex hull based model selection method. Behavior Research Methods, 45, 1-15.\ Ceulemans E, and Kiers H A L (2006). Selecting among three-mode principal component models of different types and complexities: A numerical convex hull based method. British Journal of Mathematical \& Statistical Psychology, 59, 133-150.

## Examples

```
data(happy)
tunelocal(happy, ca3type="CA3", boots=FALSE)
```


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