Package 'AnnuityRIR'

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Title Annuity Random Interest Rates

Depends R (>= 3.2.5), mc2d

Imports tseries, EnvStats, fitdistrplus, actuar, stats

Suggests MASS

Description Annuity Random Interest Rates proposes different techniques for the approximation of the present and final value of a unitary annuity-due or annuity-immediate considering interest rate as a random variable. Cruz Rambaud et al. (2017) <doi:10.1007/978-3-319-54819-7_16>. Cruz Rambaud et al. (2015) <doi:10.23755/rm.v28i1.25>.

License GPL (>= 2)

URL https://www.r-project.org

BugReports https://github.com/fabriziomaturo/AnnuityRIR

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R topics documented:

`beta_parameters` *Compute the parameters of the beta distribution and plot normalized data.*

Description

Compute the parameters of the beta distribution and plot normalized data.

Usage

```
beta_parameters(data)
```

Arguments

`data`	A vector of interest rates.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Source

Cruz Rambaud, S.; Maturo, F. and Sánchez Pérez A. M. (2015): "Approach of the value of an annuity when non-central moments of the capitalization factor are known: an R application with interest rates following normal and beta distributions". *Ratio Mathematica*, 28(1), pp. 15-30. doi: 10.23755/rm.v28i1.25.

Examples

```
# example 1
data=c(0.00,-0.05,-0.05,-0.06,-0.06,0.02,-0.06,-0.05,-0.04,-0.05,
-0.03,-0.06,0.04,-0.05,-0.08,-0.05,-0.12,-0.03,-0.05,-0.04,-0.06)
beta_parameters(data)

# example 2
data<-rnorm(n=200,m=0.075,sd=0.2)
beta_parameters(data)
```

FV_post_artan *Compute the final expected value of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate), valued at the rate X, using the tetraparametric function approach.*

Description

Compute the final expected value of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate), valued at the rate X, using the tetraparametric function approach.

Usage

```
FV_post_artan(data,years)
```

Arguments

data	A vector of interest rates.
years	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Source

Cruz Rambaud, S.; Maturo, F. and Sánchez Pérez A. M. (2017): Expected present and final value of an annuity when some non-central moments of the capitalization factor are unknown: Theory and an application using R. In Š. Hošková-Mayerová, *et al.* (Eds.), *Mathematical-Statistical Models and Qualitative Theories for Economic and Social Sciences* (pp. 233-248). Springer, Cham. doi:10.1007/978-3-319-54819-7_16.

Examples

```
#example 1
data=c(0.298,0.255,0.212,0.180,0.165,0.163,0.167,0.161,0.154,
0.128,0.079,0.059,0.042,-0.008,-0.012,-0.002)
FV_post_artan(data,6)

# example 2
data<-rnorm(n=30,m=0.03,sd=0.01)
FV_post_artan(data,10)
```

FV_post_beta_kmom — *Compute the final expected value of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate), valued at the rate X, using the estimated moments of the beta distribution.*

Description

Compute the final expected value of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate), valued at the rate X, using the estimated moments of the beta distribution.

Usage

```
FV_post_beta_kmom(data,years)
```

Arguments

`data`	A vector of interest rates.
`years`	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Source

Cruz Rambaud, S.; Maturo, F. and Sánchez Pérez A. M. (2015): “Approach of the value of an annuity when non-central moments of the capitalization factor are known: an R application with interest rates following normal and beta distributions”. *Ratio Mathematica*, 28(1), pp. 15-30. doi: 10.23755/rm.v28i1.25.

Examples

```
# example 1
data=c(0.00,-0.05,-0.05,-0.06,-0.06,0.02,-0.06,-0.05,-0.04,-0.05,
-0.03,-0.06,0.04,-0.05,-0.08,-0.05,-0.12,-0.03,-0.05,-0.04,-0.06)
FV_post_beta_kmom(data,8)

# example 2
data<-rnorm(n=200,m=0.075,sd=0.2)
FV_post_beta_kmom(data,8)
```

FV_post_mood — *Compute the final expected value of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate), valued at the rate X, using the method of Mood* et al.

Description

Compute the final expected value of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate), valued at the rate X, using the method of Mood *et al.*

Usage

```
FV_post_mood(data,years)
```

Arguments

data	A vector of interest rates.
years	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Source

Cruz Rambaud, S.; Maturo, F. and Sánchez Pérez A. M. (2017): "Expected present and final value of an annuity when some non-central moments of the capitalization factor are unknown: Theory and an application using R". In Š. Hošková-Mayerová, *et al.* (Eds.), *Mathematical-Statistical Models and Qualitative Theories for Economic and Social Sciences* (pp. 233-248). Springer, Cham. doi:10.1007/978-3-319-54819-7_16.

Examples

```
#example 1
data=c(0.298,0.255,0.212,0.180,0.165,0.163,0.167,0.161,0.154,
0.128,0.079,0.059,0.042,-0.008,-0.012,-0.002)
FV_post_mood(data,6)

# example 2
data<-rnorm(n=30,m=0.03,sd=0.01)
FV_post_mood(data,10)
```

`FV_post_norm_kmom` *Compute the final expected value of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate), valued at the rate X, using the estimated moments of the normal distribution.*

Description

Compute the final expected value of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate), valued at the rate X, using the estimated moments of the normal distribution.

