Package ‘Benchmarking’

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Description Methods for frontier analysis, Data Envelopment Analysis (DEA), under different technology assumptions (fdh, vrs, drs, crs, irs, add/frh, and fdh+), and using different efficiency measures (input based, output based, hyperbolic graph, additive, super, and directional efficiency). Peers and slacks are available, partial price information can be included, and optimal cost, revenue and profit can be calculated. Evaluation of mergers is also supported. Methods for graphing the technology sets are also included. There is also support comparative methods based on Stochastic Frontier Analyses (SFA). In general, the methods can be used to solve not only standard models, but also many other model variants. It complements the book, Bogetoft and Otto, Benchmarking with DEA, SFA, and R, Springer-Verlag, 2011, but can of course also be used as a stand-alone package.

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`Benchmarking-package` *Data Envelopment Analyses (DEA) and Stochastic Frontier Analyses (SFA) – Model Estimations and Efficiency Measuring*

Description

The Benchmarking package contains methods to estimate technologies and measure efficiencies using DEA and SFA. Data Envelopment Analysis (DEA) are supported under different technology assumptions (fdh, vrs, drs, crs, irs, add), and using different efficiency measures (input based, output based, hyperbolic graph, additive, super, directional). Peers are available, partial price information can be included, and optimal cost, revenue and profit can be calculated. Evaluation of mergers are also supported. A comparative method for estimating stochastic frontier function (SFA) efficiencies is included. The methods can solve not only standard models, but also many other model variants, and they can be modified to solve new models.

The package also support simple plots of DEA technologies with two goods; either as a transformation curve (2 outputs), an isoquant (2 inputs), or a production function (1 input and 1 output). When more inputs and outputs are available they are aggregated using weights (prices, relative prices).

The package complements the book, Bogetoft and Otto, *Benchmarking with DEA, SFA, and R*, Springer-Verlag 2011, but can of course also be used as a stand-alone package.

Details

Package:	Benchmarking
Type:	Package
Version:	0.27 ($Revision: 157 $)
Date:	$Date: 2015-07-09 11:40:20 +0200 (to, 09 jul 2015) $
License:	Copyright

`dea`	DEA input or output efficience measures, peers, lambdas and slacks
`dea.dual`	Dual weights (prices), including restrictions on weights
`dea.direct`	Directional efficiency
`sdea`	Super efficiency.
`dea.add`	Additive efficiency; sum of slacks in DEA technology.
`mea`	Multidirectional efficiency analysis or potential improvements.
`eff`	Efficiency from an object returned from any of the dea or sfa functions.
`slack`	Slacks in DEA models
`excess`	Calculates excess input or output compared to DEA frontier.
`peers`	get the peers for each firm.
`dea.boot`	Bootstrap DEA models
`cost.opt`	Optimal input for given output and prices.
`revenue.opt`	Optimal output for given input and prices.
`profit.opt`	Optimal input and output for given input and output prices.
`dea.plot`	Graphs of DEA technologies under alternative technology assumptions.
`dea.plot.frontier`	Specialized for 1 input and 1 output.
`dea.plot.isoquant`	Specialized for 2 inputs.
`dea.plot.transform`	Specialized for 2 outputs.
`eladder`	Efficiency ladder for a single firm.
`eladder.plot`	Plot efficiency ladder for a single firm.
`make.merge`	Make an aggregation matrix to perform mergers.
`dea.merge`	Decompose efficiency from a merger of firms
`sfa`	Stochastic frontier analysis, production, distance, and cost function (SFA)
`outlier.ap`	Detection of outliers
`eff.dens`	Estimate and plot kernel density of efficiencies
`critValue`	Critical values calculated from bootstrap DEA models.
`typeIerror`	Probability of a type I error for a test in bootstrap DEA models.

Note

The interface for the methods are very much like the interface to the methods in the package **FEAR** (Wilson 2008). One change is that the data now are transposed to reflect how data is usually available in applications, i.e. we have firms on rows, and inputs and output in the columns. Also, the argument for the options `RTS` and `ORIENTATION` can be given as memotechnical strings, and there are more options to control output.

The input and output matrices can contain negative numbers, and the methods can thereby manage restricted or fixed input or output.

The return is not just the efficiency, but also slacks, dual values (shadow prices), peers, and lambdas (weights).

Author(s)

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References

Bogetoft and Otto; *Benchmarking with DEA, SFA, and R*; Springer 2011

Paul W. Wilson (2008), "FEAR 1.0: A Software Package for Frontier Efficiency Analysis with R," *Socio-Economic Planning Sciences* 42, 247–254

Examples

```
# Plot of different technologies
x <- matrix(c(100,200,300,500),ncol=1,dimnames=list(LETTERS[1:4],"x"))
y <- matrix(c(75,100,300,400),ncol=1,dimnames=list(LETTERS[1:4],"y"))
dea.plot(x,y,RTS="vrs",ORIENTATION="in-out",txt=rownames(x))
dea.plot(x,y,RTS="drs",ORIENTATION="in-out",add=TRUE,lty="dashed",lwd=2)
dea.plot(x,y,RTS="crs",ORIENTATION="in-out",add=TRUE,lty="dotted")

dea.plot(x,y,RTS="fdh",ORIENTATION="in-out",txt=rownames(x),main="fdh")
dea.plot(x,y,RTS="irs",ORIENTATION="in-out",txt=TRUE,main="irs")
dea.plot(x,y,RTS="irs2",ORIENTATION="in-out",txt=rownames(x),main="irs2")
dea.plot(x,y,RTS="add",ORIENTATION="in-out",txt=rownames(x),main="add")

#  A quick frontier with 1 input and 1 output
dea.plot(x,y, main="Basic plot of frontier")

# Calculating efficiency
dea(x,y, RTS="vrs", ORIENTATION="in")
e <- dea(x,y, RTS="vrs", ORIENTATION="in")
e
eff(e)
peers(e)
peers(e, NAMES=TRUE)
print(peers(e, NAMES=TRUE), quote=FALSE)
lambda(e)
summary(e)
```

```
# Calculating super efficiency
esuper <- sdea(x,y, RTS="vrs", ORIENTATION="in")
esuper
print(peers(esuper,NAMES=TRUE),quote=FALSE)
# Technology for super efficiency for firm number 3/C
# Note that drop=FALSE is necessary for XREF and YREF to be matrices
# when one of the dimensions is or is reduced to 1.
e3 <- dea(x,y, XREF=x[-3,,drop=FALSE], YREF=y[-3,,drop=FALSE])
dea.plot(x[-3],y[-3],RTS="vrs",ORIENTATION="in-out",txt=LETTERS[c(1,2,4)])
points(x[3],y[3],cex=2)
text(x[3],y[3],LETTERS[3],adj=c(-.75,.75))
e3 <- dea(x,y, XREF=x[-3,,drop=FALSE], YREF=y[-3,,drop=FALSE])
eff(e3)
peers(e3)
print(peers(e3,NAMES=TRUE),quote=FALSE)
lambda(e3)
e3$lambda

# Taking care of slacks
x <- matrix(c(100,200,300,500,100,600),ncol=1,
        dimnames=list(LETTERS[1:6],"x"))
y <- matrix(c(75,100,300,400,50,400),ncol=1,
        dimnames=list(LETTERS[1:6],"y"))

# Phase one, calculate efficiency
e <- dea(x,y)
print(e)
peers(e)
lambda(e)
# Phase two, calculate slacks (maximize sum of slacks)
sl <- slack(x,y,e)
data.frame(sl$sx,sl$sy)
peers(sl)
lambda(sl)
sl$lambda
summary(sl)

# The two phases in one function call
e2 <- dea(x,y,SLACK=TRUE)
print(e2)
data.frame(eff(e2),e2$slack,e2$sx,e2$sy,lambda(e2))
peers(e2)
lambda(e2)
e2$lambda
```

`charnes1981` *Data: Charnes et al. (1981): Program follow through*

Description

The data set is from an US federally sponsored program for providing remedial assistance to disadvantaged primary school students. The firms are 70 school sites, and data are from entire sites. The variables consists of results from three different kind of tests, a reading score, `y1`, a math score, `y2`, and a self–esteem score, `y3`, which are considered outputs in the model, and five different variables considered to be inputs, the education level of the mother, `x1`, the highest occupation of a family member, `x2`, parental visits to school, `x3`, time spent with children in school-related topics, `x4`, and the number of teachers at the site, `x5`.

Usage

```
data(charnes1981)
```

Format

A data frame with 70 school sites with the following variables.

- `firm` school site number
- `x1` education level of the mother
- `x2` highest occupation of a family member
- `x3` parental visits to school
- `x4` time spent with children in school-related topics
- `x5` the number of teachers at the site
- `y1` reading score
- `y2` math score
- `y3` self–esteem score
- `pft` =1 if in program (program follow through) and =0 if not in program
- `name` Site name

Details

The command `data(charnes1981)` will create a data frame named `charnes1981` with the above data.

Beside input and output varianles there is further information in the data set, that the first 50 school sites followed the program and that the last 20 are the results for sites not following the program. This is showed by the variable `pft`.

Note

Data as `.csv` are loaded by the command `data` using `read.table(...,header=TRUE,sep=";")` such that this file is a semicolon separated file and not a comma separated file.

Therefore, to read the file from a script the command must be `read.csv("charnes1981.csv",sep=";")` or `read.csv2("charnes1981.csv")`.

Thus the data can be read either as `charnes1981 <-`
`read.csv2(paste(.Library,"Benchmarking/data","charnes1981.csv",sep ="/"))`
or as `data(charnes1981)` if the package **Benchmarking** is loaded. In both cases the data will be in the data frame `charnes1981`.

Source

Charnes, Cooper, and Rhodes, “Evaluating Program and Managerial Efficiency: An Application of Data Envelopment Analysis to Program Follow Through”, *Management Science*, volume 27, number 6, June 1981, pages 668–697.

Examples

```
data(charnes1981)
x <- with(charnes1981, cbind(x1,x2,x3,x4,x5))
y <- with(charnes1981, cbind(y1,y2,y3))

# Farrell inpout efficiency; vrs technology
e <- dea(x,y)
# The number of times each peer is a peer
np <- get.number.peers(e)
# Peers that are peers for more than 20 schools, and the number of
# times they are peers
np[which(np[,2]>20),]

# Plot first input against first output and emphasize the peers that
# are peers for more than 20 schools in the model with five inputs and
# three outputs
inp <- np[which(np[,2]>20),1]
dea.plot(x[,1],y[,1])
points(x[inp,1], y[inp,1], pch=16, col="red")
```

---

cost.opt — *DEA optimal cost, revenue, and profit*

Description

Estimates the input and/or output vector(s) that minimize cost, maximize revenue or maximize profit in the context of a DEA technology

Usage

```
cost.opt(XREF, YREF, W, YOBS=NULL, RTS="vrs", param=NULL,
         TRANSPOSE=FALSE, LP=FALSE, LPK = NULL)

revenue.opt(XREF, YREF, P, XOBS=NULL, RTS="vrs",  param=NULL,
            TRANSPOSE = FALSE, LP = FALSE, LPK = NULL)

profit.opt(XREF, YREF, W, P, RTS = "vrs",  param=NULL,
           TRANSPOSE = FALSE, LP = FALSE, LPK = NULL)
```

Arguments

Input and output matrices are in the same form as for the method dea.

XREF	Input of the firms defining the technology, a K x m matrix of observations of K firms with m inputs (firm x input). In case TRANSPOSE=TRUE the input matrix is transposed as input x firm.
YREF	output of the firms defining the technology, a K x n matrix of observations of K firms with n outputs (firm x input). In case TRANSPOSE=TRUE the output matrix is transposed as output x firm.
W	Input prices as a matrix. Either same prices for all firms or individual prices for all firms; i.e. either a 1 x m or a K x m matrix for K firms and m inputs
P	Output prices as a matrix. Either same prices for all firms or individual prices for all firms; i.e. either a 1 x n or K x n matrix for K firms and n outputs
XOBS	The input for which an optimal, revenue maximizing, output vector is to be calculated. Defaults is XREF. Same form as XREF
YOBS	The output for which an optimal, cost minimizing input vector is to be calculated. Defaults is YREF. Same form as YREF
RTS	A text string or a number defining the underlying DEA technology / returns to scale assumption.

0	fdh	Free disposability hull, no convexity assumption
1	vrs	Variable returns to scale, convexity and free disposability
2	drs	Decreasing returns to scale, convexity, down-scaling and free disposability
3	crs	Constant returns to scale, convexity and free disposability
4	irs	Increasing returns to scale, (up-scaling, but not down-scaling), convexity and free disposability
5	add	Additivity (scaling up and down, but only with integers), and free disposability
6	fdh+	A combination of free disposability and restricted or local constant return to scale

param	Possible parameters. At the moment only used for RTS="fdh+" to set low and high values for restrictions on lambda; see the section details and examples in dea for its use. Future versions might also use param for other purposes.
TRANSPOSE	Input and output matrices are treated as firms times goods for the default value TRANSPOSE=FALSE corresponding to the standard in R for statistical models. When TRUE data matrices, quantities and prices, are transposed to goods times firms matrices.
LP	Only for debugging. If LP=TRUE then input and output for the LP program are written to standard output for each unit.
LPK	When LPK=k then a mps file is written for firm k; it can be used as input to an alternative LP solver to check the results.

Details

The LP optimization problem is formulated in Bogetoft and Otto (2011, pp 35 and 102) and is solved by the LP method in the package **lpSolveAPI**.