Usage

```
FV_post_norm_kmom(data,years)
```

Arguments

`data`	A vector of interest rates.
`years`	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Source

Cruz Rambaud, S.; Maturo, F. and Sánchez Pérez A. M. (2015): "Approach of the value of an annuity when non-central moments of the capitalization factor are known: an R application with interest rates following normal and beta distributions". *Ratio Mathematica*, 28(1), pp. 15-30. doi: 10.23755/rm.v28i1.25.

Examples

```
# example 1
data = c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
data=data/100
FV_post_norm_kmom(data,8)


# example 1
data<-rnorm(n=200,m=0.075,sd=0.2)
norm_test_jb(data) #test data
FV_post_norm_kmom(data,8)
```

FV_post_quad — *Compute the final expected value of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate), valued at the rate X, using the quadratic discount method.*

Description

Compute the final expected value of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate), valued at the rate X, using the quadratic discount method.

Usage

```
FV_post_quad(data,years)
```

Arguments

data	A vector of interest rates.
years	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Source

Cruz Rambaud, S.; Maturo, F. and Sánchez Pérez A. M. (2017): "Expected present and final value of an annuity when some non-central moments of the capitalization factor are unknown: Theory and an application using R". In Š. Hošková-Mayerová, *et al.* (Eds.), *Mathematical-Statistical Models and Qualitative Theories for Economic and Social Sciences* (pp. 233-248). Springer, Cham. doi:10.1007/978-3-319-54819-7_16.

Examples

```
#example 1
data=c(0.298,0.255,0.212,0.180,0.165,0.163,0.167,0.161,0.154,
0.128,0.079,0.059,0.042,-0.008,-0.012,-0.002)
FV_post_quad(data,8)

# example 2
data<-rnorm(n=30,m=0.03,sd=0.01)
FV_post_quad(data,10)
```

`FV_pre_artan` *Compute the final expected value of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate X, using the tetraparametric function approach.*

Description

Compute the final expected value of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate X, using the tetraparametric function approach.

Usage

```
FV_pre_artan(data,years)
```

Arguments

`data`	A vector of interest rates.
`years`	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Source

Cruz Rambaud, S.; Maturo, F. and Sánchez Pérez A. M. (2017): "Expected present and final value of an annuity when some non-central moments of the capitalization factor are unknown: Theory and an application using R". In Š. Hošková-Mayerová, *et al.* (Eds.), *Mathematical-Statistical Models and Qualitative Theories for Economic and Social Sciences* (pp. 233-248). Springer, Cham. doi:10.1007/978-3-319-54819-7_16.

Examples

```
#example 1
data=c(0.298,0.255,0.212,0.180,0.165,0.163,0.167,0.161,0.154,
0.128,0.079,0.059,0.042,-0.008,-0.012,-0.002)
FV_pre_artan(data,6)

# example 2
data<-rnorm(n=30,m=0.03,sd=0.01)
FV_pre_artan(data,10)
```

`FV_pre_beta_kmom` *Compute the final expected value of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate X, using the estimated moments of the beta distribution.*

Description

Compute the final expected value of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate X, using the estimated moments of the beta distribution.

Usage

```
FV_pre_beta_kmom(data,years)
```

Arguments

`data`	A vector of interest rates.
`years`	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Source

Cruz Rambaud, S.; Maturo, F. and Sánchez Pérez A. M. (2015): “Approach of the value of an annuity when non-central moments of the capitalization factor are known: an R application with interest rates following normal and beta distributions”. *Ratio Mathematica*, 28(1), pp. 15-30. doi: 10.23755/rm.v28i1.25.

Examples

```
# example 1
data=c(0.00,-0.05,-0.05,-0.06,-0.06,0.02,-0.06,-0.05,-0.04,-0.05,
-0.03,-0.06,0.04,-0.05,-0.08,-0.05,-0.12, -0.03,-0.05,-0.04,-0.06)
FV_pre_beta_kmom(data,8)

# example 2
data<-rnorm(n=200,m=0.075,sd=0.2)
FV_pre_beta_kmom(data,8)
```

FV_pre_mood — *Compute the final expected value of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate X, using the method of Mood* et al.

Description

Compute the final expected value of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate X, using the method of Mood *et al.*

Usage

```
FV_pre_mood(data,years)
```

Arguments

data	A vector of interest rates.
years	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Source

Cruz Rambaud, S.; Maturo, F. and Sánchez Pérez A. M. (2017): "Expected present and final value of an annuity when some non-central moments of the capitalization factor are unknown: Theory and an application using R". In Š. Hošková-Mayerová, *et al.* (Eds.), *Mathematical-Statistical Models and Qualitative Theories for Economic and Social Sciences* (pp. 233-248). Springer, Cham. doi:10.1007/978-3-319-54819-7_16.

Examples

```
#example 1
data=c(0.298,0.255,0.212,0.180,0.165,0.163,0.167,0.161,0.154,
0.128,0.079,0.059,0.042,-0.008,-0.012,-0.002)
FV_pre_mood(data,6)

# example 2
data<-rnorm(n=30,m=0.03,sd=0.01)
FV_pre_mood(data,10)
```

`FV_pre_norm_kmom` *Compute the final expected value of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate X, using the estimated moments of the normal distribution.*

Description

Compute the final expected value of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate X, using the estimated moments of the normal distribution.

Usage

```
FV_pre_norm_kmom(data,years)
```

Arguments

`data`	A vector of interest rates.
`years`	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Source

Cruz Rambaud, S.; Maturo, F. and Sánchez Pérez A. M. (2015): “Approach of the value of an annuity when non-central moments of the capitalization factor are known: an R application with interest rates following normal and beta distributions”. *Ratio Mathematica*, 28(1), pp. 15-30. doi: 10.23755/rm.v28i1.25.

Examples

```
# example 1
data<-rnorm(n=30,m=0.03,sd=0.01)
norm_test_jb(data) #test data
FV_pre_norm_kmom(data,8)

# example 1
data<-rnorm(n=200,m=0.075,sd=0.2)
norm_test_jb(data) #test data
FV_pre_norm_kmom(data,8)
```

FV_pre_quad *Compute the final expected value of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate X, using the quadratic discount method.*

Description

Compute the final expected value of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate X, using the quadratic discount method.

Usage

```
FV_pre_quad(data,years)
```

Arguments

data	A vector of interest rates.
years	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Source

Cruz Rambaud, S.; Maturo, F. and Sánchez Pérez A. M. (2017): "Expected present and final value of an annuity when some non-central moments of the capitalization factor are unknown: Theory and an application using R". In Š. Hošková-Mayerová, *et al.* (Eds.), *Mathematical-Statistical Models and Qualitative Theories for Economic and Social Sciences* (pp. 233-248). Springer, Cham. doi:10.1007/978-3-319-54819-7_16.

Examples

```
#example 1
data=c(0.298,0.255,0.212,0.180,0.165,0.163,0.167,0.161,0.154,
0.128,0.079,0.059,0.042,-0.008,-0.012,-0.002)
FV_pre_quad(data,6)

# example 2
data<-rnorm(n=30,m=0.03,sd=0.01)
FV_pre_quad(data,10)
```

`moment` *Compute the exact moments of a distribution.*

Description

Compute the exact moments of a distribution.

Usage

```
moment(x,order,central, absolute, na.rm)
```

Arguments

`x`	A vector X of interest rates.
`order`	The order of moment that should be computed. Default is 1.
`central`	If central moments are to be computed. Default is "FALSE".
`absolute`	If absolute moments are to be computed. Default is "FALSE".
`na.rm`	If missing values should be removed. Default is "FALSE".

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Source

Cruz Rambaud, S.; Maturo, F. and Sánchez Pérez A. M. (2015): “Approach of the value of an annuity when non-central moments of the capitalization factor are known: an R application with interest rates following normal and beta distributions”. *Ratio Mathematica*, 28(1), pp. 15-30. doi: 10.23755/rm.v28i1.25.

Examples

```
#example 1
data=c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
moment(data,3)
```

norm_mom *Fit the data to a normal curve and compute the moments of the normal distribution according to the definition (as integral).*

Description

Fit the data to a normal curve and compute the moments of the normal distribution according to the definition (as integral).

Usage

```
norm_mom(data,order)
```

Arguments

data	A vector X of interest rates.
order	The order of moment that should be computed.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Source

Cruz Rambaud, S.; Maturo, F. and Sánchez Pérez A. M. (2015): "Approach of the value of an annuity when non-central moments of the capitalization factor are known: an R application with interest rates following normal and beta distributions". *Ratio Mathematica*, 28(1), pp. 15-30. doi: 10.23755/rm.v28i1.25.

Examples

```
#example 1
data=c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
data=data/100
norm_mom(data,5)
```

norm_test_jb — *Compute the Jarque-Bera test for checking the assumption of normality of the interest rates distribution and returns the parameters of the fitted normal distribution.*

Description

Compute the Jarque-Bera test for checking the assumption of normality of the interest rates distribution and returns the parameters of the fitted normal distribution.

Usage

```
norm_test_jb(data)
```

Arguments

`data`	A vector of interest rates.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Source

Cruz Rambaud, S.; Maturo, F. and Sánchez Pérez A. M. (2015): "Approach of the value of an annuity when non-central moments of the capitalization factor are known: an R application with interest rates following normal and beta distributions". *Ratio Mathematica*, 28(1), pp. 15-30. doi: 10.23755/rm.v28i1.25.

Examples

```
#example 1
data=c(0.298,0.255,0.212,0.180,0.165,0.163,0.167,0.161,
0.154,0.128,0.079,0.059,0.042,-0.008,-0.012,-0.002)
norm_test_jb(data)

# example 2
data<-rnorm(n=30,m=0.03,sd=0.01)
norm_test_jb(data)

# example 3
data=runif(999, min = 0, max = 1)
norm_test_jb(data)

# example 4
data=c(0.00,-0.05,-0.05,-0.06,-0.06,0.02,-0.06,-0.05,-0.04,-0.05,
-0.03,-0.06,0.04,-0.05,-0.08,-0.05,-0.12,-0.03,-0.05,-0.04,-0.06)
norm_test_jb(data)
```

`plot_FVs_post` *Plot the final expected values of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate), valued at the rate X, using different approaches.*

Description

Plot the final expected values of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate), valued at the rate X, using different approaches.