The methods print and summary are working for cost.opt, revenue.opt, and profit.opt

Value

The values returned are the optimal input, and/or optimal output. When saved in an object the following components are available:

`xopt`	The optimal input, returned as a matrix by `cost.opt` and `profit.cost`.
`yopt`	The optimal output, returned as a matrix by `revenue.opt` and `profit.cost`.
`cost`	The optimal/minimal cost.
`revenue`	The optimal/maximal revenue
`profit`	The optimal/maximal profit
`lambda`	The peer weights that determines the technology, a matrix. Each row is the lambdas for the firm corresponding to that row; for the vrs technology the rows sum to 1. A column shows for a given firm how other firms are compared to this firm; i.e. peers are firms with a positive element in their columns.

Note

The index for peer units can be returned by the method `peers` and the weights are returned in `lambda`. Note that the peers now are the firms for the optimal input and/or output allocation, not just the technical efficient firms.

Author(s)

Peter Bogetoft and Lars Otto <larsot23@gmail.com>

References

Bogetoft and Otto; *Benchmarking with DEA, SFA, and R*; Springer 2011

See Also

Paul W. Wilson (2008), "FEAR 1.0: A Software Package for Frontier Efficiency Analysis with R," *Socio-Economic Planning Sciences* 42, 247–254

Examples

```
x <- matrix(c(2,12, 2,8, 5,5, 10,4, 10,6, 3,13), ncol=2, byrow=TRUE)
y <- matrix(1,nrow=dim(x)[1],ncol=1)
w <- matrix(c(1.5, 1),ncol=2)

txt <- LETTERS[1:dim(x)[1]]
dea.plot(x[,1],x[,2], ORIENTATION="in",  cex=1.25)
text(x[,1],x[,2],txt,adj=c(-.7,-.2),cex=1.25)

# technical efficiency
te <- dea(x,y,RTS="vrs")
xopt <- cost.opt(x,y,w,RTS=1)
cobs <- x %*% t(w)
copt <- xopt$x %*% t(w)
```

```
# cost efficiency
ce <- copt/cobs
# allocaltive efficiency
ae <- ce/te$eff
data.frame("ce"=ce,"te"=te$eff,"ae"=ae)
print(cbind("ce"=c(ce),"te"=te$eff,"ae"=c(ae)),digits=2)

# isocost line in the technology plot
abline(a=copt[1]/w[2], b=-w[1]/w[2], lty="dashed")
```

---

critValue *Critical values from bootstrapped DEA models*

Description

Calculates critical value for test using bootstrap output in DEA models

Usage

```
critValue(s, alpha=0.05)
```

Arguments

`s`	Vector with calculated values of the statistic for each of the `NREP` bootstraps; NREP is from `boot.sw98`
`alpha`	The size of the test

Details

Needs bootstrapped values of the test statistic

Value

Returns the critical value

Author(s)

Peter Bogetoft and Lars Otto <`larsot23@gmail.com`>

See Also

`boot.sw98` in **FEAR**, Paul W. Wilson (2008), "FEAR 1.0: A Software Package for Frontier Efficiency Analysis with R," *Socio-Economic Planning Sciences* 42, 247–254

Examples

```
# The critical value for two-sided test in normal distribution found
# by simulation.
x <- rnorm(1000000)
critValue(x,.975)
```

dea *DEA efficiency*

Description

Estimates a DEA frontier and calculates efficiency measures a la Farrell.

Usage

```
dea(X, Y, RTS="vrs", ORIENTATION="in", XREF=NULL, YREF=NULL,
    FRONT.IDX=NULL, SLACK=FALSE, DUAL=FALSE, DIRECT=NULL, param=NULL,
    TRANSPOSE=FALSE, FAST=FALSE, LP=FALSE, CONTROL=NULL, LPK=NULL)

## S3 method for class 'Farrell'
print(x, digits=4, ...)
## S3 method for class 'Farrell'
summary(object, digits=4, ...)
```

Arguments

`X` Inputs of firms to be evaluated, a K x m matrix of observations of K firms with m inputs (firm x input). In case `TRANSPOSE=TRUE` the input matrix is transposed to input x firm.

`Y` Outputs of firms to be evaluated, a K x n matrix of observations of K firms with n outputs (firm x input). In case `TRANSPOSE=TRUE` the output matrix is transposed to output x firm.

`RTS` Text string or a number defining the underlying DEA technology / returns to scale assumption.

0	fdh	Free disposability hull, no convexity assumption
1	vrs	Variable returns to scale, convexity and free disposability
2	drs	Decreasing returns to scale, convexity, down-scaling and free disposability
3	crs	Constant returns to scale, convexity and free disposability
4	irs	Increasing returns to scale, (up-scaling, but not down-scaling), convexity and free disposability
5	irs2	Increasing returns to scale (up-scaling, but not down-scaling), additivity, and free disposability
6	add	Additivity (scaling up and down, but only with integers), and free disposability; also known af replicability and fr
7	fdh+	A combination of free disposability and restricted or local constant return to scale
10	vrs+	As vrs, but with restrictions on the individual lambdas via `param`

`ORIENTATION` Input efficiency "in" (1), output efficiency "out" (2), and graph efficiency "graph" (3). For use with `DIRECT`, an additional option is "in-out" (0).

`XREF` Inputs of the firms determining the technology, defaults to `X`

`YREF` Outputs of the firms determining the technology, defaults to `Y`

`FRONT.IDX` Index for firms determining the technology

`SLACK` Calculate slack in a phase II calculation by an intern call of the function `slack`. Note that the precision for calculating slacks for orientation graph is low.

DUAL — Calculate dual variables, i.e. shadow prices; not calculated for orientation graph as that is not an LP problem.

DIRECT — Directional efficiency, `DIRECT` is either a scalar, an array, or a matrix with non-negative elements.

If the argument is a scalar, the direction is (1,1,...,1) times the scalar; the value of the efficiency depends on the scalar as well as on the unit of measurements.

If the argument is an array, this is used for the direction for every firm; the length of the array must correspond to the number of inputs and/or outputs depending on the `ORIENTATION`.

If the argument is a matrix then different directions are used for each firm. The dimensions depends on the `ORIENTATION` (and `TRANSPOSE`), the number of firms must correspond to the number of firms in `X` and `Y`.

`DIRECT` must not be used in connection with `DIRECTION="graph"`.

param — Possible parameters. At the moment only used for RTS="fdh+" to set low and high values for restrictions on lambda; see the section details and examples for its use. Future versions might also use `param` for other purposes.

TRANSPOSE — Input and output matrices are treated as firms times goods matrices for the default value `TRANSPOSE=FALSE` corresponding to the standard in R for statistical models. When `TRUE` data matrices are transposed to good times firms matrices as is normally used in LP formulation of the problem.

LP — Only for debugging. If `LP=TRUE` then input and output for the LP program are written to standard output for each unit.

FAST — Only calculate efficiencies and just return them as a vector, i.e. no lambda or other output. The return when using FAST cannot be used as input for `slack` and `peers`.

CONTROL — Possible controls to **lpSolveAPI**, see the documentation for that package.

... — Optional parameters for the print and summary methods.

object, x — An object of class Farrell (returned by the function `dea`) – R code uses ‘`object`’ and ‘`x`’ alternating for generic methods.

digits — digits in printed output, handled by format in print.

LPK — when `LPK=k` then a mps file is written for firm `k`; it can be used as input to an alternative LP solver to check the results.

Details

The return from `dea` and `sdea` is an object of class Farrell. The efficiency in `dea` is calculated by the LP method in the package **lpSolveAPI**. Slacks can be calculated either in the call of `dea` using the option `SLACK=TRUE` or in a following call to the function `slack`.

The directional efficiency when the argument `DIRECT` is used, depends on the unit of measurement and is not restricted to be less than 1 (or greater than 1 for output efficiency) and is therefore completely different from the Farrell efficiency.

The crs factor in RTS="fdh+" that sets the lower and upper bound can be changed by the argument `param` that will set the lower and upper bound to 1-param and 1+param; the default value is param=.15. The value must be greater than or equal to 0 and strictly less than 1. A value of 0 corresponds to RTS="fdh". To get an asymmetric interval set param to a 2 dimentional array with values for the low and high end for interval, for instance `param=c(.8,1.15)`. The FDH+ technology set is described in Bogetoft and Otto (2011) pages 73–74.

The technology RTS="vrs+" uses the parameter `param` to set restrictions on lambda, the convexity parameters. The elements of param are `param=(low,high,sum_low,sum_high)` where "low" and "high" are restrictions on the individual lambda and "sum_low" and "sum_high" are restrictions on the sum of lambdas. The individual lambda must be in the interval from low to high or be zero. With one parameter the restrictions set are `(param,1+1-(param),1,1)`, with two parameters `(param[1],param[2],1,1)`, and with four parameters `(param[1],param[2],param[3],param[4])`. The resulting technology set is not necessarily convex.

The graph orientated efficiency is calculated by bisection between feasible and infeasible values of G. The precision in the result is less than for the other orientations.

When the argument `DIRECT=d` is used then the returned value `e` for input orientation is the exces input measured in `d` units of measurements, i.e. $x - ed$, and for output orientation $y + ed$. The directional efficency can be restricted to inputs (`ORIENTAION="in"`), restricted to outputs (`ORIENTAION="out"`), or both include inputs and output directions (`ORIENTAION="in-out"`). Dirctional efficiency is discussed on pages 31–35 and 121–127 in Bogetoft and Otto (2011).

Value

The results are returned in a Farrell object with the following components. The last three components in the list are only part of the object when `SLACK=TRUE`.

`eff`	The efficiencies. Note when DIRECT is used then the efficencies are not Farrell efficiencies but rather exces values in DIRECT units of measurement
`lambda`	The lambdas, i.e. the weight of the peers, for each firm
`objval`	The objective value as returned from the LP program; normally the same as eff, but for `slack` it is the the sum of the slacks.
`RTS`	The return to scale assumption as in the option `RTS` in the call
`ORIENTATION`	The efficiency orientation as in the call
`TRANSPOSE`	As in the call
`slack`	A logical vector where the component for a firm is `TRUE` if the sums of slacks for the corresponding firm is positive. Only calculated in dea when option `SLACK=TRUE`
`sum`	A vector with sums of the slacks for each firm. Only calculated in dea when option `SLACK=TRUE`
`sx`	A matrix for input slacks for each firm, only calculated if the option `SLACK` is `TRUE` or returned from the method `slack`
`sy`	A matrix for output slack, see `sx`
`ux`	Dual variable for input, only calculated if `DUAL` is `TRUE`.
`vy`	Dual variable for output, only calculated if `DUAL` is `TRUE`.

Note

The arguments X, Y, XREF, and YREF are supposed to be matrices or numerical data frames that in the function will be converted to matrices. When subsetting a matrix or data frame to just one column then the class of the resulting object/variable is no longer a matrix or a data frame, but just a numeric (array, vector). Therefore, in this case a numeric input that is not a matrix nor a data frame is transformed to a 1 column matrix, and here the use of the argument TRANSPOSE=TRUE gives an error.

The dual values are not unique for extreme points (firms on the boundary with an efficiency of 1) and therefore the calculated dual values for these firms can depend on the order of firms in the reference technology. The same lack of uniqueness also make the peers for some firms depend on the order of firms in the reference technology.

To calucalte slack use the argument SLACK=TRUE or use the function slack directly.

When there is slack, and slack is not taken into consideration, then the peers for a firm with slack might depend on the order of firms in the data set; this is a property of the LP algorithm used to solve the problem.

To handle fixed, non-discretionary inputs, one can let it appear as negative output in an input-based mode, and reversely for fixed, non-discretionary outputs. Fixed inputs (outputs) can also be handled by directional efficiency; set the direction, the argument DIRECT, equal to the variable, discretionary inputs (outputs) and 0 for the fixed inputs (outputs).

When the the argument DIRECT=X is used the then the returned effiency is equal to 1 minus the Farrell efficiency for input orientation and to the Farrell effiency minus 1 for output orientation.

To use matrices X and Y prepared for the methods in the package **FEAR** (Wilson 2008) set the options TRANSPOSE=TRUE; for consistency with **FEAR** the options RTS and ORIENTATION also accepts numbers as in **FEAR**.

The tolerance that lambda is zero or one is 1e-7, the default value of 'epsint' in the package lpSolveAPI, i.e. values closer than 1e-7 from zero or one are set to respective integer value. The 'epsint' is the tolerance that is used to determine whether a floating-point number is in fact an in teger. The same tolerance is used for efficiency value near one.

Author(s)

Peter Bogetoft and Lars Otto <larsot23@gmail.com>

References

Peter Bogetoft and Lars Otto; *Benchmarking with DEA, SFA, and R*; Springer 2011

See Also

Paul W. Wilson (2008), "FEAR 1.0: A Software Package for Frontier Efficiency Analysis with R," *Socio-Economic Planning Sciences* 42, 247–254

Examples

```
x <- matrix(c(100,200,300,500,100,200,600),ncol=1)
y <- matrix(c(75,100,300,400,25,50,400),ncol=1)
dea.plot.frontier(x,y,txt=TRUE)
```

```
e <- dea(x,y)
eff(e)
print(e)
summary(e)
lambda(e)

# Input savings potential for each firm
(1-eff(e)) * x
(1-e$eff) * x

# calculate slacks
el <- dea(x,y,SLACK=TRUE)
data.frame(e$eff,el$eff,el$slack,el$sx,el$sy)

# Fully efficient units, eff==1 and no slack
which(eff(e) == 1 & !el$slack)

# fdh+ with limits in the interval [.7, 1.2]
dea(x,y,RTS="fdh+", param=c(.7,1.2))
```

---

dea.add *Additive DEA model*

Description

Calculates additive efficiency as sum of input and output slacks within different DEA models

Usage

```
dea.add(X, Y, RTS="vrs", XREF=NULL, YREF=NULL,
        FRONT.IDX=NULL, param=NULL, TRANSPOSE=FALSE, LP=FALSE)
```

Arguments

X	Inputs of firms to be evaluated, a K x m matrix of observations of K firms with m inputs (firm x input). In case `TRANSPOSE=TRUE` the input matrix is transposed to input x firm.
Y	Outputs of firms to be evaluated, a K x n matrix of observations of K firms with n outputs (firm x input). In case `TRANSPOSE=TRUE` the output matrix is transposed to output x firm.
RTS	Text string or a number defining the underlying DEA technology / returns to scale assumption.