Usage

```
plot_FVs_post(data,years,lwd,lty1,lty2,lty3)
```

Arguments

data	A vector of interest rates.
years	The number of years of the income. Default is 10 years.
lwd	The width of the curve. Default is 1.5.
lty1	The style of the curve for the "arctan" approximation. Default is 1.
lty2	The style of the curve for the "cubic" approximation. Default is 2.
lty3	The style of the curve for the "mood" approximation. Default is 3.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
#example 1
data = c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
data=data/100
plot_FVs_post(data)

# example 2
data<-rnorm(n=30,m=0.03,sd=0.003)
plot_FVs_post(data)
```

`plot_FVs_pre` *Plot the final expected values of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate X, using different approaches.*

Description

Plot the final expected values of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate X, using different approaches.

Usage

```
plot_FVs_pre(data,years,lwd,lty1,lty2,lty3)
```

Arguments

`data`	A vector of interest rates.
`years`	The number of years of the income. Default is 10 years.
`lwd`	The width of the curve. Default is 1.5.
`lty1`	The style of the curve for the "arctan" approximation. Default is 1.
`lty2`	The style of the curve for the "cubic" approximation. Default is 2.
`lty3`	The style of the curve for the "mood" approximation. Default is 3.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
#example 1
data = c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
data=data/100
plot_FVs_pre(data)

# example 2
data<-rnorm(n=30,m=0.03,sd=0.003)
plot_FVs_pre(data)
```

`plot_FV_post_beta_kmom`

Plot the final expected value of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate), valued at the rate X, using the estimated moments of the beta distribution.

Description

Plot the final expected value of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate), valued at the rate X, using the estimated moments of the beta distribution.

Usage

```
plot_FV_post_beta_kmom(data,years,lwd,lty)
```

Arguments

`data`	A vector of interest rates.
`years`	The number of years of the income. Default is 10 years.
`lwd`	The width of the curve. Default is 1.5.
`lty`	The style of the curve. Default is 1.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
# example 1
data<-runif(34, 0,1)
plot_FV_post_beta_kmom(data,8)
```

`plot_FV_post_norm_kmom`

Plot the final expected value of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate), valued at the rate X, using the estimated moments of the normal distribution.

Description

Plot the final expected value of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate), valued at the rate X, using the estimated moments of the normal distribution.

Usage

```
plot_FV_post_norm_kmom(data,years,lwd,lty)
```

Arguments

`data`	A vector of interest rates.
`years`	The number of years of the income. Default is 10 years.
`lwd`	The width of the curve. Default is 1.5.
`lty`	The style of the curve. Default is 1.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
# example 1
data<-rnorm(n=30,m=0.03,sd=0.01)
plot_FV_post_norm_kmom(data,8)

# example 2
data<-rnorm(n=200,m=0.075,sd=0.2)
plot_FV_post_norm_kmom(data,8)
```

`plot_FV_pre_beta_kmom` *Plot the final expected value of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate X, using the estimated moments of the beta distribution.*

Description

Plot the final expected value of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate X, using the estimated moments of the beta distribution.

Usage

```
plot_FV_pre_beta_kmom(data,years,lwd,lty)
```

Arguments

`data`	A vector of interest rates.
`years`	The number of years of the income. Default is 10 years.
`lwd`	The width of the curve. Default is 1.5.
`lty`	The style of the curve. Default is 1.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
# example 1
data<-runif(34, 0,1)
plot_FV_pre_beta_kmom(data,8)
```

`plot_FV_pre_norm_kmom` *Plot the final expected value of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate X, using the estimated moments of the normal distribution.*

Description

Plot the final expected value of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate X, using the estimated moments of the normal distribution.

Usage

```
plot_FV_pre_norm_kmom(data,years,lwd,lty)
```

Arguments

`data`	A vector of interest rates.
`years`	The number of years of the income. Default is 10 years.
`lwd`	The width of the curve. Default is 1.5.
`lty`	The style of the curve. Default is 1.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
# example 1
data<-rnorm(n=30,m=0.03,sd=0.01)
plot_FV_pre_norm_kmom(data,8)


# example 2
data<-rnorm(n=200,m=0.075,sd=0.2)
plot_FV_pre_norm_kmom(data,8)
```

`plot_PVs_post` *Plot the present expected values of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate), valued at the rate X, using different approaches.*

Description

Plot the present expected values of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate), valued at the rate X, using different approaches.

Usage

```
plot_PVs_post(data,years,lwd,lty1,lty2,lty3,lty4,lty5,lty6)
```

Arguments

`data`	A vector of interest rates.
`years`	The number of years of the income. Default is 10 years.
`lwd`	The width of the curve. Default is 1.5.
`lty1`	The style of the curve for the "arctan" approximation. Default is 1.
`lty2`	The style of the curve for the "cubic" approximation. Default is 2.
`lty3`	The style of the curve for the "mood with positive moments" approximation. Default is 3.
`lty4`	The style of the curve for the "mood with negative moments" approximation. Default is 4.
`lty5`	The style of the curve for the exact value. Default is 5.
`lty6`	The style of the curve for "triangular distribution" approximation. Default is 6.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
# example 1
data = c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
data=data/100
plot_PVs_post(data)

# example 2
data<-rnorm(n=30,m=0.03,sd=0.003)
plot_PVs_post(data)
```

`plot_PVs_pre` *Plot the present expected values of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate X, using different approaches.*

Description

Plot the present expected values of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate X, using different approaches.

Usage