0	fdh	Free disposability hull, no convexity assumption
1	vrs	Variable returns to scale, convexity and free disposability

2	drs	Decreasing returns to scale, convexity, down-scaling and free disposability
3	crs	Constant returns to scale, convexity and free disposability
4	irs	Increasing returns to scale, (up-scaling, but not down-scaling), convexity and free disposability
5	add	Additivity (scaling up and down, but only with integers), and free disposability

`XREF` Inputs of the firms determining the technology, defaults to `X`

`YREF` Outputs of the firms determining the technology, defaults to `Y`

`FRONT.IDX` Index for firms determining the technology

`param` Possible parameters. At the moment only used for RTS="fdh+" to set low and high values for restrictions on lambda; see the section details and examples for its use. Future versions might also use `param` for other purposes.

`TRANSPOSE` Input and output matrices are treated as firms times goods matrices for the default value `TRANSPOSE=FALSE` corresponding to the standard in R for statistical models. When `TRUE` data matrices are transposed to good times firms matrices as is normally used in LP formulation of the problem.

`LP` Only for debugging. If `LP=TRUE` then input and output for the LP program are written to standard output for each unit.

Details

The sum of the slacks is maximized in a LP formulation of the DEA technology. The sum of the slacks can be seen as distance to the frontier when you only move parallel to the axes of inputs and outputs, i.e. not a usual Euclidean distance, but what is also known as an L1 norm.

Since it is the sum of slacks that is calculated, there is no exogenous ORIENTATION in the problem. Rather, there is generally both an input and an output direction in the slacks. The model considers the input excess and output shortfall simultaneously and finds a point on the frontier that is most distant to the point being evaluated.

Value

`sum` Sum of all slacks for each firm, `sum=sum(sx)+sum(sy)`.

`slack` A non-NULL vector of logical variables, `TRUE` if there is slack for the corresponding firm, and `FALSE` if the there is no slack, i.e. the sum of slacks is zero.

`sx` A matrix of input slacks for each firm

`sy` A matrix of output slack for each firm

`lambda` The lambdas, i.e. the weights of the peers for each firm

Note

This is neither a Farrell nor a Shephard like efficiency.

The value of the slacks depends on the scaling of the different inputs and outputs. Therefore the values are not independent of how the input and output are measured.

Author(s)

Peter Bogetoft and Lars Otto `<larsot23@gmail.com>`

Source

Corresponds to Eqs. 4.34-4.38 in Cooper et al. (2007)

References

Bogetoft and Otto; *Benchmarking with DEA, SFA, and R*; Springer 2011

Cooper, Seiford, and Tone; *Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software*; Second edition, Springer 2007

Examples

```
x <- matrix(c(2,3,2,4,6,5,6,8),ncol=1)
y <- matrix(c(1,3,2,3,5,2,3,5),ncol=1)
dea.plot.frontier(x,y,txt=1:dim(x)[1])

sb <- dea.add(x,y,RTS="vrs")
data.frame("sx"=sb$sx,"sy"=sb$sy,"sum"=sb$sum,"slack"=sb$slack)
```

---

dea.boot — *Bootstrap DEA models*

Description

The function `dea.boot` bootstrap DEA models and returns bootstrap of Farrell efficiencies. This function is slower than the `boot.sw89` from the package **FEAR**. The faster function `boot.fear` is a wrapper for `boot.sw89` from the package **FEAR** returning results directly as Farrell measures.

Usage

```
dea.boot(X, Y, NREP = 200, EFF = NULL, RTS = "vrs", ORIENTATION="in",
         alpha = 0.05, XREF = NULL, YREF = NULL, FRONT.IDX=NULL, EREF = NULL,
         DIRECT = NULL, TRANSPOSE = FALSE, SHEPHARD.INPUT = TRUE, LP)

boot.fear(X, Y, NREP = 200, EFF = NULL, RTS = "vrs", ORIENTATION = "in",
         alpha = 0.05, XREF = NULL, YREF = NULL, EREF = NULL)
```

Arguments

`X`	Inputs of firms to be evaluated, a K x m matrix of observations of K firms with m inputs (firm x input)
`Y`	Outputs of firms to be evaluated, a K x n matrix of observations of K firms with n outputs (firm x input).
`NREP`	Number of bootstrap replicats
`EFF`	Efficiencies for (X,Y) relative to the technology generated from (XREF,YREF).
`RTS`	The returns to scale assumptions as in `dea`, only works for "vrs", "drs", and "crs"; more to come.

ORIENTATION	Input efficiency "in" (1), output efficiency "out" (2), and graph efficiency "graph" (3).
alpha	One minus the size of the confidence interval for the bias corrected efficiencies
XREF	Inputs of the firms determining the technology, defaults to X.
YREF	Outputs of the firms determining the technology, defaults to Y.
FRONT.IDX	Index for firms determining the technology.
EREF	Efficiencies for the firms in XREF, YREF.
DIRECT	Does not yet work and is therefore not used.
TRANSPOSE	Input and output matrices are K x m and K x n for the default value TRANSPOSE=FALSE; this is standard in R for statistical models. When TRANSPOSE=TRUE data matrices are m x K and n x K.
SHEPHARD.INPUT	The bootstrap of the Farrell input efficiencies is done as a Shephard input distance function, the inverse Farrell input efficiency. The option is only relevant for input and graph directions.
LP	Only for debugging purposes.

Details

The details are lightly explained in Bogetoft and Otto (2011) Chap. 6, and with more mathematical details in Dario and Simar (2007) Sect. 3.4 and in Simar and Wilson (1998).

The bootstrap at the moment does not work for any kind of directional efficiency.

The returned confidence intervals are for the bias corrected efficiencies; to get confidence intervals for the uncorrected efficiencies add the biases to both upper and lower values for the intervals.

Under the default option SHEPHARD.INPUT=TRUE bias and bias corrected efficiencies are calculated for Shephard input distance function and then transformed to Farrell input efficiencies to avoid possible negative biased corrected input efficiencies. If this is not wanted use the option SHEPHARD.INPUT=FALSE. This option is only relevant for input and graph oriented directions.

Value

The returned values from both functions are as follows:

eff	Efficiencies
eff.bc	Bias-corrected efficiencies
bias	An array of bootstrap bias estimates for the K firms
conf.int	K x 2 matrix with confidence interval for the estimated efficiencies
var	An array of bootstrap variance estimates for the K firms
boot	The replica bootstrap estimates of the Farrell efficiencies, a K x NREP matrix

Note

The function `dea.boot` does not depend on the FEAR package and can therefore be used on computers where the package FEAR is not available. This, however, comes with a time penalty as it takes around 4 times longer to run compared to using FEAR directly

The returned bootstrap estimates from `FEAR::boot.sw98` of efficiencies are sorted for each firm individually.

Unfortunately, this means that the component of replicas is not the efficiencies for the same bootstrap replica, but could easily be from differencts bootstrap replicas. This also means that this function can *not* be used to bootstrap tests for statistical hypotheses where the statistics involves summing of firms efficiencies.

Author(s)

Peter Bogetoft and Lars Otto <larsot23@gmail.com>

References

Peter Bogetoft and Lars Otto; *Benchmarking with DEA, SFA, and R*; Springer 2011.

Cinzia Dario and L. Simar; *Advanced Robust and Nonparametric Methods in Efficiency Analysis*. Methodology and Applications; Springer 2007.

Leopold Simar and Paul .W. Wilson (1998), "Sensitivity analysis of efficiency scores: How to bootstrap in nonparametric frontier models", *Management Science* 44, 49–61.

Paul W. Wilson (2008), "FEAR 1.0: A Software Package for Frontier Efficiency Analysis with R," *Socio-Economic Planning Sciences* 42, 247–254

See Also

The documentation for `boot.sw98` in the package **FEAR**.

Examples

```
x <- matrix(c(100,200,300,500,100,200,600),ncol=1)
y <- matrix(c( 75,100,300,400, 25, 50,400),ncol=1)

e <- dea(x,y)
eff(e)

dea.plot.frontier(x,y,txt=TRUE)

#  To bootstrap for real, NREP should be at least 2000. Run the
#  following lines a couple of times with nrep=100 and see how the
#  bootstrap frontier changes from one run to the next. Try the same
#  with NREP=2000 even though is does take a longer time to run,
#  especially for dea.boot.
nrep <- 5
# nrep <- 2000

# if ( "FEAR" %in% .packages(TRUE) )  {
##  The following only works if the package FEAR is installed; it does
```

```
##  not have to be loaded.
#  b <- boot.fear(x,y, NREP=nrep)
# } else {
  b <- dea.boot(x,y, NREP=nrep)
# }

#  bias corrected frontier
dea.plot.frontier(b$eff.bc*x, y, add=TRUE, lty="dashed")
#  outer 95% confidence interval frontier for uncorrected frontier
dea.plot.frontier((b$conf.int[,1]+b$bias)*x, y, add=TRUE, lty="dotted")


## Test of hypothesis in DEA model
# Null hypothesis is that technology is CRS and the alternative is VRS
# Bogetoft and Otto (2011) pages 183--185.
ec <- dea(x,y, RTS="crs")
Ec <- eff(ec)
ev <- dea(x,y, RTS="vrs")
Ev <- eff(ev)
# The test statistic; equation (6.1)
S <- sum(Ec)/sum(Ev)

# To calculate CRS and VRS efficiencies in the same bootstrap replicas
# we reset the random number generator before each call of the
# function dea.boot.

# To get the an initial value for the random number generating process
# we save its state (seed)
save.seed <- sample.int(1e9,1)

# The bootstrap and calculate CRS and VRS under the assumption that
# the true technology is CRS (the null hypothesis) and such that the
# results correponds to the case where CRS and VRS are calculated for
# the same reference set of firms; to make this happen we set the
# random number generator to the same state before the calls.
set.seed(save.seed)
bc <- dea.boot(x,y, nrep,, RTS="crs")
set.seed(save.seed)
bv <- dea.boot(x,y, nrep,, RTS="vrs", XREF=x,YREF=y, EREF=ec$eff)

# Calculate the statistic for each bootstrap replica
bs <- colSums(bc$boot)/colSums(bv$boot)
# The critical value for the test (default size \code{alpha} of test is 5%)
critValue(bs, alpha=.1)
S
# Accept the hypothesis at 10% level?
critValue(bs, alpha=.1) <= S

# The probability of observering a smaller value of S when the
# hypothesis is true; the p--value.
typeIerror(S, bs)
# Accept the hypothesis at size level 10%?
typeIerror(S, bs) >= .10
```

dea.direct *Directional efficiency*

Description

Directional efficiency rescaled to an interpretation a la Farrell efficiency and the corresponding peer importance (lambda).

Usage

```
dea.direct(X, Y, DIRECT, RTS = "vrs", ORIENTATION = "in",
          XREF = NULL, YREF = NULL, FRONT.IDX = NULL,
          SLACK = FALSE, param=NULL, TRANSPOSE = FALSE)
```

Arguments

`X` Inputs of firms to be evaluated, a K x m matrix of observations of K firms with m inputs (firm x input)

`Y` Outputs of firms to be evaluated, a K x n matrix of observations of K firms with n outputs (firm x input).

`DIRECT` Directional efficiency, `DIRECT` is either a scalar, an array, or a matrix with non-negative elements.

If the argument is a scalar, the direction is (1,1,...,1) times the scalar; the value of the efficiency depends on the scalar as well as on the unit of measurements.

If the argument an array, this is used for the direction for every firm; the length of the array must correspond to the number of inputs and/or outputs depending on the `ORIENTATION`.

If the argument is a matrix then different directions are used for each firm. The dimensions depends on the `ORIENTATION` (and `TRANSPOSE`), the number of firms must correspond to the number of firms in `X` and `Y`.

`DIRECT` must not be used in connection with `DIRECTION="graph"`.

`RTS` Text string or a number defining the underlying DEA technology / returns to scale assumption.

0	fdh	Free disposability hull, no convexity assumption
1	vrs	Variable returns to scale, convexity and free disposability
2	drs	Decreasing returns to scale (down-scaling, but not up-scaling), convexity, and free disposability
3	crs	Constant returns to scale, convexity and free disposability
4	irs	Increasing returns to scale (up-scaling, but not down-scaling), convexity, and free disposability
6	add	Additivity (scaling up and down, but only with integers), and free disposability
7	fdh+	A combination of free disposability and restricted or local constant return to scale

`ORIENTATION` Input efficiency "in" (1), output efficiency "out" (2), and graph efficiency "graph" (3). For use with `DIRECT`, an additional option is "in-out" (0).

`XREF` Inputs of the firms determining the technology, defaults to `X`.