```
plot_PVs_pre(data,years,lwd,lty1,lty2,lty3,lty4,lty5,lty6)
```

Arguments

`data`	A vector of interest rates.
`years`	The number of years of the income. Default is 10 years.
`lwd`	The width of the curve. Default is 1.5.
`lty1`	The style of the curve for the "arctan" approximation. Default is 1.
`lty2`	The style of the curve for the "cubic" approximation. Default is 2.
`lty3`	The style of the curve for the "mood with positive moments" approximation. Default is 3.
`lty4`	The style of the curve for the "mood with negative moments" approximation. Default is 4.
`lty5`	The style of the curve for the exact value. Default is 5.
`lty6`	The style of the curve for "triangular distribution" approximation. Default is 6.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
# example 1
data = c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
data=data/100
plot_PVs_pre(data)

# example 2
data<-rnorm(n=30,m=0.03,sd=0.003)
plot_PVs_pre(data)
```

PV_post_artan   *Compute present expected value of an n-payment annuity, with payments of 1 unit each, made at the end of every year (annuity-immediate), valued at the rate X, using the tetraparametric function approach.*

Description

Compute present expected value of an n-payment annuity, with payments of 1 unit each, made at the end of every year (annuity-immediate), valued at the rate X, using the tetraparametric function approach.

Usage

```
PV_post_artan(data,years)
```

Arguments

data	A vector of interest rates.
years	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Source

Cruz Rambaud, S.; Maturo, F. and Sánchez Pérez A. M. (2017): "Expected present and final value of an annuity when some non-central moments of the capitalization factor are unknown: Theory and an application using R". In Š. Hošková-Mayerová, *et al.* (Eds.), *Mathematical-Statistical Models and Qualitative Theories for Economic and Social Sciences* (pp. 233-248). Springer, Cham. doi:10.1007/978-3-319-54819-7_16.

Examples

```
#example 1
data=c(0.298,0.255,0.212,0.180,0.165,0.163,0.167,0.161,0.154,
0.128,0.079,0.059,0.042,-0.008,-0.012,-0.002)
PV_post_artan(data)

# example 2
data<-rnorm(n=30,m=0.03,sd=0.01)
PV_post_artan(data)

# example 3
data<-rnorm(n=30,m=0.03,sd=0.2)
PV_post_artan(data)
```

PV_post_cubic — *Compute the present expected value of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-due), valued at the rate X, using the cubic discount method.*

Description

Compute the present expected value of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-due), valued at the rate X, using the cubic discount method.

Usage

```
PV_post_cubic(data,years)
```

Arguments

`data`	A vector of interest rates.
`years`	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
#example 1
data=c(0.298,0.255,0.212,0.180,0.165,0.163,0.167,0.161,0.154,
0.128,0.079,0.059,0.042,-0.008,-0.012,-0.002)
PV_post_cubic(data)

#example 2
data<-rnorm(n=30,m=0.03,sd=0.01)
PV_post_cubic(data)
```

```
# example 3
data = c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
data=data/100
PV_post_cubic(data)
```

PV_post_exact *Computes the present value of an annuity-immediate considering only non-central moments of negative orders.*

Description

Computes the present value of an annuity-immediate considering only non-central moments of negative orders.

Usage

```
PV_post_exact(data,years)
```

Arguments

data	A vector of interest rates.
years	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
# example 1
data=c(0.0177, 0.0185, 0.0185, 0.0184, 0.0184, 0.0183, 0.0185, 0.0185, 0.0188, 0.0185,
0.0180, 0.0184, 0.0191, 0.0185, 0.0184, 0.0185, 0.0186, 0.0185, 0.0188, 0.0186)
PV_post_exact(data,10)
```

`PV_post_mood_nm` *Compute the present expected value of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate), valued at the rate X, with the method of Mood* et al. *using some negative moments of the distribution.*

Description

Compute the present expected value of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate), valued at the rate X, with the method of Mood *et al.* using some negative moments of the distribution.

Usage

```
PV_post_mood_nm(data,years)
```

Arguments

`data`	A vector of interest rates.
`years`	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Source

Mood, A. M.; Graybill, F. A. and Boes, D. C. (1974). *Introduction to the Theory of Statistics* (3rd Ed.). New York: McGraw Hill.

Rice, J. A. (1995). *Mathematical Statistics and Data Analysis* (2nd Ed.). California: Ed. Duxbury Press.

Examples

```
#example 1
data=c(0.298,0.255,0.212,0.180,0.165,0.163,0.167,0.161,0.154,
0.128,0.079,0.059,0.042,-0.008,-0.012,-0.002)
PV_post_mood_nm(data)

# example 2
data<-rnorm(n=30,m=0.03,sd=0.01)
PV_post_mood_nm(data)

# example 3
data = c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
data=data/100
PV_post_mood_nm(data)
```

PV_post_mood_pm *Compute the present expected value of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate), valued at the rate X, with the method of Mood* et al. *using some positive moments of the distribution.*

Description

Compute the present expected value of an n-payment annuity, with payments of 1 unit each made at the end of every year (annuity-immediate), valued at the rate X, with the method of Mood *et al.* using some positive moments of the distribution.

Usage

```
PV_post_mood_pm(data,years)
```

Arguments

`data`	A vector of interest rates.
`years`	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Source

Mood, A. M.; Graybill, F. A. and Boes, D. C. (1974). *Introduction to the Theory of Statistics* (3rd Ed.). New York: McGraw Hill.

Rice, J. A. (1995). *Mathematical Statistics and Data Analysis* (2nd Ed.). California: Ed. Duxbury Press.

Cruz Rambaud, S.; Maturo, F. and Sánchez Pérez A. M. (2017): "Expected present and final value of an annuity when some non-central moments of the capitalization factor are unknown: Theory and an application using R". In Š. Hošková-Mayerová, *et al.* (Eds.), *Mathematical-Statistical Models and Qualitative Theories for Economic and Social Sciences* (pp. 233-248). Springer, Cham. doi:10.1007/978-3-319-54819-7_16.

Examples

```
#example 1
data=c(0.298,0.255,0.212,0.180,0.165,0.163,0.167,0.161,0.154,
0.128,0.079,0.059,0.042,-0.008,-0.012,-0.002)
PV_post_mood_pm(data)

# example 2
data<-rnorm(n=30,m=0.03,sd=0.01)
PV_post_mood_pm(data)
```

```
# example 3
data = c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
data=data/100
PV_post_mood_pm(data)
```

`PV_post_triang_3` *Compute the present value of an annuity-immediate considering only non-central moments of negative orders. The calculation is performed by using the function triangular_moments_3 for the moments greater than -2 (in absolute value).*

Description

Compute the present value of an annuity-immediate considering only non-central moments of negative orders. The calculation is performed by using the function triangular_moments_3 for the moments greater than -2 (in absolute value).

Usage

```
PV_post_triang_3(data,years)
```

Arguments

`data`	A vector of interest rates expressed as percentages.
`years`	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
data=c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
PV_pre_triang_3(data,10)
```

PV_post_triang_dis *Compute the present value of an annuity-immediate considering only non-central moments of negative orders. The calculation is performed by using the moments of the fitted triangular distribution of the random variable "capitalization factor" U (which are obtained from the definition of negative moment of a continuous random variable).*

Description

Compute the present value of an annuity-immediate considering only non-central moments of negative orders. The calculation is performed by using the moments of the fitted triangular distribution of the random variable "capitalization factor" U (which are obtained from the definition of negative moment of a continuous random variable).

Usage

```
PV_post_triang_dis(data,years)
```

Arguments

data	A vector of interest rates expressed as percentages.
years	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
data=c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
PV_post_triang_dis(data,10)
```

PV_pre_artan *Compute the present expected value of an n-payment annuity, with payments of 1 unit each, made at the beginning of every year (annuity-due), valued at the rate X, using the tetraparametric function approach.*

Description

Compute the present expected value of an n-payment annuity, with payments of 1 unit each, made at the beginning of every year (annuity-due), valued at the rate X, using the tetraparametric function approach.

Usage

```
PV_pre_artan(data,years)
```

Arguments

data	A vector of interest rates.
years	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Source

Cruz Rambaud, S.; Maturo, F. and Sánchez Pérez A. M. (2017): "Expected present and final value of an annuity when some non-central moments of the capitalization factor are unknown: Theory and an application using R". In Š. Hošková-Mayerová, *et al.* (Eds.), *Mathematical-Statistical Models and Qualitative Theories for Economic and Social Sciences* (pp. 233-248). Springer, Cham. doi:10.1007/978-3-319-54819-7_16.

Examples

```
#example 1
data=c(0.298,0.255,0.212,0.180,0.165,0.163,0.167,0.161,0.154,0.128,
0.079,0.059,0.042,-0.008,-0.012,-0.002)
PV_pre_artan(data)

# example 2
data<-rnorm(n=30,m=0.03,sd=0.01)
PV_pre_artan(data)
```

`PV_pre_cubic` *Compute the present expected value of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate X, using the cubic discount method.*

Description

Compute the present expected value of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate X, using the cubic discount method.

Usage

```
PV_pre_cubic(data,years)
```

Arguments

data	A vector of interest rates.
years	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
#example 1
data=c(0.298,0.255,0.212,0.180,0.165,0.163,0.167,0.161,0.154,
0.128,0.079,0.059,0.042,-0.008,-0.012,-0.002)
PV_pre_cubic(data)

#example 2
data<-rnorm(n=30,m=0.03,sd=0.01)
PV_pre_cubic(data)

# example 3
data = c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
data=data/100
PV_pre_cubic(data)
```

PV_pre_exact — *Compute the present value of an annuity-due considering only non-central moments of negative orders.*

Description

Compute the present value of an annuity-due considering only non-central moments of negative orders.

Usage

```
PV_pre_exact(data,years)
```

Arguments

data	A vector of interest rates.
years	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
# example 1
data=c(0.0177, 0.0185, 0.0185, 0.0184, 0.0184, 0.0183, 0.0185, 0.0185, 0.0188,
0.0185, 0.0180, 0.0184, 0.0191, 0.0185, 0.0184, 0.0185, 0.0186, 0.0185, 0.0188, 0.0186)
PV_pre_exact(data,10)
```

PV_pre_mood_nm — *Compute the present expected value of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate X, with the method of Mood* et al. *using some negative moments of the distribution.*

Description

Compute the present expected value of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate X, with the method of Mood *et al.* using some negative moments of the distribution.