YREF	Outputs of the firms determining the technology, defaults to Y.
FRONT.IDX	Index for firms determining the technology.
SLACK	See dea and slack.
param	Possible parameters. At the moment only used for RTS="fdh+" to set low and high values for restrictions on lambda; see the section details and examples in dea for its use. Future versions might also use param for other purposes.
TRANSPOSE	see dea

Details

When the argument DIRECT=d is used then component objval of the returned object for input orientation is the maximum value of e where for input orientation $x - ed$, and for output orientation $y + ed$ are in the generated technology set. The returned component eff is for input $1 - ed/X$ and for output $1 + ed/Y$ to make the interpretation as for a Farrell efficiency. Note that when the direction is not proportional to X or Y the returned eff are different for different inputs or outputs and eff is a matrix and not just an array. The directional efficiency can be restricted to inputs (ORIENTATION="in"), restricted to outputs (ORIENTATION="out"), or both include inputs and output directions (ORIENTATION="in-out"). Directional efficiency is discussed on pages 31–35 and 121–127 in Bogetoft and Otto (2011).

The Farrell efficiency interpretation is the ratio by which a firm can proportionally reduce all inputs (or expand all outputs) without producing less outputs (using more inputs). The directional efficiecies have the same interpretation expect that the direction is not proportional to the inputs (or outputs) and therefore the different inputs may have different reduction ratios, the efficiency is an array and not just a number.

Value

The results are returned in a Farrell object with the following components. The method slack only returns the three components in the list relevant for slacks.

eff	The Farrell efficiencies. Note that the the efficiencies are calculated to have the same interpretations as Farrell efficiencies. eff is a matrix if there are more than 1 good.
lambda	The lambdas, i.e. the weight of the peers, for each firm
objval	The objective value as returned from the LP program; the objval are excess values in DIRECT units of measurement.
RTS	The return to scale assumption as in the option RTS in the call
ORIENTATION	The efficiency orientation as in the call
TRANSPOSE	As in the call
slack	A vector with sums of the slacks for each firm. Only calculated in dea when option SLACK=TRUE
sx	A matrix for input slacks for each firm, only calculated if the option SLACK is TRUE or returned from the method slack
sy	A matrix for output slack, see sx

Note

To handle fixed, non-discretionary inputs, one can let it appear as negative output in an input-based mode, and reversely for fixed, non-discretionary outputs. Fixed inputs (outputs) can also be handled by directional efficiency; set the direction, the argument `DIRECT`, equal to the variable, discretionary inputs (outputs) and 0 for the fixed inputs (outputs).

When the the argument `DIRECT=X` is used the then the returned efficiency is equal to 1 minus the Farrell efficiency for input orientation and equal to the Farrell efficiency minus 1 for output orientation.

Author(s)

Peter Bogetoft and Lars Otto <larsot23@gmail.com>

References

Directional efficiency is discussed on pages 31–35 and 121–127 in Bogetoft and Otto (2011).

Peter Bogetoft and Lars Otto; *Benchmarking with DEA, SFA, and R*; Springer 2011

See Also

dea

Examples

```
# Directional efficiency
x <- matrix(c(2,5 , 1,2 , 2,2 , 3,2 , 3,1 , 4,1), ncol=2,byrow=TRUE)
y <- matrix(1,nrow=dim(x)[1])
dea.plot.isoquant(x[,1], x[,2],txt=1:dim(x)[1])

E <- dea(x,y)
z <- c(1,1)
e <- dea.direct(x,y,DIRECT=z)
data.frame(Farrell=E$eff, Perform=e$eff, objval=e$objval)
# The direction
arrows(x[,1], x[,2], (x-z)[,1], (x-z)[,2], lty="dashed")
# The efficiency (e$objval) along the direction
segments(x[,1], x[,2], (x-e$objval*z)[,1], (x-e$objval*z)[,2], lwd=2)



# Different directions
x1 <- c(.5, 1, 2, 4, 3, 1)
x2 <- c(4,  2, 1,.5, 2, 4)
x <- cbind(x1,x2)
y <- matrix(1,nrow=dim(x)[1])
dir1 <- c(1,.25)
dir2 <- c(.25, 4)
dir3 <- c(1,4)
e <- dea(x,y)
e1 <- dea.direct(x,y,DIRECT=dir1)
```

```
e2 <- dea.direct(x,y,DIRECT=dir2)
e3 <- dea.direct(x,y,DIRECT=dir3)
data.frame(e=eff(e),e1=e1$eff,e2=e2$eff,e3=e3$eff)[6,]

# Technology and directions for all firms
dea.plot.isoquant(x[,1], x[,2],txt=1:dim(x)[1])
arrows(x[,1], x[,2],  x[,1]-dir1[1], x[,2]-dir1[2],lty="dashed")
segments(x[,1], x[,2],
    x[,1]-e1$objval*dir1[1], x[,2]-e1$objval*dir1[2],lwd=2)
# slack for direction 1
dsl1 <- slack(x,y,e1)
cbind(E=e$eff,e1$eff,dsl1$sx,dsl1$sy, sum=dsl1$sum)



# Technology and directions for firm 6,
# Figure 2.6 page 32 in Bogetoft & Otto (2011)
dea.plot.isoquant(x1,x2,lwd=1.5, txt=TRUE)
arrows(x[6,1], x[6,2],  x[6,1]-dir1[1], x[6,2]-dir1[2],lty="dashed")
arrows(x[6,1], x[6,2],  x[6,1]-dir2[1], x[6,2]-dir2[2],lty="dashed")
arrows(x[6,1], x[6,2],  x[6,1]-dir3[1], x[6,2]-dir3[2],lty="dashed")
segments(x[6,1], x[6,2],
    x[6,1]-e1$objval[6]*dir1[1], x[6,2]-e1$objval[6]*dir1[2],lwd=2)
segments(x[6,1], x[6,2],
    x[6,1]-e2$objval[6]*dir2[1], x[6,2]-e2$objval[6]*dir2[2],lwd=2)
segments(x[6,1], x[6,2],
    x[6,1]-e3$objval[6]*dir3[1], x[6,2]-e3$objval[6]*dir3[2],lwd=2)
```

---

dea.dual *Dual DEA models and assurance regions*

Description

Solution of dual DEA models, possibly with partial value information given as restrictions on the ratios (assurance regions)

Usage

```
dea.dual(X, Y, RTS = "vrs", ORIENTATION = "in",
         XREF = NULL, YREF = NULL,
         FRONT.IDX = NULL, DUAL = NULL, DIRECT=NULL,
         TRANSPOSE = FALSE, LP = FALSE, CONTROL=NULL, LPK=NULL)
```

Arguments

X   Inputs of firms to be evaluated, a K x m matrix of observations of K firms with m inputs (firm x input). In case `TRANSPOSE=TRUE` the input matrix is transposed to input x firm.

Y — Outputs of firms to be evaluated, a K x n matrix of observations of K firms with n outputs (firm x input). In case `TRANSPOSE=TRUE` the output matrix is transposed to output x firm.

RTS — A text string or a number defining the underlying DEA technology / returns to scale assumption.

1	vrs	Variable returns to scale, convexity and free disposability
2	drs	Decreasing returns to scale, convexity, down-scaling and free disposability
3	crs	Constant returns to scale, convexity and free disposability
4	irs	Increasing returns to scale, (up-scaling, but not down-scaling), convexity and free disposability.

ORIENTATION — Input efficiency "in" (1), output efficiency "out" (2), and graph efficiency "graph" (3) (not yet implemented). For use with `DIRECT` an additional option is "in-out" (0). In this case, "graph" is not feasible

XREF — Input of the firms determining the technology, defaults to `X`

YREF — Output of the firms determining the technology, defaults to `Y`

FRONT.IDX — Index for firms determining the technology

DUAL — Matrix of order "number of input plus number of outputs minus 2" times 2. The first column is the lower bound and the second column is the upper bound for the restrictions on the multiplier ratios. The ratios are relative to the first input and the first output, respectively. This implyies that there is no restriction for neither the first input nor the first output so that the number of restrictions is two less than the total number of inputs and outputs.

DIRECT — Directional efficiency, `DIRECT` is either a scalar, an array, or a matrix with non-negative elements.

NB *Not yet implemented*

TRANSPOSE — Input and output matrices are treated as firms times goods for the default value `TRANSPOSE=FALSE` corresponding to the standard in R for statistical models. When `TRUE` data matrices shall be transposed to good times firms matrices as is normally used in LP formulation of the problem.

LP — Only for debugging. If `LP=TRUE` then input and output for the LP program are written to standard output for each unit.

CONTROL — Possible controls to lpSolveAPI, see the documentation for that package.

LPK — When `LPK=k` then a mps file is written for firm `k`; it can be used as input to an alternative LP solver just to check the our results.

Details

Solved as an LP program using the package lpSolveAPI. The method `dea.dual.dea` calls the method `dea` with the option `DUAL=TRUE`.

Value

eff — The efficiencies

objval — The objective value as returned from the LP problem, normally the same as eff

RTS	The return to scale assumption as in the option RTS in the call
ORIENTATION	The efficiency orientation as in the call
TRANSPOSE	As in the call
u	Dual values, prices, for inputs
v	Dual values, prices, for outputs
gamma	The values of gamma, the shadow price(s) for returns to scale restriction
sol	Solution of all variables as one component, sol=c(u,v,gamma).

Note

Note that the dual values are not unique for extreme points in the technology set. In this case the value of the calculated dual variable can depend on the order of the complete efficient firms.

Author(s)

Peter Bogetoft and Lars Otto <larsot23@gmail.com>

References

Bogetoft and Otto; *Benchmarking with DEA, SFA, and R*; Springer 2011. Sect. 5.10: Partial value information

See Also

dea

Examples

```
x <- matrix(c(2,5 , 1,2 , 2,2 , 3,2 , 3,1 , 4,1), ncol=2,byrow=TRUE)
y <- matrix(1,nrow=dim(x)[1])
dea.plot.isoquant(x[,1],x[,2],txt=1:dim(x)[1])
segments(0,0, x[,1], x[,2], lty="dotted")


e <- dea(x,y,RTS="crs",SLACK=TRUE)
ed <- dea.dual(x,y,RTS="crs")
print(cbind("e"=e$eff,"ed"=ed$eff, peers(e), lambda(e),
            e$sx, e$sy, ed$u, ed$v), digits=3)

dual <- matrix(c(.5, 2.5), nrow=dim(x)[2]+dim(y)[2]-2, ncol=2, byrow=TRUE)
er <- dea.dual(x,y,RTS="crs", DUAL=dual)
print(cbind("e"=e$eff,"ar"=er$eff, lambda(e), e$sx, e$sy, er$u,
            "ratio"=er$u[,2]/er$u[,1],er$v),digits=3)
```

dea.merge *Estimate potential merger gains and their decompositions*

Description

Calculate and decompose potential gains from mergers of similar firms (horizontal integration).

Usage

```
dea.merge(X, Y, M, RTS = "vrs", ORIENTATION = "in",
          XREF = NULL, YREF = NULL, FRONT.IDX = NULL, TRANSPOSE=FALSE)
```

Arguments

	Most of the arguments correspond to the arguments in `dea`, with K firms, m inputs, and n outputs.
`X`	K times m matrix as in `dea`
`Y`	K times n matrix as in `dea`
`M`	Kg times K matrix where each row defines a merger by the firms (collums) included; matrix as returned from method `make.merge`
`RTS`	as in `dea`
`ORIENTATION`	as in `dea`
`XREF`	as in `dea`
`YREF`	as in `dea`
`FRONT.IDX`	as in `dea`
`TRANSPOSE`	as in `dea`

Details

The K firms are merged into Kg new, merged firms.

The decomposition is summarized on page 275 and in table 9.1 page 276 in Bogetoft and Otto (2011) and is based on Bogetoft and Wang (2005)

Value

`Eff`	Overall efficiencies of mergers, Kg vector
`Estar`	Adjusted overall efficiencies of mergers after the removal of individual learning, Kg vector
`learning`	Learning effects, Kg vector
`harmony`	Harmony (scope) effects, Kg vector
`size`	Size (scale) effects, Kg vector

Author(s)

Peter Bogetoft and Lars Otto <larsot23@gmail.com>

References

Bogetoft and Otto; *Benchmarking with DEA, SFA, and R*; chapter 9; Springer 2011.

Bogetoft and Wang; "Estimating the Potential Gains from Mergers"; *Journal of Productivity Analysis*, 23, pp. 145-171, 2005.

See Also

dea and make.merge

Examples

```
x <- matrix(c(100,200,300,500),ncol=1,dimnames=list(LETTERS[1:4],"x"))
y <- matrix(c(75,100,300,400),ncol=1,dimnames=list(LETTERS[1:4], "y"))

dea.plot.frontier(x,y,RTS="vrs",txt=LETTERS[1:length(x)],
xlim=c(0,1000),ylim=c(0,1000) )
dea.plot.frontier(x,y,RTS="drs", add=TRUE, lty="dashed", lwd=2)
dea.plot.frontier(x,y,RTS="crs", add=TRUE, lty="dotted")

dea(x,y,RTS="crs")
M <- make.merge(list(c(1,2), c(3,4)), X=x)
xmer <- M %*% x
ymer <- M %*% y
points(xmer,ymer,pch=8)
text(xmer,ymer,labels=c("A+B","C+D"),pos=4)
dea.merge(x,y,M, RTS="vrs")
dea.merge(x,y,M, RTS="crs")
```

---

dea.plot *Plot of DEA technologies*

Description

Draw a graph of a DEA technology. Designed for two goods illustrations, either isoquant (2 inputs), transformation curve (2 outputs), or a production function (1 input and 1 output). If the number of good is larger than 2 then aggregation occur, either simple or weighted.

Usage

```
dea.plot(x, y, RTS="vrs", ORIENTATION="in-out", txt=NULL, add=FALSE,
         wx=NULL, wy=NULL, TRANSPOSE=FALSE, fex=1, GRID=FALSE,
         RANGE=FALSE, param=NULL, ..., xlim, ylim, xlab, ylab)
```

```
dea.plot.frontier(x, y, RTS="vrs",...)

dea.plot.isoquant(x1, x2, RTS="vrs",...)

dea.plot.transform(y1, y2, RTS="vrs",...)
```

Arguments

`x`	The good illustrated on the first axis. If there are more than 1 input then inputs are just summed or, if `wx` is present, a weighted sum of inputs is used.
`y`	The good illustrated on the second axis. If there are more than 1 output then outputs are just summed or, if `wy` is present, a weighted sum of outputs is used.
`x1, y1`	The good illustrated on the first axis
`x2, y2`	The good illustrated on the second axis
`RTS`	Underlying DEA model / assumptions about returns to scale: "fdh" (0), "vrs" (1), "drs" (2), "crs" (3), "irs" (4), "irs2" (5) (irs without convexity), "add" (6), and "fdh+" (7). Numbers in parenthesis can also be used as values for `RTS`
`ORIENTATION`	Input-output graph of 1 input and 1 output is "in-out" (0), graph of 2 inputs is "in" (1), and graph of 2 outputs is "out" (2).
`txt`	`txt` is an array to label the observations. If `txt=TRUE` the observations are labeled by the observation number or rownames if there are any.
`add`	For `add=T` the technology is drawn on top of an existing graph. With the default `add=F`, a new graph is made.
`wx`	Weight to aggregate the first axis if there are more than 1 good behind the first axis.
`wy`	Weights to aggregate for the second axis if there are more than 1 good behind the second the second axis.
`TRANSPOSE`	Only relevant for more than 1 good for each axis, see `dea` for a description of this option.
`GRID`	If `GRIF=TRUE` a gray grid is put on the plot.
`...`	Usual options for the methods `plot`, `lines`, and `abline` etc.
`fex`	Relative size of the text/labels on observations; corresponds to `cex`, but only changes the size of the text.
`RANGE`	A logical variable, if `RANGE=TRUE` the limits for the graph is the range of the variables; zero is always included. Default is `RANGE=FALSE` when the range is from zero to the max values. Relevant if some values are negative.
`param`	Possible parameters. At the moment only used for RTS="fdh+"; see the section details and examples for its use. Future versions might also use `param` for other purposes.
`xlim`	Possible limits `c(x1,x2)` for the first axis
`ylim`	Possible limits `c(y1,y2)` for the second axis
`xlab`	Possible label for the x-axis
`ylab`	Possible label for the y-axis

Details

The method `dea.plot` is the general plotting method. The the 3 others are specialized versions for frontiers (1 input and 1 output), isoquant curves (2 inputs for given outputs), and transformation curves (2 outputs for given inputs) obtained by using the argument `ORIENTATION`.

The crs factor in RTS="fdh+" that sets the lower and upper bound can be changed by the argument `param` that will set the lower and upper bound to 1-param and 1+param; the default value is param=.15. The value must be greater than or equal to 0 and strictly less than 1. A value of 0 corresponds to RTS="fdh". The FDH+ technology set is described in Bogetoft and Otto (2011) pages 72–73.

Value

No return, uses the original graphing system.

Note

If there are more than 1 good for the arguments `x` and `y` then the goods are just summed or, if `wx` or `wy` are present, weighted sum of goods are used. In this case the use of the command `identify` must be calles as `dea.plot(rowSums(x),rowSums(y))`.

Warning If you use this facility to plot multi input and multi output then the plot may deceive you as fully multi efficient firms are not necessarely placed on the two dimensional frontier.

Note that `RTS="add"` and `RTS="fdh+"` only works for `ORIENTATION="in-out"` (0).

Author(s)

Peter Bogetoft and Lars Otto `<larsot23@gmail.com>`

References

Peter Bogetoft and Lars Otto; *Benchmarking with DEA, SFA, and R*; Springer 2011

Paul Murrell; *R Graphics*; Chapman & Hall 2006

See Also

The documentation for the function `plot` and Murrell (2006) for further options and on customizing plots.

Examples

```
x <- matrix(c(100,200,300,500,600,100),ncol=1)
y <- matrix(c(75,100,300,400,400,50),ncol=1)

dea.plot(x,y,RTS="vrs",ORIENTATION="in-out",txt=LETTERS[1:length(x)])
dea.plot(x,y,RTS="crs",ORIENTATION="in-out",add=TRUE,lty="dashed")

dea.plot.frontier(x,y,txt=1:dim(x)[1])

n <- 10
```

```
x <- matrix(1:n,,1)
y <- matrix(x^(1.6) + abs(rnorm(n)),,1)
dea.plot.frontier(x,y,RTS="irs",txt=1:n)
dea.plot.frontier(x,y,RTS="irs2",add=TRUE,lty="dotted")

# Two different forms of irs: irs and irs2, and two different ways to
# make a frontier
id <- sample(1:n,30,replace=TRUE)
dea.plot(x[id],y[id],RTS="irs",ORIENTATION="in-out")
dea.plot.frontier(x[id],y[id],RTS="irs2")

# Difference between the FDH technology and the additive
# FRH technology
x <- matrix(c(100,220,300,520,600,100),ncol=1)
y <- matrix(c(75,100,300,400,400,50),ncol=1)
dea.plot(x,y,RTS="fdh",ORIENTATION="in-out",txt=LETTERS[1:length(x)])
dea.plot(x,y,RTS="add",ORIENTATION="in-out",add=TRUE,lty="dashed",lwd=2)
dea.plot(x,y,RTS="fdh+",ORIENTATION="in-out",add=TRUE,
                            lty="dotted",lwd=3,col="red")

# Use of parameter in FDH+
dea.plot(x,y,RTS="fdh",ORIENTATION="in-out",txt=LETTERS[1:length(x)])
dea.plot(x,y,RTS="fdh+",ORIENTATION="in-out",add=TRUE,lty="dashed")
dea.plot(x,y,RTS="fdh+",ORIENTATION="in-out",add=TRUE,lty="dotted",param=.5)
```

`eff, efficiencies` *Calculate efficiencies for Farrell and sfa object*

Description

Calculate efficiencies for Farrell and sfa object. For a sfa there are several types

Usage

```
eff( object, ... )
efficiencies( object, ... )
## Default S3 method:
efficiencies( object, ... )
## S3 method for class 'Farrell'
efficiencies(object, type = "Farrell", ...)
## S3 method for class 'Farrell'
eff(object, type = "Farrell", ...)
## S3 method for class 'sfa'
efficiencies(object, type = "BC", ...)
## S3 method for class 'sfa'
eff(object, type = "BC", ...)
```

Arguments

`object`	A Farrell object returned from a DEA function like dea, sdea, or mea or an sfa object returned from the function sfa.
`type`	The type of efficiencies to be calculated. For a Farrell object the possibilities are "Farrell" efficiency or "Shephard" efficiency. For a sfa object the possibilities are "BC", "Mode", "J", or "add".
`...`	Further arguments ...

Details

The possible types for class `Farrell` (an object returned from `dea` et al. are "Farrell" and "Shephard".

The possible types for class `sfa` efficiencies are

BC Efficiencies estimated by minimizing the mean square error; Eq. (7.21) in Bogetoft and Otto (2011, 219) and Battese and Coelli (1988, 392)

Mode Efficiencies estimates using the conditional mode approach; Bogetoft and Otto (2011, 219), Jondrow et al. (1982, 235).

J Efficiencies estimates using the conditional mean approach Jondrow et al. (1982, 235).

add Efficiency in the additive model, Bogetoft and Otto (2011, 219)

Value

The efficiencies are returned as an array.

Note

For the Farrell object the orientation is determined by the calculations that led to the object and can not be changed here.

Author(s)

Peter Bogetoft and Lars Otto <larsot23@gmail.com>

References

Bogetoft and Otto; *Benchmarking with DEA, SFA, and R*, Springer 2011

See Also

`dea` and `sfa`.

Examples

```
##---- Should be DIRECTLY executable !! ----
##-- ==>  Define data, use random,
##--or do  help(data=index)  for the standard data sets.
```

eff.dens *Estimate and plot density of efficiencies*

Description

A method to estimate and plot kernel estimate of (Farrell) efficiencies taken into consideration that efficiencies are bounded either above (input direction) or below (output direction).

Usage

```
eff.dens(eff, bw = "nrd0")

eff.dens.plot(obj, bw = "nrd0", ..., xlim, ylim, xlab, ylab)
```

Arguments

`eff`	Either a list of (Farrell) efficiencies or a Farrell object returned from the method `dea`.
`bw`	Bandwith, look at the documentation of `density` for an explanation.
`obj`	Either an array of efficiencies or a list returned from `eff.dens`.
`...`	Further arguments to the `plot` method like line type and line width.
`xlim`	Range on the x-axis; usualy not needed, just use the defaults.
`ylim`	Range on the x-axis; usualy not needed, just use the defaults.
`xlab`	Label for the x-axis.
`ylab`	Label for the y-axis.

Details

The calculation is based on a reflection method (Silverman 1986, 30) using the default window kernel and defult bandwidth (window width) in the method `density`.

The method `eff.dens.plot` plot the density directly, and `eff.dens` just estimate the numerical density, and the result can then either be plotted by `plot`, corresponds to `eff.dens.plot`, or by lines as an overlay on an existing plot.

Value

The return from `eff.dens` is a list `list(x,y)` with efficiencies and the corresponding density values.

Note

The input efficiency is also bounded below by 0, but for normal firms an efficiency at 0 will not happen, i.e. the boundary is not effective, and therefore this boundary is not taken into consideration.

Author(s)

Peter Bogetoft and Lars Otto <larsot23@gmail.com>

References

B.W. Silverman (1986), *Density Estimation for Statistics and Data Analysis*, Chapman and Hall, London.

Examples

```
e <- 1 - rnorm(100)
e[e>1] <- 1
e <- e[e>0]
eff.dens.plot(e)

hist(e, breaks=15, freq=FALSE, xlab="Efficiency", main="")
den <- eff.dens(e)
lines(den,lw=2)
```

---

eladder *Efficiency ladder for a single firm*

Description

How the efficiency changes as the most influential peer is removed sequentially one at a time

Usage

```
eladder(n, X, Y, RTS = "vrs", ORIENTATION = "in",
        XREF=NULL, YREF=NULL, DIRECT = NULL, param=NULL, MAXELAD=NULL)
eladder.plot(elad, peer, TRIM = NULL, ...)
```

Arguments

n	The number of the firm where the ladder is calculated
X	Inputs of firms to be evaluated, a K x m matrix of observations of K firms with m inputs (firm x input). In case TRANSPOSE=TRUE the input matrix is transposed to input x firm.
Y	Outputs of firms to be evaluated, a K x n matrix of observations of K firms with n outputs (firm x input). In case TRANSPOSE=TRUE the output matrix is transposed to output x firm.
RTS	Text string or a number defining the underlying DEA technology / returns to scale assumption, se the possible values for dea.
ORIENTATION	Input efficiency "in" (1), output efficiency "out" (2), and graph efficiency "graph" (3). For use with DIRECT, an additional option is "in-out" (0).
XREF	Inputs of the firms determining the technology, defaults to X

YREF	Outputs of the firms determining the technology, defaults to Y
DIRECT	Directional efficiency, DIRECT is either a scalar, an array, or a matrix with non-negative elements. See dea for a further description of this argument.
param	Possible parameters. At the moment only used for RTS="fdh+" to set low and high values for restrictions on lambda; see the section details and examples in dea for its use. Future versions might also use param for other purposes.
MAXELAD	The maximum number of influential peers to remove.
elad	The sequence of efficiencies returned from eladder.
peer	The sequence of peers returned from eladder.
TRIM	The number of characters for the name of the peers on the axis in the plot.
...	Usual options for the method plot.

Details

The function eladder calculates how the efficiency for a firm changes when the most influential peer is removed sequentially one at a time. Somewhere in the sequence the firm becomes efficient and are itself removed from the set of firms generating the technology (or the only firm left) and thereafter the efficiencies are super-efficiencies.

Value

The object returned from eladder is a list with components

eff	The sequence of efficiencies when the peer with the largest value of lambda has been removed.
peer	The sequence of removed peers corresponding to the largest values of lambda as index in the X rows.

Note

When the number of firms is large then the number of influential peers will also be large and the names or numbers of the peers on the x-axis might be squeeze together and be illegible. In this case restrict the number of influential peers to be removed.

The efficiency step ladder is discussed in Essay 4 of Dag Fjeld Edvardsens's Ph.D. thesis from 2004.

Author(s)

Peter Bogetoft and Lars Otto <larsot23@gmail.com>

References

Dag Fjeld Edvardsen; *Four Essays on the Measurement of Productive Efficiency*; University of Gothenburg 2004; http://hdl.handle.net/2077/2923

Examples

```
data(charnes1981)
x <- with(charnes1981, cbind(x1,x2,x3,x4,x5))
y <- with(charnes1981, cbind(y1,y2,y3))

# Choose the firm for analysis, we choose 'Tacoma'
n <- which(charnes1981$name=="Tacoma")[1]

el <- eladder(n, x, y, RTS="crs")
eladder.plot(el$eff, el$peer)

# Restrict to 20 most influential peers for 'Tacoma' and use names
# instead of number
eladder.plot(el$eff[1:20], charnes1981$name[el$peer][1:20])

# Truncate the names of the peers and put a title on top
eladder.plot(el$eff[1:20], charnes1981$name[el$peer][1:20], TRIM=5)
title("Eladder for Tacoma")
```

`excess` *Excess input compared over frontier input*

Description

Excess input compared over frontier input and/or less output than frontier/transformation/optimal output.