Usage

```
PV_pre_mood_nm(data,years)
```

Arguments

`data`	A vector of interest rates.
`years`	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
#example 1
data=c(0.298,0.255,0.212,0.180,0.165,0.163,0.167,0.161,0.154,
0.128,0.079,0.059,0.042,-0.008,-0.012,-0.002)
PV_pre_mood_nm(data)

# example 2
data<-rnorm(n=30,m=0.03,sd=0.01)
PV_pre_mood_nm(data)

# example 3
data = c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,
```

```
1.84,1.85,1.86,1.85,1.88,1.86)
data=data/100
PV_pre_mood_nm(data)
```

`PV_pre_mood_pm` *Compute the present expected value of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate X, with the method of Mood* et al. *using some positive moments of the distribution.*

Description

Compute the present expected value of an n-payment annuity, with payments of 1 unit each made at the beginning of every year (annuity-due), valued at the rate X, with the method of Mood *et al.* using some positive moments of the distribution.

Usage

```
PV_pre_mood_pm(data,years)
```

Arguments

`data`	A vector of interest rates.
`years`	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Source

Cruz Rambaud, S.; Maturo, F. and Sánchez Pérez A. M. (2017): "Expected present and final value of an annuity when some non-central moments of the capitalization factor are unknown: Theory and an application using R". In Š. Hošková-Mayerová, *et al.* (Eds.), *Mathematical-Statistical Models and Qualitative Theories for Economic and Social Sciences* (pp. 233-248). Springer, Cham. doi:10.1007/978-3-319-54819-7_16.

Examples

```
#example 1
data=c(0.298,0.255,0.212,0.180,0.165,0.163,0.167,0.161,0.154,
0.128,0.079,0.059,0.042,-0.008,-0.012,-0.002)
PV_pre_mood_pm(data)

# example 2
data<-rnorm(n=30,m=0.3,sd=0.01)
PV_pre_mood_pm(data)
```

```
# example 3
data = c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
data=data/100
PV_pre_mood_pm(data)
```

`PV_pre_triang_3` *Compute the present value of an annuity-due considering only non-central moments of negative orders. The calculation is performed by using the function $triangular\_moments\_3$ for the moments greater than -2 (in absolute value).*

Description

Compute the present value of an annuity-due considering only non-central moments of negative orders. The calculation is performed by using the function $triangular\_moments\_3$ for the moments greater than -2 (in absolute value).

Usage

```
PV_pre_triang_3(data,years)
```

Arguments

`data`	A vector of interest rates expressed as percentages.
`years`	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
data=c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
PV_pre_triang_3(data,10)
```

`PV_pre_triang_dis` *Compute the present value of an annuity-due considering only non-central moments of negative orders. The calculation is performed by using the moments of the fitted triangular distribution of the random variable "capitalization factor" U (which are obtained from the definition of negative moment of a continuous random variable)*

Description

Compute the present value of an annuity-due considering only non-central moments of negative orders. The calculation is performed by using the moments of the fitted triangular distribution of the random variable "capitalization factor" U (which are obtained from the definition of negative moment of a continuous random variable)

Usage

```
PV_pre_triang_dis(data,years)
```

Arguments

`data`	A vector of interest rates expressed as percentages.
`years`	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
data=c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
PV_pre_triang_dis(data,10)
```

`triangular_moments_3` *Compute the negatives moments (different from orders 1 and 2) of the fitted triangular distribution of the random variable X.*

Description

Compute the negatives moments (different from orders 1 and 2) of the fitted triangular distribution of the random variable X.

Usage

```
triangular_moments_3(data,order)
```

Arguments

`data`	A vector X of interest rates.
`order`	The order of moment that should be computed.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
#example 1
data=c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
triangular_moments_3(data,3)
triangular_moments_3(data,4)

# example 2 - first 10 negative moments of fitted triangular distribution
#(an example from normal distributed simulated data)
data<-rnorm(n=200,m=0.75,sd=0.2)
triangular_parameters(data)
first10negmoments=rep(NA,10)  #except first and second
for (i in 3:10) first10negmoments[i]=triangular_moments_3(data,i)
first10negmoments
```

---

triangular_moments_3_U

Compute the negatives moments (different from orders 1 and 2) of the fitted triangular distribution of the random variable "capitalization factor" U.

Description

Compute the negatives moments (different from orders 1 and 2) of the fitted triangular distribution of the random variable "capitalization factor" U.

Usage

```
triangular_moments_3_U(data,order)
```

Arguments

`data`	A vector X of interest rates.
`order`	The order of moment that should be computed.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
#example 1
data=c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
triangular_moments_3_U(data,3)
triangular_moments_3_U(data,4)

# example 2 - first 10 negative moments of fitted triangular distribution
#(an example from normal distributed simulated data)
data<-rnorm(n=200,m=0.75,sd=0.2)
triangular_parameters(data)
first10negmoments=rep(NA,10)  #except first and second
for (i in 3:10) first10negmoments[i]=triangular_moments_3_U(data,i)
first10negmoments
```

---

triangular_moments_dis

Compute the negative moments of the fitted triangular distribution of the random variable X according to the definition (as integral).

Description

Compute the negative moments of the fitted triangular distribution of the random variable X according to the definition (as integral).

Usage

```
triangular_moments_dis(data,order)
```

Arguments

`data`	A vector of interest rates as percentage.
`order`	The order of moment of the triangular distribution

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
# example 1
data=c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
triangular_moments_dis(data,1)
triangular_moments_dis(data,2)
triangular_moments_dis(data,3)
triangular_moments_dis(data,4)

# example 2 - first 10 negative moments of fitted triangular distribution
#(an example from normal distributed simulated data)
data<-rnorm(n=200,m=0.75,sd=0.2)
triangular_parameters(data)
first10negmoments=rep(NA,10)
for (i in 1:10) first10negmoments[i]=triangular_moments_dis(data,i)
first10negmoments
```

---

triangular_moments_dis_U

Compute the negative moments of the fitted triangular distribution of the random variable "capitalization factor" U according to the definition (as integral).