Usage

```
excess(object, X = NULL, Y = NULL)
```

Arguments

`object`	A Farrell object as returned from functions like dea, dea.direct, linksdea, and mea.
`X`	Input matrix, only neccesary for ordinary input Farrell efficiency
`Y`	Ouput matrix , only neccesary for ordinary output Farrell efficiency

Details

For Farrell input efficiency E the exess input is $(1-E)X$ and for Farrell ouput efficiency F the missing output is $(F-1)Y$.

Notice that the excess calculated does not include any slackvalues. In case slacks are present and calculated it might be more appropriate to add slack; i.e. to use `excess(object,X,Y) + slack(X,Y,object)`.

For directional efficiency e in the direction D the excess input is eD.

If a firm is outside the technology set, as could be the case when calculating super-efficiencies, the Farrell input efficiency is larger then 1 and then the excess values are negative.

Value

Return a matrix with exces input and/or less output.

Author(s)

Peter Bogeroft and Lars Otto <larsot23@gmail.com>

References

Peter Bogetoft and Lars Otto; *Benchmarking with DEA, SFA, and R*; Springer 2011

Examples

```
x <- matrix(c(100,200,300,500,100,200,600),ncol=1)
y <- matrix(c(75,100,300,400,25,50,400),ncol=1)

e <- dea(x,y)
excess(e,x)
x - eff(e) * x

e <- dea(x,y, ORIENTATION="graph")
excess(e, x, y)
x - eff(e) * x
1/eff(e) * y -y

me <- mea(x,y)
excess(me)
```

---

lambda — *Lambdas or the weight of the peers*

Description

The lambdas, i.e. the weight of the peers, for each firm.

Usage

```
lambda(object, KEEPREF = FALSE)
lambda.print(x, KEEPREF = FALSE, ...)
```

Arguments

`object,x`	A Farrell object as returned from `dea` et al.
`KEEPREF`	if `TRUE` then all firms are kept as reference firms even though they have all zero weights (lambda); might come handy if one needs to calculate `X` x `lambda` such that the firms in `X` and `lambda` agree. If `FALSE`, the default, then only weight for the peers are in the matrix `lambda`.
`...`	Optional parameters for the print method.

Details

Only returns the the lambdas for firms that appear as a peer, i.e. only lambdas for firms where at least one element of the lambda is positive.

Value

The return is a matrix with the firms as rows and the peers as columns.

Author(s)

Peter Bogetoft and Lars Otto <larsot23@gmail.com>

See Also

dea

Examples

```
##---- Should be DIRECTLY executable !! ----
##-- ==>  Define data, use random,
##--or do  help(data=index)  for the standard data sets.
```

---

make.merge *Make an aggregation matrix to perform mergers*

Description

Make an aggregation matrix to perform mergers of firms. The matrix can be post multiplied (matrix multiplication) to input and output matrices to make merged input and output matrices.

Usage

```
make.merge(grp, nFirm = NULL, X = NULL, names = NULL)
```

Arguments

grp	Either a list of length Kg for Kg firms after mergers; each component of the list is a (named) list with the firm numbers or names going into this merger. Or a factor of length K with Kg levels where where each level determines a merger; to exclude firms for mergers set the factor value to NA.
nFirm	Number of firms before the mergers
X	A matrix of inputs or outputs where the rows corresponds to the number of original (starting) firms
names	A list with names of all firms, only needed if the mergers are given as a list of names, i.e. grp is a list of names.

Details

Either nFirm or X must be present; if both are present then nFirm must be equal to the number of rows in X, the number of firms.

When X is an input matrix of dimension K x m, K firms and m inputs, and M <-make.merge(gr,K) then M %*% X is the input matrix for the merged firms.

Value

Returns an aggregation matrix of dimension Kg times K where rows corresponds to new merged firms and columns are 1 for firms to be included and 0 for firms to be excluded in the the given merger as defined by the row.

Note

The argument TRANSPOSE has not been implemented for this function. If you need transposed matrices you must transpose the merger matrix yourself. If you define mergers via factors there is no need to transpose in the arguments; just do not use X in the arguments.

Author(s)

Peter Bogetoft and Lars Otto <larsot23@gmail.com>

See Also

dea.merge

Examples

```
# To merge firms 1,3, and 5; and 2 and 4 of 7 firms into 2 new firms
# the aggregation matrix is M; not all firms are involved in a merger.
M <- make.merge(list(c(1,3,5), c(2,4)),7)
print(M)

# Merge 1 and 2, and 4 and 5, and leave 3 alone, total of 5 firms.
# Using a list
M1 <- make.merge(list(c(1,2), c(4,5)), nFirm=5)
print(M1)

# Using a factor
fgr <- factor(c("en","en",NA,"to","to"))
M2 <- make.merge(fgr)
print(M2)

# Name of mergers
M3 <- make.merge(list(AB=c("A","B"), DE=c("D","E")), names=LETTERS[1:5])
print(M3)

# No name of mergers
M4 <- make.merge(list(c("A","B"), c("D","E")), names=LETTERS[1:5])
print(M4)
```

malmq *Malmquist index*

Description

Estimates Malmquist indices for productivity and its decomposition beteween two periods. The units in the two periods does not have to be exactly the same, but the Malmquist index is only calculated for units present in both periods.

Usage

```
malmq(X0, Y0, ID0 = NULL, X1, Y1, ID1 = NULL, RTS = "vrs", ORIENTATION = "in",
SLACK = FALSE, DUAL = FALSE, DIRECT = NULL, param = NULL, TRANSPOSE = FALSE,
FAST = TRUE, LP = FALSE, CONTROL = NULL, LPK = NULL)
```

Arguments

`X0`	Inputs of firms in period 0, a K0 x m matrix of observations of K0 firms with m inputs (firm x input).
`Y0`	Outputs of firms in period 0, a K0 x n matrix of observations of K0 firms with n outputs (firm x input).
`ID0`	Index for firms in period 0; could be numbers or labels. Length K0.
`X1`	Inputs of firms in period 1, a K1 x m matrix of observations of K1 firms with m inputs (firm x input).
`Y1`	Outputs of firms in period 1, a K1 x n matrix of observations of K1 firms with n outputs (firm x input).
`ID1`	Index for firms in period 0; could be numbers or labels. Length K0.
`RTS`	Returns to scale assumption as in dea.
`ORIENTATION`	Input efficiency "in" (1), output efficiency "out" (2), and graph efficiency "graph" (3) as in dea.
`SLACK`	See dea.
`DUAL`	See dea.
`DIRECT`	See dea.
`param`	See dea.
`TRANSPOSE`	See dea.
`FAST`	See dea.
`LP`	See dea.
`CONTROL`	See dea.
`LPK`	See dea.

Details

The *index for technical changes* `tc` is calculated as `sqrt(e10/e11 * e00/e01)` where `e<s><t>` is the efficience for period `s` when the refenrece technology is for period `t`, i.e. determined from the observations for period `t` and `XREF=X_t,YREF=Y_t`, as is the option for the function `dea`.

The *Malmquist index for productivity* `mq` is calculates as `sqrt(e10/e00 * e11/e01)` and the *index for change in efficiency* `ec` is `e11/e00`. Note that `mq = tc * ec`.

Value

`m`	Malmquist index for productivity.
`tc`	Index for technoligy change.
`ec`	Index for efficiency change.
`mq`	Malmquist index for productivity; same as `m`.
`id`	Index for firms present in both period 0 and period 1.
`id0`	Index for firms in period 0 that are also in period 1.
`id1`	Index for firms in period 1 that are also in period 0.
`e00`	The efficiencies for period 0 with reference technoligy from period 0.
`e10`	The efficiencies for period 1 with reference technoligy from period 0.
`e11`	The efficiencies for period 1 with reference technoligy from period 1.
`e01`	The efficiencies for period 0 with reference technoligy from period 1.

Note

The calculations of efficiencies are only done for units present in both periods.

Author(s)

Peter Bogetoft and Lars Otto <larsot23@gmail.com>

References

Peter Bogetoft and Lars Otto; *Benchmarking with DEA, SFA, and R*; Springer 2011

See Also

dea

Examples

```
x0 <- matrix(c(10, 28, 30, 60),ncol=1)
y0 <- matrix(c(5, 7, 10, 15),ncol=1)
x1 <- matrix(c(12, 26, 16, 60 ),ncol=1)
y1 <- matrix(c(6, 8, 9, 15 ),ncol=1)

dea.plot(x0, y0, RTS="vrs", txt=TRUE)
dea.plot(x1, y1, RTS="vrs", add=TRUE, col="red")
```

```
points(x1, y1, col="red", pch=16)
text(x1, y1, 1:dim(x1)[1], col="red", adj=-1)

m <- malmq(x0,y0,,x1,y1,,RTS="vrs")
print("Malmquist index for change in productivity, technoligy change:")
print(m$mq)
print("Index for change of frontier:")
print(m$tc)
```

---

malmquist — *Malmquist index for fimrs in a panel*

Description

Estimate Malmquist index for firms in a panel data set. The data set does not need to be balanced.

Usage

```
malmquist(X, Y, ID, TIME, RTS = "vrs", ORIENTATION = "in", SLACK = FALSE, DUAL = FALSE,
DIRECT = NULL, param = NULL, TRANSPOSE = FALSE, FAST = TRUE, LP = FALSE,
CONTROL = NULL, LPK = NULL)
```

Arguments

X	Inputs of firms in many periods, a (T*K) x m matrix of observations of K firms with m outputs (firm x input) in at the most T periods.
Y	Outputs of firms in many periods, a (T*K) x n matrix of observations of K0 firms with n outputs (firm x input) in at the most T periods.
ID	Identifier for the firms in rows of X and Y.
TIME	Array with period number for each row in the input maxtrix X and output matrixY
RTS	Returns to scale assumption as in dea.
ORIENTATION	Input efficiency "in" (1), output efficiency "out" (2), and graph efficiency "graph" (3) as in dea.
SLACK	See dea.
DUAL	See dea.
DIRECT	See dea.
param	See dea.
TRANSPOSE	See dea.
FAST	See dea.
LP	See dea.
CONTROL	See dea.
LPK	See dea.

Details

Malmquist uses malmq for the calculations of the necessary efficiencies, and the returned indeces are as in malmq. The data must be a long data set with regards to TIME and ID; se the example below.

Note that the calculated index are index comparing a period and the previous period. To compare the development over time the indices must be turned into a chain index as shown in the example below.

Value

m	Malmquist indicies, an array of length T*K in the order of ID and TIME, i.e. the order of the rows of X.
tc	Technical change indices, an array of length T*K.
ec	Efficiency indices, an array of length T*K.
id	Index for firms as ID
time	Index for time as TIME
e00	The efficiencies for period 0 with reference technoligy from period 0.
e10	The efficiencies for period 1 with reference technoligy from period 0.
e11	The efficiencies for period 1 with reference technoligy from period 1.
e01	The efficiencies for period 0 with reference technoligy from period 1.

Note

The lagged values e11 are not neccesary equal to values of e00 as the reference technology for the two periods could be generated by different units, if the units in different time periodes are not the same.

Author(s)

Peter Bogetoft and Lars Otto <larsot23@gmail.com>

References

Peter Bogetoft and Lars Otto; *Benchmarking with DEA, SFA, and R*; Springer 2011

See Also

dea, malmq

Examples

```
x0 <- matrix(c(10, 28, 30, 60),ncol=1)
y0 <- matrix(c(5, 7, 10, 15),ncol=1)
x1 <- matrix(c(12, 26, 16, 60 ),ncol=1)
y1 <- matrix(c(6, 8, 9, 15 ),ncol=1)
x2 <- matrix(c(13, 26, 15, 60 ),ncol=1)
y2 <- matrix(c(7, 9, 10, 15 ),ncol=1)
```

```
dea.plot(x0, y0, RTS="vrs", txt=TRUE)
dea.plot(x1, y1, RTS="vrs", add=TRUE, col="red")
dea.plot(x2, y2, RTS="vrs", add=TRUE, col="blue")
points(x1, y1, col="red", pch=16)
# points(x2, y2, col="blue", pch=17)
text(x1, y1, 1:dim(x1)[1], col="red", adj=-1)
text(x2, y2, 1:dim(x1)[1], col="blue", adj=-1)
legend("bottomright", legend=c("Period 0", "Period 1", "Period 2"),
 col=c("black", "red", "blue"), lty=1, pch=c(1,16, 17),  bty="n")

X <- rbind(x0, x1, x2)
Y <- rbind(y0, y1, y2)
# Make ID and TIME variables one way or another
ID <- rep(1:dim(x1)[1], 3)
# TIME <- c(rep(0,dim(x1)[1]), rep(1,dim(x1)[1]), rep(2,dim(x1)[1]))
TIME <- gl(3, dim(x1)[1], labels=0:2)
# This is how the data for malmquist must look like
data.frame(TIME, ID, X, Y)
mq <- malmquist(X,Y, ID, TIME=TIME)
data.frame(TIME, ID, X, Y, mq$e00, mq$e01, mq$e10, mq$e11, mq$m, mq$tc)[order(ID, TIME),]

# How to make the Malmquist indices to a chain index
# Make data.frame with indices
DM <- data.frame(TIME, ID, m=mq$m, tc=mq$tc, ec=mq$ec)
# Set missing index for first period to 1, the base
DM[DM$TIME==0, c("m","tc", "ec")] <- 1
# Make chain index of the individual indices
AD <- aggregate(cbind(m=DM$m), by=list(ID=DM$ID), cumprod)
# Compare chain index to original index
data.frame(ID, TIME, m=c(AD$m), DM$m)
```

---

mea *MEA multi-directional efficiency analysis*

Description

Potential improvements PI or multi-directional efficiency analysis. The result is an exces value measures by the direction.