Description

Compute the negative moments of the fitted triangular distribution of the random variable "capitalization factor" U according to the definition (as integral).

Usage

```
triangular_moments_dis_U(data,order)
```

Arguments

`data`	A vector of interest rates as percentage.
`order`	The order of moment of the triangular distribution

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
# example 1
data=c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
triangular_moments_dis_U(data,1)
triangular_moments_dis_U(data,2)
triangular_moments_dis_U(data,3)
triangular_moments_dis_U(data,4)

# example 2 - first 10 negative moments of fitted triangular distribution
#(an example from normal distributed simulated data)
data<-rnorm(n=200,m=0.75,sd=0.2)
triangular_parameters(data)
first10negmoments=rep(NA,10)
for (i in 1:10) first10negmoments[i]=triangular_moments_dis_U(data,i)
first10negmoments
```

---

triangular_parameters *Compute the parameters and plot the fitted triangular distribution of the random variable X.*

Description

Compute the parameters and plot the fitted triangular distribution of the random variable X.

Usage

```
triangular_parameters(data)
```

Arguments

data	A vector of interest rates.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
# example 1
data=c(0.00,-0.05,-0.05,-0.06,-0.06,0.02,-0.06,-0.05,-0.04,-0.05,
-0.03,-0.06,0.04,-0.05,-0.08,-0.05,-0.12,-0.03,-0.05,-0.04,-0.06)
triangular_parameters(data)
```

```
# example 2
data<-rnorm(n=200,m=0.75,sd=0.2)
triangular_parameters(data)


# example 3
data = c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
triangular_parameters(data)
```

---

triangular_parameters_U

Return the parameters of the fitted triangular distribution of the random variable "capitalization factor" U.

Description

Return the parameters of the fitted triangular distribution of the random variable "capitalization factor" U.

Usage

```
triangular_parameters_U(data)
```

Arguments

`data`	A vector of interest rates expressed as percentage.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
# example 1
data = c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
triangular_parameters_U(data)
```

`variance_drv` *Compute the variance of the present value of an annuity using "discrete random variable" approach.*

Description

Compute the variance of the present value of an annuity using "discrete random variable" approach.

Usage

```
variance_drv(data,years)
```

Arguments

`data`	A vector X of interest rates.
`years`	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
# example 1
data = c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
data=data/100
variance_drv(data)
```

`variance_post_mood_nm` *Compute the variance of the present value of an annuity-immediate using the Mood* et al. *approximation and some non-central moments of negative order.*

Description

Compute the variance of the present value of an annuity-immediate using the Mood *et al.* approximation and some non-central moments of negative order.

Usage

```
variance_post_mood_nm(data,years)
```

Arguments

data	A vector X of interest rates.
years	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
# example 1
data = c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
data=data/100
variance_post_mood_nm(data)
```

---

variance_post_mood_pm *Compute the variance of the present value of an annuity-immediate using the Mood* et al. *approximation and some non-central moments of positive order.*

Description

Compute the variance of the present value of an annuity-immediate using the Mood *et al.* approximation and some non-central moments of positive order.

Usage

```
variance_post_mood_pm(data,years)
```

Arguments

data	A vector X of interest rates.
years	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
# example 1
data = c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
data=data/100
variance_post_mood_pm(data)
```

`variance_pre_mood_nm` *Compute the variance of the present value of an annuity-due using the Mood* et al. *approximation and some non-central moments of negative order.*

Description

Compute the variance of the present value of an annuity-due using the Mood *et al.* approximation and some non-central moments of negative order.

Usage

```
variance_pre_mood_nm(data,years)
```

Arguments

`data`	A vector X of interest rates.
`years`	The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
# example 1
data = c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
data=data/100
variance_pre_mood_nm(data)
```

`variance_pre_mood_pm` *Compute the variance of the present value of an annuity-due using the Mood* et al. *approximation and some non-central moments of positive order.*

Description

Compute the variance of the present value of an annuity-due using the Mood *et al.* approximation and some non-central moments of positive order.

Usage

```
variance_pre_mood_pm(data,years)
```

Arguments

`data` A vector X of interest rates.

`years` The number of years of the income. Default is 10 years.

Author(s)

Salvador Cruz Rambaud, Fabrizio Maturo, Ana María Sánchez Pérez

Examples

```
# example 1
data = c(1.77,1.85,1.85,1.84,1.84,1.83,1.85,1.85,1.88,1.85,1.80,1.84,1.91,1.85,1.84,1.85,
1.86,1.85,1.88,1.86)
data=data/100
variance_pre_mood_pm(data)
```

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