The direction is determined by the direction corresponding to the minimum input/maximum direction each good can be changed when they are changed one at a time.

Usage

```
mea(X, Y, RTS = "vrs", ORIENTATION = "in", XREF = NULL, YREF = NULL,
    FRONT.IDX = NULL, param=NULL, TRANSPOSE = FALSE,
    LP = FALSE, CONTROL = NULL, LPK = NULL)
mea.lines(N, X, Y, ORIENTATION="in")
```

Arguments

`X`	K times m matrix with K firms and m inputs as in `dea`
`Y`	K times n matrix with K firms and n outputs as in `dea`
`RTS`	Text string or a number defining the underlying DEA technology / returns to scale assumption.

0	fdh	Free disposability hull, no convexity assumption
1	vrs	Variable returns to scale, convexity and free disposability
2	drs	Decreasing returns to scale, convexity, down-scaling and free disposability
3	crs	Constant returns to scale, convexity and free disposability
4	irs	Increasing returns to scale, (up-scaling, but not down-scaling), convexity and free disposability
6	add	Additivity (scaling up and down, but only with integers), and free disposability
7	fdh+	A combination of free disposability and restricted or local constant return to scale

`ORIENTATION`	Input efficiency "in" (1) or output efficiency "out" (2), and also the additional option "in-out" (0) for both input and output direction.
`XREF`	Inputs of the firms determining the technology, defaults to `X`
`YREF`	Outputs of the firms determining the technology, defaults to `Y`
`FRONT.IDX`	Index for firms determining the technology
`param`	Possible parameters. At the moment only used for RTS="fdh+" to set low and high values for restrictions on lambda; see the section details and examples in `dea` for its use. Future versions might also use `param` for other purposes.
`TRANSPOSE`	as in `dea`
`LP`	as in `dea`
`CONTROL`	as in `dea`
`LPK`	as in `dea`
`N`	Number of firms where directional lines are to be drawn on an all raeady ecisting frontier plot (dea.plot.frontier)

Details

Details can be found in Bogetoft and Otto (2011, 121–124).

This method is for input directional efficiency only interesting when there are 2 or more inputs, and for output only when there are 2 or more outputs.

Value

The results are returned in a Farrell object with the following components.

`eff`	Exces value in DIRECT units of measurement, this is *not* Farrell efficiency
`lambda`	The lambdas, i.e. the weight of the peers, for each firm
`objval`	The objective value as returned from the LP program; normally the same as eff
`RTS`	The return to scale assumption as in the option `RTS` in the call

`ORIENTATION`	The efficiency orientation as in the call
`direct`	A K times m|n|m+n matrix with directions for each firm: the number of columns depends on wether it is input, output og in-out orientated.
`TRANSPOSE`	As in the call

Note

The calculation is done in `dea` after a calculation of the dirction that then is used in the argument `DIRECT`. The calulation of the direction is done in a series LP programs, one for each good in the direction.

Author(s)

Peter Bogetoft and Lars Otto <larsot23@gmail.com>

References

Peter Bogetoft and Lars Otto; *Benchmarking with DEA, SFA, and R*; Springer 2011

See Also

`dea` and the argument `DIRECT`.

Examples

```
X <- matrix(c(2, 2, 5, 10, 10, 3,    12, 8, 5, 4, 6,12), ncol=2)
Y <- matrix(rep(1,dim(X)[1]), ncol=1)

dea.plot.isoquant(X[,1], X[,2],txt=1:dim(X)[1])
mea.lines(c(5,6),X,Y)

me <- mea(X,Y)
me
peers(me)
# MEA potential saving in inputs, exces inputs
eff(me) * me$direct
me$eff *  me$direct

# Compare to traditionally Farrell efficiency
e <- dea(X,Y)
e
peers(e)
# Farrell potential saving in inputs, exces inputs
(1-eff(e)) * X
```

milkProd *Data: Milk producers*

Description

Data colected from Danish milk producers.

Usage

```
data(milkProd)
```

Format

A data frame with 108 observations on the following 5 variables.

`farmNo` farm number

`milk` Output of milk, kg

`energy` Energy expenses

`vet` Veterinary expenses

`cows` Number of cows

Note

Data as `.csv` are loaded by the command `data` using `read.table(...,header = TRUE,sep = ";")` such that this file is a semicolon separated file and not a comma separated file.

Source

Accounting and buissiness check data

Examples

```
data(milkProd)
y <- with(milkProd, cbind(milk))
x <- with(milkProd, cbind(energy, vet, cows))
```

`norWood2004` *Data: Forestry in Norway*

Description

A data set for 113 farmers in forestry in Norway.

Usage

```
data(norWood2004)
```

Format

A data frame with 113 observations on the following 7 variables.

`firm` firm number

`m` Variable cost

`x` Woodland, value of forrest and land

`y` Profit

`z1` Secondary income from ordinary farming

`z3` Age of forrest owner

`z6` Whether there is a long-term plan =1 or not =0

Details

Collected from farmers in forestry.

Note

Data as `.csv` are loaded by the command `data` using `read.table(...,header=TRUE,sep=";")` such that this file is a semicolon separated file and not a comma separated file.

Source

Norwegian Agricultural Economics Research Institute.

Examples

```
data(norWood2004)
## maybe str(norWood2004) ; plot(norWood2004) ...
```

`outlier.ap` *Detection of outliers in benchmark models*

Description

The functions implements the Wilson (1993) outlier detection method using only R functions.

Usage

```
outlier.ap(X, Y, NDEL = 3, NLEN = 25, TRANSPOSE = FALSE)

outlier.ap.plot(ratio, NLEN = 25, xlab = "r", ylab = "Log ratio",
                ..., ylim)
```

Arguments

X	Input as a firms times goods matrix, see TRANSPOSE.
Y	Output as a firms times goods matrix, see TRANSPOSE.
NDEL	The maximum number of firms to be considered as a group of outliers, i.e. the maximum number of firms to be deleted.
NLEN	The number of ratios to save for each level or removal, the number of rows in ratio used.
TRANSPOSE	Input and output matrices are treated as firms times goods matrices for the default value TRANSPOSE=FALSE corresponding to the standard in R for statistical models. When TRUE data matrices are transposed to good times firms matrices as is normally used in LP formulation of the problem.
ratio	The ratio component from the list as output from outlier.ap.
xlab	Label for the x-axis.
ylab	Label for the y-axis
ylim	The y limits (y1,y2) of the plot, an array/vector of length 2.
...	Usual options for the methods plot and lines.

Details

An implementation of the method in Wilson (1993) using only R functions and especially the function det to calculate $R^{(i)}_{\min}$.

An elementary presentation of the method is found in Bogetoft and Otto (2011), Sect. 5.13 on outliers.

Value

ratio	A min(NLEN,K) x NDEL matrix with the log-ratios to be plotted.
imat	A NDEL x NDEL matrix with indicies for deleted firms.
r0	A NDEL array with the minimum value R^i of the for each number of deleted firms.

Note

The function `outlier.ap` is extremely slow and for `NDEL` larger than 3 or 4 it might be advisable to use the function `ap` from the package **FEAR**.

The name of the returned components are the same as for `ap` in the package **FEAR**.

Author(s)

Peter Bogetoft and Lars Otto <larsot23@gmail.com>

References

Bogetoft and Otto; *Benchmarking with DEA, SFA, and R*; Springer 2011

Wilson (1993), "Detecing outliers in deterministic nonparametric frontier models with multiple outputs," *Journal of Business and Economic Statistics* 11, 319-323.

Wilson (2008), "FEAR 1.0: A Software Package for Frontier Efficiency Analysis with R," *Socio-Economic Planning Sciences* 42, 247–254

See Also

The function `ap` in the package **FEAR**.

Examples

```
n <- 25
x <- matrix(rnorm(n))
y <- .5 + 2.5*x + 2*rnorm(25)
tap <- outlier.ap(x,y, NDEL=2)
print(cbind(tap$imat,tap$rmin), na.print="", digit=2)
outlier.ap.plot(tap$ratio)
```

---

peers — *Find peer firms and units*

Description

The function `peers` finds for each firm its peers, `get.number.peers` finds for each peer the number of times this peer apears as a peer, and `get.which.peers` determines for one or more peers the firms they apear as peers for. Also include a function `get.peers.lambda` to calculate for fimrs the importances (lambdas) of peers.

Usage

```
peers(object, NAMES = FALSE, N=1:dim(object$lambda)[1], LAMBDA=0)
get.number.peers(object, NAMES = FALSE, N=1:dim(object$lambda)[2], LAMBDA=0)
get.which.peers(object, N = 1:dim(object$lambda)[2], LAMBDA=0)
get.peers.lambda(object,  N=1:dim(object$lambda)[1], LAMBDA=0)
```

Arguments

`object`	An object of class Farrell as returned by the functions `dea`, `dea.direct` et al.
`NAMES`	If true then names for the peers are returned if names are available otherwise the unit index numbers are used. If `NAMES` is a list of names with length equal to the number of units then it is used as names for peers.
`N`	The firm(s) or peer(s) for which to get the results.
`LAMBDA`	Minimum weight for extracted peers, i.e. the extracted peers have lambda values larger than `LAMBDA`.

Details

The returned values are index of the firms and can be used by itself, but can also by used as an index for a variable with names of the firms.

The `peers` returns a matrix with numbers for the peers for each firm; for firms with efficiency 1 the peers are just the firm itself. If there is slack in the evaluation of a firm with efficiency 1, this can be found with a call to `slack`, either directly or by the argument `SLACK` when a function `dea` was called to generate the Farrell `object`.

The `get.number.peers` returns the number of firms that a peer serves as a peer for.

The `get.peers.lambda` returns a list of firms with the peers and corresponding value af lambda.

Value

The return values are firm numbers. If the argument `NAMES=TRUE` is used in the function `peers` the return is a list of names of the peers if names for the firms are avilable as rownames.

Note

Peers are defines as firms where the coresponding lambdas are positive.

Note that peers might change between a Farrell object return from `dea` with `SLACK=FALSE` and a call with `SLACK=TRUE` or a following call to the function `slack` because a peer on the frontier with slack might by the call to `dea` be seen as a peer for itself whereas this will not happen when slacks are calculated.

Author(s)

Peter Bogetoft and Lars Otto <larsot23@gmail.com>

References

Peter Bogetoft and Lars Otto; *Benchmarking with DEA, SFA, and R*; Springer 2011. Sect. 4.6 page 93

See Also

`dea`

Examples

```
x <- matrix(c(100,200,300,500,100,200,600),ncol=1)
y <- matrix(c(75,100,300,400,25,50,400),ncol=1)

e <- dea(x,y)
peers(e)
get.number.peers(e)

# Who are the firms that firm 1 and 4 is peers for
get.which.peers(e, c(1,4))
```

---

pigdata — *Data: Multi-output pig producers*

Description

Input and output data for 248 pig producers that also produces crop, i.e. a multi–output data set.

Usage

```
data(pigdata)
```

Format

A data frame with 248 observations on the following 16 variables.

`firm` Serial number for pig producer

`x1` Input fertilizer

`x2` Input feedstuf

`x3` Input land

`x4` Input labour

`x5` Input machinery

`x6` Input other capital

`y2` Output crop

`y4` Output pig

`w1` Price of fertilizer

`w2` Price of feedstuf

`w3` Price of land

`w4` Price of labour

`w5` Price of michenery

`w6` Price of other capital

`p2` Price of pig

`p4` Price of crop

`cost` Total cost, w1*x1+...+w6*x6.

`rev` Total revenue, p2*y2+p4*y4.

Details

In raising pigs, most farmers also produce crops to feed the pigs. Labor and capital are used not just directly for pig-raising but also on the field.

Note

Data as `.csv` are loaded by the command `data` using `read.table(...,header = TRUE,sep = ";")` such that this file is a semicolon separated file and not a comma separated file.

Source

Farmers accounting data converted to index.

Examples

```
data(pigdata)
## maybe str(pigdata) ; plot(pigdata) ...
```

---

projekt — *Data: Milk producers*

Description

Accounting and production data for 101 milk producing farmers.

Usage

```
data(projekt)
```

Format

A data frame with 101 observations on the following 14 variables.

`numb` Serial number for the milk producer

`cows` Number of cows

`vet` Veterinary expences

`unitCost` Unit cost, variable cost

`capCost` Capacity cost

`fixedCost` Fixed cost

`milkPerCow` Milk per cow, kg

`quota` Milk quota

`fatPct` Fat percent in milk

`protPct` Protein percent in milk

`cellCount` Cell count for milk

`race` Race for cows, a factor with levels `jersey`, `large`, and `mixed`

`type` Type of production, conventional or organic, a factor with levels `conv` `orga`

`age` Age of the farmer

Details

Data is a mix of accounting data and production controls.

Note

Data as `.csv` are loaded by the command `data` using `read.table(...,header = TRUE,sep = ";")` such that this file is a semicolon separated file and not a comma separated file.

Source

Collected from farmers.

Examples

```
data(projekt)
## maybe str(projekt) ; plot(projekt) ...
```

---

sdea — *Super efficiency*

Description

The method `sdea` calculates super-effciency and returns the same class of object as `dea`.

Usage

```
sdea(X, Y, RTS = "vrs", ORIENTATION = "in", DIRECT = NULL, param = NULL,
     TRANSPOSE = FALSE, LP = FALSE)
```

Arguments

`X` Inputs of firms to be evaluated, a K x m matrix of observations of K firms with m inputs (firm x input). In case `TRANSPOSE=TRUE` the input matrix is transposed to input x firm.

`Y` Outputs of firms to be evaluated, a K x n matrix of observations of K firms with n outputs (firm x input). In case `TRANSPOSE=TRUE` the output matrix is transposed to output x firm.

`RTS` Text string or a number defining the underlying DEA technology / returns to scale assumption; the same values as for `dea`.

0	fdh	Free disposability hull, no convexity assumption
1	vrs	Variable returns to scale, convexity and free disposability
2	drs	Decreasing returns to scale, convexity, down-scaling and free disposability
3	crs	Constant returns to scale, convexity and free disposability
4	irs	Increasing returns to scale, (up-scaling, but not down-scaling), convexity and free disposability
5	irs2	Increasing returns to scale (up-scaling, but not down-scaling), additivity, and free disposability
6	add	Additivity (scaling up and down, but only with integers), and free disposability
7	fdh+	A combination of free disposability and restricted or local constant return to scale

ORIENTATION	Input efficiency "in" (1), output efficiency "out" (2), and graph efficiency "graph" (3). For use with DIRECT, an additional option is "in-out" (0).
DIRECT	Directional efficiency, DIRECT is either a scalar, an array, or a matrix with non-negative elements. If the argument is a scalar, the direction is (1,1,...,1) times the scalar; the value of the efficiency depends on the scalar as well as on the unit of measurements. If the argument an array, this is used for the direction for every firm; the length of the array must correspond to the number of inputs and/or outputs depending on the ORIENTATION. If the argument is a matrix then different directions are used for each firm. The dimensions depends on the ORIENTATION, the number of firms must correspond to the number of firms in X and Y. DIRECT must not be used in connection with DIRECTION="graph".
param	Argument is at present only used when RTS="fdh+", see dea for a description.
TRANSPOSE	See the description in dea.
LP	Only for debugging, see the description in dea.

Details

Super-efficiency measures are constructed by avoiding that the evaluated firm can help span the technology; ie. if the firm in qestuen is a firm on the frontier in a normal dea approach then this firm in super efficiency might be outside the technology set.

Value

The object returned is a Farrell object with the component described in dea. The relevant components are

eff	The efficiencies. Note when DIRECT is used then the efficencies are not Farrell efficiencies but rather exces values in DIRECT units of measurement
lambda	The lambdas, i.e. the weight of the peers, for each firm
objval	The objective value as returned from the LP program; normally the same as eff.
RTS	The return to scale assumption as in the option RTS in the call
ORIENTATION	The efficiency orientation as in the call

Note

Calculation of slacks for super efficiency should be done by using the option SLACK=TRUE in the call of the method sdea. If the two phases are done in two steps as first a call to sdea and then a call to slacks the user must make sure to set the reference technology to the one corresponding to super-efficiency in the call to slack and this requires a loop with calls to slack.

Author(s)

Peter Bogetoft and Lars Otto <larsot23@gmail.com>

References

Peter Bogetoft and Lars Otto; *Benchmarking with DEA, SFA, and R*; Springer 2011. Sect. 5.2 page 115

P Andersen and NC Petersen; "A procedure for ranking efficient units in data envelopment analysis"; *Management Science* 1993 39(10):1261–1264

See Also

dea

Examples

```
x <- matrix(c(100,200,300,500,100,200,600),ncol=1)
y <- matrix(c(75,100,300,400,25,50,400),ncol=1)
se <- sdea(x,y)
se

# Leave out firm 3 as a determining firm of the technology set
n <- 3
dea.plot.frontier(x[-n], y[-n], txt=(1:dim(x)[1])[-n])
# Plot and label firm 3
points(x[n],y[n],cex=1.25,pch=16)
text(x[n],y[n],n,adj=c(-.75,.75))
```

---

sfa *Stochastic frontier estimation*

Description

Estimate a stochastic frontier production or cost function using a maximum likelihood method.

Usage

```
sfa(x, y, beta0 = NULL, lambda0 = 1, resfun = ebeta,
    TRANSPOSE = FALSE, DEBUG=FALSE,
    control=list(), hessian=2)

sfa.cost(W, Y, COST, beta0 = NULL, lambda0 = 1, resfun = ebeta,
    TRANSPOSE = FALSE, DEBUG=FALSE,
    control=list(), hessian=2)



te.sfa(object)
teBC.sfa(object)
teMode.sfa(object)
teJ.sfa(object)
```

```
te.add.sfa(object)

sigma2u.sfa(object)
sigma2v.sfa(object)
sigma2.sfa(object)

lambda.sfa(object)
```

Arguments

`x`	Input as a K x m matrix of observations on m inputs from K firms; (firm x input); MUST be a matrix. No constant for the intercept should be included in x as it is added by default.
`y`	Output; K times 1 matrix (one output)
`Y`	Output; K times n matrix for m outputs; only to be used in cost function estimation.
`W`	Input prices as a K x m matrix.
`COST`	Cost as a K array for the K firms
`beta0`	Optional initial parameter values
`lambda0`	Optional initial ratio of variances
`resfun`	Function to calculate the residuals, default is a linear model with an intercept. Must be called as `resfun(x,y,parm)` where `parm=c(beta,lambda)` or `parm=c(beta)`, and return the residuals as an array of length corresponding to the length of output `y`.
`TRANSPOSE`	If TRUE, data is transposed, i.e. input is now m x K matrix
`DEBUG`	Set to TRUE to get various debugging information written on the console
`control`	List of control parameters to `ucminf`
`hessian`	How the Hessian is delivered, see the ucminf documentation
`object`	Object of class 'sfa' as output from the function `sfa`

Details

The optimization is done by the R method `ucminf` from the package with the same name. The efficiency terms are assumed to be half–normal distributed.

Changing the maximum steplength, the trust rgion, might be important, and this can be done by the option 'control= list(stepmax=0.1)'. The default value is 0.1 and that value is suitable for parameters around 1; for smaller parameters a lower value should be used. Notice that the step length is updated by the optimizing program and thus must be set for every call of the function sfa if it is to be set.

The generic functions `print.sfa`, `summary.sfa`, `fitted.sfa`, `residuals.sfa`, `logLik.sfa`, and `coef.sfa` all work as expected.

The methods `te.sfa`, `teMode.sfa` etc. calculates the efficiency corresponding to different methods

Value

The values returned from `sfa` is the same as for `ucminf`, i.e. a list with components plus some especially relevant for sfa:

`par`	The best set of parameters found `c(beta,lambda)`.
`value`	The value of minus log-likelihood function corresponding to 'par'.
`beta`	The parameters for the function
`sigma2`	The estimate of the total variance
`lambda`	The estimate of lambda
`N`	The number of observations
`df`	The degrees of freedom for the model
`residuals`	The residuals as a K times 1 matrix/vector, can also be obtained by `residuals(sfa-object)`
`fitted.values`	Fitted values
`vcov`	The variance-covarians matrix for all estimated parameters incl. lambda
`convergence`	An integer code. '0' indicates successful convergence. Some of the error codes taken from `ucminf` are '1' Stopped by small gradient (grtol). '2' Stopped by small step (xtol). '3' Stopped by function evaluation limit (maxeval). '4' Stopped by zero step from line search More codes are found in `ucminf`
`message`	A character string giving any additional information returned by the optimizer, or 'NULL'.
`o`	The object returned by `ucminf`, for further information on this see `ucminf`

Note

Calculation of technical efficiencies for each unit can be done by the method te.sfa as shown in the examples.

`te.sfa(sfaObject),teBC.sfa(sfaObject)`: Efficiencies estimated by minimizing the mean square error; Eq. (7.21) in Bogetoft and Otto (2011, 219) and Battese and Coelli (1988, 392)

`teMode.sfa(sfaObject),te1.sfa(sfaObject)`: Efficiencies estimates using the conditional mode approach; Bogetoft and Otto (2011, 219), Jondrow et al. (1982, 235).

`teJ.sfa(sfaObject),te2.sfa(sfaObject)`: Efficiencies estimates using the conditional mean approach Jondrow et al. (1982, 235).

`te.add.sfa(sfaObject)` Efficiency in the additive model, Bogetoft and Otto (2011, 219)

The variance pf the distribution of efficiency can be calculated by `sigma2u.sfa(sfaObject)`, the variance of the random error by `sigma2v.sfa(sfaObject)`, and the total variance (sum of variances of efficiency and random noise) by `sigma2.sfa`.

The ratio of variances of the efficiency and the random noise can be found from the method `lambda.sfa`

The generic method `summary` prints the parameters, standard errors, t-values, and a few more statistics from the optimization.

Author(s)

Peter Bogetoft and Lars Otto <larsot23@gmail.com>

References

Bogetoft and Otto; *Benchmarking with DEA, SFA, and R*, Springer 2011; chapters 7 and 8.

See Also

See the method `ucminf` for the possible optimization methods and further options to use in the option `control`.

The method `sfa` in the package **frontier** gives another way to estimate stochastic production functions.

Examples

```
# Example from the book by Coelli et al.
# d <- read.csv("c:/0work/rpack/front41Data.csv", header = TRUE, sep = ",")
# x <- cbind(log(d$capital), log(d$labour))
# y <- matrix(log(d$output))

n <- 50
x1 <- 1:50 + rnorm(50,0,10)
x2 <- 100 + rnorm(50,0,10)
x <- cbind(x1,x2)
y <- 0.5 + 1.5*x1 + 2*x2 + rnorm(n,0,1) - pmax(0,rnorm(n,0,1))
sfa(x,y)
summary(sfa(x,y))


# Estimate efficiency for each unit
o <- sfa(x,y)
eff(o)

te <- te.sfa(o)
teM <- teMode.sfa(o)
teJ <- teJ.sfa(o)
cbind(eff(o),te,Mode=eff(o, type="Mode"),teM,teJ)[1:10,]


sigma2.sfa(o)       # Estimated varians
lambda.sfa(o)       # Estimated lambda
```

---

slack — *Calculate slack in an efficiency analysis*

Description

Slacks are calculated after taking the efficiency into consideration.

Usage

```
slack(X, Y, e, XREF = NULL, YREF = NULL, FRONT.IDX = NULL, LP = FALSE)
```

Arguments

X	Inputs of firms to be evaluated, a K x m matrix of observations of K firms with m inputs (firm x input).
Y	Outputs of firms to be evaluated, a K x n matrix of observations of K firms with n outputs (firm x input).
e	A Farrell object as returned from dea et al.
XREF	Inputs of the firms determining the technology, defaults to X
YREF	Outputs of the firms determining the technology, defaults to Y
FRONT.IDX	Index for firms determining the technology
LP	Set TRUE for debugging.

Details

Slacks are calculated in a LP problem where the sum of all slacks are maximied after correction for efficiency. The for calculating slacks for orientation graph is low because of the low precision in the calculated graph efficiency.

Value

The result is returned as the Farrell object used as the argument in the call of the function with the following added components:

slack	A logical vector where the component for a firm is TRUE if the sums of slacks for the corresponding firm is positive. Only calculated in dea when option SLACK=TRUE
sum	A vector with sums of the slacks for each firm. Only calculated in dea when option SLACK=TRUE
sx	A matrix for input slacks for each firm, only calculated if the option SLACK is TRUE or returned from the method slack
sy	A matrix for output slack, see sx

Author(s)

Peter Bogetoft and Lars Otto <larsot23@gmail.com>

References

Peter Bogetoft and Lars Otto; *Benchmarking with DEA, SFA, and R*; Springer 2011. Sect. 5.6 page 127.

WW Cooper, LM Seiford, and K Tone; *Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software*, 2nd edn. Springer 2007 .

Examples

```
x <- matrix(c(100,200,300,500,100,200,600),ncol=1)
y <- matrix(c(75,100,300,400,25,50,400),ncol=1)
dea.plot.frontier(x,y,txt=1:dim(x)[1])

e <- dea(x,y)
eff(e)

# calculate slacks
sl <- slack(x,y,e)
data.frame(e$eff,sl$slack,sl$sx,sl$sy)
```

---

typeIerror — *Probability of type I error for test in a bootstrap DEA model*

Description

Calculates the probability of a type I error for a test in bootstrapped DEA models.

Usage

```
typeIerror(shat,s)
```

Arguments

`shat`	The value of the statistic for which the probability of a type I error is to be calculated
`s`	Vector with calculated values of the statistic for each of the `NREP` bootstraps; NREP is from `dea.boot`

Details

Needs bootstrapped values of the test statistic

Value

Returns the probability of a type I error

Author(s)

Peter Bogetoft and Lars Otto <larsot23@gmail.com>

See Also

`boot.sw98` in **FEAR**, Paul W. Wilson (2008), "FEAR 1.0: A Software Package for Frontier Efficiency Analysis with R," *Socio-Economic Planning Sciences* 42, 247–254

Examples

```
# Probability of getting something larger than 1.96 in 10000 random
# standard normal variates.
x <- rnorm(10000)
typeIerror(1.96,x)
```

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